Title: Helical edge resistance introduced by charge disorder

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Abstract: Electron puddles created by doping of a 2D topological insulator may violate the ideal helical edge conductance. Because of a long electron dwelling time, even a single puddle may lead to a significant inelastic backscattering. We find the resulting correction to the perfect edge conductance. Generalizing to multiple puddles, we assess the dependence of the helical edge resistance on temperature and on the doping level. Puddles with odd electron number carry a spin and lead to a logarithmically-weak temperature dependence of the resistivity of a long edge. The developed theory provides a framework for analyzing the results of the past and ongoing electron transport experiments with 2D topological insulators

Effect of Charge Disorder on the Conduction of a Helical Edge

L.I. Glazman

Yale University

PI– April2014

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Outline

- Elementary mechanisms of electron backscattering in helical edge

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- Less elementary mechanisms of electron backscattering (scattering off a single electron puddle)
- Strong vs. weak disorder: statistics of charge puddles and their effect on conduction, estimates for HgTe/CdTe

Electron backscattering in helical edge

states $\psi_\sigma(k)$ and $\psi_{-\sigma}(-k)$

are orthogonal spinors (Kramers doublet)

No finite matrix elements of a potential $U(\vec{r}) \cdot \delta_{\sigma\sigma'}$

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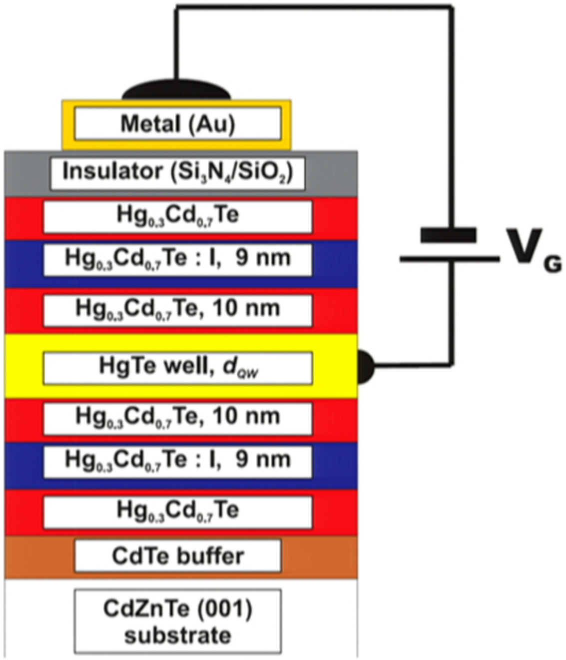
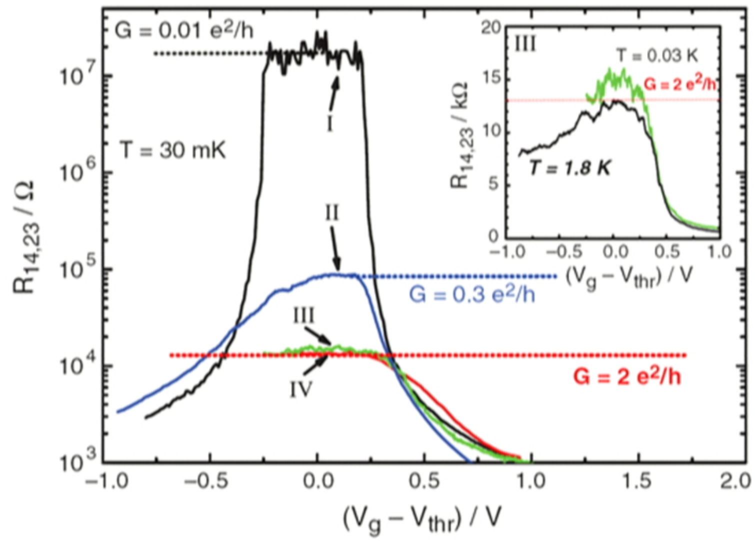
No elastic backscattering off a TR-conserving perturbation
$\rightarrow$ no conductance correction at any T

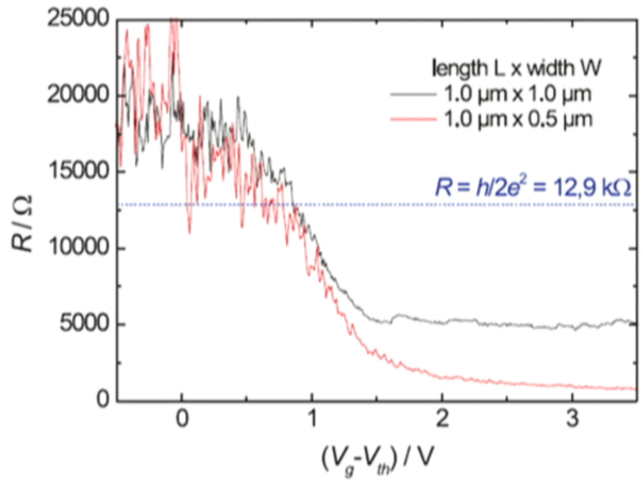
Experiments (Molenkamp group, 2007)



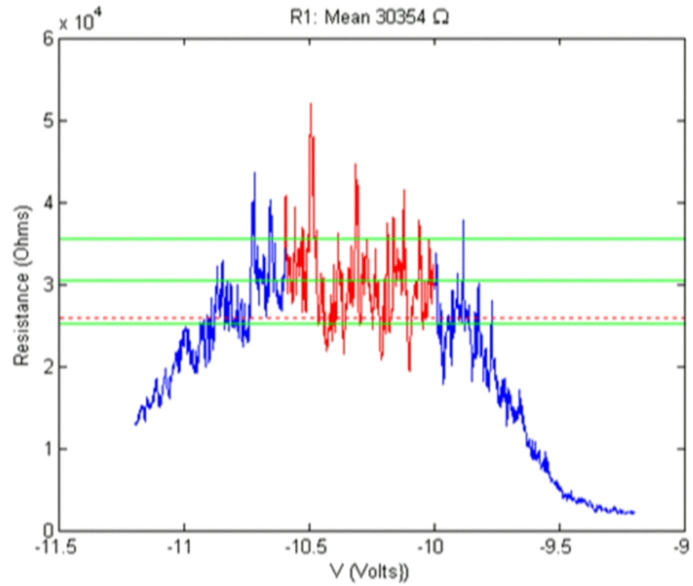


Approximate quantization is observed only for ~1μm samples

Conductance fluctuations in experiment



Molenkamp group 2007



Yacoby group 2012

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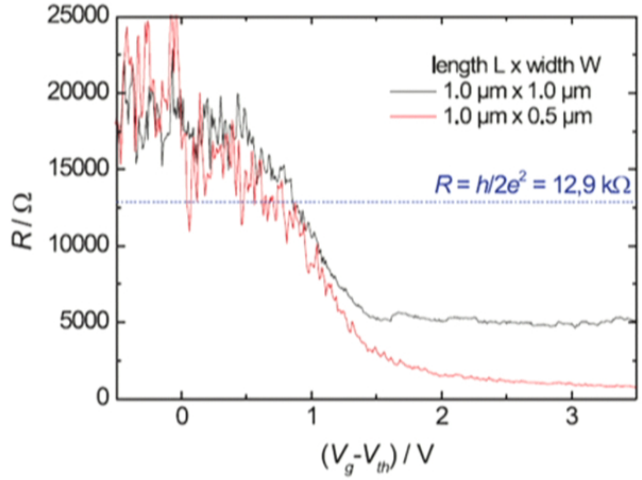

