Title: Quantum Spacetime Engineering

Date: Apr 22, 2014 03:20 PM

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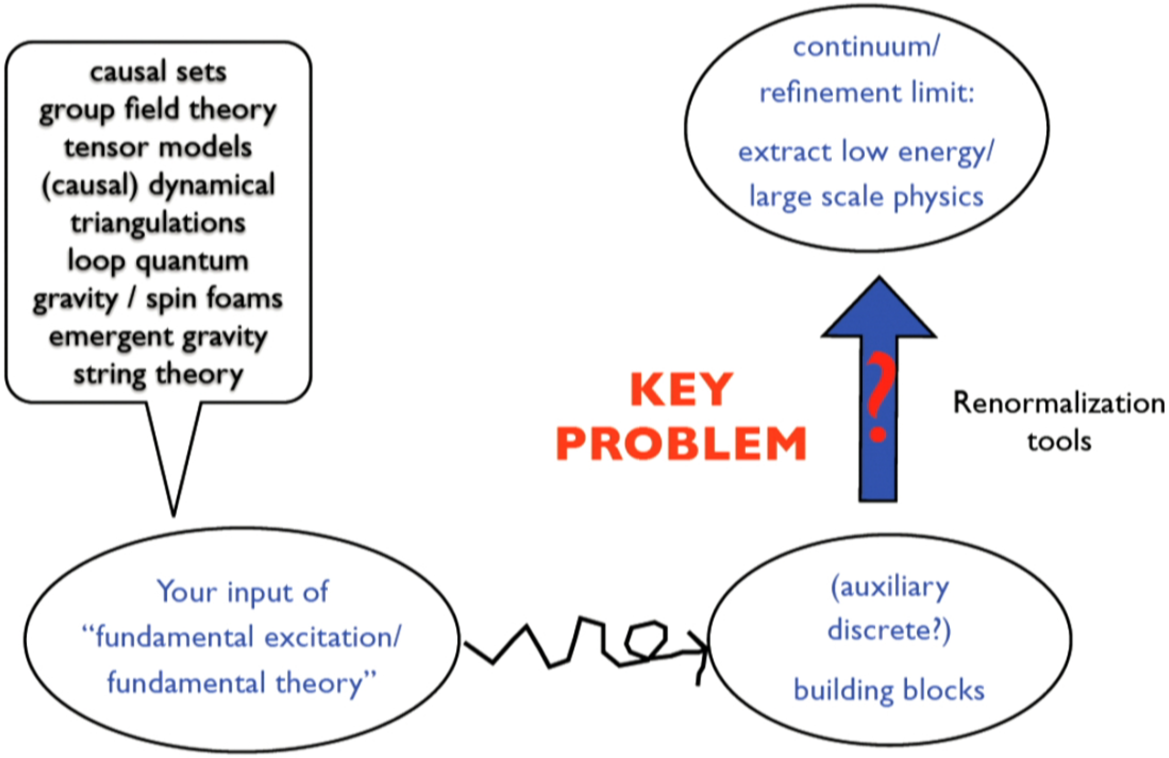
Abstract: Given (a set of) fundamental models of quantum space time, for instance spin foam models, we aim to understand the large scale physics encoded in these fundamental models. Renormalization and coarse graining address this issue and help to understand how large scale physics depends on parameters in the fundamental models.

I will review recent work on coarse graining and renormalization of spin foam and analogue models, revealing possible large scale phases, depending on parameters of the microscopic models. I will explain how these phases are connected to topological field theories and possible vacua for the theory of quantum gravity, e.g. loop quantum gravity. I outline how these different vacua are connected to different representations of the observable algebra, that is different Hilbert spaces, and how this allows to expand the theory around different vacuum states.

Bottom-up design



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Overview

Motivation.

Space time from atoms?

Conceptual: How to construct the continuum limit?

Emergence of scale.

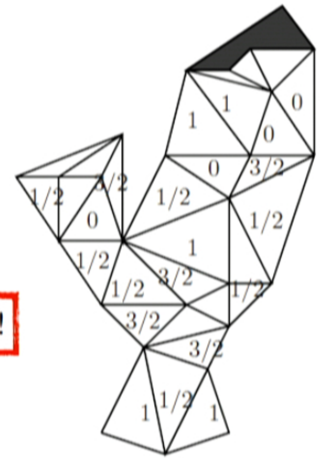
Application to spin foam / spin nets.

Mapping out the phase diagram.

Application to loop quantum gravity.

New representation and vacuum for loop quantum gravity.

Spin foams: path integral approach



$$\int \exp(iS(\text{geom}))\ \mathcal{D}\text{geom}$$

We keep the i !

years of research: Reisenberger, Rovelli, Barrett-Crane, Barbieri, Freidel-Krasnov, Engle-Pereira-Rovelli, Livine, ...

