

Title: Quantum Computing with Noninteracting Particles

Date: Apr 02, 2014 04:00 PM

URL: <http://pirsa.org/14040077>

Abstract: We introduce an abstract model of computation corresponding to an experiment in which identical, non-interacting bosons are sent through a non-adaptive linear circuit before being measured. We show that despite the very limited nature of the model, an exact classical simulation would imply a collapse of the polynomial hierarchy. Moreover, under plausible conjectures, a "noisy" approximate simulation would do the same. This gives evidence that quantum computers can sample a distribution that classical computers cannot even approximate, even when restricted to use no entanglement except that arising from particles being identical. We briefly discuss experimental prospects for realizing this model. This talk is based on The Computational Complexity of Linear Optics [STOC '11], which is joint work with Scott Aaronson.

Noninteracting Particle Model

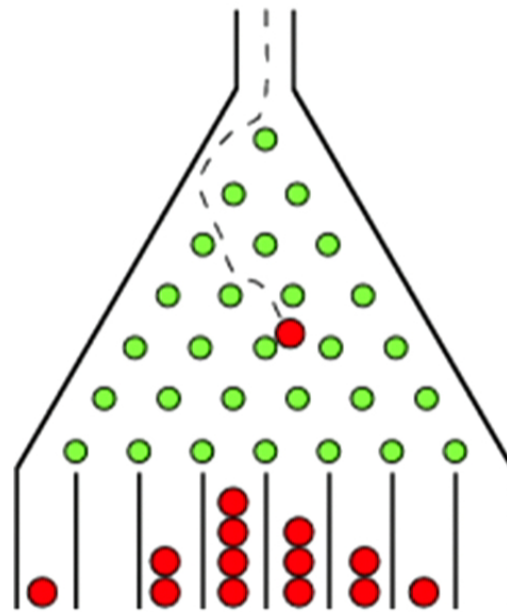
- Weak model of QC
 - Restricted entanglement and coupling
 - Likely not universal
- Why do we care?
 - Quantum gains with less
 - Already beyond classical

Noninteracting Particle Model

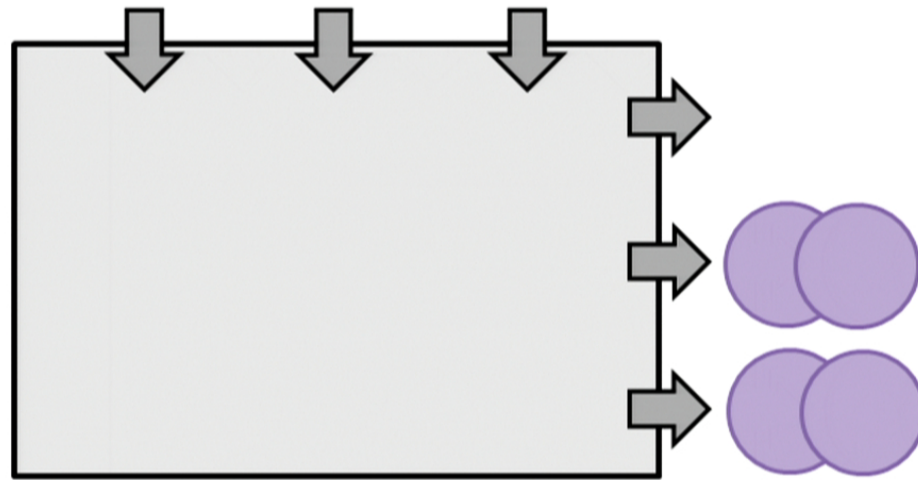
- Weak model of QC
 - Restricted entanglement and coupling
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- Why do we care?
 - Quantum gains with less
 - Already beyond classical
 - Easier to build



