Title: Renormalization of group field theories: motivations and a brief review

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Abstract: Group field theories are tensorial models enriched with group-theoretic data in order to define proper field theories of quantum geometry. They can be understood as a second quantised (Fock space) reformulation of loop quantum gravity kinematics and dynamics. The renormalization group provides, as a in any quantum field theory, a key tool to select well-defined models, to unravel the impact of quantum effects on the dynamics across different scales, and to study the continuum limit. Beside introducing the general formalism and clarifying the relation to other approaches, we will motivate the renormalisation group analysis of group field theories and review recent developments in this direction.

Group field theories

Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \to \mathbb{C}$

relevant classical phase space for "GFT quanta": $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$

can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$

can be defined for any (Lie) group and dimension d, any signature, ...

very general framework; interest rests on specific models/use

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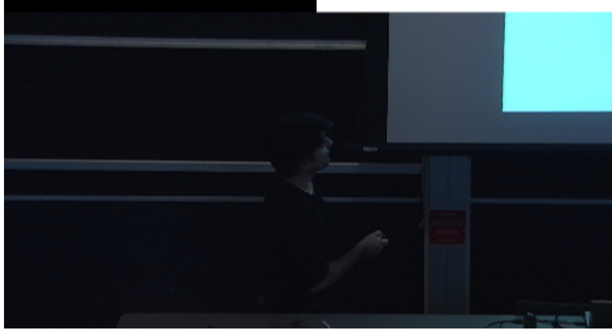

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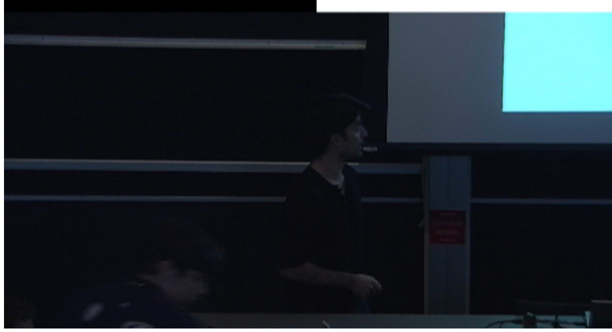
Group field theories