→ discretization / quantization/ reformulation into generalized lattice gauge theory

$$\sum_{\text{geom labels}} \mathcal{A}(\text{ geom labels})$$

with local (i.e. factorizing) amplitude:

$$\mathcal{A}(\text{ geom labels}) \;=\; \prod_{\text{fund building blocks}} \mathcal{A}_{fbb}(\text{ geom labels})$$

5

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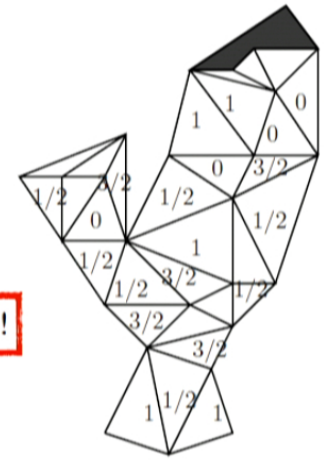

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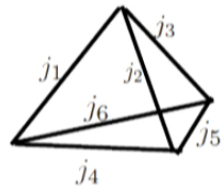
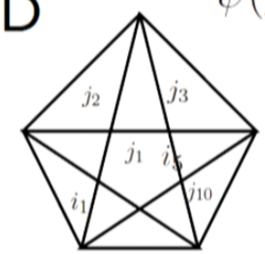
Space time atoms? Relation to gravity action

3D



sum over orientations of space time atoms

4D



$$\psi(j) \underset{j>>1}{\sim} \exp(iS_{\text{discr grav}}) + \exp(-iS_{\text{discr grav}})$$

Large j (semi-classical) limit for single building blocks gives discrete GR action (for flat building blocks)!

[Ponzano-Regge,..., Barrett et al, Conrady-Freidel, ...]

ve think of these as atoms (small)?

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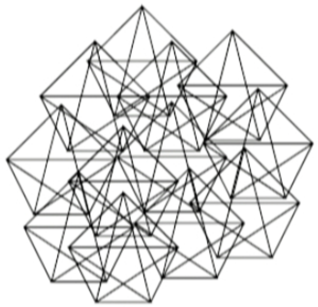
Many space time atoms?



Is there a phase describing smooth space time?

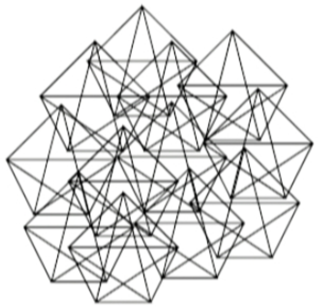
Do we get General Relativity at large scales?

What are the phases of spin foam theories (and in loop quantum gravity)?

But what are large scales?

7

Many space time atoms?



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Coarse graining / Renormalization

- needed to deal with many degrees of freedom in a conceivable way

  (even if your model is finite: Ising model)

- essential for effective computations (truncations)

~~Requires~~ Provides notion of scale and (re-) ordering of degrees of freedom according to these scales.

Depending on question need only consider subset of degrees of freedom.

But: Non-localities

- (Real space) coarse graining produces non-local couplings (Ising model)
- Main problem for real space approaches (until a few years ago)
- basically only local truncations considered (Migdal-Kadanoff methods)

- Momentum space renormalization: completely non-local
- however does give perfect scale decomposition for free theories

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Can we avoid non-localities?

coarse graining ~ change of triangulation ~ diffeomorphism

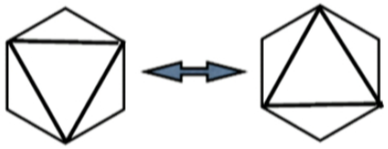
[....,Pfeiffer 04,...]

coarse graining fixed point ~ triangulation invariant ~ diffeomorphism symmetry

[Bahr, BD 09, Bahr, BD, Steinhaus 11]

Non-localities cannot be avoided

3-3 move



•Allowing non-local amplitudes the requirement of triangulation invariance becomes much less restrictive

Defining

A(Left): = A(Right)

gives invariance

•How to glue simplices / regions with non-local amplitudes?

•How can we deal with / control non-local couplings?

Simplices are too simplistic!

Need a new framework.

Its here! [BD 12, BD, Steinhaus 13]

(inspired by tensor network renormalization and LQG kinematical Hilbert space construction)

Simplices just represent the simplest (coarsest) possible boundaries.

Need to allow more general (refined) boundaries. These are central in the new framework.

