Title: Quantum Computing with Noninteracting Particles

Date: Apr 02, 2014 04:00 PM

URL: http://pirsa.org/14040077

Abstract: We introduce an abstract model of computation corresponding to an experiment in which identical, non-interacting bosons are sent through a non-adaptive linear circuit before being measured. We show that despite the very limited nature of the model, an exact classical simulation would imply a collapse of the polynomial hierarchy. Moreover, under plausible conjectures, a "noisy" approximate simulation would do the same. This gives evidence that quantum computers can sample a distribution that classical computers cannot even approximate, even when restricted to use no entanglement except that arising from particles being identical. We briefly discuss experimental prospects for realizing this model. This talk is based on The Computational Complexity of Linear Optics [STOC '11], which is joint work with Scott Aaronson.

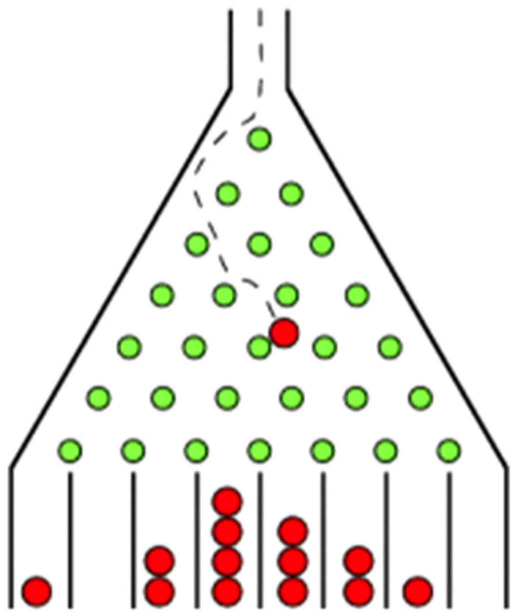
Noninteracting Particle Model

- Weak model of QC
  - Restricted entanglement and coupling
  - Likely not universal
- Why do we care?
  - Quantum gains with less
  - Already beyond classical

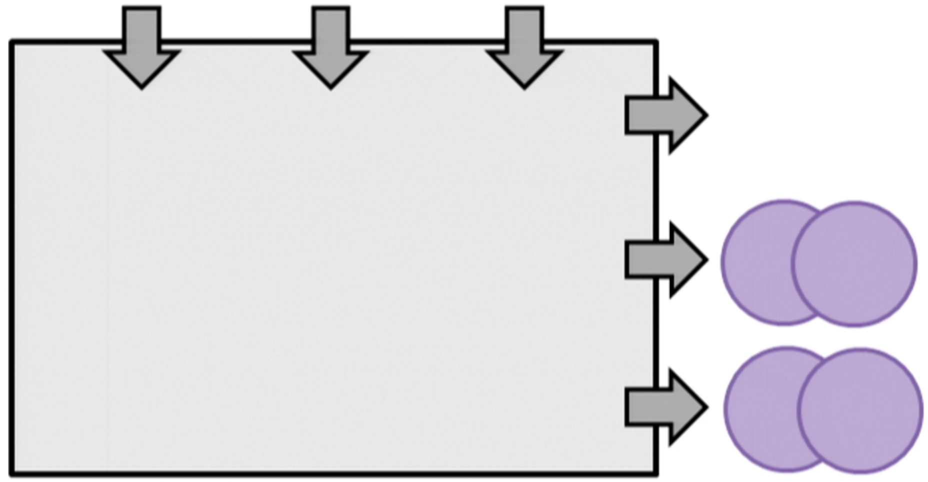
Noninteracting Particle Model

- Weak model of QC
  - Restricted entanglement and coupling
  - Likely not universal
- Why do we care?
  - Quantum gains with less
  - Already beyond classical
  - Easier to build

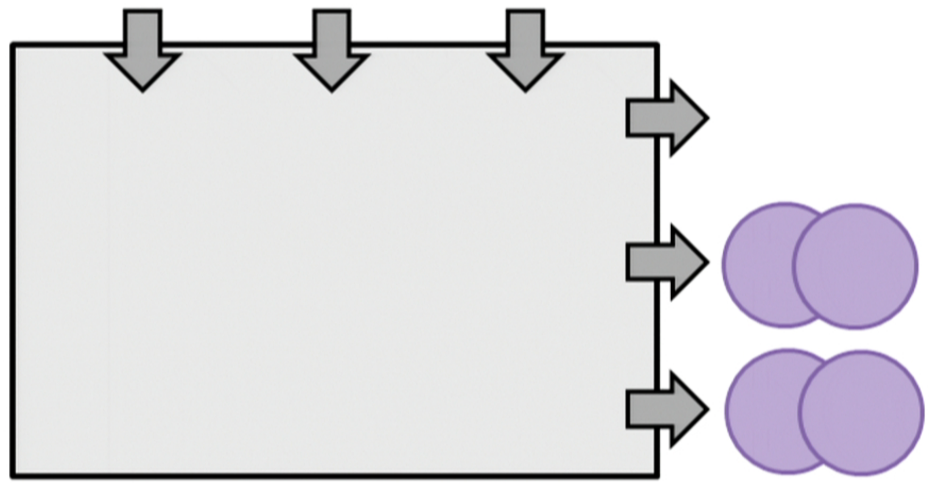
Balls and Slots



Balls and Slots



Balls and Slots



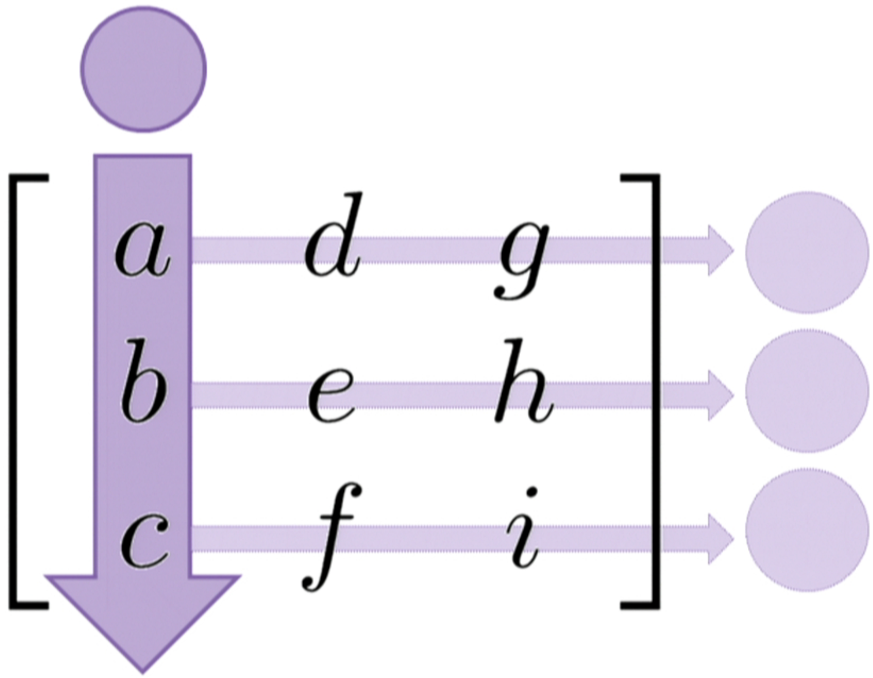
Transition Matrix

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Transition Matrix

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Transition Matrix



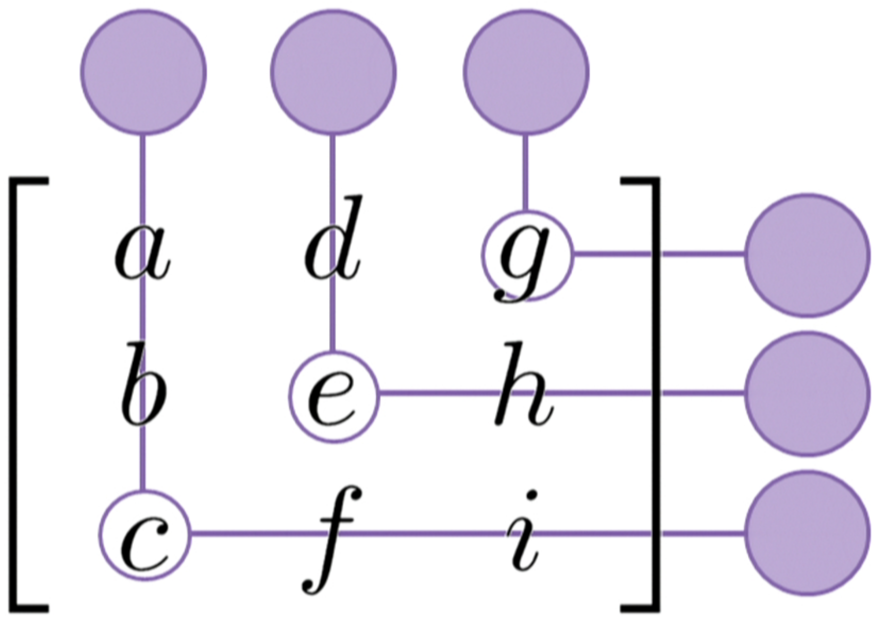
$$a, b, c \geq 0$$
$$a + b + c = 1$$

A Transition Probability

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

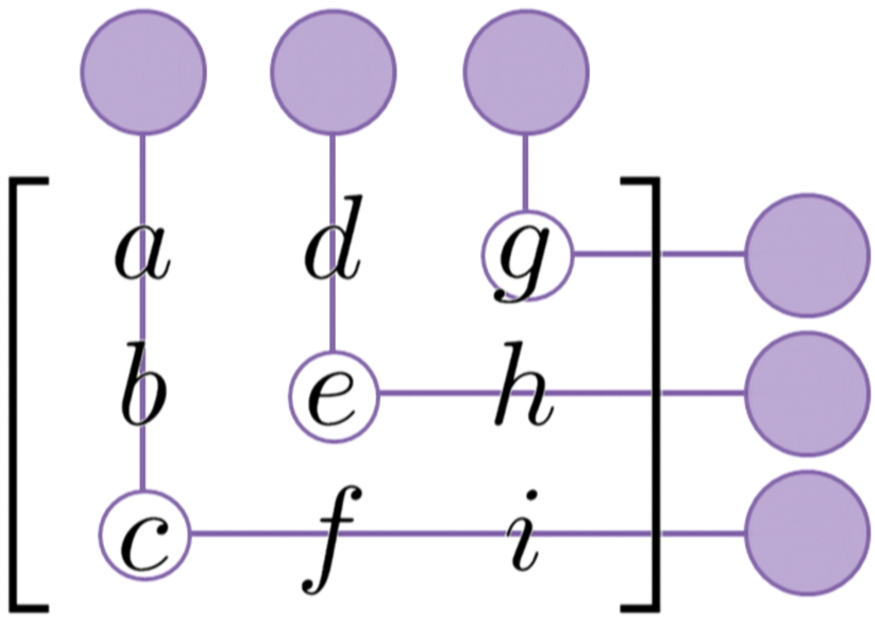
?

