

Title: 13/14 PSI - Explorations in Condensed Matter - Lecture 15

Date: Apr 04, 2014 10:15 AM

URL: <http://pirsa.org/14040067>

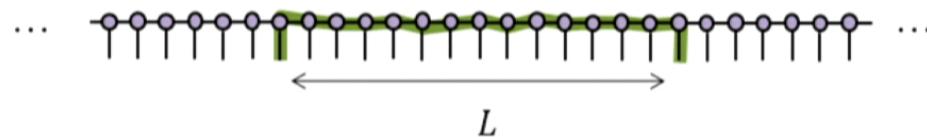
Abstract:

2014 PSI course
Explorations in Condensed Matter
(lecture 15)

Guifre Vidal

CORRELATIONS and DISTANCE

matrix product state (MPS)

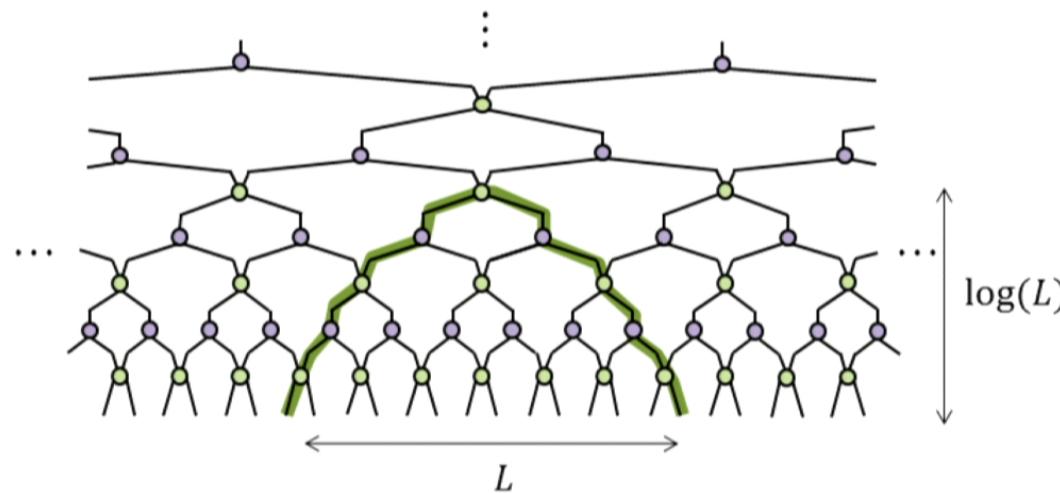


exponential correlators

$$C(L) \approx e^{-L/\xi}$$

$$[C(L) \approx \lambda^L]$$

multi-scale entanglement renormalization ansatz (MERA)



polynomial correlators

$$C(L) \approx L^{-p}$$

$$[C(L) \approx \lambda^{\log(L)}]$$

D=1 spatial dimensions

matrix product state
(MPS)

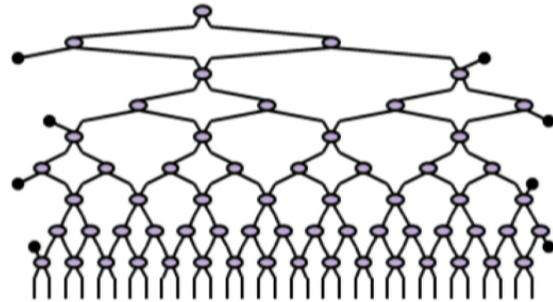


$$C(L) \approx e^{-L/\xi}$$

$$S_L \approx \text{const } (= L^{D-1})$$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=1 spatial dimensions

matrix product state
(MPS)

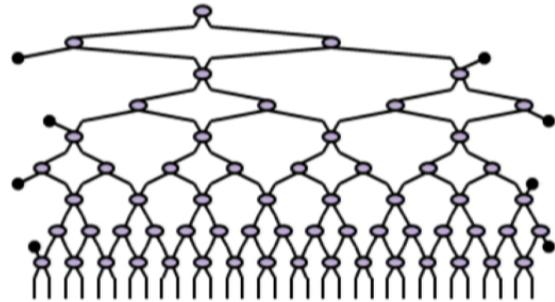


$$C(L) \approx e^{-L/\xi}$$

$$S_L \approx \text{const } (= L^{D-1})$$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



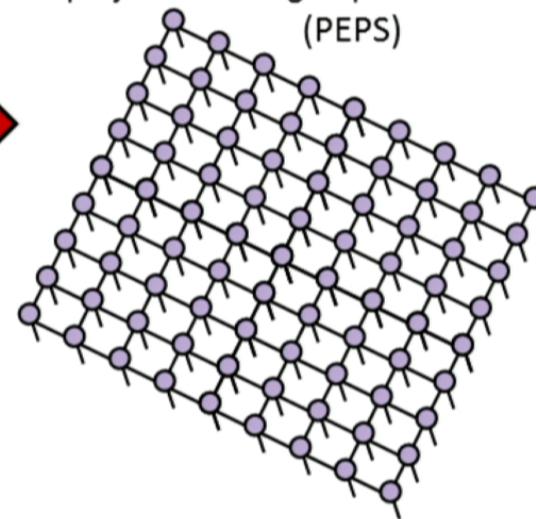
$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=2 spatial dimensions

projected entangled pair states
(PEPS)



