Title: What you always wanted to know about CDT, but did not have time to read about in our papers

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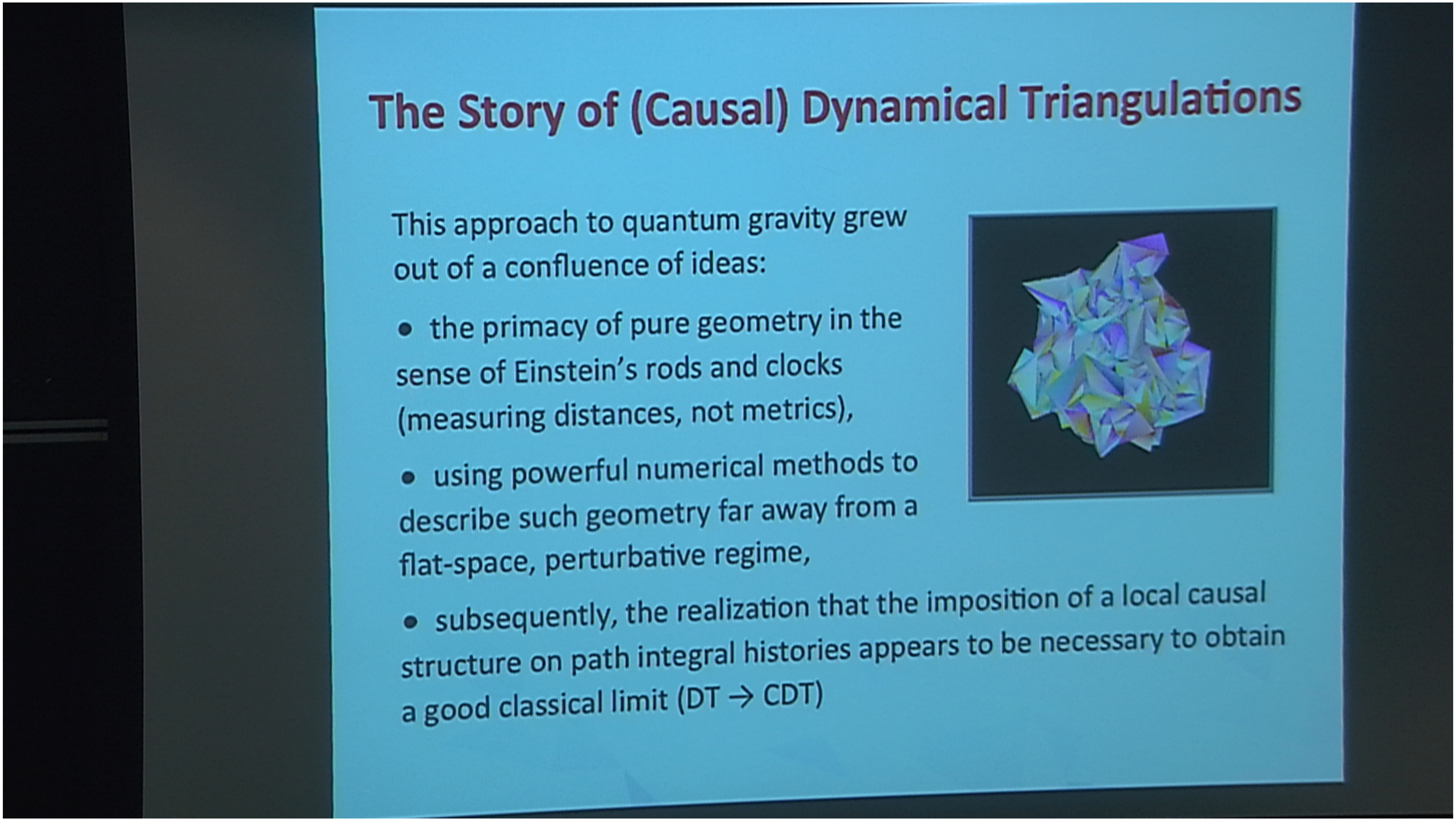
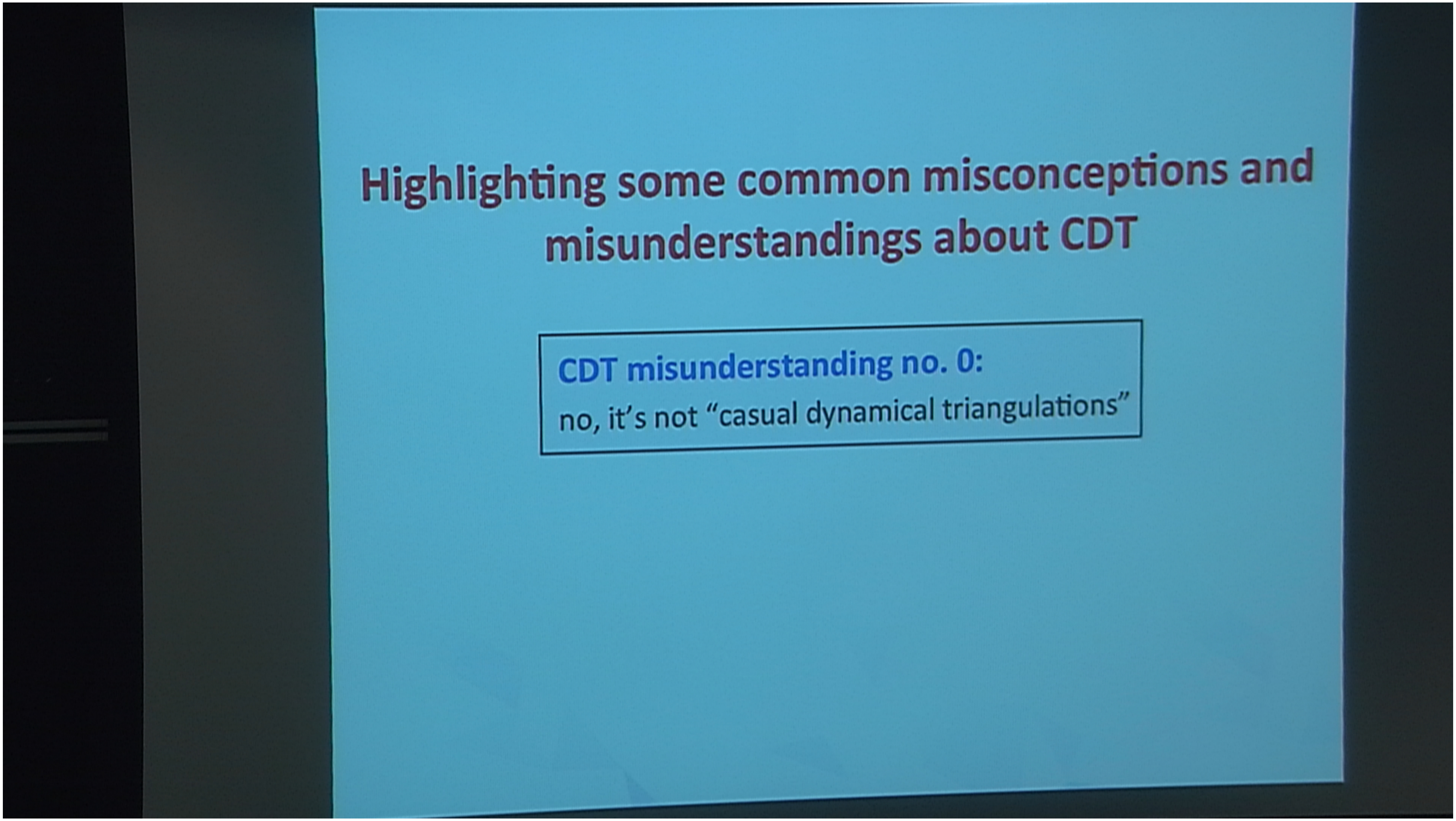
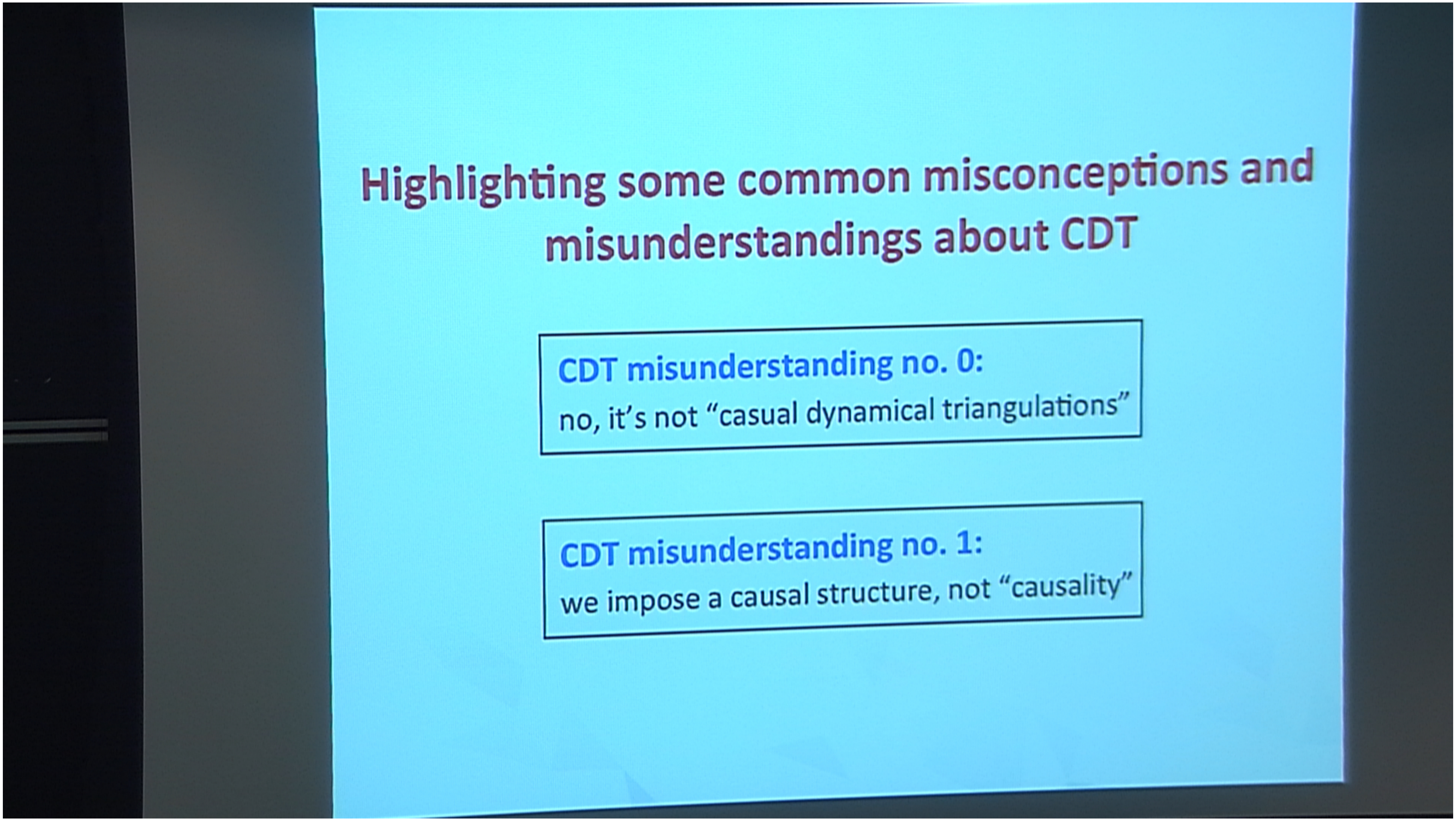
Abstract: I will review the approach of Causal Dynamical Triangulations to nonperturbative quantum gravity, high-lighting some frequently mis- or ununderstood features, emphasizing recent developments and discussing some interesting open issues.