A Transition Probability



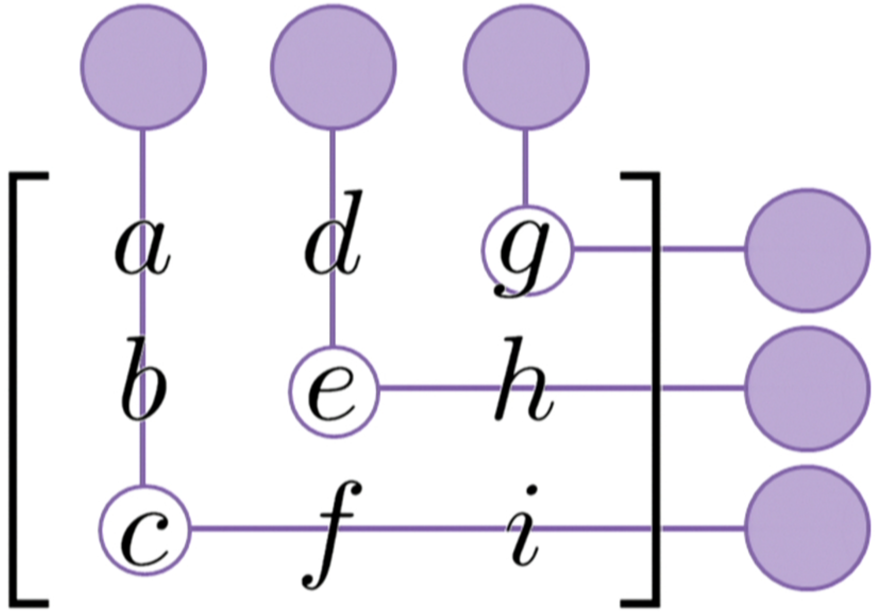
$$\Pr\left[\text{one per slot}\right] = aei + afh + bdi + bfg + cdh + ceg$$

A Transition Probability



$$\Pr\left[\text{one per slot}\right] = aei + afh + bdi + bfg + cdh + ceg$$

A Transition Probability



$$\Pr[\text{one per slot}] = aei + afh + bdi + bfg + cdh + ceg$$
$$= \text{perm}(M)$$

Probabilities for Classical Analogue

$$\Pr\left[\text{one per slot} \to \text{one per slot}\right] =$$

Probabilities for Classical Analogue

- What about other transitions?

Probabilities for Classical Analogue

- What about other transitions?

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \quad ? \qquad \text{perm}\begin{bmatrix} a & g \\ b & h \end{bmatrix}$$

Probabilities for Classical Analogue

- What about other transitions?

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} ? \qquad \text{perm}\begin{bmatrix} a & g \\ b & h \end{bmatrix}$$

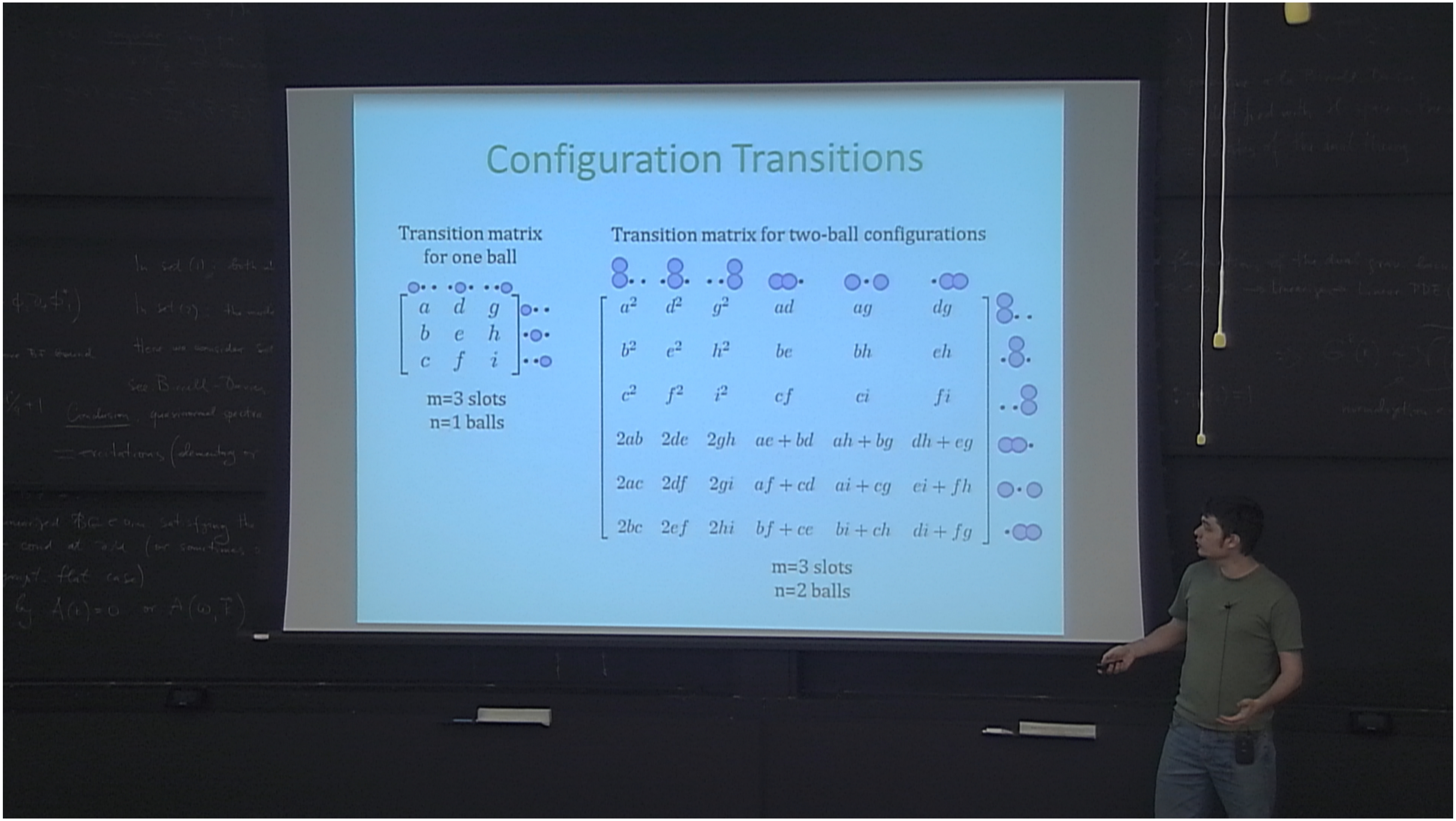
$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} ? \qquad \frac{1}{2}\text{perm}\begin{bmatrix} a & a & g \\ b & b & h \\ b & b & h \end{bmatrix}$$

Probabilities for Classical Analogue

- What about other transitions?

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \; ? \qquad \text{perm}\begin{bmatrix} a & g \\ b & h \end{bmatrix}$$

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \; ? \qquad \frac{1}{2}\text{perm}\begin{bmatrix} a & a & g \\ b & b & h \\ b & b & h \end{bmatrix}$$


Configuration Transitions
Transition matrix for one ball
a d g
b e h
c f i
m=3 slots
n=1 balls
Transition matrix for two-ball configurations
a² d² g² ad ag dg
b² e² h² be bh eh
c² f² i² cf ci fi
2ab 2de 2gh ae + bd ah + bg dh + eg
2ac 2df 2gi af + cd ai + cg ei + fh
2bc 2ef 2hi bf + ce bi + ch di + fg
m=3 slots
n=2 balls

