Title: Recent developments in asymptotic safety: tests and properties

Date: Apr 22, 2014 09:10 AM

URL: http://pirsa.org/14040085

Abstract: The talk will review recent tests of the asymptotic safety conjecture within functional renormalisation group studies and progress in understanding the properties that such a fixed point would have.

Recent developments in asymptotic safety: tests and properties

RG approaches to QG, Perimeter, 22/4/14

Tim Morris,
Physics & Astronomy,
University of Southampton, UK.

GR:

- $g_{\mu\nu}$ is a field

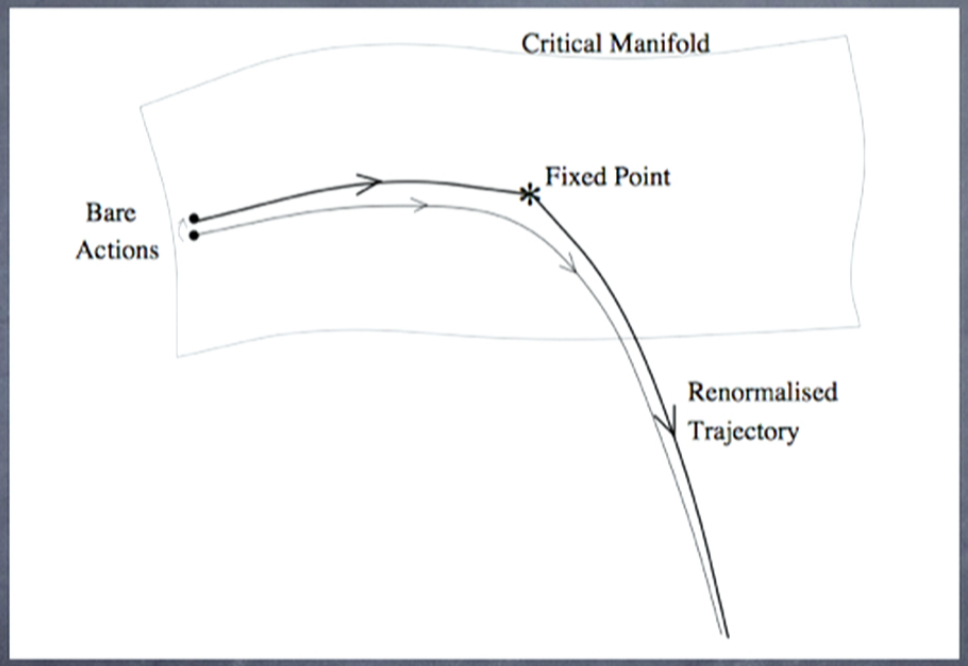
QM:

- which must be consistent with QM

$\downarrow$

- Effective quantum field theory

Wilsonian RG



TRM, Prog. Theor. Phys. Suppl. **131** (1998) 395.

G is irrelevant about Gaussian FP.

So what is the UV completion?

Non-perturbative UV FP

- Higgs? K.G. Wilson & J. Kogut, Phys. Rep. **12C** (1974) 75
- Large N Gross-Neveu model (four-fermi) in <4 dimensions K.G. Wilson, Phys. Rev. D10 (1973) 2911

Gravity: asymptotic Safety

- Gravity D=2+ε S. Weinberg, in *Hawking, S.W., Israel, W.: General Relativity* (1979) 790-831; Proc. Int. School of Subnuclear physics, Erice (1976).
- Large N L. Smolin, Nucl. Phys. **B208** (1982) 439; R. Percacci, Phys. Rev. **D73** (2006) 041501(R)

Exact RG...

K.G. Wilson & J. Kogut, Phys. Rep. **12C** (1974) 75; F.J. Wegner & A. Houghton, Phys. Rev. **A8** (1973) 401

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\,\mathrm{tr}\,\left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\,.$$

C. Wetterich, Phys. Lett. **B301** (1993) 90; T.R. Morris, Int. J. Mod. Phys. **A9** (1994) 2411.

Momentum dependent mass term (IR cutoff):

$$\frac{1}{2}\varphi\cdot\mathcal{R}\cdot\varphi$$

suppresses momenta $p < k$

- $k \to 0$ gives full Legendre effective action.
- If $k \to \infty$ exists then continuum limit constructed

1) Global Flows in Quantum Gravity
By Nicolai Christiansen, Benjamin Knorr, Jan M. Pawlowski, Andreas Rodigast.
arXiv:1403.1232 [hep-th].
2) RG flows of Quantum Einstein Gravity on maximally symmetric spaces
By Maximilian Demmel, Frank Saueressig, Omar Zanusso.

... applied to QG

By C. Wetterich.
arXiv:1401.5313 [astro-ph.CO].
4) Black holes within Asymptotic Safety
By Benjamin Koch, Frank Saueressig.
arXiv:1401.4452 [hep-th].
10.1142/S0217751X1430011 7.
Int.J.Mod.Phys. A29 (2014) 8, 1430011.
5) The local potential approximation in the background field formalism
By I. Hamzaan Bridle, Juergen A. Dietz, Tim R. Morris.
JHEP 1403 (2014) 093.
6) The numerically [illegible] renormalization group
By I.G. Márián, U.D. Jentschura, I. Nandori.
10.1088/0954-3899/41/5/055001.
J.Phys. G41 (2014) 055001.
7) Matter matters in asymptotically safe quantum gravity
By Pietro Donà, Astrid Eichhorn, Roberto Percacci.
arXiv:1311.2898 [hep-th].
8) Brans-Dicke theory in the local potential approximation
By Dario Benedetti, Filippo Guarnieri.
arXiv:1311.1081 [hep-th].
9) A Stable Extension with(out) Gravity
By Oleg Antipin, Jens Krog, Matin Mojaza, Francesco Sannino.
arXiv:1311.1092 [hep-ph].
10) Black holes and running couplings: A comparison of two complementary approaches
By Benjamin Koch, Carlos Contreras, Paola Rioseco, Frank Saueressig.
arXiv:1311.1121 [hep-th].
11) Asymptotically Safe Starobinsky Inflation
By Edmund J. Copeland, Christoph Rahmede, Ippocratis D. Saltas.
arXiv:1311.0881 [gr-qc].
12) Non-perturbative quantum gravity: a conformal perspective
By T. G. Budd.
13) Fermions in gravity with local spin-base invariance

M. Reuter, Phys. Rev. **D57** (1998) 971

Reviews...

