Title: New Exact Results in Calabi-Yau, D-branes, and Orientifolds

Date: Apr 15, 2014 10:00 AM

URL: http://pirsa.org/14040081

Abstract: We compute the exact two-sphere, disk and real projective plane partition functions of two-dimensional supersymmetric theories using the localization technique. From these new results, we will attack old and new important problems in the string theory on Calabi-Yau spaces, and D-branes and Orientifold planes therein.

New Exact Results in Calabi-Yau, D-branes & Orientifolds

SUNGJAY LEE

UNIVERSITY OF CHICAGO

based on: Doroud,Gomis,Le Floch, S.L., arXiv:1206.2609
Gomis,S.L.,arXiv:1210.6022
Kim,S.L.,Yi,arXiv:1310.4505

Perimeter Institute for Theoretical Physics

April 15th, 2014

Recent Developments

Sphere-Partition Function	Superconformal Index
S^4 : [Pestun]	**$S^3 \times S^1$**: [Romelsberger]
S^3 : [Kapustin,Willet,Yaakov] [Jafferis] [Hama,Hosomichi,**S.L**]	**$S^2 \times S^1$**: [Kim] [Imamura,Yokohama]

AGT correspondence, F-theorem, Test of Dualities and so on

S^2 : [Doroud,Gomis,Le Floch,**S.L**] [Benini,Cremonesi]	**$S^1 \times S^1$**: [Witten] [Benini,Eadger,Hori, Tachikawa]
D_2 : [Honda,Okuda][Hori,Romo]	
RP^2 : [Kim, **S.L**,Yi]	

Attack some basic questions in 2D (SUSY) theories

Recent Developments

Sphere-Partition Function	Superconformal Index
S^4 : [Pestun]	**S^3 x S^1**: [Romelsberger]
S^3 : [Kapustin,Willet,Yaakov]	**S^2 x S^1**: [Kim]
[Jafferis]	[Imamura,Yokohama]
[Hama,Hosomichi,**S.L**]	

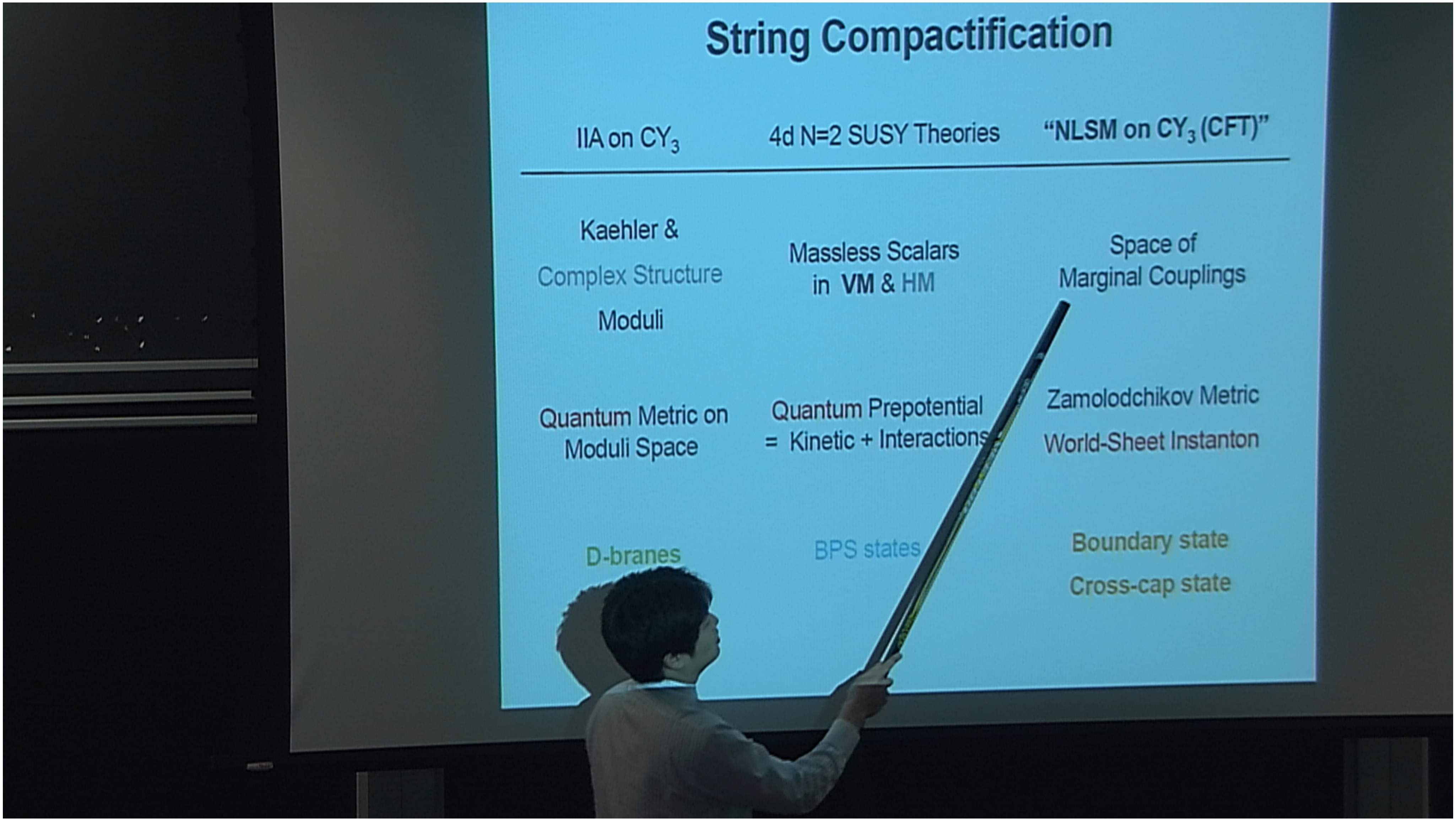
AGT correspondence, F-theorem, Test of Dualities and so on

S^2 : [Doroud,Gomis,Le Floch,**S.L**]	**S^1 x S^1**: [Witten]
[Benini,Cremonesi]	[Benini,Eadger,Hori, Tachikawa]
D_2 : [Honda,Okuda][Hori,Romo]	
RP^2 : [Kim, **S.L**,Yi]	

Attack some basic questions in 2D (SUSY) theories

String Compactification

IIA on CY_3	4d N=2 SUSY Theories	**“NLSM on CY_3 (CFT)”**
Kaehler & Complex Structure Moduli	Massless Scalars in **VM** & **HM**	Space of Marginal Couplings
Quantum Metric on Moduli Space	Quantum Prepotential = Kinetic + Interactions	Zamolodchikov Metric World-Sheet Instanton
D-branes O-planes	BPS states	**Boundary state** **Cross-cap state**


String Compactification
IIA on CY3
4d N=2 SUSY Theories
"NLSM on CY3 (CFT)"
Kaehler &
Complex Structure
Moduli
Massless Scalars
in VM & HM
Space of
Marginal Couplings
Quantum Metric on
Moduli Space
Quantum Prepotential
= Kinetic + Interactions
Zamolodchikov Metric
World-Sheet Instanton
D-branes
BPS states
Boundary state
Cross-cap state

New Exact Results

2D SUSY gauge theories (**GLSM**) flowing to CY_3 sigma models are useful

- Kahler moduli of CY_3: complexified FI parameters $\tau = \frac{\theta}{2\pi} + i\xi$
- Complex structure moduli of CY_3: parameters in superpotential W

New Exact Results: Using the localization technique,

$Z_{S^2}(\mathrm{GLSM}) = e^{-K(\tau,\bar{\tau})}$	Kahler potential for Kahler moduli space	
$Z_{D_2}(\mathrm{GLSM}) = \mathcal{Z}_{\text{D-brane}}$	Central charge of D-brane	**EXACT** in α'-corrections including the world-sheet instanton effects
$Z_{\mathbb{RP}^2}(\mathrm{GLSM}) = \mathcal{Z}_{\text{O-plane}}$	Central charge of O-plane	

New Exact Results

2D SUSY gauge theories (**GLSM**) flowing to CY_3 sigma models are useful

- Kahler moduli of CY_3: complexified FI parameters $\tau = \frac{\theta}{2\pi} + i\xi$
- Complex structure moduli of CY_3: parameters in superpotential W