Configuration Transitions

Transition matrix
for one ball

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

m=3 slots
n=1 balls

Transition matrix for two-ball configurations

$$\begin{bmatrix} a^2 & d^2 & g^2 & ad & ag & dg \\ b^2 & e^2 & h^2 & be & bh & eh \\ c^2 & f^2 & i^2 & cf & ci & fi \\ 2ab & 2de & 2gh & ae+bd & ah+bg & dh+eg \\ 2ac & 2df & 2gi & af+cd & ai+cg & ei+fh \\ 2bc & 2ef & 2hi & bf+ce & bi+ch & di+fg \end{bmatrix}$$

m=3 slots
n=2 balls

Configuration Transitions

Transition matrix for one ball

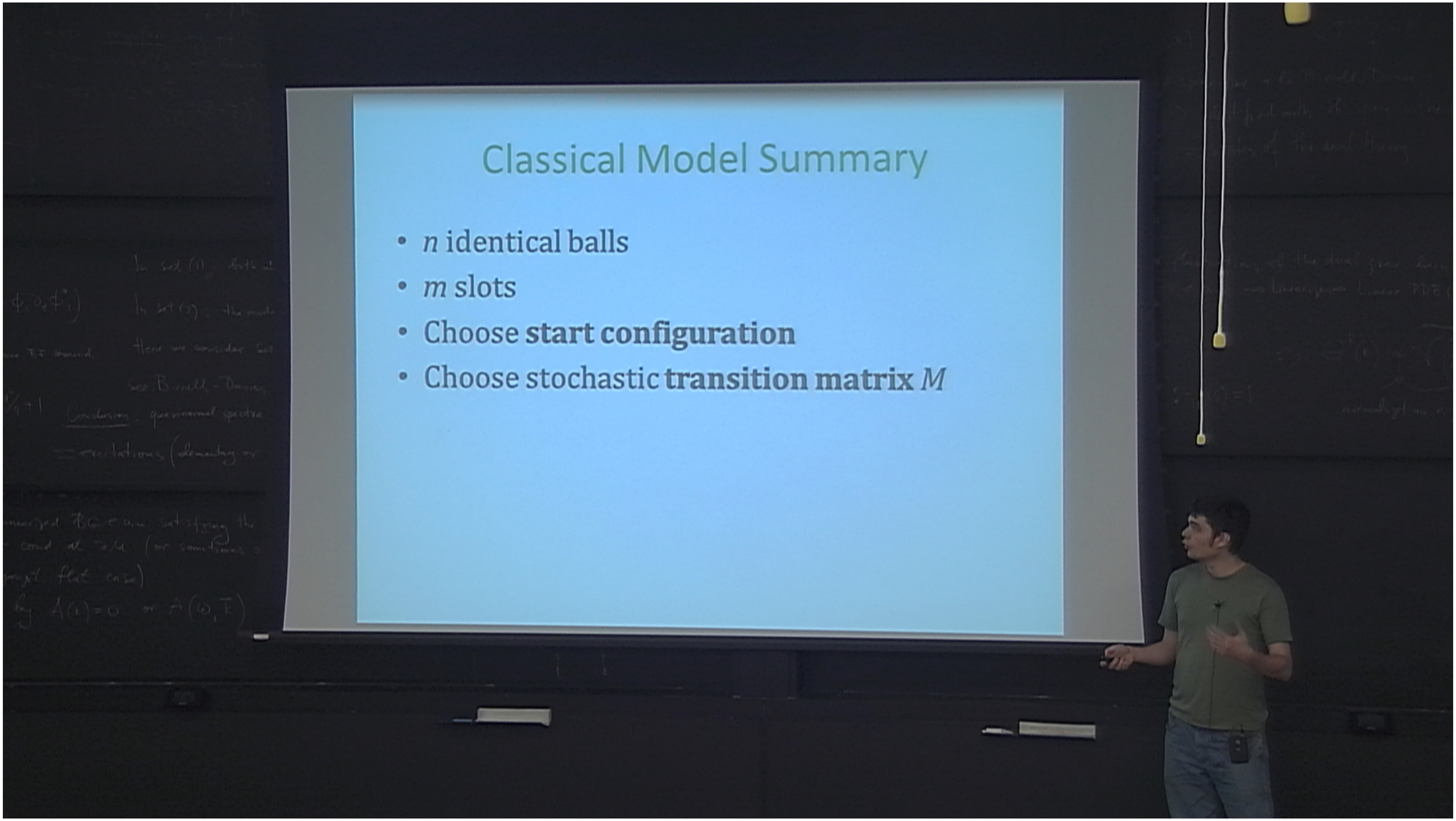
$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

m=3 slots
n=1 balls

Transition matrix for two-ball configurations

$$\begin{bmatrix} a^2 & d^2 & g^2 & ad & ag & dg \\ b^2 & e^2 & h^2 & be & bh & eh \\ c^2 & f^2 & i^2 & cf & ci & fi \\ 2ab & 2de & 2gh & ae+bd & ah+bg & dh+eg \\ 2ac & 2df & 2gi & af+cd & ai+cg & ei+fh \\ 2bc & 2ef & 2hi & bf+ce & bi+ch & di+fg \end{bmatrix}$$

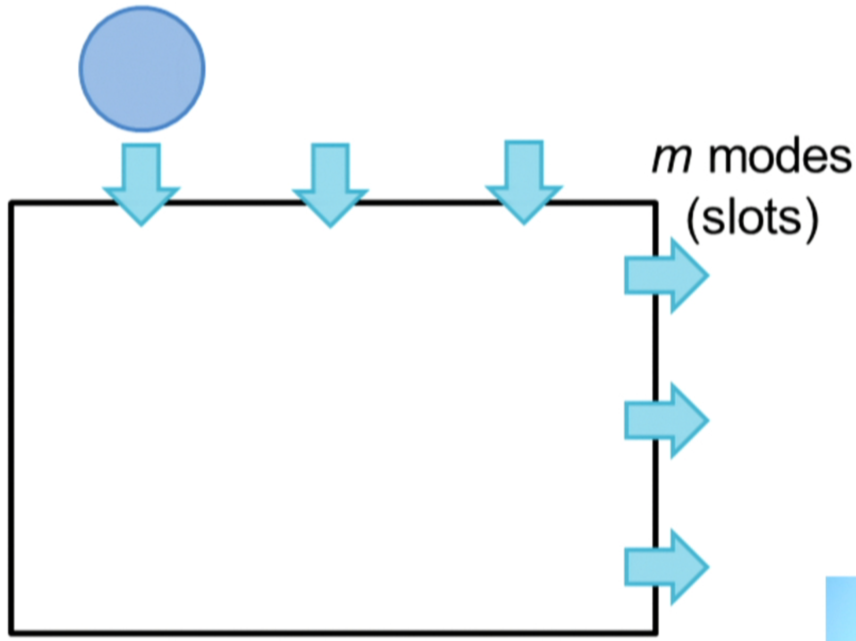
m=3 slots
n=2 balls


Classical Model Summary
• n identical balls
• m slots
• Choose start configuration
• Choose stochastic transition matrix M

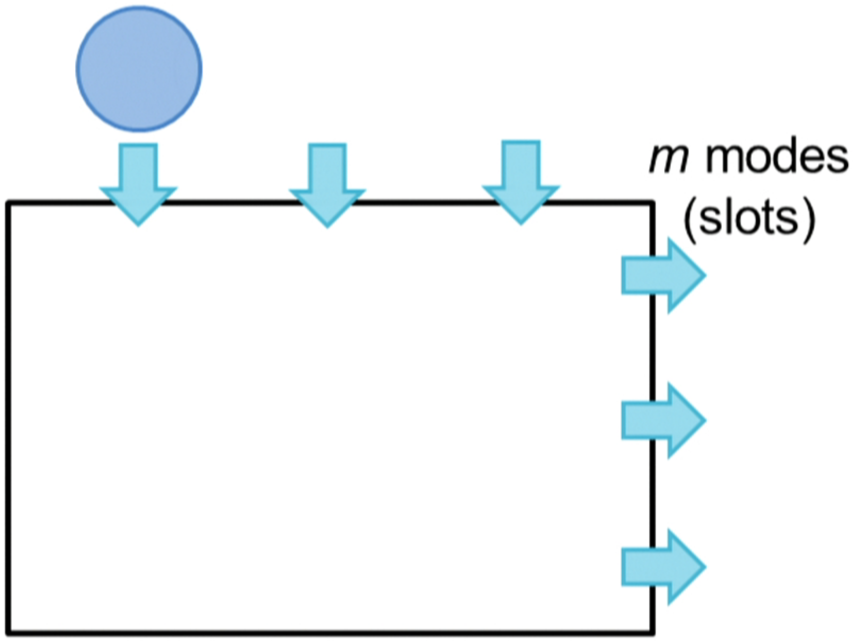
Classical Model Summary

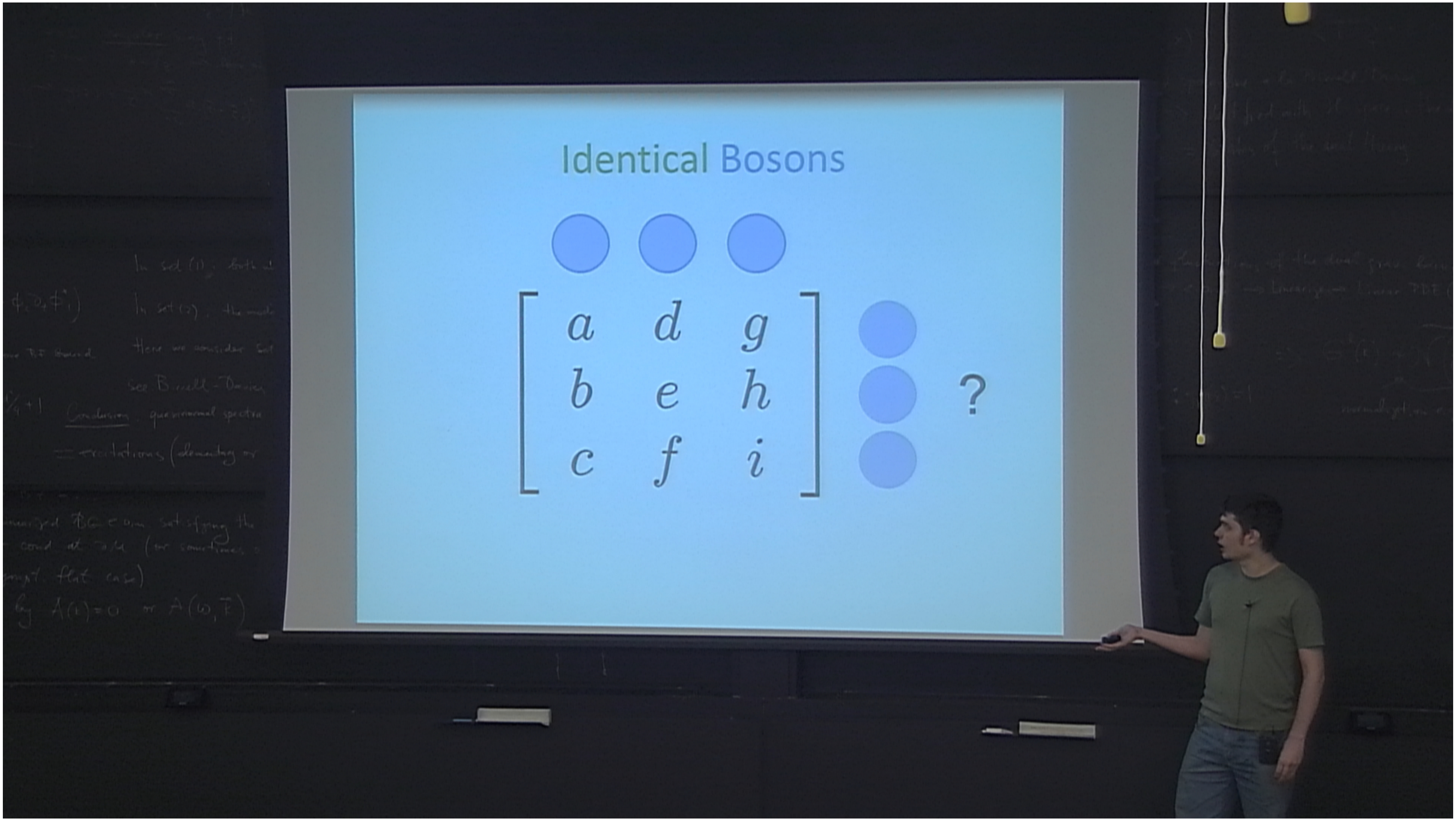
- n identical balls
- m slots
- Choose **start configuration**
- Choose stochastic **transition matrix** M
- Move each ball as per M
- Look at resulting configuration

