Title: Universal problems

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Abstract: Arguments that gravity cannot be a local renormalizable quantum field theory come from both field theory lore and black hole physics. Two current approaches to quantum gravity, asymptotic safety and Horava-Lifshitz gravity, both of which treat quantum gravity as a local renormalizable QFT, are explicitly constructed to counter field theory arguments about the non-renormalizability of gravity. However, any proposed renormalizable theory of quantum gravity must also answer black hole physics based counter-arguments. Formulating these arguments concretely requires understanding black hole solutions and thermodynamics in these theories. For Horava-Lifshitz gravity this entails understanding the thermodynamics of universal horizons. I describe the current status of universal horizon physics and which aspects are/are not still in tension with the fundamental premise of renormalizability.

A definition

Universal horizon: boundary of a spacetime region which cannot be connected to spatial infinity by any causally allowed curve.

And...?

Great. Why should I care?

An easy multiple choice question

Fundamentally, quantum gravity

- is ☐
- should be ☐
- may be ☐
- should not be ☐
- is not ☐

a 4d renormalizable local quantum field theory.

Should not be – what are the arguments?

The textbook argument **against:** perturbative non-renormalizability

$$S = (16\,\pi G)^{-1} \int \sqrt{-g}\, d^4x\, R$$

Perturbation theory about flat space

$$\kappa = \sqrt{8\pi G},\, g_{ab} = \eta_{ab} + \kappa h_{ab}$$

Algebra

$$S \sim \frac{1}{2} \int d^4x [(\partial h)^2 + \kappa (\partial h)^2 h]$$

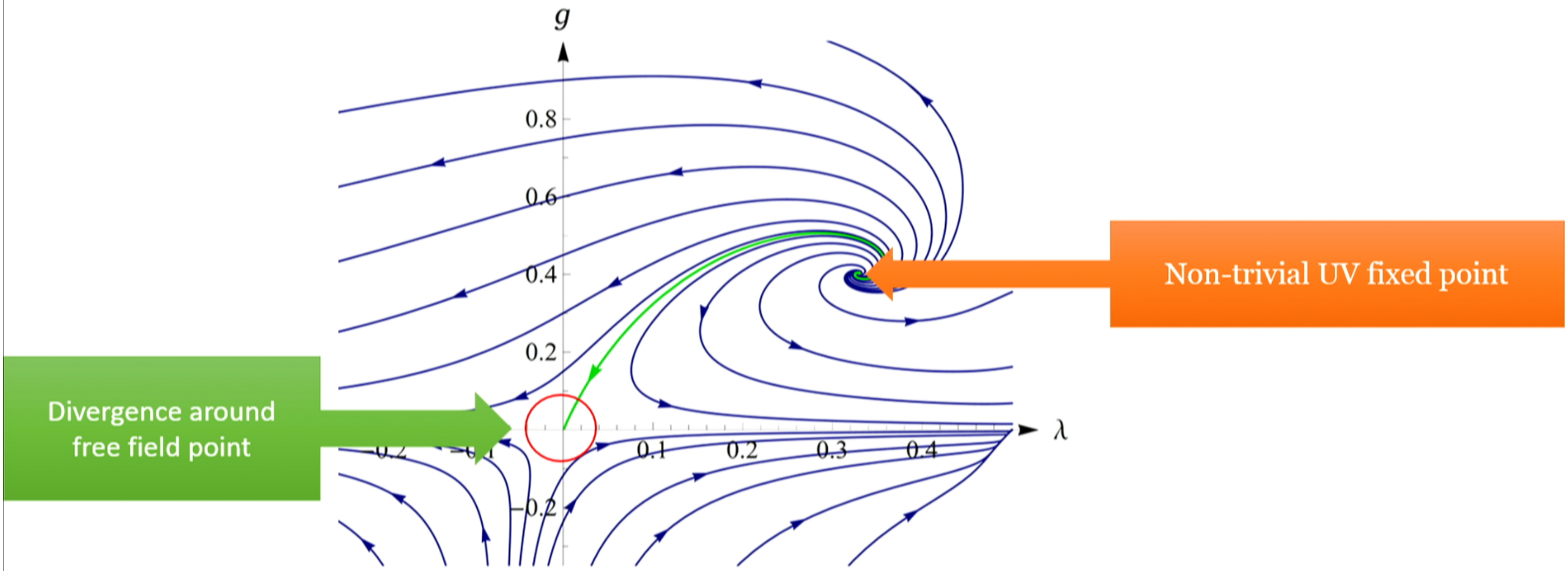
Perturbation about free field theory by *irrelevant* operator.

Should not be – what are the arguments?

Theory valid at all scales $\Longleftrightarrow$ Operators finite in far UV

Finite IR irrelevant operators → Divergent operators in far UV

Asymptotic safety: a "non-trivial" possible resolution.