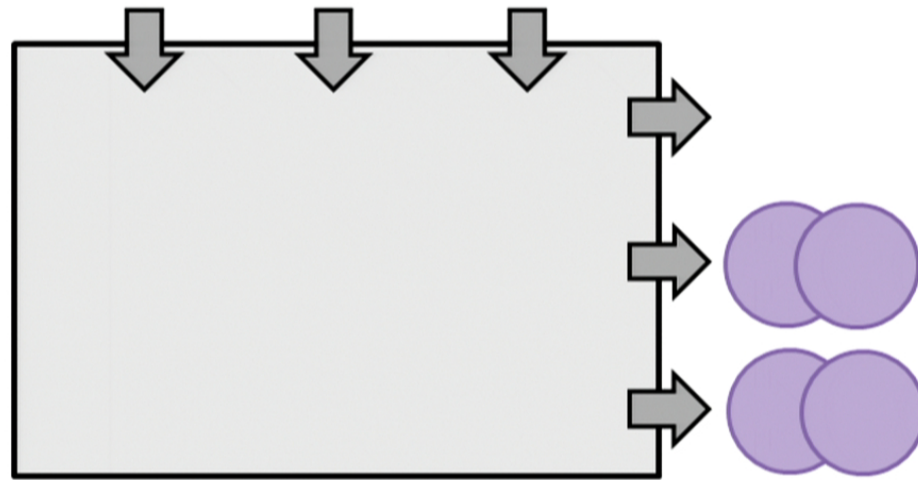
Balls and Slots



Balls and Slots



Balls and Slots



Transition Matrix

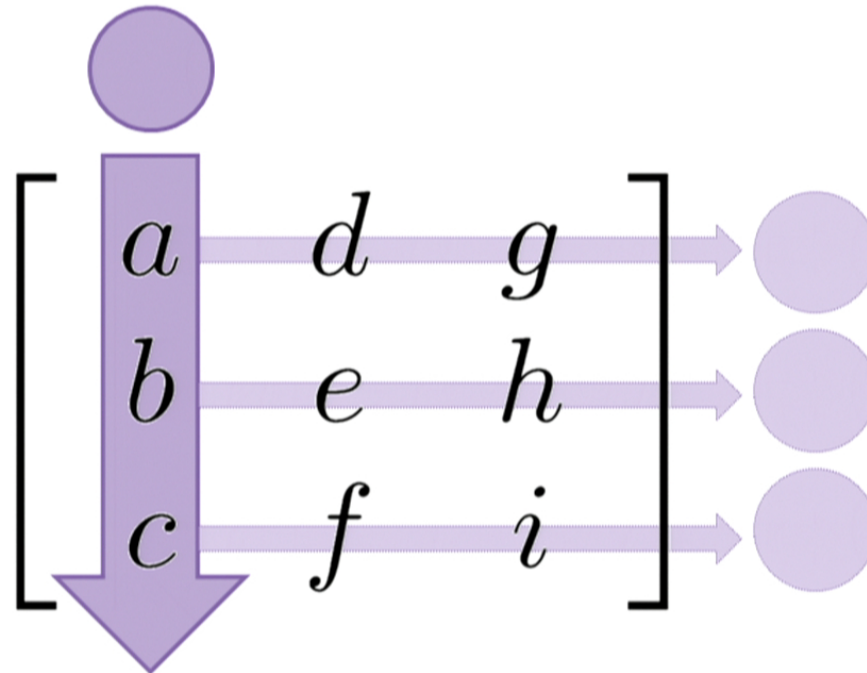
$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Transition Matrix



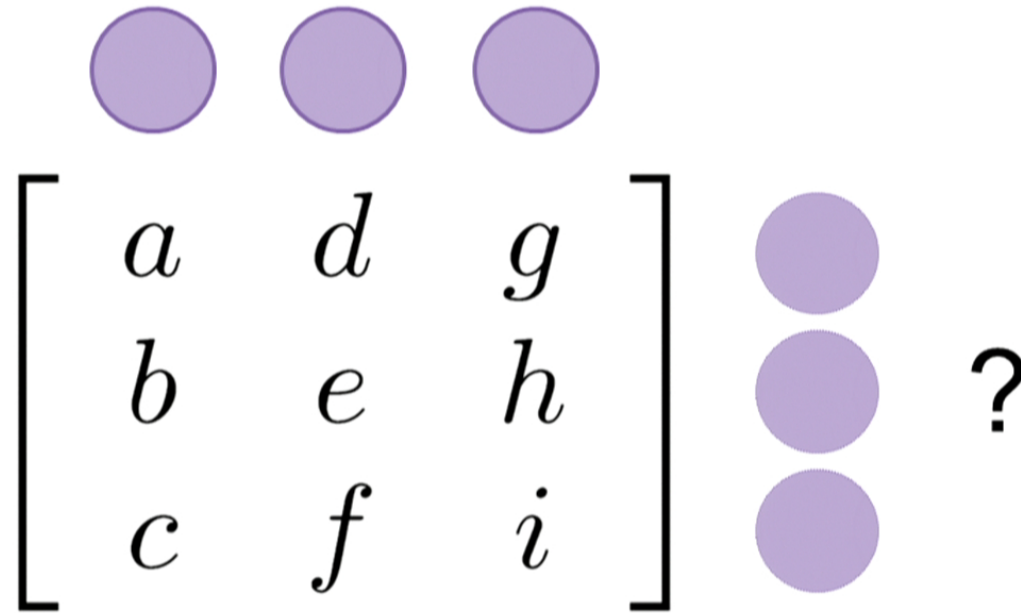
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Transition Matrix