Identical Bosons



Identical Bosons




Identical Bosons
a d g
b e h
c f i
?

Identical Bosons

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

?

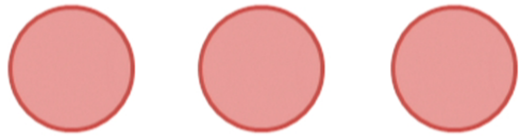
Identical Bosons

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

?

$$\begin{aligned} \text{Am}\,[\text{one per slot}] &= aei + afh + bdi + bfg + cdh + ceg \\ &= \text{perm}\,(M) \end{aligned}$$

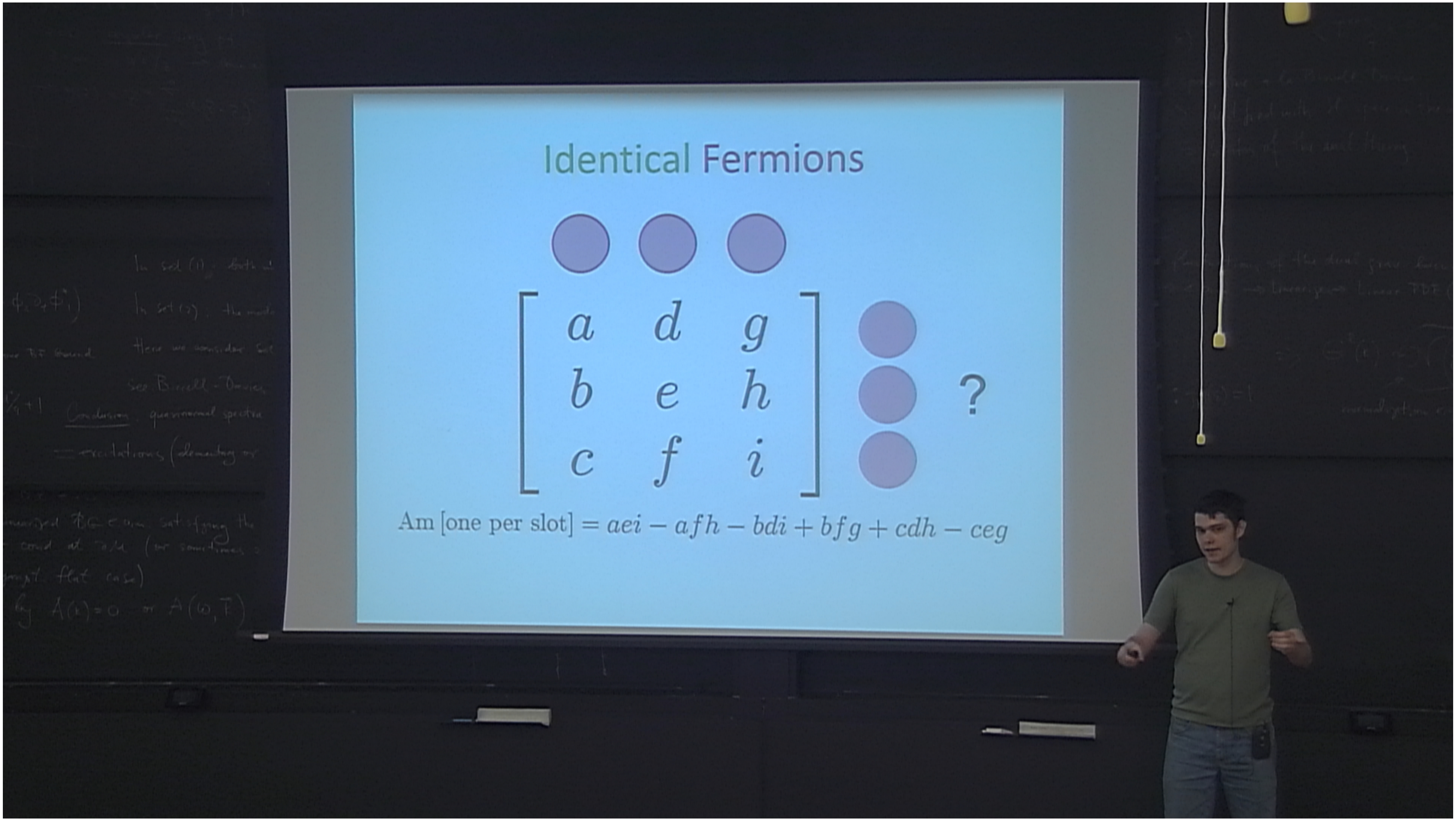
Identical Fermions

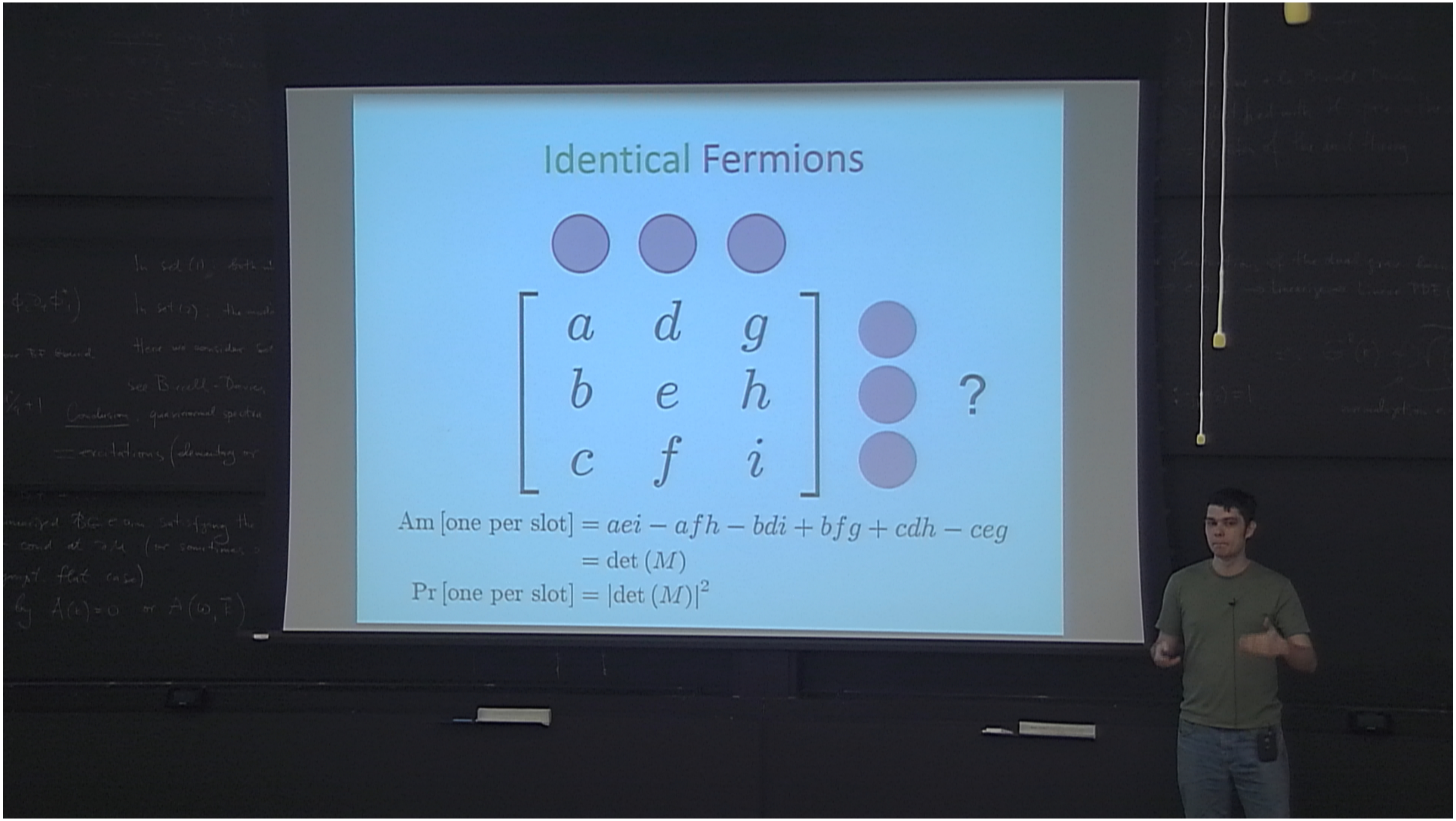


$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

?