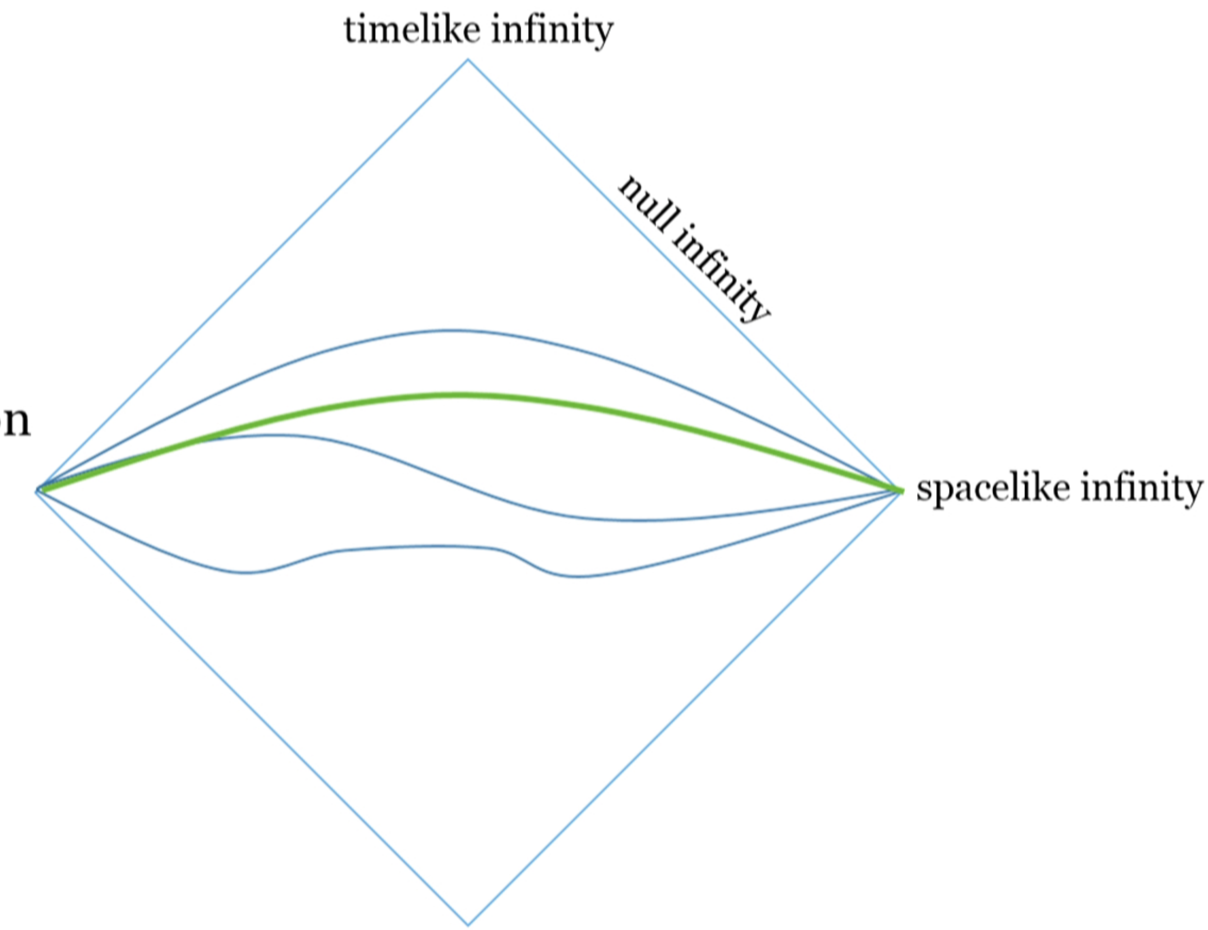


Getting around the field theory arguments

Horava-Lifshitz theory

Horava: 0901.3775

There exists a preferred foliation in spacetime.



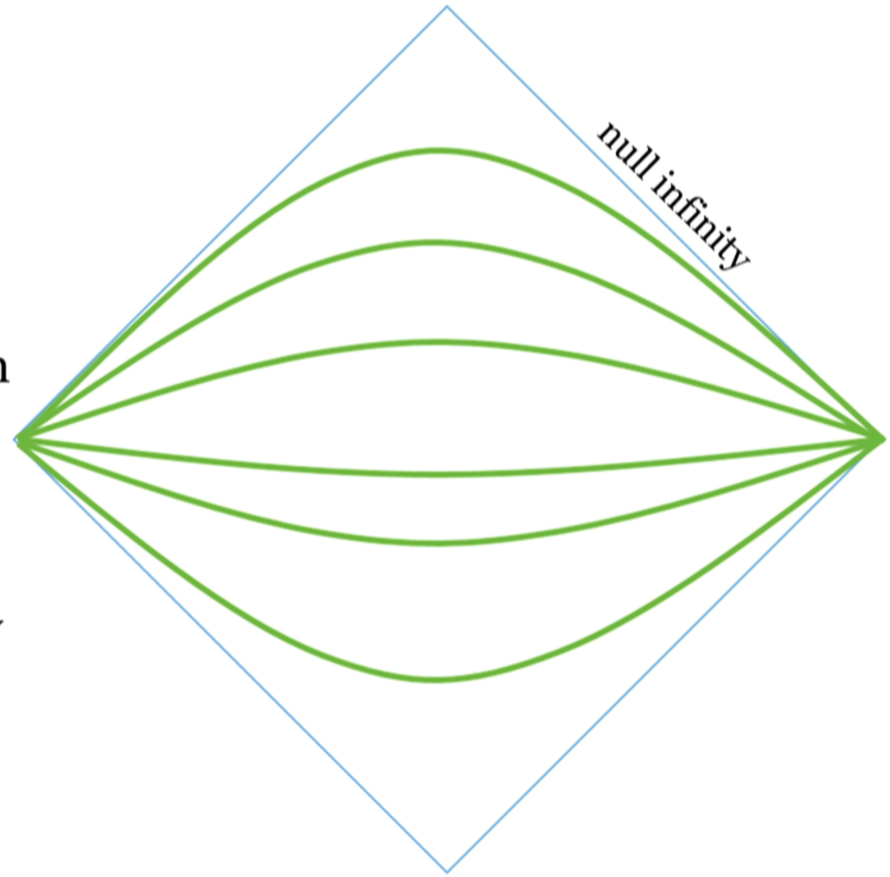
Getting around the field theory arguments

Horava-Lifshitz theory

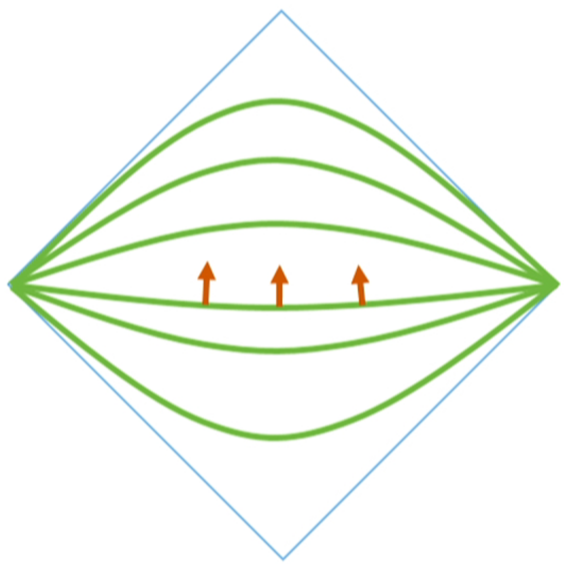
There exists a preferred foliation in spacetime.