D=1 spatial dimensions

matrix product state
(MPS)

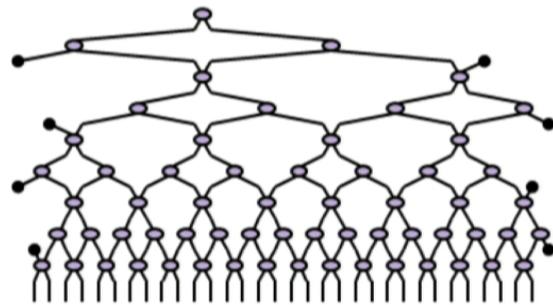


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multi-scale entanglement renormalization ansatz
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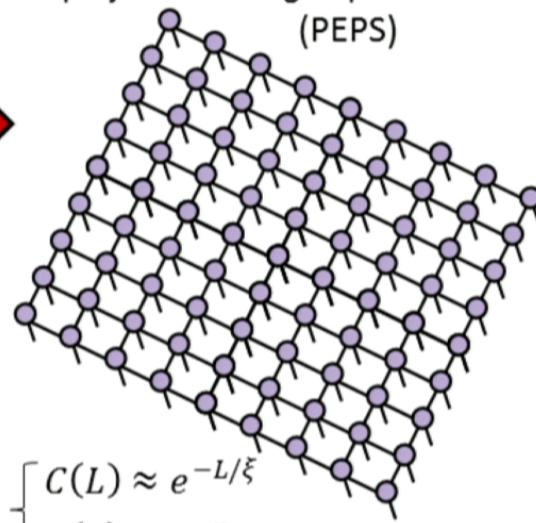
$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=2 spatial dimensions

projected entangled pair states
(PEPS)

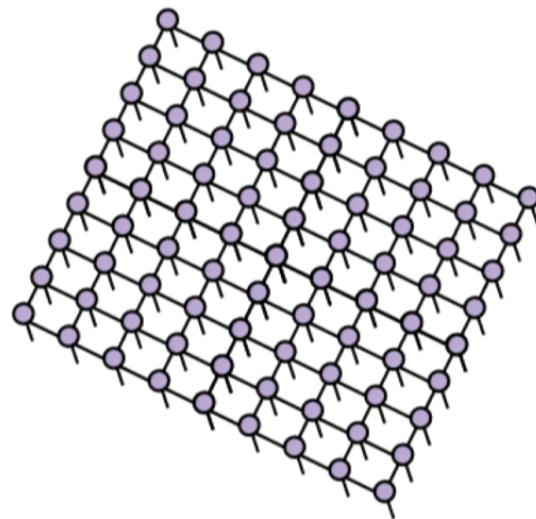


$$\begin{cases} C(L) \approx e^{-L/\xi} \\ C(L) \approx L^{-p} \end{cases}$$

$$S_L \approx L \quad (= L^{D-1})$$

Lecture 14-Tensor networks in two and more dimensions

Projected entangled pair states (PEPS)



exponential or polynomial correlations

$$C(L) \approx e^{-L/\xi}$$

$$C(L) \approx L^{-p}$$

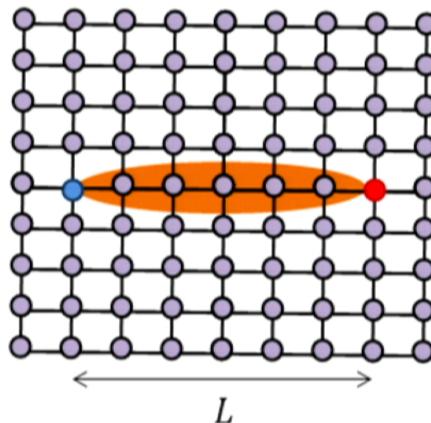
boundary law for entanglement entropy

$$S_L \approx L \quad (= L^{D-1})$$

Projected entangled pair states (PEPS)

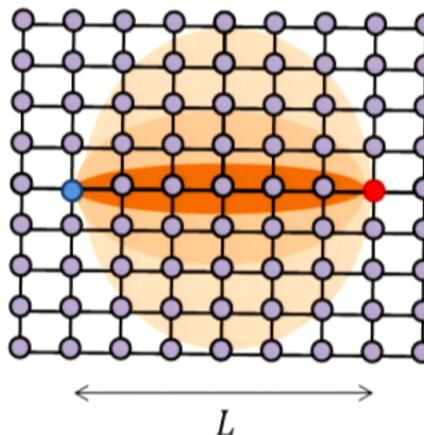
Correlations:

$$\langle \Psi | \hat{o}(\vec{x}) \hat{o}'(\vec{y}) | \Psi \rangle$$



exponential correlations

$$C(L) \approx e^{-L/\xi}$$



polynomial correlations

$$C(L) \approx L^{-p}$$

D=1 spatial dimensions

matrix product state
(MPS)

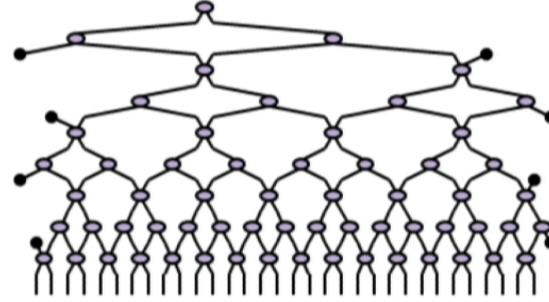


$$C(L) \approx e^{-L/\xi}$$

$$S_L \approx \text{const } (= L^{D-1})$$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