- circumvents non-localities (shifting these inside the bulk)
- provides a notion of scale via coarser/refined boundaries

General boundary formalism

[Oeckl 03+]

here applied to a priori discrete boundaries

- discrete boundary and associated boundary Hilbert space

$\mathcal{H}_\Delta = \otimes_3 \mathcal{H}_{site}$ $\quad \mathcal{H}_\square = \otimes_4 \mathcal{H}_{site}$ $\quad \mathcal{H}_\circ = \otimes_{10} \mathcal{H}_{site}$

- central: Amplitude map $\quad A_b : \mathcal{H}_b \mapsto \mathbb{C}$ $\quad$ encodes dynamics of the system

- Often identification: $\quad A_b(j) := A_b(\psi_j)$

← Spin network basis, elements labelled by j

(of course one can use any other basis)

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How to formulate a continuum theory of quantum gravity

(aka the coarse graining fixed point)

[BD, Steinhaus 13]

New framework

- amplitude maps for a partially ordered set of boundaries

$$A_b : \mathcal{H}_b \mapsto \mathbb{C}$$

- no reference to bulk triangulation necessary!

- boundary discretizations are partially ordered:

coarser $b < b'$ finer

- embedding maps for boundary Hilbert spaces

$$\iota_{bb'} : \quad \mathcal{H}_b \to \mathcal{H}_{b'}$$

relate boundary amplitude maps:

$$A_b(\psi_b) \; = \; A_{b'}(\iota_{bb'}(\psi_b))$$

Old

- amplitude map for simplex

(can be glued for larger regions)

- bulk triangulation invariance

- relation between different boundary triangulations?

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Dynamical cylindrical consistency

[Bahr 11] [BD 12]

$$A_b(\psi_b) \;=\; A_{b'}(\iota_{bb'}(\psi_b))$$

Calculation involving only coarse data gives the same result as the computation that uses the embedding of the coarse data into finer data.

(i.e. we deal with effective amplitudes)

We can obtain continuum result by doing calculation only involving coarse data.

Perfect mirroring of continuum dynamics into discrete/ coarse data.

Embedding maps are essential: provide ordering of degrees of freedom:

$\mathrm{Im}(\iota_{bb'})$ states describing coarser boundary data (in finer Hilbert space)

$(\mathrm{Im}(\iota_{bb'}))^{\perp}$ states not relevant for coarse observables

Notion of scale: states ordered into coarser and finer via (images of) embedding maps.

What are good embedding maps?

Should simplify the process of finding cylindrically consistent amplitudes (via coarse graining).

In fact tensor network algorithms [Levin, Nave 06, Gu, Wen 09] construct embedding maps from dynamics of the system.

Two principles:

- coarse states = low energy states, coarsest state = vacuum

(motivated by transfer operator diagonalization techniques, where one truncates to the lowest energy states)

Tensor network algorithms implement a (localized) version of this truncation.

(good for gapped phases)

- 'radial' time evolution [BD, Hoehn 12,13][BD, Steinhaus 13] [BD, Hoehn, Jacobson w.i.p.]

(related to entanglement renormalization [Vidal 05])

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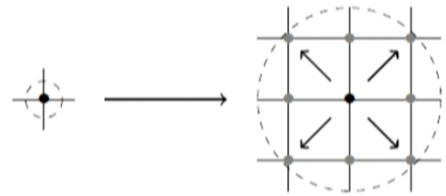
Radial time evolution and coarse states



Consider a radial time evolution from a "smaller" to a "larger" Hilbert space.

(Incorporated naturally in systems based on simplicial discretizations, such as spin foams).

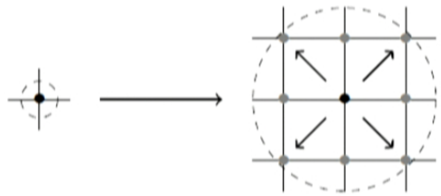
This time evolution itself defines an embedding map:
The image of the time evolution defines coarser states in the larger Hilbert space.

Conjecture: This definition leads to states describing coarse excitations.
(Radial) time evolution can be used to define useful embedding maps.
[BD, Steinhaus 13]

[also BD, Hoehn 11,13, Hoehn 14, BD, Hoehn, Jacobson wip]

This definition would lead in general to non-local embedding maps.
Regains indeed notion of Fourier basis for free theories.
Suitable for description of phase transitions.

Radial time evolution and coarse states



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The coarse graining fixed point

- should provide us with embedding maps
- cylindrically consistent amplitudes with respect to these embedding maps

- encodes physics of all scales at once (perfect discretization, expect diffeomorphism symmetry)

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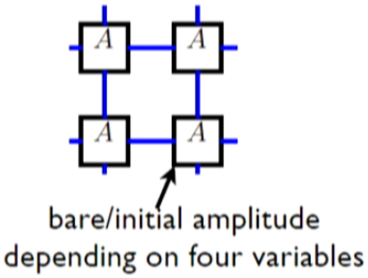
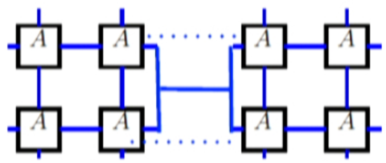
Tensor network renormalization methods

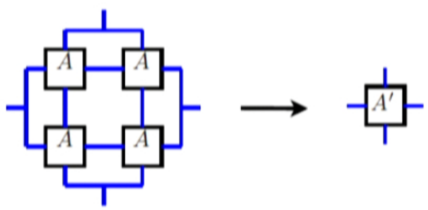
(using local truncation method)



Contract initial amplitudes (sum over bulk variables). Obtain "effective amplitude" with more boundary variables.