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

"combinatorial non-locality"
in pairing of field arguments



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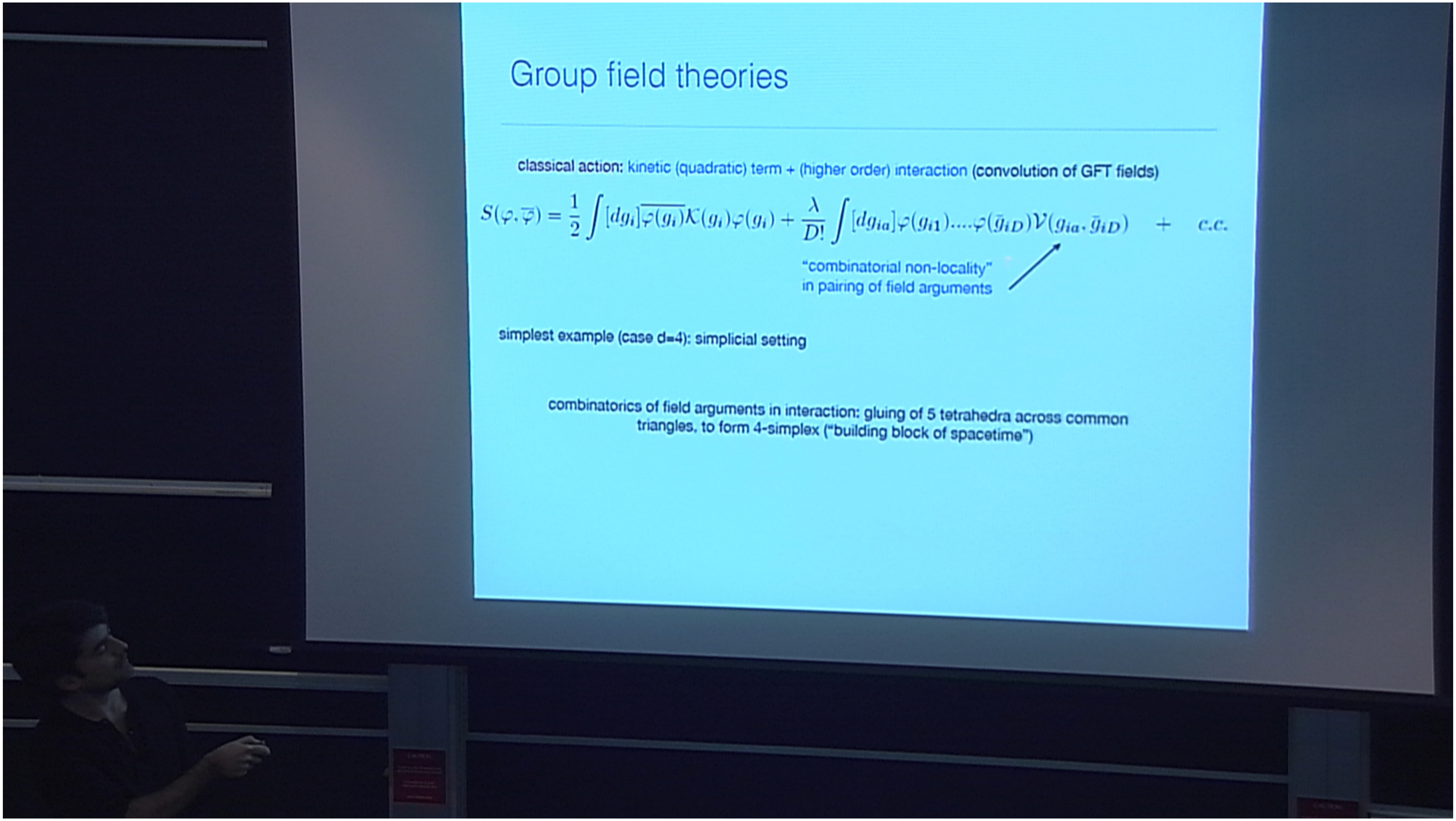
simplest example (case d=4): simplicial setting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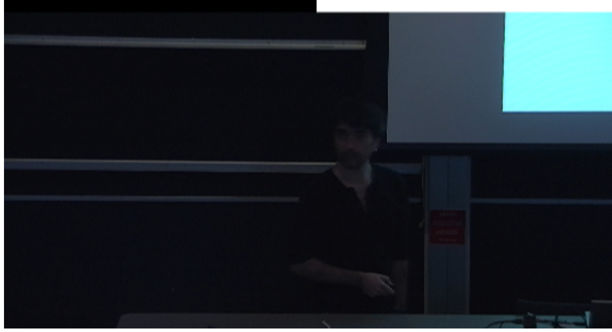
simplest example (case d=4): simplicial setting
combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common
triangles, to form 4-simplex ("building block of spacetime")

Group field theories

Feynman perturbative expansion around trivial vacuum

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$



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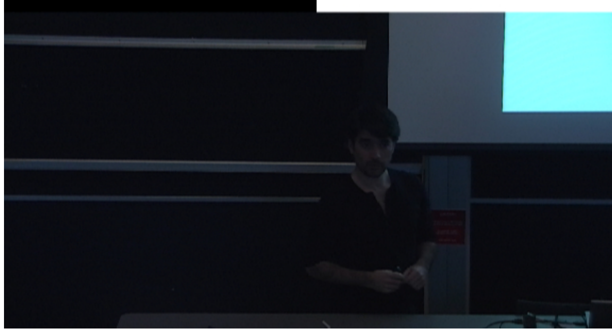
Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

(within the class specified by the chosen combinatorics)
(simplicial case: simplicial complexes obtained by gluing d-simplices in arbitrary ways)



Group Field Theories

and others

Group field theories and tensor models

see talk by Razvan

same combinatorics (of states/observables and histories/Feynman diagrams), additional group-theoretic data

example: d=3

dropping group/algebra data
(or restricting to finite group)

$$\varphi(g_1, g_2, g_3) : G^{\times 3} \to \mathbb{C} \longrightarrow \begin{array}{l} T_{ijk} : \mathbb{Z}_N^{\times 3} \to \mathbb{C} \\ T_{ijk} : X^{\times 3} \to \mathbb{C} \end{array} \qquad X = 1, 2, ..., N$$