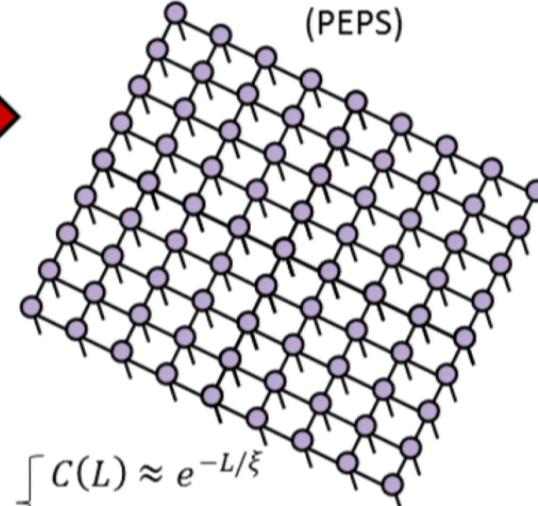
$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=2 spatial dimensions

projected entangled pair states
(PEPS)



$$C(L) \approx e^{-L/\xi}$$

$$C(L) \approx L^{-p}$$

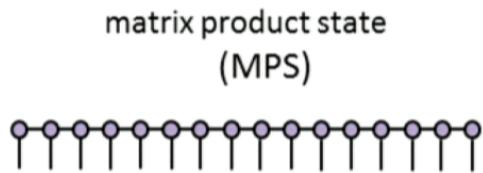
$$S_L \approx L \quad (= L^{D-1})$$

2D MERA

$$C(L) \approx L^{-p}$$

$$S_L \approx L \quad (= L^{D-1})$$

D=1 spatial dimensions

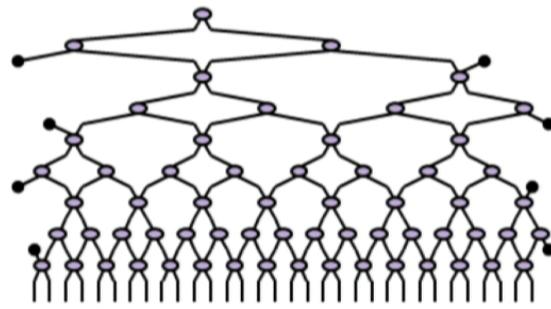


$$C(L) \approx e^{-L/\xi}$$

$$S_L \approx \text{const } (= L^{D-1})$$

(gapped systems)

multi-scale entanglement renormalization ansatz (MERA)