UV theory has Lifshitz symmetry

$$t \to b^z t, x \to bx$$



Dynamical Horava-Lifshitz theory

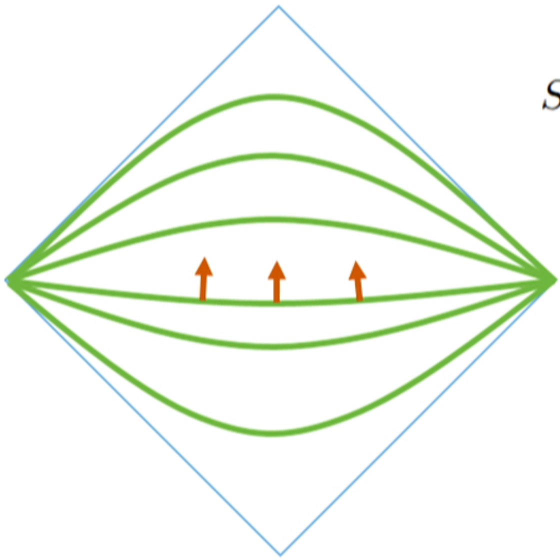


Dynamical foliation given by time function U.

$$u^a := \frac{\nabla^a U}{\sqrt{-\nabla_b U \nabla^b U}}$$

3+1 split, due to reduced symmetry more terms...

Dynamical Horava-Lifshitz theory



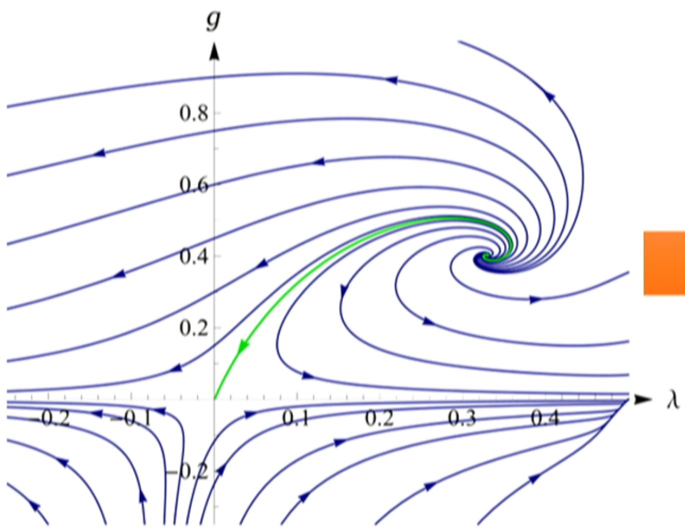
$$S_{np} = \frac{M_{\rm pl}^2}{2}\int d^3x dt N\sqrt{g}\left\{K^{ij}K_{ij} - \lambda K^2 + \xi R + \eta\, a_i a^i + \frac{1}{M_A^2}L_4 + \frac{1}{M_B^4}L_6\right\}$$

$N = lapse$
$g_{ab} = spatial\ metric$
$K_{ij} = extrinsic\ curvature\ of\ U\ hypersurface$
$R = 3d\ Ricci\ scalar$
$a_i = acceleration\ of\ u^a$

Changes UV divergence structure without introducing ghosts by permitting higher spatial derivatives in propagators without higher time derivatives.

Counting states

Required property: both theories happy and well-behaved in UV with fixed point



β functions vanish: scale invariance!
CFT or Lifshitz QFT.

$$S_{QFT} \propto E^{\frac{3}{3+z}}$$

Asymptotic safety: z=1
Horava-Lifshitz: z>1, tunable

We already have a state counting mechanism

Black holes allow us to count states as well!

Four laws of BH mechanics (Schwarzschild version)

0. The surface gravity κ is constant on a stationary horizon.
1. $\delta M = \frac{\kappa}{8\pi}\delta A$
2. $\delta A \geq 0$
3. If $\kappa > 0$ initially, one cannot reach a black hole state with $\kappa = 0$.

+

Hawking radiation

$$T = \frac{\kappa}{2\pi}$$

=

Black hole thermodynamics

In particular

$$S \propto A$$

Lots of effort to match QG black hole states to Bekenstein-Hawking entropy

Mismatched state counting

If there exist BH's, and BH entropy counts the UV density of states, then

$$S_{QFT} \propto S_{BH} \propto E^2$$

and, last I checked,

$$\boxed{E^{\frac{3}{3+z}} \neq E^2}$$

UV QG state counting must be compatible with corresponding black hole state counting for any QG theory.

c.f. Shomer, 2007

Lot of ifs

If there exist BH's, and BH entropy counts the UV density of states, then

We must understand black hole physics in these theories!

Asymptotic safety	Horava-Lifshitz gravity
• Koch, Saueressig: 1401.4452, 1306.1546	• Lu, Mei, Pope: 0904.1595
• Falls, Litim: 1212.1821, 1002.0260	• Kehagias, Sfetsos: 0905.0477
• Cai, Easson:1007.1317	• Park: 0905.4480
• Basu, DM: 1006.0718	• Blas, Sibiryakov: 1110.2195
• Bonnano, Reuter: hep-th/0002196	• Eling, Jacobson: gr-qc/0604088
• More…	• Barausse, Jacobson, Sotiriou:1104.2889
	• Berglund, Bhattacharyya, DM: 1210.4940, 1202.4497
	• Saravani, Afshordi, Mann: 1310.4143
	• Janiszewski, Karch: 1401.1463,1401.6479
	• Lin, Shu, Wang, Wu: 1404.3413

The overall picture

	Knowns		Unknowns
We usually do this	BH Thermo (GR + QFT) Sometimes "Knowns" contains Unknowns	→	QG (not necessarily QFT)
But this generates information too	QG as QFT (AS, HL)	→	BH Thermo (non-standard, and maybe not our universe)

Framework

<u>Assumptions</u>

- Dynamical, non-projectable version of HL.
- Infrared limit of HL.
- Do NOT work in a limit where HL is "almost" GR unless we have to.
- Notion of causality exists.
- Allow matter sector to be Lifshitz in UV.
- Spherical symmetry and staticity.

Can work in Einstein-aether theory, a theory of gravity coupled to a timelike unit vector field.