Find an approximation (embedding map) that would minimize the error as compared to full summation (dotted lines). For instance using singular value decomposition, keeping only the largest ones.
Leads to field redefinition, and ordering of fields into more and less relevant.



Use embedding maps to define coarse grained amplitude with the same (as initial) number of boundary variables.

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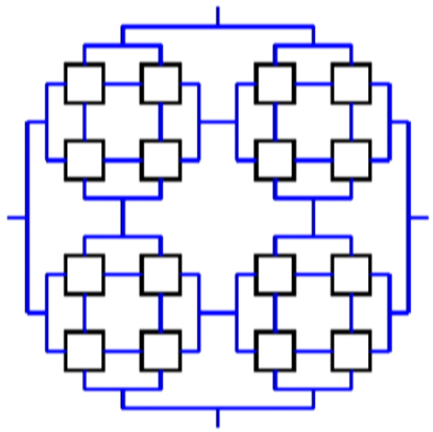
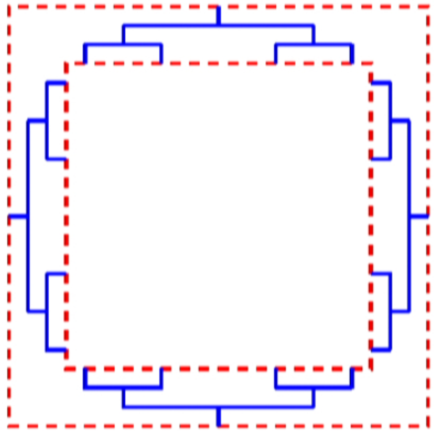
Tensor network renormalization methods

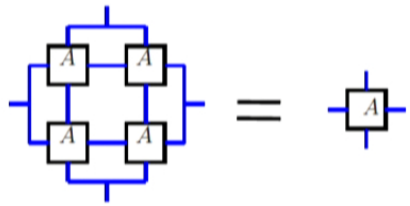


Iteration. Summation over bulk variables are truncated using the embedding maps.

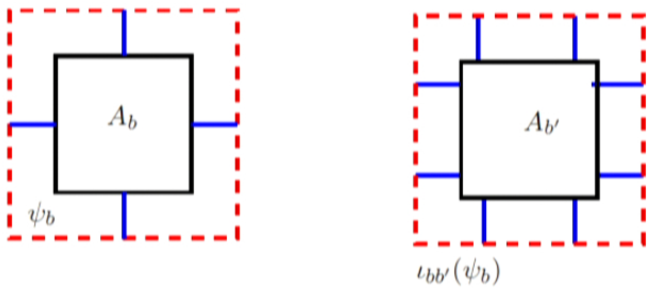


Associated embedding maps for boundary Hilbert space.

Fixed points give cylindrical consistent amplitudes



Condition for cylindrically consistent amplitudes



satisfied for

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Application to spin foams / spin nets

[BD, Eckert, Martin-Benito 2011,
Bahr, BD, Hellmann, Kaminski 2012,
BD, Martin-Benito, Schnetter 2013,
BD, Kaminski 2013,
BD, Martin-Benito, Steinhaus 2013]

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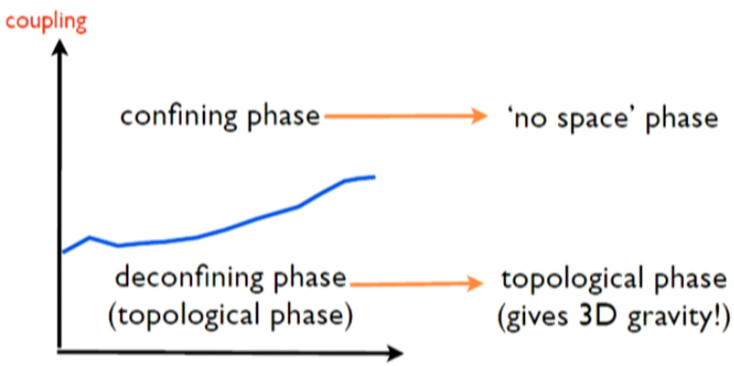
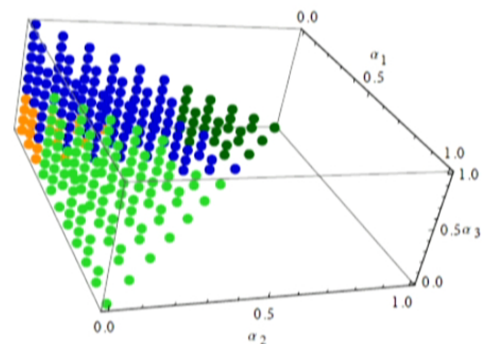
Space time from spin foams?