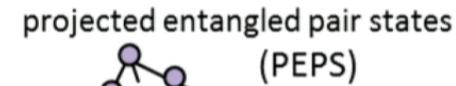


$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=2 spatial dimensions



$$C(L) \approx e^{-L/\xi}$$

$$C(L) \approx L^{-p}$$

$$S_L \approx L \quad (= L^{D-1})$$

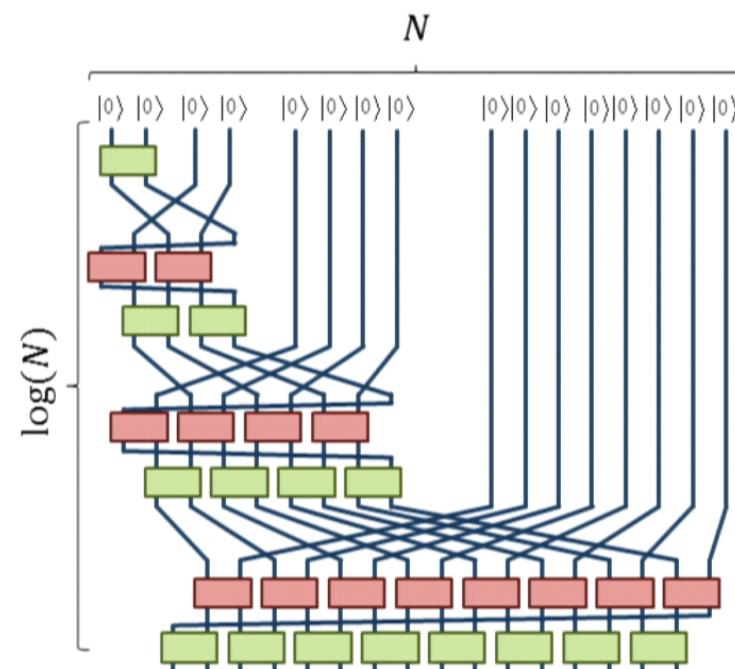
2D MERA

$$C(L) \approx L^{-p}$$

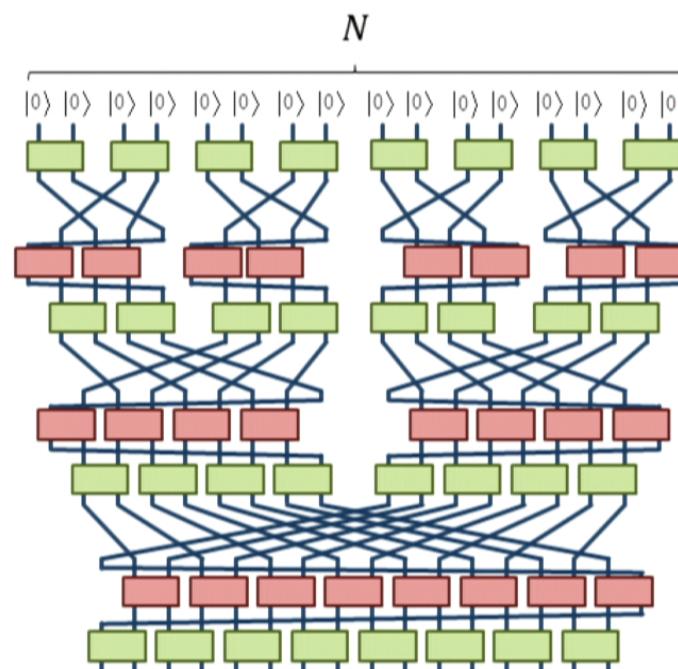
$$S_L \approx L \quad (= L^{D-1})$$

Branching MERA

MERA

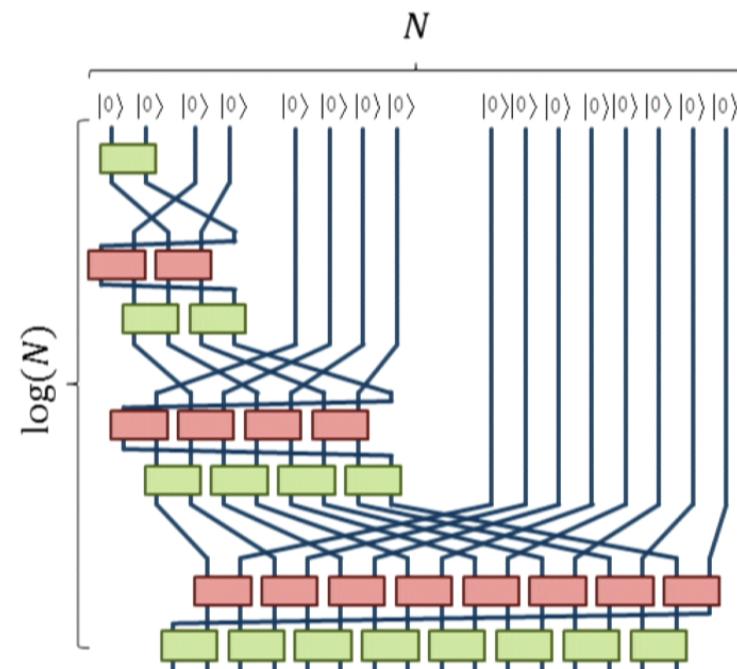


branching MERA

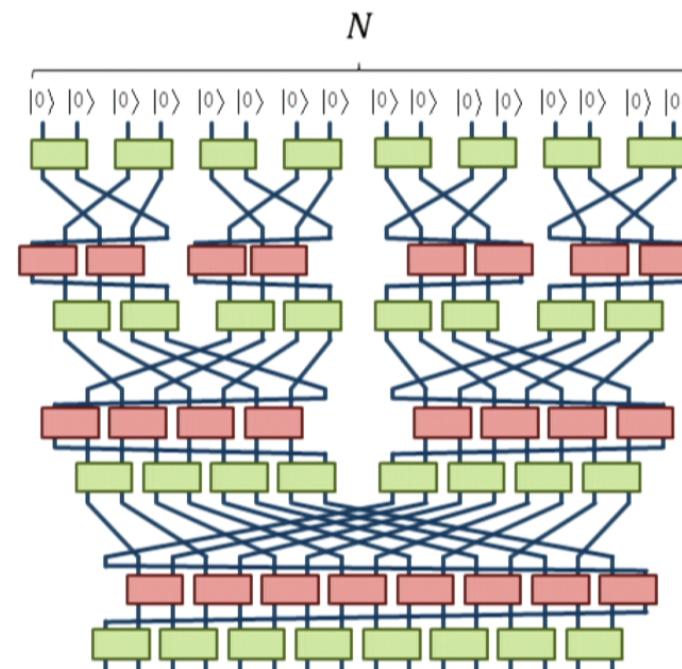


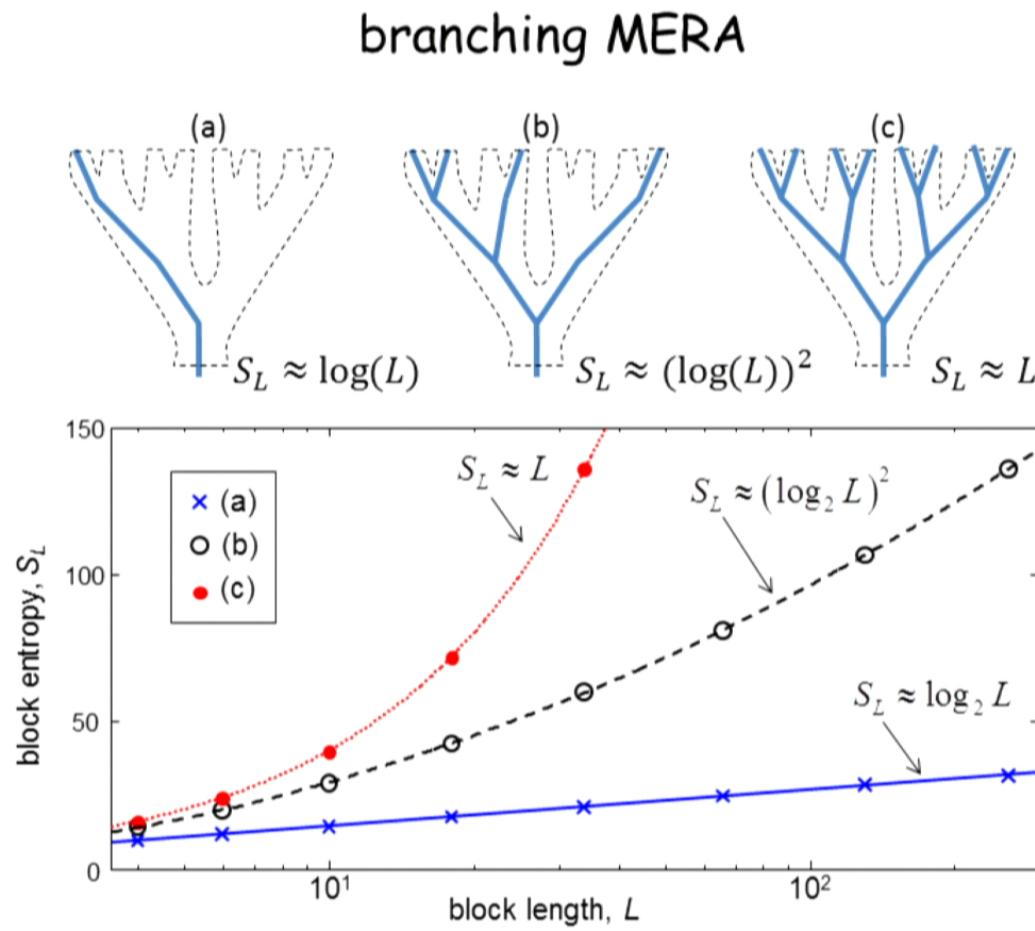
Branching MERA

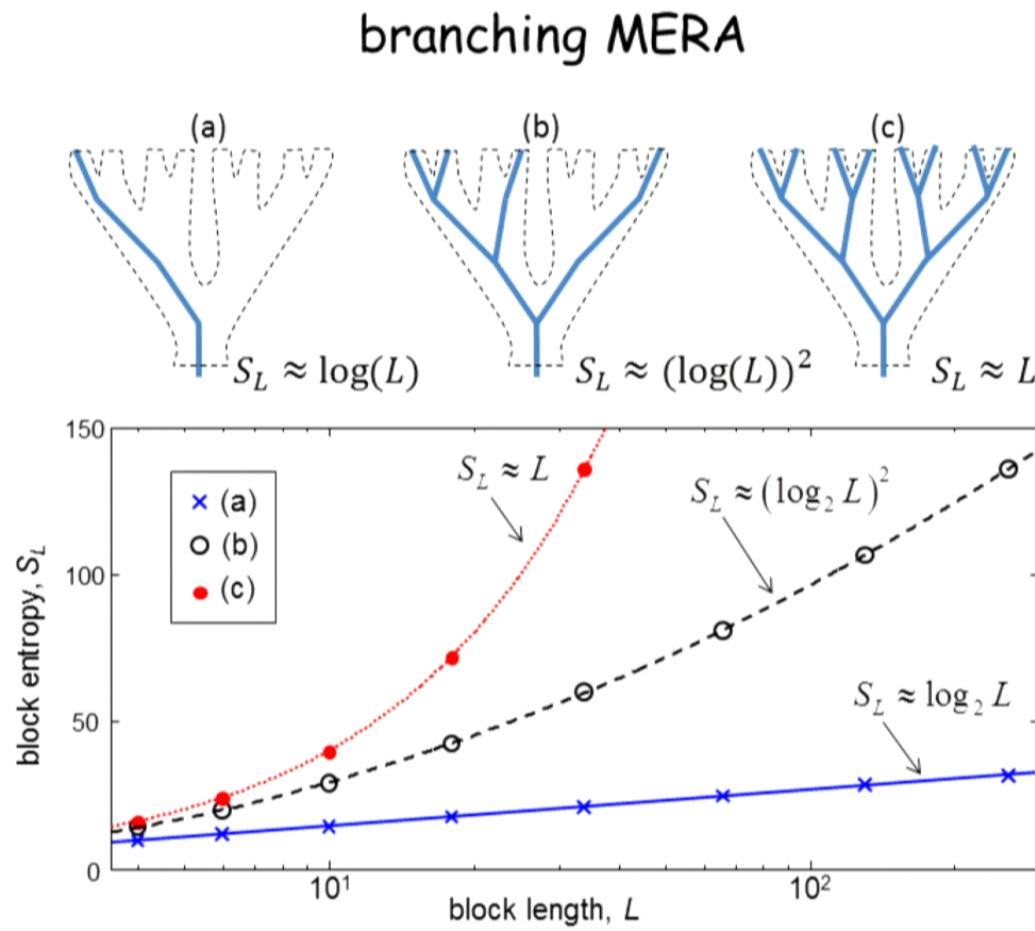
MERA



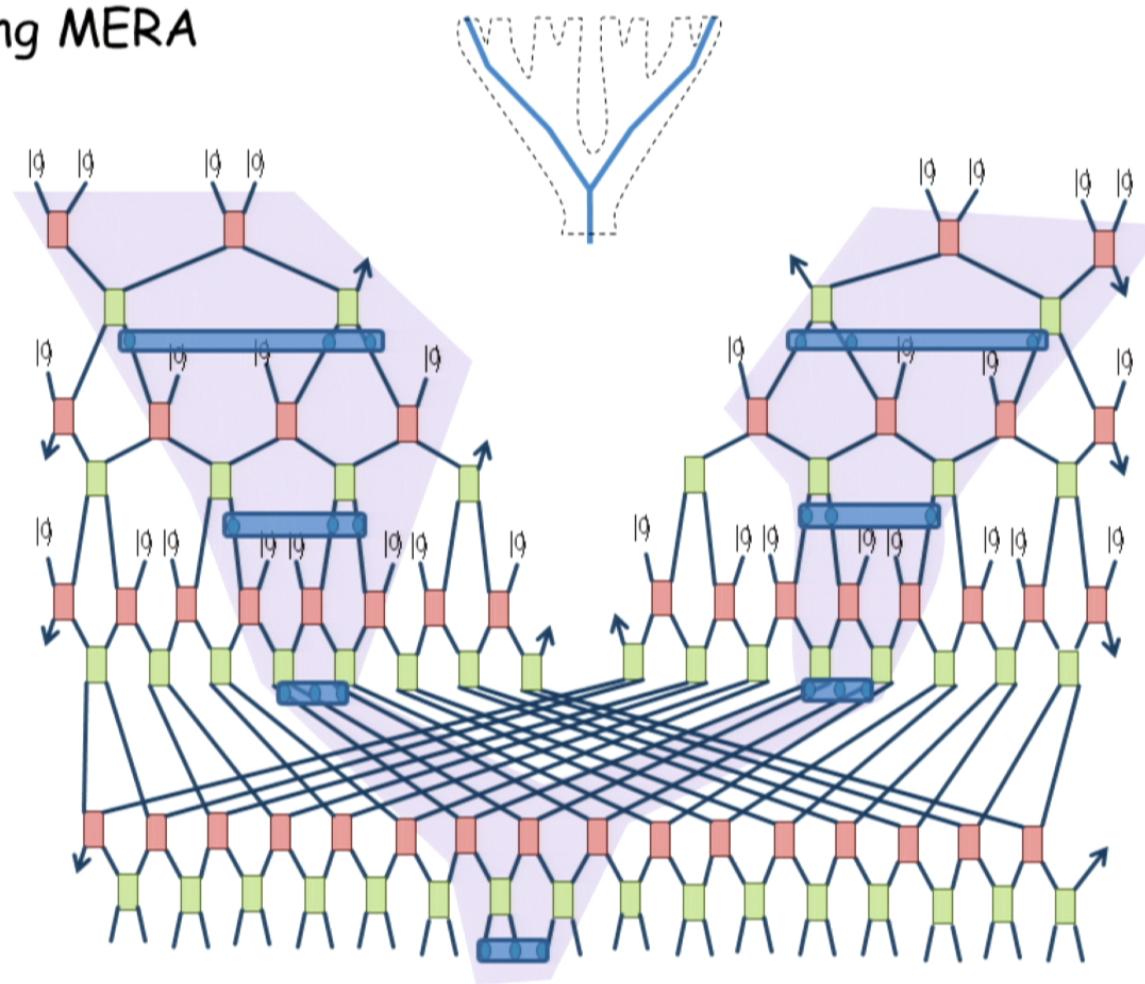
branching MERA





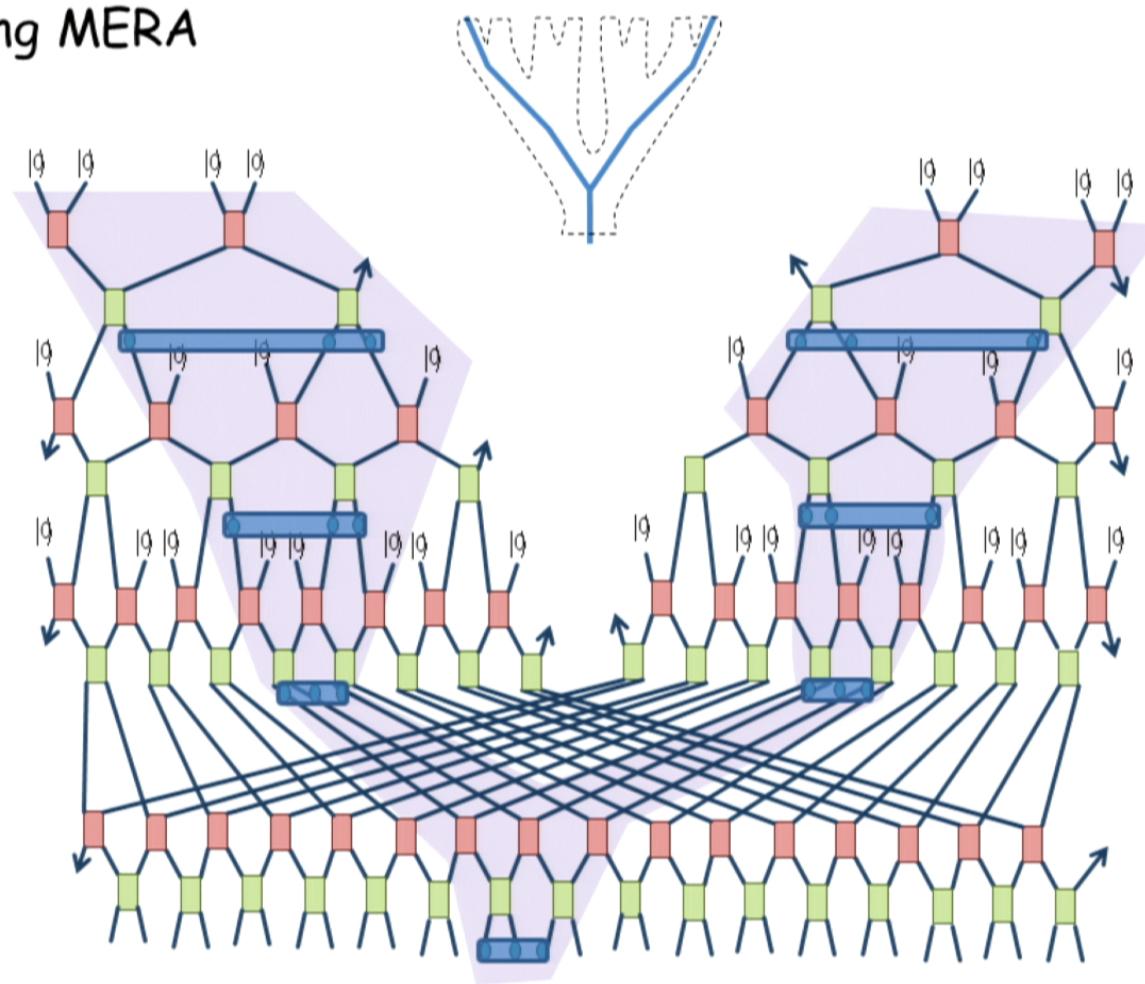