The Story of (Causal) Dynamical Triangulations
This approach to quantum gravity grew out of a confluence of ideas:
• the primacy of pure geometry in the sense of Einstein's rods and clocks (measuring distances, not metrics),
• using powerful numerical methods to describe such geometry far away from a flat-space, perturbative regime,
• subsequently, the realization that the imposition of a local causal structure on path integral histories appears to be necessary to obtain a good classical limit (DT → CDT)

Highlighting some common misconceptions and misunderstandings about CDT


Highlighting some common misconceptions and misunderstandings about CDT
CDT misunderstanding no. 0:
no, it's not "casual dynamical triangulations"


Highlighting some common misconceptions and misunderstandings about CDT
CDT misunderstanding no. 0:
no, it's not "casual dynamical triangulations"
CDT misunderstanding no. 1:
we impose a causal structure, not "causality"

Timeline of main results in 4d CDT

2000/01: set-up of 3&4d Lorentzian DT quantum gravity, building blocks, gluing rules, action, Wick rotation, transfer matrix, reflection positivity JA, JJ & RL, PRL 85 (2000) 924; NPB 610 (2001) 347

2004: emergence of a macroscopic 4d world from CDT, global scaling properties JA, JJ & RL, PRL 93 (2004) 131301

2004: reconstruction of minisuperspace action, relation with wave function of the universe JA, JJ & RL, PLB 607 (2005) 205

2005: spectral and Hausdorff dimension of the dynamically generated universe, geometry of spatial slices, phase diagram JA, JJ & RL, PRL 95 (2005) 171301; PRD 72 (2005) 064014

2007/08: Planckian birth of de Sitter universe, matching of volume profile and quantum fluctuations JA, AG, JJ & RL, PRL 100 (2008) 091304; PRD 78 (2008) 063544

Timeline of main results in 4d CDT

2010: relation with Horava-Lifshitz gravity JA, AG, SJ, JJ & RL, PLB 690 (2010) 413