New Exact Results: Using the localization technique,

$Z_{S^2}(\mathrm{GLSM}) = e^{-K(\tau,\bar{\tau})}$	Kahler potential for Kahler moduli space	**EXACT** in α'-corrections including the world-sheet instanton effects
$Z_{D_2}(\mathrm{GLSM}) = \mathcal{Z}_{\text{D-brane}}$	Central charge of D-brane	
$Z_{\mathbb{RP}^2}(\mathrm{GLSM}) = \mathcal{Z}_{\text{O-plane}}$	Central charge of O-plane	

New Exact Results

2D SUSY gauge theories (**GLSM**) flowing to CY_3 sigma models are useful

- Kahler moduli of CY_3: complexified FI parameters $\tau = \frac{\theta}{2\pi} + i\xi$
- Complex structure moduli of CY_3: parameters in superpotential W

New Exact Results: Using the localization technique,

$Z_{S^2}(\mathrm{GLSM}) = e^{-K(\tau,\bar{\tau})}$	Kahler potential for Kahler moduli space	**EXACT** in α'-corrections including the world-sheet instanton effects
$Z_{D_2}(\mathrm{GLSM}) = \mathcal{Z}_{\text{D-brane}}$	Central charge of D-brane	
$Z_{\mathbb{RP}^2}(\mathrm{GLSM}) = \mathcal{Z}_{\text{O-plane}}$	Central charge of O-plane	

Motivation

Does the conventional expression of the RR-charge need a modification ?

Topological Coupling: minimal coupling to RR gauge fields C

$$S_{\rm WZ} = \int_M C \wedge Q_{\rm RR}$$

- Long history to find the correct RR-charge:

$$I(a,b) = \int_M Q_{\rm RR}(\mathcal{F}_a, R) \wedge Q_{\rm RR}(-\mathcal{F}_b, -R)$$

Dirac Charge Quantization

[Brunner,Douglas,Lawrence,Romelsberger]

Motivation

Does the conventional expression of the RR-charge need a modification ?

Central Charge (Tension) of D-brane & O-plane in Calabi-Yau space

In large volume limit

$$Z = \int_M e^{-(B+iJ)} \wedge Q_{\mathrm{RR}}$$

$$Q^D_{\mathrm{RR}} = \mathrm{ch}(\mathcal{F}) \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}$$

$$Q^{O_p}_{\mathrm{RR}} = \pm 2^{p-4} \sqrt{\frac{\mathcal{L}(R_T/4)}{\mathcal{L}(R_N/4)}}$$

Recently, many mathematicians point out that this formula needs to be modified

$$Z^D = \int_M e^{-(B+iJ)} \mathrm{ch}(\mathcal{F}) \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$

[Libgober][Iritani]
[Katzarkov,Kontsevich,Pantev]...

What about Orientifolds ?

Need a similar modification ?

New characteristic class ?

Phyiscal understanding ?

N=(2,2) SUSY on S²

SUSY on Two-Sphere: SU(2|1)

Subalgebra of N=(2,2) SCA

Bosonic subalgebra:

- SU(2): rotational symmetry of S^2
- U(1): vector U(1) R-symmetry

NB: axial U(1) R-symmetry is broken unless the theory is conformal

Parametrized by Killing spinors $(\epsilon, \bar{\epsilon})$ satisfying

$$\nabla_i \epsilon = +\frac{1}{2l}\gamma_i \gamma^3 \epsilon \qquad \nabla_i \bar{\epsilon} = -\frac{1}{2l}\gamma_i \gamma^3 \bar{\epsilon}$$

Localization

Start with a following path-integral

$$Z[t] = \int \mathcal{D}\Phi \; e^{-S[\Phi]-tQ.V[\Phi]}$$

$$Q.S[\phi] = 0$$
$$Q^2 = J$$

- $S[\Phi]$: action of a theory we want to study
- The term V is invariant under J, $J.V[\Phi] = 0$

Supersymmetry tells us

$Z[0]$	=	$Z[\infty]$
$S[\Phi]$		$S_{\text{def}} = Q.V[\Phi]$
Quantum		**Semi-Classical**
Hard to evaluate		Easy to evaluate (**Gaussian Integral**)

Localization

Start with a following path-integral

$$Z[t] = \int \mathcal{D}\Phi \; e^{-S[\Phi]-tQ.V[\Phi]}$$

$$Q.S[\phi] = 0$$
$$Q^2 = J$$

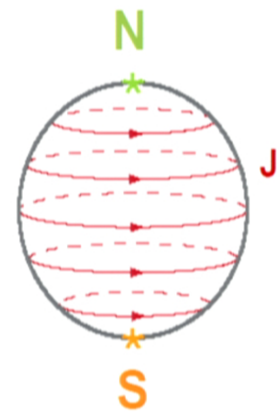
- $S[\Phi]$: action of a theory we want to study
- The term V is invariant under J, $J.V[\Phi] = 0$

Supersymmetry tells us

$Z[0]$	=	$Z[\infty]$
$S[\Phi]$		$S_{\rm def} = Q.V[\Phi]$
Quantum		Semi-Classical
Hard to evaluate		Easy to evaluate (Gaussian Integral)

Exact S² Partition Function

Localization Scheme



- Choice of supercharge : $\mathcal{Q}^2 = J + \frac{R}{2}$

- Q-exact deformation : Given the above choice,

$$\underbrace{\mathcal{L}_{\text{v.m.}} = \mathcal{Q}V_{\text{v.m.}} \quad \mathcal{L}_{\text{c.m.}} = \mathcal{Q}V_{\text{c.m.}} \quad \mathcal{L}_{\text{t.c.m.}} = \mathcal{Q}V_{\text{t.c.m.}}}_{\text{Kinetic Lagrangians: Q-exact deformations}} \quad \underbrace{\mathcal{L}_{\mathcal{W}} = \mathcal{Q}V_{\mathcal{W}}}_{\text{Superpotential}}$$

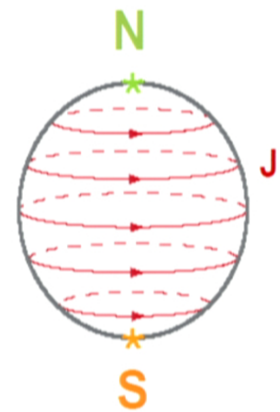
Decoupling Theorem: S² partition function is independent of

(1) gauge coupling constant

(2) parameters in superpotential $\mathcal{W}(\phi)$

Exact S² Partition Function

Localization Scheme



- Choice of supercharge : $\mathcal{Q}^2 = J + \frac{R}{2}$

- Q-exact deformation : Given the above choice,

$$\underbrace{\mathcal{L}_{\text{v.m.}} = \mathcal{Q}V_{\text{v.m.}} \quad \mathcal{L}_{\text{c.m.}} = \mathcal{Q}V_{\text{c.m.}} \quad \mathcal{L}_{\text{t.c.m.}} = \mathcal{Q}V_{\text{t.c.m.}}}_{\text{Kinetic Lagrangians: Q-exact deformations}} \quad \underbrace{\mathcal{L}_{\mathcal{W}} = \mathcal{Q}V_{\mathcal{W}}}_{\text{Superpotential}}$$

Decoupling Theorem: S² partition function is independent of

(1) gauge coupling constant

(2) parameters in superpotential $\mathcal{W}(\phi)$

Exact S² Partition Function

S² Partition Function [Doroud,Gomis,Le Floch,**S.L**][Benini,Cremonesi]

$$Z^{\text{GLSM}}_{S^2} = \frac{(l\Lambda)^{c/3}}{|W|} \sum_B \int_{\mathfrak{t}} d^r\sigma \; e^{-4\pi i \xi_{\text{ren}}\sigma + i\theta B} \times Z^{\text{reg}}_{\text{1-loop}}(\sigma)$$

$$Z_{\text{v.m.}} = \prod_{\alpha \in \Delta^+} \left[\left(\frac{\alpha \cdot B}{2} \right)^2 + (\alpha \cdot l\sigma)^2 \right]$$

$$Z_{\text{c.m.}} = \prod_{\rho \in \mathbf{R}} \frac{\Gamma\left(\frac{q}{2} - il(\rho\cdot\sigma + m) - \frac{\rho\cdot B}{2}\right)}{\Gamma\left(1 - \frac{q}{2} + il(\rho\cdot\sigma + m) - \frac{\rho\cdot B}{2}\right)}$$