$$\text{Am [one per slot]} = aei - afh - bdi + bfg + cdh - ceg$$

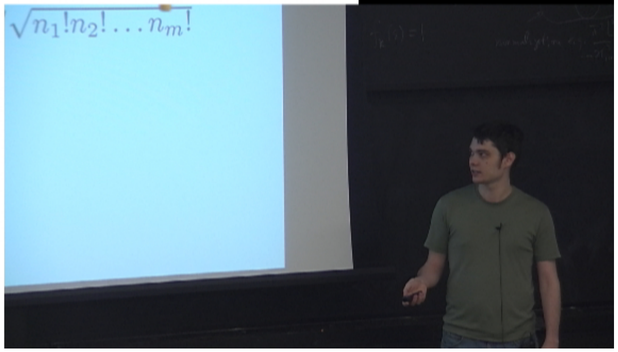

Identical Fermions
$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$
?
Am [one per slot] = $aei - afh - bdi + bfg + cdh - ceg$



Identical Fermions
a d g
b e h
c f i
?
$\mathrm{Am}\,[\text{one per slot}] = aei - afh - bdi + bfg + cdh - ceg$
$= \det(M)$
$\Pr\,[\text{one per slot}] = |\det(M)|^2$

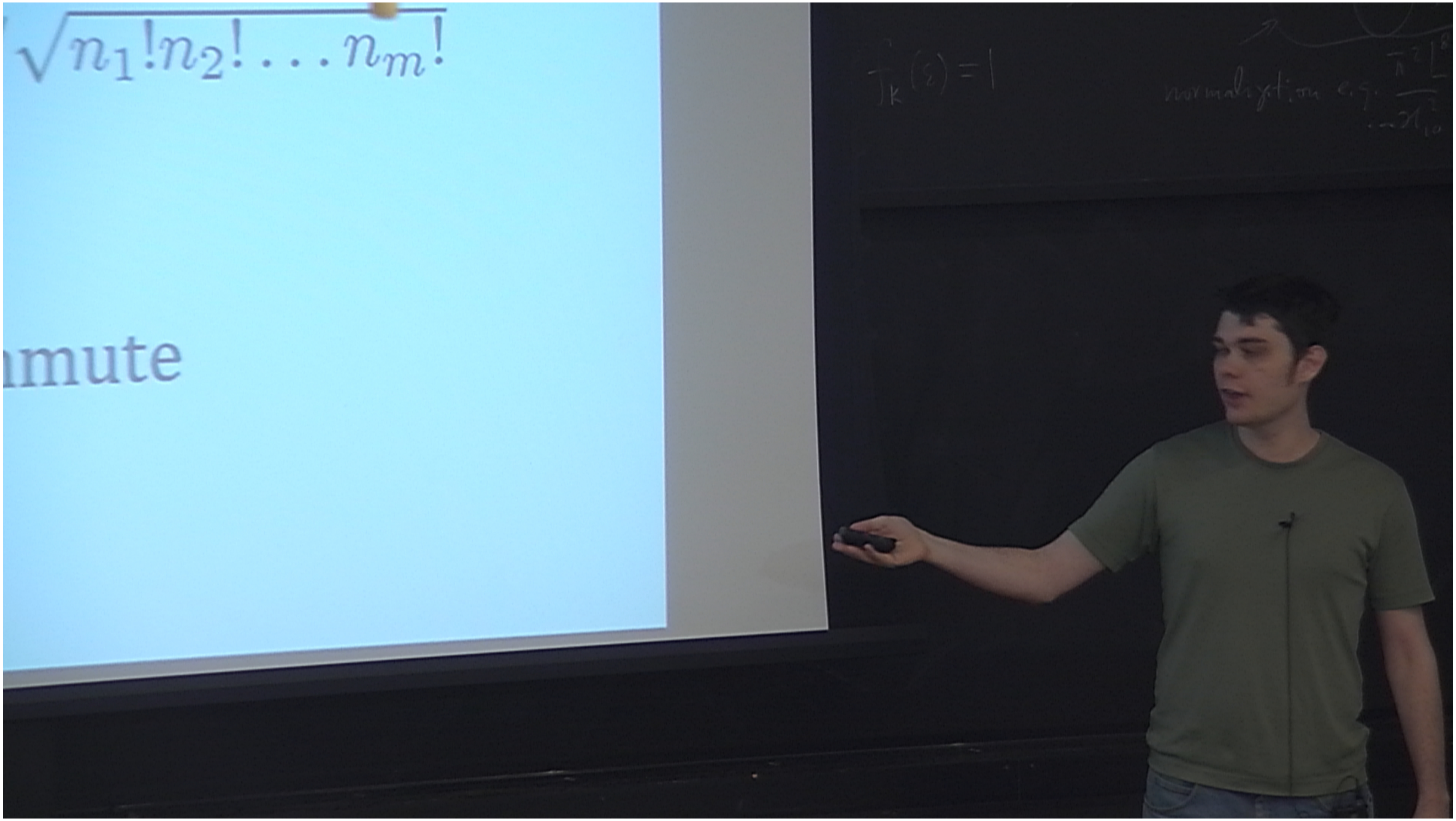
Algebraic Formalism

- Modes are single-particle basis states
  - Formal variables $x_1, \ldots, x_m$
- Configurations are multi-particle basis states
  - Monomials $x_1^{n_1} x_2^{n_2} \ldots x_m^{n_m} / \sqrt{n_1! n_2! \ldots n_m!}$



Algebraic Formalism

- Modes are single-particle basis states
  - Formal variables $x_1, \ldots, x_m$
- Configurations are multi-particle basis states
  - Monomials $x_1^{n_1} x_2^{n_2} \ldots x_m^{n_m} / \sqrt{n_1! n_2! \ldots n_m!}$
- Identical bosons commute
  - $x_i x_j = x_j x_i$


$\sqrt{n_1!n_2!\ldots n_m!}$
mute

Algebraic Formalism

- Modes are single-particle basis states
  - Formal variables $x_1, \ldots, x_m$
- Configurations are multi-particle basis states
  - Monomials $x_1^{n_1} x_2^{n_2} \ldots x_m^{n_m} / \sqrt{n_1! n_2! \ldots n_m!}$
- Identical bosons commute
  - $x_i x_j = x_j x_i$
- Identical fermions anticommute
  - $x_i x_j = -x_j x_i$
  - $x_i^2 = 0$

Example: Hadamarding Bosons

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Example: Hadamarding Bosons

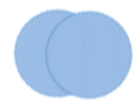
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$xy \rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\left[\frac{x^2}{\sqrt{2}}\right] - \frac{1}{\sqrt{2}}\left[\frac{y^2}{\sqrt{2}}\right]$$