2010: coupling a point-like mass IK, RL & PR, CQG 27 (2010) 185025

2010/11: fine-grained geometry and volume profile JA, AG, JJ & RL, PLB 690 (2010) 420; JA, AG, JJ, RL, JGS & TT, NPB 849 (2011) 144

2011/12: first- and second-order phase transitions JA, SJ, JJ & RL, PRL 107 (2011) 211303; JA, SJ, JJ & RL, PRD 85 (2012) 124044

2013: CDT without preferred foliation SJ & RL, PLB 724 (2013) 155; PRD 88 (2013) 044055

Collaborators: JA = Jan Ambjørn, AG = Andrzej Görlich, SJ = Samo Jordan, JJ = Jerzy Jurkiewicz, IK = Igor Khavkine, PR = Paul Reska, JGS = Jakub Gisbert-Studnicki, TT = Tomasz Trzesniewski

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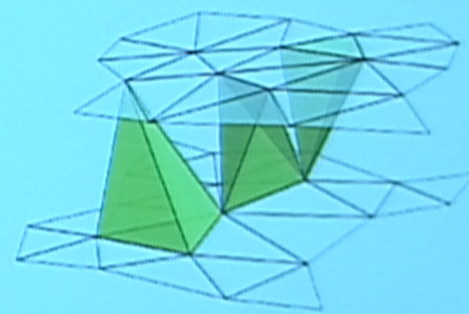
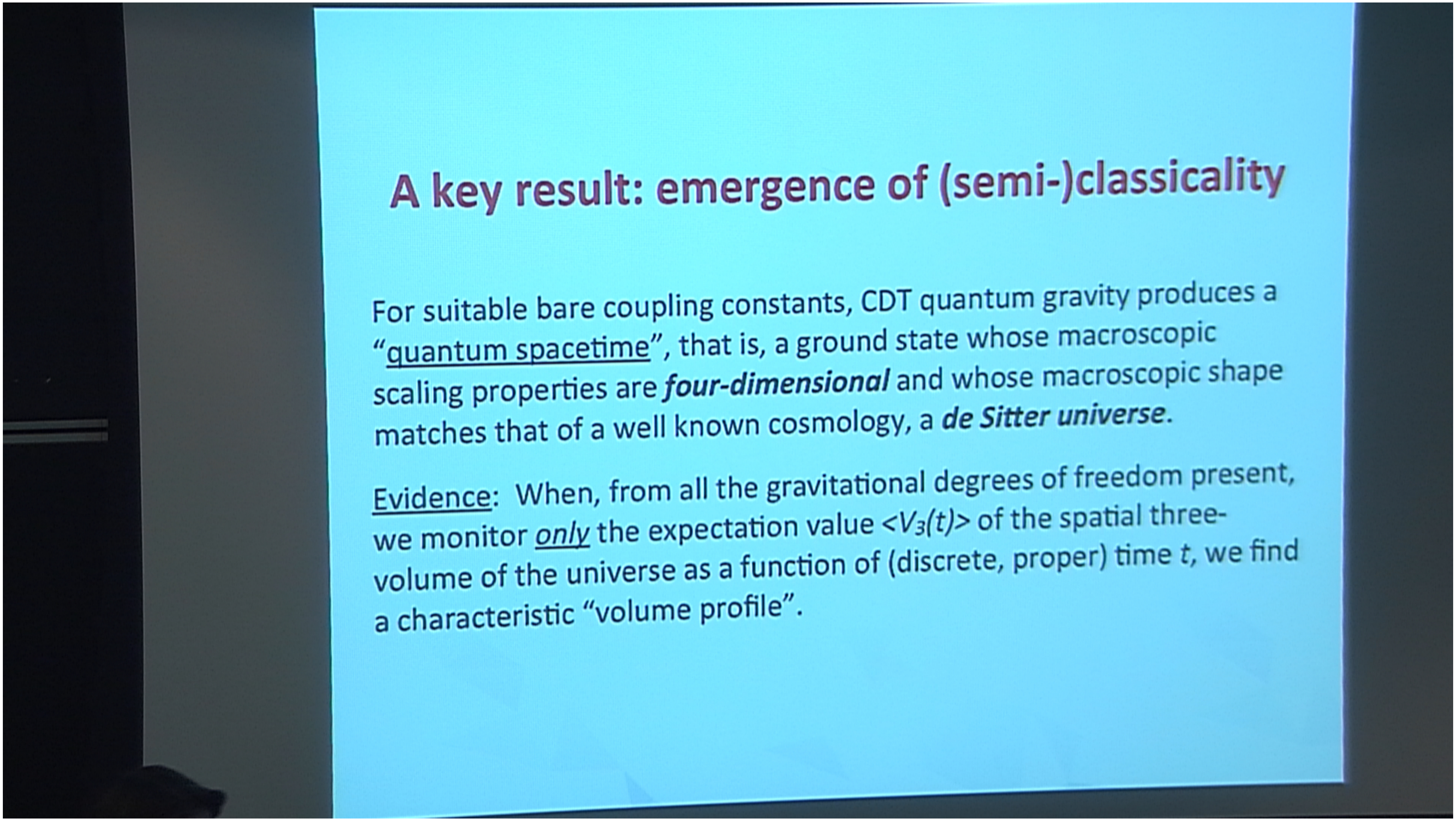
Key properties and ingredients of the CDT approach to quantum gravity

- CDT uses few ingredients/priors:
  - path integral/quantum superposition principle
  - locality and causal structure (this is *not* Euclidean quantum gravity)
  - notion of (proper) time
  - Wick rotation
  - standard tools of quantum field theory

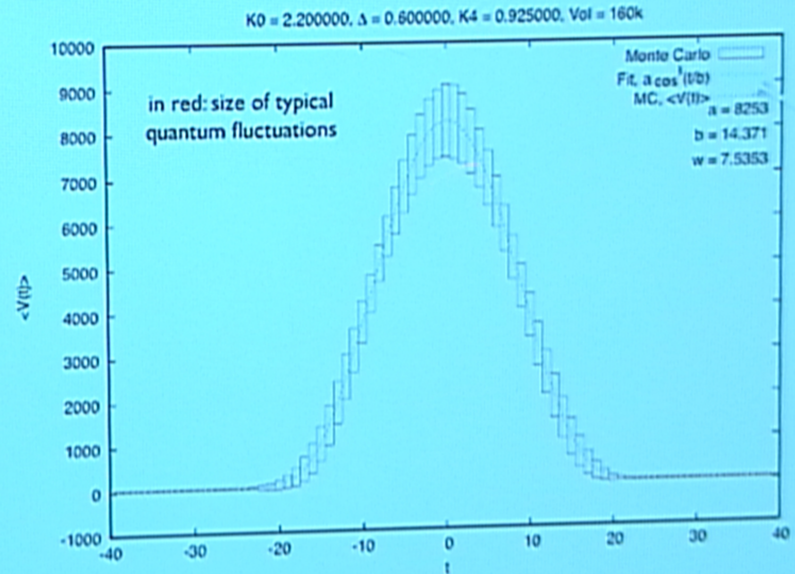


piece of causal triangulation

- "conservative" configuration space, close to that of GR: piecewise flat geometries (the triangulations)
- Few free parameters (Λ, G_N, Δ)
- Robustness of construction; universality
- Crucial: nonperturbative computational tools to extract quantitative results


A key result: emergence of (semi-)classicality
For suitable bare coupling constants, CDT quantum gravity produces a "quantum spacetime", that is, a ground state whose macroscopic scaling properties are four-dimensional and whose macroscopic shape matches that of a well known cosmology, a de Sitter universe.
Evidence: When, from all the gravitational degrees of freedom present, we monitor only the expectation value <V3(t)> of the spatial three-volume of the universe as a function of (discrete, proper) time t, we find a characteristic "volume profile".