In particular: *the regular, static, and spherically symmetric black hole solution spaces of HL and EA are equivalent.*

Jacobson, 1001.4823.
Bhattacharyya, DM, in prep

Horava-Lifshitz and Einstein-Aether

Einstein aether theory:

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int \sqrt{-g}\,(-R + L_{æ})\, d^4x$$

$$L_{æ} = -M^{\alpha\beta\mu\nu}\nabla_\alpha u_\mu \nabla_\beta u_\nu \quad M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta}g^{\mu\nu} + c_2 g^{\alpha\mu}g^{\beta\nu} + c_3 g^{\alpha\nu}g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

Assume aether is hypersurface orthogonal.

$$u^a := \frac{\nabla^a U}{\sqrt{-\nabla_b U \nabla^b U}}$$

Dynamical, non-projectable HL theory in IR:

$$S = \frac{M_{\rm pl}^2}{2}\int dt d^3x\, N\sqrt{g}(K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta\, a_i a^i)$$

$$\frac{1}{8\pi M_{\rm pl}^2 G_{æ}} = \xi = \frac{1}{1-c_{13}}, \quad \lambda = \frac{1+c_2}{1-c_{13}}, \quad \eta = \frac{c_{14}}{1-c_{13}}$$

Matter sector

Scalar field, no interaction terms

Low energy speed squared

U hypersurface derivative

$$\mathscr{L} = -\frac{s_\phi^2}{2}\mathrm{g}^{ab}_{(\phi)}(\nabla_a\phi)(\nabla_b\phi) - \frac{(\vec{\nabla}^2\phi)^2}{2k_0^2}$$

Dimensionful constant (z=2)

Assume z=2 Lifshitz behavior for simplicity.

In principle any z could be chosen (subject to analyticity and stability requirements).

$$\mathrm{g}^{ab}_{(\phi)} = \mathrm{g}^{ab} - (s_\phi^{-2} - 1)u^a u^b$$

Flat space mode dispersion

Aether-metric mode	Dispersion
Transverse	$\omega^2 = \dfrac{k^2}{1 - c_{13}}$
Vector	$\omega^2 = \dfrac{c_1 - \frac{1}{2}c_1^2 + \frac{1}{2}c_3^2}{c_{14}(1 - c_{13})} k^2$
Trace	$\omega^2 = (c_{123}/c_{14}(2 - c_{14})(2(1 + c_2)^2 - c_{123}(1 + c_2 + c_{123}))k^2$

Scalar field mode	Dispersion
Scalar	$\omega^2 = s_\phi^2 k^2 + \dfrac{k^4}{k_0^2}$

All fields propagate to the future in U time.
No closed causal curves. No ghosts. Yay!

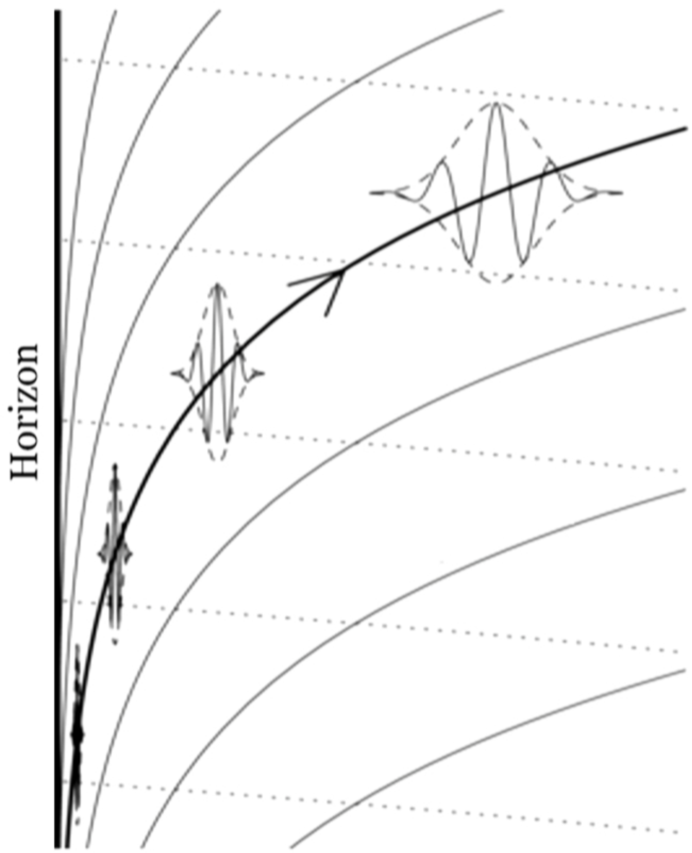
Flat space mode dispersion in aether frame

Aether-metric mode	**Dispersion**
Transverse	$\omega^2 = \dfrac{k^2}{1 - c_{13}}$
Vector	$\omega^2 = \dfrac{c_1 - \frac{1}{2}c_1^2 + \frac{1}{2}c_3^2}{c_{14}(1 - c_{13})} k^2$
Trace	$\omega^2 = (c_{123}/c_{14}(2 - c_{14})(2(1 + c_2)^2 - c_{123}(1 + c_2 + c_{123}))k^2$

Scalar field mode	**Dispersion**
Scalar	$\omega^2 = s_\phi^2 k^2 + \dfrac{k^4}{k_0^2}$

Why this asymmetry?

Cause that's how it works!



Black hole thermodynamics: IR gravity, but UV matter

1. Matter and gravity really are both Lifshitz in UV, but can consistently truncate gravity sector as it is an IR solution.

2. Cannot consistently truncate matter sector as physics demands assumptions about UV modes.

3. You need higher derivative terms for a matter field if you want to consistently be UV Lifshitz.

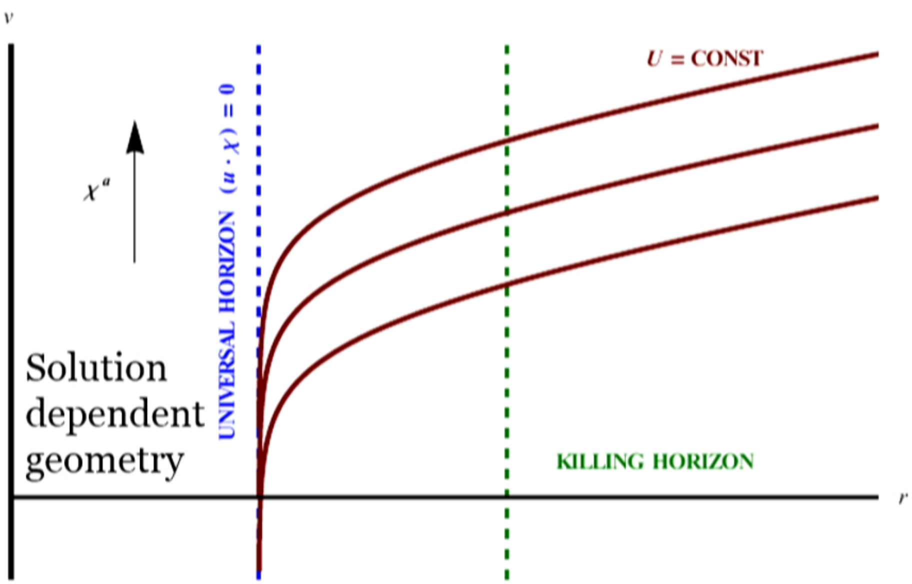
4. Different speeds for different fields and no higher derivative terms is not UV Lifshitz, but random Lorentz violation.

