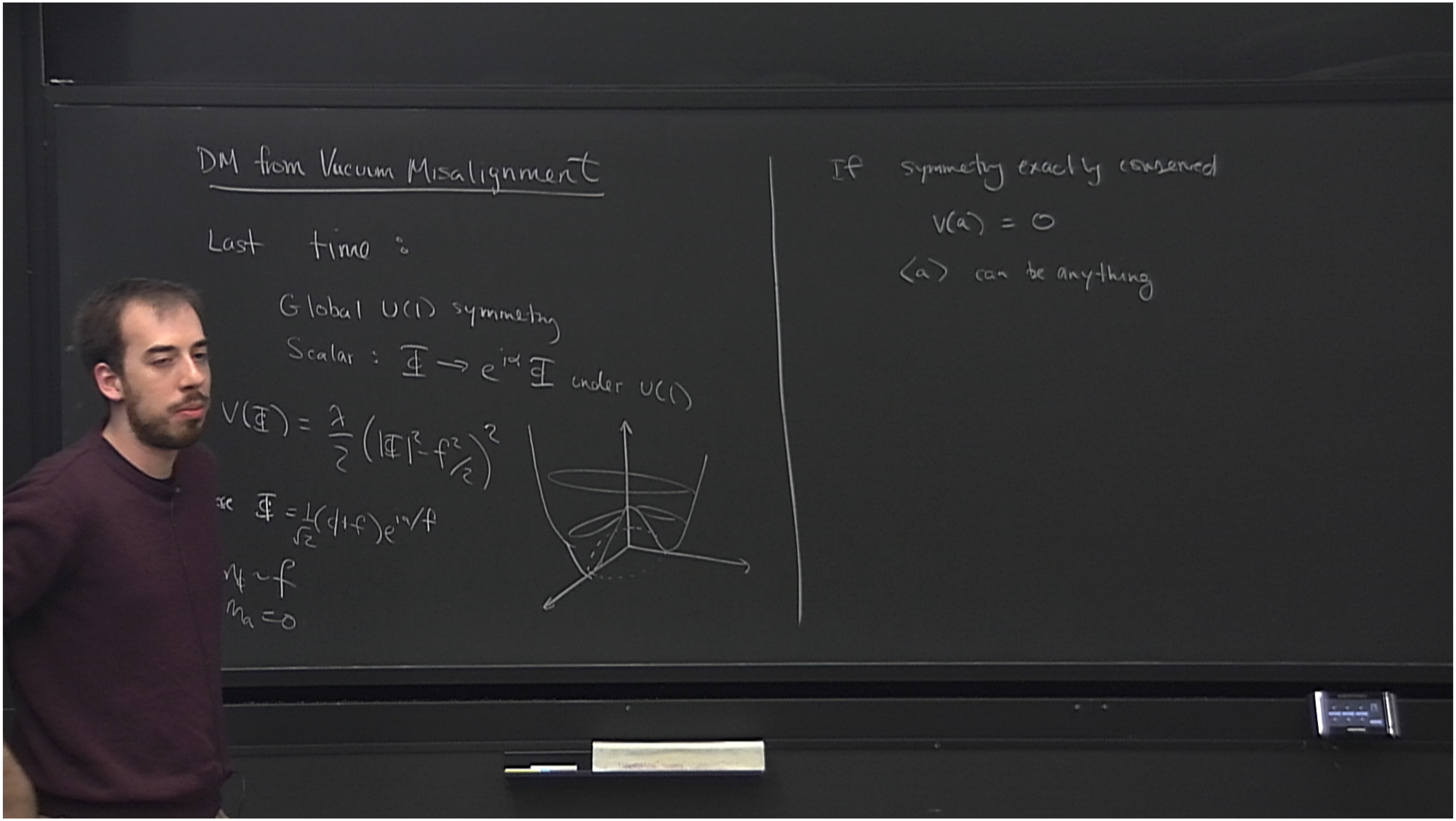
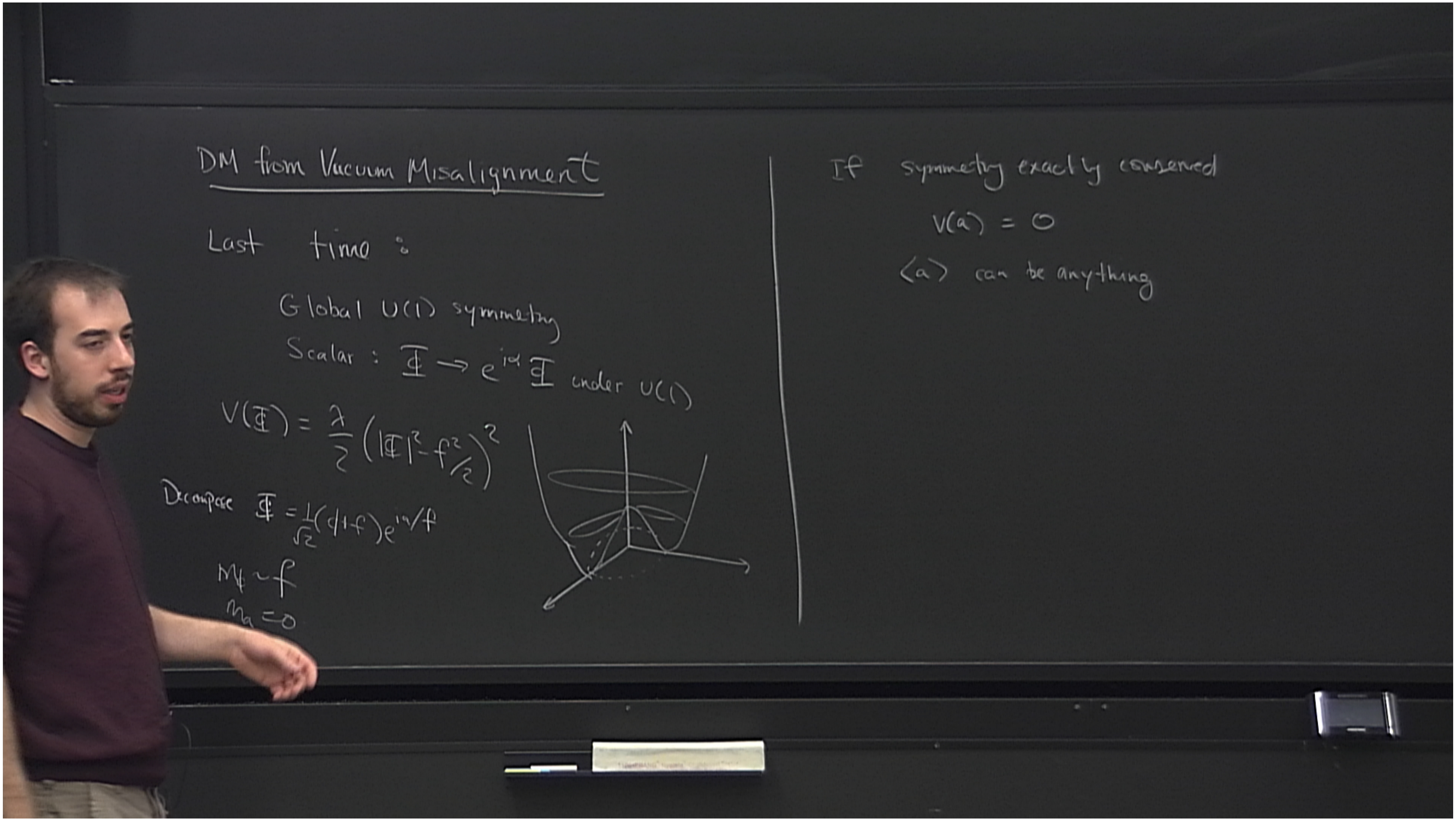
Title: 13/14 PSI - Explorations in Particle Theory - Lecture 11

Date: Apr 22, 2014 09:00 AM

URL: http://pirsa.org/14040068

Abstract:


DM from Vacuum Misalignment
Last time:
Global U(1) symmetry
Scalar: $\Phi \to e^{i\alpha}\Phi$ under U(1)
$V(\Phi) = \frac{\lambda}{2}\left(|\Phi|^2 - f^2/2\right)^2$
$\Phi = \frac{1}{\sqrt{2}}(\phi + f)e^{ia/f}$
$m_\phi \sim f$
$m_a = 0$
If symmetry exactly conserved
$V(a) = 0$
$\langle a \rangle$ can be anything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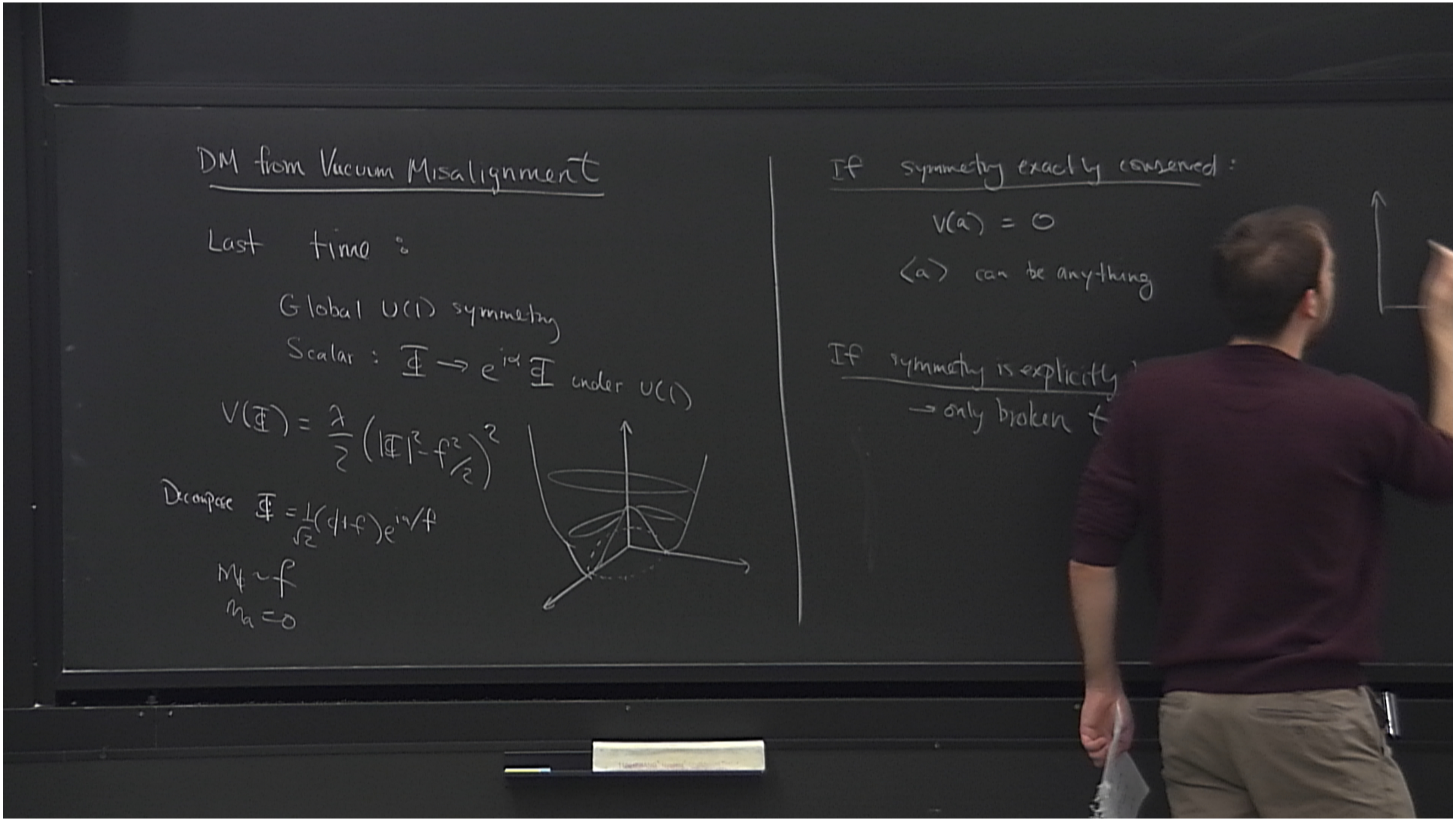
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Scalar : Φ → e^{iα} Φ under U(1)
V(Φ) = λ/2 (|Φ|² − f²/2)²
Decompose Φ = 1/√2 (φ+f) e^{ia/f}
m_φ ~ f
m_a = 0
If symmetry exactly conserved :
V(a) = 0
⟨a⟩ can be anything
If symmetry is explicitly
→ only broken t

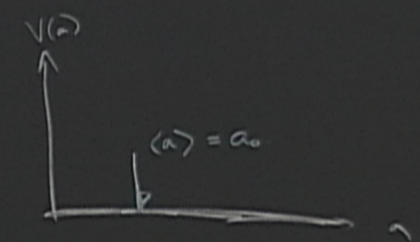
If symmetry exactly conserved:

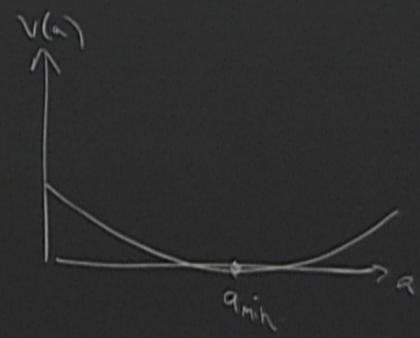
$V(a) = 0$

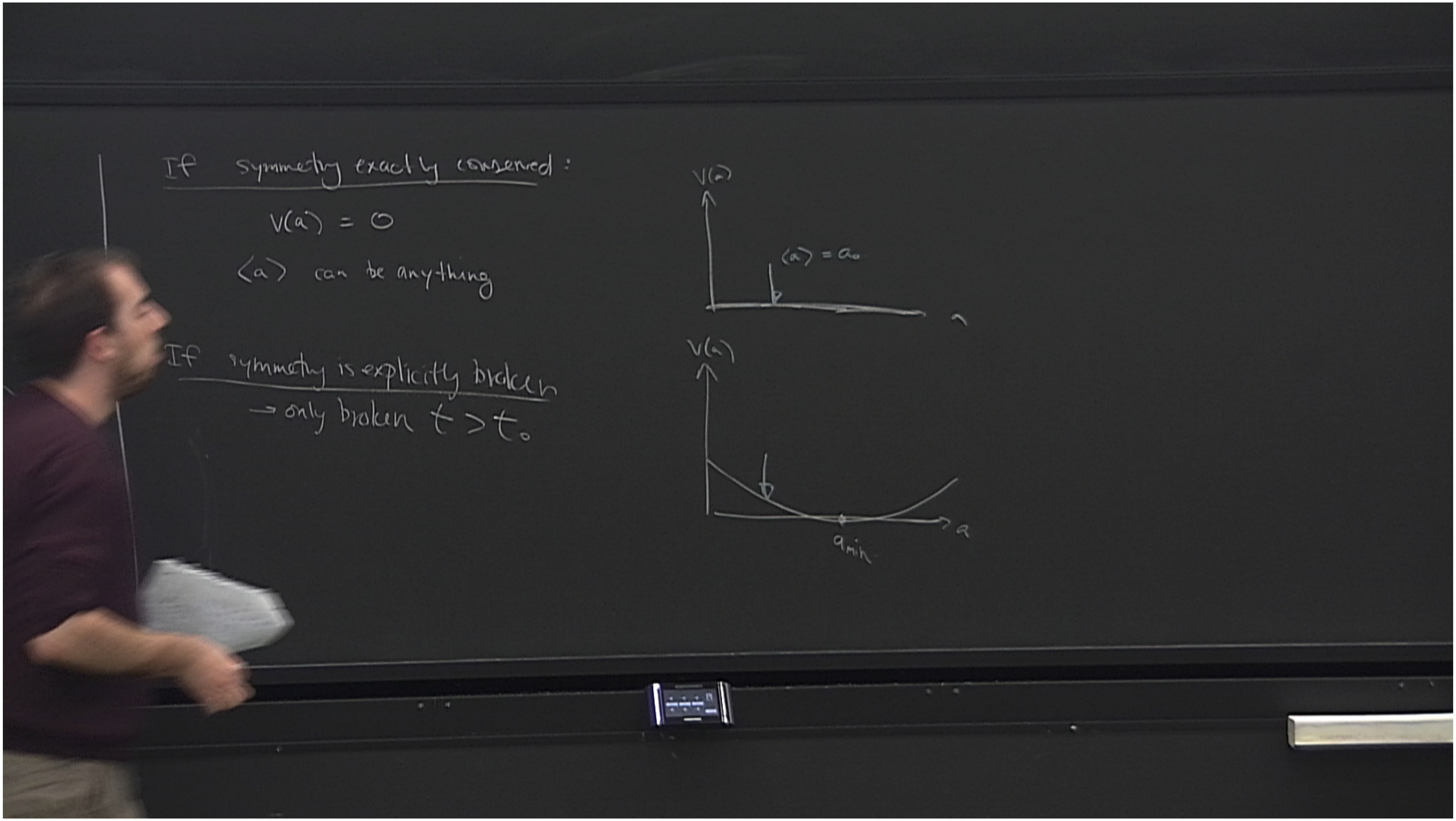
$\langle a \rangle$ can be anything

If symmetry is explicitly broken

$\rightarrow$ only broken $t > t_0$






If symmetry exactly conserved :
V(a) = 0
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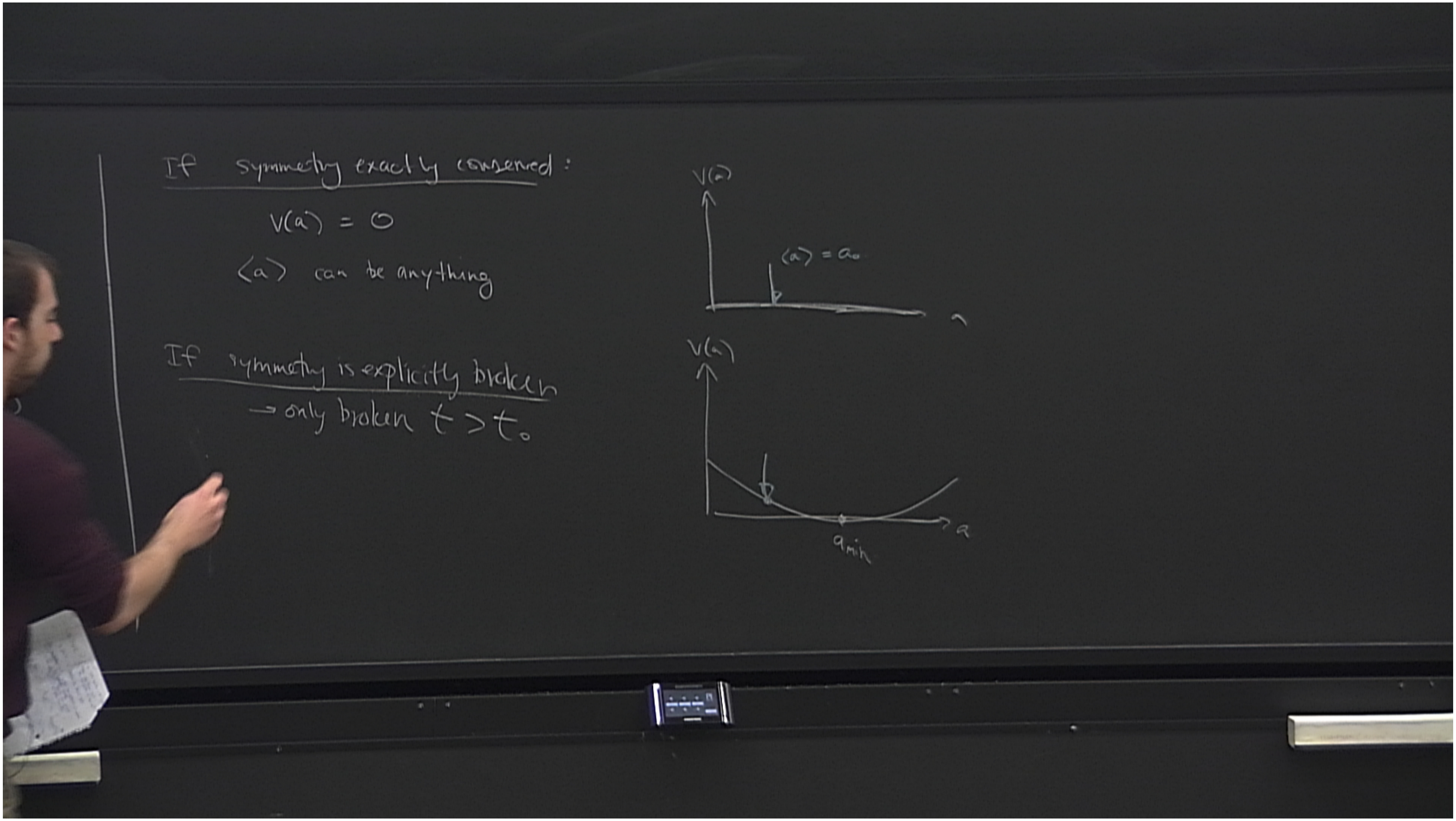
If symmetry is explicitly broken
→ only broken t > t₀
V(a)
⟨a⟩ = a₀
V(a)
a
a_min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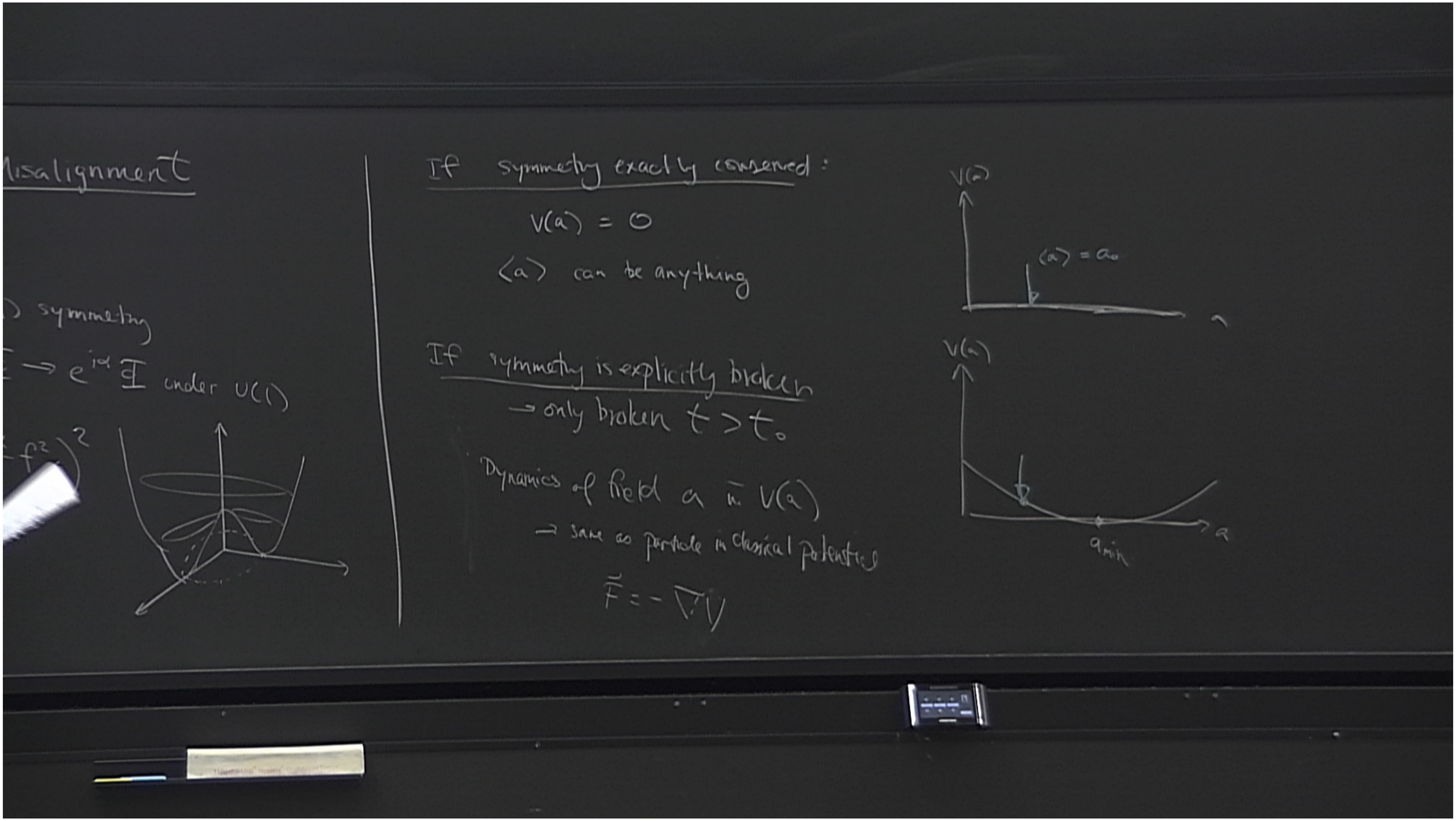
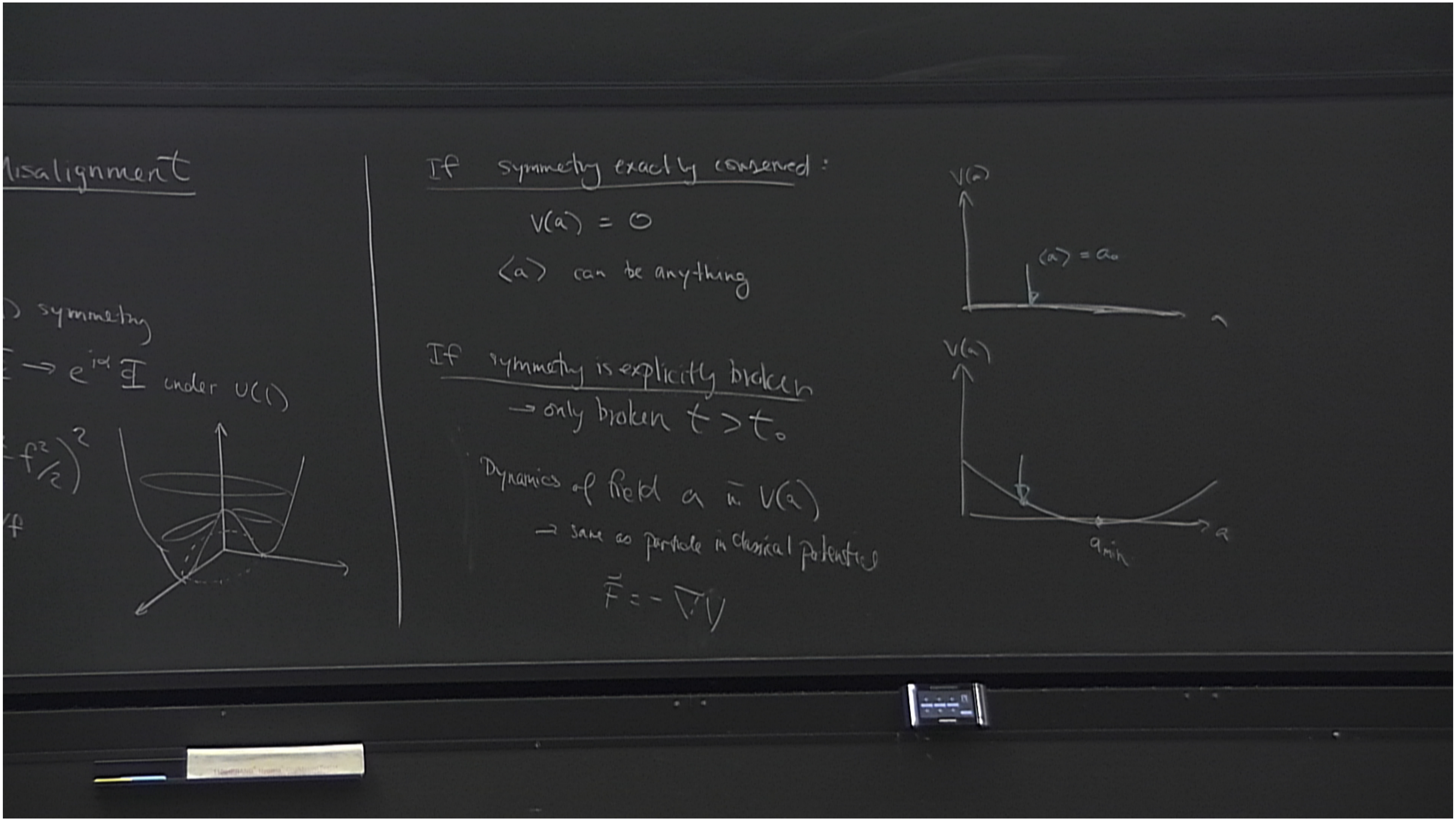