ξ : FI parameter θ : theta angle

W : Weyl group   **r** : rank of G

B : Flux on S²   q : U(1) R charge

- Central charge (scale anomaly)

$$\frac{c}{3} = \underbrace{\sum_i \dim[\mathbf{R}_i](1-q_i)}_{\text{chiral multiplets}} - \underbrace{\dim[G]}_{\text{vector multiplet}} = \text{Tr}_f[R]$$

[Silverstein,Witten]
[Hori,Tong]

Exact S^2 Partition Function

What does S^2 Partition Function compute ?

$$Z_{S^2}(\tau,\bar{\tau}) = e^{-K(\tau,\bar{\tau})} \qquad \tau = \frac{\theta}{2\pi} + i\xi$$

- Exact (in α') Kahler potential for the quantum Kahler moduli space of CY
- Do not need to rely on mirror symmetry
- Conjectured by [Jockers,Kumar,Lapan,Morrison,Romo]

Exact S² Partition Function

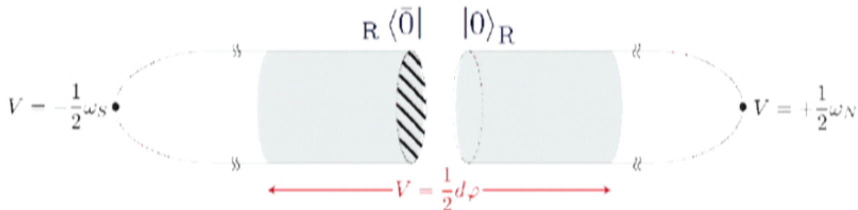
Sketch of Proof [Gomis,**S.L**]

$$Z_{S^2} \underset{[1]}{=} Z_{S^2_b} \overset{\substack{b = l/\tilde{l} \\ b \to 0}}{\underset{[2]}{=}} {}_{\mathrm{R}}\langle \bar{0}|0\rangle_{\mathrm{R}} \underset{\text{tt* eqn}}{=} e^{-K(\vec{\tau},\bar{\vec{\tau}})}$$

[Cecotti,Vafa]

[1] Partition function on S^2_b: independent of squashing parameter **b**

[2] Infinite squashing limit:



A-SUSY **B-SUSY** **$\overline{\text{A}}$-SUSY**

SUSY Theories on D_2

Boundary Data

SUSY Theories on D_2 = SUSY Theories on S^2 + Boundary Data

[1] Boundary Condition: Neumann (N) or Dirichlet (D)

[2] Boundary Interaction: to preserve 2 supercharges

- Chan-Paton Vector Space (V):
- Tachyon Profile ***Q***

$$\mathcal{Q}^2 = \mathcal{W}\ \mathbf{1}_{\mathcal{V}}$$

Matrix Factorization:
NOT UNIQUE

- Boundary Interaction (Tachyon Condensation)

$$\left\{\mathcal{Q}, \mathcal{Q}^\dagger\right\} = V_{bd}(\phi)\ \mathbf{1}_{\mathcal{V}}$$

D-Branes

Tangential: Neumann

$$D_\theta \phi^T(\theta = \pi/2, \varphi) = 0$$

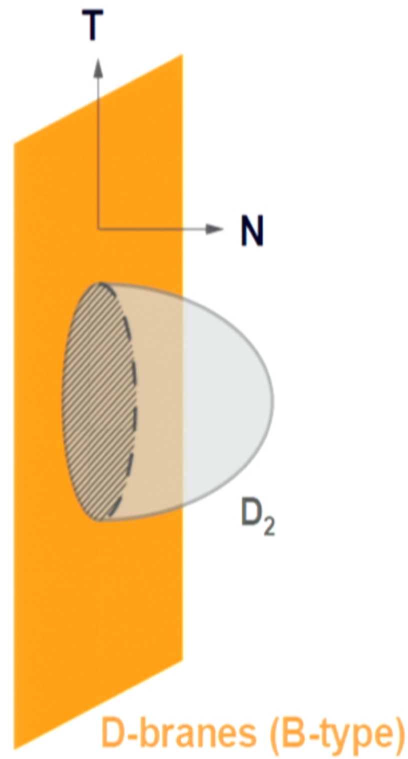
Normal: two equivalent descriptions

[1] Dirichlet

$$\phi^N(\theta = \pi/2, \varphi) = 0$$

[2] Neumann + Tachyon Condensation

$$V_{bd}(\phi^N = 0) = 0$$



D-Branes

Tangential: Neumann

$$D_\theta \phi^T(\theta = \pi/2, \varphi) = 0$$

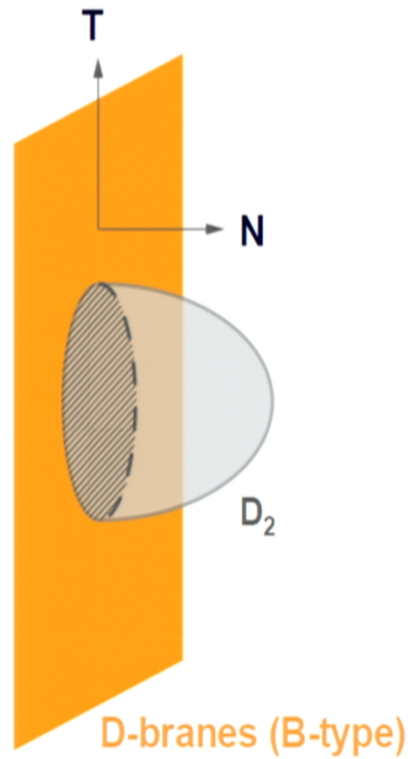
Normal: two equivalent descriptions

[1] Dirichlet

$$\phi^N(\theta = \pi/2, \varphi) = 0$$

[2] Neumann + Tachyon Condensation

$$V_{bd}(\phi^N = 0) = 0$$



D-Branes

Tangential: Neumann

$$D_\theta \phi^T(\theta = \pi/2, \varphi) = 0$$

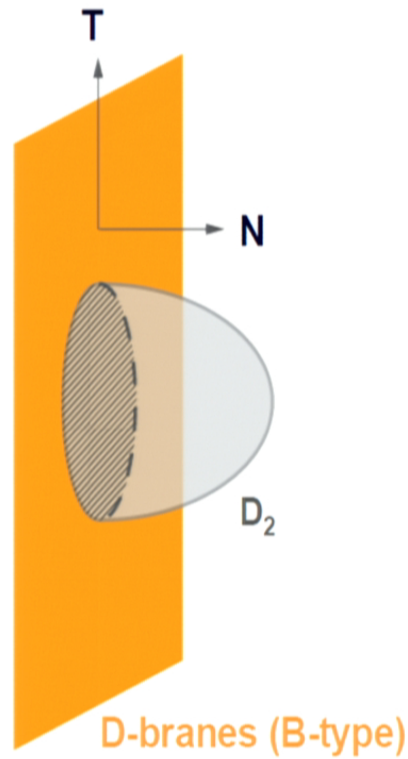
Normal: two equivalent descriptions

[1] Dirichlet

$$\phi^N(\theta = \pi/2, \varphi) = 0$$

[2] Neumann + Tachyon Condensation

$$V_{bd}(\phi^N = 0) = 0$$



D-Branes

Tangential: Neumann

$$D_\theta \phi^T(\theta = \pi/2, \varphi) = 0$$

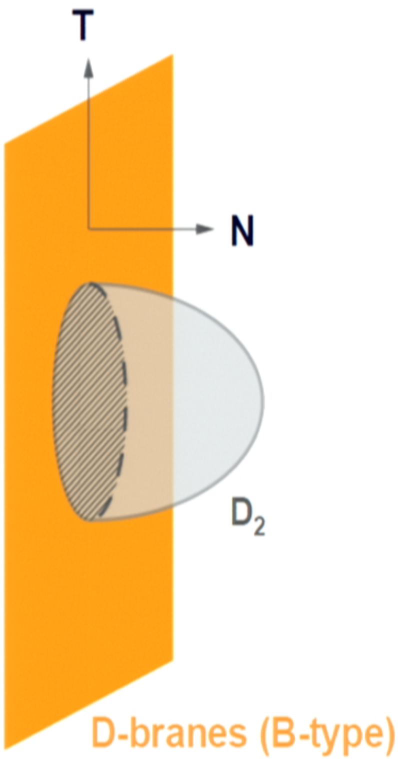
Normal: two equivalent descriptions

[1] Dirichlet

$$\phi^N(\theta = \pi/2, \varphi) = 0$$

[2] Neumann + Tachyon Condensation

$$V_{bd}(\phi^N = 0) = 0$$



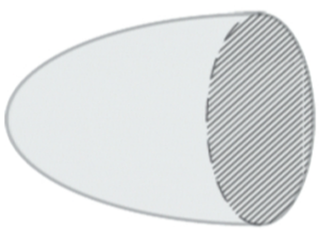
Exact D² Partition Function

Hemi-Sphere Partition Function [Hori,Romo] [Honda,Okuda] $\tau = \frac{\theta}{2\pi} + i\xi$