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all results of tensor models apply to GFTs as well

in particular:

- large-N expansion

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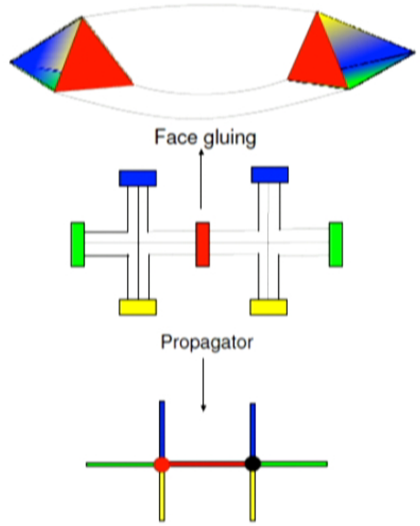

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- use of colours to encode cellular topology:

example: d=3 ———> 4 fields

Every PL d-pseudomanifold M can be represented by a (d+1)-colored graph G

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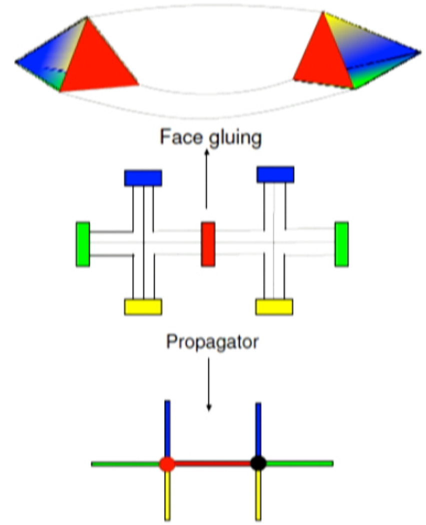

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GFT/tensor models locality

from “simplicial” to “tracial” locality

$$e^{-N^D F_N(\lambda,\bar\lambda)} = Z_N(\lambda,\bar\lambda) = \int d\bar\psi\, d\psi\; e^{-S(\psi,\bar\psi)}\,,$$

$$S(\psi,\bar\psi) = \sum_{i=0}^{D}\sum_{n} \bar\psi^i_{\vec n_i}\psi^i_{\vec n_i} + \frac{\lambda}{N^{D(D-1)/4}}\sum_n \prod_{i=0}^{D}\psi^i_{\vec n_i} + \frac{\bar\lambda}{N^{D(D-1)/4}}\sum_n \prod_{i=0}^{D}\bar\psi^i_{\vec n_i}$$

integrating out all but one colored field

$$Z = \int d\psi^D d\bar\psi^D\; e^{-S^D(\psi^D,\bar\psi^D)}$$

$$S^D(\psi^D,\bar\psi^D) = \sum \bar\psi^D_{\vec n_D}\psi^D_{\vec n_D} + \sum_{\mathcal{B}^{\hat D}} \frac{(\lambda\bar\lambda)^p}{\mathrm{Sym}(\mathcal{B}^{\hat D})}\; \mathrm{Tr}_{\mathcal{B}^{\hat D}}[\bar\psi^D,\psi^D]\; N^{-\frac{D(D-1)}{2}p+\mathcal{F}_{\mathcal{B}^{\hat D}}}$$

“bubble” or “trace” invariants

colors now associated to arguments of (un-symmetrized) field

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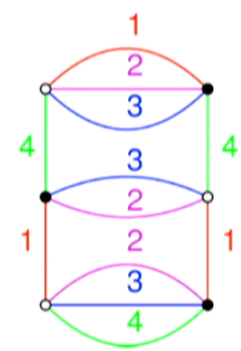

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- same combinatorics, but more algebraic and geometric structures: proper QFTs
  - "more gravity-conscious model building" in 3d and 4d
  - different scaling behaviour and renormalization
  - new symmetries (new universality classes?)
- link with other approaches (and all the corresponding results and insights):
  - loop quantum gravity and spin foam models
  - categorical state sums
  - simplicial quantum gravity
- more interesting effective physics?