Misalignment
symmetry
under U(1)
If symmetry exactly conserved:
V(a) = 0
⟨a⟩ can be anything
If symmetry is explicitly broken
→ only broken t > t0
Dynamics of field a in V(a)
→ same as particle in classical potential
F = - ∇V
V(a)
⟨a⟩ = a0
a
a min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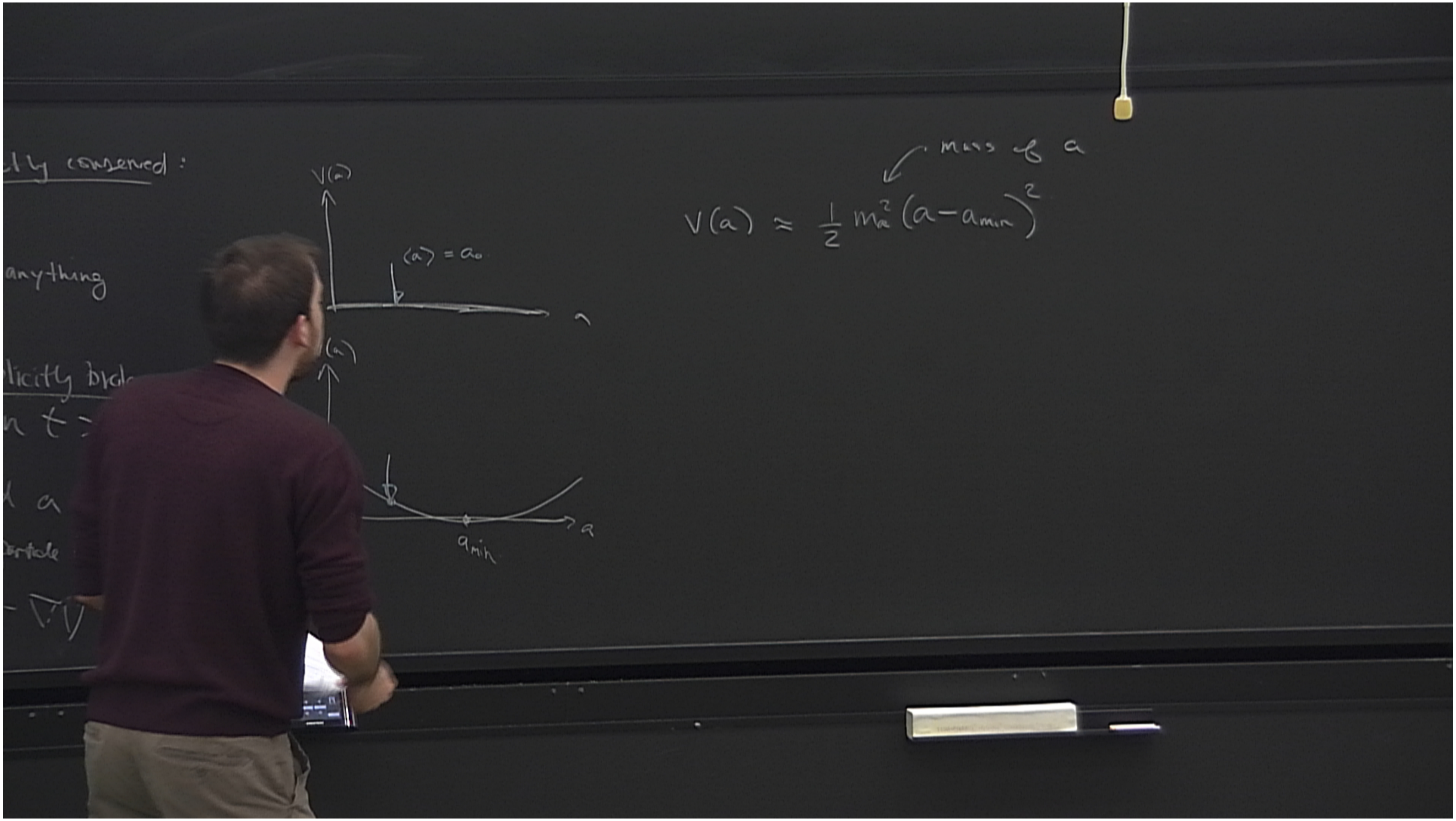
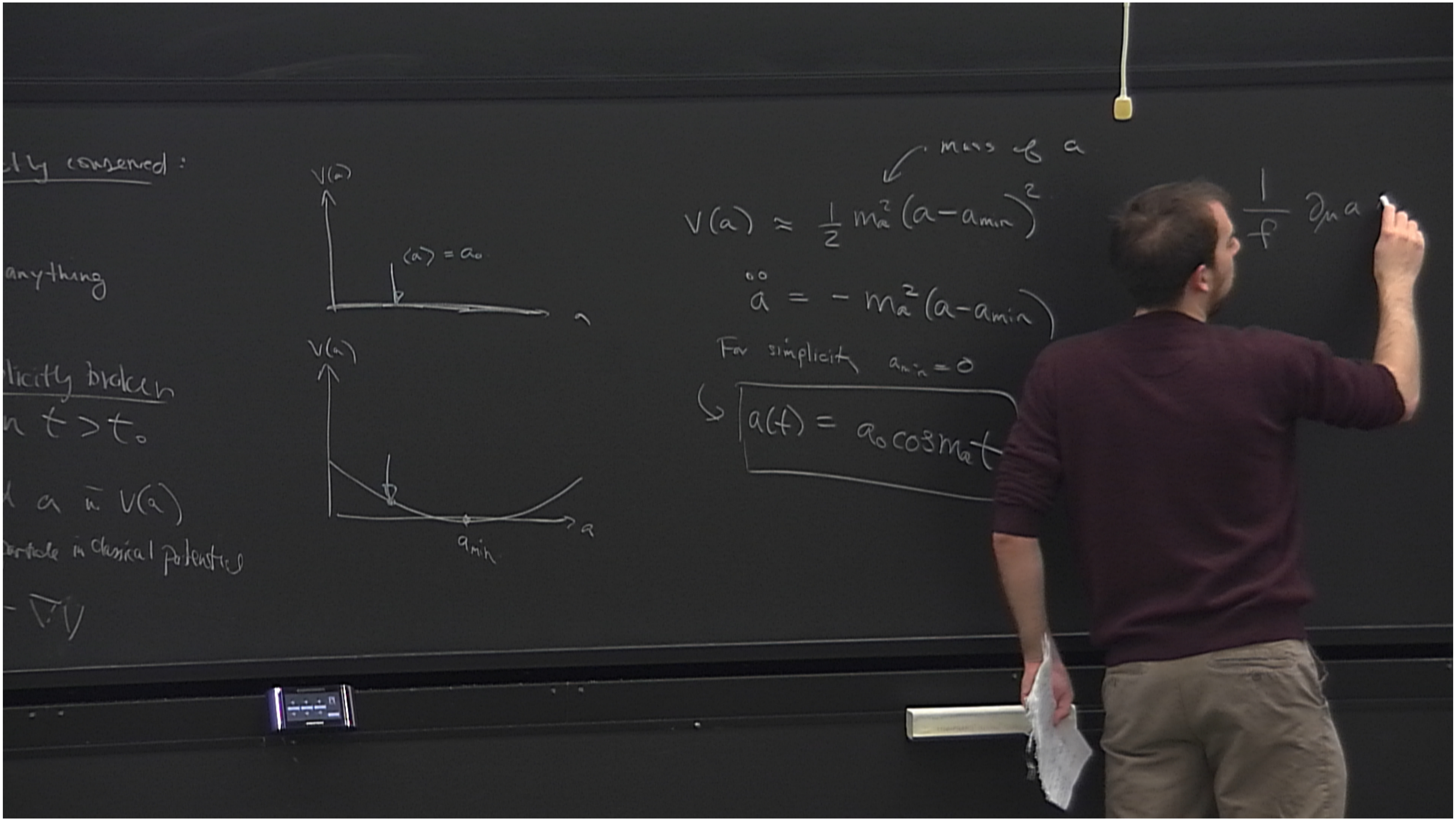

mass of a
$V(a) \simeq \frac{1}{2} m_a^2 (a - a_{min})^2$
V(a)
$\langle a \rangle = a_0$
anything
a_{min}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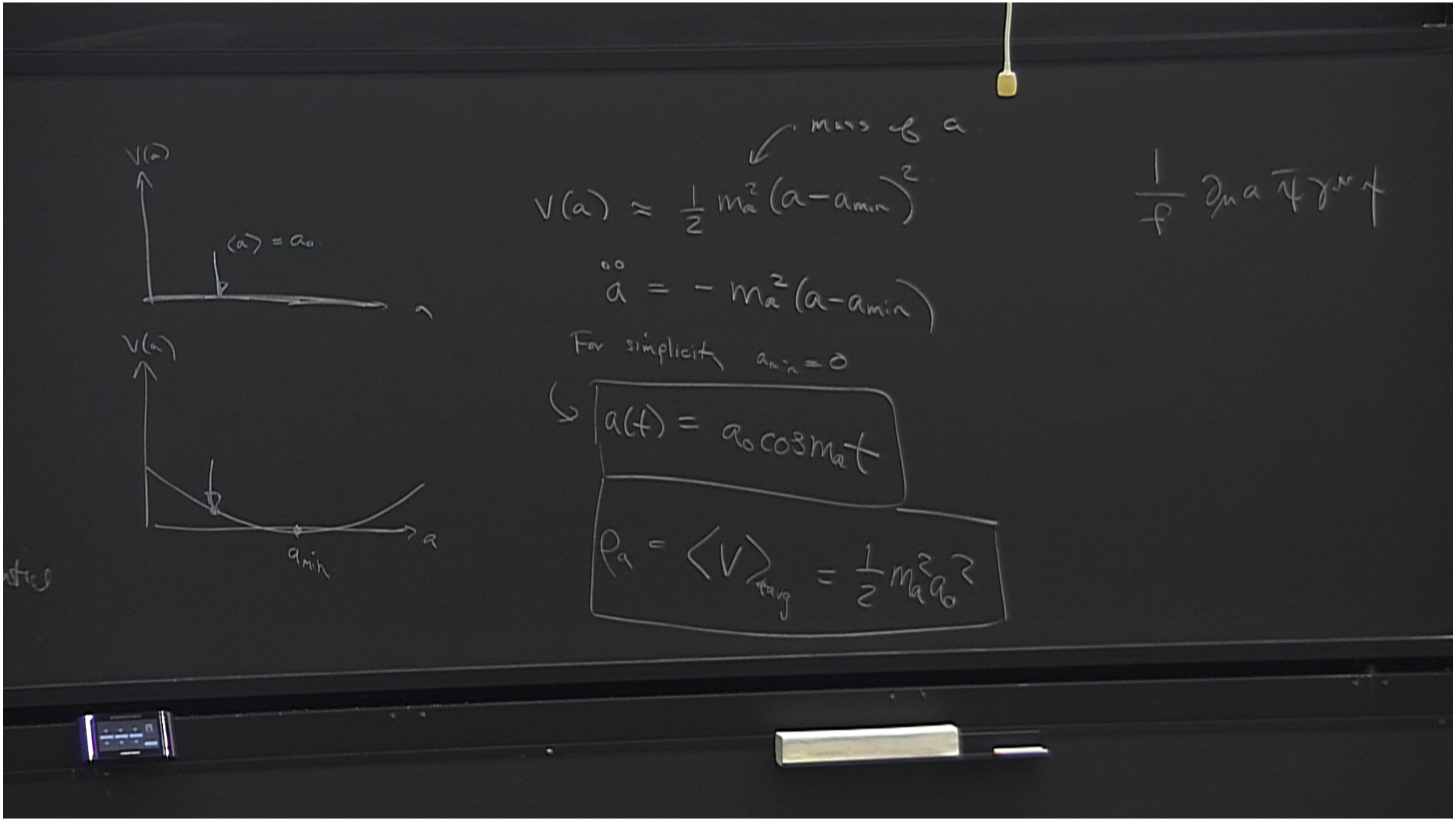
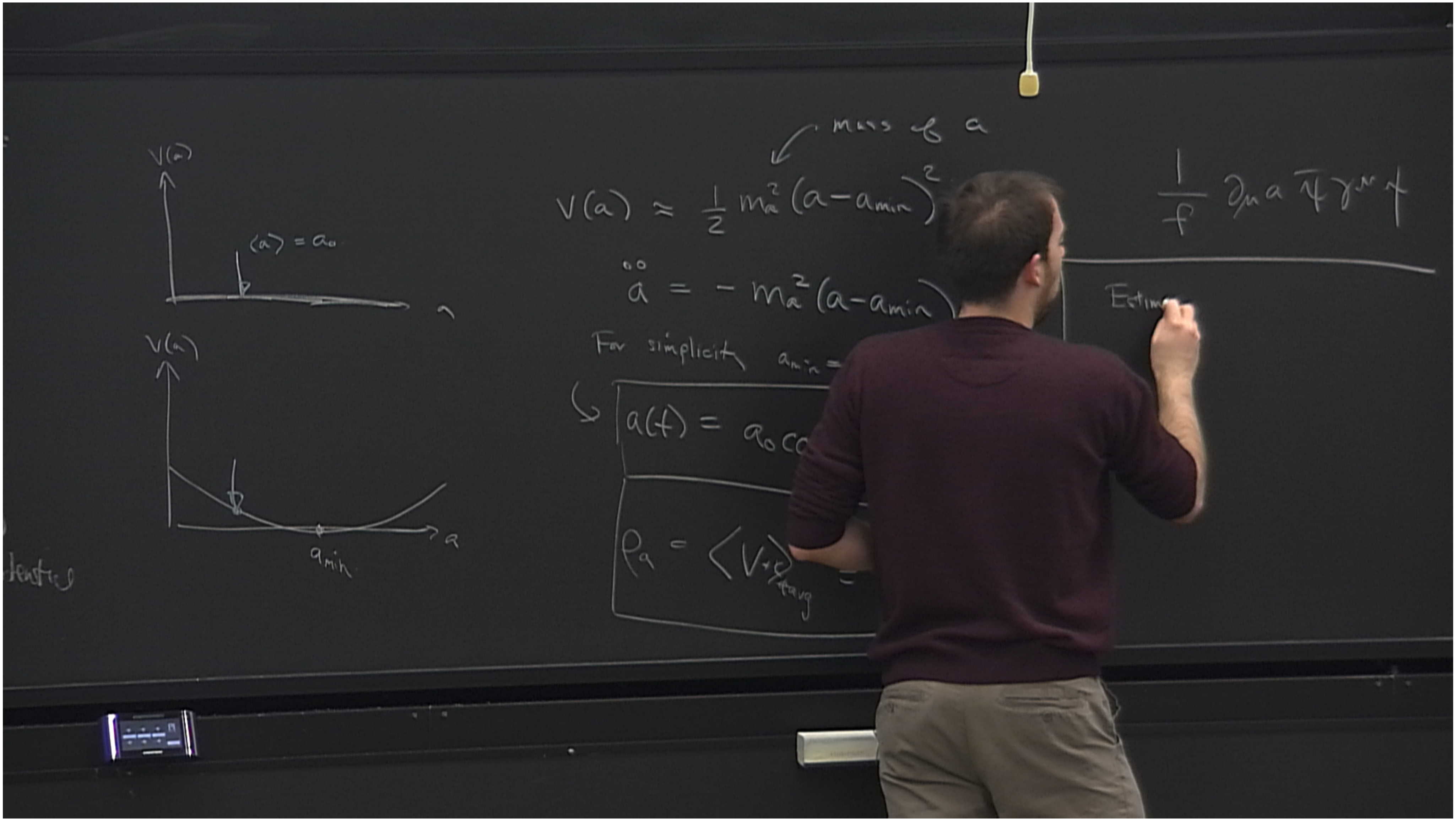
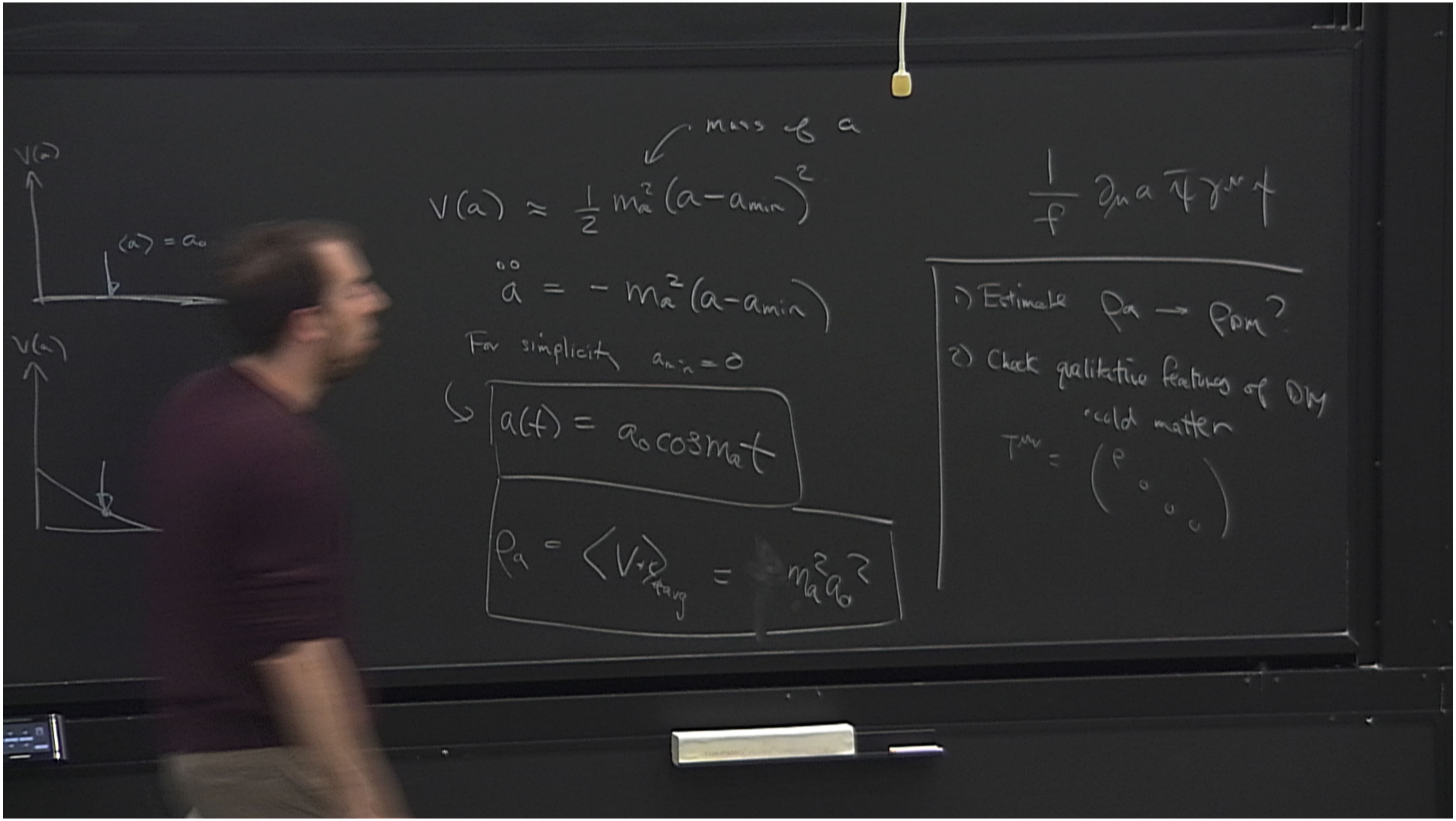
anything
licity broken
n t > t₀
d a in V(a)
particle in classical potential
V(a)
⟨a⟩ = a₀
a
V(a)
a_min
a
mass of a
V(a) ≈ 1/2 m_a² (a − a_min)²
ä = − m_a² (a − a_min)
For simplicity a_min = 0
a(t) = a₀ cos m_a t
1/f ∂_μ a