$$Z_{D_2}(\tau, \mathfrak{B}) = \frac{1}{|W|} \int d^r\sigma \; e^{-2\pi\tau \mathrm{tr}\sigma} \mathrm{Tr}_{\mathcal{V}} \left[e^{2\pi\rho_*(\sigma) + i\pi r_*} \right] Z_{\text{1-loop}}(\sigma)$$

$$Z_{\text{1-loop}}(\sigma) = \underbrace{\prod_{\alpha \in \Delta^+} \sigma \cdot \sigma \sinh \alpha \cdot \sigma}_{\text{Vector multiplets}} \times \underbrace{\prod_a \prod_{w_a \in \mathbb{R}_a} \Gamma\left[\frac{q_a}{2} - i w_a \cdot \sigma \right]}_{\text{Chiral matter multiplet}}$$

What does it compute ?



$= \; {}_R\langle \bar{0} | \mathfrak{B} \rangle_R \; =$ central charge of D-branes

[Ooguri,Oz,Yin]

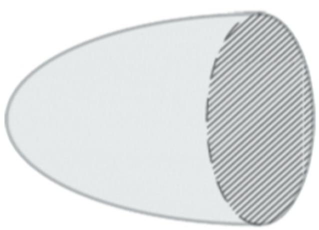
Exact D^2 Partition Function

Hemi-Sphere Partition Function [Hori,Romo] [Honda,Okuda] $\tau = \frac{\theta}{2\pi} + i\xi$

$$Z_{D_2}(\tau, \mathfrak{B}) = \frac{1}{|W|}\int d^r\sigma\; e^{-2\pi\tau \mathrm{tr}\sigma}\mathrm{Tr}_{\mathcal{V}}\left[e^{2\pi\rho_*(\sigma)+i\pi r_*}\right] Z_{1\text{-loop}}(\sigma)$$

$$Z_{1\text{-loop}}(\sigma) = \underbrace{\prod_{\alpha\in\Delta^+} \sigma\cdot\sigma \sinh \alpha\cdot\sigma}_{\text{Vector multiplets}} \times \underbrace{\prod_{a}\prod_{w_a\in\mathbb{R}_a} \Gamma\left[\frac{q_a}{2} - i w_a\cdot\sigma\right]}_{\text{Chiral matter multiplet}}$$

What does it compute ?



= $_R\langle\bar{0}|\mathfrak{B}\rangle_R$ = central charge of D-branes

[Ooguri,Oz,Yin]

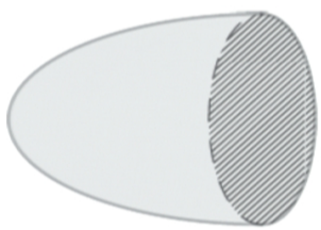
Exact D² Partition Function

Hemi-Sphere Partition Function [Hori,Romo] [Honda,Okuda] $\tau = \frac{\theta}{2\pi} + i\xi$

$$Z_{D_2}(\tau, \mathfrak{B}) = \frac{1}{|W|} \int d^r\sigma \; e^{-2\pi\tau \mathrm{tr}\sigma} \mathrm{Tr}_{\mathcal{V}} \left[e^{2\pi\rho_*(\sigma) + i\pi r_*} \right] Z_{\text{1-loop}}(\sigma)$$

$$Z_{\text{1-loop}}(\sigma) = \underbrace{\prod_{\alpha\in\Delta^+} \sigma\cdot\sigma \sinh\alpha\cdot\sigma}_{\text{Vector multiplets}} \times \underbrace{\prod_a \prod_{w_a\in\mathbb{R}_a} \Gamma\left[\frac{q_a}{2} - i w_a\cdot\sigma\right]}_{\text{Chiral matter multiplet}}$$

What does it compute ?



$= \; {}_R\langle\bar{0}|\mathfrak{B}\rangle_R \; =$ central charge of D-branes

[Ooguri,Oz,Yin]

Canonical Example

CY_{N-2} Hypersurface in CP^{N-1}

2d N=(2,2) SUSY gauge theory with **G=U(1)** gauge group, coupled to

N chiral multiplets X_a (a=1,2,..,N) of electric charge +1

a chiral multiplets P of electric charge - N

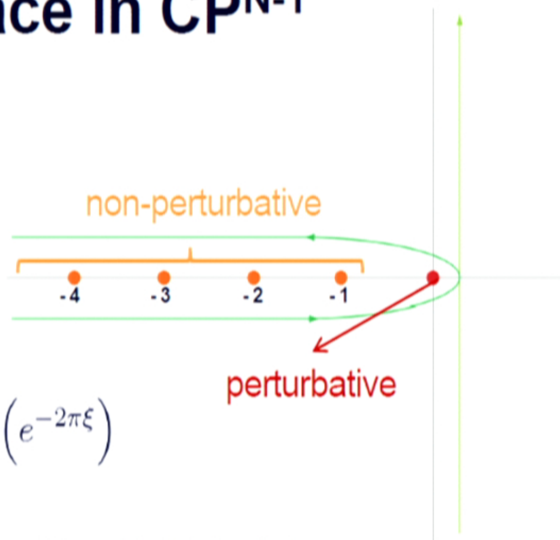
with superpotential $W = P \cdot G_N(X)$

$G_N(x)$: homogeneous polynomial of degree N

e.g. N=5: **Quintic Threefold**

CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}



$$\mathcal{Z}_D \simeq \int_{0^+ - i\infty}^{0^+ + i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$

$\xi > 0$

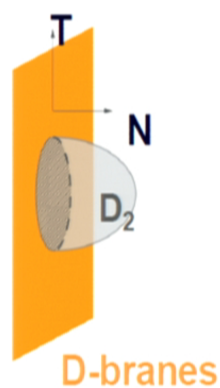
$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$

$\xi \to \infty$

$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X) \qquad \int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l D-brane in CY_{N-2}

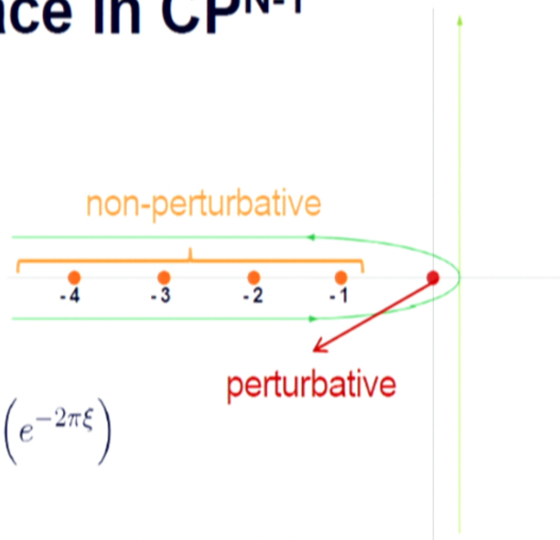


$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$

CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}

$$\mathcal{Z}_D \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$



$\xi > 0$

$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$

$\xi \to \infty$

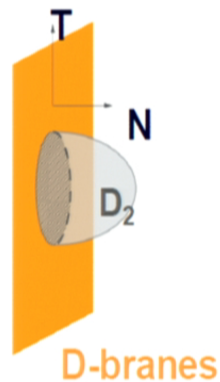
$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X)$$