GFTs, loop quantum gravity, spin foam models

appropriate conditions on GFT fields or GFT dynamics (and choice of data)
turn GFT Feynman amplitudes into lattice gauge theories

gauge invariance of GFT fields under diagonal action of group G

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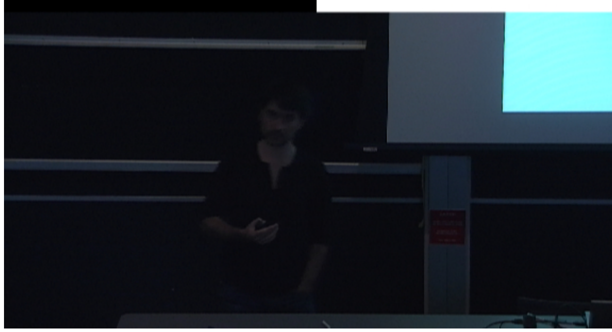
example: d=3 $\varphi_\ell : SO(3)^3/SO(3) \rightarrow \mathbb{R}$

$\forall h \in \mathrm{SO}(3), \quad \varphi_\ell(hg_1, hg_2, hg_3) = \varphi_\ell(g_1, g_2, g_3)$

+ simplicial interaction with only delta functions

can be computed in different (equivalent) representations (group, spin, Lie algebra)



GFTs, loop quantum gravity, spin foam models

the same results hold for any dimension d and any Lie group G: GFT formulation of BF theory

more involved choices of GFT dynamics give different lattice (gauge) theories

GFT models of 4d gravity:

based on classical formulation of GR as BF theory + (simplicity) constraints

start from GFT formulation of 4d BF theory
+ impose simplicity constraints (geometricity of simplicial structures)

(Barbieri, Baez, Barrett, Crane, Engle, Pereira, Freidel, Krasnov, Rovelli, Livine, Speziale, Baratin, Oriti, ...)

the result is to effectively define field on subspace of Lorentz group

same combinatorics of Feynman diagrams = simplicial complexes

same duality between spin foam models ~ lattice gravity path integral

all current spin foam models have a GFT formulation

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"geometricity operator" = simplicity constraints + gauge invariance:

$$G \triangleright \phi \equiv C \triangleright S^{\beta} \triangleright \phi = S^{\beta} \triangleright C \triangleright \phi \equiv \Psi$$

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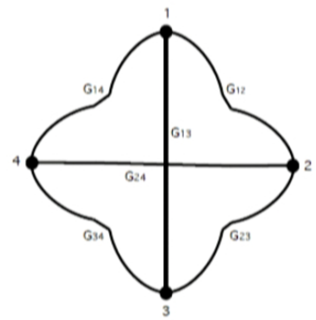
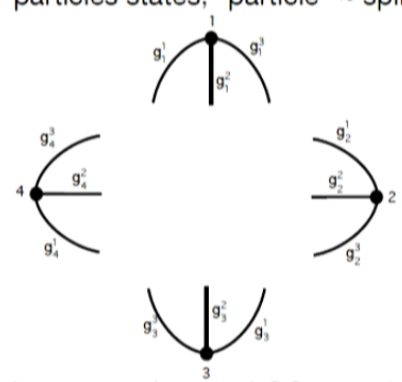
connection between LQG and GFT can be obtained even more directly

second quantized version of Loop Quantum Gravity (adapted to simplicial context),
but dynamics not derived from canonical quantization of GR

(DO, 1310.7786 [gr-qc])

(LQG spin network states ~ many-particles states, "particle" ~ spin network vertex)





GFT Hilbert space = Fock space of open spin network vertices - contains any LQG state (all spin networks)

any LQG observable has a 2nd quantised, GFT counterpart

choice of LQG dynamics (Hamiltonian constraint operator) translates into choice of GFT action

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QFT methods (i.e. GFT reformulation of LQG and spin foam models) useful to address physics of large numbers of LQG d.o.f.s, i.e. many and refined graphs (continuum limit)

(superpositions of "many-vertices" states, refinement as creation of new vertices, etc)