mass of a
$V(a) \simeq \frac{1}{2} m_a^2 (a - a_{min})^2$
$\ddot{a} = - m_a^2 (a - a_{min})$
For simplicity $a_{min} = 0$
$a(t) = a_0 \cos m_a t$
$\rho_a = \langle V \rangle_{t\,avg} = \frac{1}{2} m_a^2 a_0^2$
$\frac{1}{f} \partial_\mu a \bar{\psi} \gamma^\mu \psi$
$V(a)$
$\langle a \rangle = a_0$
a_{min}



mass of a
V(a)
⟨a⟩ = a₀
V(a)
a
a_min
V(a) ≃ ½ m_a² (a − a_min)²
ä = − m_a² (a − a_min)
For simplicity a_min =
a(t) = a₀ co
ρ_a = ⟨V+
1/f ∂_μ a ψ̄ γ^μ ψ
Estim


mass of a
V(a)
⟨a⟩ = a₀
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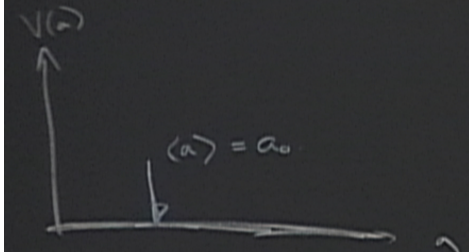
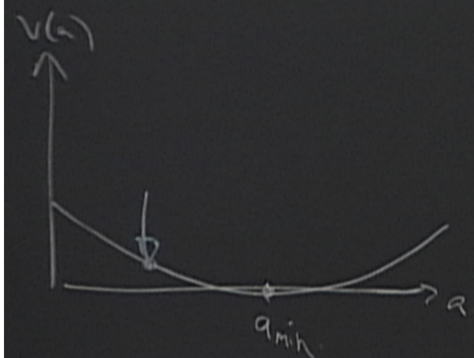
V(a) ≃ ½ m_a² (a − a_min)²
ä = − m_a² (a − a_min)
For simplicity a_min = 0
a(t) = a₀ cos m_a t
ρ_a = ⟨V⟩_t avg = m_a² a₀ ?
1/f ∂_μ a ψ̄ γ^μ ψ
1) Estimate ρ_a → ρ_DM ?
2) Check qualitative features of DM
cold matter
T^μν =





mass of a

$$V(a) \simeq \frac{1}{2} m_a^2 (a - a_{min})^2$$

$$\ddot{a} = - m_a^2 (a - a_{min})$$

For simplicity $a_{min} = 0$

$$\boxed{a(t) = a_0 \cos m_a t}$$

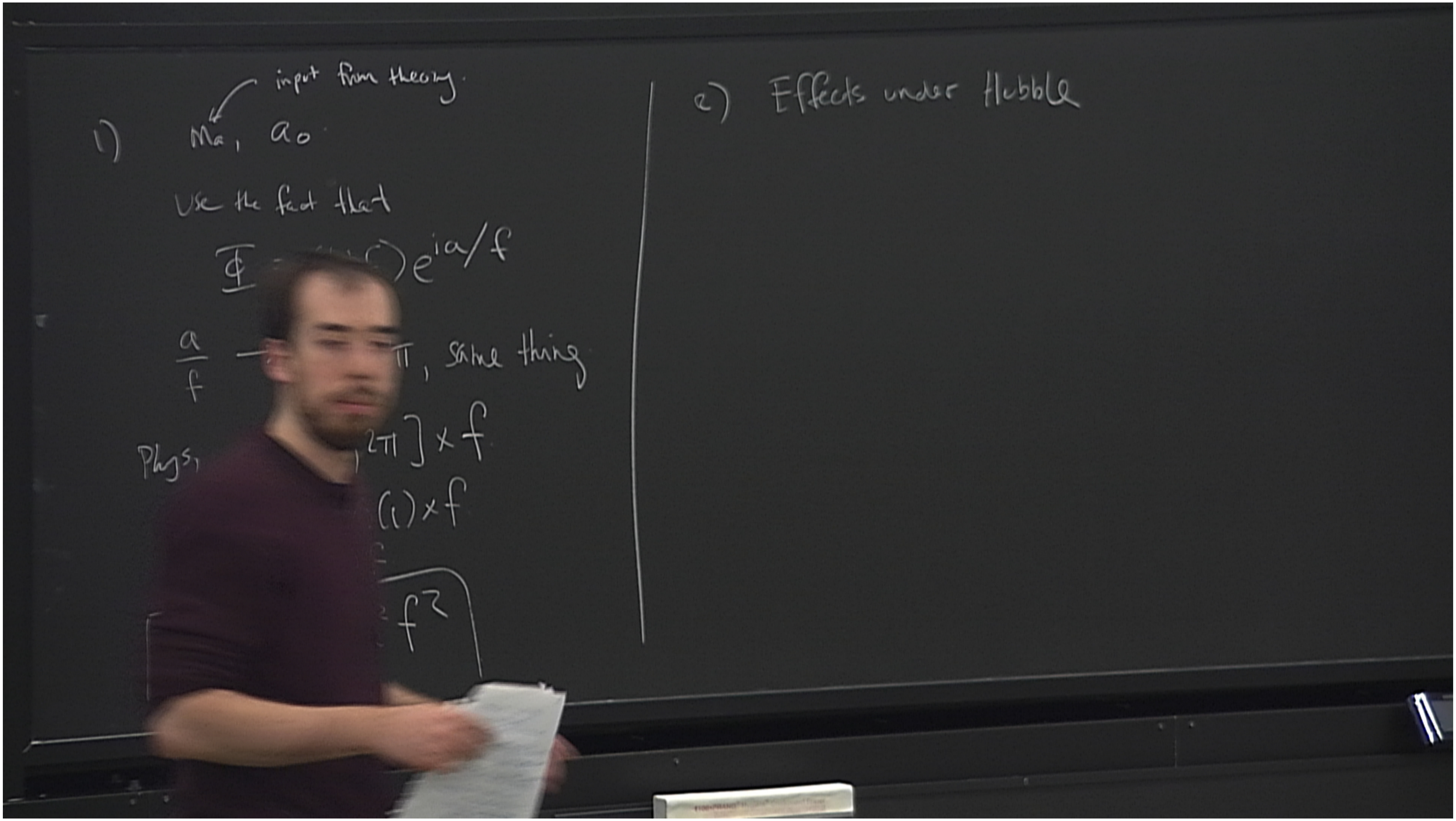
$$\boxed{\rho_a = \langle V + K \rangle_{t\,avg} = m_a^2 a_0^2 ?}$$

$$\frac{1}{f} \partial_\mu a \bar{\psi} \gamma^\mu \psi$$

1) Estimate $\rho_a \to \rho_{DM}$?

2) Check qualitative features of DM "cold matter"

$$T^{\mu\nu} = \begin{pmatrix} \rho & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \Updownarrow \rho_m \sim \left(\frac{1}{a}\right)^3$$


input from theory
1) ma, a0
Use the fact that
same thing
Phys,
2) Effects under Hubble

1) m_a, a_0 input from theory.

Use the fact that

$$\Phi = (\phi + f)e^{ia/f}$$

$$\frac{a}{f} \longrightarrow \frac{a}{f} + 2\pi, \text{ same thing}$$

Phys: $a \in [0, 2\pi] \times f$

$$\mathcal{O}(1) \times f$$

$$a_0 = \mathcal{O}(1) f$$

$$\boxed{\rho_a = \mathcal{O}(1)\, m_a^2 f^2}$$

2) Effects under Hubble

↳ classical field e.o.m.

in FRW background

input from theory.

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Use the fact that

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↳ classical field e.o.m.
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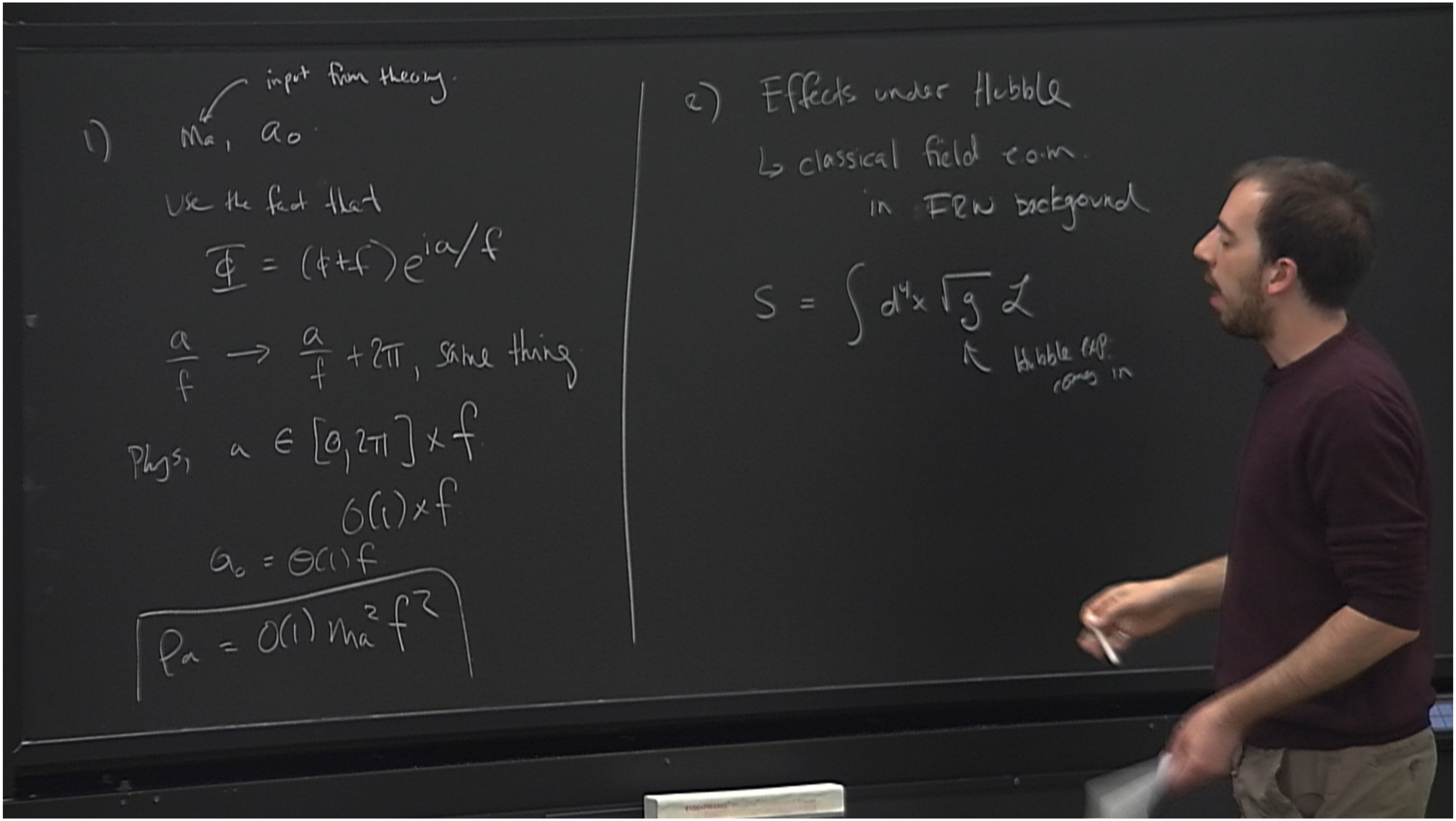
$$S = \int d^4x \sqrt{g}\, \mathcal{L}$$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Hubble exp. comes in

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Hubble exp. comes in

$$\frac{\delta S}{\delta \phi} = 0$$

$$\downarrow$$

$$\boxed{\ddot{a} + 3H\dot{a} + m^2 a = 0}$$

1) m_a, a_0

input from theory.

Use the fact that

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$$\frac{a}{f} \to \frac{a}{f} + 2\pi, \text{ same thing}$$

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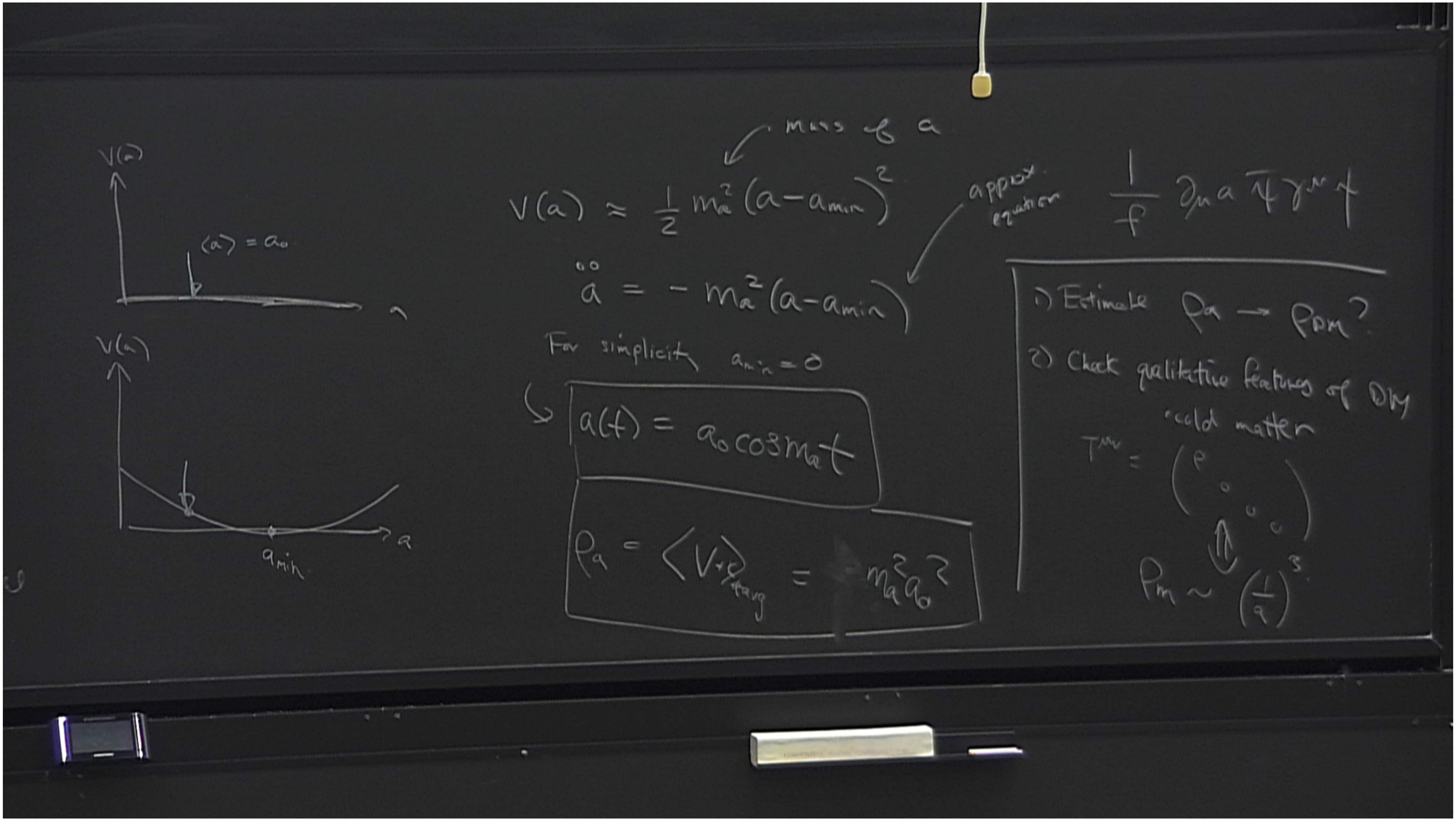
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approx. equation
$\frac{1}{f} \partial_\mu a \bar{\psi} \gamma^\mu \psi$
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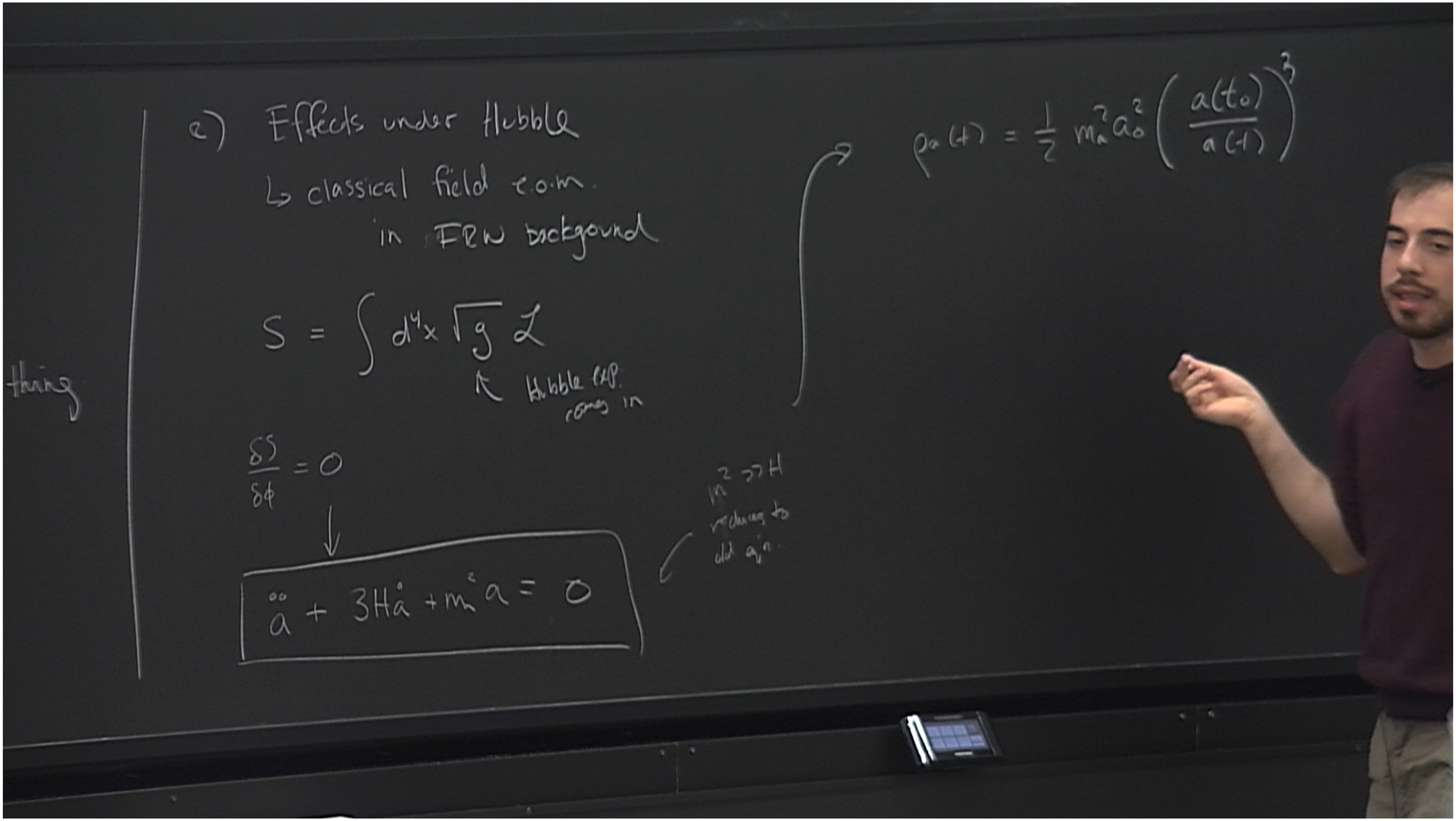
$\rho_m \sim \left(\frac{1}{a}\right)^3$