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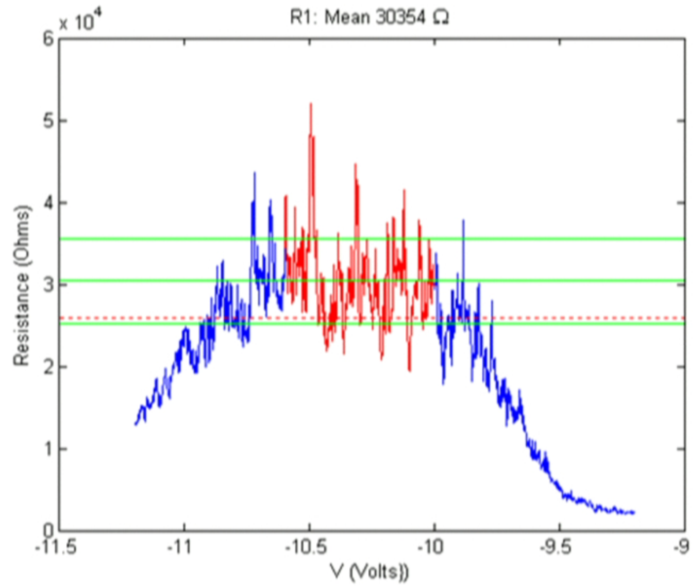

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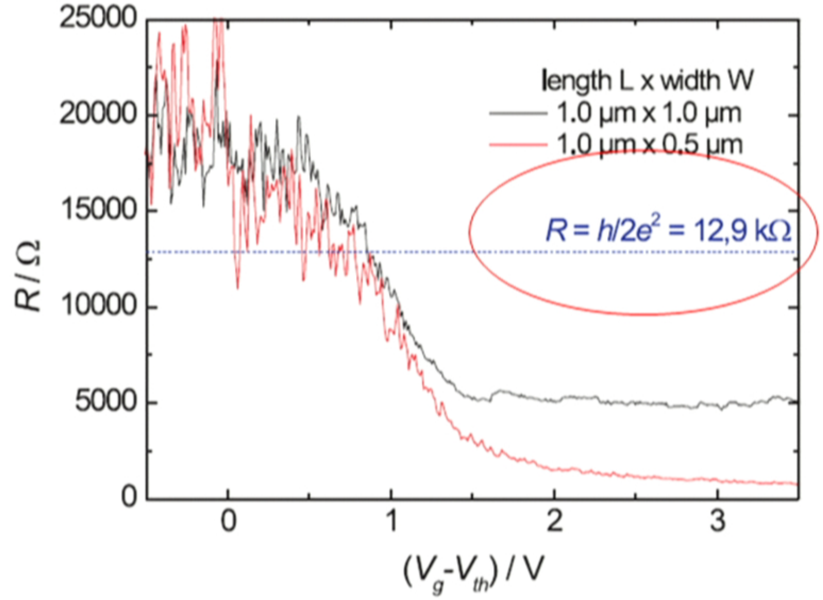

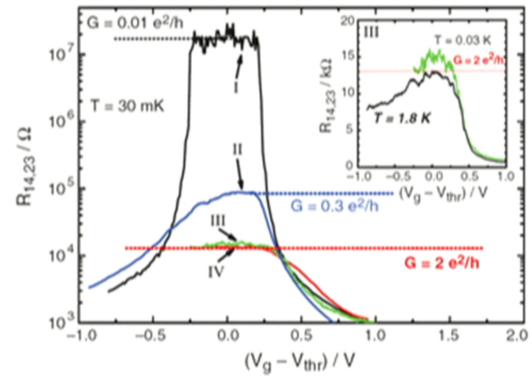

Sample II:
Length X Width ~20μm X 13μm

For two devices with $L = 1\ \mu$m, the longitudinal resistance in the insulating regime is close to $h/(2e^2)$.

Electron backscattering in helical edge

No finite matrix elements of a potential $U(\vec{r}) \cdot \delta_{\sigma\sigma'}$

between states $\psi_\sigma(k)$ and $\psi_{-\sigma}(-k)$

which are orthogonal spinors (Kramers doublet)

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Backscattering in s_z-conserving models

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Minimal TR-invariant interaction leading to backscattering

$$u\psi^\dagger_{L\downarrow}\partial_x\psi^\dagger_{L\downarrow}\psi_{R\uparrow}\partial_x\psi_{R\uparrow} + \mathrm{h.c.}$$

(2 particles backscattered)

(Kane, Mele, PRL 2005 ,Bernevig, Zhang, PRL 2006; Xu, Moore, PRB 2006)

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Fermi Golden Rule ("impurity", no momentum conservation):

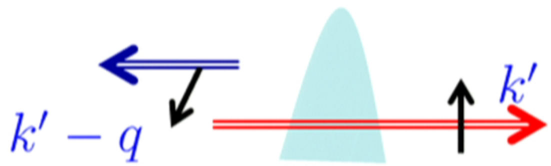
$$\Delta G \propto |u|^2 T^6 \;\; \propto |u|^2 \;\; T^2 \;\; \cdot T^2 \;\; \cdot T^2$$

Pauli (L);Pauli (R); Phase space

Backscattering without s_z -conservation

(T. Schmidt,S. Rachel, F.von Oppen, LG – PRL2012)

(spin texture in k-space)



$$H^{\text{eff}}_{V,\text{int}} \propto \frac{k_F U_0 V_0}{k_0^2 v_F} \sum_{kk'qq'} (k - k')\psi^\dagger_{+,k+q+q'}\psi^\dagger_{-,k'-q}\psi_{+,k'}\psi_{+,k} + (\psi_+ \leftrightarrow \psi_-) + \text{h.c.}$$

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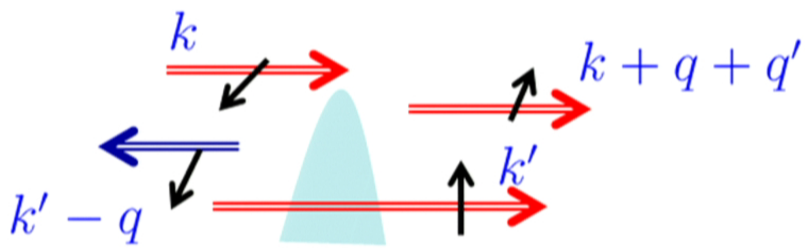


(spin texture in k-space)

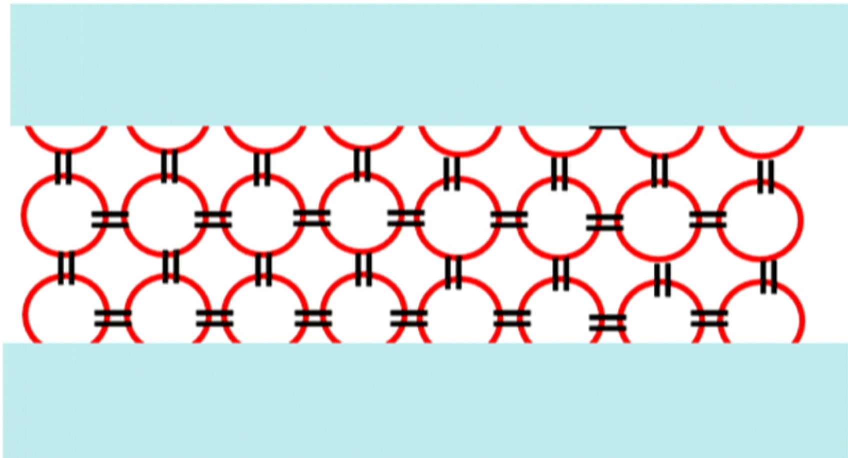
$$u\psi^\dagger_{L\downarrow}\partial_x\psi^\dagger_{L\downarrow}\psi_{R\uparrow}\partial_x\psi_{R\uparrow} + \text{h.c.} \longrightarrow \psi^\dagger_R\partial_x\psi^\dagger_R\psi_R\psi_L$$