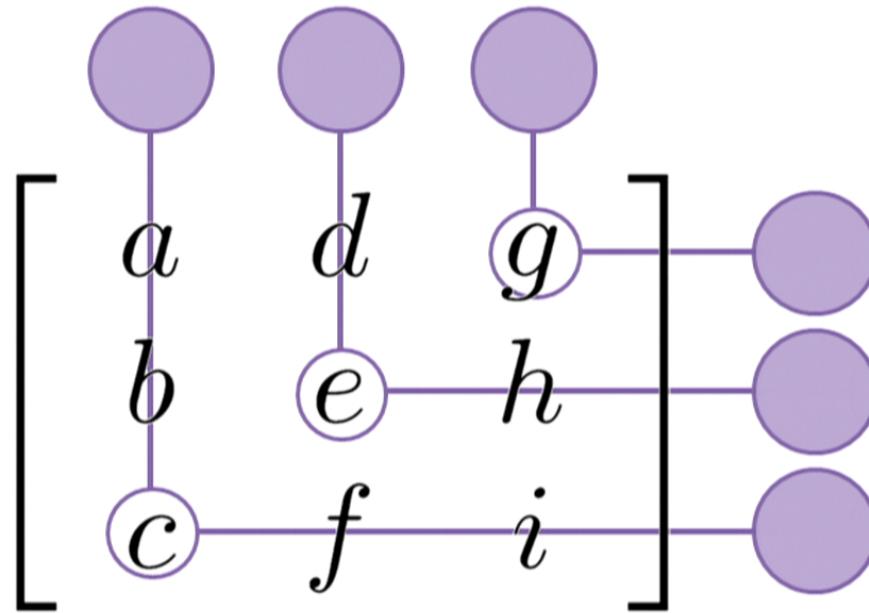


$$a, b, c \geq 0$$
$$a + b + c = 1$$

A Transition Probability

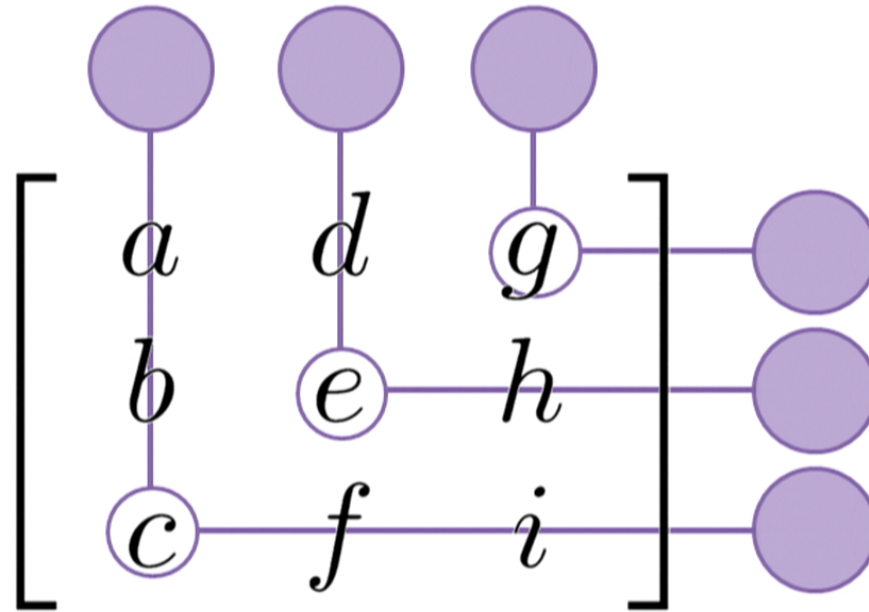


A Transition Probability



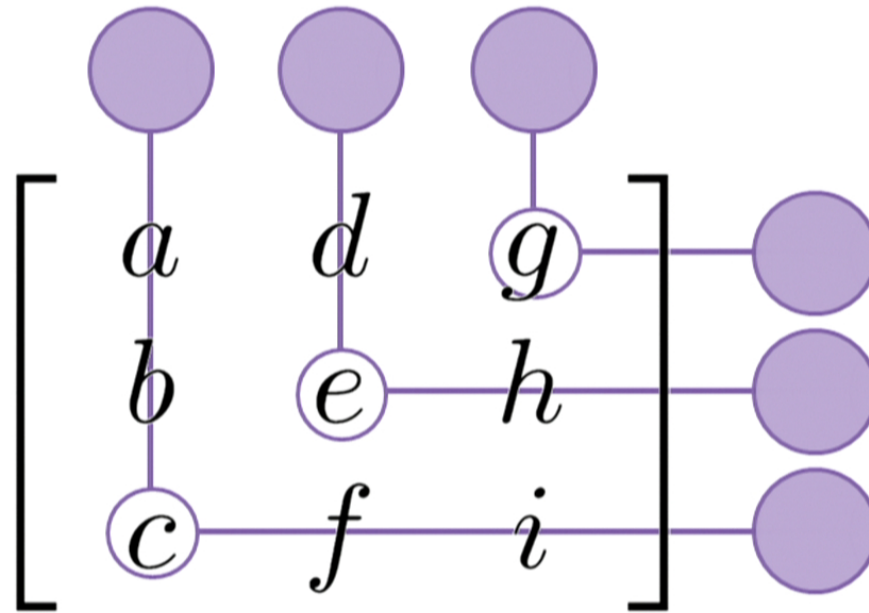
$$\Pr[\text{one per slot}] = aei + afh + bdi + bfg + cdh + ceg$$

A Transition Probability



$$\Pr[\text{one per slot}] = aei + afh + bdi + bfg + cdh + ceg$$

A Transition Probability



$$\begin{aligned} \Pr[\text{one per slot}] &= aei + afh + bdi + bfg + cdh + ceg \\ &= \text{perm}(M) \end{aligned}$$

Probabilities for Classical Analogue

$\Pr [\text{one per slot} \rightarrow \text{one per slot}] =$

Probabilities for Classical Analogue

- What about other transitions?



Probabilities for Classical Analogue

- What about other transitions?

$$\begin{array}{c} \text{●} \quad \quad \quad \text{●} \\ \left[\begin{array}{ccc} a & d & g \\ b & e & h \\ c & f & i \end{array} \right] \begin{array}{c} \text{●} \\ \text{●} \end{array} ? \end{array} \quad \text{perm} \left[\begin{array}{cc} a & g \\ b & h \end{array} \right]$$

Probabilities for Classical Analogue

- What about other transitions?

$$\begin{array}{c} \bullet \\ \left[\begin{array}{ccc} a & d & g \\ b & e & h \\ c & f & i \end{array} \right] \end{array} \begin{array}{c} \bullet \\ \bullet \\ ? \end{array} \quad \text{perm} \begin{bmatrix} a & g \\ b & h \end{bmatrix}$$

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Probabilities for Classical Analogue

- What about other transitions?

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Configuration Transitions

Transition matrix
for one ball

$$\begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ a & d & g & \bullet & \bullet \\ b & e & h & \bullet & \bullet \\ c & f & i & \bullet & \bullet \end{bmatrix}$$

m=3 slots
n=1 balls

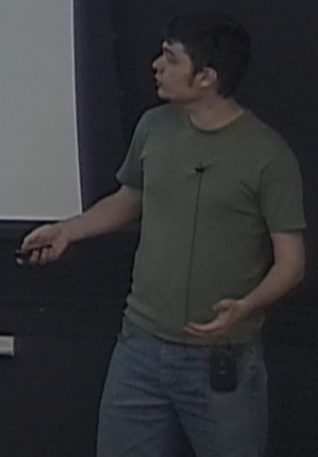
Transition matrix for two-ball configurations

$$\begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ a^2 & d^2 & g^2 & ad & ag & dg \\ b^2 & e^2 & h^2 & be & bh & eh \\ c^2 & f^2 & i^2 & cf & ci & fi \\ 2ab & 2de & 2gh & ae+bd & ah+bg & dh+eg \\ 2ac & 2df & 2gi & af+cd & ai+cg & ei+fh \\ 2bc & 2ef & 2hi & bf+ce & bi+ch & di+fg \end{bmatrix}$$

m=3 slots
n=2 balls

In set (1), both 2
In set (2), the number
Here we consider set
See Brillouin-Din
Condensing, quonormal spectra
excitations (elementary or
merged Bloch wave satisfying the
cond at side (or sometimes a
empty flat case)
by $A(\omega) = 0$ or $A(\omega, T)$

of the dual group
linearized linear PDE
normalization



Configuration Transitions

Transition matrix
for one ball

$$\begin{array}{c}
 \bullet \dots \bullet \dots \bullet \\
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m=3 slots
n=2 balls