Spin foams can be described as generalized lattice gauge theories.
However feature a far more complicated structure, reminiscent of higher dimensional version of spin chains / golden chains.

Lattice gauge theory



Spin foams?



Indications of a much richer phase structure!

[BD, Martin-Benito, Schnetter 2013, BD, Martin-Benito, Steinhaus 2013]

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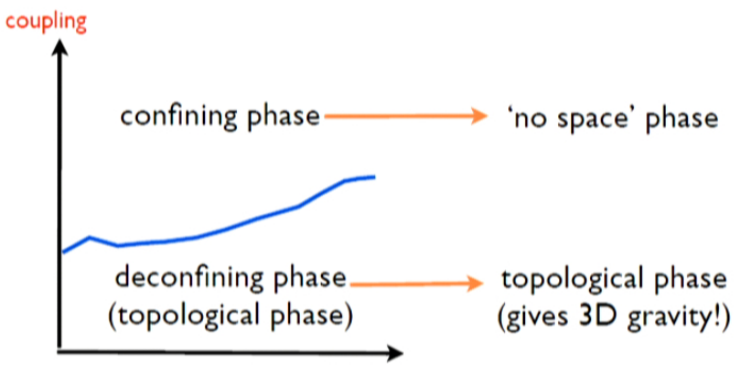

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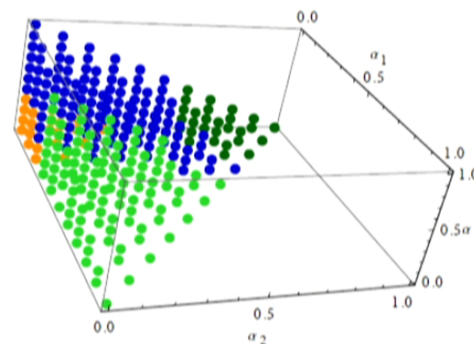

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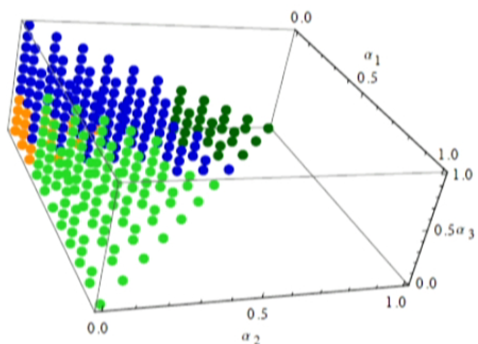
From spin foams to spin net models

- 4D spin foam models are very complicated
- devised analogue models capturing the essential dynamical ingredients of spin foams [similar to 4D lattice gauge and 2D spin system duality]
- these spin nets can actually be interpreted as spin foams based on very special discretizations
- unlike spin foams the spin net models are non-trivial in 2D
- investigated the phase diagrams for such models (with quantum group structure)
- these phase diagrams have a very rich structure



Interpretation: different phases describe uncoupled space time atoms (green) and coupled space time atoms (orange,blue).

Each phase corresponds to a topological field theory. Can be classified. [BD, Kaminski 13]

Towards spin foams coarse graining

- So far encouraging results for 'spin foam analogue' models.

Conclusions:

- Relevant parameters related to (SU(2)) intertwiners

  leading to rich phase spaces. This is expected from the gravity dynamics.
- Need to implement a weak version of discretization independence to uncover these rich phase spaces (escape the two lattice gauge theory phases).
- Positive indication for finding a geometric phase (transition) in spin foams.

- Are now looking at actual spin foam models in higher dimensions.
- Need to develop tensor network tools (applicable to lattice gauge theories) to this end.

Application to loop quantum gravity:

Coarse states, vacua and

new representations of loop quantum gravity

[BD, Geiller 2014,
BD, Geiller, to appear]

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Loop quantum gravity

So far the theory is based on the

Ashtekar-Lewandowski representation based on a (AL) vacuum describing

[Ashtekar, Isham, Lewandowski 92+]

- zero volume spatial geometry
- maximal uncertainty in conjugated variable

Based on simple embedding map:

attach j=0 (zero spatial geometry) to additonal edges.