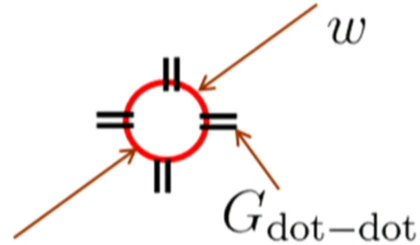
$$\Delta G_{V,\text{int}} \propto T^4 \quad \propto T^2 \quad \cdot T^2$$

Pauli (R); Phase space

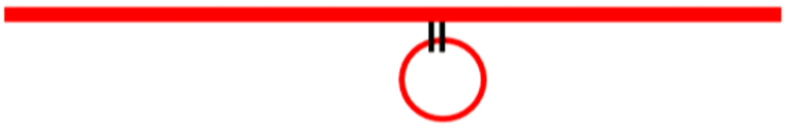
$$\Delta G \sim G_q(\hbar v/E_g)n_{\text{imp}}(T/E_g)^4$$

Puddles in a Doped Semiconductor

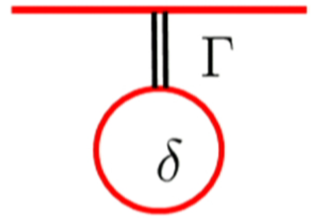




Backscattering off a Puddle



New physics: helical edge scattering off a dot



Vayrynen, Goldstein, LG -- Phys. Rev. Lett. **110**, 216402 (2013)

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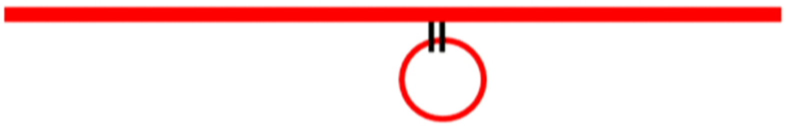

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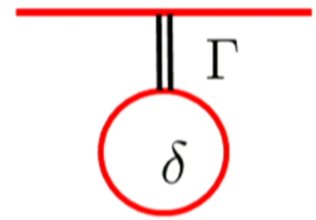

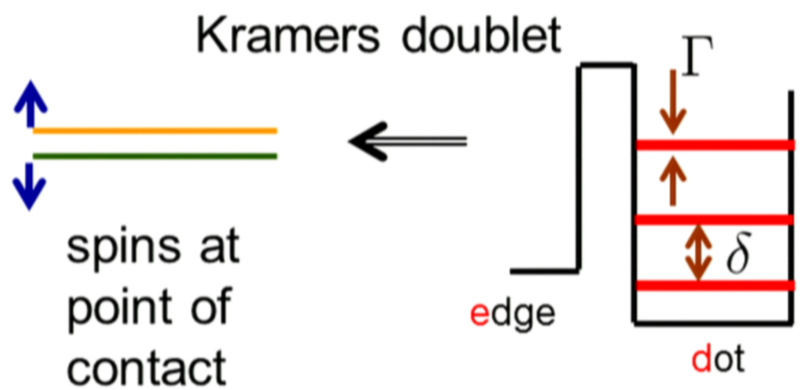

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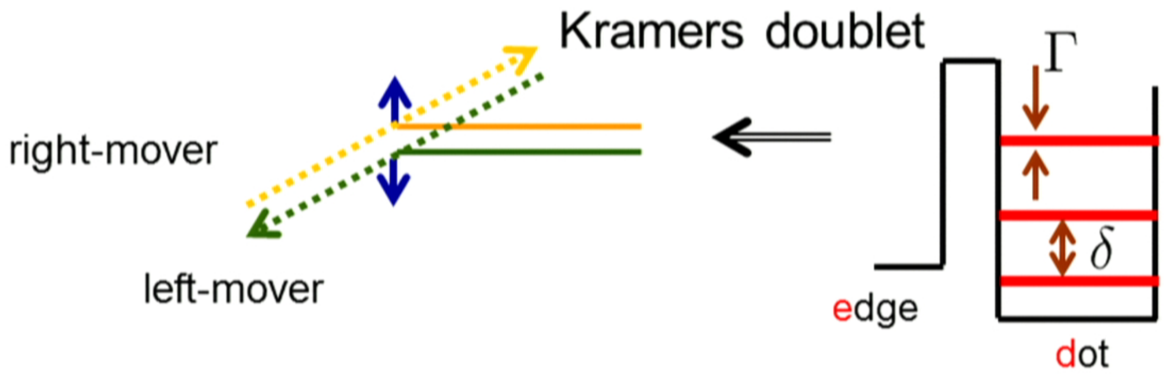
Weak-Interaction Limit



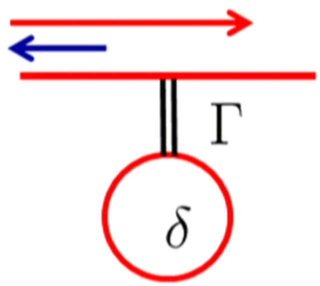
Two-particle scattering cross-section, e.g, $S_{RR\to LR}(E_1, E_2; E_3, E_4)$ in the lowest non-vanishing order (V^2):

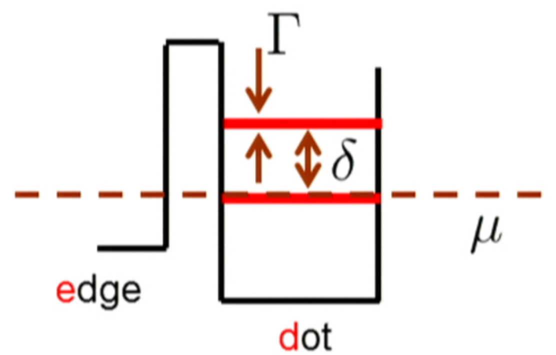
$$= \frac{2\pi}{\pi^4}\delta(E_1 + E_2 - E_3 - E_4)$$

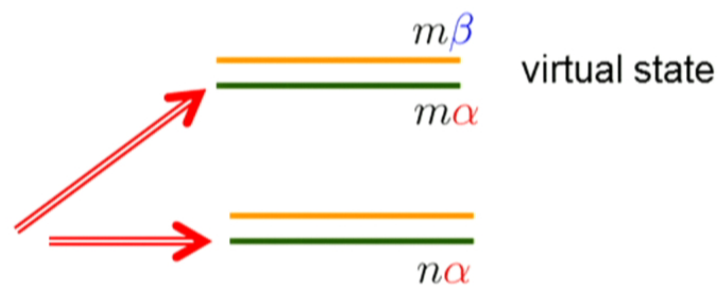
$$\sum_{m_1 m_2 m_3 m_4} \sum_{n_1 n_2 n_3 n_4} \langle m_1 \uparrow, m_2 \uparrow |\hat{V}| m_3 \uparrow, m_4 \downarrow\rangle \langle n_1 \uparrow, n_2 \uparrow |\hat{V}| n_3 \uparrow, n_4 \downarrow\rangle^*$$

$$\mathrm{Im}G^R_{m_1 n_1}(E_1)\mathrm{Im}G^R_{m_2 n_2}(E_2)\mathrm{Im}G^R_{n_3 m_3}(E_3)\mathrm{Im}G^R_{n_4 m_4}(E_4).$$

Two Narrow Levels, Lowest Temp.