Configuration Transitions




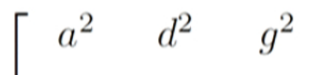

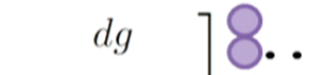
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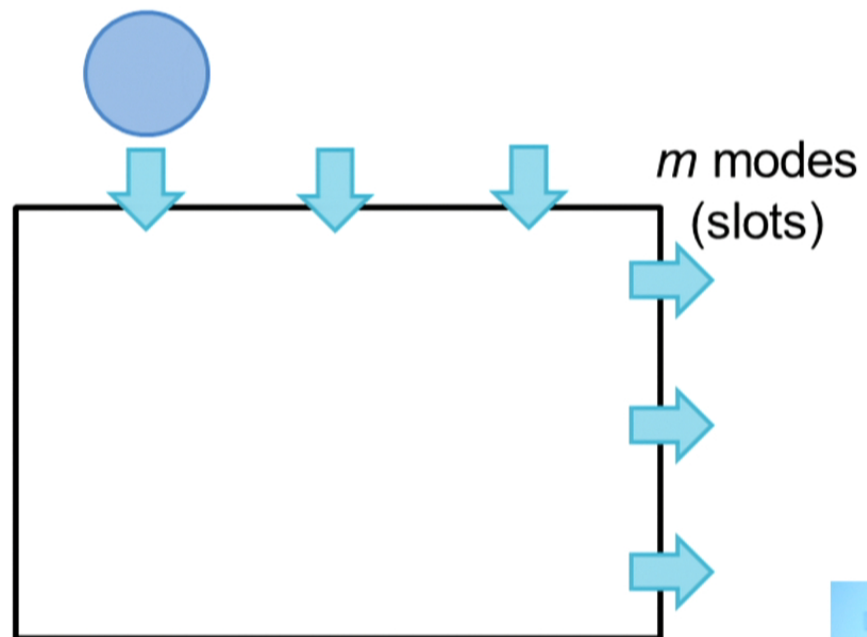
Classical Model Summary

- n identical balls
- m slots
- Choose **start configuration**
- Choose stochastic **transition matrix M**

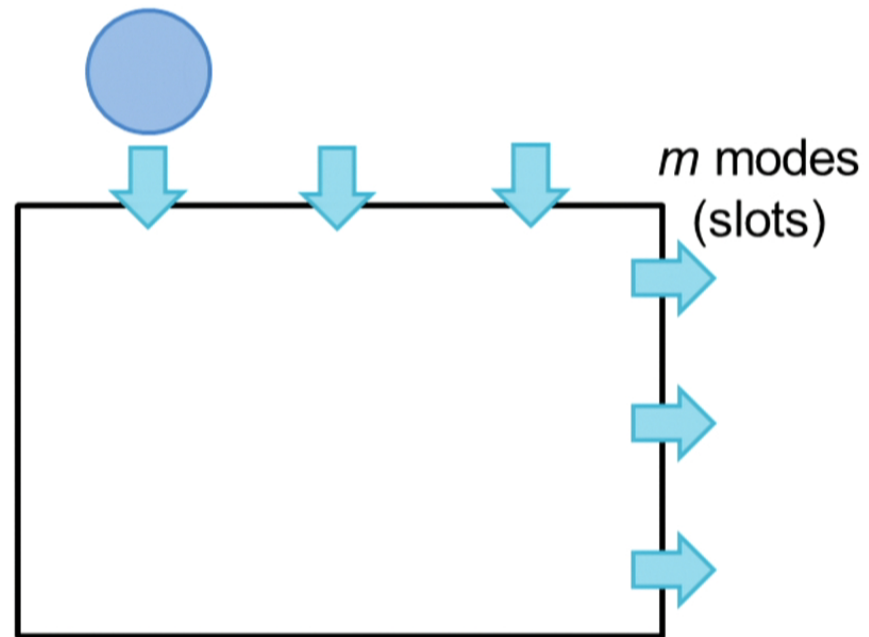
Classical Model Summary

- n identical balls
- m slots
- Choose **start configuration**
- Choose stochastic **transition matrix** M
- Move each ball as per M
- Look at resulting configuration