thing
2) Effects under Hubble
↳ classical field e.o.m.
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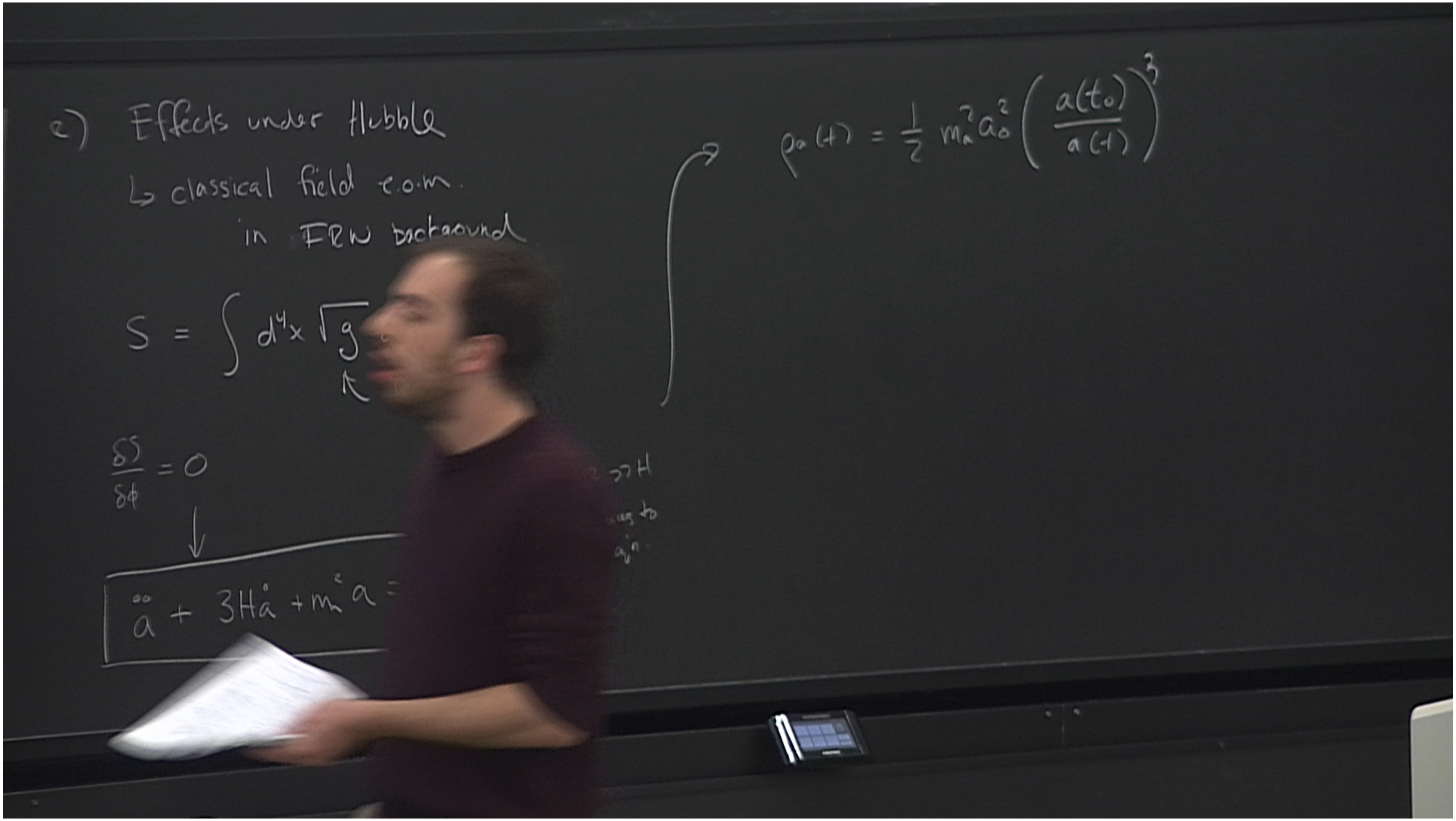
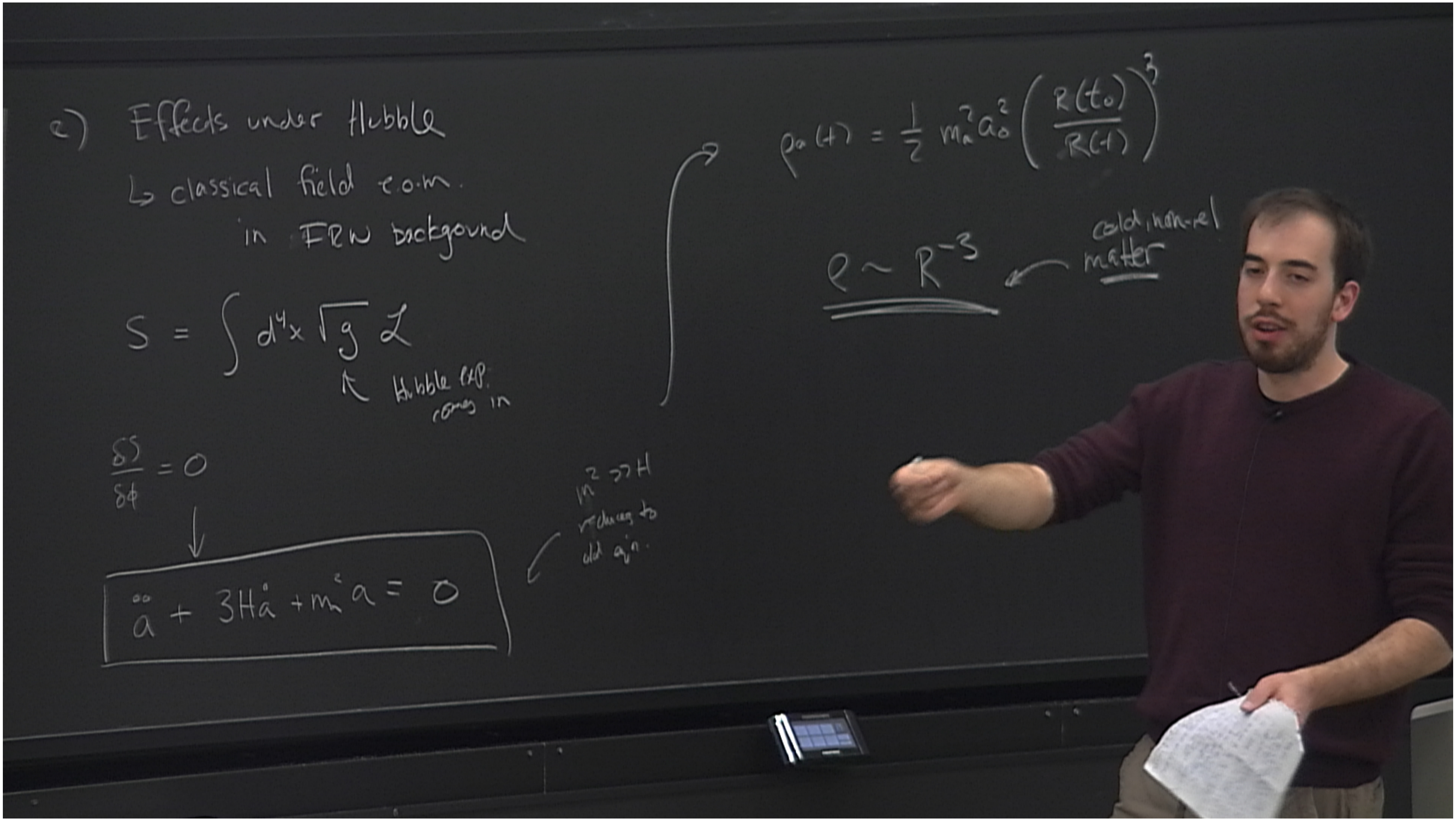
Hubble exp. comes in
$\frac{\delta S}{\delta \phi} = 0$
$\boxed{\ddot{a} + 3H\dot{a} + m_a^2 a = 0}$
$m^2 >> H$
reducing to
old eq^n.
$\rho_a(t) = \frac{1}{2} m_a^2 a_0^2 \left(\frac{a(t_0)}{a(t)} \right)^3$


c) Effects under Hubble
↳ classical field e.o.m.
in FRW background
$S = \int d^4x \sqrt{g}$
$\frac{\delta S}{\delta \phi} = 0$
$\ddot{a} + 3H\dot{a} + m_a^2 a =$
$\rho_a(t) = \frac{1}{2} m_a^2 a_0^2 \left(\frac{a(t_0)}{a(t)} \right)^3$


c) Effects under Hubble
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in FRW background
$S = \int d^4x \sqrt{g}\, \mathcal{L}$
Hubble exp. comes in
$\frac{\delta S}{\delta \phi} = 0$
$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$
$m^2 >> H$
reduces to
old eqn.
$\rho_a(t) = \frac{1}{2} m_a^2 a_0^2 \left(\frac{R(t_0)}{R(t)}\right)^3$
$\rho \sim R^{-3}$
cold, non-rel
matter

2) Effects under Hubble

↳ classical field e.o.m.
in FRW background

$$S = \int d^4x \sqrt{g}\, \mathcal{L}$$

Hubble exp. coming in

$$\frac{\delta S}{\delta \phi} = 0$$

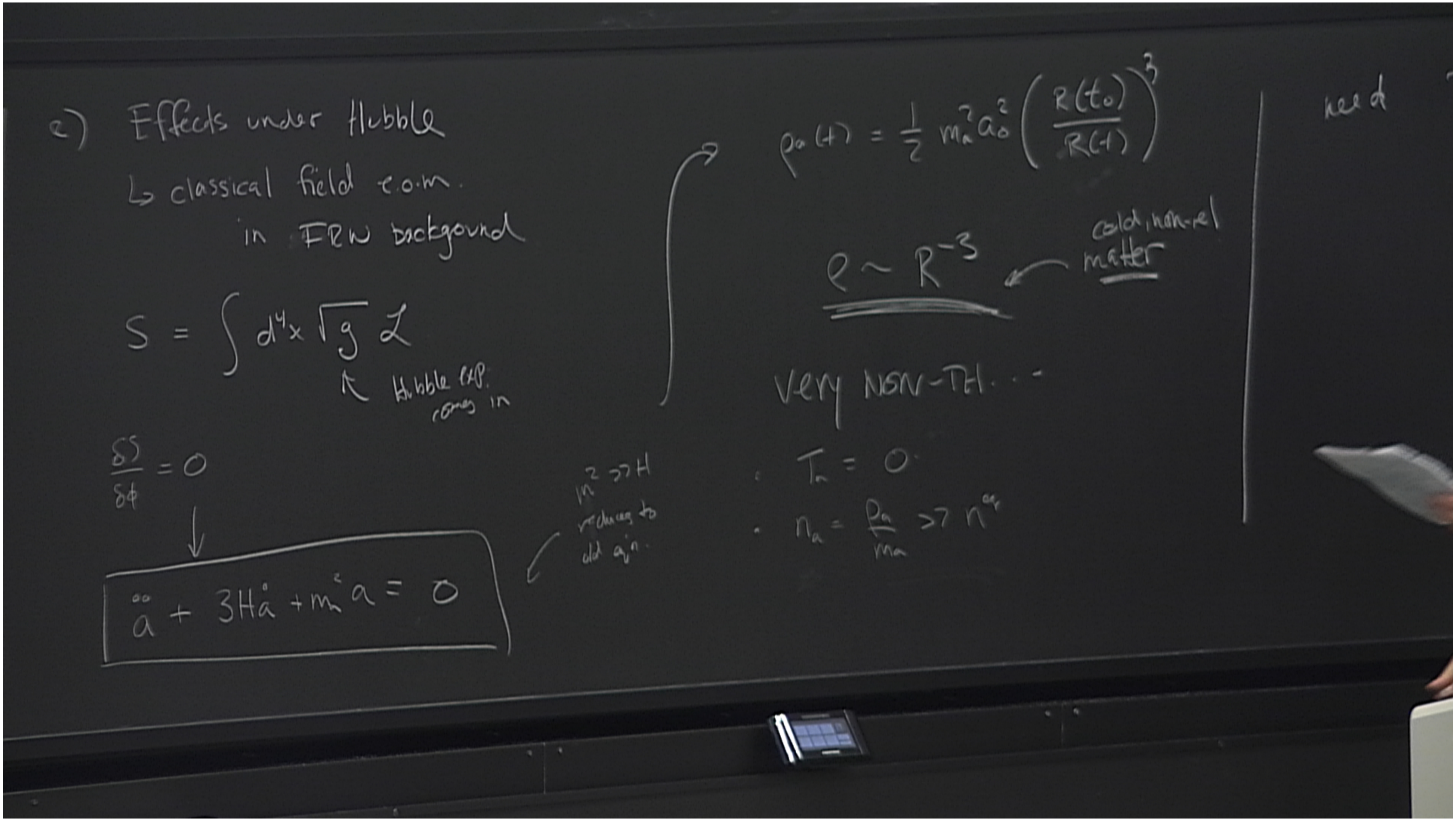
$$\boxed{\ddot{a} + 3H\dot{a} + m_a^2 a = 0}$$

$m^2 \gg H$ reducing to old eqn.

$$\rho_a(t) = \frac{1}{2} m_a^2 a_0^2 \left(\frac{R(t_0)}{R(t)} \right)^3$$

$$\rho \sim R^{-3}$$

cold, non-rel matter


c) Effects under Hubble
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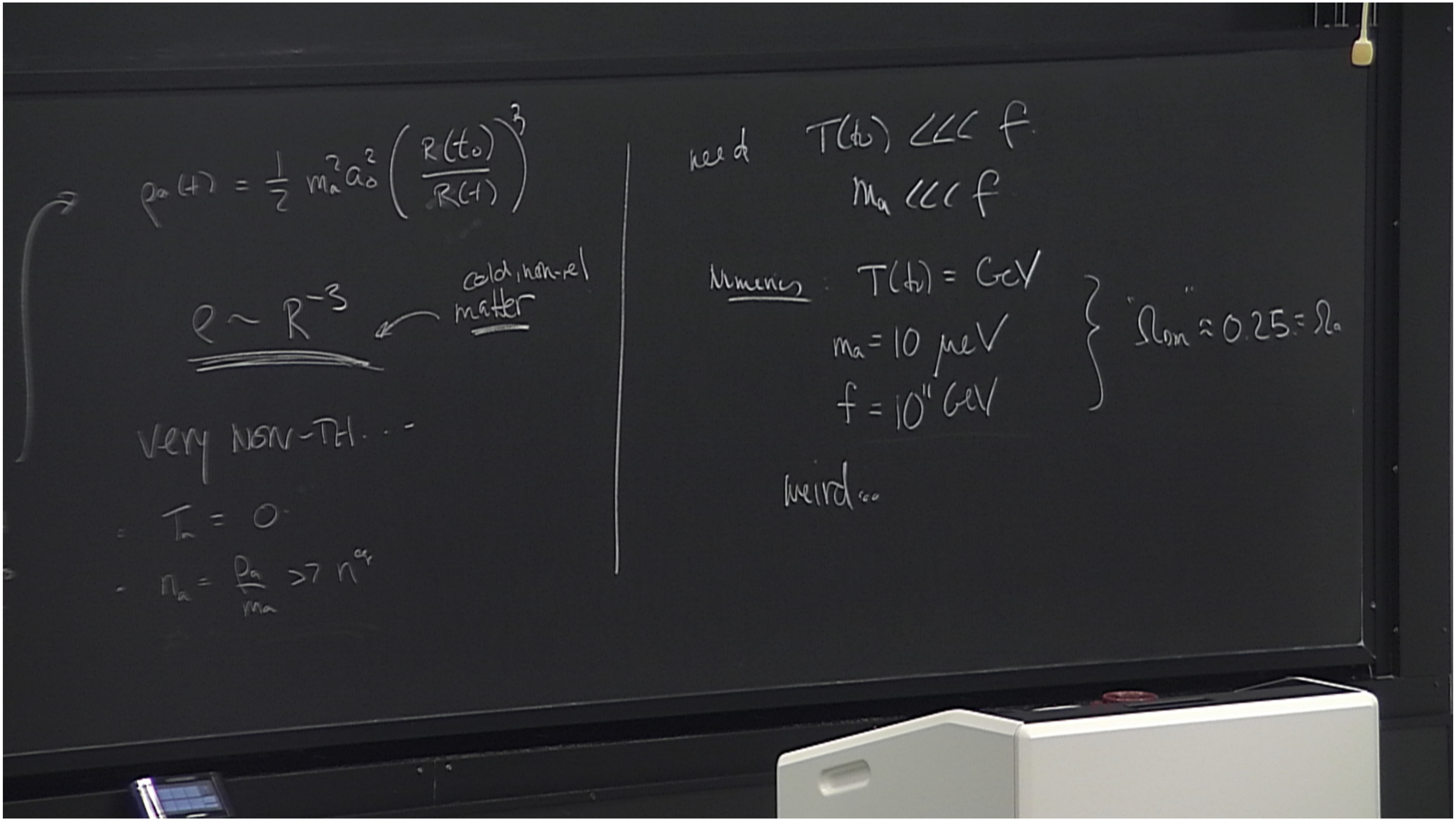
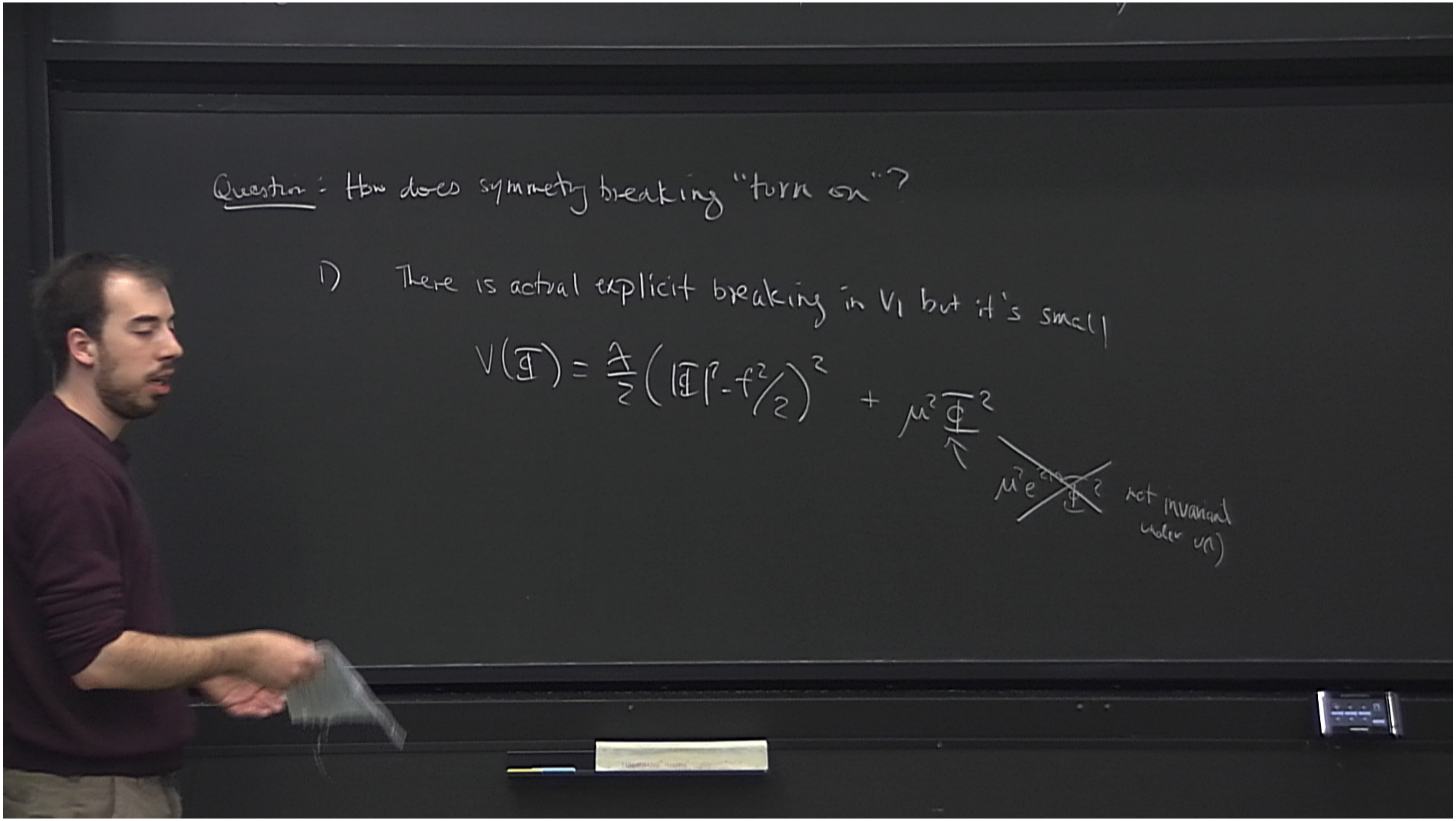
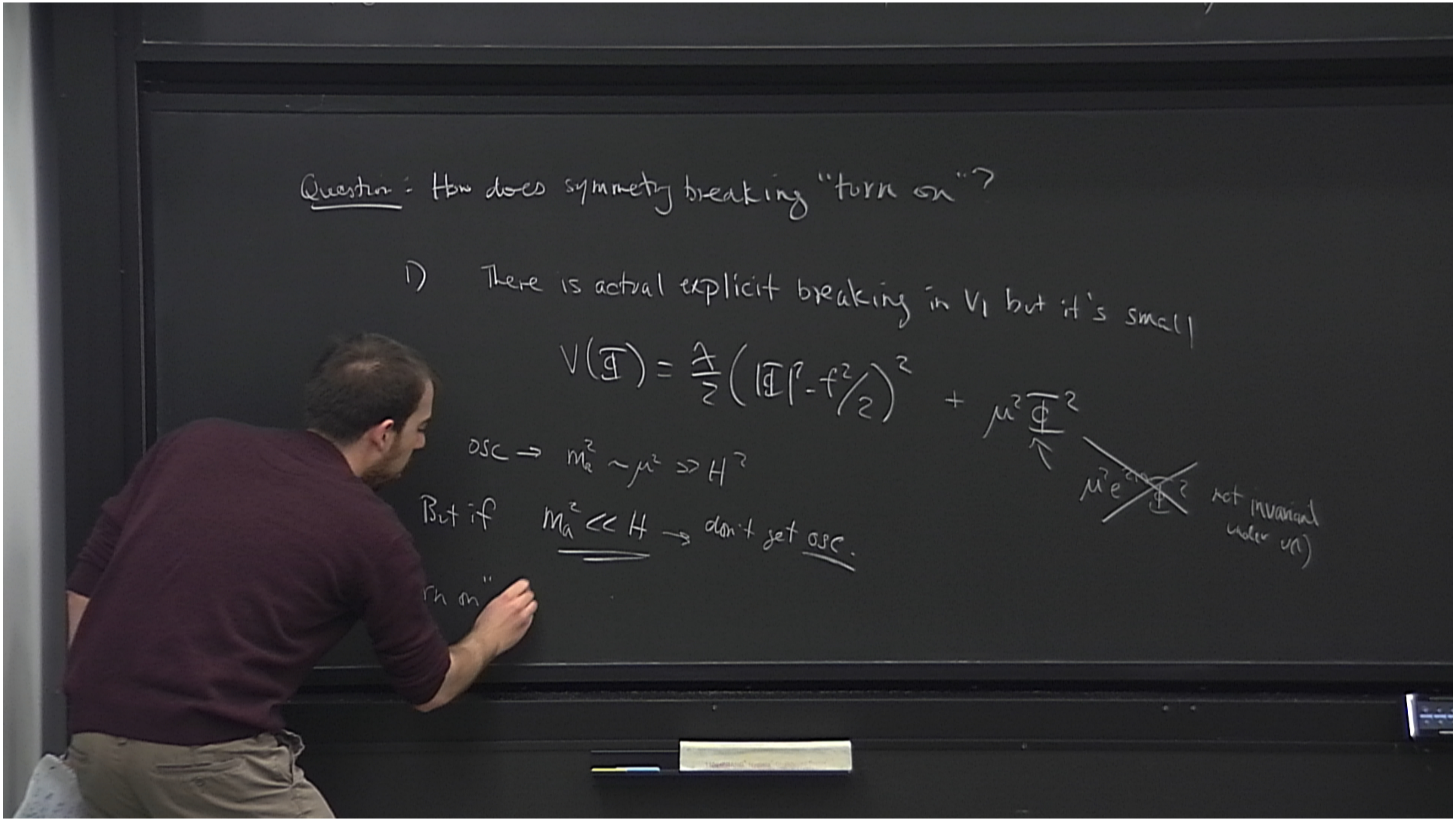
S = ∫ d^4x √g 𝓛
Hubble exp. comes in
δS/δφ = 0
ä + 3Hȧ + m_a² a = 0
m² >> H
reducing to
old eq'n.
ρ_a(t) = 1/2 m_a² a_0² (R(t_0)/R(t))^3
ρ ~ R^-3
cold, non-rel
matter
very non-TH...
T_a = 0
n_a = ρ_a/m_a >> n^eq
need


ρa(t) = 1/2 ma² a0² (R(t0)/R(t))³
ρ ~ R⁻³
cold, non-rel matter
very non-TH...
Ta = 0
na = ρa/ma >> n^eq
need T(t0) <<< f
ma <<< f
Numerics: T(t0) = GeV
ma = 10 μeV
f = 10¹¹ GeV
"ΩDM" ≈ 0.25 = Ωa
weird...