Many Levels, RMT
Scales for the matrix elements of (screened) Coulomb interaction: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Off-diagonal (lead to scattering):

$$V_{n\alpha,m\alpha;k\beta,l\alpha} \sim \frac{\delta}{g}$$

$$g = E_T/\delta \gg 1$$

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Thouless energy

Diagonal (charging):

$$V_{n\alpha,n\alpha;m\beta,m\beta} \sim \frac{\text{spacer}}{\text{Bohr radius}} \cdot \delta$$

$$\gtrsim 1$$

Modification of the peak value, neglecting the charging:

$$\frac{\Delta G}{G_q} \sim \frac{1}{g^2} \cdot \frac{T^4}{\Gamma^4}, \quad T \lesssim \Gamma; \qquad \frac{\Delta G}{G_q} \sim \frac{1}{g^2} \cdot \frac{\Gamma}{T}, \quad \delta \gg T \gtrsim \Gamma$$

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Modification of the peak value, with the charging - estimates:

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Many Levels, RMT, Charging, odd-e

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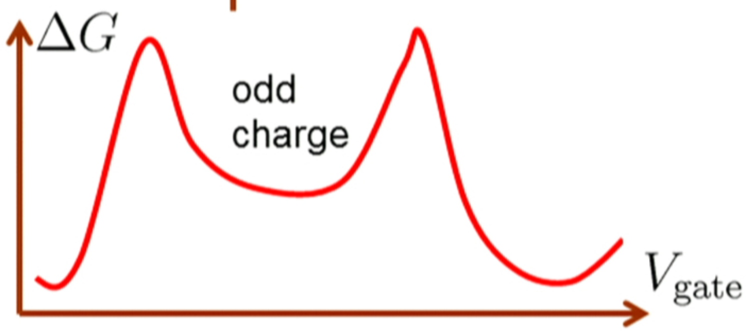
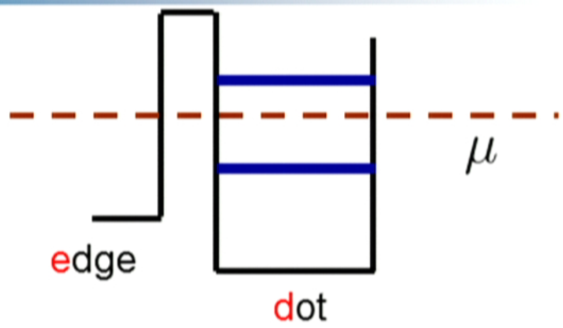
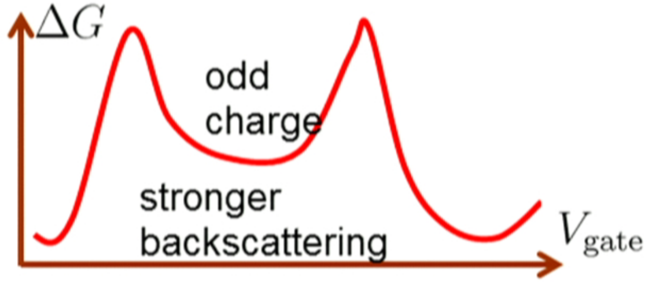
Even-odd charge asymmetry of the valleys:



RMT, odd-e, Kondo





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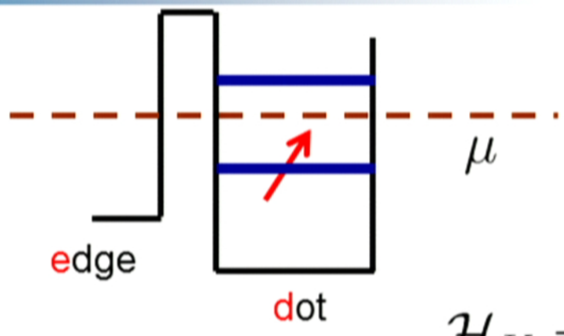

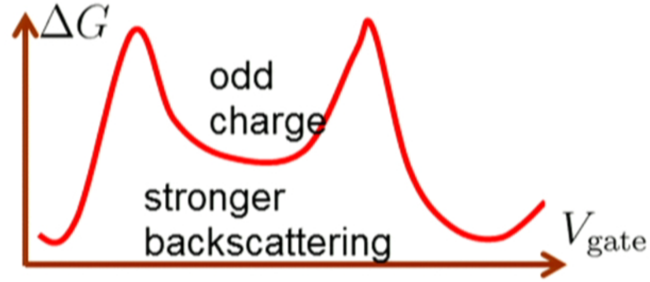

$$\mathcal{H}_K = J\mathbf{S}\cdot\mathbf{s}$$

Single level (Anderson imp. model)-- isotropic exchange, enough for Kondo eff. at $T<T_K(V_{gate})$ (Maciejko et al, 2009) but not enough for backscattering (Tanaka et al 2011).

conservation law: $\partial_t\left(\hat{N}_R - \hat{N}_L - \hat{S}_z\right) = 0$

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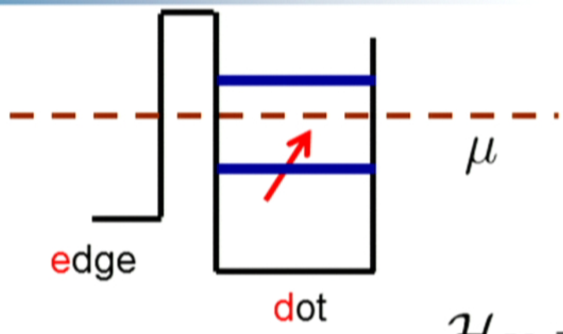

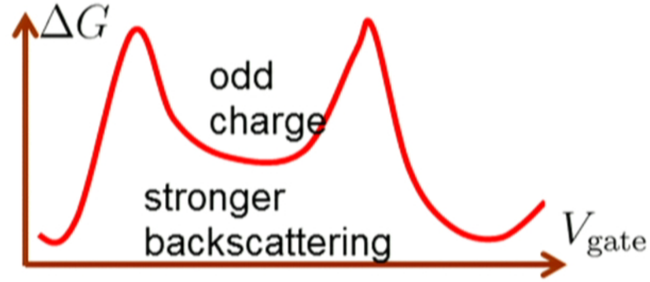

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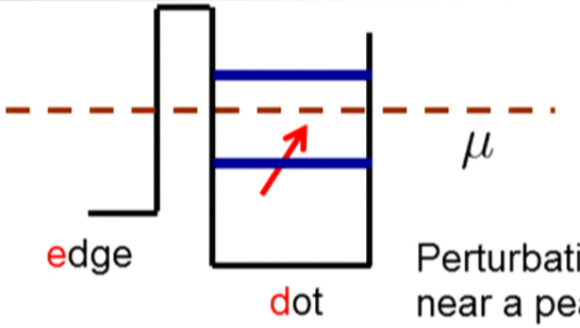
$$\overline{\partial_t\langle\hat{N}_R - \hat{N}_L\rangle} = \overline{\partial_t\langle\hat{S}_z\rangle} = 0$$

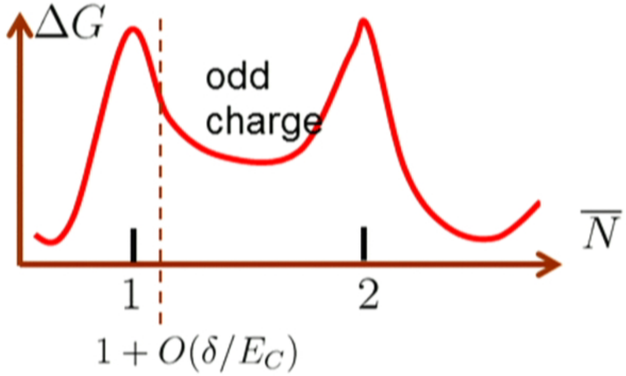
More levels – off-diag. exchange matrix elements allowing for backscattering (Tanaka et al 2011).