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New loop quantum gravity representation

geometric variables: $\{A\,,\,E\} = \delta$

connection | flux: spatial geometry

[BD, Geiller 2014,
BD, Geiller to appear]

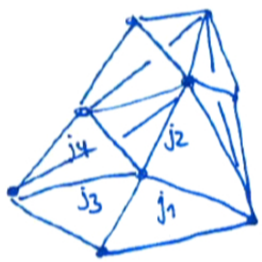
Ashtekar - Lewandowski representation (90's)

$\psi_{vac}(A) \equiv 1 \quad , \qquad E \equiv 0$

[Koslowski: shifting E vacuum value]

peaked on degenerate (spatial) geometry
maximal uncertainty in connection

excitations:
spin network states supported on graphs



(representation)
labels for edges

BF (topological) theory representation

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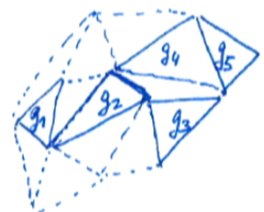
peaked on flat connections
maximal uncertainty in spatial geometry

excitations:
flux states supported on (d-1) D-surfaces



(group) labels
for faces

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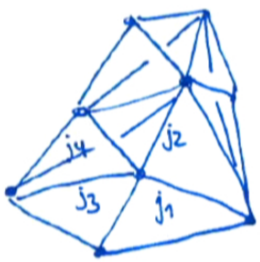

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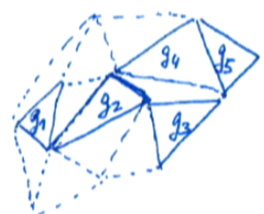

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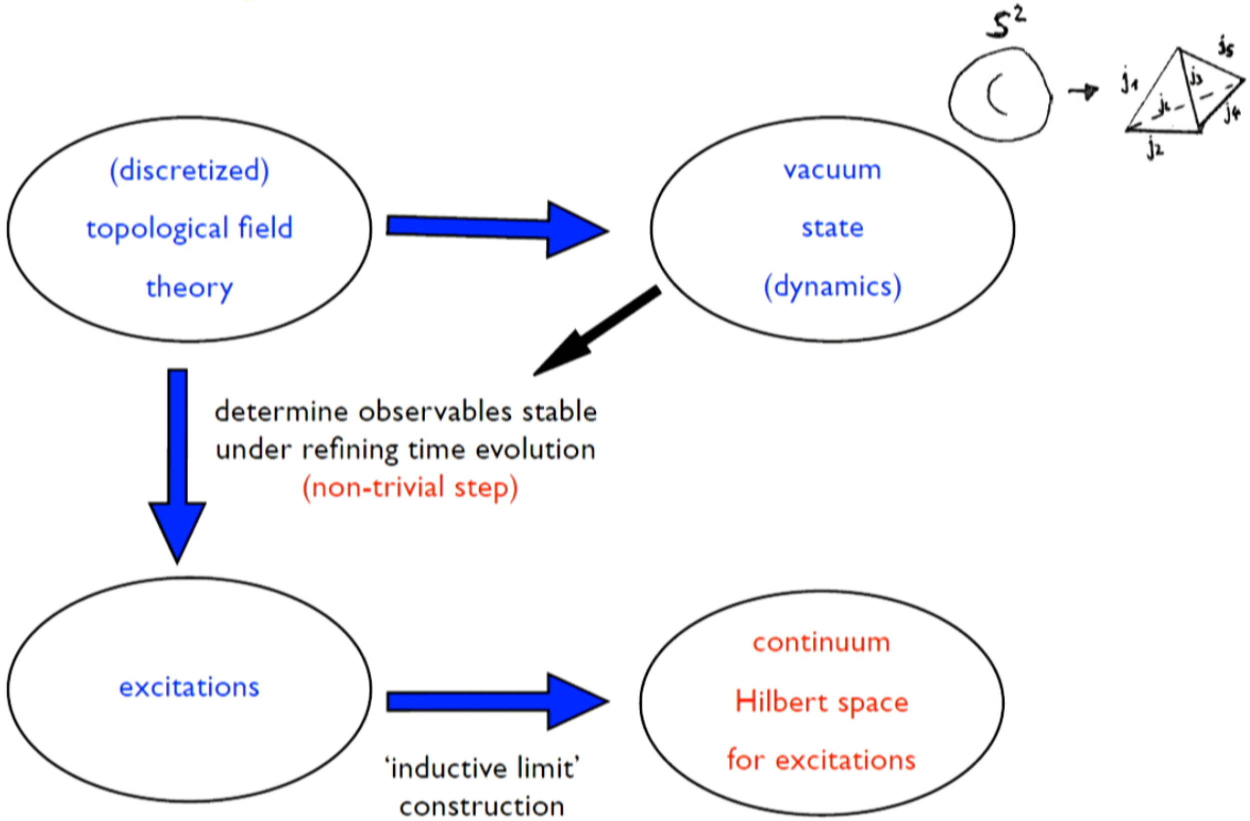
Time evolution as embedding maps