1. making sense of quantum dynamics and LQG partition function (correlations)
2. understanding LQG phase structure
3. extracting effective continuum dynamics

A new direction for Quantum Gravity

new (non-geometric, non-spatio-temporal) physical degrees of freedom (“building blocks”) for space-time

GFTs are a formulation of LQG/spin foams that is most suited to tackle this problem, thanks to QFT tools
richer content, thus richer effective continuum dynamics (and more tools to study it) than tensor models

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LQG, spin foams and tensor models should learn (and are learning) to move along N-direction
and extract the effective continuum space-time dynamics in the limit

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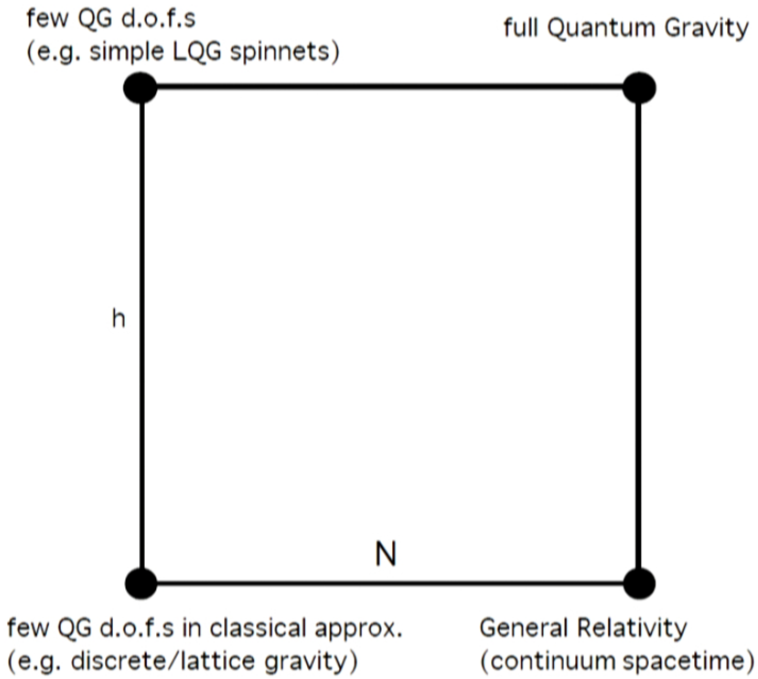
four (independent) directions: c, h, G, N

Moving along the N-direction: the case of QG

assume relativistic and gravitational setting: $c \sim 1$, $G \sim 1$



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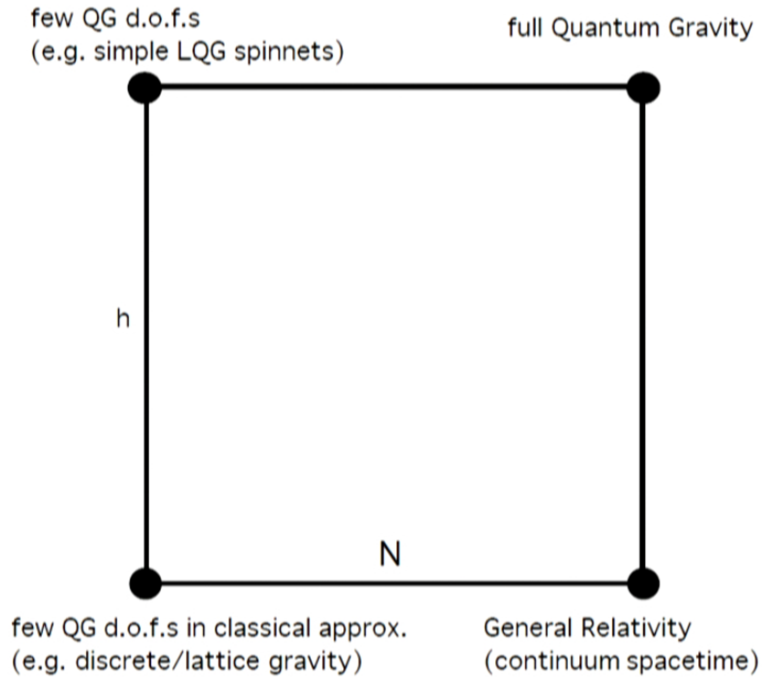
N-direction: continuum approximation

h-direction: classical approximation

very different!