M. Niedermaier & M. Reuter, Living Rev. Rel. **9** (2006) 5;

R. Percacci, in *Oriti, D (ed.)* arXiv:0709.3851;

D.F. Litim, arXiv:0810.3675;

M. Reuter & F. Saueressig, New J. Phys. **14** (2012) 055022

Einstein-Hilbert truncation

$$\Gamma \sim \frac{1}{16\pi G}\int d^4x\,\sqrt{g}\,\{-R+2\Lambda\}$$

$$G := k^2 G_{phys}(k)\,, \quad \Lambda := \Lambda_{phys}/k^2, \quad t := \ln(k/\mu)$$

E.g. using sharp cutoff:

$$\begin{aligned}
\partial_t \Lambda &= -(2-\eta)\Lambda - \frac{G}{\pi}\left[5\ln(1-2\Lambda) - 2\zeta(3) + \frac{5}{2}\eta\right], \\
\partial_t G &= (2+\eta)G\,, \\
\eta &= -\frac{2G}{6\pi+5G}\left[\frac{18}{1-2\Lambda} + 5\ln(1-2\Lambda) - \zeta(2) + 6\right].
\end{aligned}$$

M. Reuter & F. Saueressig, Phys. Rev. **D65** (2002) 065016, New J. Phys. **14** (2012) 055022

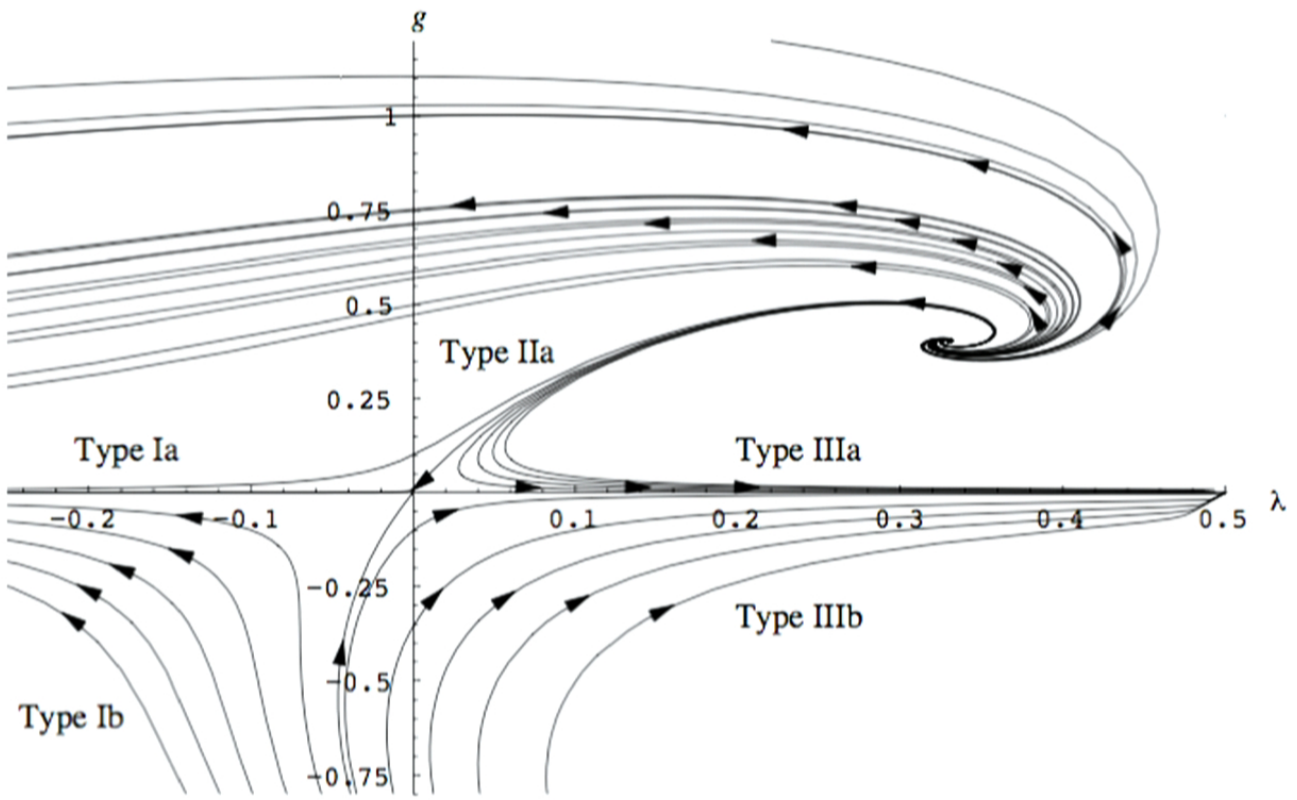
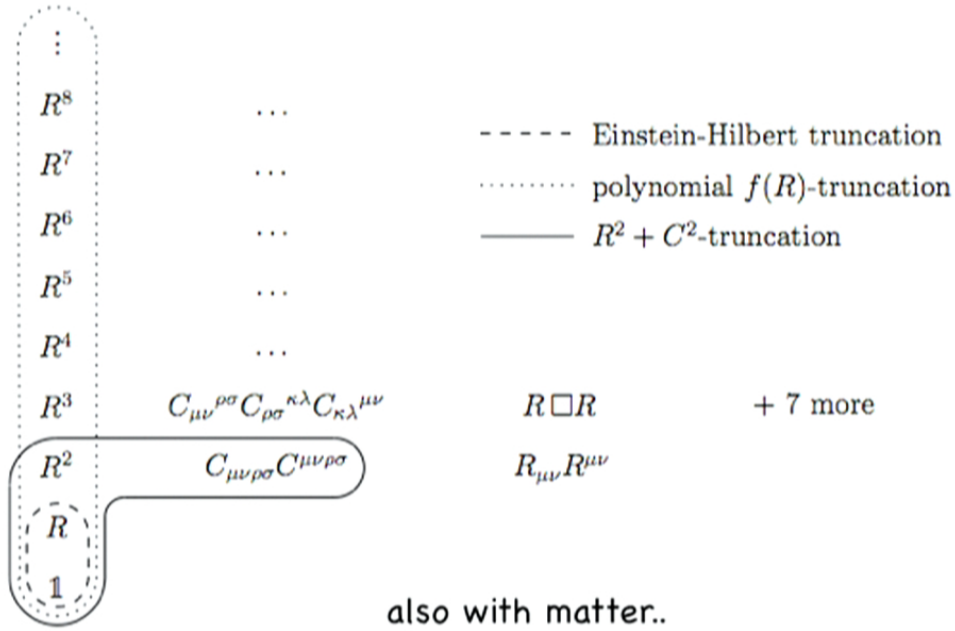


Figure 4: RG flow in the g-λ−plane. The arrows point in the direction of increasing coarse graining, i.e., of decreasing k. (From [14].)