Question: How does symmetry breaking "turn on"?
1) There is actual explicit breaking in V, but it's small
$V(\Phi) = \frac{\lambda}{2}\left(|\Phi|^2 - f^2/2\right)^2 + \mu^2 \Phi^2$
$\cancel{\mu^2 e^{2i\alpha} \Phi^2}$ not invariant under $U(1)$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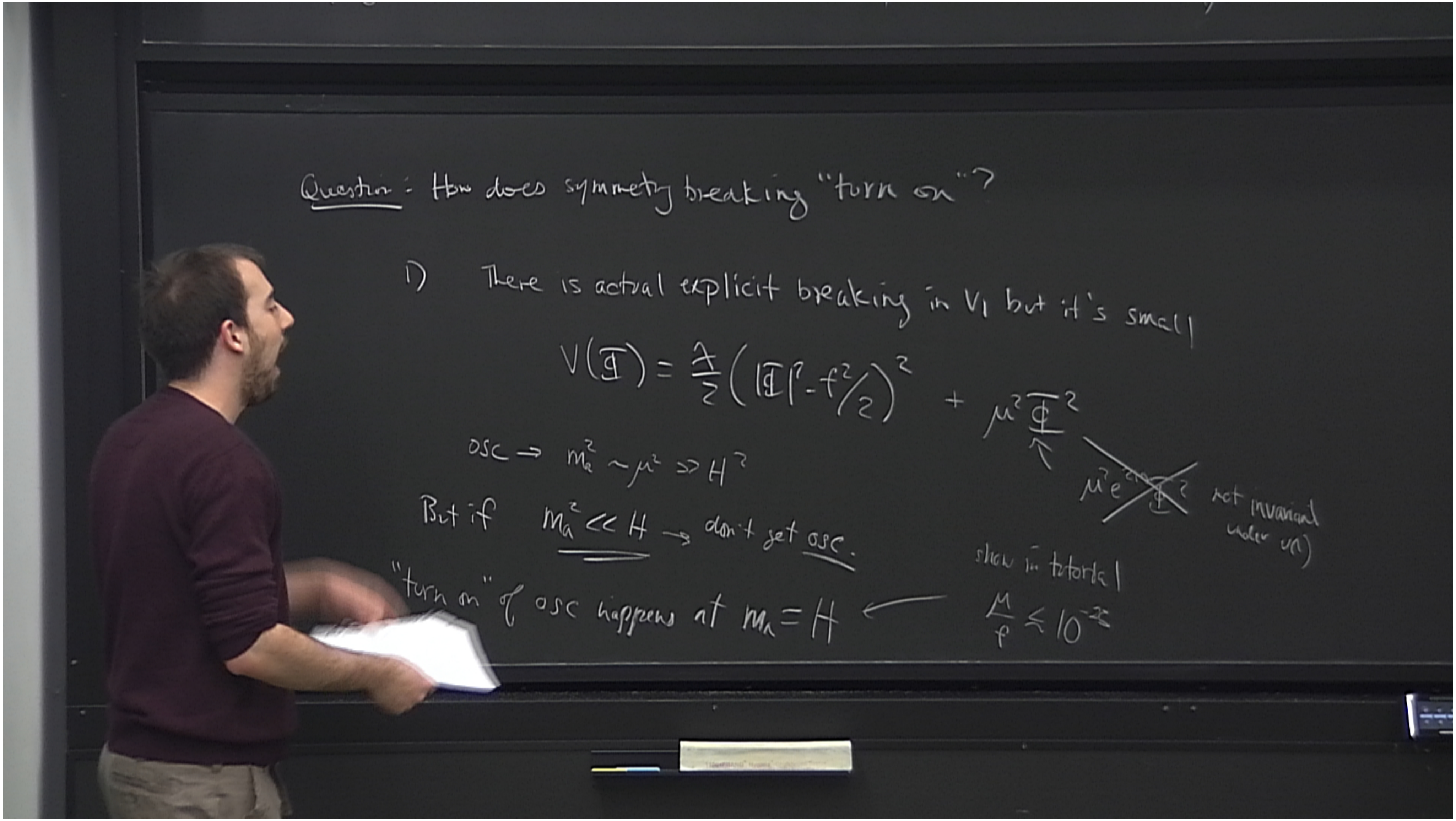
$\cancel{\mu^2 e^{2i\alpha} \Phi^2}$
not invariant under $U(1)$
osc $\to$ $m_a^2 \sim \mu^2 \gg H^2$
But if $m_a^2 \ll H$ $\to$ don't get osc.
rn on"


Question: How does symmetry breaking "turn on"?
1) There is actual explicit breaking in V, but it's small
$V(\Phi) = \frac{\lambda}{2}\left(|\Phi|^2 - f^2/2\right)^2 + \mu^2 \Phi^2$
$\cancel{\mu^2 e^{2i\theta} \Phi^2}$ not invariant under U(1)
osc → $m_a^2 \sim \mu^2 \gg H^2$
But if $m_a^2 \ll H$ → don't get osc.
"turn on" of osc happens at $m_a = H$
show in tutorial
$\frac{\mu}{f} \lesssim 10^{-3}$

Question: How does symmetry breaking "turn on"?

1) There is actual explicit breaking in V, but it's small

$$V(\Phi) = \frac{\lambda}{2}\left(|\Phi|^2 - f^2/2\right)^2 + \mu^2 \Phi^2$$

$$\cancel{\mu^2 e^{2i\alpha} \Phi^2}$$ not invariant under $U(1)$

osc $\Rightarrow$ $m_a^2 \sim \mu^2 \gg H^2$

But if $m_a^2 \ll H$ $\rightarrow$ don't get osc.

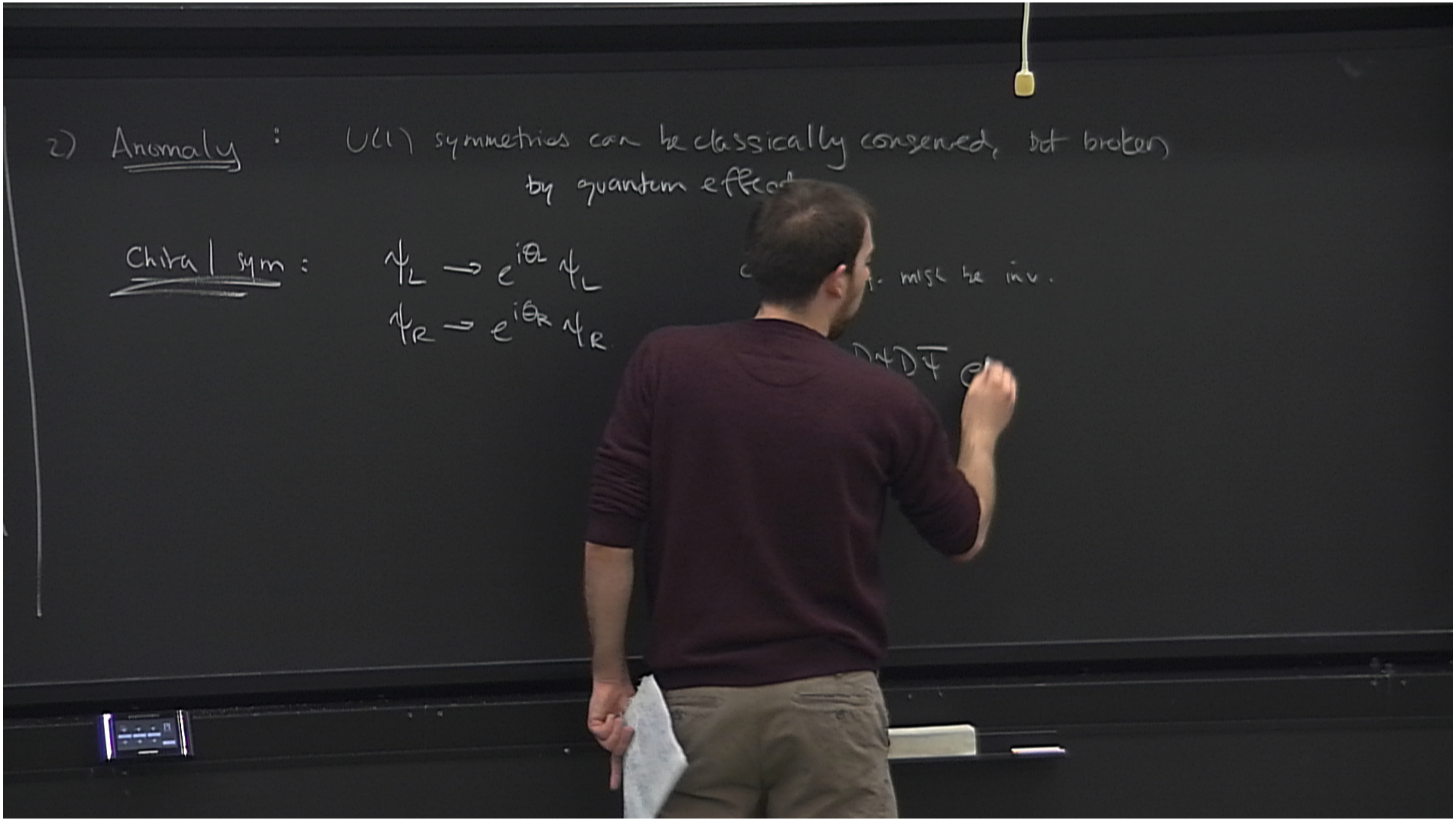
"turn on" of osc happens at $m_a = H$ $\leftarrow$ show in tutorial

$$\frac{\mu}{f} \lesssim 10^{-25}$$

2) Ar

2) Anomaly : $U(1)$ symmetries can be classically conserved, but broken by quantum effects.

Chiral sym :


2) Anomaly : U(1) symmetries can be classically conserved, but broken
by quantum effect
Chiral sym : $\psi_L \to e^{i\theta_L} \psi_L$
$\psi_R \to e^{i\theta_R} \psi_R$
must be inv.

2) Anomaly : U(1) symmetries can be classically conserved, but broken by quantum effects.

Chiral sym : $\psi_L \to e^{i\theta_L} \psi_L$ classical Lag. must be inv.

$\psi_R \to e^{i\theta_R} \psi_R$

$$Z = \int \underbrace{D\psi D\bar{\psi}}_{\text{not inv.}} e^{i \overbrace{S_{\text{clas}}}^{\text{inv.}}}$$

2) <u>Anomaly</u>: $U(1)$ symmetries can be classically conserved, but broken by quantum effects.

<u>Chiral sym</u>:

$$\psi_L \to e^{i\theta_L}\psi_L$$

$$\psi_R \to e^{i\theta_R}\psi_R$$

classical Lag. must be inv.

$$Z = \int \underbrace{D\psi D\bar{\psi}}_{\text{not inv.}}\, e^{i\overbrace{S_{class}}^{\text{inv.}}}$$

Effect of anomaly: $\mathcal{L}_{class} \to \mathcal{L}_{class} + \frac{\alpha_G}{4\pi}(\theta_L - \theta_R)\,\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$

2) Anomaly: U(1) symmetries can be classically conserved, but broken by quantum effects.

Chiral sym:

$$\psi_L \to e^{i\theta_L}\psi_L$$
$$\psi_R \to e^{i\theta_R}\psi_R$$

classical Lag. must be inv.

$$Z = \int \underbrace{D\psi D\bar{\psi}}_{\text{not inv.}}\, e^{i\overbrace{S_{class}}^{\text{inv.}}}$$

Effect of anomaly: $\mathcal{L}_{class} \to \mathcal{L}_{class} + \frac{\alpha_G}{4\pi}(\theta_L - \theta_R)\,\mathrm{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$ $\quad \tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$

↳ shift symmetry broken → Goldstone acquires a mass

How does anomaly → m_a??

2) Anomaly : U(1) symmetries can be classically conserved, but broken by quantum effects.

Chiral sym : $\psi_L \to e^{i\theta_L}\psi_L$ classical Lag. must be inv.

$\psi_R \to e^{i\theta_R}\psi_R$

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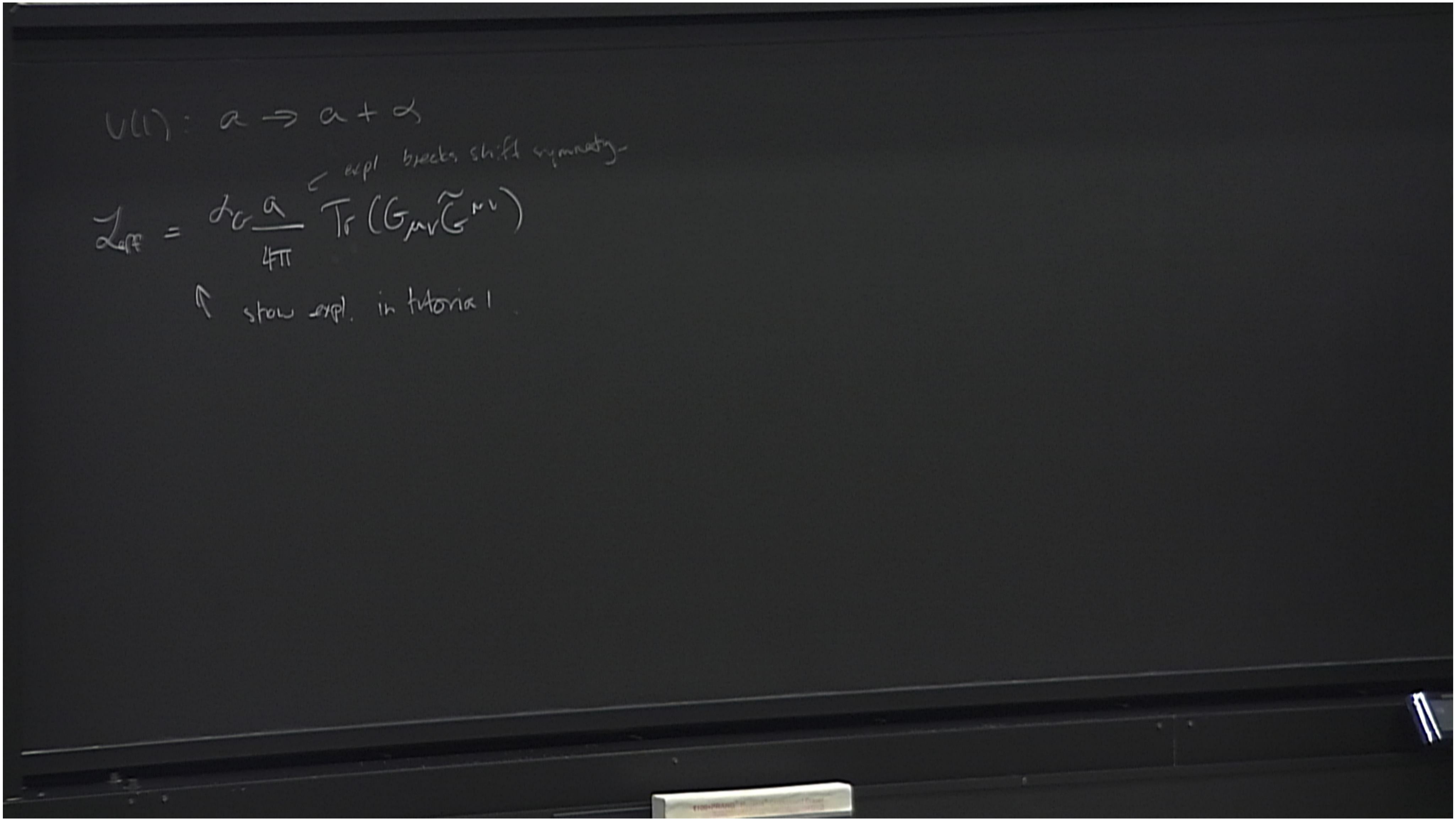
↳ shift symmetry broken → Goldstone acquires a mass

How does anomaly → m_a??

$U(1): a \to a + \alpha$

$$\mathcal{L}_{eff} = \alpha_G \frac{a}{4\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

↑ show expl. in tutorial


$U(1): a \to a + \alpha$
expl breaks shift symmetry
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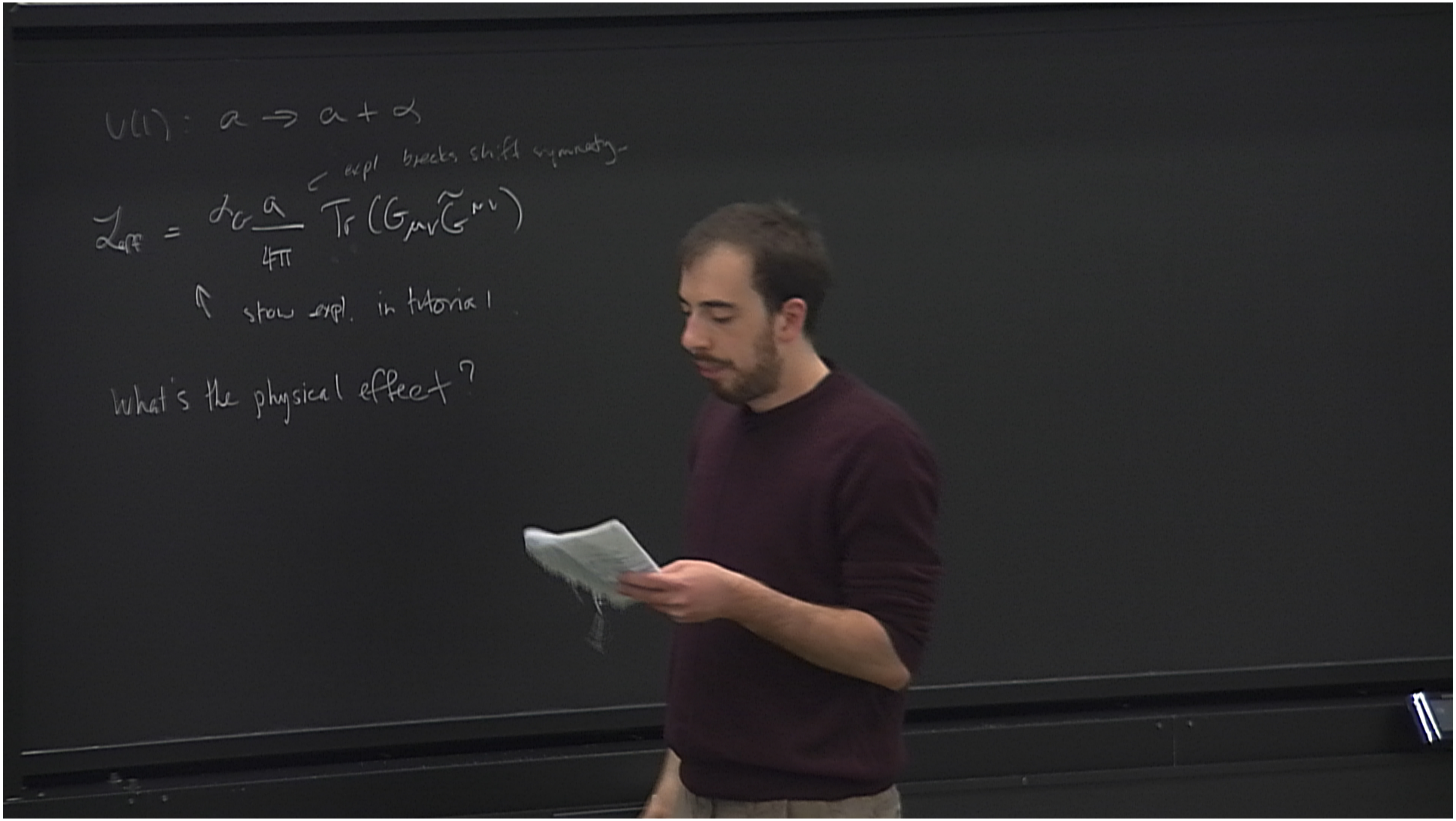
show expl. in tutorial



$U(1): a \Rightarrow a + \alpha$
expl breaks shift symmetry.
$\mathcal{L}_{eff} = \alpha_G \frac{a}{4\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$
show expl. in tutorial
What's the physical effect?

$U(1): a \Rightarrow a + \alpha$

← expl breaks shift symmetry

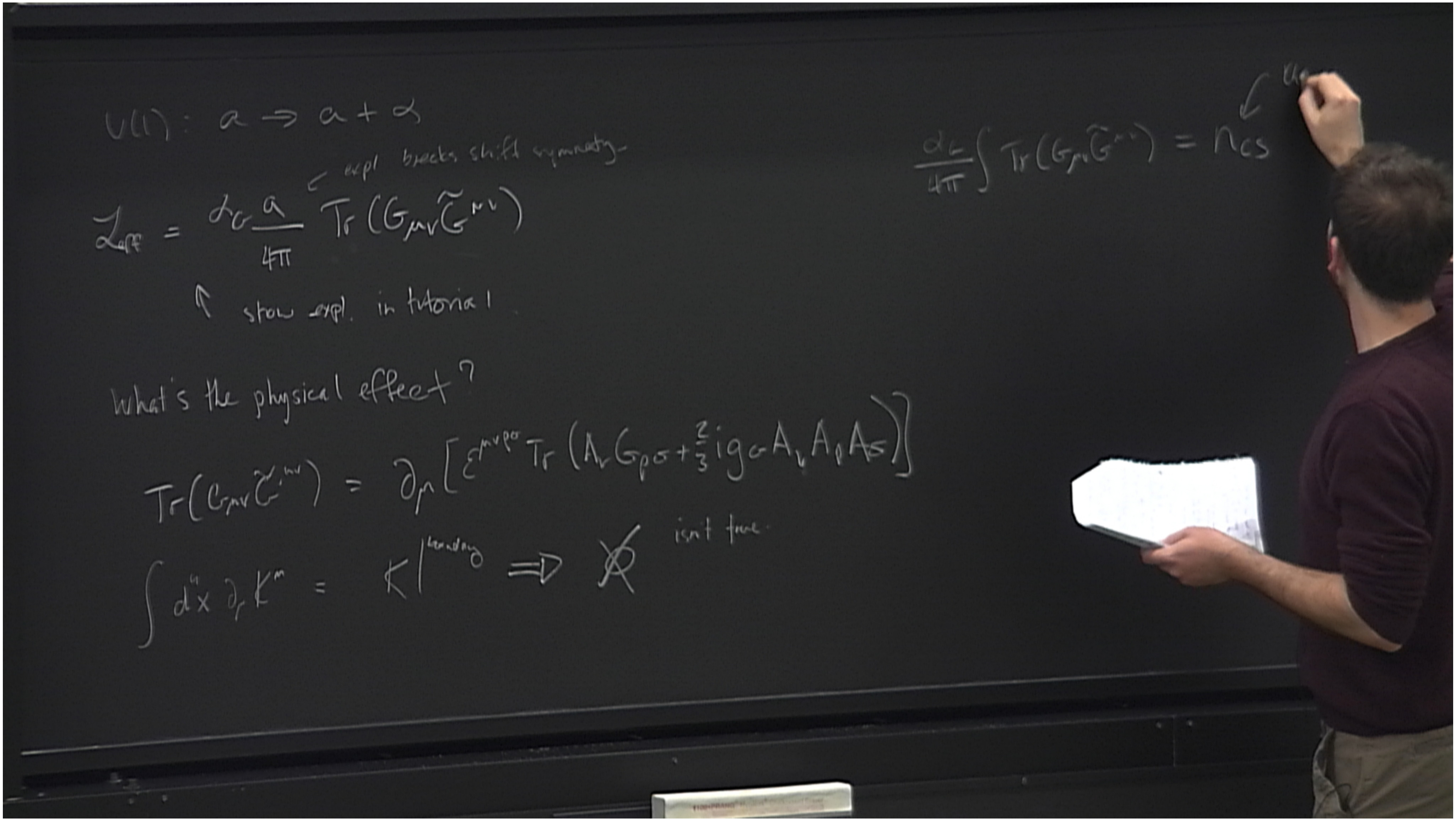
$$\mathcal{L}_{eff} = \alpha_G \frac{a}{4\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

↑ show expl. in tutorial

What's the physical effect?

$$\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) = \partial_\mu\left[\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(A_\nu G_{\rho\sigma} + \tfrac{2}{3} i g_G A_\nu A_\rho A_\sigma\right)\right]$$

$$\int d^4x\, \partial_\mu K^\mu = K\Big|^{\text{boundary}} \Rightarrow 0$$


$U(1): a \Rightarrow a + \alpha$
expl breaks shift symmetry.
$\mathcal{L}_{eff} = \alpha_G \frac{a}{4\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$
show expl. in tutorial
What's the physical effect?
$\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) = \partial_\mu\left[\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(A_\nu G_{\rho\sigma} + \frac{2}{3} i g_G A_\nu A_\rho A_\sigma\right)\right]$
$\int d^4x\, \partial_\mu K^\mu = K|^{\text{boundary}} = 0 \Rightarrow \cancel{\alpha}$ isn't true.
$\frac{\alpha_G}{4\pi}\int \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) = n_{CS}$

$U(1): a \Rightarrow a + \alpha$

explicit breaks shift symmetry

$$\mathcal{L}_{eff} = \alpha_G \frac{a}{4\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

show expl. in tutorial

Chern-Simons number

$$\frac{\alpha_G}{4\pi}\int \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) = n_{CS}$$

What's the physical effect?

$$\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) = \partial_\mu\left[\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(A_\nu G_{\rho\sigma} + \frac{2}{3} i g_G A_\nu A_\rho A_\sigma\right)\right]$$

$$\int d^4x\, \partial_\mu K^\mu = K\Big|^{\text{boundary}} 0 \Longrightarrow \cancel{\alpha}$$

isn't true.