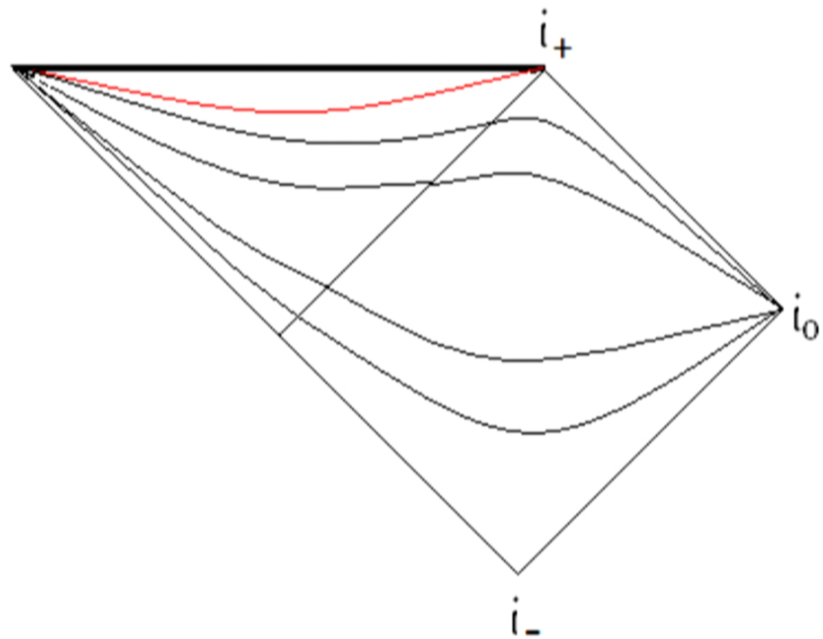
Regular vacuum solutions

Eddington-Finkelstein coordinates $\mathrm{d}s^2 = -e(r)\mathrm{d}v^2 + 2f(r)\mathrm{d}v\mathrm{d}r + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$

$$\mathcal{G}_{ab} = \mathcal{T}^{æ}_{ab}, \qquad Æ_a = 0, \qquad u^2 = -1$$

Typical asymptotically flat static vacuum solution





Regular vacuum solutions

A brief history of regular black hole solutions

1. A number of asymptotically flat numerical solutions found first.
(Eling, Jacobson, Barausse, Sotiriou, 2007,2011)

2. Two regular (from UH on out) static, asymptotically flat analytic solutions
(Berglund, Bhattacharyya, DM, 2011,2012)

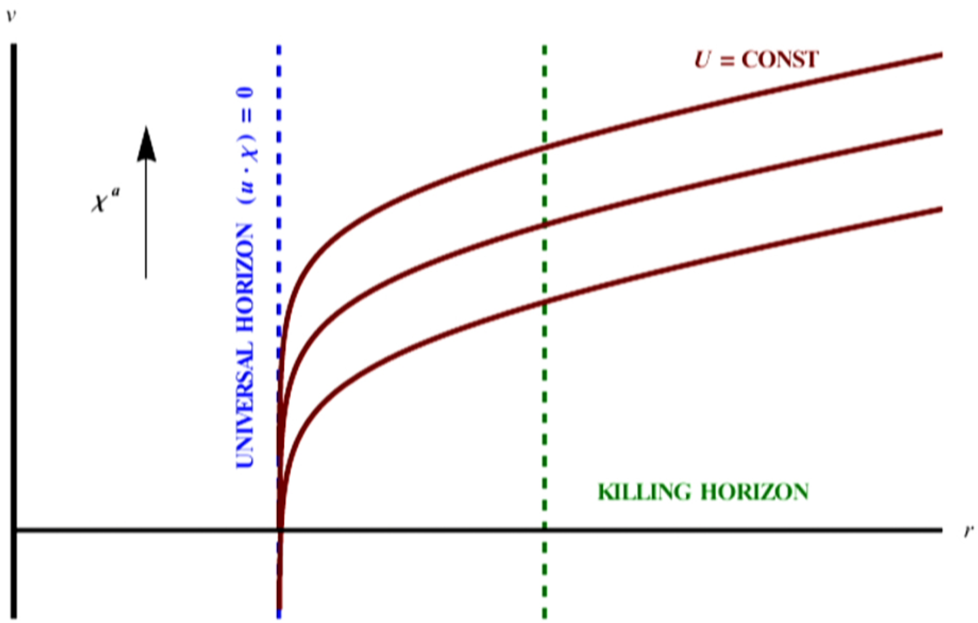
$$c_{14} = 0: e(r) = 1 - \frac{r_0}{r} - \frac{F(c_i) r_0^4}{r^4}, f(r) = 1$$

$$c_{123} = 0: e(r) = 1 - \frac{r_0}{r} - \frac{G(c_i) r_0^2}{r^2}, f(r) = 1$$

3. Collapsing solution with dynamical UH formation and varied asymptotic b.c.
(Saravani, Afshordi, Mann, 2013)

4. Solutions galore (Lifshitz, AdS asymptotics etc.)
(Lin, Shu, Wang, Wu, 4 days ago)

Which is the right horizon for black hole thermodynamics?