M. Reuter & F. Saueressig, Phys. Rev. **D65** (2002) 065016, New J. Phys. **14** (2012) 055022

Less severe truncations



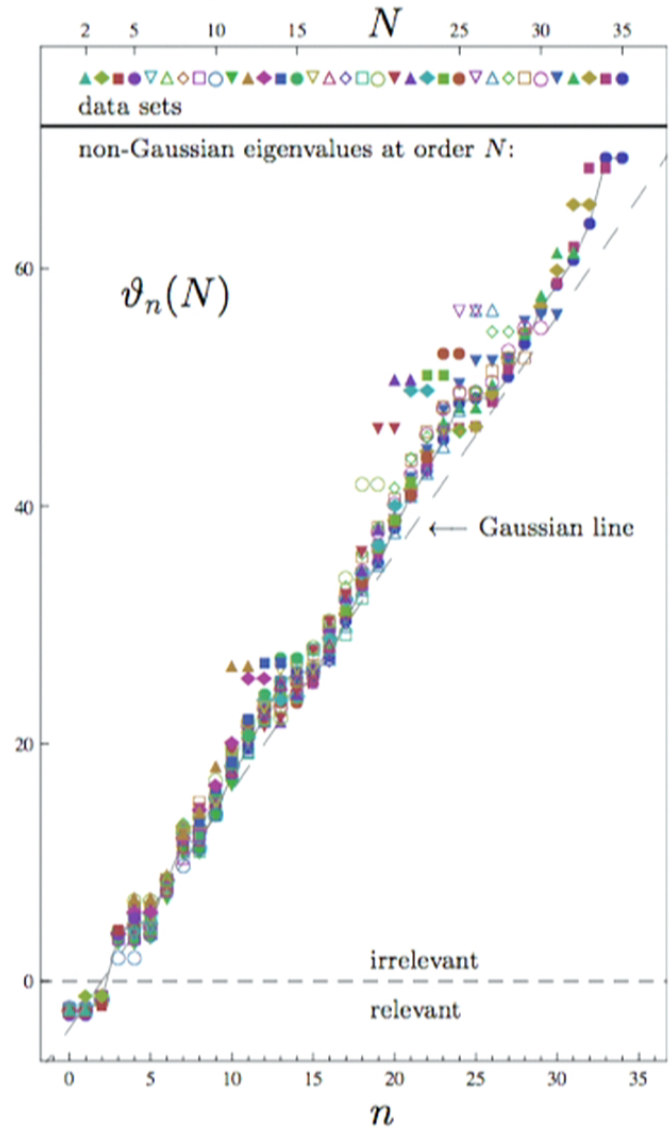
also with matter..

Figure 3: Overview of the various truncations employed in the systematic exploration of the theory space of QEG. The lines indicate the interaction monomials contained in the various truncation ansätze for $\bar{\Gamma}_k[g]$, eq. (4.4). All truncations have confirmed the existence of a non-trivial UV fixed point of the gravitational RG flow.

M. Reuter & F. Saueressig, New J. Phys. **14** (2012) 055022; M. Reuter, F. Saueressig, O. Lauscher, D. Benedetti, P. F. Machado, A. Codello, R. Percacci, C. Rahmede, M. Nierdermaier, K. Groh, S. Rechenberger, O. Zanusso ...

Figure 1: The complete sets of eigenvalues at the ultraviolet fixed point (8) for all N, sorted by magnitude. The results at the highest order ($N = 35$) are linked by a line to guide the eye. The long dashed line indicates Gaussian scaling. The inset (upper panel) relates the data sets at approximation order N with the symbols used in the lower panel.

K. Falls, D.F. Litim, K. Nikolakopoulos & C. Rahmede, arXiv:1301.4191



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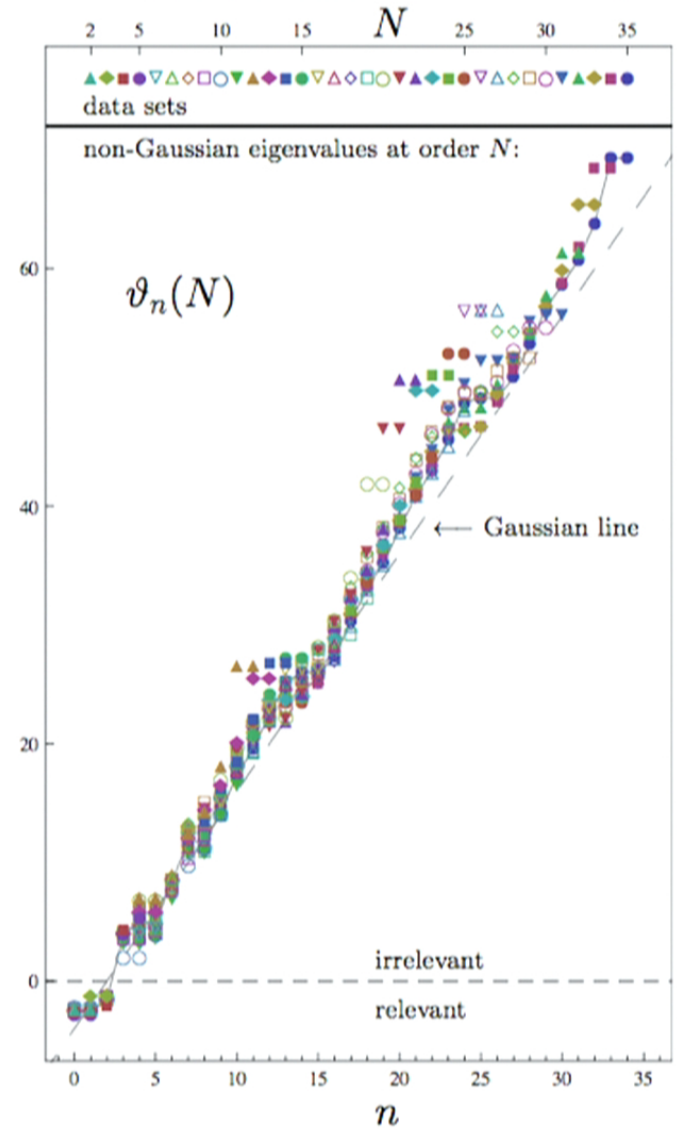
K. Falls, D.F. Litim, K. Nikolakopoulos & C. Rahmede, arXiv:1301.4191

Go beyond polynomial truncations to explore

$$\bar{R} \sim O(1)$$



Split into background + fluctuation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$Z \sim \int \mathcal{D}h \; e^{-S[\bar{g}+h]}$$

Intuitively results should be background independent

Split into background + fluctuation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

IR cutoff: $\mathcal{R} \sim \mathcal{R}(-\bar{\nabla}^2/k^2)$

Impose Landau gauge: D. Litim & J. Pawlowski, Phys. Lett. **B435** (1998) 181

$$\bar{\nabla}^\mu h_{\mu\nu} = \frac{1}{4}\bar{\nabla}_\nu h$$

Ghosts get IR cutoff too, so BRS invariance recovered only in the limit k→0 (if we're careful).