$$\mathcal{H}_K = J_{ij}S_i s_j$$

RMT, odd-e, Kondo, backscattering



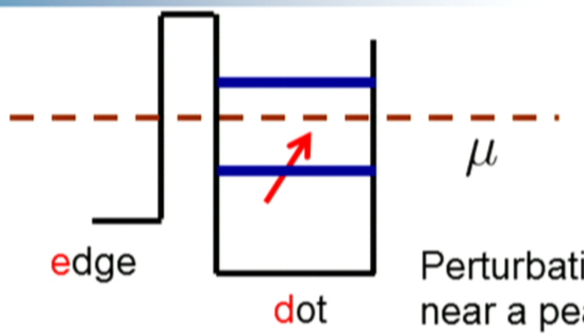
Perturbative treatment near a peak



RG+Kinetic equation:

$$\frac{\Delta G}{G_q} \sim \frac{\Gamma^2}{g^2\delta^2} \cdot \frac{\ln^2(T/T_K)}{\ln^2(\delta/T_K)}, \quad \delta \gg T \gtrsim T_K$$

RMT, odd-e, Kondo, backscattering



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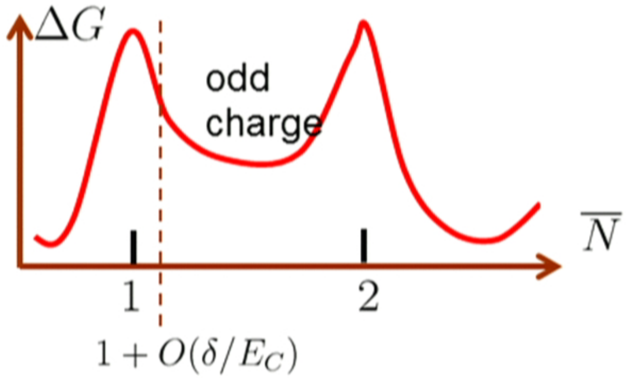

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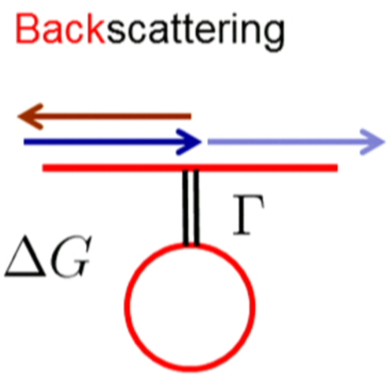
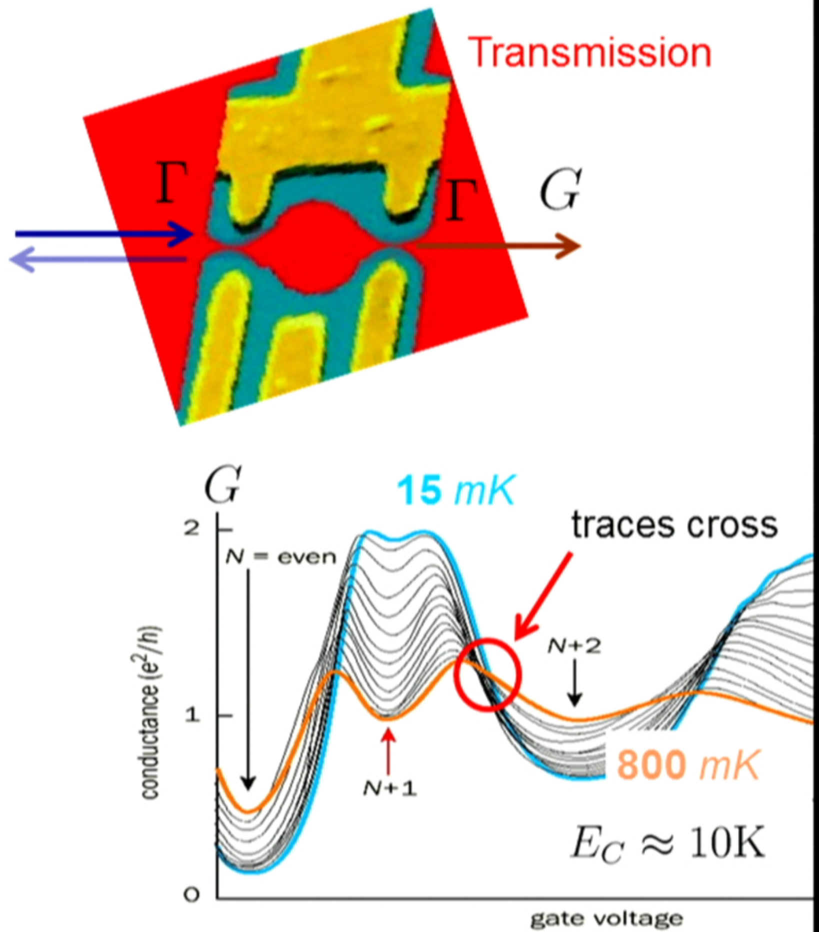
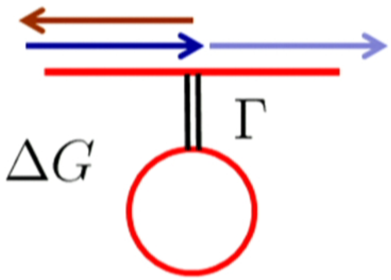
New *vs.* "Old" Quantum Dots Physics