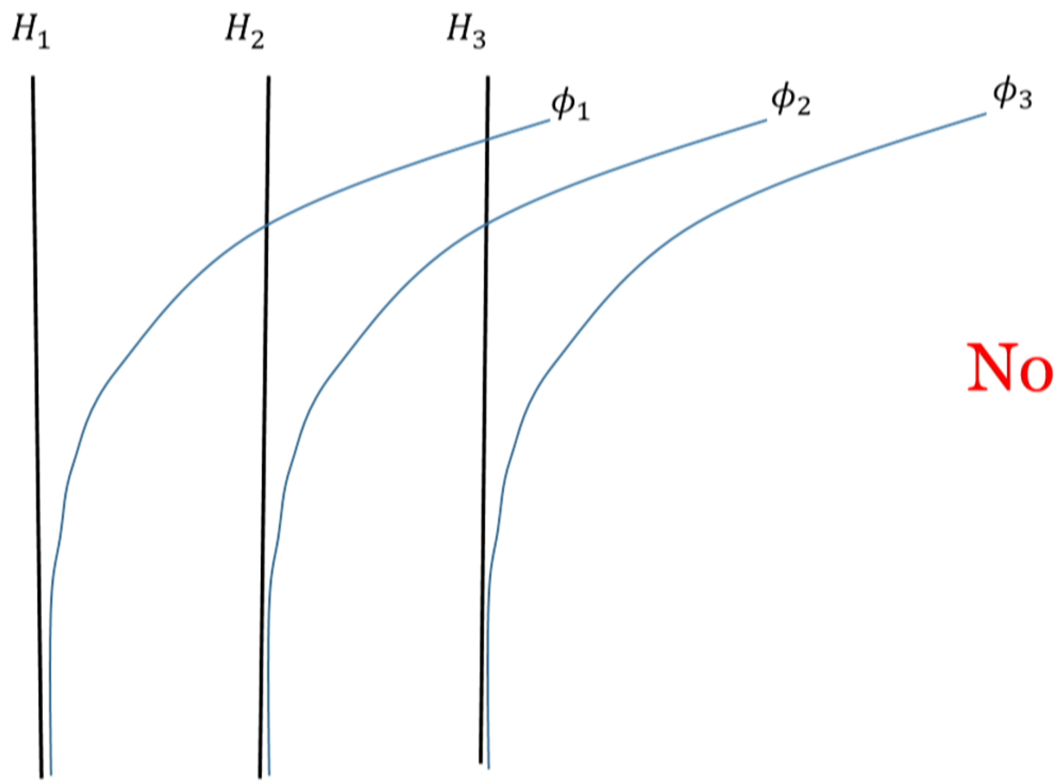


Bekenstein's argument: Toss matter into the center. The *causal* boundary must have an entropy if the second law is not to be violated.

The universal horizon, not the Killing horizon is the right spot to apply thermodynamics.

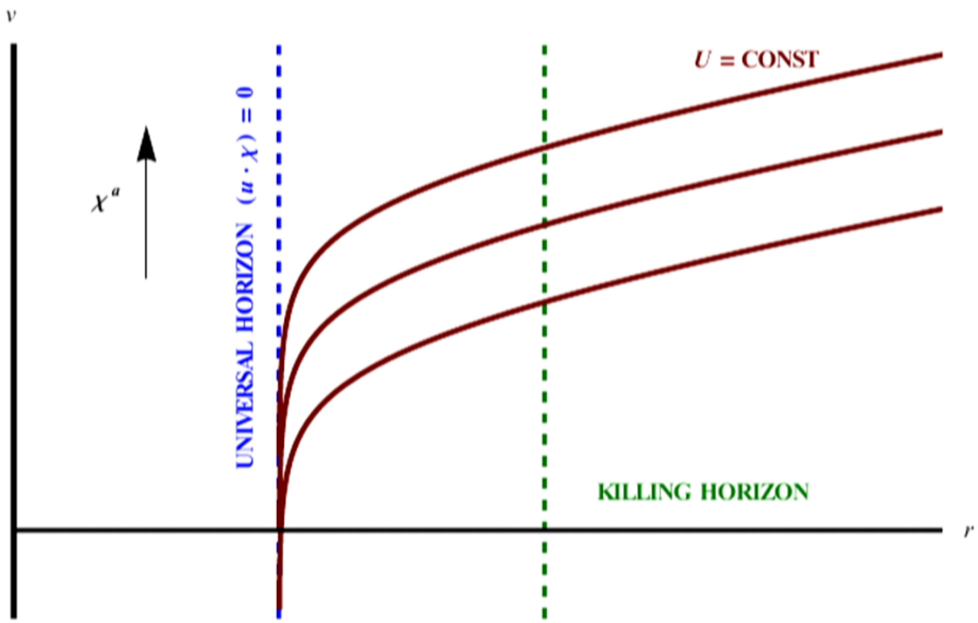
Random Lorentz violation

Just Lorentz violating dimension four IR matter terms for fields $\phi_1, \phi_2, \phi_3, \ldots$ $L_n = -\frac{s_{\phi_n}^2}{2} g_{\phi_n}^{ab} (\nabla_a \phi_n)(\nabla_b \phi_n)$



No unique causal boundary

Which is the right horizon for black hole thermodynamics?