The quantitative evidence for de Sitter space



fit to $\cos^3(t/b)$

The average volume profile $\langle V_3(t) \rangle$, as function of Euclidean proper time $t = i\tau$, perfectly matches that of a Euclidean *de Sitter space*, with scale factor $a(t)^2$ given by

$$ds^2 = dt^2 + a(t)^2 d\Omega^2_{(3)} = dt^2 + c^2 \cos^2\left(\frac{t}{c}\right) d\Omega^2_{(3)}$$

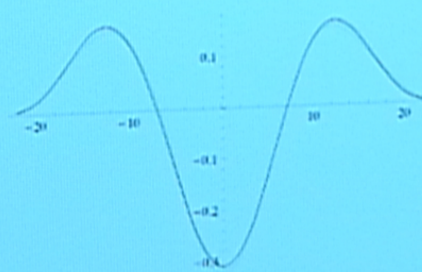
← volume el. S^3

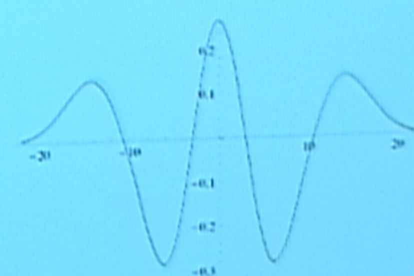
Quantum volume fluctuations around de Sitter

In addition, expanding the continuum minisuperspace action around the classical de Sitter solution $V_3^{dS}(t)$, with $\delta V_3(t)=V_3(t)-V_3^{dS}(t)$,

$$S_{\rm eu}(V_3) = S(V_3^{\rm dS}) + \kappa \int dt\ \delta V_3(t) \hat{H} \delta V_3(t)$$

the eigenmodes of the quadratic operator $\hat{H}$ match well with those extracted from the simulations:

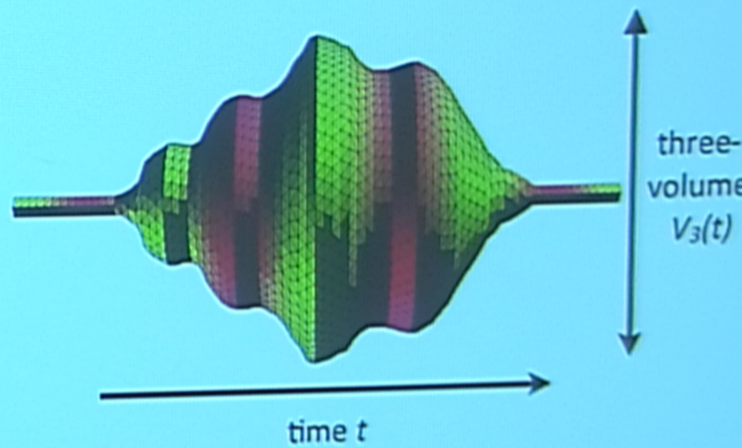


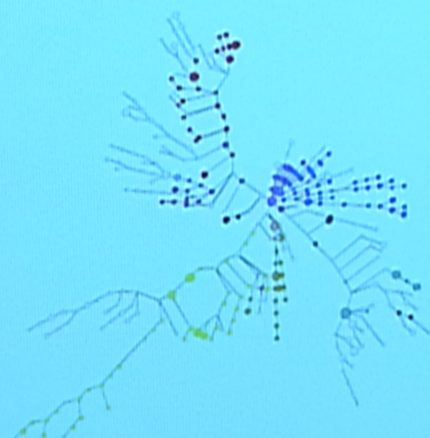


(N.B.: no further fitting necessary)

CDT misunderstanding no. 2:

the local geometry of this quantum universe is close to that of a smooth, constant-curvature de Sitter space





minimal-neck tree structure of spatial slices

- the universe is only about 15 ℓ_{Pl} across and its local geometry is strongly quantum-fluctuating, with fractal properties
- only(?) the three-volume behaves semi-classically
- no good notion of local curvature in the nonperturbative regime

CDT misunderstanding no. 3:

One obtains a semiclassical result because curvature fluctuations are suppressed in CDT and the path integral is simply dominated by the classical extremum.

- no; even integrated curvatures diverge in the continuum limit $N \to \infty$
- note that GR action is unbounded below! generically encounter a "conformal catastrophe" when $\kappa = 1/G_N$ is too large

Starting from the Wick-rotated path integral,

$$\int \mathcal{D}g \; \mathrm{e}^{-S_{eu}^{\mathrm{bare}}}, \quad S_{eu}^{\mathrm{bare}} = \kappa \int dt \, \Big(-\frac{\dot{V}_3^2}{V_3} - V_3^{1/3} + \dots \Big)$$

by integrating out everything *but* the global conformal mode $V_3(t)$, we obtain an effective dynamics for $V_3(t)$, given by

$$\mathrm{e}^{-S_{eu}^{\mathrm{eff}}}, \quad S_{eu}^{\mathrm{eff}} = \kappa \int dt \, \Big(+\frac{\dot{V}_3^2}{V_3} + V_3^{1/3} + \dots \Big)$$

This is entirely due to nonperturbative, entropic effects (the PI measure).

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Wick rotation in CDT

The Wick rotation of CDT is an analytic continuation in the anisotropy parameter α in the Einstein-Hilbert action S^{bare} of piecewise flat spacetimes *T*. - For illustration, consider CDT in 1+1 dimensions:

elementary Minkowskian building block in 1+1d

- spacelike edge, squared length = a^2
- timelike edge, squared length = $-\alpha\, a^2$, $\alpha > 0$

$$S^{bare} = S^{bare}[\text{connectivity of } T;\, a,\, \alpha]$$

The continuation $\alpha \mapsto -\alpha$ turns Feynman amplitudes into Boltzmann weights,

$$e^{iS^{\text{bare}}[T]} \mapsto e^{-S^{\text{bare}}_{eu}[T_{eu}]}$$

This is needed to establish convergence properties of the path integral and to perform Monte Carlo simulations.