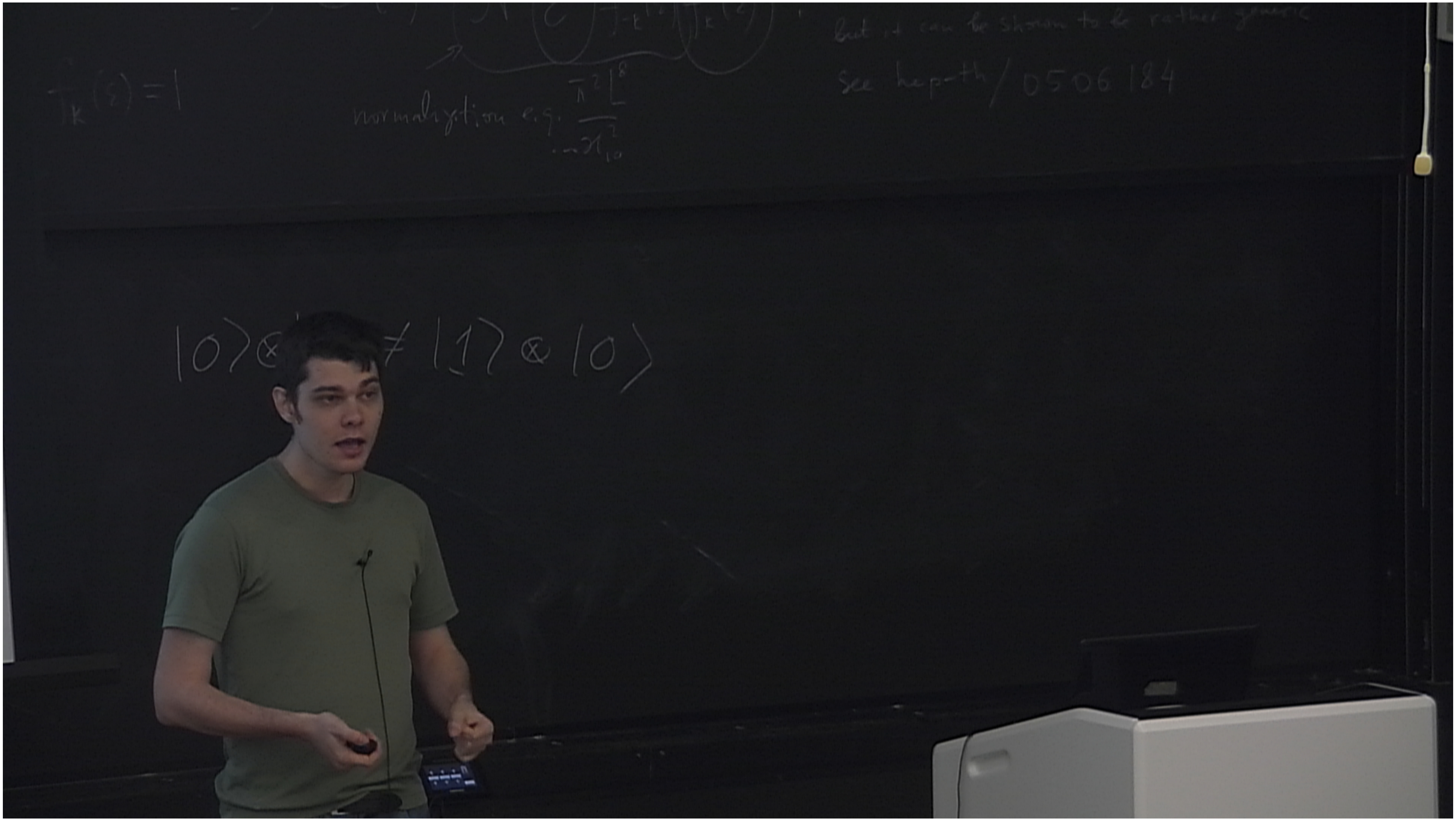
Example: Hadamarding Bosons

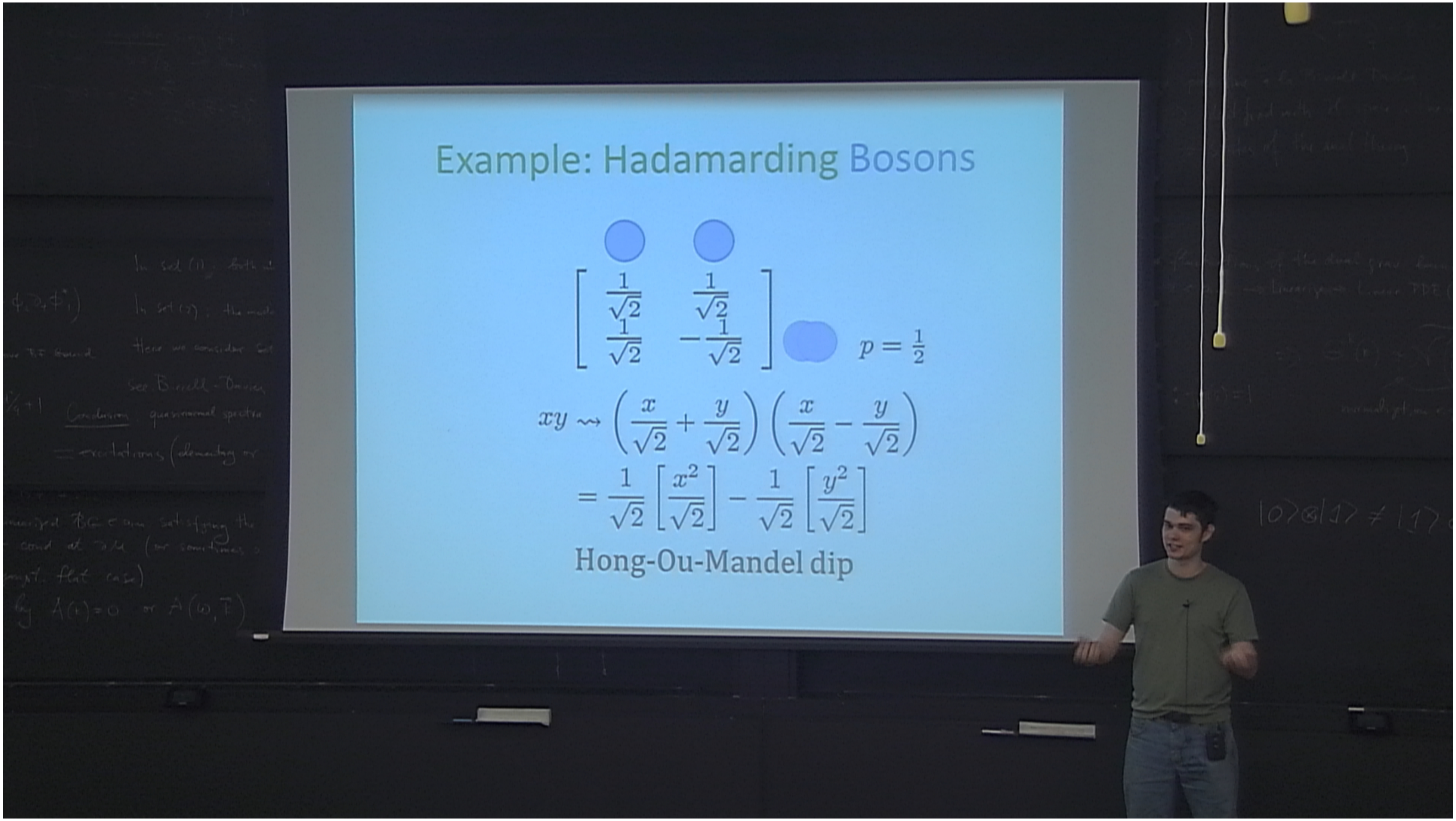
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