no reason they expect that they commute!

nor that the path is one-to-one

(—> universality vs different phases)

Renormalization of (tensorial) GFTs: motivations

Renormalization Group is crucial tool (mathematical, conceptual, physical)

renormalization is not about "curing or hiding divergences", but
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in continuum spacetime physics, “number of d.o.f.s” translates to energy/distance scale,
because of background geometry

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in specific GFT case: fundamental formulation of QG d.o.f.s given by a QFT, defined perturbatively around the "no-space" vacuum - need to prove consistency of the theory ("while moving along the N-direction"):

perturbative GFT renormalizability

if achieved (and GR emerges in continuum limit): a renormalizable quantum field theory of gravity
(full background independence, as a QFT for the non-spatio-temporal "atoms of space")

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Renormalization of (tensorial) GFTs: motivations

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- for a non-spatio-temporal QG system (e.g. LQG in GFT formulation),

what are the macroscopic phases?
which one is effectively described by a smooth geometry with matter fields? which one do we live in?

idea of "geometrogenesis" in LQG/GFT : continuum geometric physics in new (condensate?) phase

in canonical LQG context:
T. Koslowski, 0709.3465 [gr-qc]

in covariant SF/GFT context:
DO, 0710.3276 [gr-qc]

Renormalization of (tensorial) GFTs: motivations

continuum phases and phase transitions

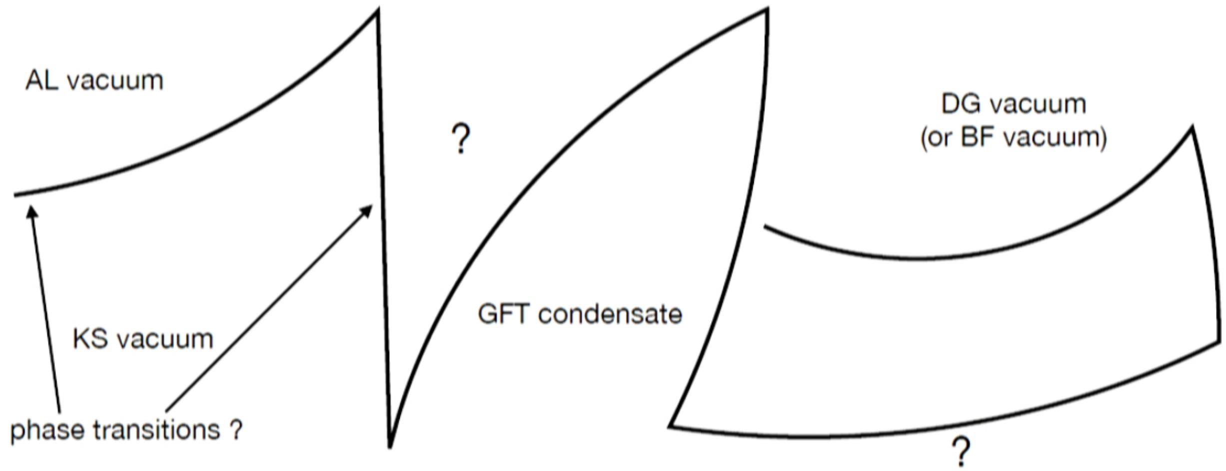
J. Lewandowski, A. Okolow, H. Sahlmannm T. Thiemann'06
C. Fleischack, '06

T. Koslowski, H. Sahlmann, 1109.4688 [gr-qc]

B. Dittrich, M. Geiller, 1401.6441 [gr-qc]

S. Gielen, DO, L. Sindoni, 1303.3576 [gr-qc], 1311.1238 [gr-qc]

Loop Quantum Gravity (and GFT)