$$\int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

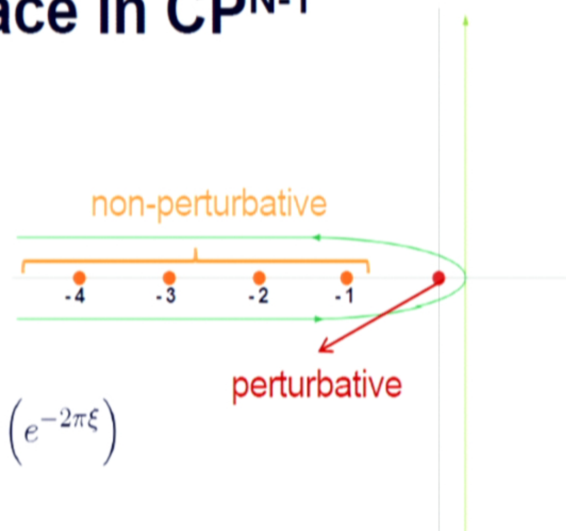
Lower-Dim'l D-brane in CY_{N-2}

$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$



CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}



$$\mathcal{Z}_D \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$

$\xi > 0$

$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$

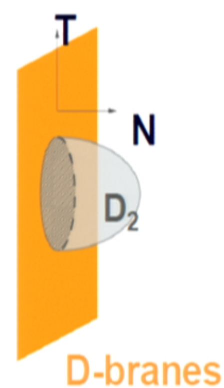
$\xi \to \infty$

$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X) \qquad \int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l D-brane in CY_{N-2}

$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$



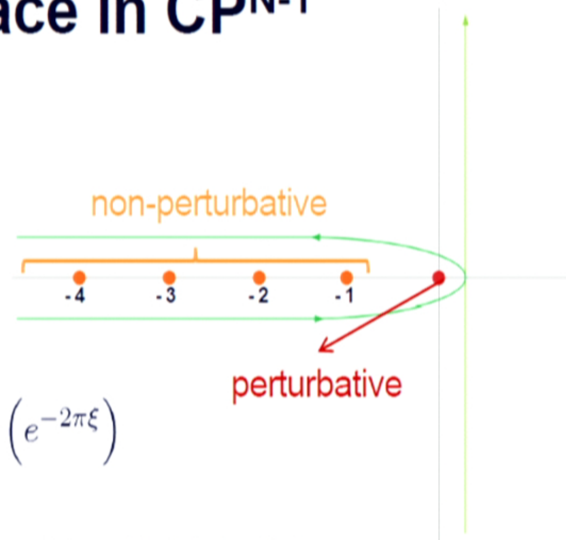
CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}

$$\mathcal{Z}_D \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$

$\xi > 0$

$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$



$\xi \to \infty$

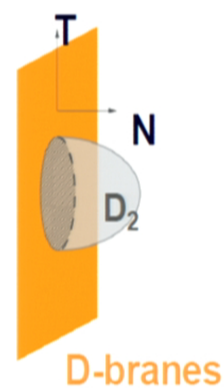
$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X)$$

$$\int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l D-brane in CY_{N-2}

$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$



D-Branes

Tangential: Neumann

$$D_\theta \phi^T(\theta = \pi/2, \varphi) = 0$$

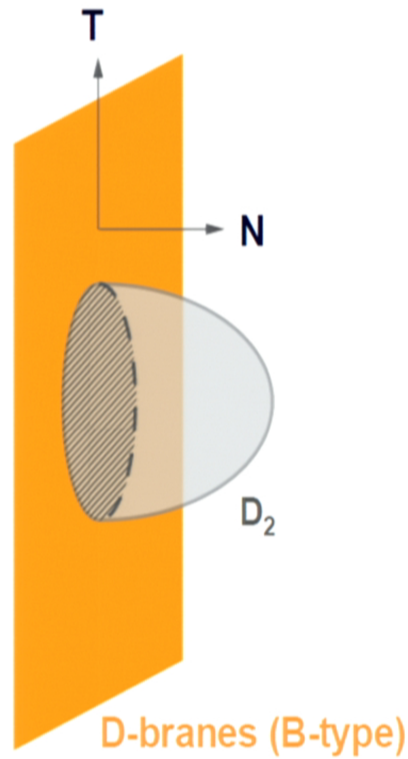
Normal: two equivalent descriptions

[1] Dirichlet

$$\phi^N(\theta = \pi/2, \varphi) = 0$$

[2] Neumann + Tachyon Condensation

$$V_{bd}(\phi^N = 0) = 0$$



D-Branes

Tangential: Neumann

$$D_\theta \phi^T (\theta = \pi/2, \varphi) = 0$$

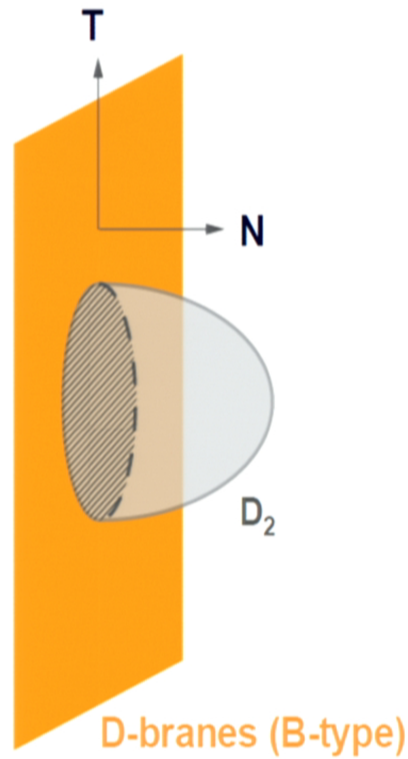
Normal: two equivalent descriptions

[1] Dirichlet

$$\phi^N (\theta = \pi/2, \varphi) = 0$$

[2] Neumann + Tachyon Condensation

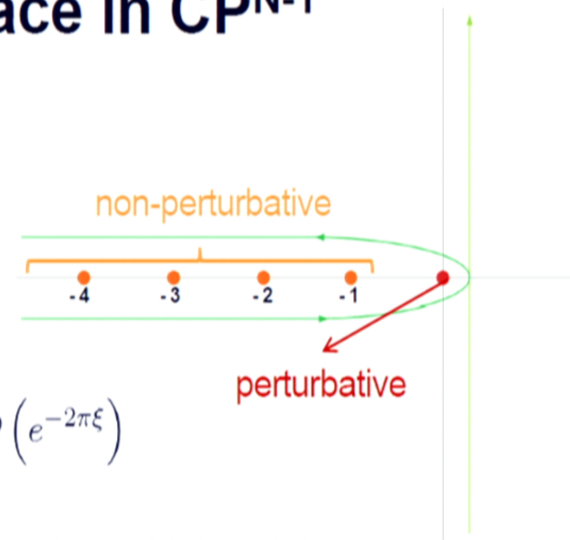
$$V_{bd}(\phi^N = 0) = 0$$



CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}

$$\mathcal{Z}_D \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$



$\xi > 0$

$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$

$\xi \to \infty$

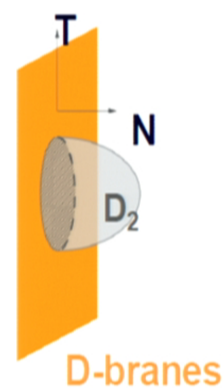
$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X)$$

$$\int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l D-brane in CY_{N-2}

$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$



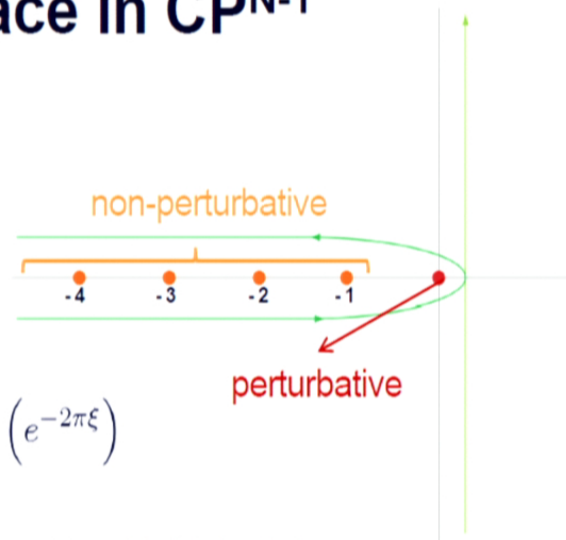
CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}