$U(1): a \Rightarrow a + \alpha$

expl breaks shift symmetry

$$\mathcal{L}_{eff} = \alpha_G \frac{a}{4\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

show expl. in tutorial

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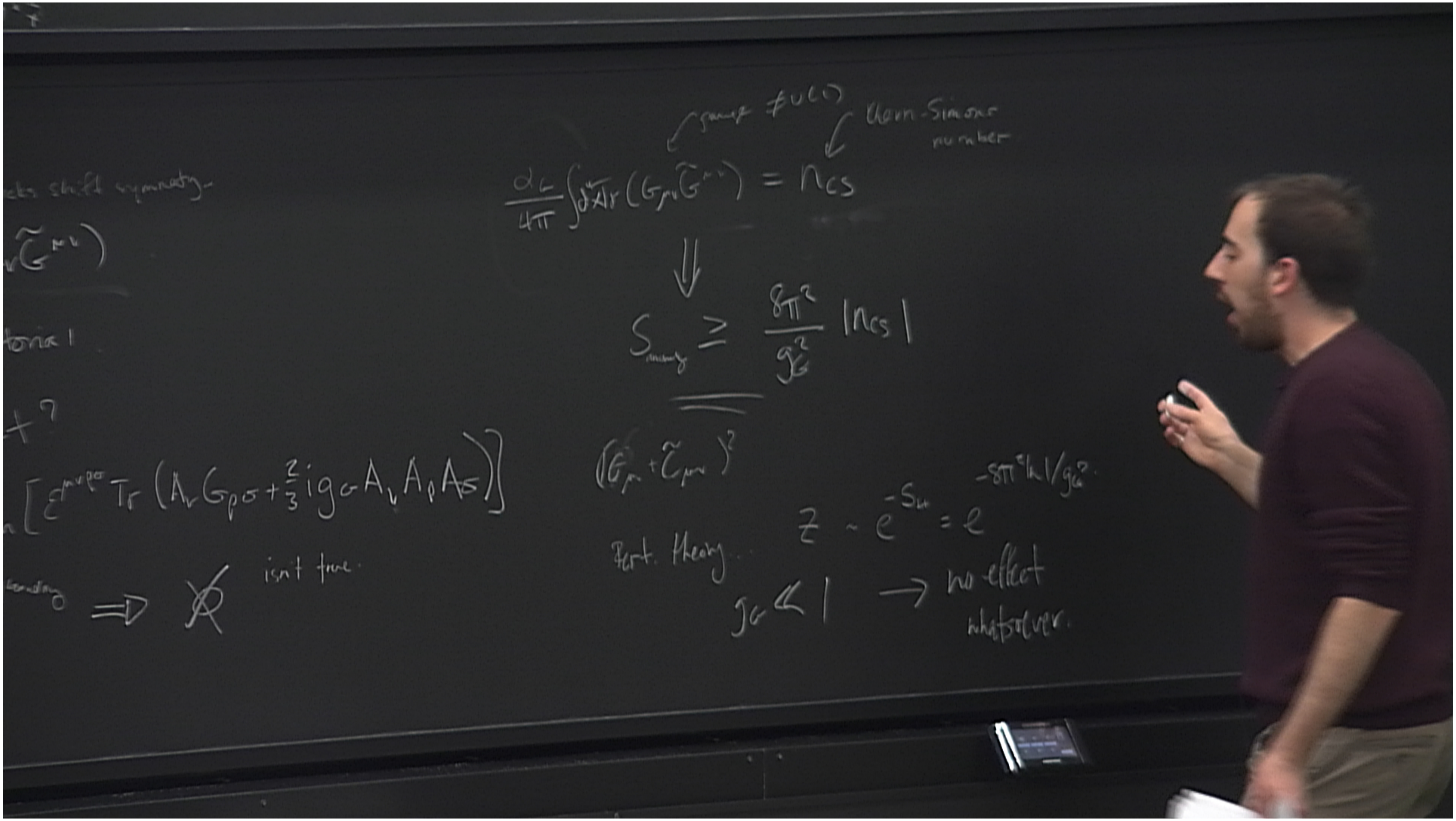
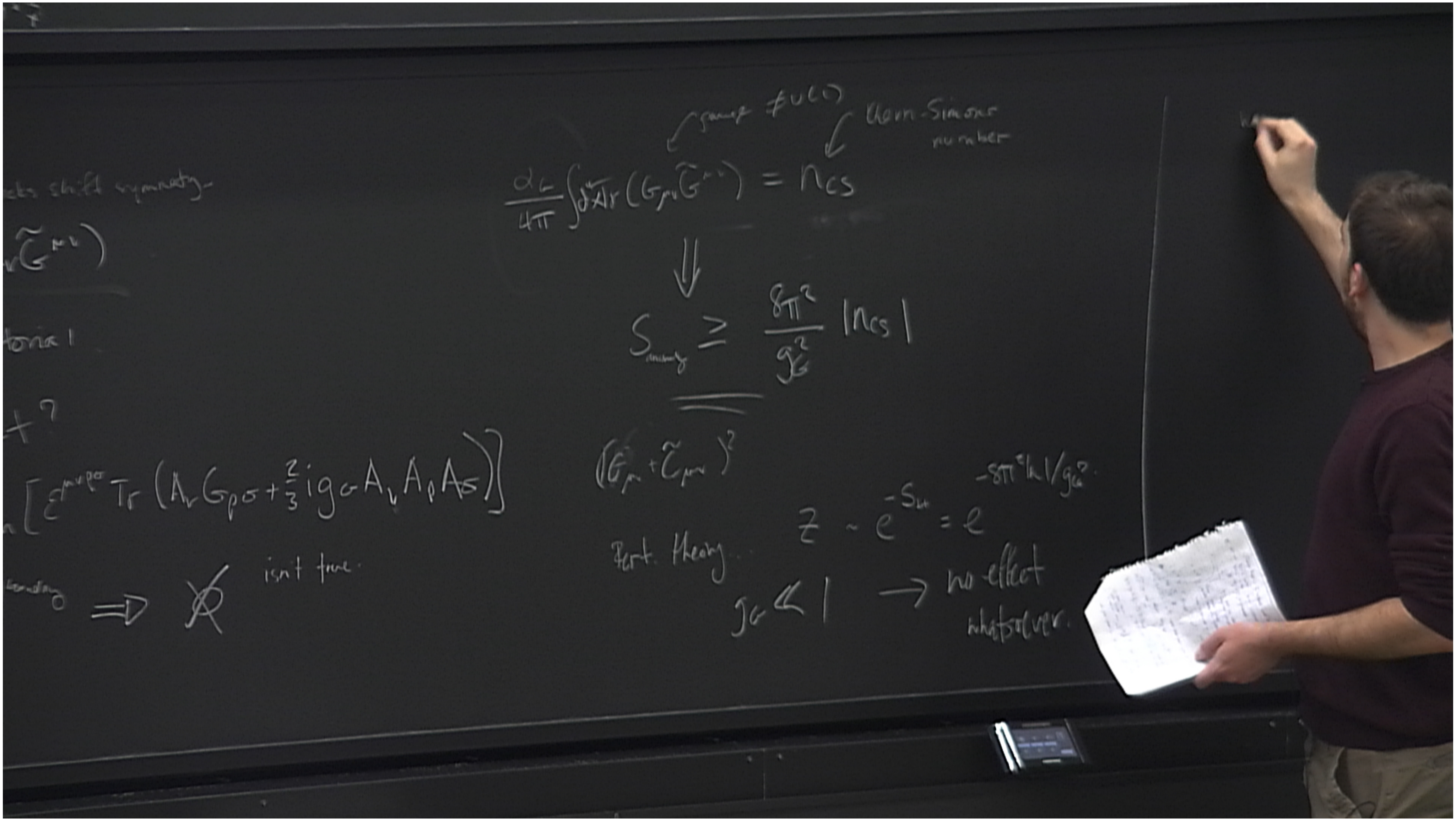
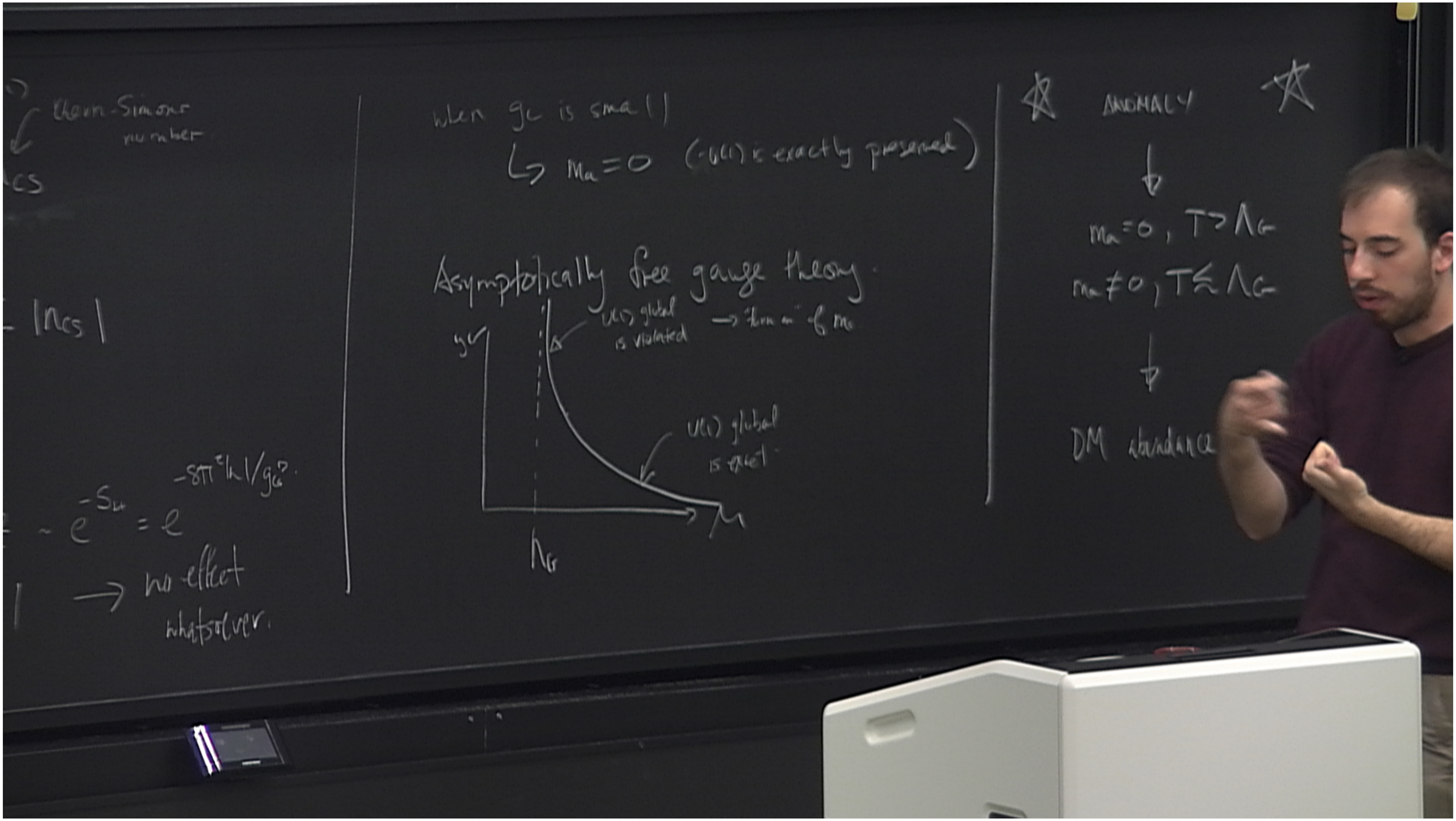
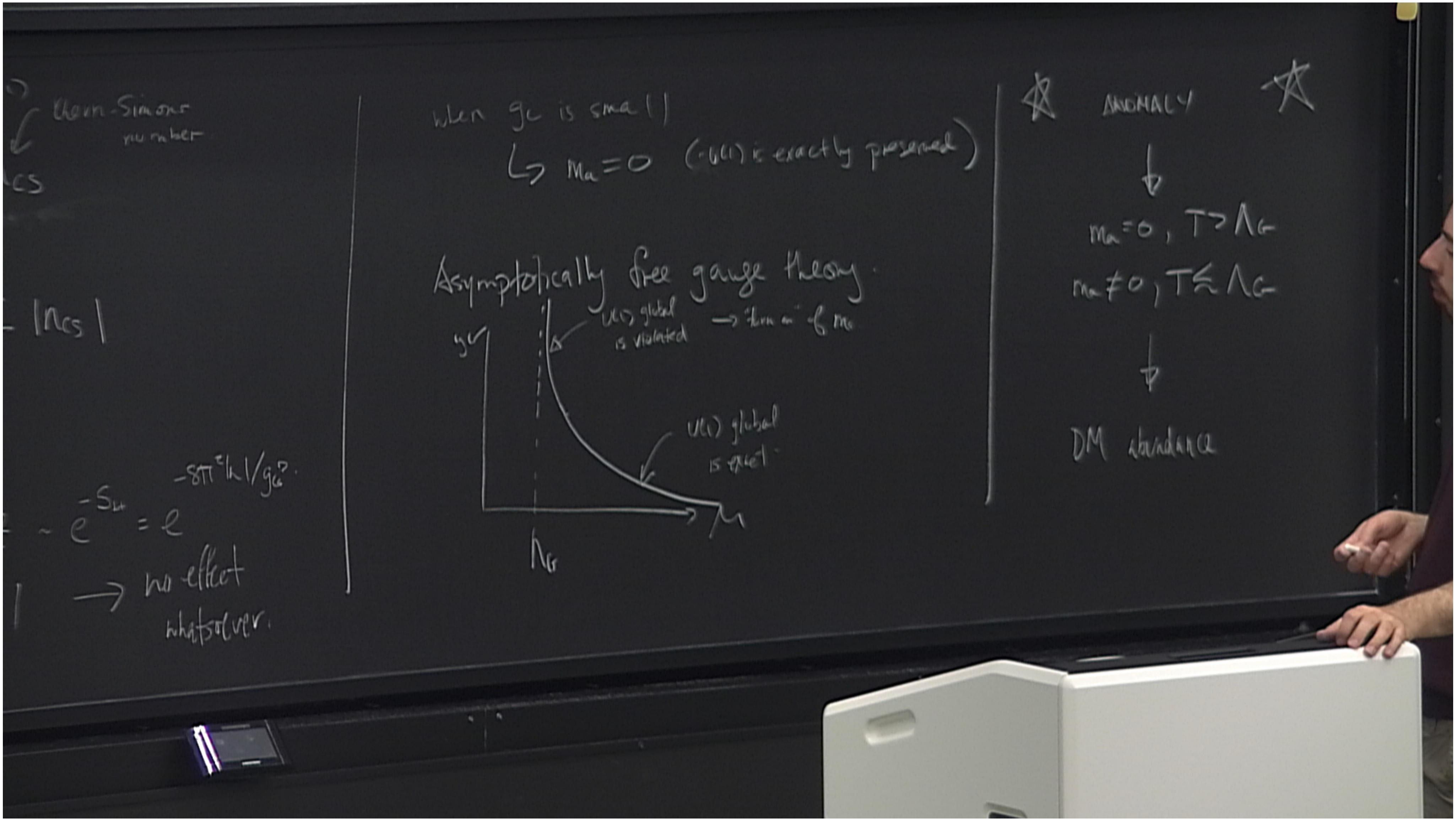
isn't true.


Chern-Simons number
$\frac{\alpha_G}{4\pi}\int d^4x \, Tr(G_{\mu\nu}\tilde{G}^{\mu\nu}) = n_{cs}$
$S \geq \frac{8\pi^2}{g_G^2} |n_{cs}|$
$(G_{\mu\nu} + \tilde{G}_{\mu\nu})^2$
$Z \sim e^{-S} = e^{-8\pi^2 n/g_G^2}$
Pert. theory...
$g_G \ll 1 \rightarrow$ no effect whatsoever.
isn't true.
$\epsilon^{\mu\nu\rho\sigma} Tr(A_\nu G_{\rho\sigma} + \frac{2}{3} i g_G A_\nu A_\rho A_\sigma)$


gauge ≠ U(1)
Chern-Simons number
$\frac{\alpha_G}{4\pi}\int d^4x\, Tr(G_{\mu\nu}\tilde{G}^{\mu\nu}) = n_{CS}$
$S_{instanton} \geq \frac{8\pi^2}{g_G^2} |n_{CS}|$
$(G_{\mu\nu} + \tilde{G}_{\mu\nu})^2$
$Z \sim e^{-S_{in}} = e^{-8\pi^2 n/g_G^2}$
Pert. theory...
$g_G \ll 1 \rightarrow$ no effect whatsoever.
isn't true.
$\epsilon^{\mu\nu\rho\sigma} Tr(A_\nu G_{\rho\sigma} + \frac{2}{3} i g_G A_\nu A_\rho A_\sigma)$