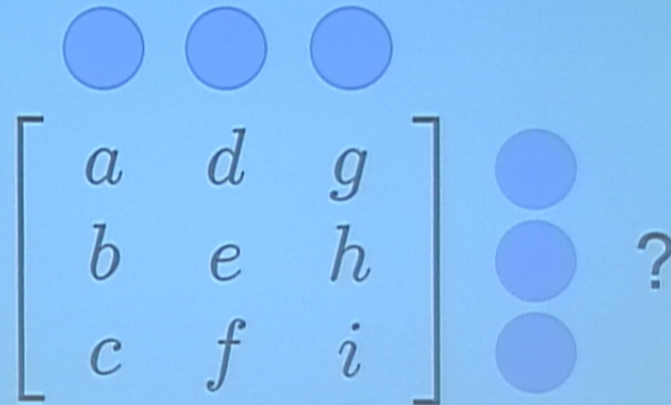
Identical Bosons



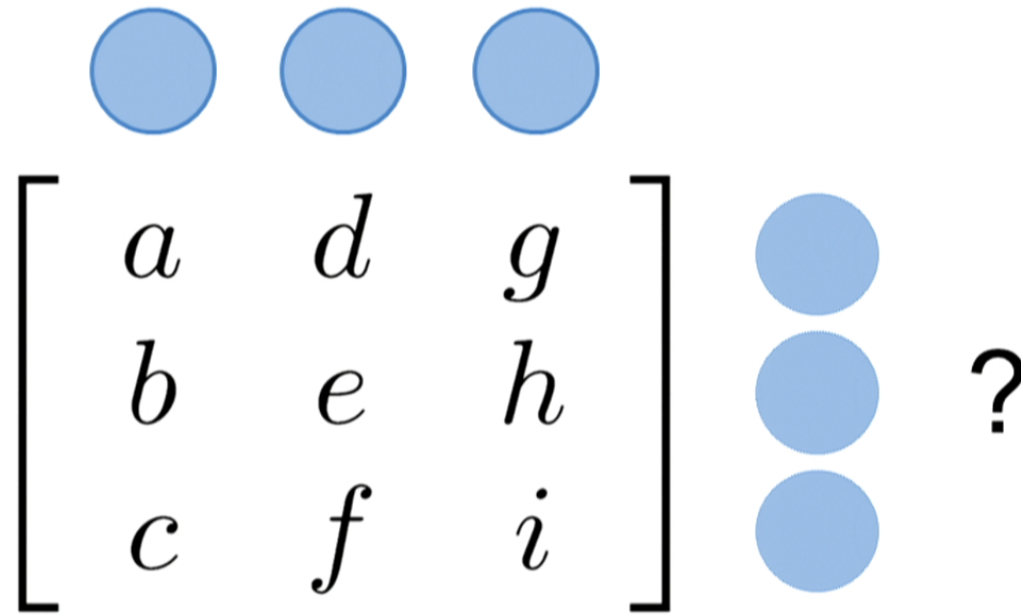
Identical Bosons



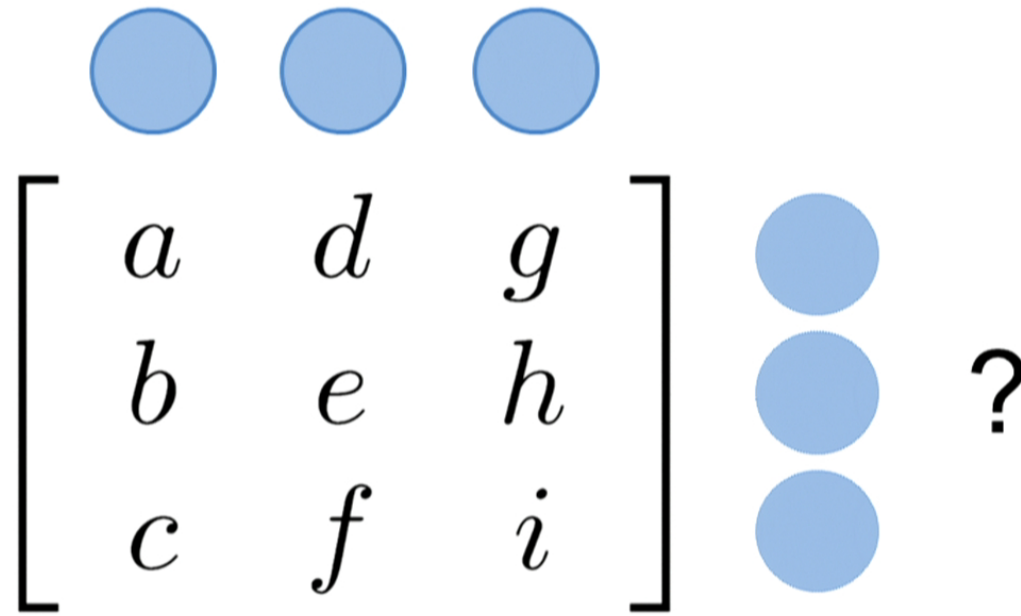
Identical Bosons



Identical Bosons

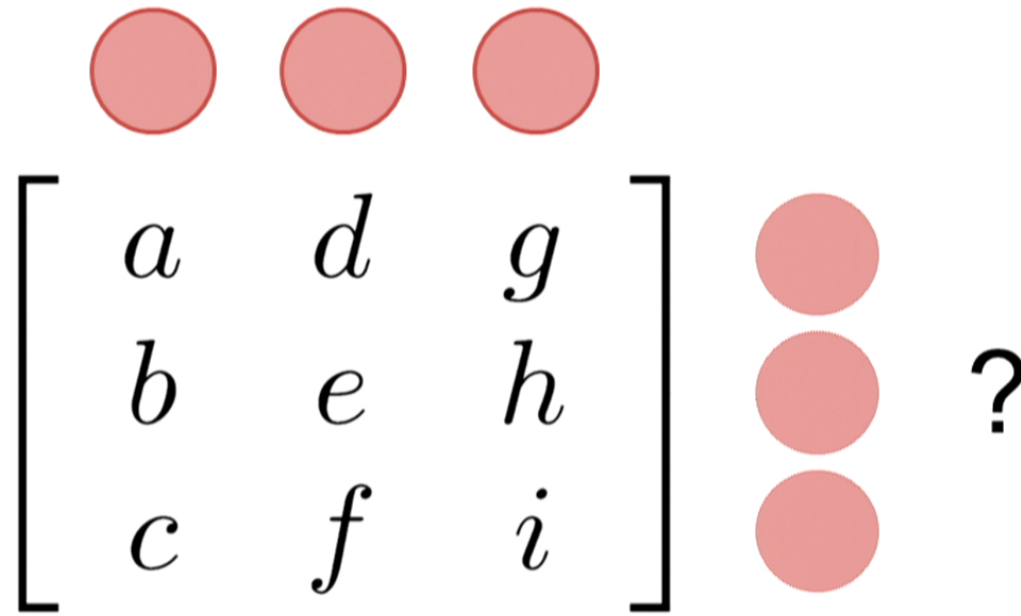


Identical Bosons



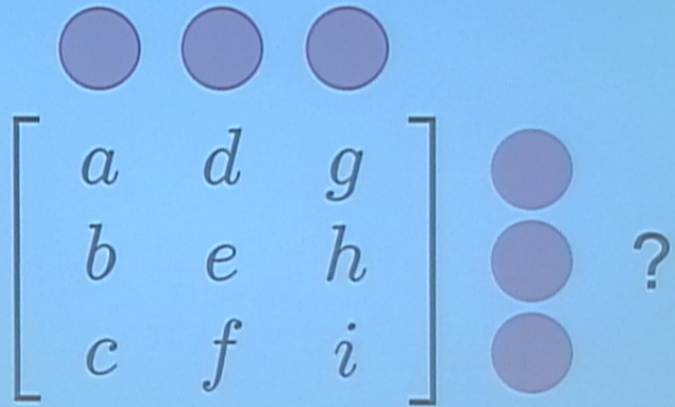
$$\begin{aligned} \text{Am [one per slot]} &= aei + afh + bdi + bfg + cdh + ceg \\ &= \text{perm}(M) \end{aligned}$$