TT decomposition (ghosts similarly):

$$h_{\mu\nu} = h^T_{\mu\nu} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma + \frac{1}{4}\bar{g}_{\mu\nu}\bar{h}$$

Jacobian ⇒ auxiliary fields & they get IR cutoff too.

M. Reuter, Phys. Rev. **D57** (1998) 971

IR cutoffs (basic idea):

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\,\mathrm{tr}\,\left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\,.$$

$$\mathcal{R} \sim \mathcal{R}_L = k^2 r(-\bar{\nabla}^2/k^2) = (k^2 + \bar{\nabla}^2)\theta(k^2 + \bar{\nabla}^2)$$

D.F. Litim, Phys. Rev. **D64** (2001) 105007

Effectively in inverse 2-pt: $-\bar{\nabla}^2 + \mathcal{R}_L \equiv k^2$

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D.F. Litim, Phys. Rev. **D64** (2001) 105007

Effectively in inverse 2-pt: $-\bar{\nabla}^2 + \mathcal{R}_L \equiv k^2$

Adaptive IR cutoffs:

On constant scalar curvature $\bar{R}$ background...

$$\frac{\delta^2\Gamma}{\delta\sigma\delta\sigma} \sim \left(-\bar{\nabla}^2 - \frac{\bar{R}}{3}\right)^2 (-\bar{\nabla}^2)$$

$$\mathcal{R} \sim \left(-\bar{\nabla}^2 + \mathcal{R}_L - \frac{\bar{R}}{3}\right)^2 (-\bar{\nabla}^2 + \mathcal{R}_L) - \frac{\delta^2\Gamma}{\delta\sigma\delta\sigma}$$

so effectively:

$$\frac{\delta^2\Gamma}{\delta\sigma\delta\sigma} + \mathcal{R} \equiv \left(k^2 - \frac{\bar{R}}{3}\right)^2 k^2$$

$$h_{\mu\nu} = h^T_{\mu\nu} + \bar{\nabla}_\mu\xi_\nu + \bar{\nabla}_\nu\xi_\mu + \bar{\nabla}_\mu\bar{\nabla}_\nu\sigma + \frac{1}{4}\bar{g}_{\mu\nu}\bar{h}$$

Performing the space-time trace

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\int_x \left\{ \left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi} \right]^{-1} \frac{\partial}{\partial k}\mathcal{R} \right\}(x,x)\,.$$

- heat kernel expansion
- short distance (small $\bar{R}$) expansion
- background independence
- $\theta(k^2 + \bar{\nabla}^2)$ $\Longrightarrow$ finite number of terms
- E.g. on 4-sphere => coeffs polynomial in $\bar{R}$

Performing the space-time trace

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\sum_{\lambda_i<k}\left\langle i\left|\left[\mathcal{R}+\frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\right|i\right\rangle .$$

- heat kernel expansion
- short distance (small $\bar{R}$) expansion
- background independence
- $\theta(k^2+\bar{\nabla}^2)$ $\Longrightarrow$ finite number of terms
- E.g. on 4-sphere => coeffs polynomial in $\bar{R}$
- But **asymptotic** so terms are neglected
- E.g. on 4-sphere really "staircase"

$$-\bar{\nabla}^2\,|i\rangle = \lambda_i^2\,|i\rangle$$

D. Benedetti and F. Caravelli, JHEP 1206 (2012) 017.
J. A. Dietz & T.R. Morris, JHEP 1301 (2013) 108.

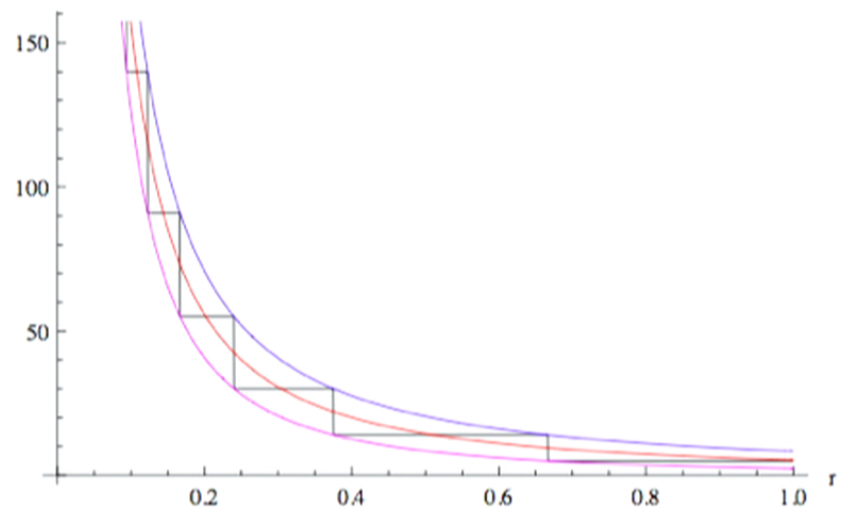
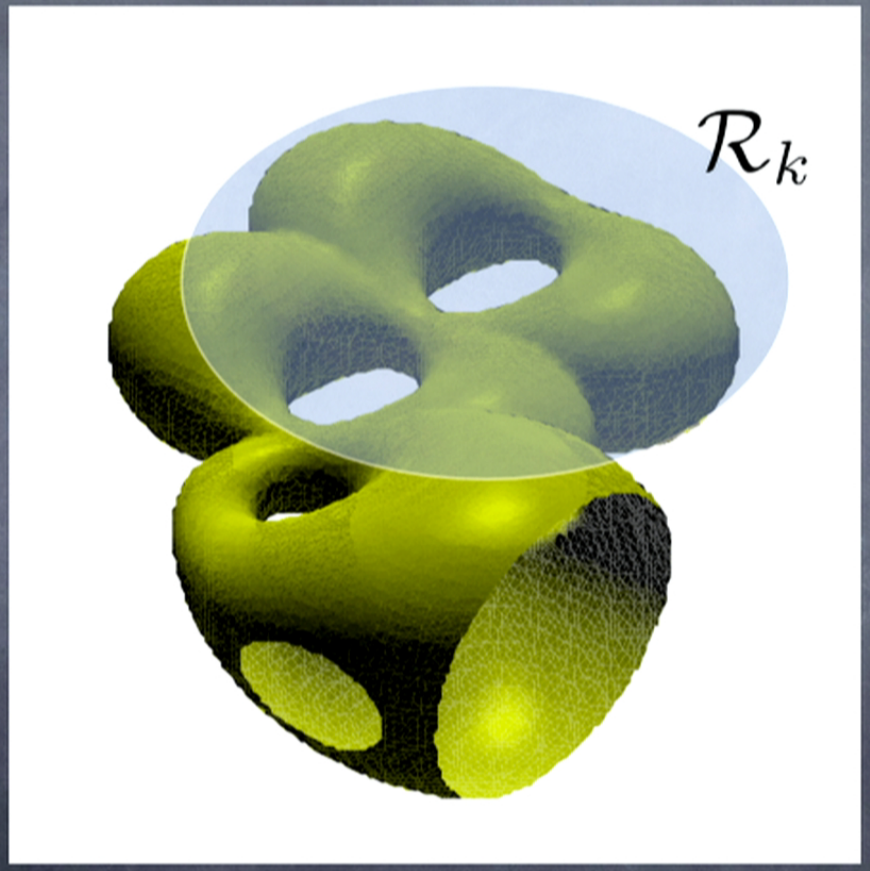