- essential for this construction: use time evolution map of BF theory as embedding map
- now coarsest state is the BF vacuum F=0
- finer states allow for more and more excitations, i.e. curvature

Advantages

- This new construction allows to expand loop quantum gravity around different vacua corresponding to the different phases (fixed points) for spin foams.
- Facilitates construction of states corresponding to smooth geometries and should be useful for discussions of i.e. black hole entropy in loop quantum gravity [Sahlmann 2011].
- New representation much easier to interpret geometrically: new handle on the dynamics of the theory. [need substitute for Thiemanns (1996) quantization of Hamiltonian constraints]

Can be generalized to other TFTs



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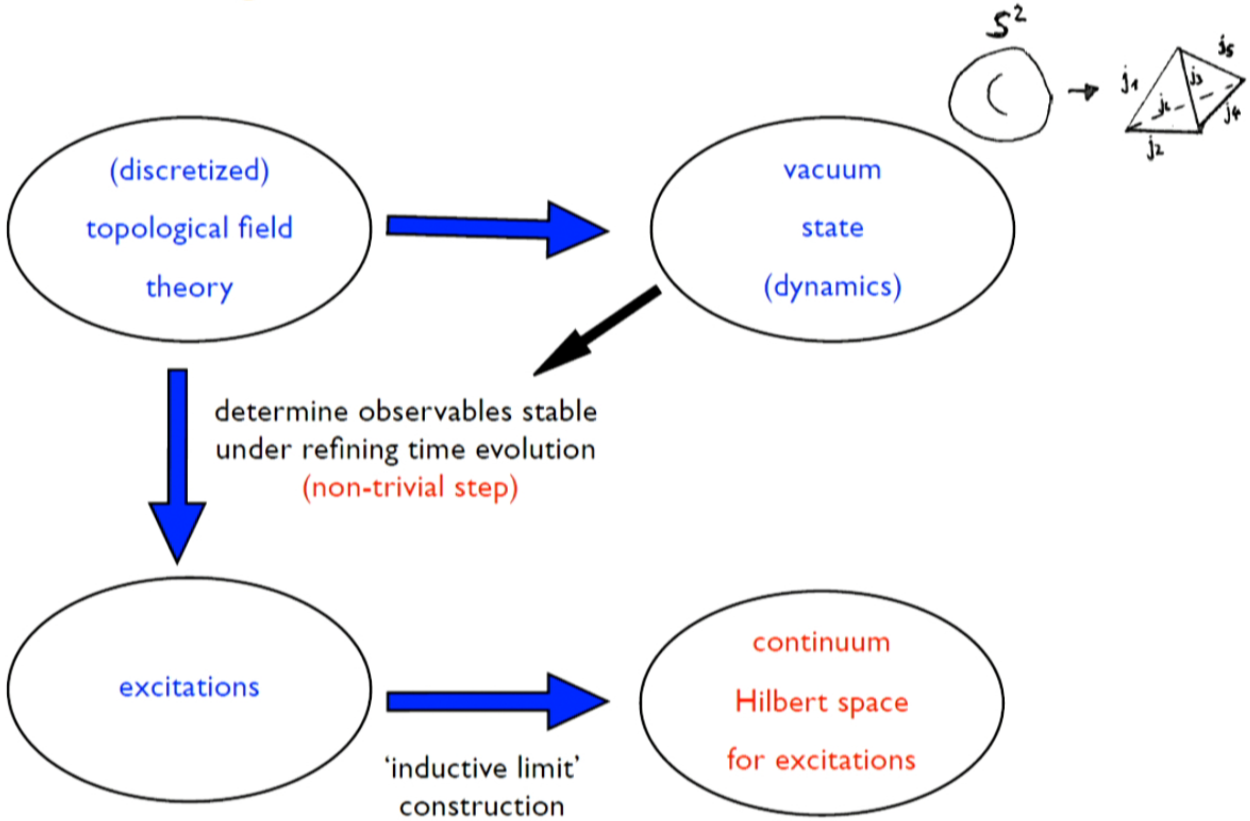