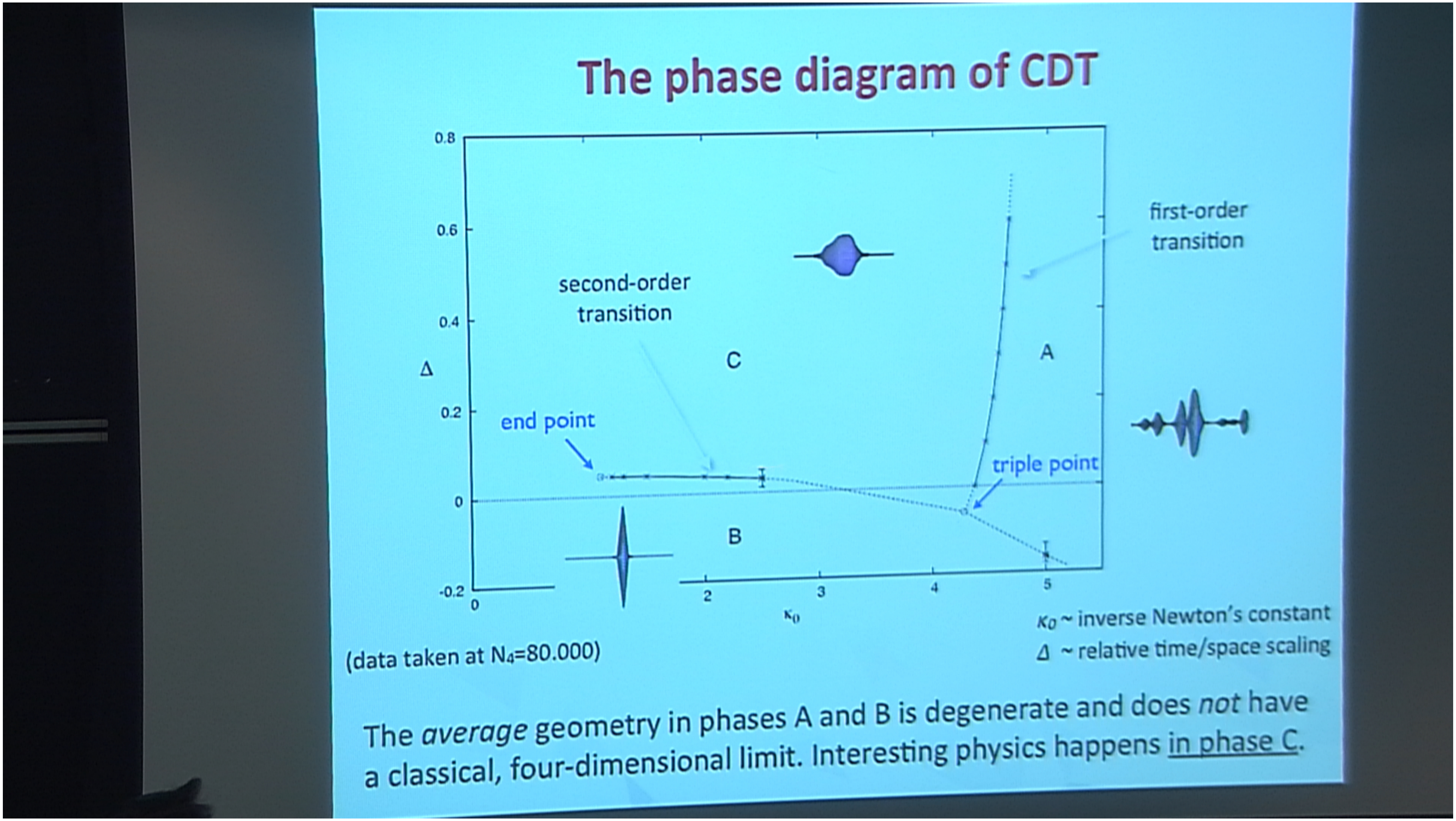
N.B.: an analogous prescription for smooth geometries is *not* known

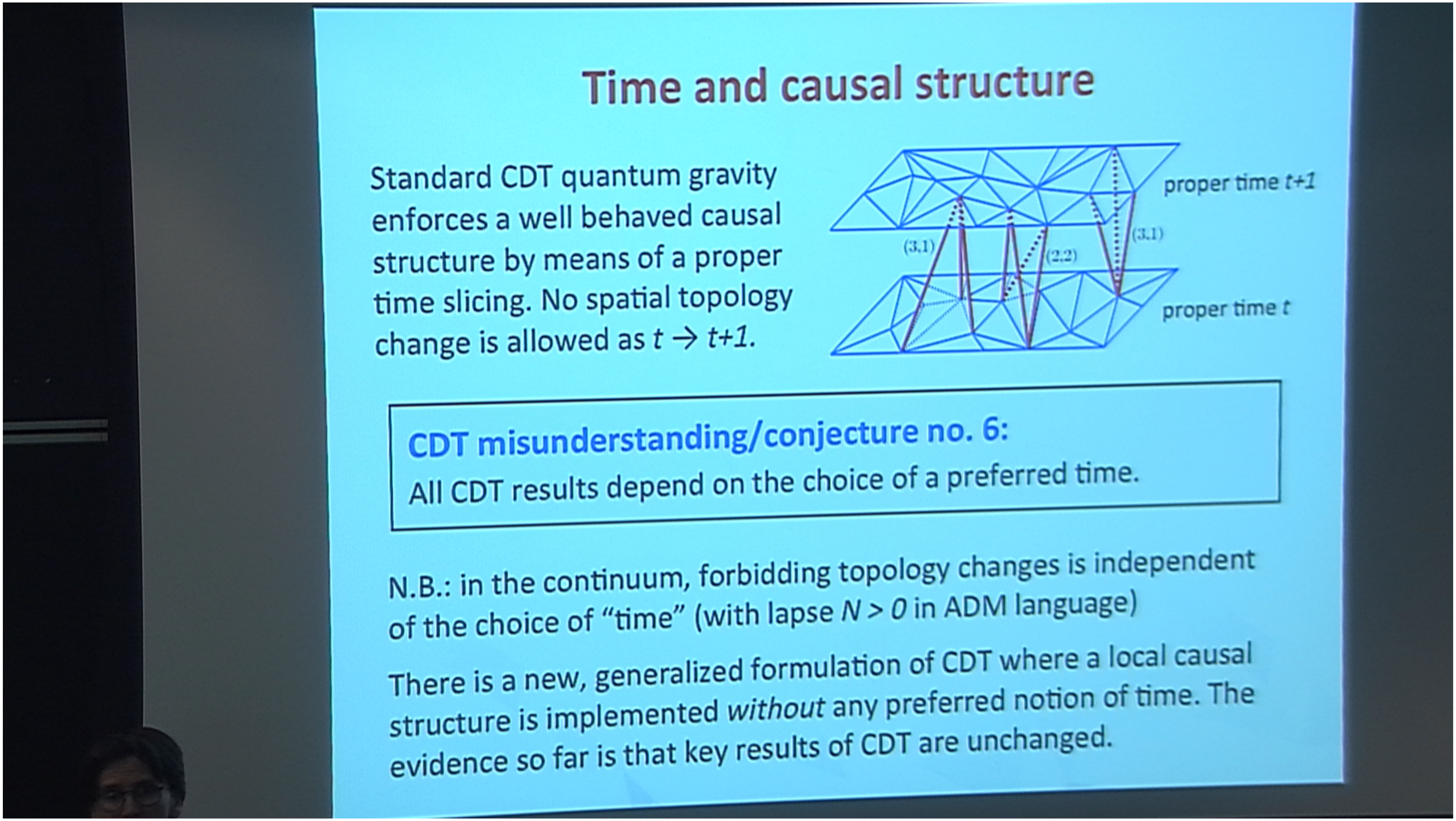
CDT misunderstanding no. 4:
After Wick-rotating, one obtains the same path integral as in Dynamical Triangulations, the Euclidean precursor of CDT.