Figure 2. Smooth approximations of the staircase function $\sum_n \theta\left(1-\frac{r}{6}n^2\right) n^4$ (black line). The curves obtained from the replacement (5.11), truncating the sum at N_r and $N_r - 1$ are given by the blue (top) and magenta (bottom) curve, respectively. The average of the two approximations resulting from eq. (5.12) gives rise to the red (middle) line.

M. Demmel, F. Saueressig & O. Zanusso, arXiv:1401.5459

$$\bar{R} > 1$$



$$\bar{R}_{phys} > k^2$$

Other typical approximations:

- k dependence of ghosts & auxiliaries neglected

- Mixed $h_{\alpha\beta}$ and $g_{\mu\nu}$ terms neglected (single field approximation)

Exceptions:

A. Eichhorn, H. Gies & M.M. Schere, Phys. Rev. **D80** (2009) 104003; K. Groh & F. Saueressig, J. Phys. **A43** (2010) 365403; A. Eichhorn & H. Gies, Phys. Rev. **D81** (2010) 104010;

E. Manrique, M. Reuter, Ann. Phys. 325 (2010) 785;
E. Manrique, M. Reuter & F. Saueressig, Ann. Phys. 326 (2011) 440 & 463;
A. Codello, G. D'Odorico & C. Pagani, arXiv:1304.4777

P. Dona, A. Eichhorn & R. Percacci, arXiv:1311.2898
D. Becker & M. Reuter, arXiv:1404.4357 (last Friday!)

Beyond polynomial truncations

E.g.

- Project on four-sphere background $(R \geq 0)$
- Effective action: $\Gamma = \int d^4x \sqrt{g}\, f(R,t)$
- Get non-linear PDE flow equation for $f(R,t)$

Fixed points: $f(R,t) \mapsto f(R)$

$$
\begin{aligned}
768\pi^2\,(2f - Rf') = \\
&\left[5R^2\theta\,(1-\tfrac{R}{3}) - (12 + 4\,R - \tfrac{61}{90}\,R^2)\right]\left[1-\tfrac{R}{3}\right]^{-1} + \Sigma \\
&+\left[10\,R^2\,\theta(1-\tfrac{R}{4}) - R^2\,\theta(1+\tfrac{R}{4}) - (36 + 6\,R - \tfrac{67}{60}\,R^2)\right]\left[1-\tfrac{R}{4}\right]^{-1} \\
&+\left[(2f' - 2Rf'')\,(10 - 5R - \tfrac{271}{36}R^2 + \tfrac{7249}{4536}R^3) + f'\,(60 - 20R - \tfrac{271}{18}R^2)\right]\left[f + f'(1-\tfrac{R}{3})\right]^{-1} \\
&+\tfrac{5R^2}{2}\left[(2f' - 2Rf'')\left\{r(-\tfrac{R}{3}) + 2r(-\tfrac{R}{6})\right\} + 2f'\theta(1+\tfrac{R}{3}) + 4f'\theta(1+\tfrac{R}{6})\right]\left[f + f'(1-\tfrac{R}{3})\right]^{-1} \\
&+\left[(2f' - 2Rf'')f'\,(6 + 3R + \tfrac{29}{60}R^2 + \tfrac{37}{1512}\,R^3)\right. \\
&-2Rf'''\,(27 - \tfrac{91}{20}R^2 - \tfrac{29}{30}\,R^3 - \tfrac{181}{3360}\,R^4) \\
&\left.+f''\,(216 - \tfrac{91}{5}R^2 - \tfrac{29}{15}R^3) + f'\,(36 + 12R + \tfrac{29}{30}R^2)\right]\left[2f + 3f'(1-\tfrac{2}{3}R) + 9f''(1-\tfrac{R}{3})^2\right]^{-1},
\end{aligned}
$$

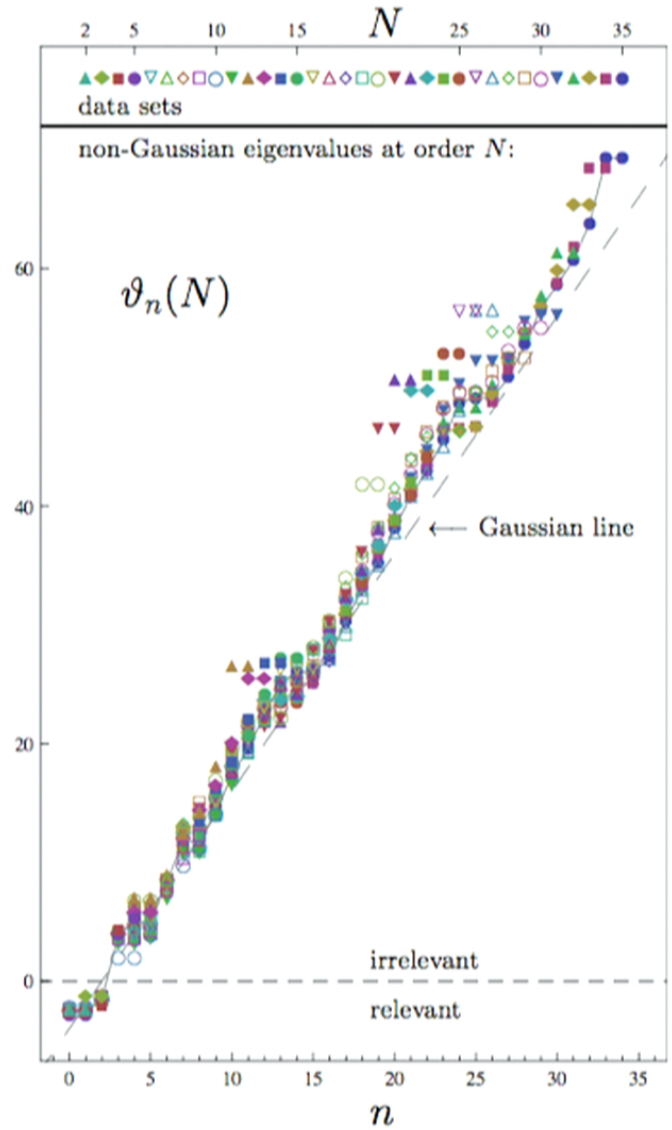
$$r(z) = (1-z)\theta(1-z) \qquad \Sigma = 10R^2\theta(1-R/3)$$

P. F. Machado & F. Saueressig, Phys. Rev. D 77 (2008) 124045

A. Codello, R. Percacci & C. Rahmede, Annals Phys. 324 (2009) 414

Figure 1: The complete sets of eigenvalues at the ultraviolet fixed point (8) for all N, sorted by magnitude. The results at the highest order ($N = 35$) are linked by a line to guide the eye. The long dashed line indicates Gaussian scaling. The inset (upper panel) relates the data sets at approximation order N with the symbols used in the lower panel.