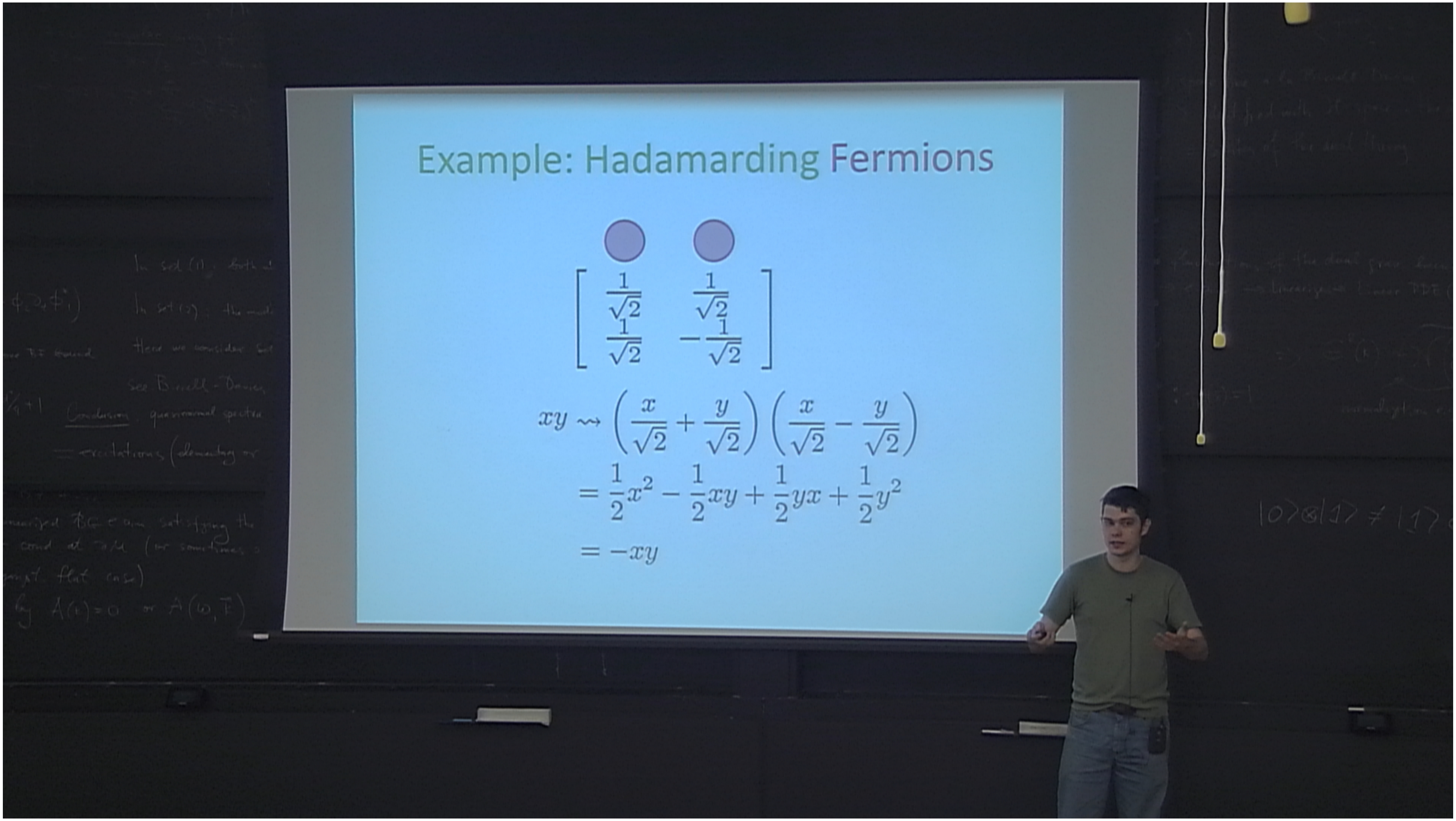


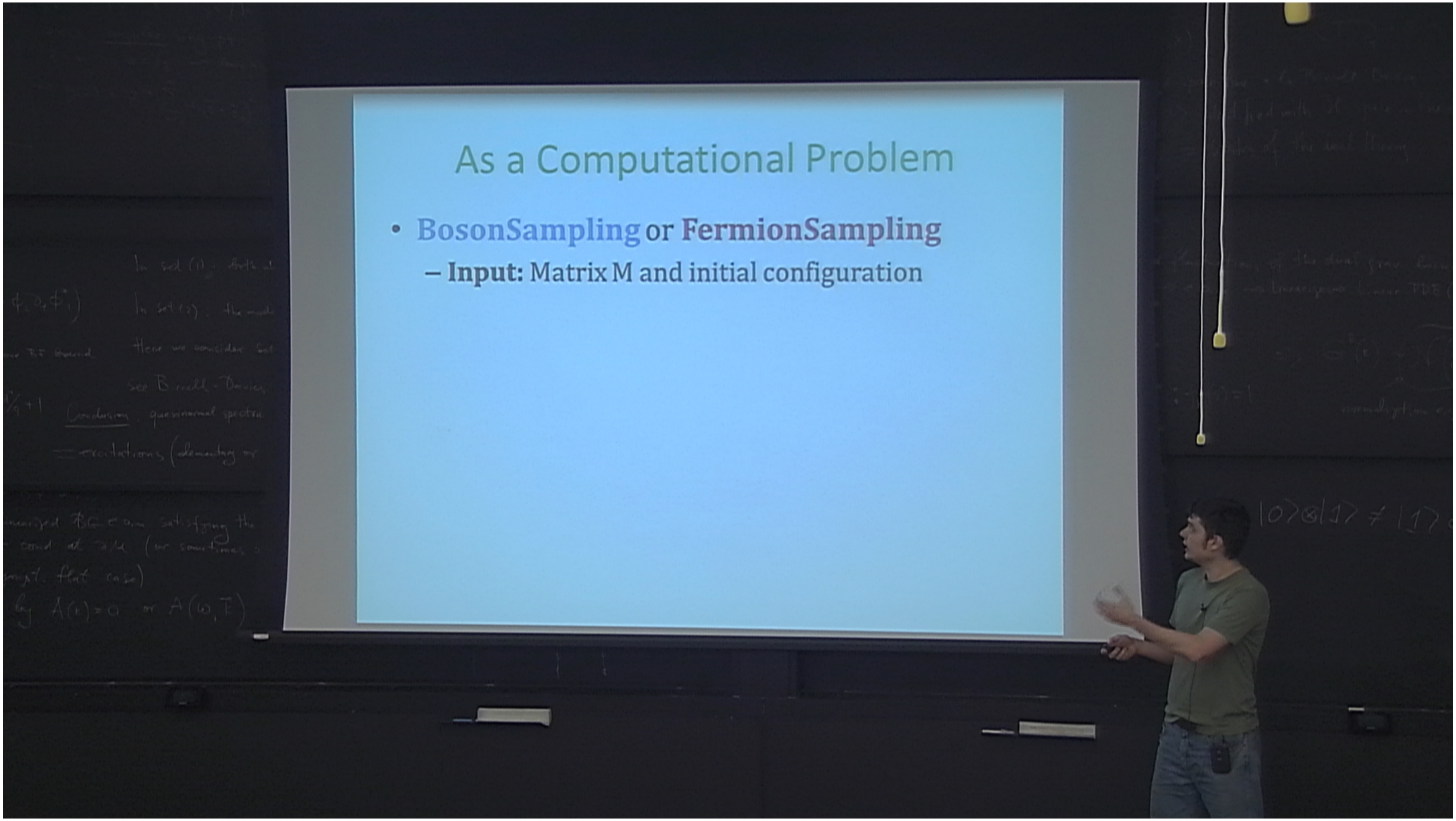
$$p = \tfrac{1}{2}$$

$$xy \rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)$$
$$= \frac{1}{\sqrt{2}}\left[\frac{x^2}{\sqrt{2}}\right] - \frac{1}{\sqrt{2}}\left[\frac{y^2}{\sqrt{2}}\right]$$


but it can be shown to be rather generic
see hep-th/0506184

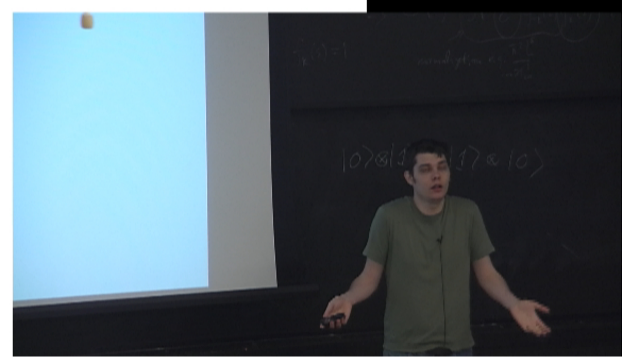

Example: Hadamarding Bosons
$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$
$p = \frac{1}{2}$
$xy \rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)$
$= \frac{1}{\sqrt{2}}\left[\frac{x^2}{\sqrt{2}}\right] - \frac{1}{\sqrt{2}}\left[\frac{y^2}{\sqrt{2}}\right]$
Hong-Ou-Mandel dip


Example: Hadamarding Fermions
$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$
$xy \rightsquigarrow \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)$
$= \frac{1}{2}x^2 - \frac{1}{2}xy + \frac{1}{2}yx + \frac{1}{2}y^2$
$= -xy$


As a Computational Problem
• BosonSampling or FermionSampling
– Input: Matrix M and initial configuration

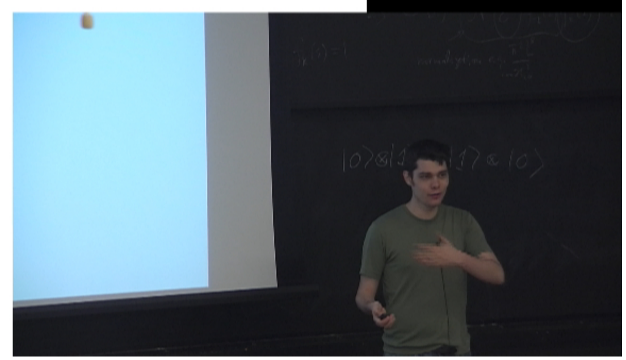
Complexity Comparison

Particle:	**Fermion**		
Function:	Det		
Matrix:	Unitary		
Compute probability:	In **P**		
Sample:			



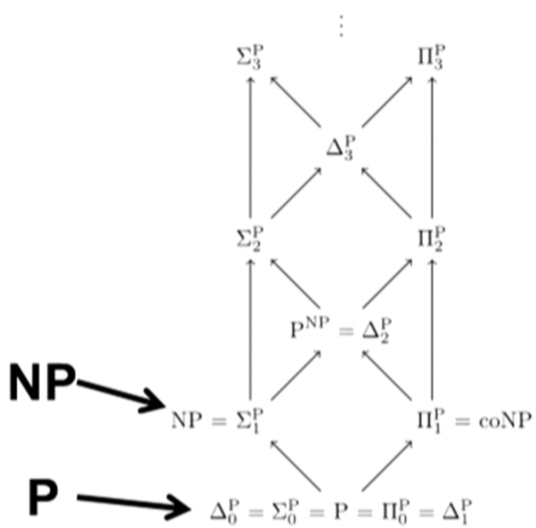
Complexity Comparison

Particle:	Fermion	Classical	
Function:	Det	Perm	
Matrix:	Unitary	Stochastic	
Compute probability:	In **P**	Approximable [JSV '01]	
Sample:	In **BPP** [Valiant '01]	In **BPP**	



Exact Result [AA' 11]

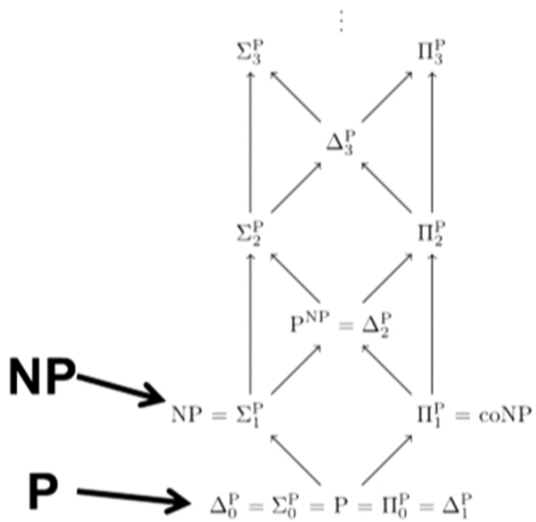
Theorem: *Identical boson sampling cannot be done in* ***BPP*** *unless* ***PH*** *collapses*



PH not collapsing generalizes **P≠NP**

Exact Result [AA' 11]

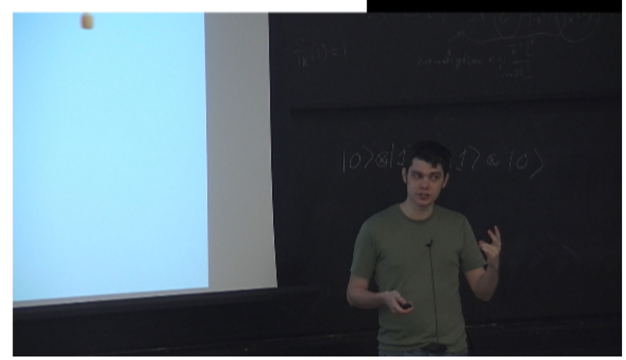
Theorem: *Identical boson sampling cannot be done in* ***BPP*** *unless* ***PH*** *collapses*



PH not collapsing generalizes **P≠NP**

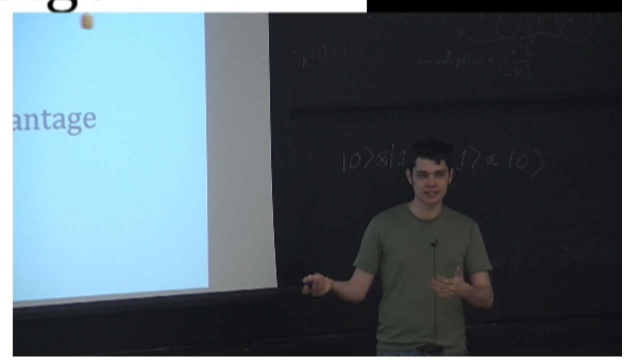
Consequences

- A quantum experiment that **can't be efficiently classically simulated**
- QC can solve a hard sampling problem using:
  - Noninteracting identical bosons