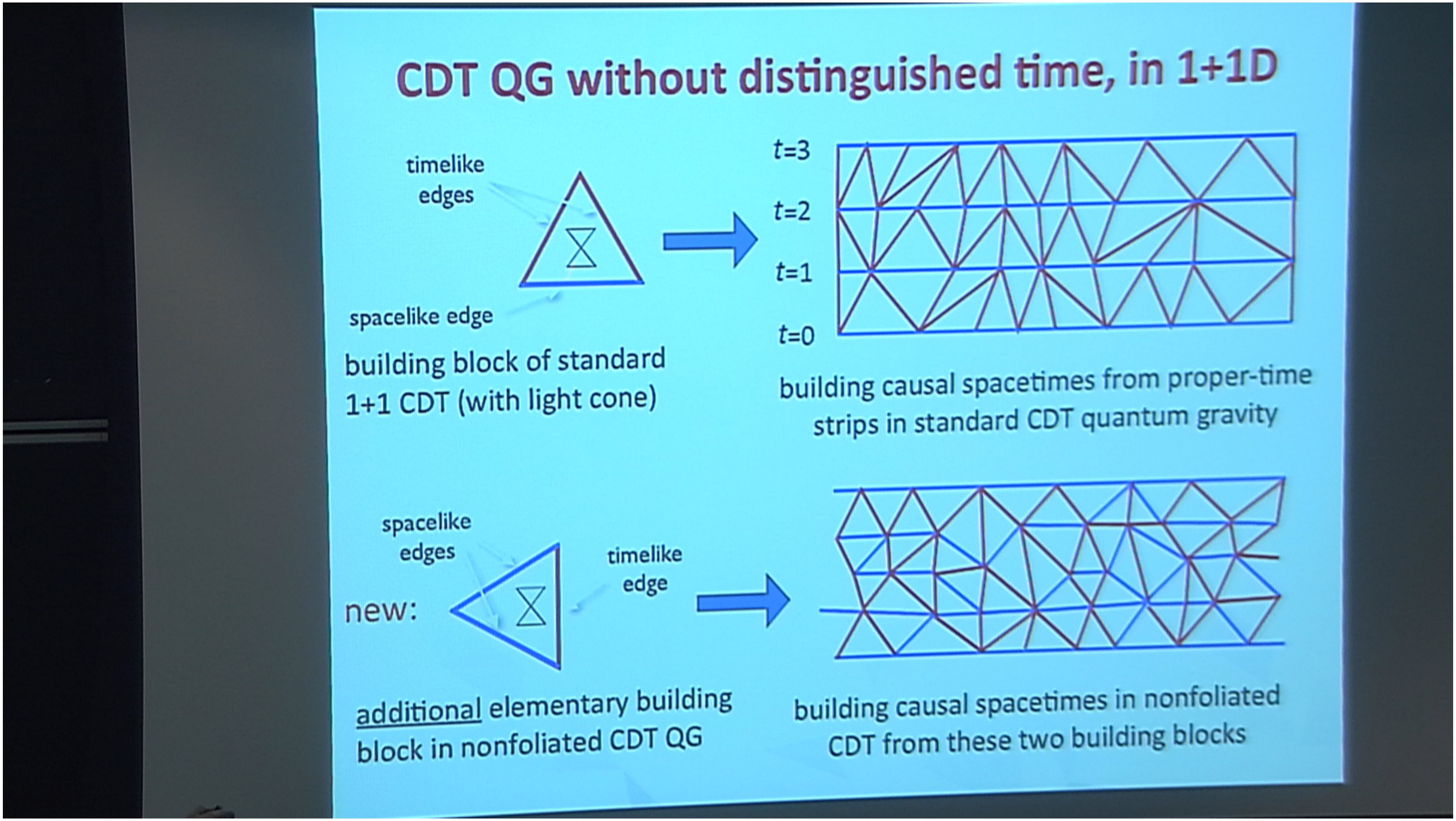
No, the Euclideanized geometries retain a "memory" of the causal gluing rules of their Lorentzian counterparts, and form a small subset of all intrinsically Euclidean Dynamical Triangulations.

CDT misunderstanding no. 5:
The Wick rotation of CDT eliminates the conformal-mode problem.

No, the Wick rotation treats the conformal mode just like any other mode. The Euclideanized action remains unbounded below in the continuum limit, and the Euclideanized path integral suffers from the usual conformal catastrophe for sufficiently large κ.


The phase diagram of CDT
first-order
transition
second-order
transition
C
A
B
Δ
end point
triple point
0.8
0.6
0.4
0.2
0
-0.2
0
2
3
4
5
κ₀
(data taken at N₄=80.000)
κ₀ ~ inverse Newton's constant
Δ ~ relative time/space scaling
The *average* geometry in phases A and B is degenerate and does *not* have a classical, four-dimensional limit. Interesting physics happens in phase C.


Time and causal structure
Standard CDT quantum gravity enforces a well behaved causal structure by means of a proper time slicing. No spatial topology change is allowed as t → t+1.
proper time t+1
(3,1)
(2,2)
(3,1)
proper time t
CDT misunderstanding/conjecture no. 6:
All CDT results depend on the choice of a preferred time.
N.B.: in the continuum, forbidding topology changes is independent of the choice of "time" (with lapse N > 0 in ADM language)
There is a new, generalized formulation of CDT where a local causal structure is implemented without any preferred notion of time. The evidence so far is that key results of CDT are unchanged.