Chern-Simons number
when gc is small
↳ ma = 0 (U(1) is exactly preserved)
Asymptotically free gauge theory.
U(1) global is violated
U(1) global is exact.
no effect whatsoever.
ANOMALY
ma = 0 , T > ΛG
ma ≠ 0 , T ≲ ΛG
DM abundance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ANOMALY
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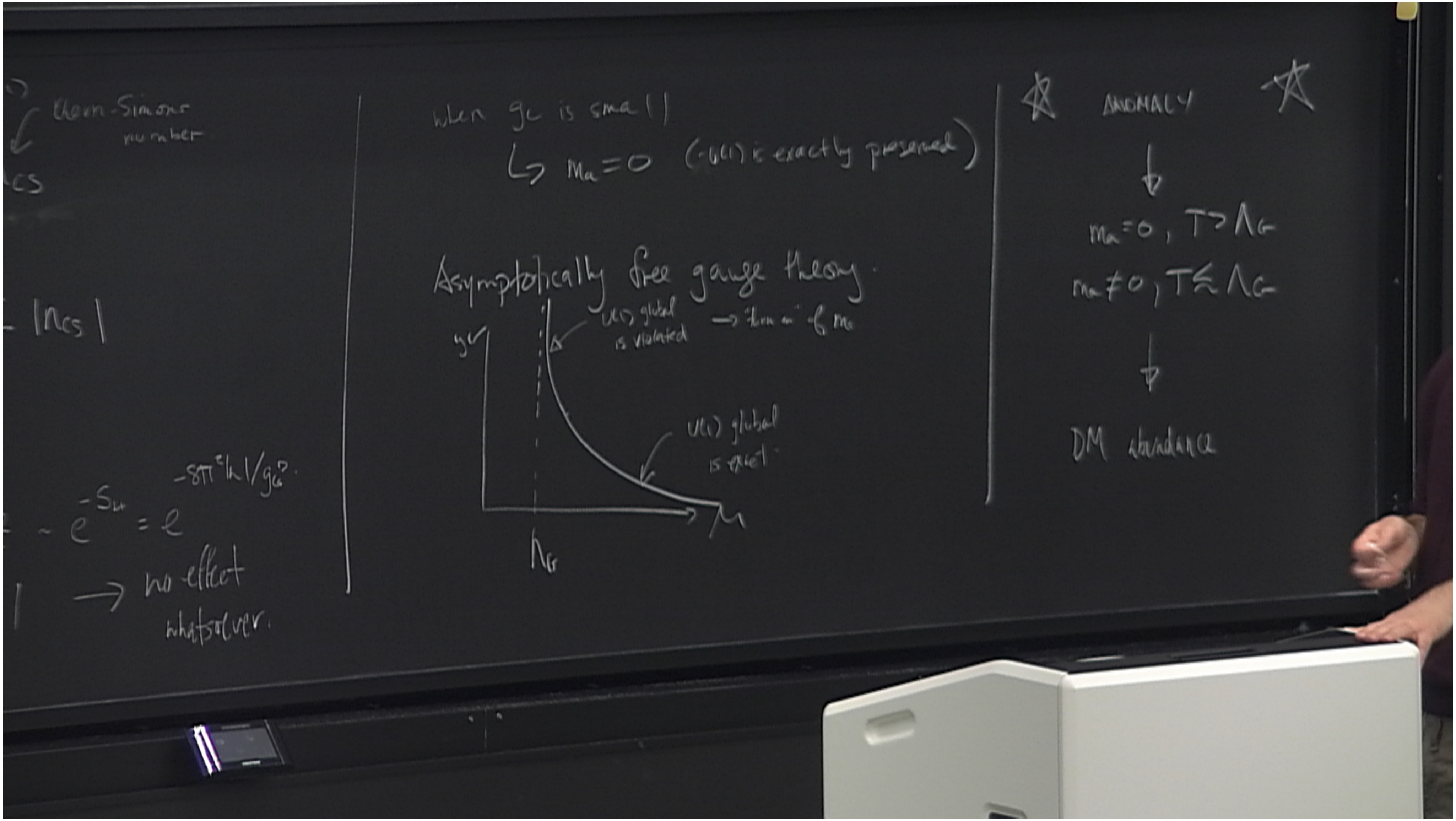
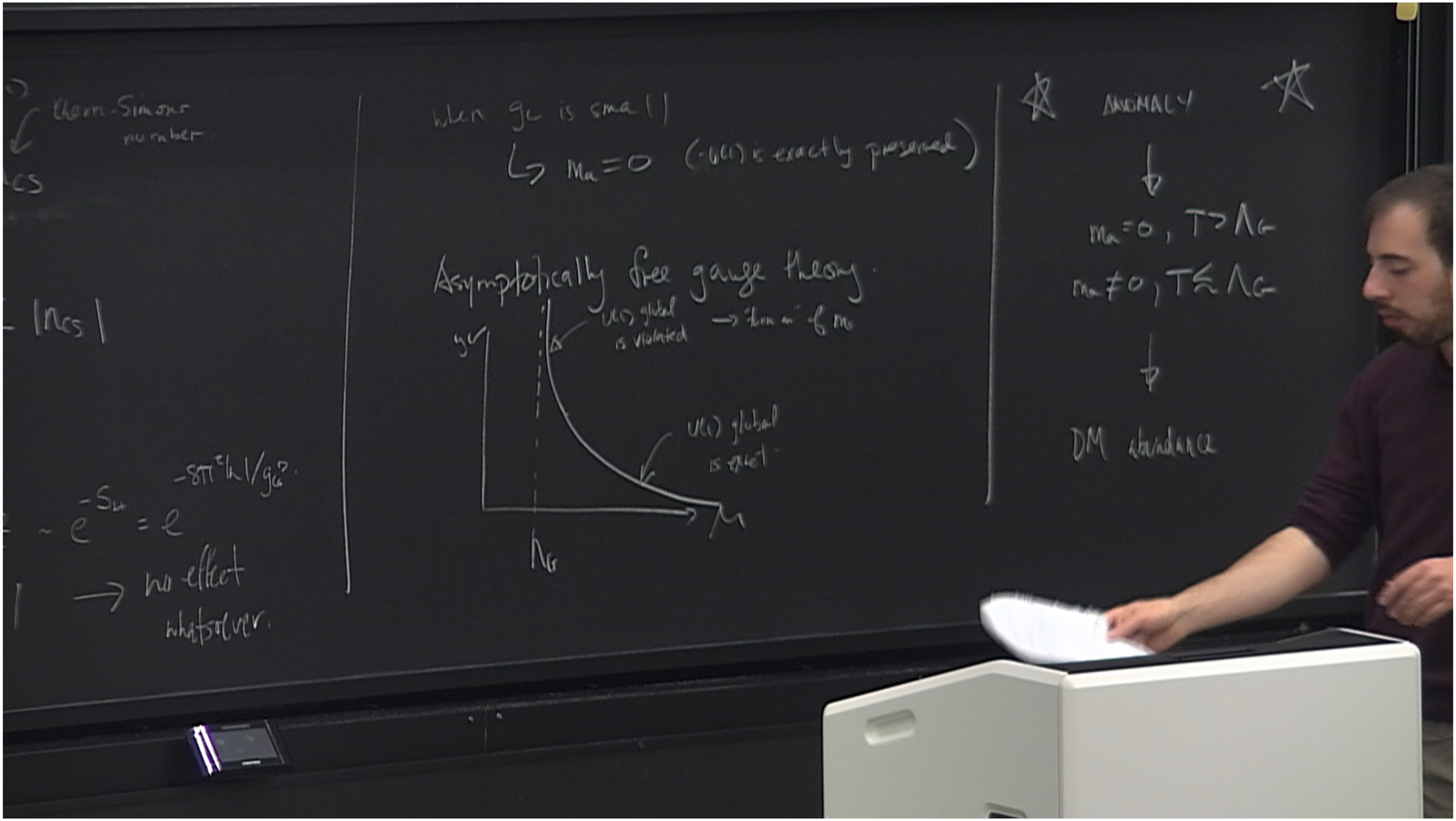
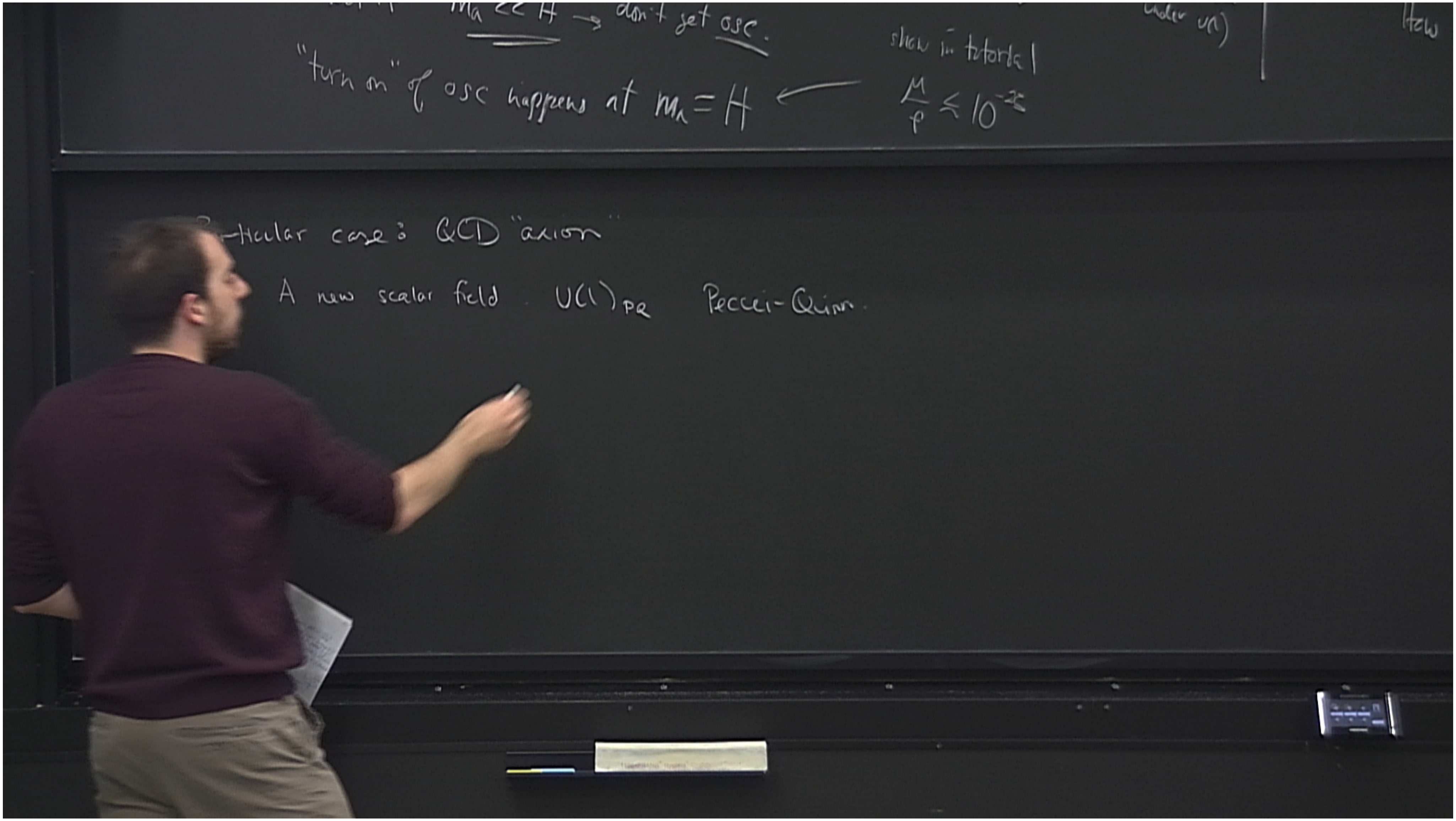
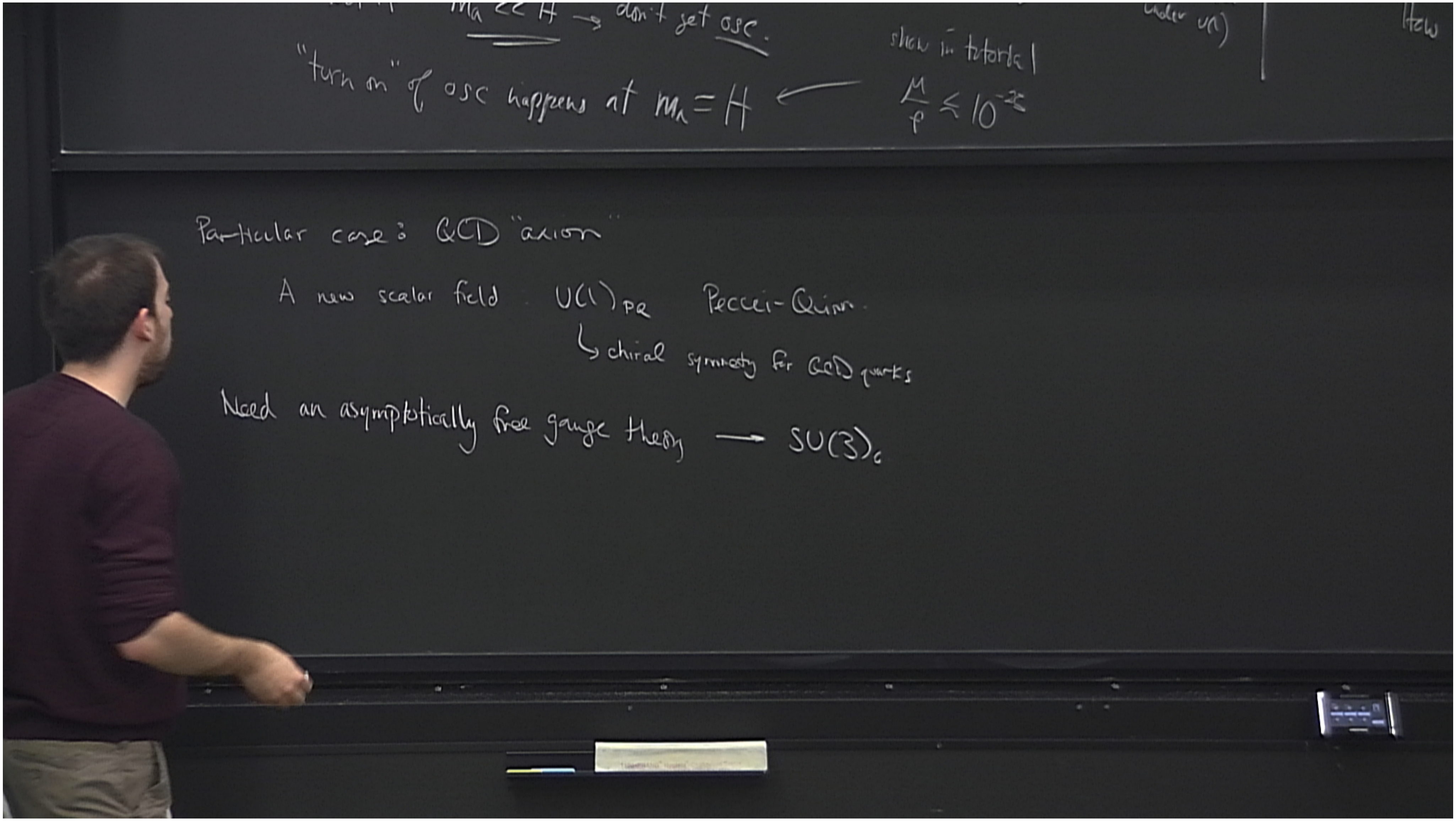
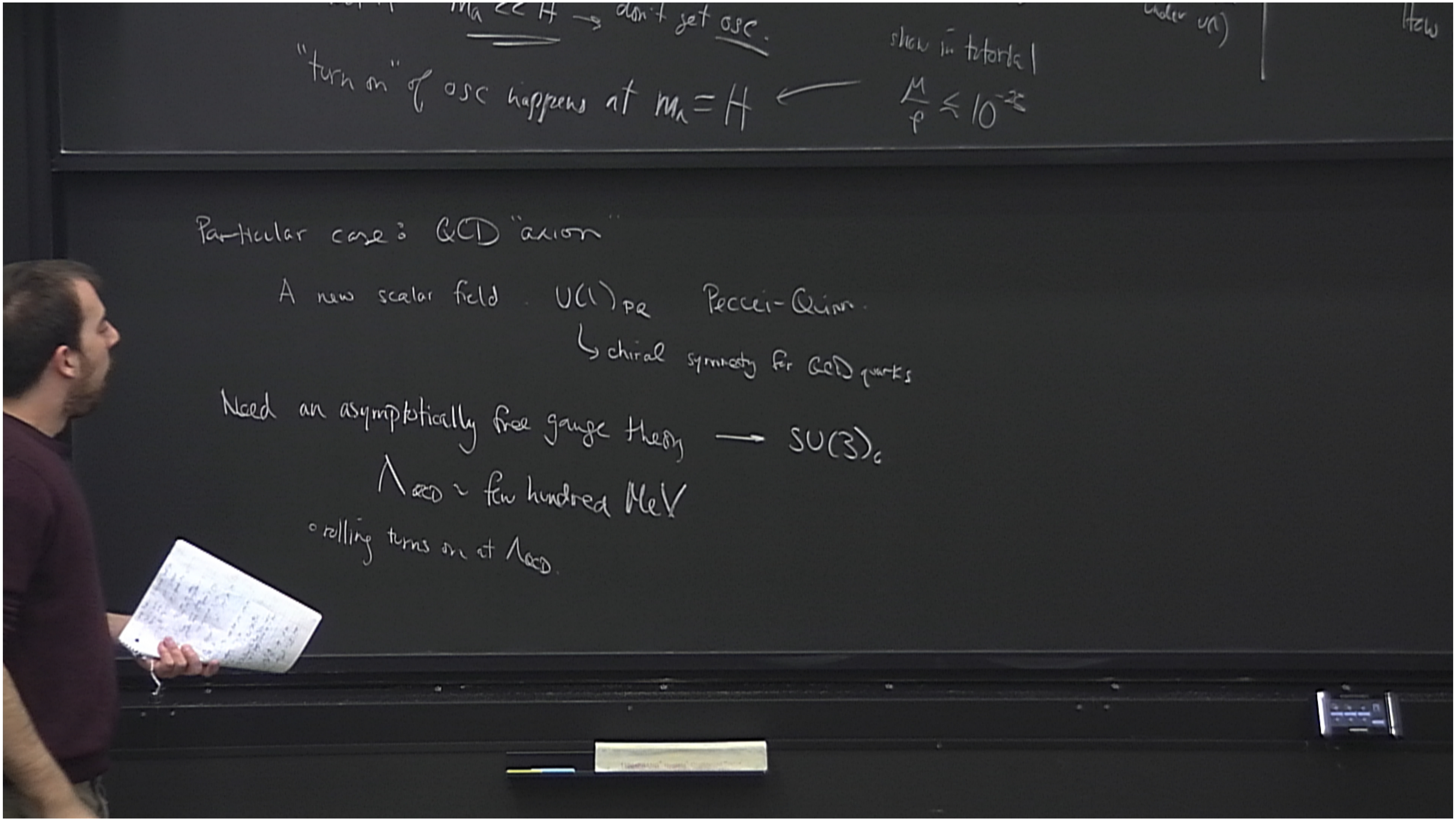

Chern-Simons number
when gc is small
(U(1) is exactly preserved)
Asymptotically free gauge theory.
U(1) global is violated
U(1) global is exact.
no effect whatsoever.
ANOMALY
DM abundance


Chern-Simons number
when g_c is small
↳ $m_a = 0$ (U(1) is exactly preserved)
Asymptotically free gauge theory.
U(1) global is violated
U(1) global is exact.
no effect whatsoever.
ANOMALY
$m_a = 0$, $T > \Lambda_G$
$m_a \neq 0$, $T \lesssim \Lambda_G$
DM abundance

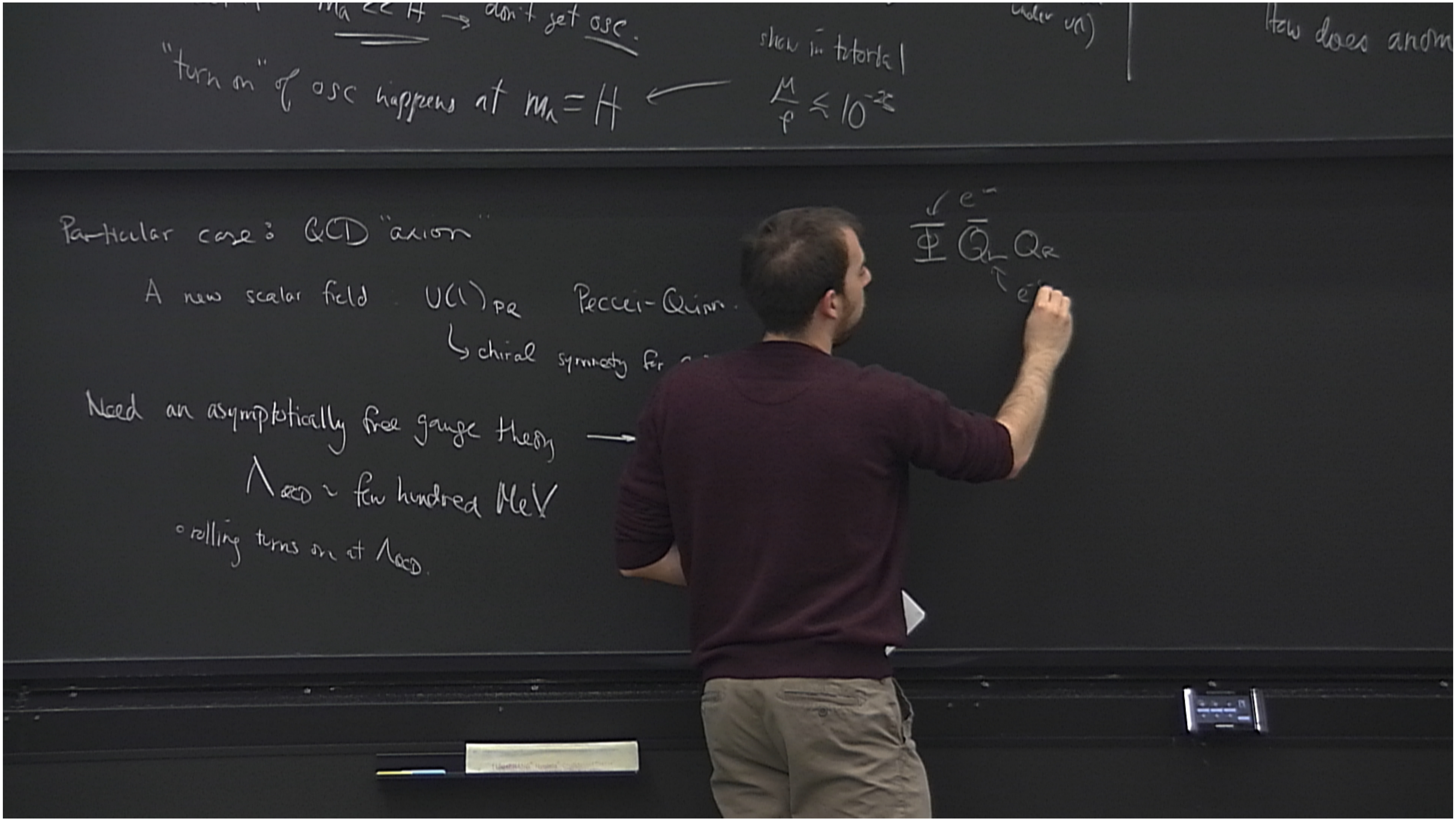


ma << H → don't get osc.
show in tutorial
"turn on" of osc happens at ma = H
-ticular case: QCD "axion"
A new scalar field . U(1)PQ Peccei-Quinn.

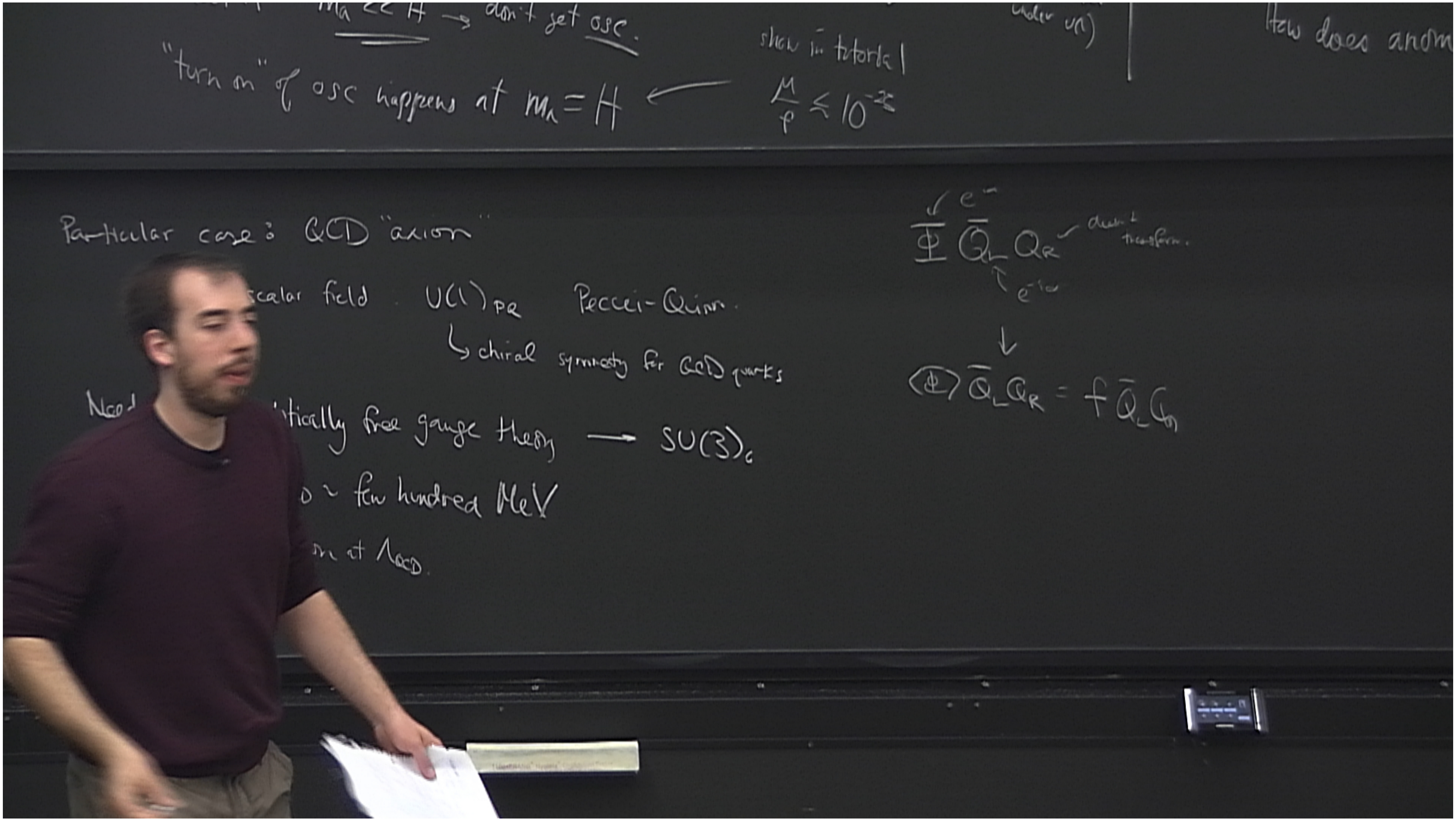

$m_a \ll H$ → don't get osc.
show in tutorial
"turn on" of osc happens at $m_a = H$ ←
Particular case: QCD "axion"
A new scalar field . $U(1)_{PQ}$ Peccei-Quinn.
↳ chiral symmetry for QCD quarks
Need an asymptotically free gauge theory → $SU(3)_c$