$$\mathcal{Z}_D \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$

$\xi > 0$

$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$



$\xi \to \infty$

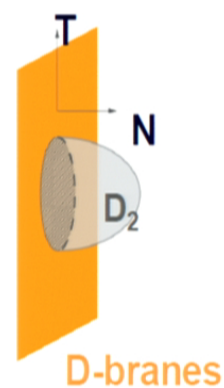
$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X)$$

$$\int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l D-brane in CY_{N-2}

$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$



SUSY Theories on RP^2

Parity Projection

SUSY Theories on RP^2 = SUSY Theories on S^2 + Parity Projection

[1] SUSY: 2 supercharges

$$\xi_\pm(\pi-\theta,\pi+\varphi) = i\xi_\mp(\theta,\varphi)$$

$$\bar{\xi}_\pm(\pi-\theta,\pi+\varphi) = -i\bar{\xi}_\mp(\theta,\varphi)$$

[2] Projection

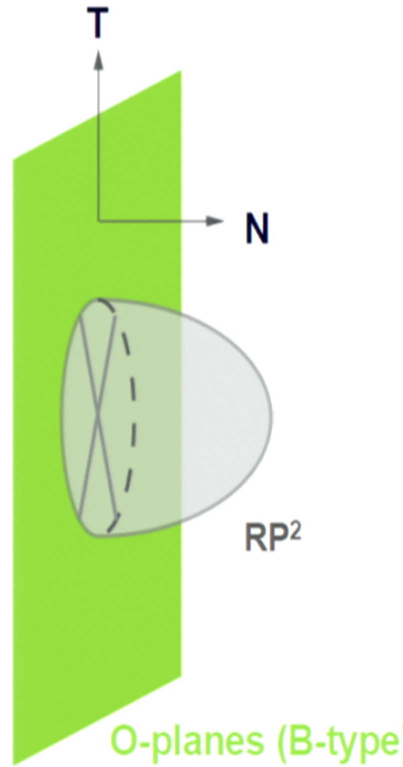
$$\phi_T(\pi-\theta,\pi+\varphi) = +\phi_T(\theta,\varphi)$$

$$\phi_N(\pi-\theta,\pi+\varphi) = -\phi_N(\theta,\varphi)$$

[3] Theta angle: $\theta = 0 \text{ or } \pi$

otherwise, the topological term breaks the parity

two values distinguish **O⁺** and **O⁻** planes



Parity Projection

SUSY Theories on RP^2 = SUSY Theories on S^2 **+ Parity Projection**

[1] SUSY: 2 supercharges

$$\xi_\pm(\pi-\theta,\pi+\varphi) = i\xi_\mp(\theta,\varphi)$$

$$\bar{\xi}_\pm(\pi-\theta,\pi+\varphi) = -i\bar{\xi}_\mp(\theta,\varphi)$$

[2] Projection

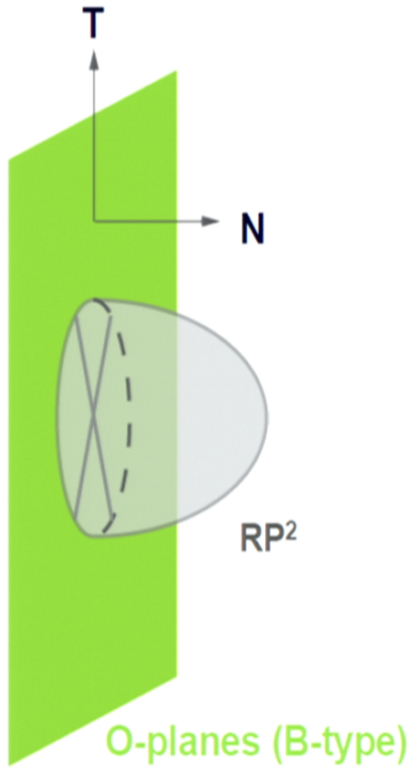
$$\phi_T(\pi-\theta,\pi+\varphi) = +\phi_T(\theta,\varphi)$$

$$\phi_N(\pi-\theta,\pi+\varphi) = -\phi_N(\theta,\varphi)$$

[3] Theta angle: $\theta = 0 \text{ or } \pi$

otherwise, the topological term breaks the parity

two values distinguish **O$^+$** and **O$^-$** planes



Exact RP² Partition Function

RP² Partition Function [H.Kim, **S.L**, P.Yi]

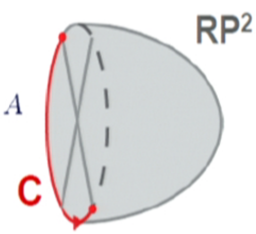
$\theta = 0$ or π

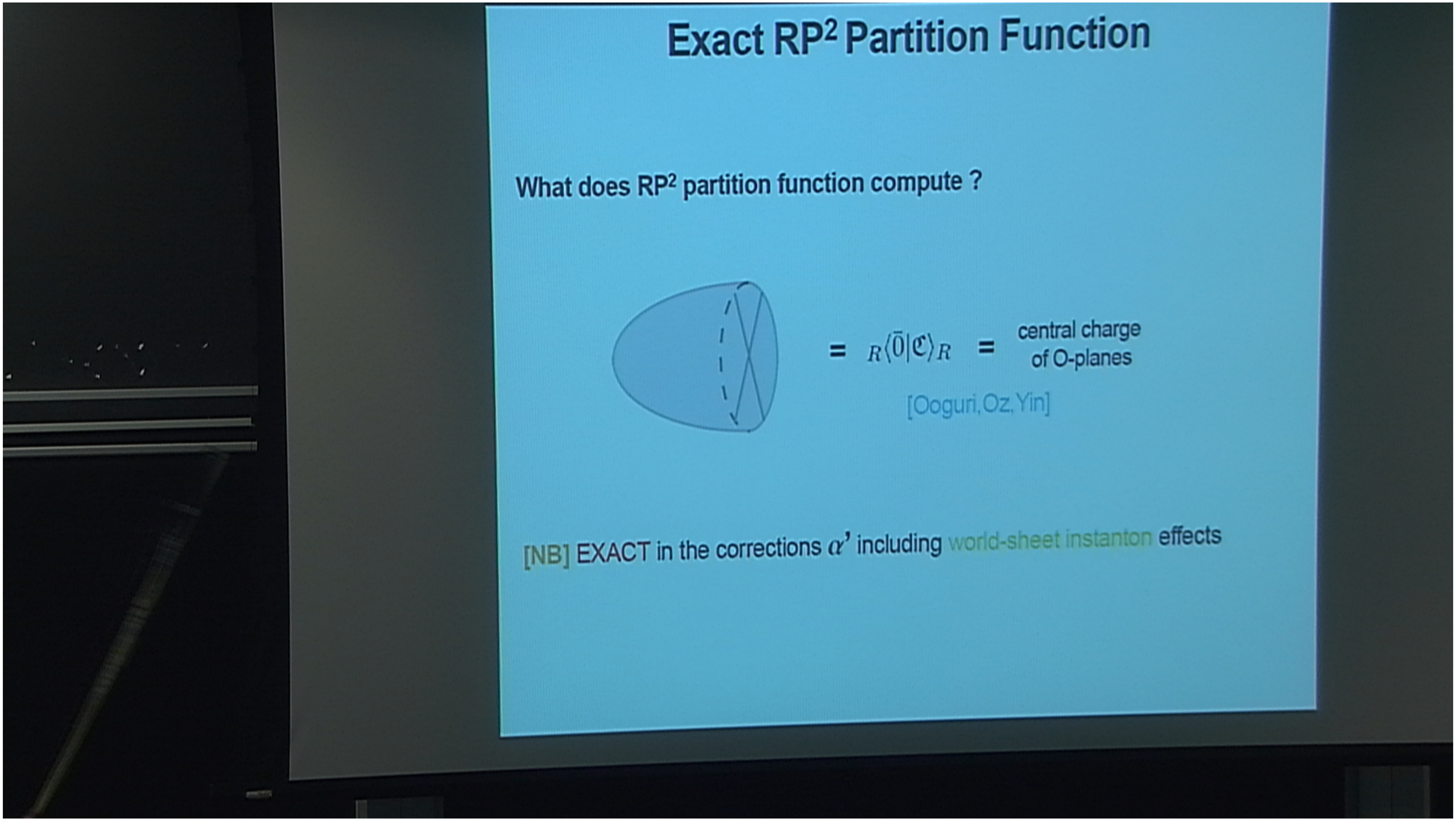
$$Z_{\mathbf{RP}^2}(\xi) = \frac{1}{|W|}\int d^r\sigma\, e^{-2\pi i\xi \mathrm{tr}\sigma}\left[Z^{\text{even}}_{\text{1-loop}} \pm Z^{\text{odd}}_{\text{1-loop}}\right]$$