K. Falls, D.F. Litim, K. Nikolakopoulos & C. Rahmede, arXiv:1301.4191



Fixed singularities and parameter space

Suppose we have normal form: $$f'''(R) = \frac{F(f, f', f'', R)}{R}$$

with fixed singularity at R=0.

Substitute: $$f(R) = a_0 + a_1 R + \frac{1}{2} a_2 R^2 + \cdots$$

=> $$\text{regular in } R \;=\; \frac{u(a_0, a_1, a_2)}{R} + \text{regular in } R$$

$u(a_0, a_1, a_2)$ is non-trivial constraint on parameters a_0, a_1, a_2

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&+\Big[(2f' - 2Rf'')\,(10 - 5R - \tfrac{271}{36}R^2 + \tfrac{7249}{4536}R^3) + f'\,(60 - 20R - \tfrac{271}{18}R^2)\Big]\,[f + f'(1-\tfrac{R}{3})]^{-1} \\
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\end{aligned}
$$

$R = 0,\ 2.0065$

$r(z) = (1-z)\theta(1-z) \qquad \Sigma = 10R^2\theta(1-R/3)$

Parameter counting => no global solutions

J. A. Dietz & TRM, JHEP 1301 (2013) 108.

D. Benedetti and F. Caravelli, JHEP 1206 (2012) 017;

D. Benedetti, New J. Phys. **14** (2012) 015005

$$768\pi^2\left(2f - Rf'\right) = \frac{40\left(Rf'' - 4f'\right)}{(R-2)f' - 2f} - 48 - 5R^2$$

$$+\frac{R\left(R^4 - 54R^2 - 54\right)f''' - \left(R^3 + 18R^2 + 12\right)\left(Rf'' - f'\right) + 18\left(R^2 + 2\right)\left(f' + 6f''\right)}{9f'' - (R-3)f' + 2f}.$$

$$R = 0,\ 7.4150$$

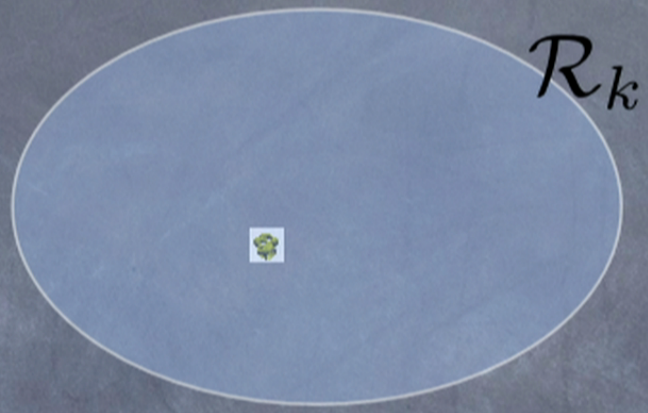
$$f(R) = A\,R^2 + R\left\{\frac{3}{2}A + B\cos\ln R^2 + C\sin\ln R^2\right\} + O(1)$$

$$\frac{121}{20}A^2 > B^2 + C^2$$

Parameter counting => lines of fixed points

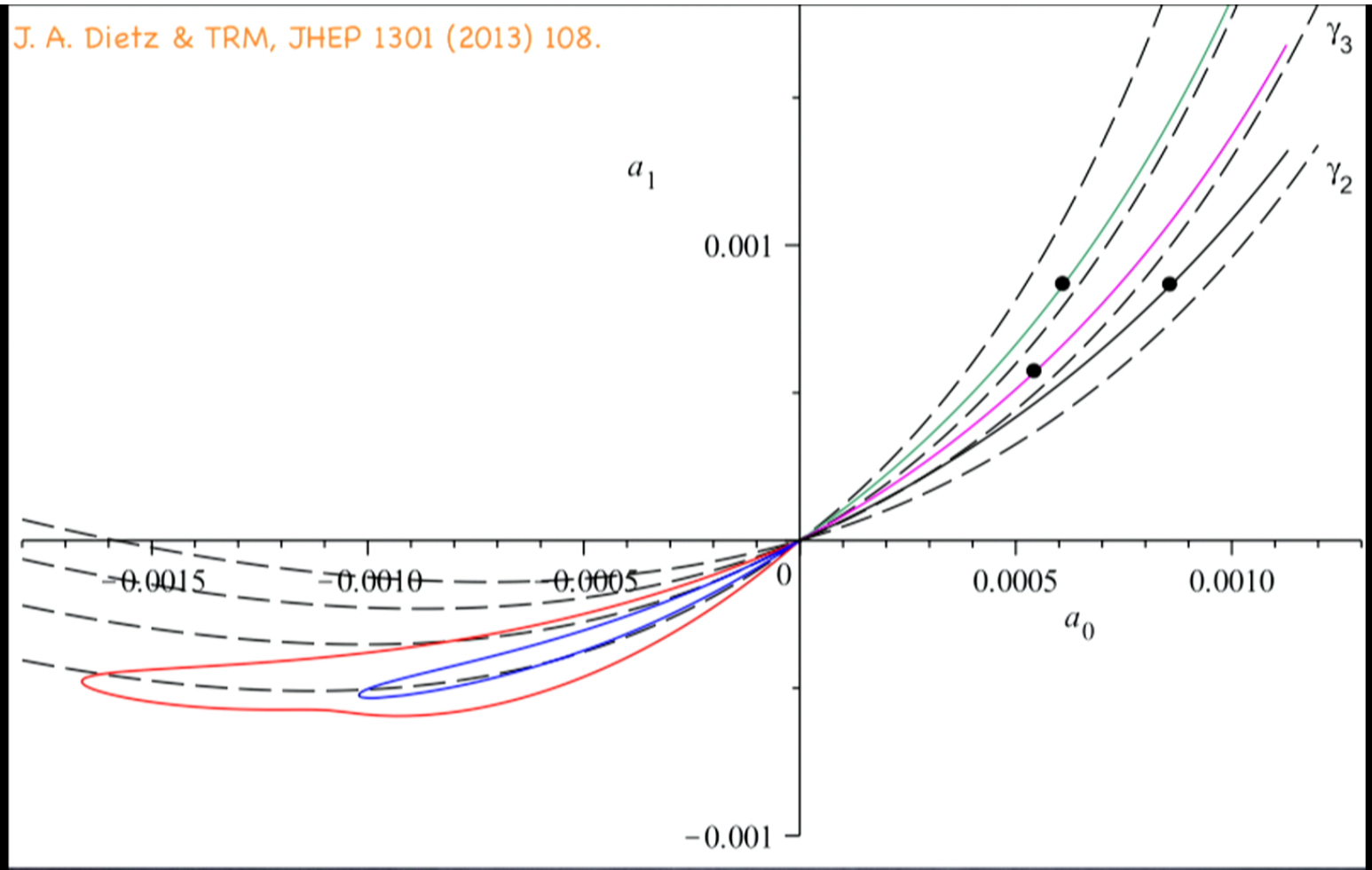
J. A. Dietz & TRM, JHEP 1301 (2013) 108.