CDT QG without distinguished time, in 1+1D
timelike
edges
spacelike edge
building block of standard
1+1 CDT (with light cone)
t=3
t=2
t=1
t=0
building causal spacetimes from proper-time
strips in standard CDT quantum gravity
spacelike
edges
timelike
edge
new:
additional elementary building
block in nonfoliated CDT QG
building causal spacetimes in nonfoliated
CDT from these two building blocks

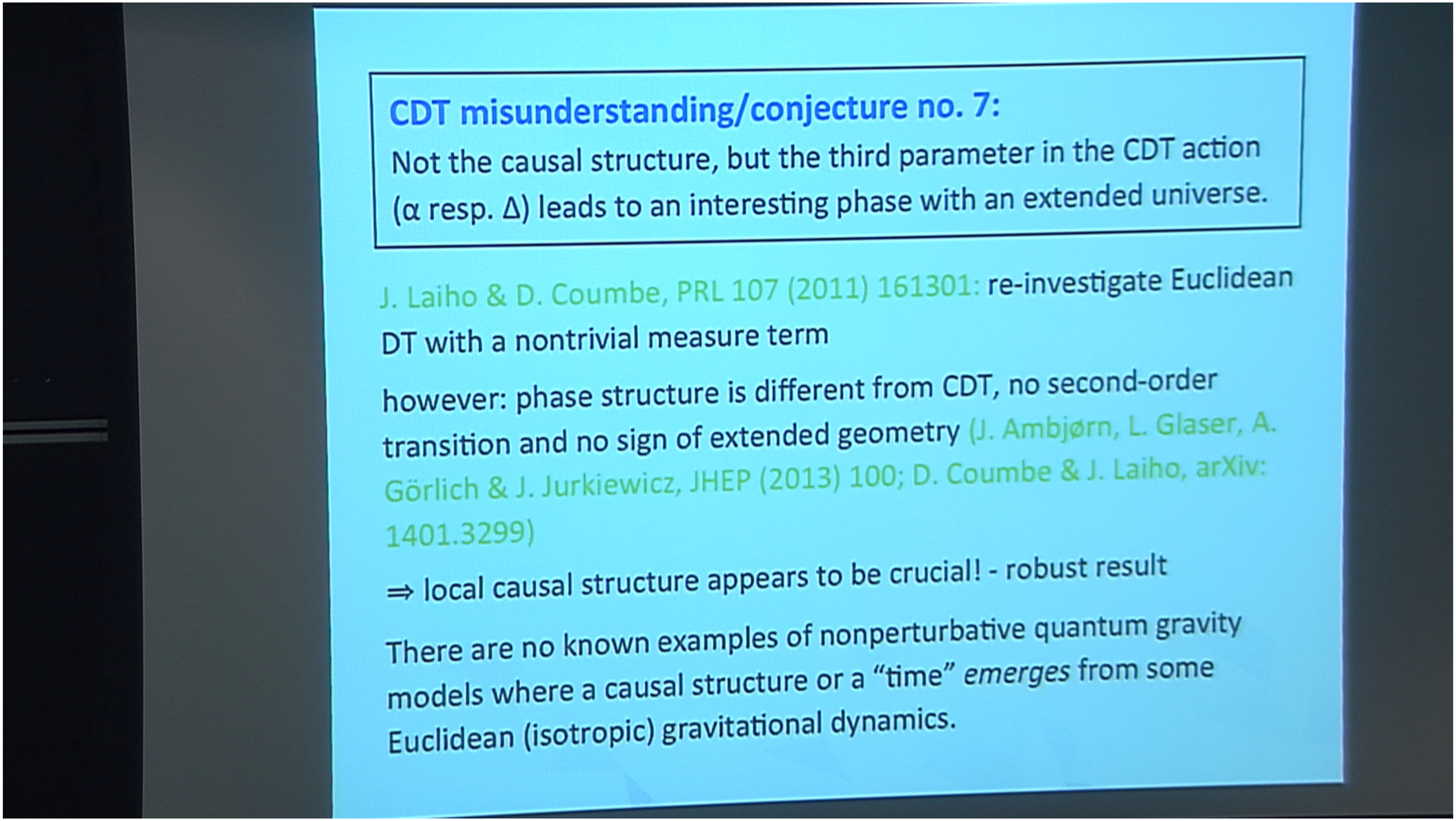
Nonfoliated CDT quantum gravity: findings

By introducing a generalized set of simplicial building blocks, one gets rid of the preferred time and associated foliation, while maintaining a well-behaved local causal (= light cone) structure everywhere. - The Wick rotation remains available!

There is numerical evidence in 1+1d that the model lies in the same universality class as standard CDT in 1+1d (work with B. Ruijl).

A careful investigation of the 2+1d model has reproduced standard results on the de Sitter-like volume profile of the universe, with respect to a "cosmological" proper time introduced *a posteriori* on the spacetime geometries in the quantum ensemble.

This strongly suggests that the distinguished foliation is *not* an essential part of CDT's "background structure", and that the nonfoliated version of CDT lies in the same universality class.


CDT misunderstanding/conjecture no. 7:
Not the causal structure, but the third parameter in the CDT action
(α resp. Δ) leads to an interesting phase with an extended universe.
J. Laiho & D. Coumbe, PRL 107 (2011) 161301: re-investigate Euclidean
DT with a nontrivial measure term
however: phase structure is different from CDT, no second-order
transition and no sign of extended geometry (J. Ambjørn, L. Glaser, A.
Görlich & J. Jurkiewicz, JHEP (2013) 100; D. Coumbe & J. Laiho, arXiv:
1401.3299)
⇒ local causal structure appears to be crucial! - robust result
There are no known examples of nonperturbative quantum gravity
models where a causal structure or a "time" emerges from some
Euclidean (isotropic) gravitational dynamics.

The role of the ensemble

(a.k.a. carrier space of the path integral ⟷ Hilbert space of states)