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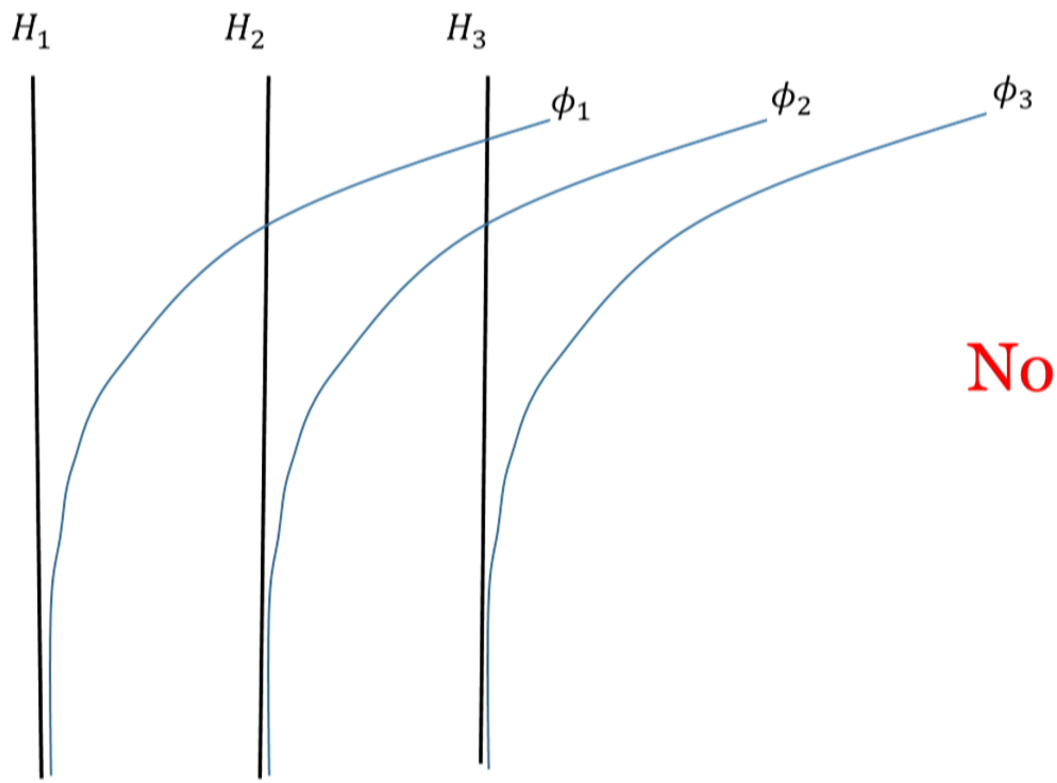
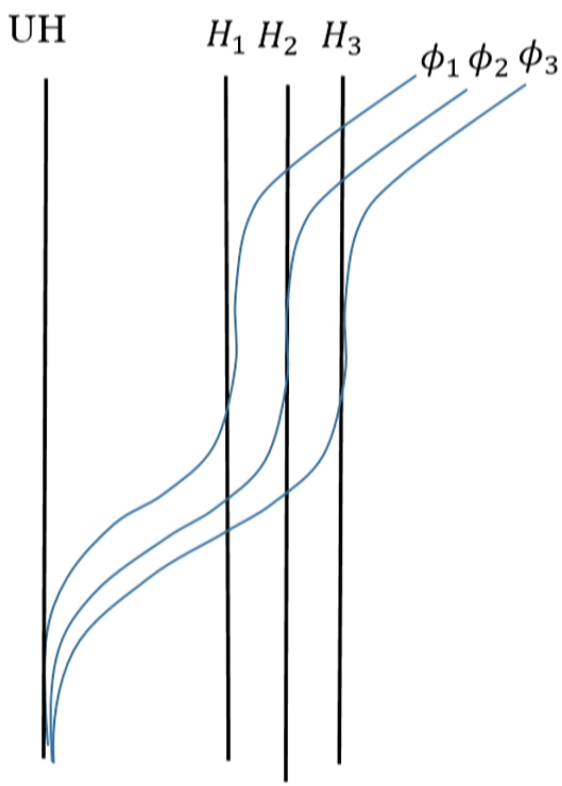


No unique causal boundary

UV Lifshitz Lorentz violation

But with full Lagrangian... $L_n = -\frac{s_{\phi_n}^2}{2} g_{\phi_n}^{ab}(\nabla_a \phi_n)(\nabla_b \phi_n) - \frac{(\vec{\nabla}^2 \phi_n)^2}{2k_{0,n}}$



Unique causal boundary

You have a chance

Show me at least a little money

How much about universal horizon thermodynamics do we know?

First law at each horizon

Via Noether at infinity and Killing horizon (Foster, gr-qc/0509121)

$$\delta M = \frac{\kappa}{8\pi G}\left[\left(1+\phi\left(c_{14}n^a{}_b - c_{13}(\delta^a_b + \frac{3}{2}h^a_b)\right)\nabla_a u^b\right)\delta A \;\; + \phi A\delta\left((c_{14}n^a{}_b - c_{123}\delta^a_b - c_{13}h^a_b)\nabla_a u^b\right)\right]$$

Via “inspired construction”/cheating (Berglund, Bhattacharyya, DM, 1202.4497) or Noether (Mohd 1309.0907) at infinity and universal horizon.

$$\delta M_{æ} = \frac{q_{\mathrm{UH}}\delta A_{\mathrm{UH}}}{8\pi G_{æ}} \qquad q_{\mathrm{UH}} = (1-c_{13})\kappa_{\mathrm{UH}} + \frac{c_{123}}{2}K_{\mathrm{UH}}|\chi|_{\mathrm{UH}}$$

$$\kappa_{UH} = \sqrt{-\frac{1}{2}\nabla_a\chi_b\nabla^a\chi^b}, K_{UH} = \nabla_a u^a$$

Radiation from universal horizon

Tunneling approach

Requirements

Vacuum: assume the infalling vacuum

No matter/aether Cerenkov radiation so $c_{123} = 0 \; or \; c_{14} = 0$
(Convenient but likely not necessary)