Identical Fermions

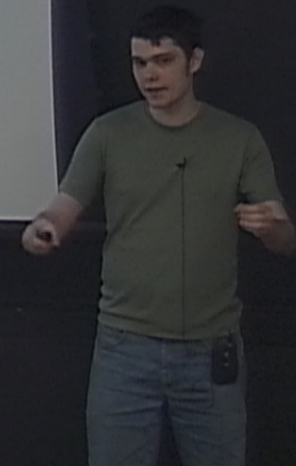


$$\text{Am [one per slot]} = aei - afh - bdi + bfg + cdh - ceg$$

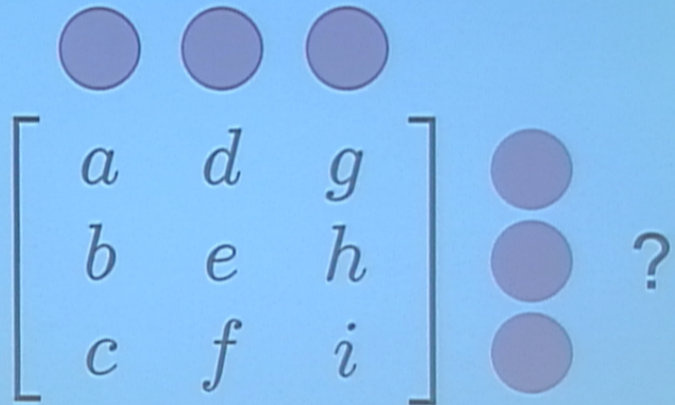
Identical Fermions



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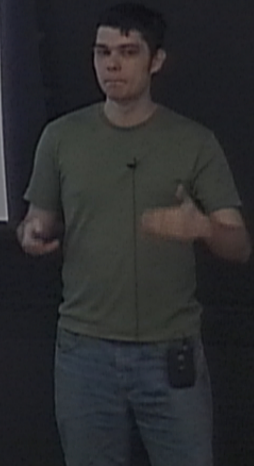


Identical Fermions



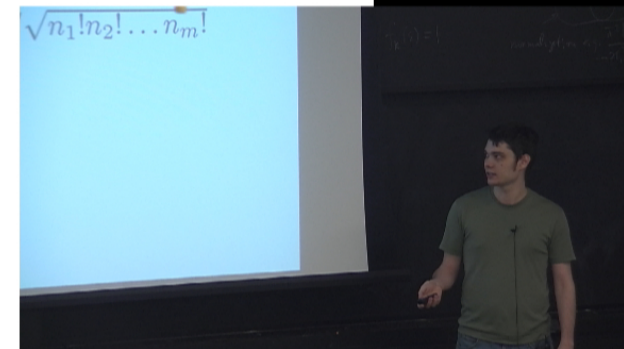
$$\text{Am [one per slot]} = aei - afh - bdi + bfg + cdh - ceg \\ = \det(M)$$

$$\text{Pr [one per slot]} = |\det(M)|^2$$



Algebraic Formalism

- Modes are single-particle basis states
 - Formal variables x_1, \dots, x_m
- Configurations are multi-particle basis states
 - Monomials $x_1^{n_1} x_2^{n_2} \dots x_m^{n_m} / \sqrt{n_1! n_2! \dots n_m!}$



Algebraic Formalism

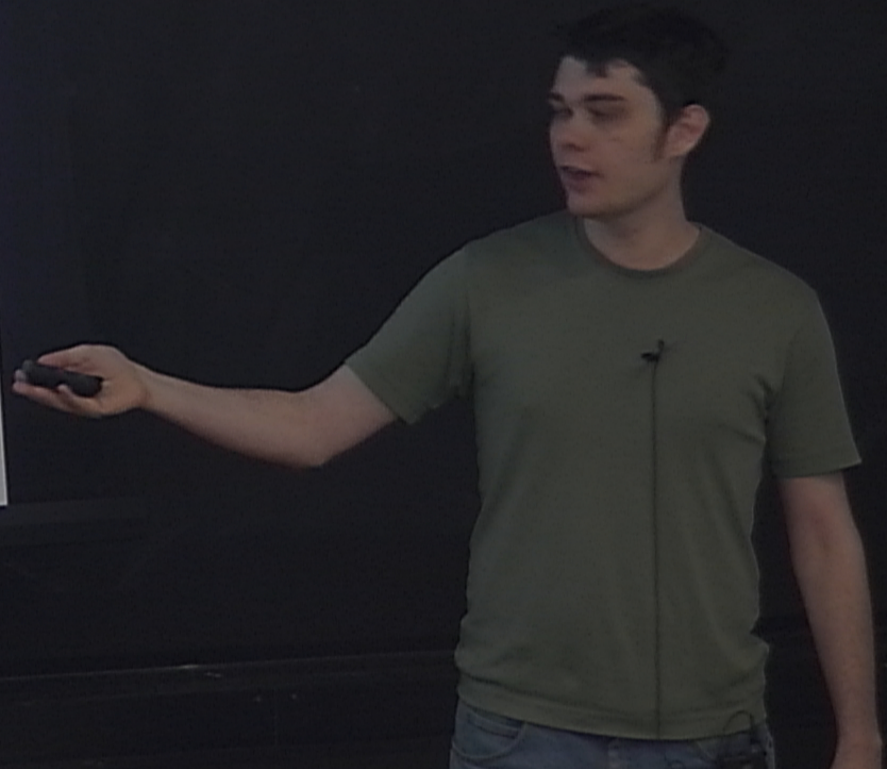
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- Identical **bosons** commute
 - $x_i x_j = x_j x_i$

$\sqrt{n_1! n_2! \dots n_m!}$

commute

$\int_k(\xi) = 1$

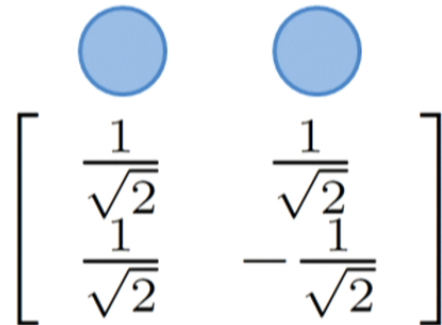
normalization eq. $\frac{1}{\sqrt{2\pi}}$



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- Modes are single-particle basis states
 - Formal variables x_1, \dots, x_m
- Configurations are multi-particle basis states
 - Monomials $x_1^{n_1} x_2^{n_2} \dots x_m^{n_m} / \sqrt{n_1! n_2! \dots n_m!}$
- Identical **bosons** commute
 - $x_i x_j = x_j x_i$
- Identical **fermions** anticommute
 - $x_i x_j = -x_j x_i$
 - $x_i^2 = 0$