$$\bar{R} \to \infty?$$



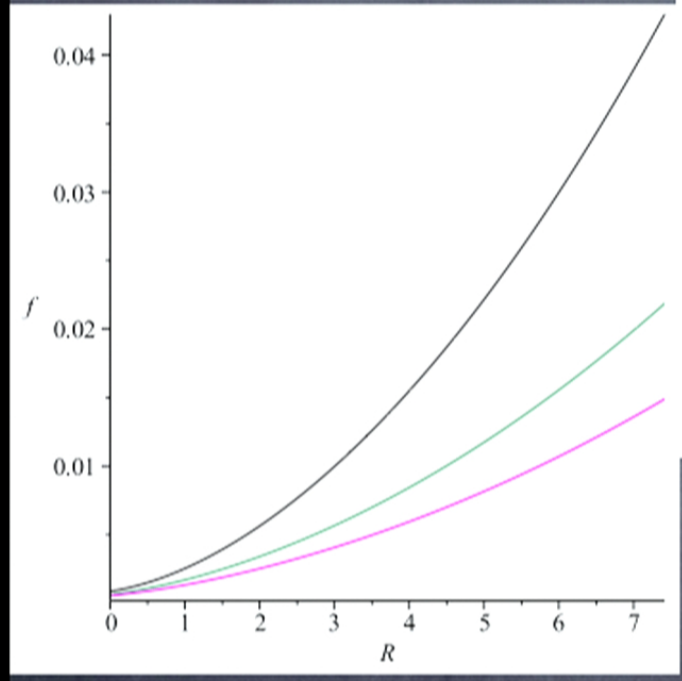
$$\bar{R}_{phys} \gg k^2$$

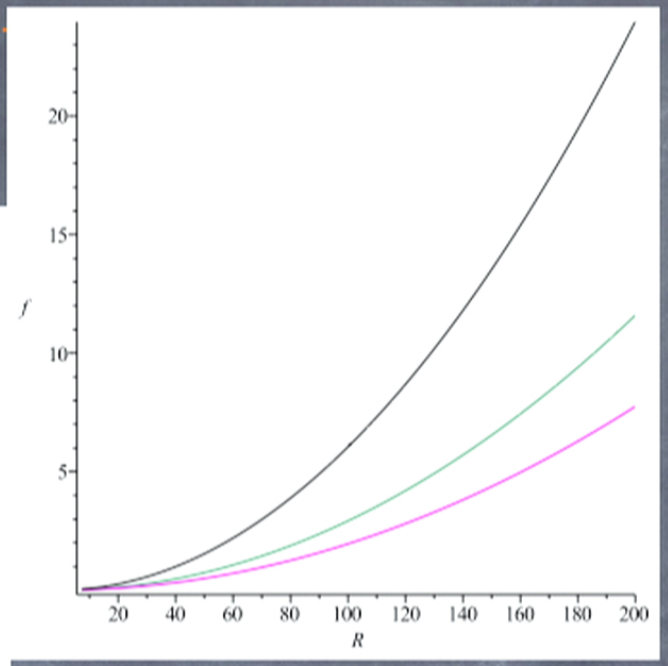
J. A. Dietz & TRM, JHEP 1301 (2013) 108.



$$f(R) = a_0 + a_1 R + \frac{1}{2} a_2(a_0, a_1) R^2 + \cdots$$

J. A. Dietz & TRM, JHEP 1301 (2013) 108.





Continuous eigenspectrum!

J. A. Dietz & T.R. Morris, JHEP 1301 (2013) 108.

Extend to -∞<R<∞

$$768\pi^2\left(2f - Rf'\right) = \frac{40\left(Rf'' - 4f'\right)}{(R-2)f' - 2f} - 48 - 5R^2$$

$$+\frac{R\left(R^4 - 54R^2 - 54\right)f''' - \left(R^3 + 18R^2 + 12\right)\left(Rf'' - f'\right) + 18\left(R^2 + 2\right)\left(f' + 6f''\right)}{9f'' - (R-3)f' + 2f}.$$

$$R = 0, \pm 7.4150$$

$$f(R) = A\,R^2 + R\left\{\frac{3}{2}A + B\cos\ln R^2 + C\sin\ln R^2\right\} + O(1)$$

$$\frac{121}{20}A^2 > B^2 + C^2$$

Parameter counting => discrete set of fixed points

Quantised Eigenoperator spectrum

J. A. Dietz & T.R. Morris, JHEP 1301 (2013) 108.

Extend to -∞<R<∞

$$768\pi^2\left(2f - Rf'\right) = \frac{40\left(Rf'' - 4f'\right)}{(R-2)f' - 2f} - 48 - 5R^2$$

$$+\frac{R\left(R^4 - 54R^2 - 54\right)f''' - \left(R^3 + 18R^2 + 12\right)\left(Rf'' - f'\right) + 18\left(R^2 + 2\right)\left(f' + 6f''\right)}{9f'' - (R-3)f' + 2f}.$$

$$R = 0, \pm 7.4150$$

$$f(R) = A\,R^2 + R\left\{\frac{3}{2}A + B\cos\ln R^2 + C\sin\ln R^2\right\} + O(1)$$

No global solutions!

$$\frac{121}{20}A^2 > B^2 + C^2$$

Parameter counting => discrete set of fixed points

Quantised Eigenoperator spectrum

Break-down of f(R) approximation

FP action $\Gamma = \int d^4x \sqrt{g}\, f(R)$

J.A. Dietz & T.R. Morris, JHEP **07** (2013) 064

Eigenoperator $\int d^4x \sqrt{g}\, v(R)$

Wegner, J. Phys. C7 (1974) 2098.

$$g_{\mu\nu}(x) \mapsto g_{\mu\nu}(x) + \varepsilon F_{\mu\nu}[g](x)$$

Eigenoperator redundant if of the form:

$$\int d^d x \sqrt{g}\, F_{\mu\nu} \left\{ \frac{1}{2} g^{\mu\nu} f - R^{\mu\nu} f' + \cancel{\nabla^\mu \nabla^\nu f'} - \cancel{g^{\mu\nu} \Box f'} \right\} .$$

$$F_{\mu\nu} = \zeta(R)\, g_{\mu\nu} \quad \Longrightarrow \quad v(R) = \zeta(R) \{2f(R) - Rf'(R)\} .$$

Does not vanish for any R≥0 $\Longrightarrow$ entire space is redundant!