Lifshitz coefficient yields chemical potential – preserves thermality

$$\mathrm{I} \propto e^{-\frac{\omega-\mu}{T_{UH}}}$$

$$\mu = -\frac{c_{ae}^2 k_0}{2N}, T_{UH} = \frac{\kappa_{UH}}{4\pi c_{ae}}$$

$$S = \frac{(1-c_{13})c_{ae}A_{UH}}{2G_{ae}}$$

Berglund, Bhattacharyya, DM:1210.4940

Killing horizon reprocessing

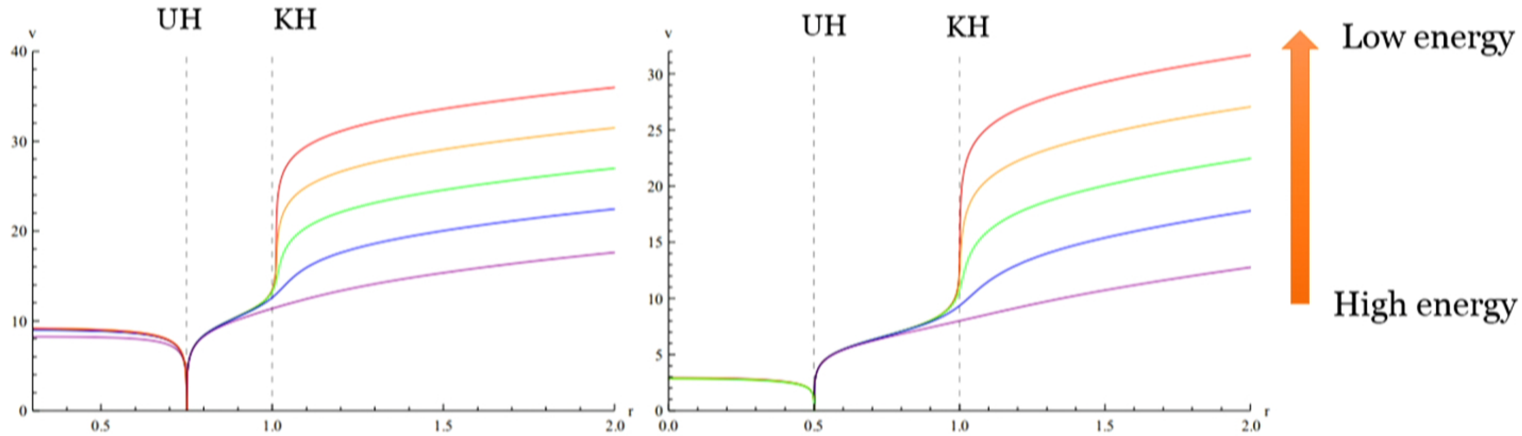


FIG. 5: Trajectories of the outgoing particle in v–r Eddington–Finkelstein coordinates. Energies of $\Omega = 0.1$ (purple), $\Omega = 10^{-2}$ (blue), $\Omega = 10^{-3}$ (green), $\Omega = 10^{-4}$ (orange) and $\Omega = 10^{-5}$ (red). For these parameters of the black hole the $c_{123} = 0$ solution (left) has Universal horizon at $r_{\rm UH} = 0.75$, while for the $c_{14} = 0$ solution (right) the Universal horizon is at $r_{\rm UH} = 0.5$. For both situations $r_{\rm KH} = 1$. Behaviour at the Universal horizon is universal while behaviour at the Killing horizon at $r_{\rm KH} = 1$ depends on energy.

(Cropp, Liberati, Mohd, Visser, 1312.0405)

Thermal spectrum modified at $\omega \ll k_0$ by scattering off Killing/IR horizon. New "greybody" factor.
Final low energy spectrum uncalculated…thermal with $T = \frac{\kappa_{KH}}{2\pi}$?

Verifying entropy calculation (Basu, DM in prep)

Rule of thumb: Gravitational question got you confused? Try it in 2+1 dimensions!

Verifying entropy calculation (Basu, DM in prep)

Leverage well understood state counting techniques of AdS3/CFT2

The 2+1 UH/AdS black hole

1. $c_{14} = 0$ required for asymp. AdS
2. Aether aligned with Killing vector at infinity

(Sotiriou et. al., Bhattacharyya, DM, Lin et. al.)

$$ds^2 = -edv^2 + 2dvdr + r^2 d\Theta^2$$

$$e(r) = \frac{r^2}{l^2} - \frac{2r_{UH}^2}{l^2} - \frac{c_{13} r_{UH}^4}{(1-c_{13}) r^2 l^2}$$

$$(u \cdot \chi) = -\frac{r}{l}\left(1 - \frac{r_{UH}}{r}\right)\left(1 + \frac{r_{UH}}{r}\right)$$

Preliminary results using 2d CFT counting at infinity do indeed indicate $S \propto r_{UH}/G_{ae}$!

We're good, right?

It all seems so promising...

Where we stand on universal horizon thermodynamics (spherical symmetry)

0. **The surface gravity is constant on a stationary horizon.**
   Yes, but it's a bit of a cheat in spherical symmetry.

1. **First law.** $\boldsymbol{\delta E = T\delta S}$
   Yes. We have thermal radiation, a first law, and corollary data about # of states that suggests thermo first law.

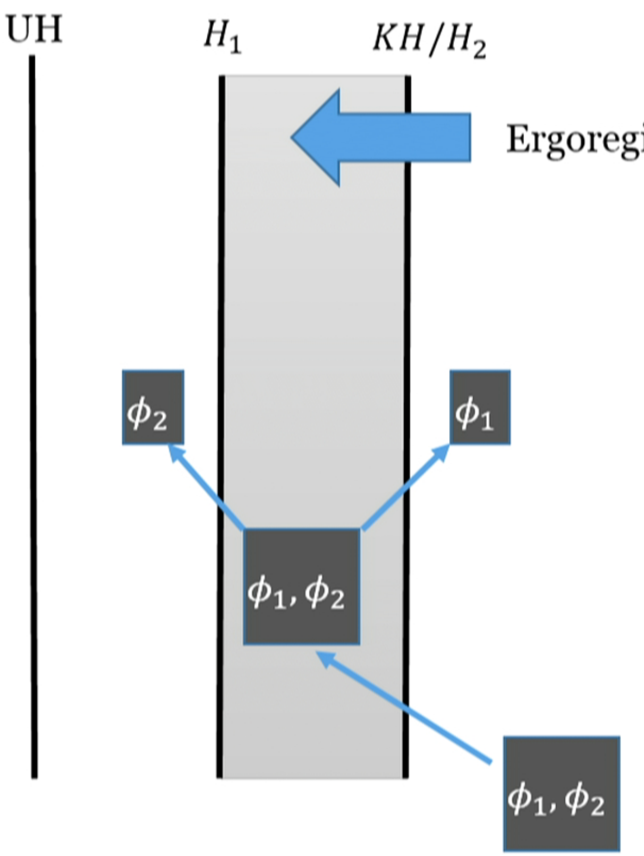
2. **Second law.** $\boldsymbol{\delta A \geq 0}$.
   Yes. However, the GSL has trouble <u>when interactions are turned on</u>.

3. **Cannot reach vanishing surface gravity in a finite number of processes.**
   Nobody's looked!

The second law and interactions

Problem: if we have two interacting scalar fields they will generically have different IR speeds

$$s_{\phi_1} > s_{\phi_2} = c$$



1. Take a system of ϕ_1 and ϕ_2 in a pure state.
2. Let it fall into ergoregion and split.
3. Can arrange this so that ϕ_2 has negative Killing energy.
4. Can arrange that no increase in entropy of outgoing ϕ_1 (stays pure).
5. Negative Killing energy goes into hole, S hole decreases, S outside stays the same.
6. Violation of GSL.

Jacobson, Wall: 0804.2720

The second law and interactions

The law that entropy always increases holds, I think, the supreme position among the laws of Nature.

If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations — then so much the worse for Maxwell's equations. If it is found to be contradicted by observation — well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

—Sir Arthur Stanley Eddington, *The Nature of the Physical World* (1927)

Take aways

Black hole physics in renormalizable QG theories must be reconciled eventually.

Black holes in Horava-Lifshitz gravity are non-standard, non Killing horizon, etc.

BH thermodynamics is coming along, but both technical and conceptual issues remain.

Questions that need answers

1. What to do about the second law?
2. What is the general/axisymmetric solution space for HL/AE theories?
3. Can one be more robust in calculating radiation from the UH?
4. Can we get more general analytic solutions?
5. What are the solutions with a UH and Lifshitz asymptotics (Lifshitz holography)?
6. Where's lunch?