$$Z^{\text{even}}_{\text{1-loop}} = \underbrace{\prod_{\alpha\in\Delta^+} \alpha\cdot\sigma \tan\left[\frac{\pi}{2}\alpha\cdot\sigma\right]}_{\text{vector multiplets}} \cdot \underbrace{\prod_{w\in\mathbf{R}} \Gamma\left[\frac{q}{2} - iw\cdot\sigma\right]\cos\left[\frac{\pi}{2}\left(\frac{q}{2} - iw\cdot\sigma\right)\right]}_{\text{chiral matter multiplet}}$$

$$Z^{\text{odd}}_{\text{1-loop}} = \prod_{\alpha\in\Delta^+} \alpha\cdot\sigma \tan\left[\frac{\pi}{2}\alpha\cdot\sigma - \frac{\alpha\cdot a}{2}\right] \cdot \prod_{w\in\mathbf{R}} \Gamma\left[\frac{q}{2} - iw\cdot\sigma\right]\cos\left[\frac{\pi}{2}\left(\frac{q}{2} - iw\cdot\sigma\right) - \frac{\omega\cdot a}{2}\right]$$

$$e^{ia} = e^{i\oint_C A}$$





Exact RP² Partition Function
What does RP² partition function compute ?
= $_R\langle\bar{0}|\mathfrak{C}\rangle_R$ = central charge of O-planes
[Ooguri,Oz,Yin]
[NB] EXACT in the corrections α' including world-sheet instanton effects

CY_{N-2} Hypersurface in CP^{N-1}

O-plane Wrapping Entire CY_{N-2}

$$\mathcal{Z}_O \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon}\cdot\Gamma(\epsilon)^N \cos^N\left[\frac{\pi}{2}\epsilon\right]\cdot\Gamma(1-N\epsilon)\sin\left[\frac{\pi}{2}N\epsilon\right]+\cdots$$

$$\sim \oint \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon}\frac{N}{e^{N-1}}\cdot\frac{\Gamma(1+N\epsilon)}{\Gamma(1-\epsilon)^N}\cdot\left(\frac{\epsilon/2}{\sin\pi\epsilon/2}\right)^N\frac{\sin\pi N\epsilon/2}{N\epsilon/2}$$

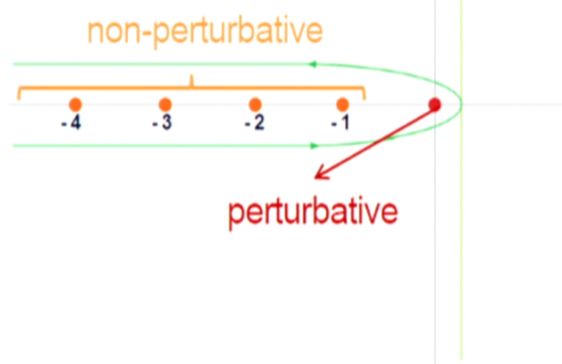
$$= \int_X e^{-iJ}\wedge\frac{\hat{A}(R_X/2)}{\hat{\Gamma}_c(-R_X)}$$

$$\int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l O-plane in CY_{N-2}

$$\mathcal{Z}_{O_p} = \pm 2^{p-4}\int_M e^{-iJ}\wedge\frac{\hat{A}(R_T/2)}{\hat{A}(R_N/2)}\frac{\hat{\Gamma}_c(R_N)}{\hat{\Gamma}_c(-R_T)}$$



Gamma Class

D-branes

$$\mathcal{Z}_D = \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$

$$= \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\hat{\Gamma}_c(x)\hat{\Gamma}_c(-x) = \hat{A}(x)$$

THE SAME FACTOR !

O-planes

$$\mathcal{Z}_{O_p} = \pm 2^{p-4} \int_M e^{-iJ} \wedge \frac{\hat{A}(R_T/2)}{\hat{A}(R_N/2)} \frac{\hat{\Gamma}_c(R_N)}{\hat{\Gamma}_c(-R_T)}$$

$$= \pm 2^{p-4} \int_M e^{-iJ} \wedge \sqrt{\frac{\mathcal{L}(R_T/4)}{\mathcal{L}(R_N/4)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\sqrt{\hat{A}(x) \cdot \mathcal{L}(x/4)} = \hat{A}(x/2)$$

Gamma Class

D-branes

$$\mathcal{Z}_D = \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$

$$= \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\hat{\Gamma}_c(x)\hat{\Gamma}_c(-x) = \hat{A}(x)$$

THE SAME FACTOR !

O-planes

$$\mathcal{Z}_{O_p} = \pm 2^{p-4} \int_M e^{-iJ} \wedge \frac{\hat{A}(R_T/2)}{\hat{A}(R_N/2)} \frac{\hat{\Gamma}_c(R_N)}{\hat{\Gamma}_c(-R_T)}$$

$$= \pm 2^{p-4} \int_M e^{-iJ} \wedge \sqrt{\frac{\mathcal{L}(R_T/4)}{\mathcal{L}(R_N/4)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\sqrt{\hat{A}(x) \cdot \mathcal{L}(x/4)} = \hat{A}(x/2)$$

Gamma Class

D-branes

$$\mathcal{Z}_D = \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$

$$= \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\hat{\Gamma}_c(x)\hat{\Gamma}_c(-x) = \hat{A}(x)$$

THE SAME FACTOR !

O-planes

$$\mathcal{Z}_{O_p} = \pm 2^{p-4} \int_M e^{-iJ} \wedge \frac{\hat{A}(R_T/2)}{\hat{A}(R_N/2)} \frac{\hat{\Gamma}_c(R_N)}{\hat{\Gamma}_c(-R_T)}$$

$$= \pm 2^{p-4} \int_M e^{-iJ} \wedge \sqrt{\frac{\mathcal{L}(R_T/4)}{\mathcal{L}(R_N/4)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\sqrt{\hat{A}(x) \cdot \mathcal{L}(x/4)} = \hat{A}(x/2)$$

Gamma Class

What are these corrections ?

$$\sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}} = \mathrm{Exp}\left[\frac{i\gamma}{2\pi}\mathrm{ch}_1(R_X) + i\sum_{k\geq 1}(-1)^k \frac{(2k)!}{(2\pi)^{2k+1}}\zeta(2k+1)\mathrm{ch}_{2k+1}(R_X)\right]$$

[1] Purely imaginary terms, starting from 6-form **$\mathrm{ch}_3(R_X)$** terms in CY

[2] Depend on the entire target space **X**, BUT do not care of the sub-manifolds M that D-branes or O-planes wrap on

[3] Should be identified as the α'- correction to the volume of X, not RR-charge

$$e^{-iJ} \rightarrow e^{-iJ+i\sum_{k\geq 1}(-1)^k \frac{2k!}{(2\pi)^{2k+1}}\zeta(2k+1)\mathrm{ch}_{2k+1}(R_X)}$$

Gamma Class

What are these corrections ?

$$\sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}} = \mathrm{Exp}\left[\frac{i\gamma}{2\pi}\mathrm{ch}_1(R_X) + i\sum_{k\geq 1}(-1)^k\frac{(2k)!}{(2\pi)^{2k+1}}\zeta(2k+1)\mathrm{ch}_{2k+1}(R_X)\right]$$

[1] Purely imaginary terms, starting from 6-form **$\mathrm{ch}_3(R_X)$** terms in CY

[2] Depend on the entire target space **X**, BUT do not care of the sub-manifolds M that D-branes or O-planes wrap on

[3] Should be identified as the α'- correction to the volume of X, not RR-charge

$$e^{-iJ} \;\rightarrow\; e^{-iJ+i\sum_{k\geq 1}(-1)^k\frac{2k!}{(2\pi)^{2k+1}}\zeta(2k+1)\mathrm{ch}_{2k+1}(R_X)}$$

Gamma Class

D-branes

$$\mathcal{Z}_D = \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$

$$= \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\hat{\Gamma}_c(x)\hat{\Gamma}_c(-x) = \hat{A}(x)$$

THE SAME FACTOR !