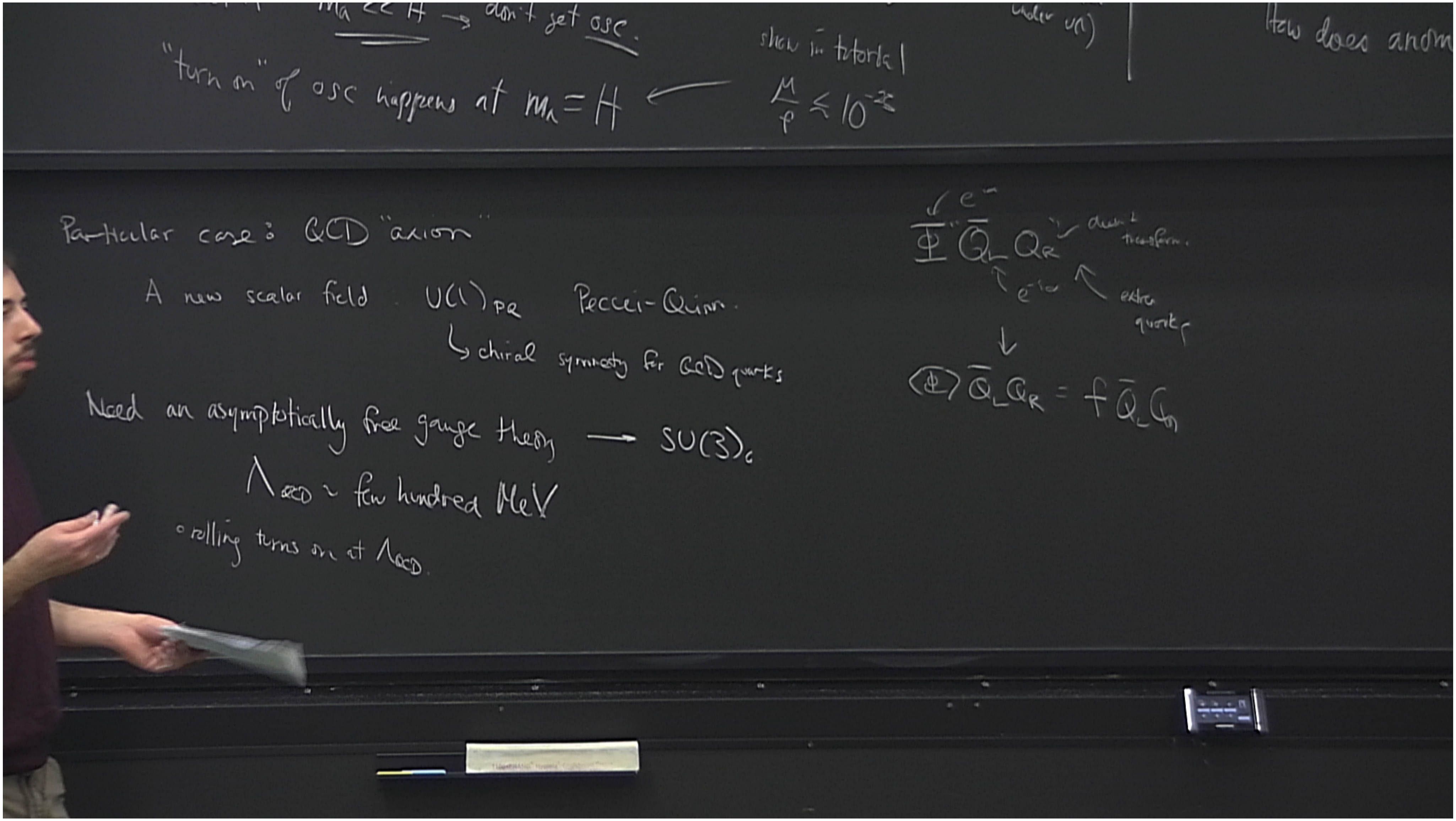


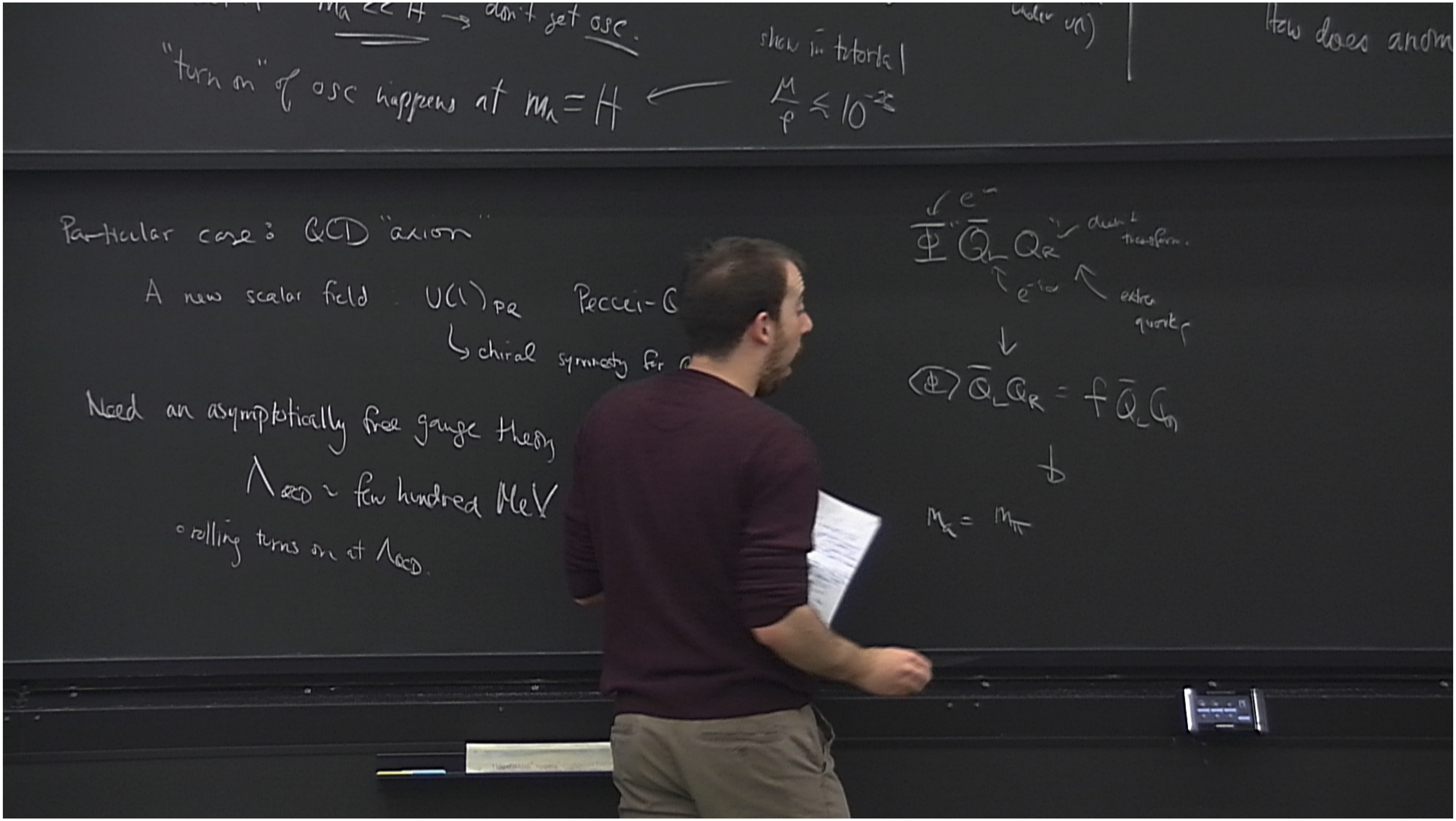
$m_a \ll H \rightarrow$ don't get osc.
under U(1)
How
show in tutorial
"turn on" of osc happens at $m_a = H$
$\frac{\mu}{\rho} \lesssim 10^{-25}$
Particular case: QCD "axion"
A new scalar field. $U(1)_{PQ}$ Peccei-Quinn.
chiral symmetry for QCD quarks
Need an asymptotically free gauge theory $\longrightarrow SU(3)_c$
$\Lambda_{QCD} \sim$ few hundred MeV
◦ rolling turns on at Λ_{QCD}.



$m_a \ll H \rightarrow$ don't get osc.
"turn on" of osc happens at $m_a = H$
show in tutorial
$\frac{\mu}{f} \lesssim 10^{-25}$
under U(1)
How does anom
Particular case: QCD "axion"
A new scalar field $U(1)_{PQ}$ Peccei-Quinn.
↳ chiral symmetry for
Need an asymptotically free gauge theory →
$\Lambda_{QCD} \sim$ few hundred MeV
rolling turns on at Λ_{QCD}.
$\Phi \bar{Q}_L Q_R$


don't get osc.
show in tutorial
"turn on" of osc happens at $m_a = H$
How does anom
Particular case: QCD "axion"
scalar field. U(1) PQ Peccei-Quinn.
chiral symmetry for QCD quarks
free gauge theory → SU(3)
few hundred MeV


don't get osc.
show in tutorial
"turn on" of osc happens at m_a = H
μ/f ≤ 10^-25
How does anom
Particular case: QCD "axion"
A new scalar field. U(1)_PQ Peccei-Quinn.
chiral symmetry for QCD quarks
Need an asymptotically free gauge theory → SU(3)_c
Λ_QCD ~ few hundred MeV
rolling turns on at Λ_QCD
Φ Q̄_L Q_R
don't transform.
extra quarks
⟨Φ⟩ Q̄_L Q_R = f Q̄_L Q_R



"turn on" of osc happens at $m_a = H$
show in tutorial
$\frac{\mu}{f} \lesssim 10^{-2}$
How does anom
Particular case: QCD "axion"
A new scalar field : $U(1)_{PQ}$ Peccei-Q
chiral symmetry for
Need an asymptotically free gauge theory
$\Lambda_{QCD} \sim$ few hundred MeV
rolling turns on at Λ_{QCD}
$\Phi \bar{Q}_L Q_R$
don't transform
extra quarks
$\langle \Phi \rangle \bar{Q}_L Q_R = f \bar{Q}_L Q_R$
$m_a = m_\pi$

under U(1)

show in tutorial

$\frac{\mu}{f} \lesssim 10^{-3}$

How does anomaly $\longrightarrow m_a$??

Goldstone acquires a mass

$\Phi \, \bar{Q}_L Q_R$

$e^{i\alpha}$

$e^{-i\alpha}$

don't transform.

extra quarks

$\downarrow$

$\langle \Phi \rangle \bar{Q}_L Q_R = f \bar{Q}_L Q_R$

$\downarrow$

$$\boxed{m_a = \frac{m_\pi f_\pi}{f}}$$

pion decay constant

$m_a = 10 \ \mu eV$

$f = 10^{11}$ GeV

Solution

show in tutorial

$\frac{\mu}{f} \lesssim 10^{-3}$

... Goldstone acquires a mass

How does anomaly $\longrightarrow m_a$??

$\Phi \bar{Q}_L Q_R$

$e^{i\alpha}$; $e^{-i\alpha}$

doesn't transform.

extra quarks

$\downarrow$

$\langle \Phi \rangle \bar{Q}_L Q_R = f \bar{Q}_L Q_R$

$$\boxed{m_a = \frac{m_\pi f_\pi}{f}}$$

pion decay constant

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Solution

show in tutorial

$\frac{\mu}{f} \lesssim 10^{-3}$

... Goldstone acquires a mass

How does anomaly $\longrightarrow m_a$??

$\bar{\Phi}\, \bar{Q}_L Q_R$ dual transform

$e^{-i\alpha}$ extra quarks

$\downarrow$

$\langle \Phi \rangle \bar{Q}_L Q_R = f \bar{Q}_L Q_R$

$$\boxed{m_a = \frac{m_\pi f_\pi}{f}}$$

pion decay constant

$m_a = 10\ \mu eV$

$f = 10^{11}\ GeV$

Solution

show in tutorial

$\frac{\mu}{f} \lesssim 10^{-3}$

under $U(1)$

... Goldstone acquires a mass

How does anomaly $\longrightarrow m_a$??

$\Phi \, \bar{Q}_L Q_R$ "$e^{i\alpha}$" chiral transform.

$e^{-i\alpha}$ extra quarks

$\downarrow$

$\langle \Phi \rangle \bar{Q}_L Q_R = f \bar{Q}_L Q_R$

$\downarrow$

$$\boxed{m_a = \frac{m_\pi f_\pi}{f}}$$

pion decay constant

$m_a = 10 \, \mu eV$

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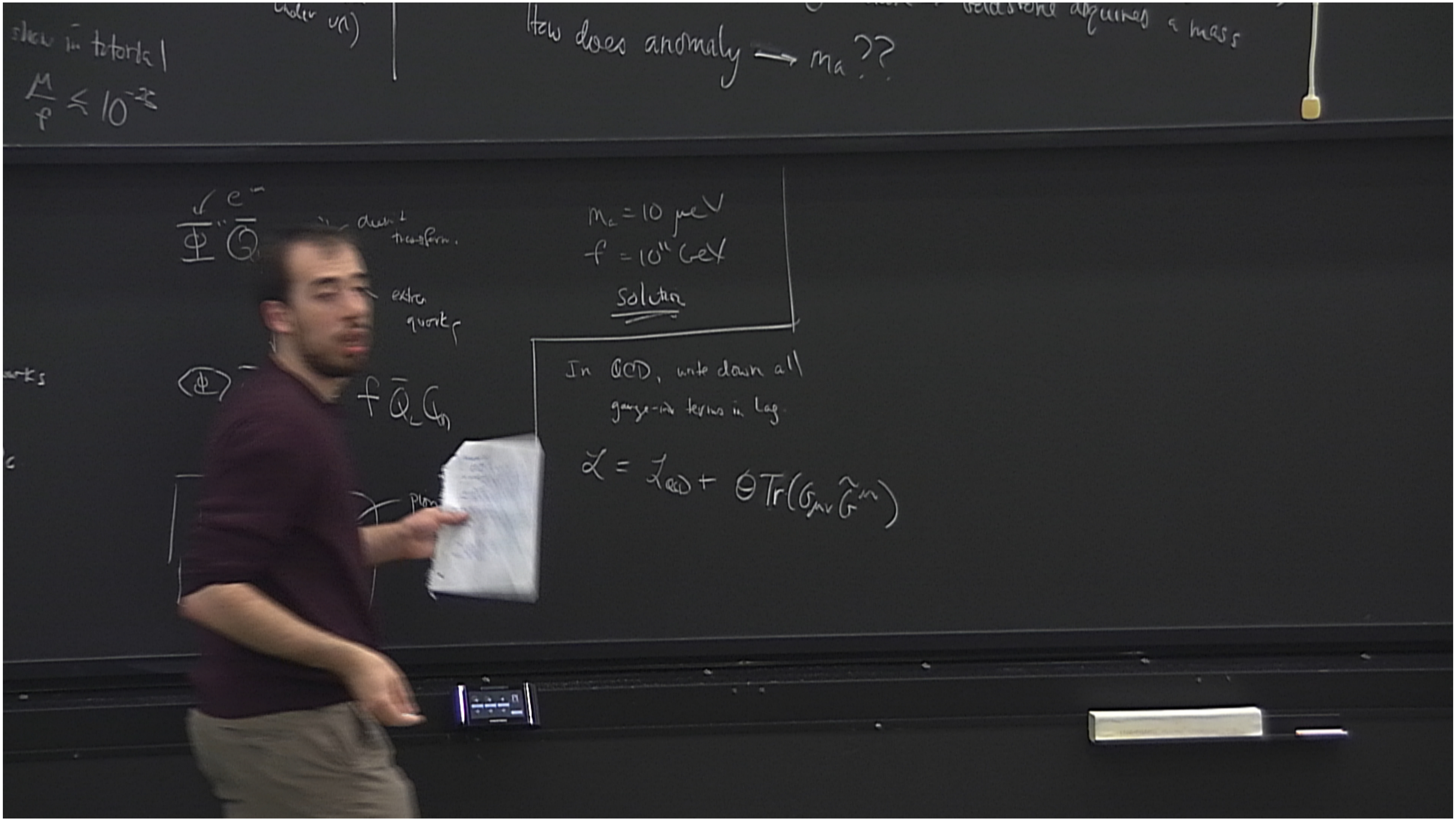
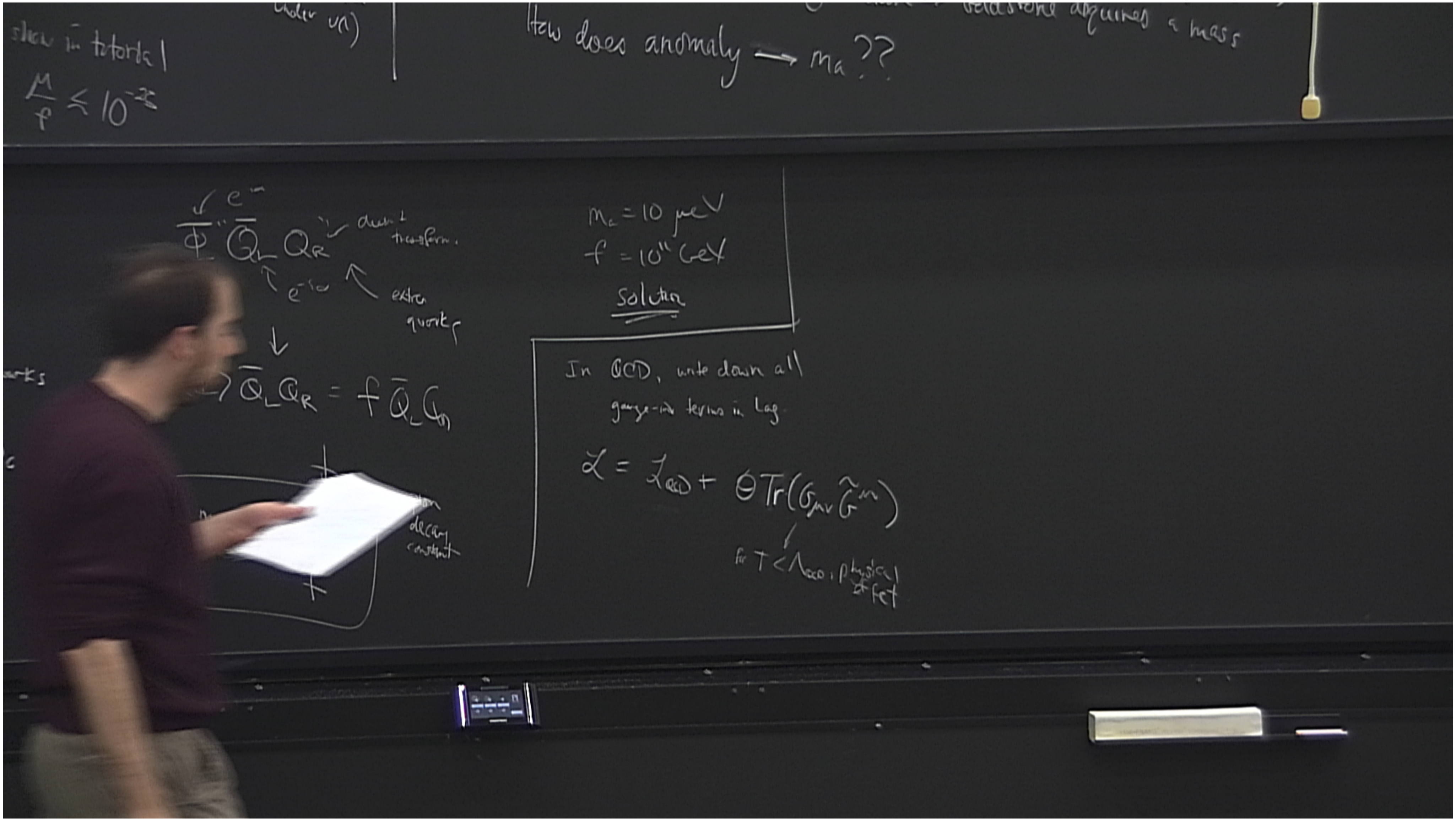
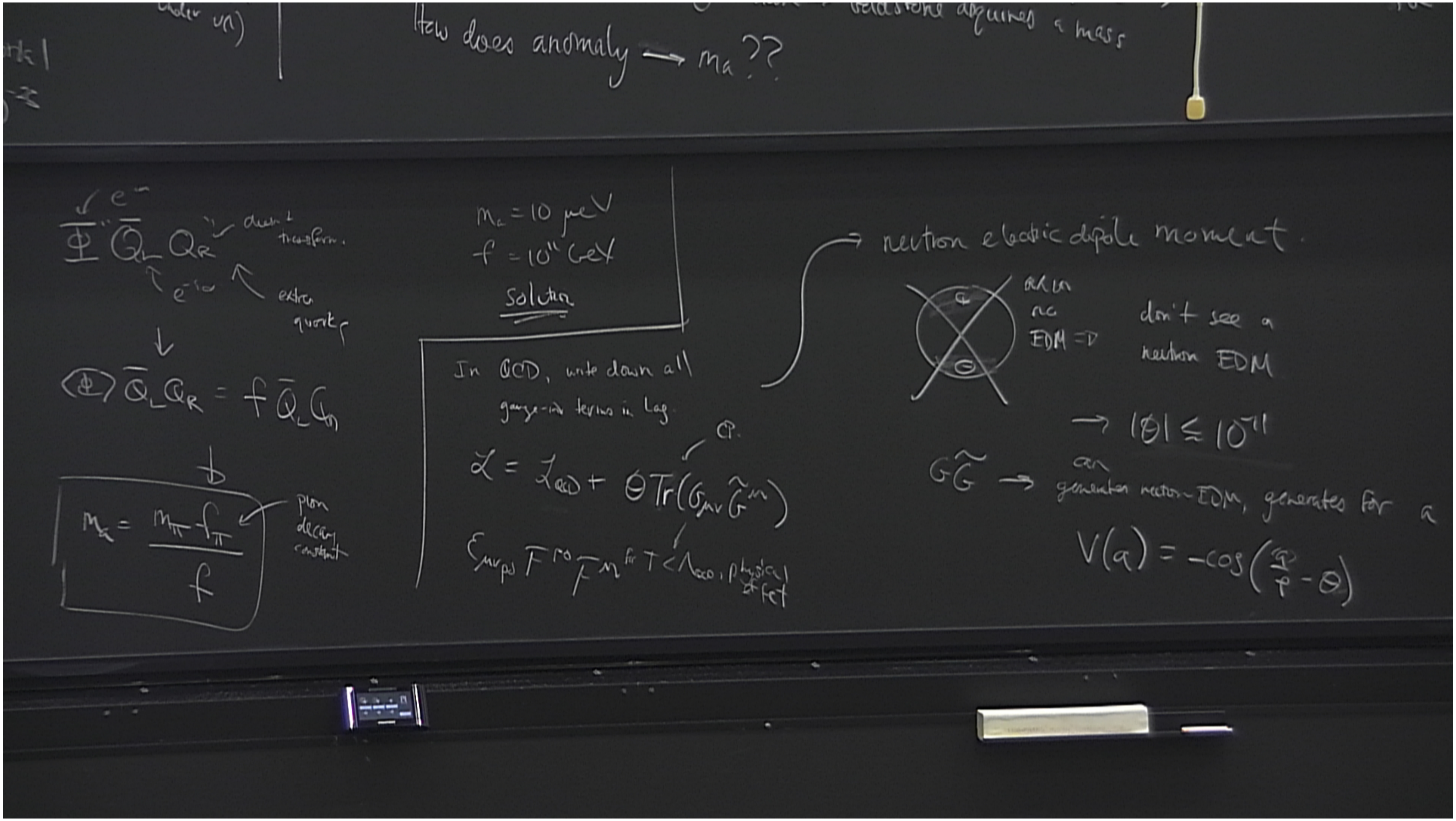
<u>Solution</u>


How does anomaly → m_a??
$\frac{\mu}{f} \lesssim 10^{-3}$
$m_a = 10\ \mu eV$
$f = 10^{11}\ GeV$
Solution
extra quarks
In QCD, write down all gauge-inv terms in Lag.
$\mathcal{L} = \mathcal{L}_{QCD} + \theta\, \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})$


show in tutorial
How does anomaly → ma ??
Goldstone acquires a mass
extra quarks
decay constant
ma = 10 μeV
f = 10^11 GeV
Solution
In QCD, write down all
gauge-inv terms in Lag.
L = LQCD + θ Tr(Gμν G̃μν)
for T < ΛQCD, physical effect


How does anomaly → m_a ??
Goldstone acquires a mass
$\bar{\Phi}\,\bar{Q}_L Q_R$ doesn't transform.
extra quarks
$\langle\Phi\rangle \bar{Q}_L Q_R = f \bar{Q}_L Q_R$
$m_a = \frac{m_\pi f_\pi}{f}$
pion decay constant
$m_a = 10\ \mu eV$
$f = 10^{11}$ GeV
Solution
In QCD, write down all gauge-inv terms in Lag.
$\mathcal{L} = \mathcal{L}_{QCD} + \theta\, Tr(G_{\mu\nu}\tilde{G}^{\mu\nu})$
CP
$\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} F^{\mu\nu}$
for $T < \Lambda_{QCD}$, physical effect
neutron electric dipole moment
don't see a neutron EDM
→ $|\theta| \leq 10^{-11}$
$G\tilde{G}$ → generates neutron-EDM, generates for a
$V(a) = -\cos\left(\frac{a}{f} - \theta\right)$