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in GFT context: need to prove dynamically the phase transition to non-degenerate (e.g. condensate) phase

possible key ingredient:

asymptotic freedom of perturbative quantum theory

perturbative Fock vacuum (AL-vacuum) is fixed point,
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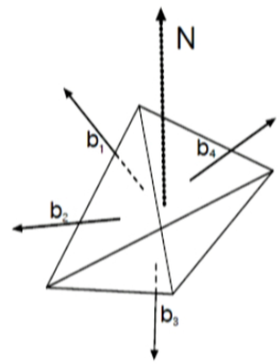
GFT Renormalization: "geometric" interpretation?

arguments of GFT field: $b_i \in \mathfrak{su}(2)$ gravity case: d=4

I b I ~ J = irrep of SU(2) ~ "area of triangles"



GFT renormalization:

- GFT "UV" cut-off N ~ J_{max}
- RG flow: J_{max} ---> small J
- (perturbative) GFT renormalizability: UV fixed point as J_{max} ---> oo
- asymptotic freedom: free theory at J_{max} ---> oo

"geometric" interpretation?

- RG flow from large areas to small areas?
- theory defined in non-geometric phase of "large" disconnected tetrahedra
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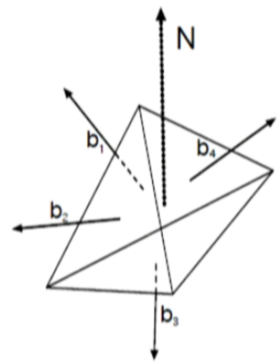

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geometric intepretation?

- CAUTION in interpreting things geometrically outside continuum geometric approx.
- expect "physical" continuum areas A ~ < J > < n >
- expect proper continuum geometric interpretation (and effective metric field) for < J > small, < n > large, A finite (not too small)

Renormalization in GFT models:

where are we?

Renormalization of (tensorial) GFTs: a brief review

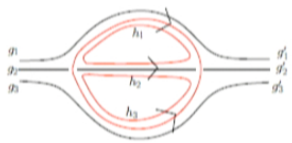
preliminary understanding:

power counting and radiative corrections in GFT models
(hard cut-off of fields, or heat-kernel regularisation of propagator, in representation space)

- 3d (non-abelian) (colored) Boulatov model (BF theory):
  - partial power counting and scaling theorems

L. Freidel, R. Gurau, DO, '09; J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, '09; J. Ben Geloun, J. Magnen, V. Rovasseau, '10 ; S. Carrozza, DO, '11,'12

  - radiative corrections of 2-point function: need for Laplacian kinetic term



J. Ben Geloun, V. Bonzom, '11

  - super-renormalizability in abelian case (with Laplacian)

J. Ben Geloun, '13

- 4d gravity models
  - radiative correction of 2-point function in EPRL-FK model

J. Ben Geloun, R. Gurau, V. Rivasseau, '10; T. Krajewski, J. Magnen, V. Rivasseau, A. Tanasa, P. Vitale, '10; A. Riello, '13

Renormalization of (tensorial) GFTs: a brief review

systematic renormalizability analysis of tensorial GFT models (crucial use of tensor models tools)

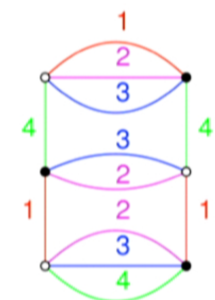
renormalization ingredients and class of models

- locality principle and soft breaking of locality:

tracial locality - tensor invariant interactions

$$S(\varphi, \overline{\varphi}) = \sum_{b \in \mathcal{B}} t_b I_b(\varphi, \overline{\varphi})$$

indexed by d-colored "bubbles"



$$\int [\mathrm{d}g_i]^{12} \varphi(g_1, g_2, g_3, g_4)\overline{\varphi}(g_1, g_2, g_3, g_5)\varphi(g_8, g_7, g_6, g_5)$$
$$\overline{\varphi}(g_8, g_9, g_{10}, g_{11})\varphi(g_{12}, g_9, g_{10}, g_{11})\overline{\varphi}(g_{12}, g_7, g_6, g_4)$$