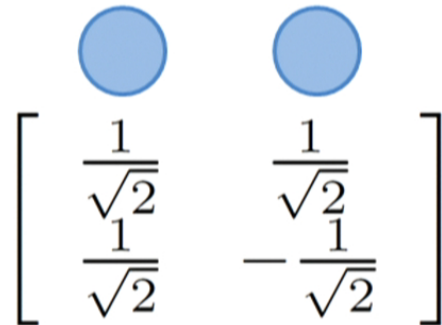
Example: Hadamarding Bosons



The diagram shows a Hadamard gate for bosons. It consists of two blue circles representing bosons at the top. Below them is a large square bracket containing a 2x2 matrix of fractions. The top-left element is $\frac{1}{\sqrt{2}}$, the top-right is $\frac{1}{\sqrt{2}}$, the bottom-left is $\frac{1}{\sqrt{2}}$, and the bottom-right is $-\frac{1}{\sqrt{2}}$.

$$\left[\begin{array}{cc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{array} \right]$$

Example: Hadamarding Bosons

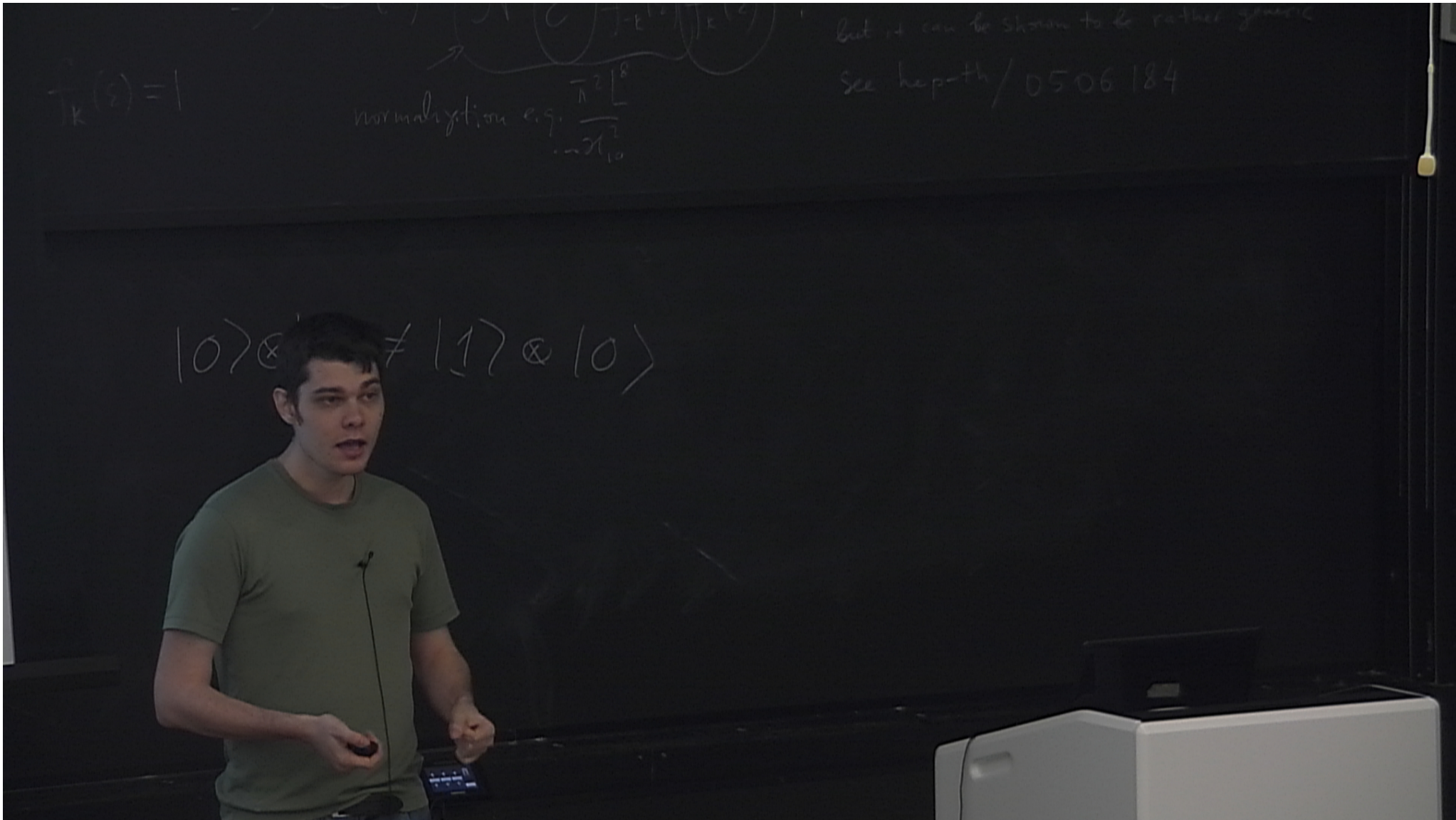

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{aligned} xy &\rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right) \left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \right) \\ &= \frac{1}{\sqrt{2}} \left[\frac{x^2}{\sqrt{2}} \right] - \frac{1}{\sqrt{2}} \left[\frac{y^2}{\sqrt{2}} \right] \end{aligned}$$

Example: Hadamarding Bosons

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad p = \frac{1}{2}$$

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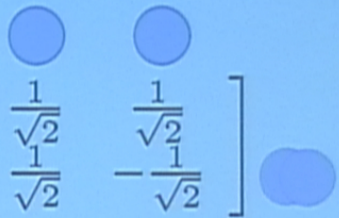
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

normalization e.g. $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

but it can be shown to be rather generic
see hep-th/0506184

$$|0\rangle \otimes |1\rangle \neq |1\rangle \otimes |0\rangle$$

Example: Hadamarding Bosons



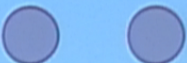
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} p = \frac{1}{2}$$

$$xy \rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right) \left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \right)$$

$$= \frac{1}{\sqrt{2}} \left[\frac{x^2}{\sqrt{2}} \right] - \frac{1}{\sqrt{2}} \left[\frac{y^2}{\sqrt{2}} \right]$$