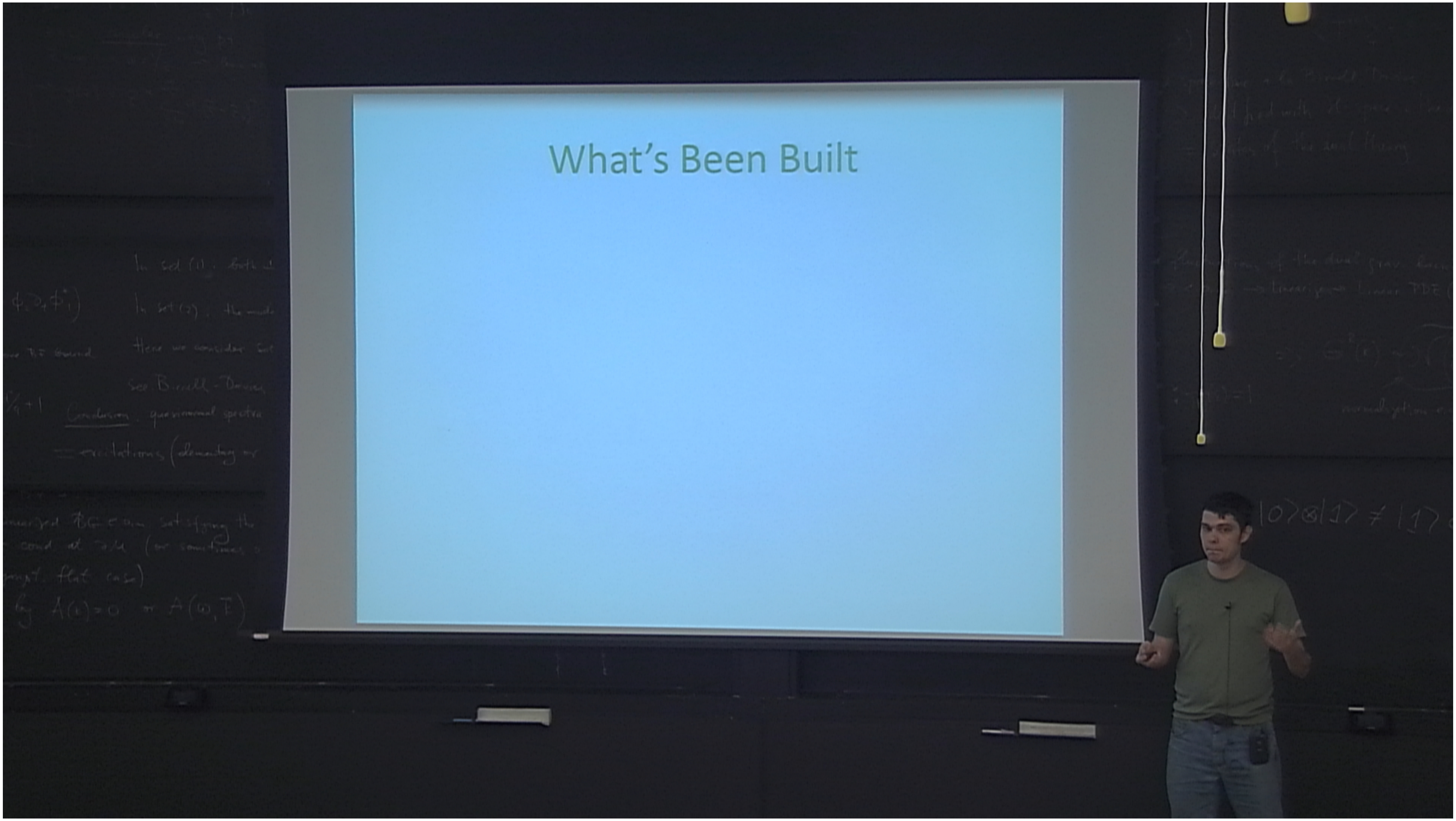
Consequences

- A quantum experiment that **can't be efficiently classically simulated**
- QC can solve a hard sampling problem using:
  - Noninteracting identical bosons
  - One-step experiment
- Strong evidence of quantum advantage



Consequences

- A quantum experiment that **can't be efficiently classically simulated**
- QC can solve a hard sampling problem using:
  - Noninteracting identical bosons
  - One-step experiment
- Strong evidence of quantum advantage


What’s Been Built

What's Been Built

- Four independent implementations

Group	Location	Number of photons	Number of output modes
M.A. Broome et al	Brisbane	3	6
J.B. Spring et al	Oxford	3	6
M. Tillmann et al	Vienna	3	5
A. Crespi et al	Rome	3	9

What's Been Built

- Four independent implementations

Group	Location	Number of photons	Number of output modes
M.A. Broome et al	Brisbane	3	6
J.B. Spring et al	Oxford	3	6
M. Tillmann et al	Vienna	3	5
A. Crespi et al	Rome	3	9

- 20 photons, 40 modes would be **beyond current computers**

What's Been Built

- Four independent implementations

Group	Location	Number of photons	Number of output modes
M.A. Broome et al	Brisbane	3	6
J.B. Spring et al	Oxford	3	6
M. Tillmann et al	Vienna	3	5
A. Crespi et al	Rome	3	9

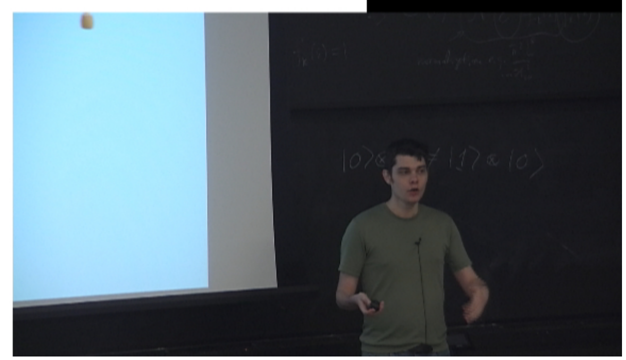
- 20 photons, 40 modes would be **beyond current computers**

Open Problems for BosonSampling

- Solve *decision* problem

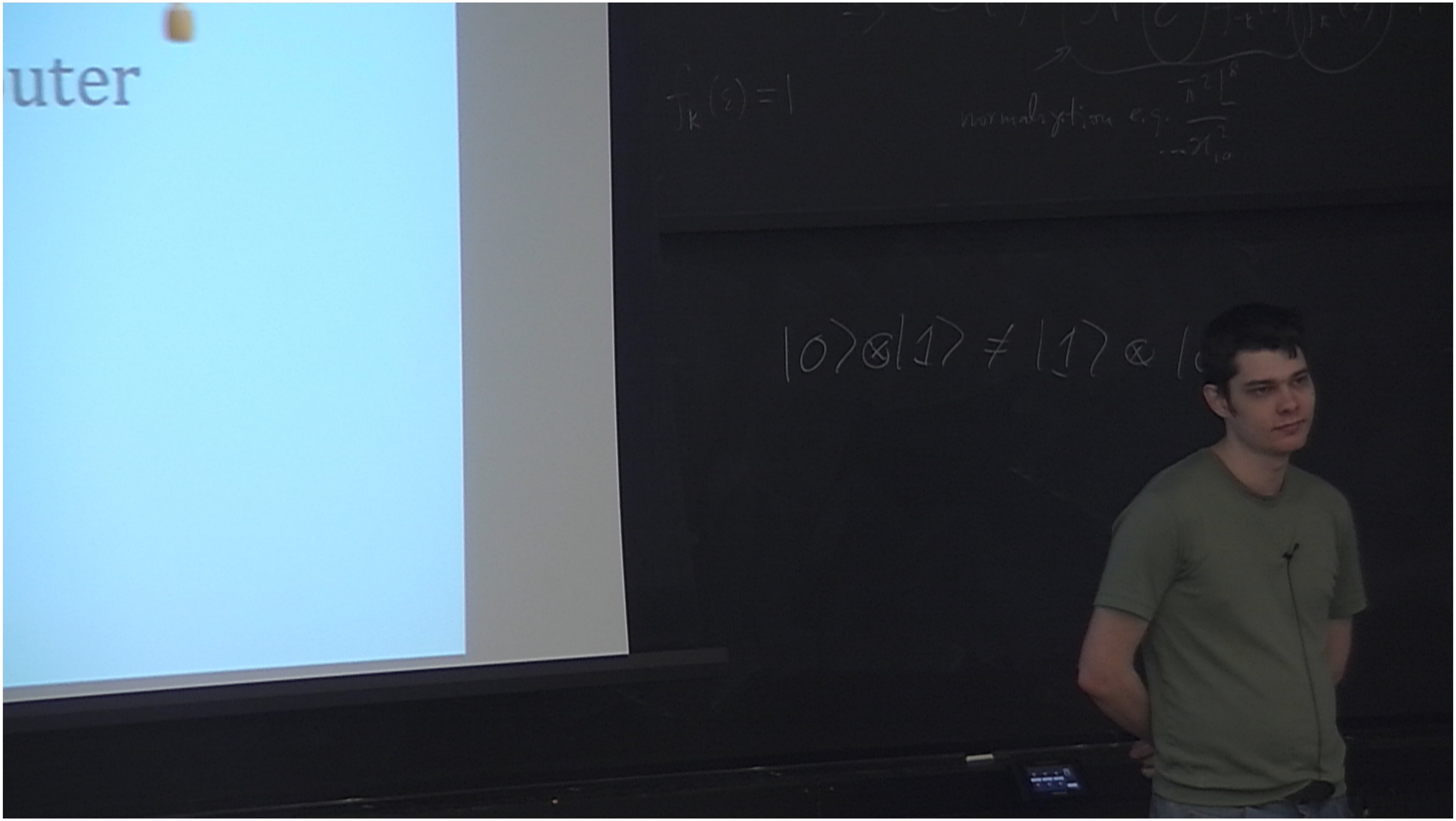
Open Problems for BosonSampling

- Solve *decision* problem
- Solve *useful* problem
- Show how to prove your QC works



Open Problems for BosonSampling

- Solve *decision* problem
- Solve *useful* problem
- Show how to prove your QC works
- Build scalable linear optics computer


uter
normalization e.g.