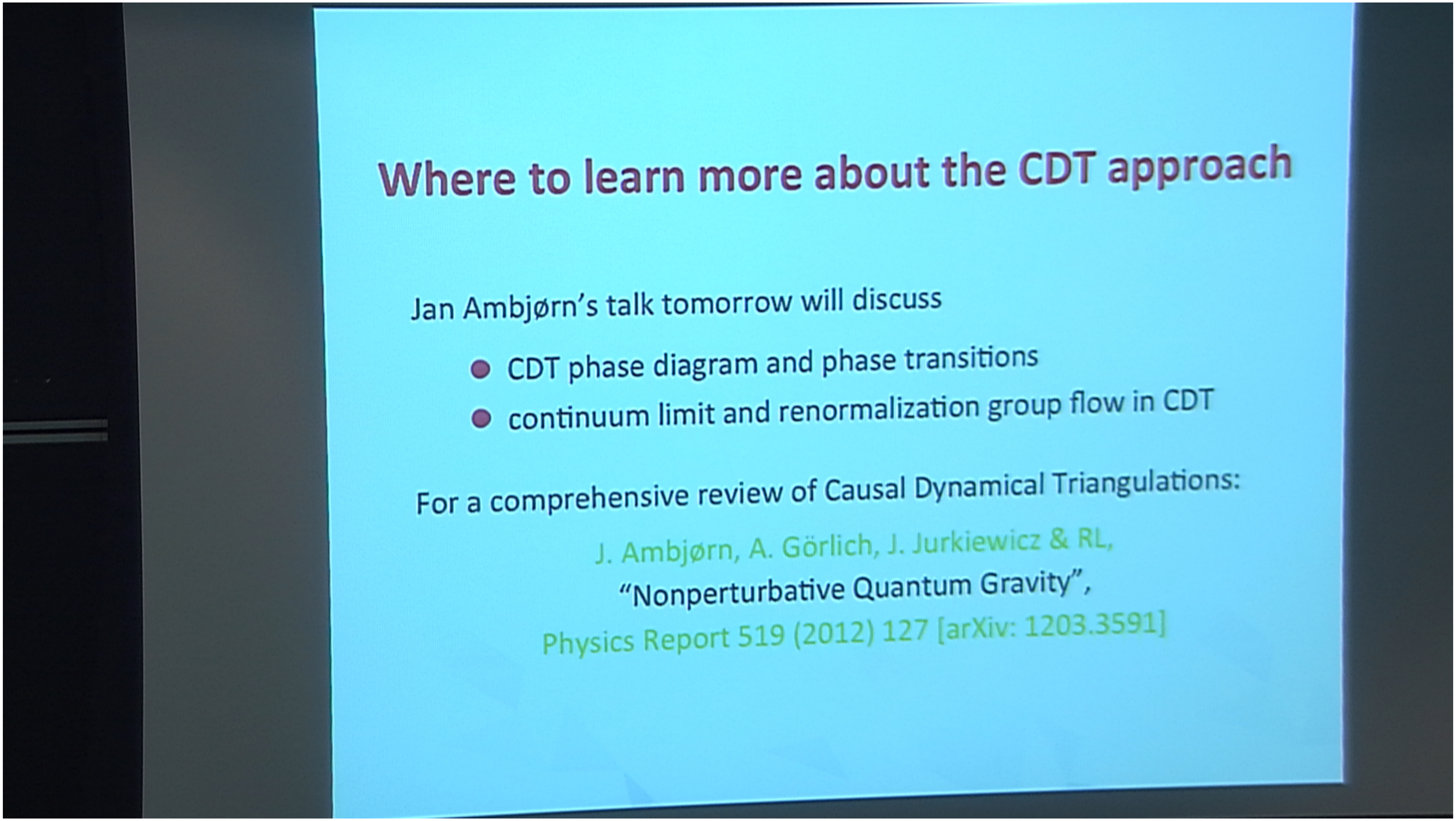
> **CDT misunderstanding no. 8:**
> The larger, the better. Let's make everything dynamical.

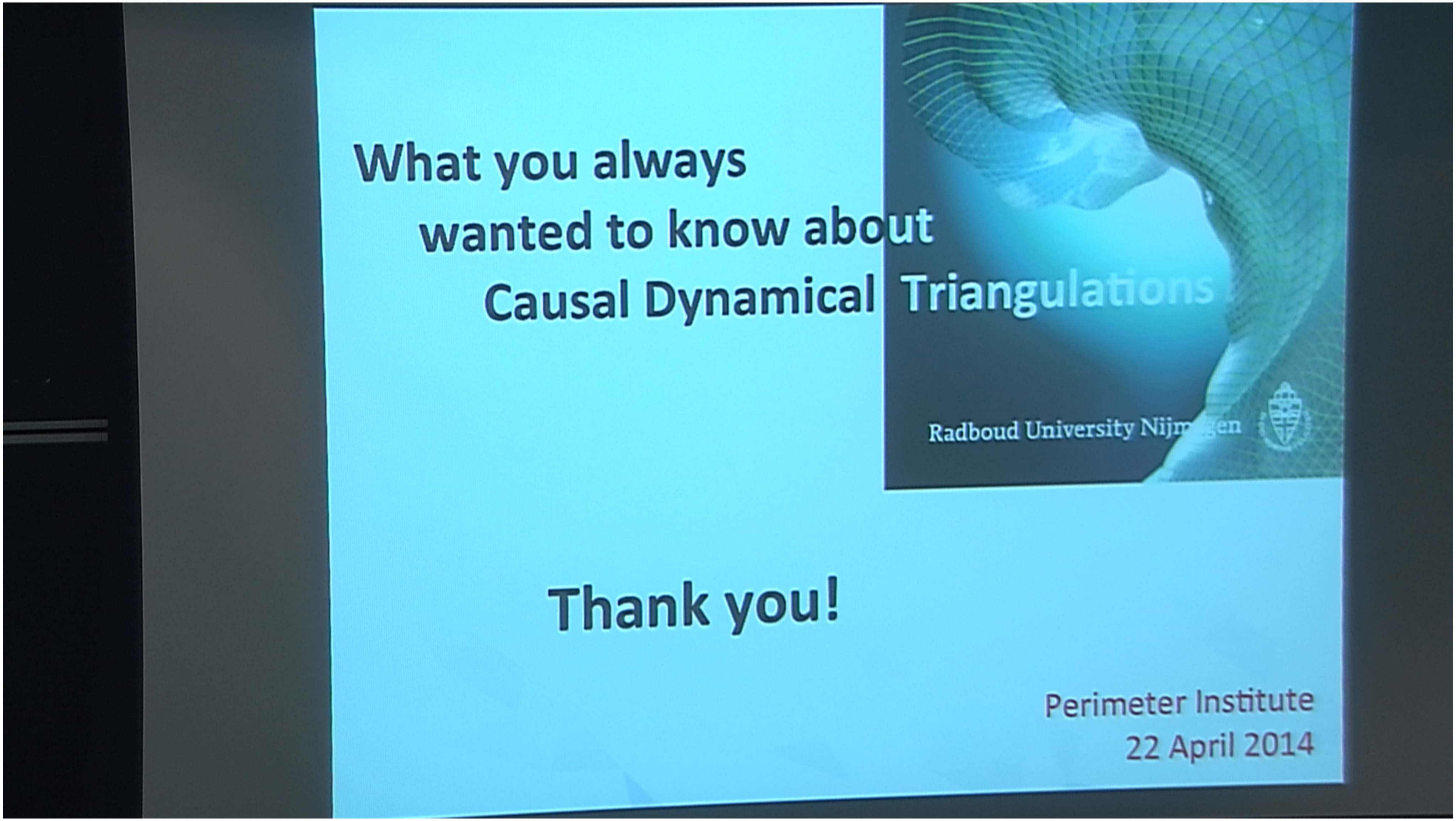
CDT ensemble is minimal, but geometric; curvature is discretized. One might consider including much more, e.g. non-manifold configurations, different topologies, ...

If the number $\boldsymbol{N}(N)$ of configurations of volume N grows superexponentially, the path integral will not be renormalizable by standard methods.

The nonperturbative dynamics will be governed by the "entropy" of these non-geometric d.o.f.; whether this has still anything to do with gravity and a classical limit exists then become wide open issues.

If a particular type of non-classical "geometry"/topology is dynamically preferred, it should already be visible from the smaller ensemble.


Where to learn more about the CDT approach
Jan Ambjørn's talk tomorrow will discuss
CDT phase diagram and phase transitions
continuum limit and renormalization group flow in CDT
For a comprehensive review of Causal Dynamical Triangulations:
J. Ambjørn, A. Görlich, J. Jurkiewicz & RL,
"Nonperturbative Quantum Gravity",
Physics Report 519 (2012) 127 [arXiv: 1203.3591]


What you always
wanted to know about
Causal Dynamical Triangulations
Radboud University Nijm
Thank you!
Perimeter Institute
22 April 2014