Split into background + fluctuation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$Z \sim \int \mathcal{D}h \; e^{-S[\bar{g}+h]}$$

Intuitively results should be background independent

Scalar field theory @ LPA

I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

$$\Gamma[\phi] = \int d^d x \left\{ \frac{1}{2} \left(\partial_\mu \phi\right)^2 + V(\phi) \right\}$$

$$\phi = \bar{\varphi} + \varphi$$

$$\mathcal{R}\left(-\partial^2, \bar{\varphi}(x)\right) = \left(k^2 + \partial^2 - h(\bar{\varphi})\right) \theta\left(k^2 + \partial^2 - h(\bar{\varphi})\right)$$

Single field approximation:

$$\partial_t V - \frac{1}{2}(d-2)\phi V' + dV = \frac{(1-h)^{d/2}}{1-h+V''} \left(1 - h - \frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\phi h'\right) \theta(1-h)$$

$\Longrightarrow$ pathologies

Keep both fields & impose Ward Identity:

$$\phi = \varphi + \bar{\varphi} \qquad \bar{\varphi} \mapsto \bar{\varphi} + \varepsilon(x) \quad \text{and} \quad \varphi \mapsto \varphi - \varepsilon(x)$$

$$\frac{\delta\Gamma}{\delta\bar{\varphi}_a} - \frac{\delta\Gamma}{\delta\varphi_a} = \frac{1}{2}\mathrm{Tr}\left[\left(\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right)^{-1}\frac{\delta\mathcal{R}}{\delta\bar{\varphi}_a}\right].$$

Reuter, Wetterich, Litim, Pawlowski... E. Manrique, M. Reuter, Ann. Phys. 325 (2010) 785; E. Manrique, M. Reuter, F. Saueressig, Ann. Phys. 326 (2011) 440 & 463; D. Becker & M. Reuter, arXiv:1404.4357

$$\partial_t V - \frac{1}{2}(d-2)\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) + dV = \frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\left(1 - h - \frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\bar{\varphi}h'\right)\theta(1-h).$$

I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

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I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

Need flow **&** broken **sWI** to determine a solution!

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$$\frac{\partial}{\partial k}\Gamma[\varphi,\bar{\varphi}] = \frac{1}{2}\,\mathrm{tr}\left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\,.$$

$$\Gamma \mapsto \Gamma + \Delta\Gamma[\bar{\varphi}] \quad \text{with} \quad \frac{\partial}{\partial k}\Delta\Gamma[\bar{\varphi}] = 0$$

I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

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$$\partial_\varphi V - \partial_{\bar{\varphi}} V = \frac{h'}{2}\frac{(1-h)^{d/2}}{1-h\partial_\varphi^2 V}\theta(1-h)$$

$$\partial_t V - \frac{1}{2}(d-2)\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) + dV = \frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\left(1-h-\frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\bar{\varphi}h'\right)\theta(1-h).$$

I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

Need flow & broken sWI to determine a solution!

$$V = (1-h)^{d/2}\hat{V}, \qquad \varphi = (1-h)^{\frac{d-2}{4}}\hat{\varphi} - \bar{\varphi}, \qquad t = \hat{t} - \ln\sqrt{1-h}$$

Keep both fields & impose Ward Identity:

$$\phi = \varphi + \bar{\varphi} \qquad \bar{\varphi} \mapsto \bar{\varphi} + \varepsilon(x) \quad \text{and} \quad \varphi \mapsto \varphi - \varepsilon(x)$$

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I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

Need flow & broken sWI to determine a solution!

$$V = (1-h)^{d/2}\hat{V}, \qquad \varphi = (1-h)^{\frac{d-2}{4}}\hat{\varphi} - \bar{\varphi}, \qquad t = \hat{t} - \ln\sqrt{1-h}$$

$$\partial_{\hat{t}}\hat{V} + d\hat{V} - \frac{1}{2}(d-2)\hat{\varphi}\partial_{\hat{\varphi}}\hat{V} = \frac{1}{1+\partial_{\hat{\varphi}}^2\hat{V}}$$

$\Longrightarrow$implements background independence!

Bimetric truncations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Matter matters: keeping separately the wavefn renormalisation of $h_{\mu\nu}$ and all other fields in an EH truncation

P. Dona, A. Eichhorn & R. Percacci, arXiv:1311.2898

TABLE IV: Fixed-point values, critical exponents and anomalous graviton dimension for specific matter content.

model	N_S	N_D	N_V	$\tilde{G}_*$	$\tilde{\Lambda}_*$	θ_1	θ_2	η_h
no matter	0	0	0	0.77	0.01	3.30	1.95	0.27
SM	4	45/2	12	1.76	-2.40	3.96	1.64	2.98
SM +dm scalar	5	45/2	12	1.87	-2.50	3.96	1.63	3.15
SM+ 3 ν's	4	24	12	2.15	-3.20	3.97	1.65	3.71
SM+3ν's + axion+dm	6	24	12	2.50	-3.62	3.96	1.63	4.28
MSSM	49	61/2	12	-	-	-	-	-
SU(5) GUT	124	24	24	-	-	-	-	-
SO(10) GUT	97	24	45	-	-	-	-	-

Full momentum dependence, keeping only $h_{\mu\nu}$

M. Christiansen, B. Knorr, J.M. Pawlowski & A. Rodigast, arXiv:1403.1232

Thursday 9am

D. Becker & M. Reuter, arXiv:1404.4357 (last Friday!)

Recovering both sWI & asymptotic safety within a double EH truncation.

-> this Friday 9am & 9:40am!

Conclusions

- Huge body of work showing asymptotic safety in various truncations/approximations
- Important to go beyond polynomial truncations to explore $\bar{R} \sim O(1)$
- New effects become visible in this regime & much more sensitive to issues with approximations. More work needed to:
- Disentangle cutoff pathologies from physics...
- Work with less drastic approximations (inc. $h_{\mu\nu}$)

Litim, Bonanno, Koslowski, Saueressig... talks