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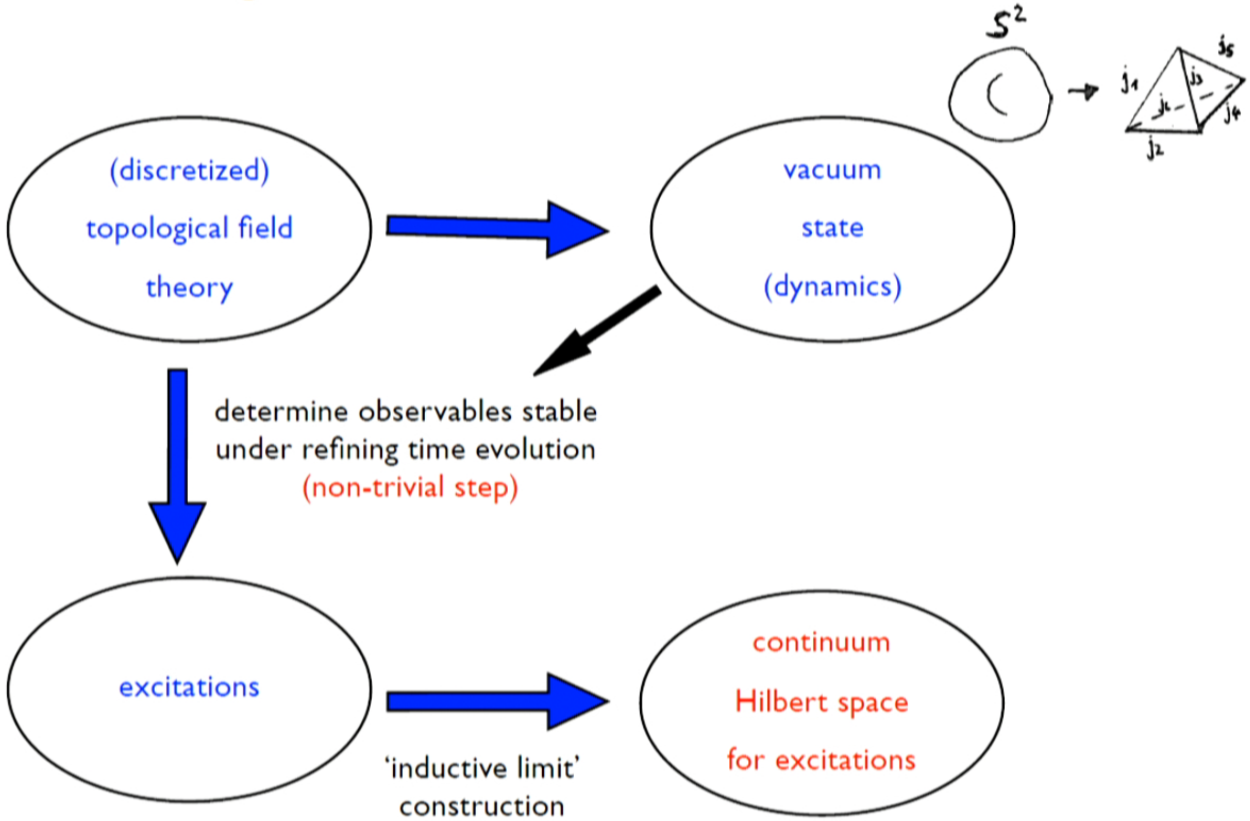
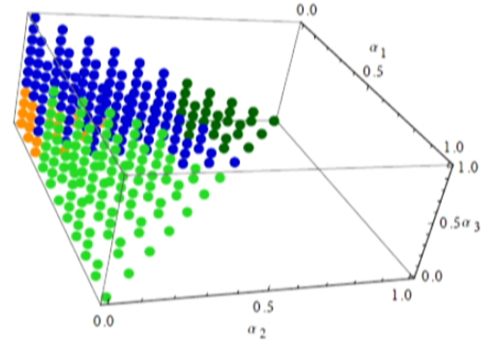


Phase transitions?



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- What about phase transitions ('non-trivial' fixed points)?
- Instead of topological theory expect conformal theory / propagating degrees of freedom.

- Expect such fixed point amplitudes to be non-local ⟶ non-local embedding maps

  (conjecture: given by time evolution)

- Do we regain triangulation independence / diffeomorphism symmetry there?

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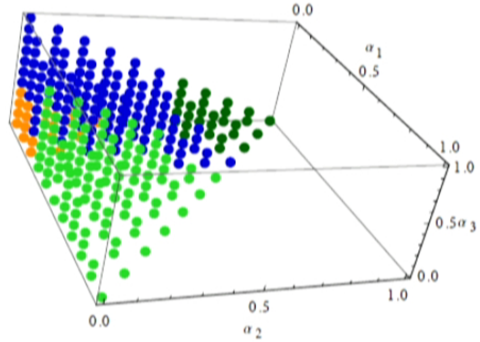

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Summary and Outlook:

Quantum Space Time Engineering

- We are on a good way to understand the the continuum limit of spin foams and loop quantum gravity.
- Conceptual: dynamical cylindrical consistency allows construction of continuum limit in terms of discrete boundaries
  - challenge: develop algorithms involving non-local embedding maps for phase transitions
- In the path integral approach (spin foams): mapping out the phase diagrams
  - connections to condensed matter physics / (new?) topological field theories and phases
  - challenge: going to higher dimensions
- In the canonical approach (loop quantum gravity): expanding around different vacua
  - facilitates construction of physical states describing extended/smooth geometries
  - each vacuum comes with its own set of excitations: investigate dynamics of these.

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