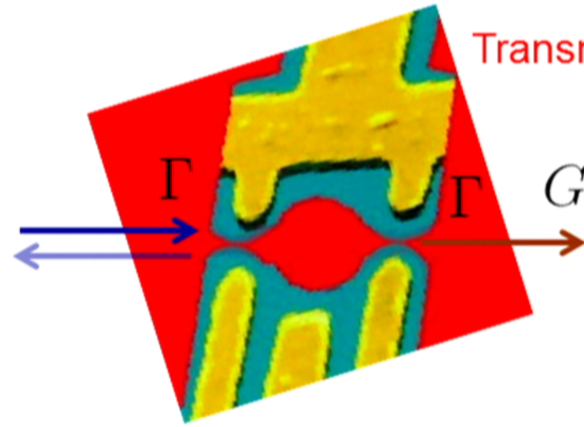


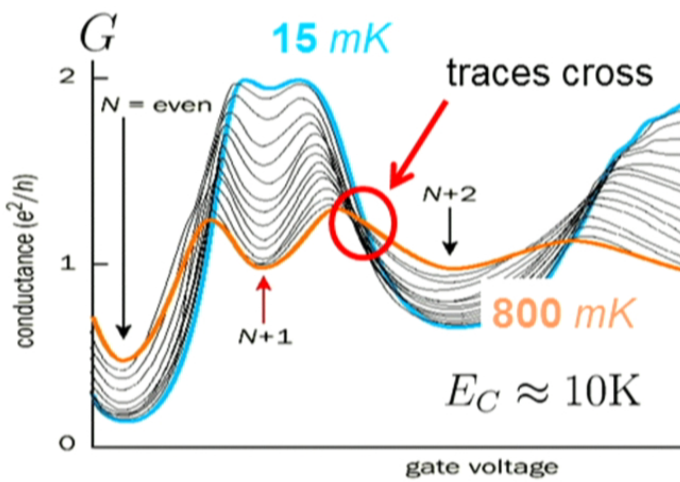
New *vs.* "Old" Quantum Dots Physics

Backscattering



Transmission





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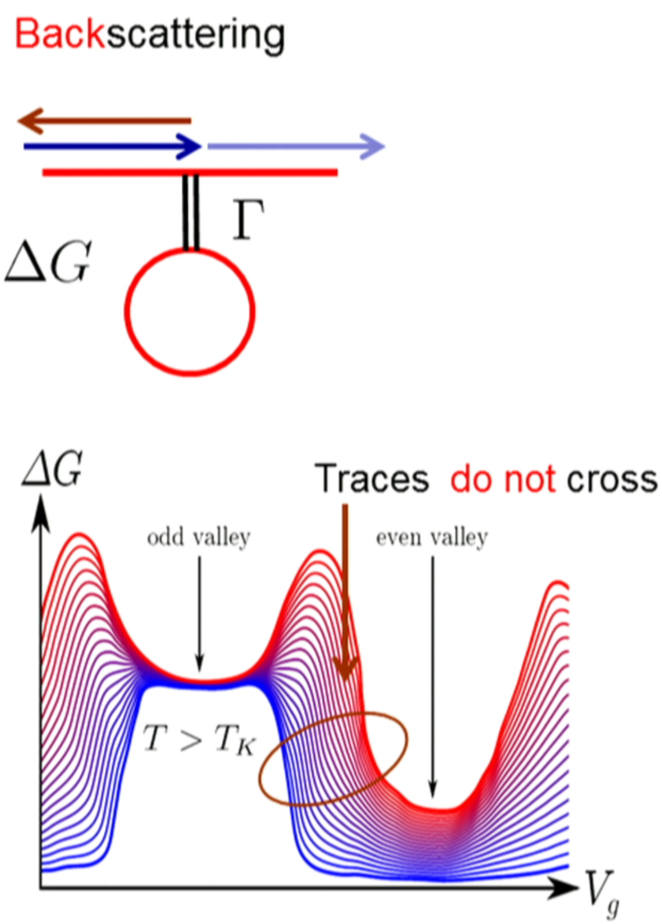
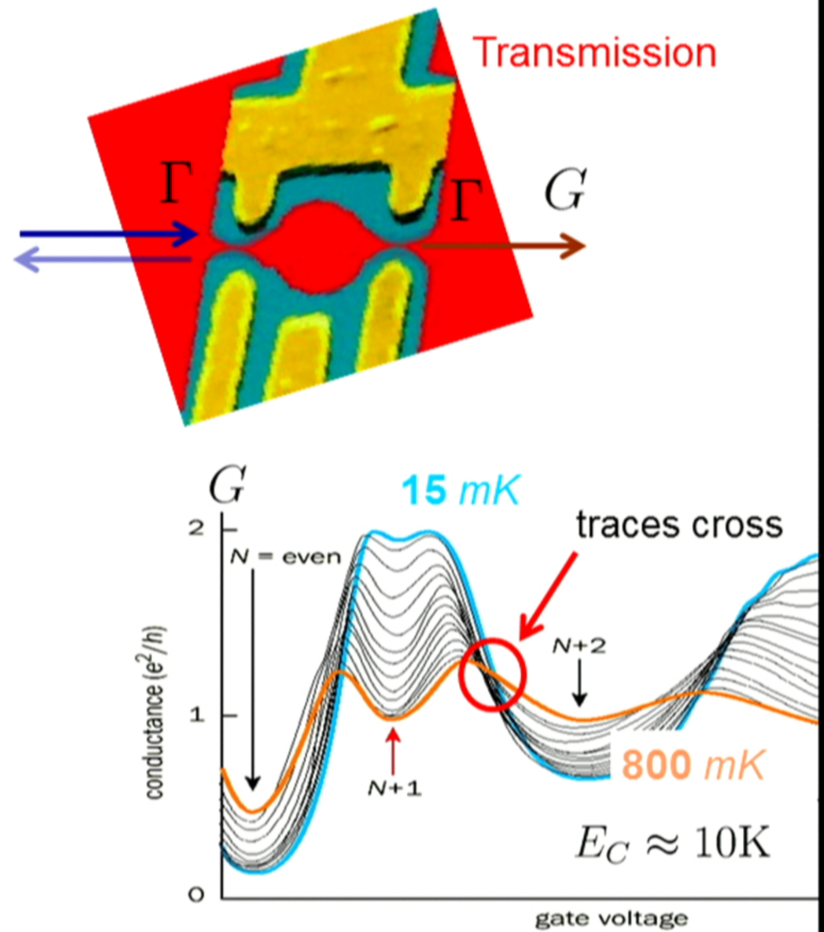
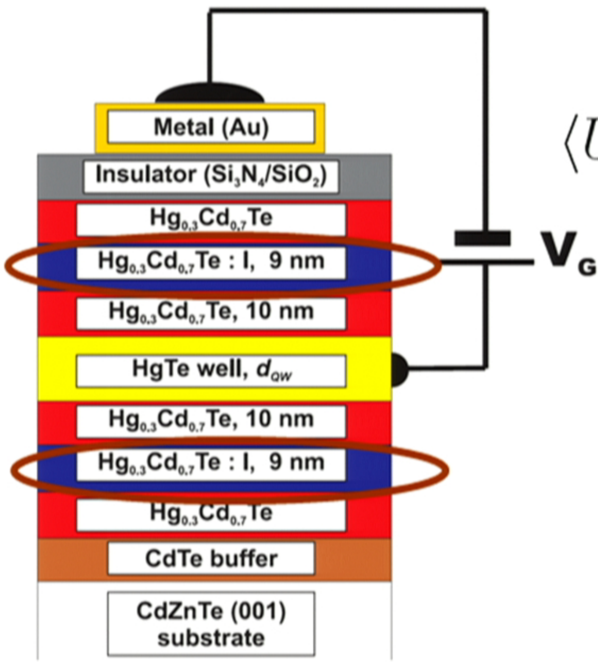




Static Charge Density Fluctuations



$$\langle U(\mathbf{r}_1)U(\mathbf{r}_2)\rangle \sim \frac{2\pi n_d e^4}{\kappa^2}\ln\frac{|\mathbf{r}_1-\mathbf{r}_2|}{d}$$

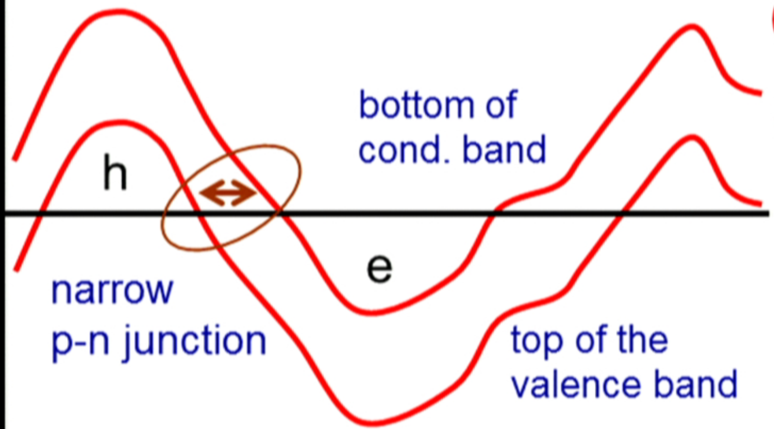
(Gergel, Suris, 1978)