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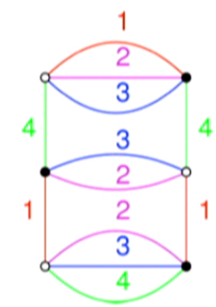

general enough class of models

equivalent to colored simplicial locality

$$\int [\mathrm{d}g_i]^{12} \varphi(g_1,g_2,g_3,g_4)\overline{\varphi}(g_1,g_2,g_3,g_5)\varphi(g_8,g_7,g_6,g_5)\overline{\varphi}(g_8,g_9,g_{10},g_{11})\varphi(g_{12},g_9,g_{10},g_{11})\overline{\varphi}(g_{12},g_7,g_6,g_4)$$

Laplacian kinetic term
(or its power "a")

propagator = $\left(m^2 - \sum_{\ell=1}^{d} \Delta_\ell\right)^{-1}$

Renormalization of (tensorial) GFTs: a brief review

many results:

J. Ben Geloun, D. Ousmane-Samary, V. Rivasseau, S. Carrozza, DO, E. Livine, F. Vignes-Tourneret, A. Tanasa, M. Raasakka, …..

- first renormalizable (abelian) TGFT model

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(requires more subtle analysis of combinatorics of diagrams, crucial role of rank of incidence matrix between edges and faces of Feynman diagrams)

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- first proof of asymptotic freedom for abelian TGFT models without gauge invariance

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- first proof of asymptotic freedom for TGFT models with gauge invariance

  S. Carrozza, '14

Results: Several Renormalizable Models

TGFT (type)	G_D	$\Phi^{k_{\max}}$	d	a	Renormalizability	UV behavior
	$U(1)$	Φ^6	4	1	Just-	AF
	$U(1)$	Φ^3	3	$\frac{1}{2}$	Just-	AF
	$U(1)$	Φ^6	3	$\frac{2}{3}$	Just-	AF
	$U(1)$	Φ^4	4	$\frac{3}{4}$	Just-	AF
	$U(1)$	Φ^4	5	1	Just-	AF
	$U(1)^2$	Φ^4	4	1	Just-	AF
	$U(1)$	Φ^{2k}	3	1	Super-	-
gi-	$U(1)$	Φ^4	6	1	Just-	AF
gi-	$U(1)$	Φ^6	5	1	Just-	AF
gi-	$SU(2)$	Φ^6	3	1	Just-	AF
gi-	$U(1)$	Φ^{2k}	4	1	Super-	-
gi-	$U(1)$	Φ^4	5	1	Super-	-
Matrix	$U(1)$	Φ^{2k}	2	$\frac{1}{2}(1-\frac{1}{k})$	Just-	($k=2$, AS$^{(\infty)}$); ($k=3$, LG)
Matrix	$U(1)^2$	Φ^{2k}	2	$1-\frac{1}{k}$	Just-	($k=2$, AS$^{(1)}$); ($k=3$, LG)
Matrix	$U(1)^3$ or $SU(2)$	Φ^6	2	1	Just-	LG
Matrix	$U(1)^3$ or $SU(2)$	Φ^4	2	$\frac{3}{4}$	Just-	AS$^{(1)}$
Matrix	$U(1)^4$	Φ^4	2	1	Just-	AS$^{(1)}$
Matrix	$U(1)$	Φ^{2k}	2	$\frac{1}{2}$	Super-	-
Matrix	$U(1)^2$	Φ^{2k}	2	1	Super-	-

Table: Updated list of tensorial renormalizable models and their features (AF $\equiv$ asymptotically free; LG $\equiv$ existence of a Landau ghost; AS$^{(\ell)}$ asymptotically safe at ℓ-loops).
a = power of Laplacian in kinetic term; d = rank of tensor = number of arguments of GFT field

Renormalization of (tensorial) GFTs: what next?

- study models not based on group manifolds (homogeneous spaces, subspaces of groups)

- include simplicity constraints

  + study renormalizability and asymptotic freedom of 4d gravity (spin foam) models (esp. causal ones)

- understand notion of “locality” without combinatorial locality: what is the physics/geometry of tensor invariance of GFT interactions?

- clarify necessity and geometric meaning of “laplacian kinetic term” or identify alternatives

- develop additional tools: Functional Renormalization Group for GFTs

  A. Eichhorn, T. Koslowski, ’13; D. Benedetti, J. Ben Geloun, DO, to appear

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