Hong-Ou-Mandel dip

Example: Hadamarding Fermions

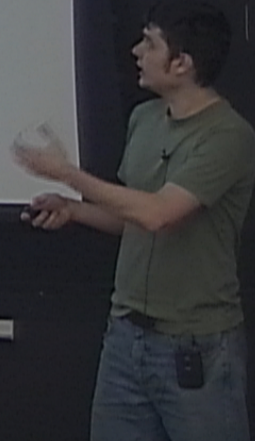

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{aligned} xy &\rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \right) \left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \right) \\ &= \frac{1}{2}x^2 - \frac{1}{2}xy + \frac{1}{2}yx + \frac{1}{2}y^2 \\ &= -xy \end{aligned}$$

As a Computational Problem

- **BosonSampling or FermionSampling**
 - **Input:** Matrix M and initial configuration

In set $(\psi_1, \psi_2, \dots, \psi_n)$
In set $(\psi_1, \psi_2, \dots, \psi_n)$
Here we consider set
see Brillouin-Dawson
Conclusions: quonormal spectra
excitations (elementary or
merged $\psi_1, \psi_2, \dots, \psi_n$ satisfying the
cond. at ψ_1 (or sometimes a
empt. flat case)
by $A(\psi) = 0$ or $A(\psi, F)$



$|0\rangle\langle 0| \neq |1\rangle\langle 1|$

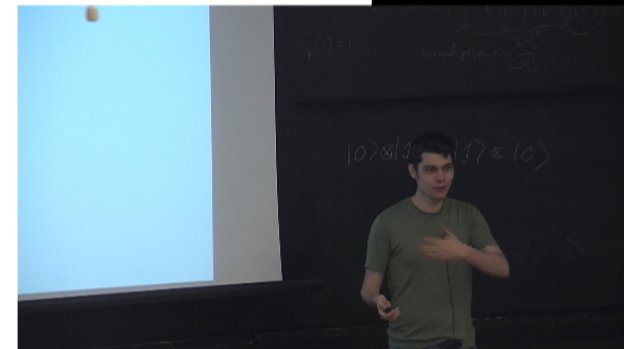
Complexity Comparison

Particle:	Fermion		
Function:	Det		
Matrix:	Unitary		
Compute probability:	In P		
Sample:			



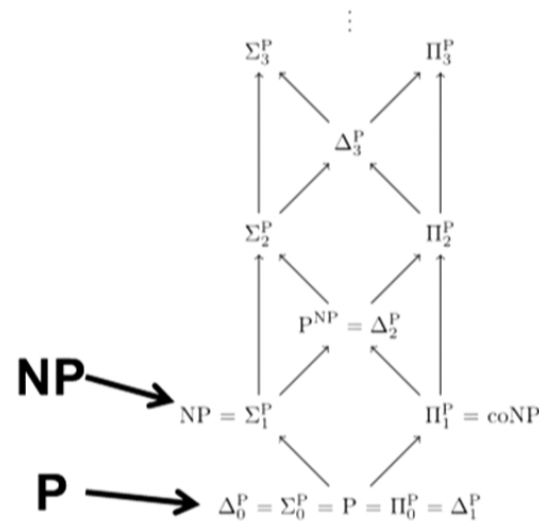
Complexity Comparison

Particle:	Fermion	Classical
Function:	Det	Perm
Matrix:	Unitary	Stochastic
Compute probability:	In P	Approximable [JSV '01]
Sample:	In BPP [Valiant '01]	In BPP



Exact Result [AA' 11]

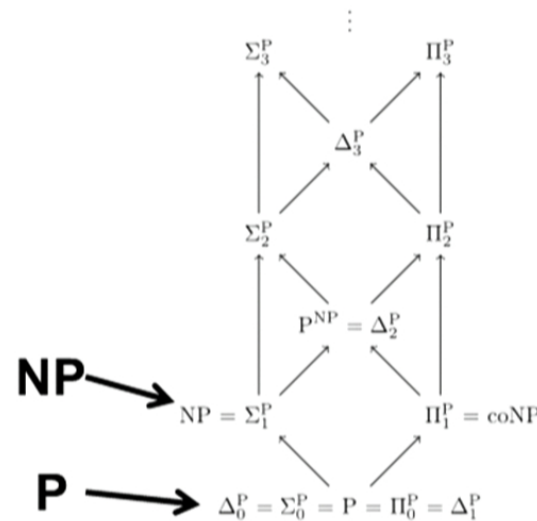
Theorem: *Identical **boson** sampling cannot be done in **BPP** unless **PH** collapses*



PH not collapsing
generalizes **P** ≠ **NP**

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Theorem: *Identical boson sampling cannot be done in **BPP** unless **PH** collapses*



PH not collapsing generalizes **P ≠ NP**



Consequences

- A quantum experiment that **can't be efficiently classically simulated**
- QC can solve a hard sampling problem using:
 - Noninteracting identical **bosons**



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- Strong evidence of quantum advantage



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What's Been Built

In set (\mathbb{C}_\pm) , both ψ_+ and ψ_- are \mathbb{R}^2 valued.
In set (\mathbb{C}_-) , the same.
Here we consider set (\mathbb{C}_+) .
See Brillouin-Dinvar.
Conclusion: quasi-moment spectra
= excitations (elementary or
emergent \mathbb{R}^2 spin) satisfying the
cond. at ω_k (or sometimes a
empt. flat case)
 $\psi_+(\omega) = 0 \Rightarrow A(\omega, \mathbb{F})$

$|0\rangle\langle 0| \neq |1\rangle\langle 1|$

What's Been Built

- Four independent implementations

Group	Location	Number of photons	Number of output modes
M.A. Broome et al	Brisbane	3	6
J.B. Spring et al	Oxford	3	6
M. Tillmann et al	Vienna	3	5
A. Crespi et al	Rome	3	9

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- Solve *decision* problem

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- Solve *decision* problem
- Solve *useful* problem
- Show how to prove your QC works



Open Problems for BosonSampling

- Solve *decision* problem
- Solve *useful* problem
- Show how to prove your QC works
- Build scalable linear optics computer



uter

$$\int_k(\xi) = 1$$

normalization e.g. $\frac{1}{\sqrt{2}}$

$$|0\rangle \otimes |1\rangle \neq |1\rangle \otimes |0\rangle$$