O-planes

$$\mathcal{Z}_{O_p} = \pm 2^{p-4} \int_M e^{-iJ} \wedge \frac{\hat{A}(R_T/2)}{\hat{A}(R_N/2)} \frac{\hat{\Gamma}_c(R_N)}{\hat{\Gamma}_c(-R_T)}$$

$$= \pm 2^{p-4} \int_M e^{-iJ} \wedge \sqrt{\frac{\mathcal{L}(R_T/4)}{\mathcal{L}(R_N/4)}} \cdot \sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}}$$

$$\sqrt{\hat{A}(x) \cdot \mathcal{L}(x/4)} = \hat{A}(x/2)$$

Gamma Class

What are these corrections ?

$$\sqrt{\frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)}} = \mathrm{Exp}\left[\frac{i\gamma}{2\pi}\mathrm{ch}_1(R_X) + i\sum_{k\geq 1}(-1)^k \frac{(2k)!}{(2\pi)^{2k+1}}\zeta(2k+1)\mathrm{ch}_{2k+1}(R_X)\right]$$

[1] Purely imaginary terms, starting from 6-form **$ch_3(R_X)$** terms in CY

[2] Depend on the entire target space **X**, BUT do not care of the sub-manifolds M that D-branes or O-planes wrap on

[3] Should be identified as the α'- correction to the volume of X, not RR-charge

$$e^{-iJ} \;\rightarrow\; e^{-iJ+i\sum_{k\geq 1}(-1)^k \frac{2k!}{(2\pi)^{2k+1}}\zeta(2k+1)\mathrm{ch}_{2k+1}(R_X)}$$

Gamma Class

Yet another support : in the large-volume limit,

$$Z_{S^2} = e^{-K(\tau,\bar{\tau})} \simeq \int_X e^{-2(B+iJ)} \cdot \frac{\hat{\Gamma}_c(R_X)}{\hat{\Gamma}_c(-R_X)} + \cdots$$

→ world-sheet instanton corrections

$$= -\frac{i}{3!} C_{ijk} (\tau - \bar{\tau})^i (\tau - \bar{\tau})^j (\tau - \bar{\tau})^k + \frac{\zeta(3)}{4\pi^3} \chi(X) + \cdots$$

CY_3

[1] Classical volume of CY_3

[2] Four-loop correction in NLSM on CY_3

[3] Prediction on the perturbative α'- correction to the volume of any CY_N
[Halverson,Jocker,Lapan,Morrison]

Gamma Class

Yet another support : in the large-volume limit,

$$Z_{S^2} = e^{-K(\tau,\bar\tau)} \simeq \int_X e^{-2(B+iJ)} \cdot \frac{\hat\Gamma_c(R_X)}{\hat\Gamma_c(-R_X)} + \cdots$$

world-sheet instanton corrections

$$\underset{\mathbf{CY_3}}{=} -\frac{i}{3!} C_{ijk}(\tau-\bar\tau)^i(\tau-\bar\tau)^j(\tau-\bar\tau)^k + \frac{\zeta(3)}{4\pi^3}\chi(X) + \cdots$$

[1] Classical volume of CY_3

[2] Four-loop correction in NLSM on CY_3

[3] Prediction on the perturbative α'- correction to the volume of any CY_N
[Halverson,Jocker,Lapan,Morrison]

Summary

Exact S^2,D_2,RP^2 partition function of GLSM

α'- exact metric for Kahler moduli of CY

α'- exact central charge for D/O wrapped on holomorphic (B) cycles

Gamma class & Quantum volume

New and direct method of computing stringy correction to L_{4D}

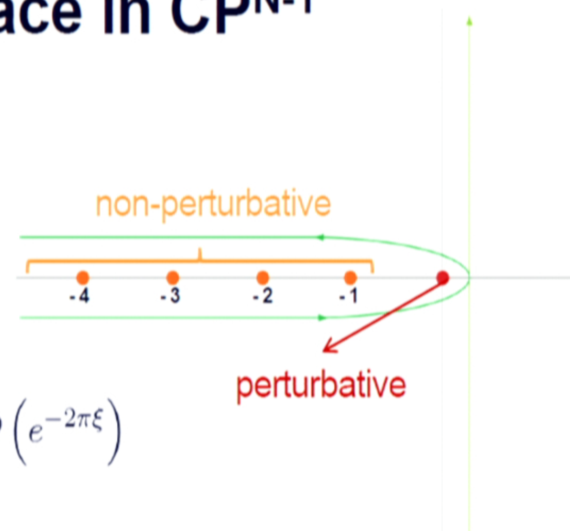
Issues associated with **Spin**^{c} world-volume resolved partially [Minasian,Moore][Freed,Witten]

Consistent to the Hori-Vafa mirror symmetry

CY_{N-2} Hypersurface in CP^{N-1}

D-brane Wrapping Entire CY_{N-2}

$$\mathcal{Z}_D \simeq \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \cdot \mathrm{ch}[\mathcal{F}] \cdot \frac{\Gamma^N(\epsilon)}{\Gamma(N\epsilon)}$$



$\xi > 0$

$$= \oint \frac{d\epsilon}{2\pi i}\, e^{-2\pi\xi\epsilon - i\theta\epsilon} \cdot \frac{N}{\epsilon^{N-1}} \cdot \frac{\Gamma(1+\epsilon)^N}{\Gamma(1+N\epsilon)} + \mathcal{O}\left(e^{-2\pi\xi}\right)$$

$\xi \to \infty$

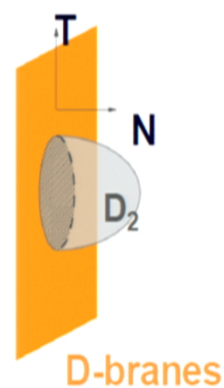
$$\simeq \int_X e^{-B-iJ} \wedge \hat{\Gamma}_c(R_X)$$

$$\int_X H^{N-2} = N \qquad J = \xi H$$

H : Toric divisor of CP^{N-1}

Lower-Dim'l D-brane in CY_{N-2}

$$\mathcal{Z}_D \sim \int_M e^{-B-iJ} \wedge \mathrm{ch}[\mathcal{F}] \wedge \frac{\hat{\Gamma}_c(R_T)}{\hat{\Gamma}_c(-R_N)}$$



Exact RP² Partition Function

RP² Partition Function [H.Kim,**S.L**,P.Yi]

$\theta = 0 \text{ or } \pi$

$$Z_{\mathbf{RP}^2}(\xi) = \frac{1}{|W|}\int d^r\sigma\, e^{-2\pi i\xi \mathrm{tr}\sigma}\left[Z^{\text{even}}_{\text{1-loop}} \pm Z^{\text{odd}}_{\text{1-loop}}\right]$$

$$Z^{\text{even}}_{\text{1-loop}} = \prod_{\alpha\in\Delta^+}\alpha\cdot\sigma\tan\left[\frac{\pi}{2}\alpha\cdot\sigma\right]\cdot\prod_{w\in\mathbf{R}}\Gamma\left[\frac{q}{2}-iw\cdot\sigma\right]\cos\left[\frac{\pi}{2}\left(\frac{q}{2}-iw\cdot\sigma\right)\right]$$

vector multiplets — chiral matter multiplet

$$Z^{\text{odd}}_{\text{1-loop}} = \prod_{\alpha\in\Delta^+}\alpha\cdot\sigma\tan\left[\frac{\pi}{2}\alpha\cdot\sigma-\frac{\alpha\cdot a}{2}\right]\cdot\prod_{w\in\mathbf{R}}\Gamma\left[\frac{q}{2}-iw\cdot\sigma\right]\cos\left[\frac{\pi}{2}\left(\frac{q}{2}-iw\cdot\sigma\right)-\frac{\omega\cdot a}{2}\right]$$

$$e^{ia} = e^{i\oint_C A}$$