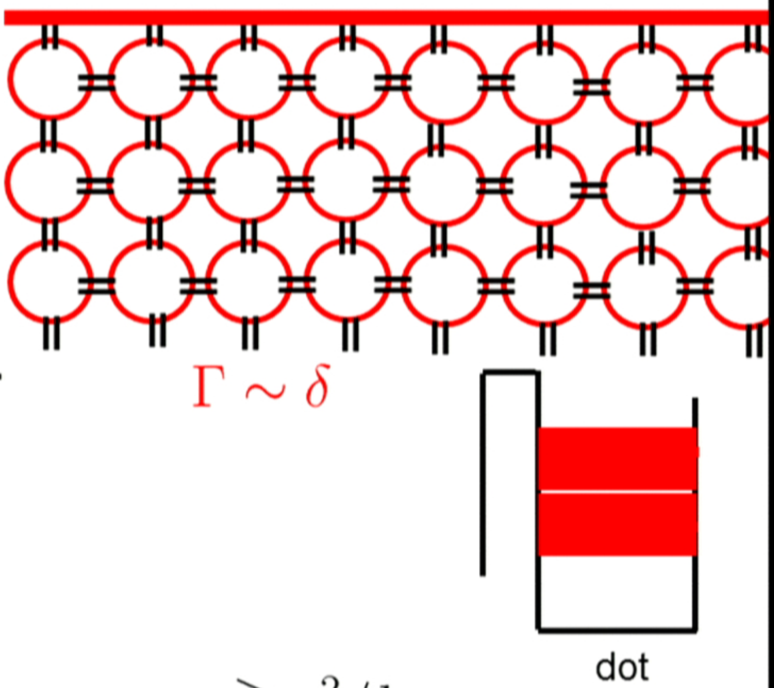
Compare $U_0 = \sqrt{\dfrac{2\pi n_d e^4}{\kappa^2}}$

with $\frac{1}{2}E_g$

Puddles at high doping density

$$n_d \gtrsim n_0 = \frac{E_g^2 \kappa^2}{8\pi e^4}$$





Strong tunneling between puddles, $\sigma_{2D} \gtrsim e^2/h$

Probably ABOVE the threshold for edge delocalization

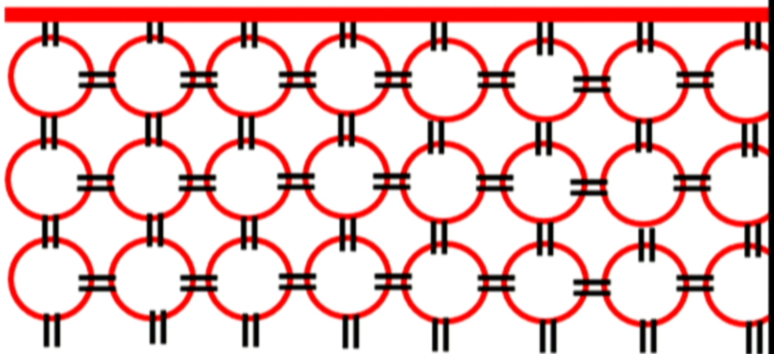
(Fu, Kane, PRL 2012)

Hopping

$$n_d \ll n_0 = \frac{E_g^2 \kappa^2}{8\pi e^4}$$

small density of puddles

$$n_p \propto \exp(-n_0/n_d)$$



Long *vs.* Short Edge

Edge length: L

density of puddles:

$$n_p \sim \left(\frac{1}{l*}\right)^2 \cdot \exp(-n_0/n_d)$$

Long *vs.* Short Edge

Edge length: L

density of puddles:

$$n_p \sim \left(\frac{1}{l*}\right)^2 \cdot \exp(-n_0/n_d)$$

$L \gg\gg n_p^{-1/2}$: macroscopic regime , incoherent addition

Long vs. Short Edge

Edge length: L

density of puddles:

$$n_p \sim \left(\frac{1}{l*}\right)^2 \cdot \exp(-n_0/n_d)$$

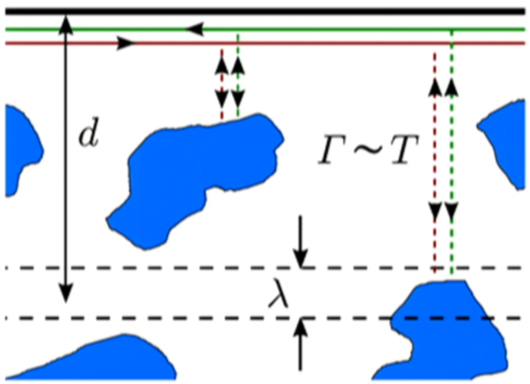
$L \gg\gg n_p^{-1/2}$: macroscopic regime , incoherent addition

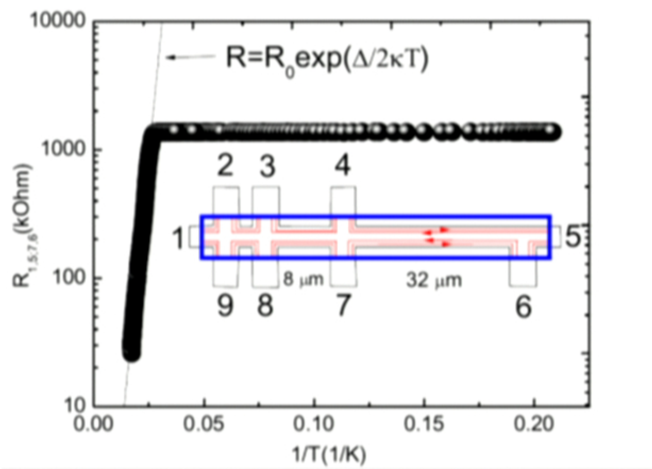
edge resistivity

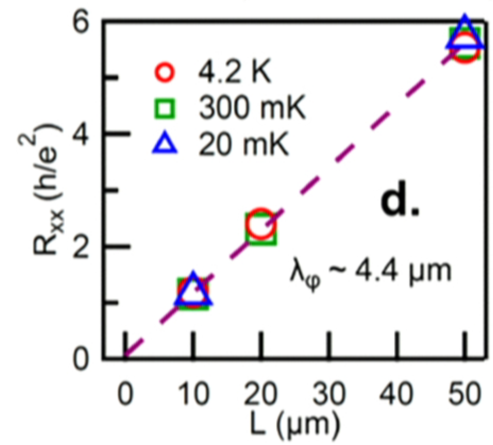
$$\rho_{\text{edge}} \propto \frac{n_p}{\ln^2(\delta/T)}, \quad T \ll \delta$$

edge resistance

$$R = \frac{1}{G} = \rho_{\text{edge}} \cdot L$$





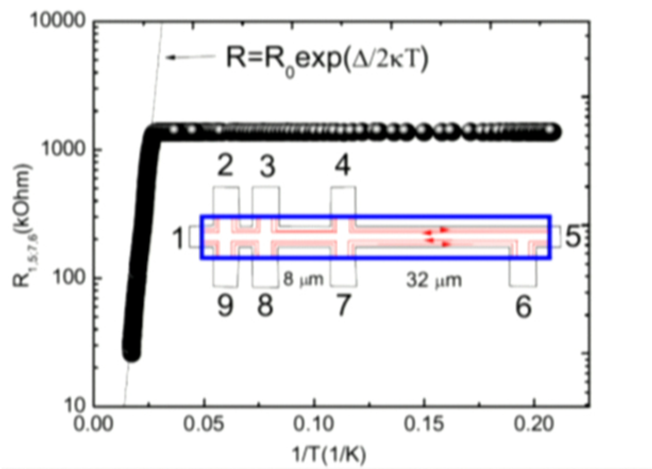


Gusev et al, arXiv1308.4356
+PRB **89**, 125305 (2014)

Du et al 1306.1925
(InAs/GaSb)

Does not seem to fit

$$\rho_{\text{edge}} \propto \left(\frac{1}{l^*}\right)^2 \cdot \exp(-n_0/n_d) \frac{1}{\ln^2(\delta/T)}$$



Gusev et al, arXiv1308.4356
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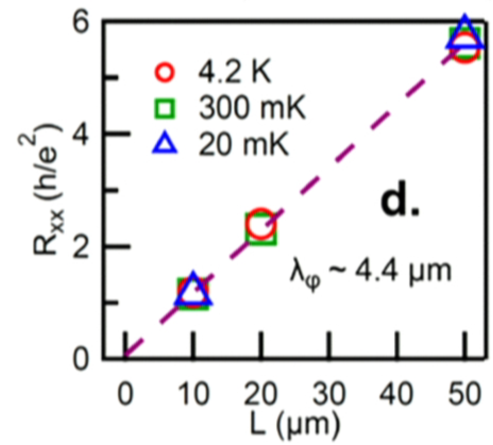

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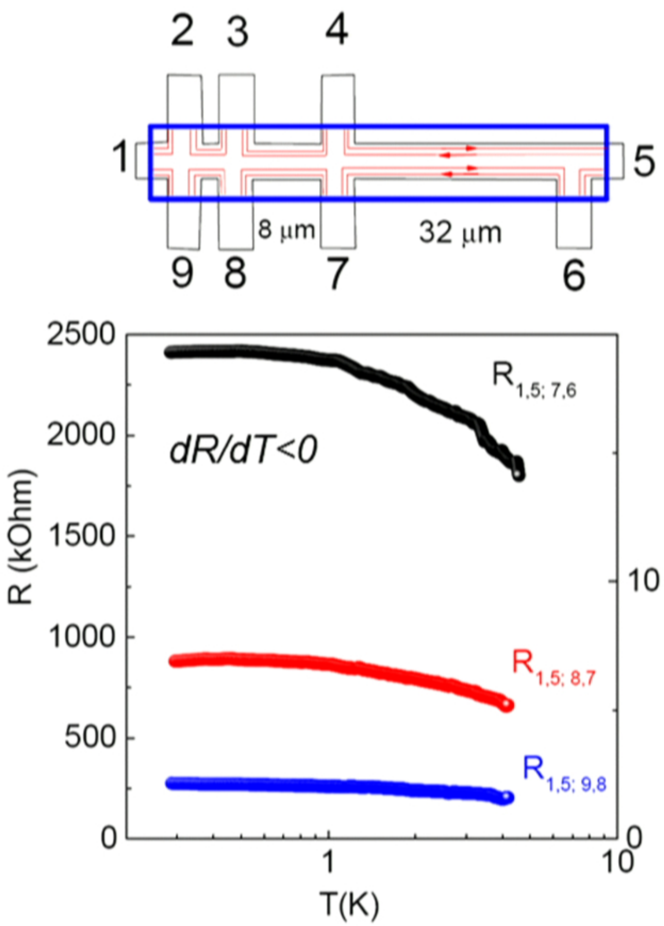
Some evidence for temperature dependence



Gusev et al, arXiv1308.4356
+PRB **89**, 125305 (2014)

apparently high donor densities,

$$n_d \gtrsim n_0 = \frac{E_g^2 \kappa^2}{8\pi e^4}$$

Long *vs.* Short Edge

Edge length: L

density of puddles:

$$n_p \sim \left(\frac{1}{l*}\right)^2 \cdot \exp(-n_0/n_d)$$

$L \ll n_p^{-1/2}$: mesoscopic regime

Average conductance close to G_q , the correction has a weak temperature dependence

Long *vs.* Short Edge

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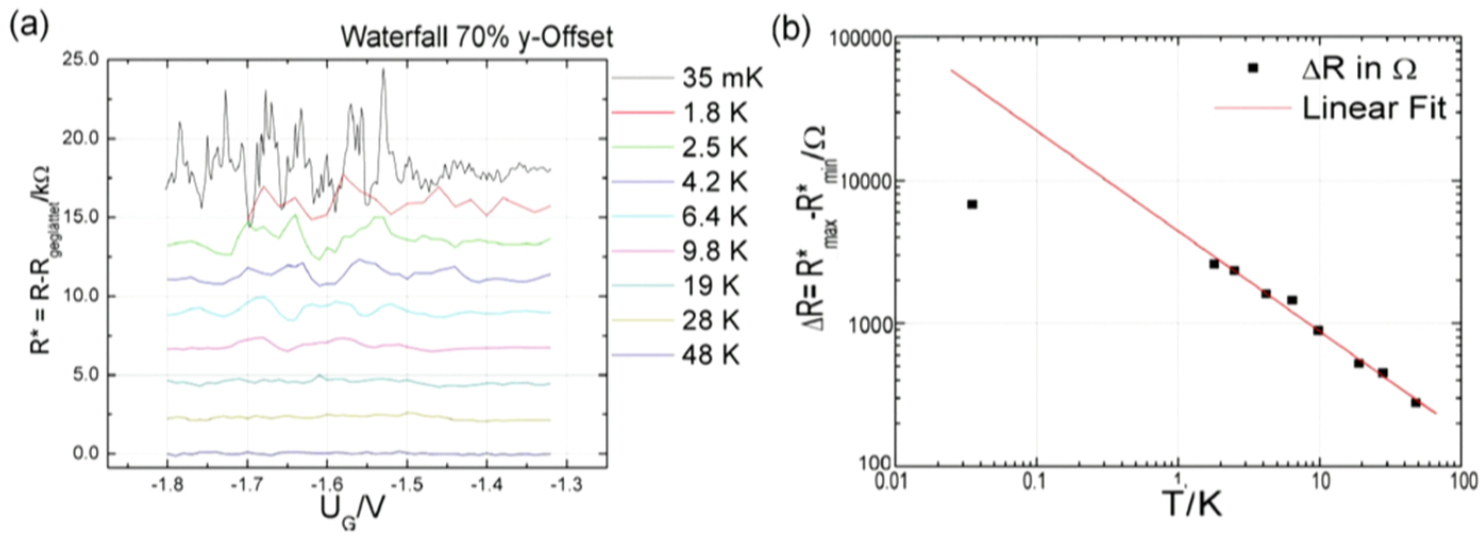
$L \ll n_p^{-1/2}$: mesoscopic regime

Average conductance close to G_q , the correction has a weak temperature dependence

Average single-puddle contribution

$$\frac{\langle \Delta G^{\mathrm{av}} \rangle}{G_q} \sim \frac{\Gamma^2}{g^2 E_C \delta}, \qquad \delta \gg T \gg \delta \cdot e^{-\pi\delta/2\Gamma}$$

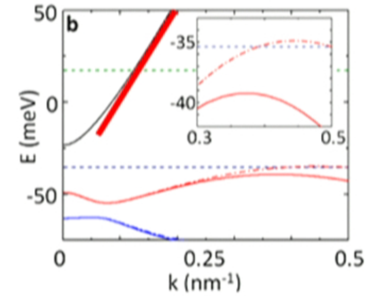
Conductance fluctuations in experiment



Molenkamp group 2013

HgTe/CdTe: Estimates, Observations

$$\kappa \approx 13\,, \quad E_g \approx 10\text{meV}\,, \quad v \approx 5.5 \cdot 10^7 \text{cm/s}$$



$$\delta \sim 1\text{mV}$$

$$n_0 = \frac{E_g^2 \kappa^2}{8\pi e^4} \approx 3 \cdot 10^{10} \text{cm}^{-2}$$

$$\alpha = \frac{e^2}{\kappa \hbar v} \approx 0.3$$

HgTe/CdTe: Estimates, Observations

$$n_0 = \frac{E_g^2 \kappa^2}{8\pi e^4} \approx 3 \cdot 10^{10} \mathrm{cm}^{-2}$$

Year of experiment	
2007	$n_d \approx 3 \cdot 10^{11} \mathrm{cm}^{-2}$
2012	$n_d \approx 3 \cdot 10^{10} \mathrm{cm}^{-2}$

Conclusions

1. Doping leads to puddles formation. Puddles are rare if doping is well below the characteristic value, $n_0^{\text{theo}} = \frac{E_g^2 \kappa^2}{8\pi e^4}$

2. Puddles backscatter electrons. Worst are the configurations carrying spin: the backscattering rate has a weak temperature dependence, $\Delta G \propto \ln^2(T/T_K^{\min})$

3. At low doping: long edge – large edge resistance; $\rho_{\text{edge}} \propto n_p/\ln^2(\delta/T)$; short edge – approx. quantization + mesoscopic fluctuations