branching MERA



$$S_L \approx \log(L) + \log(L)$$

branching MERA



$$S_L \approx \log(L) + \log(L)$$

branching MERA



D=1 spatial
dimensions

$$S_L \approx \log(L)$$

...

$$S_L \approx L$$

D>1 spatial
dimensions

$$S_L \approx L^{D-1}$$

...

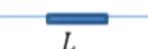
$$S_L \approx L^{D-1} \log(L)$$

...

$$S_L \approx L^D$$

Summary/outlook

Tensor networks (third week)

Dimension	gapped $\Delta > 0$	gapless no (D-1)- dimensional Fermi surface	$\Delta = 0$ (D-1)- dimensional Fermi surface
D=1 	$S_L \approx \text{const}$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 	$S_L \approx L$ PEPS	$S_L \approx L$ MERA	$S_L \approx L \log(L)$
D=3 	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ MERA	$S_L \approx L^2 \log(L)$

$S_L \approx L^{D-1}$
 boundary law $S_L \approx L^{D-1} \log(L)$
 boundary law
 with logarithmic
 correction

Summary/outlook

Tensor networks (third week)

Dimension	gapped $\Delta > 0$	gapless no (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ (D-1)- dimensional Fermi surface
D=1 	$S_L \approx \text{const}$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 	$S_L \approx L$ PEPS	$S_L \approx L$ MERA	$S_L \approx L \log(L)$ branching MERA
D=3 	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ MERA	$S_L \approx L^2 \log(L)$ branching MERA

$S_L \approx L^{D-1}$
boundary law $S_L \approx L^{D-1} \log(L)$
 boundary law with logarithmic correction

Summary/outlook

Current applications of tensor networks:

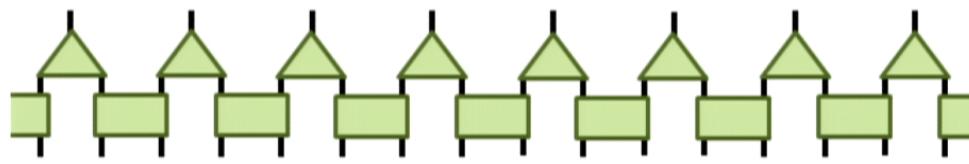
- Basis for non-perturbative numerical approaches to
 - a. frustrated antiferromagnets (2D)
 - b. interacting fermions (2D)
 - c. topologically ordered phases (2D)
 - d. quantum criticality/phase transitions
- Classification of symmetry protected gapped phases
 - a. complete classification in 1D (MPS!)
 - b. partial classification in 2D, 3D, etc
- Non-perturbative lattice realization of
 - a. (real space) renormalization group
 - b. conformal field theories
 - c. quantum field theories
 - d. the holographic principle

Summary/outlook

Current applications of tensor networks:

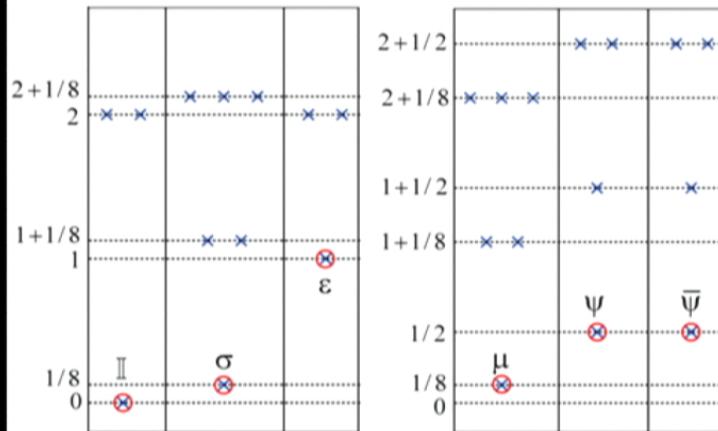
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Scaling operators/scaling dimensions



Example: operator content of quantum Ising model

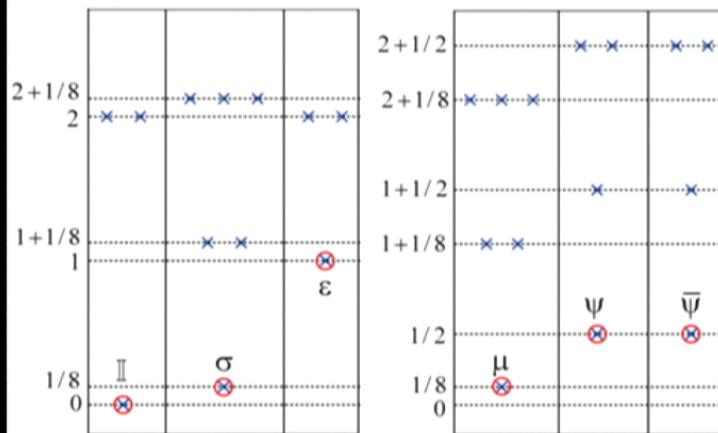
scaling operators/dimensions:



	scaling dimension (exact)	scaling dimension (MERA)	error
identity	\mathbb{I} 0	0	----
spin	σ 0.125	0.124997	0.003%
energy	ε 1	0.99993	0.007%
disorder	μ 0.125	0.1250002	0.0002%
fermions	ψ 0.5	0.5	$<10^{-8}\%$
	$\bar{\psi}$ 0.5	0.5	$<10^{-8}\%$

Example: operator content of quantum Ising model

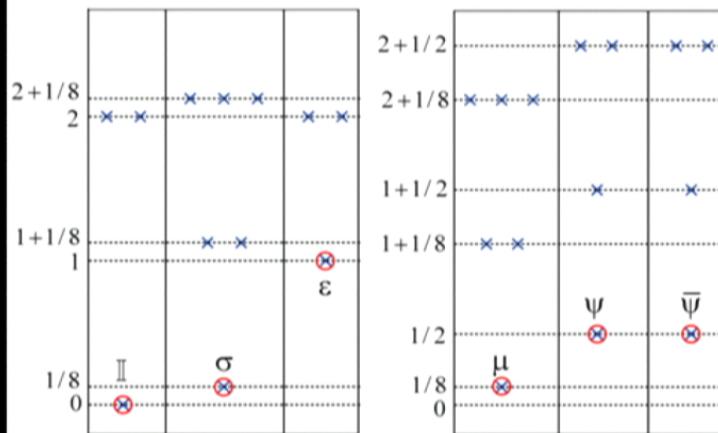
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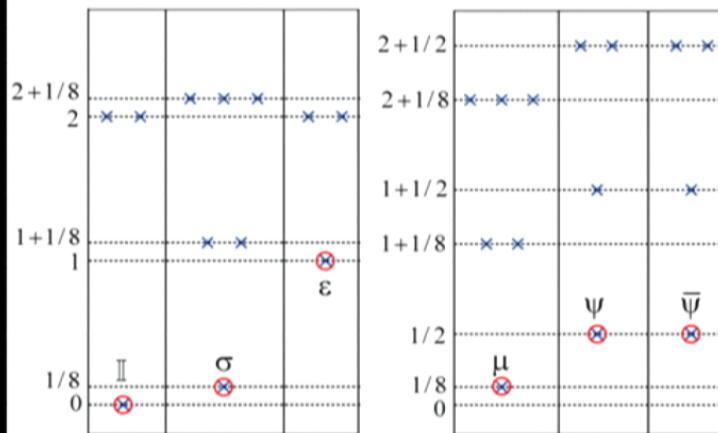
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OPE for local & non-local primary fields

$$C_{\varepsilon\sigma\sigma} = 1/2 \quad C_{\varepsilon\psi\bar{\psi}} = i$$

$$C_{\varepsilon\mu\mu} = -1/2 \quad C_{\varepsilon\bar{\psi}\psi} = -i$$

$$C_{\psi\mu\sigma} = e^{-i\pi/4} / \sqrt{2} \quad (\pm 6 \times 10^{-4})$$

$$C_{\bar{\psi}\mu\sigma} = e^{i\pi/4} / \sqrt{2}$$

OPE for local & non-local primary fields

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$$C_{\varepsilon\psi\bar{\psi}} = i$$

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\Rightarrow

$$C_{\psi\mu\sigma} = e^{-i\pi/4} / \sqrt{2}$$

$(\pm 6 \times 10^{-4})$

$$C_{\bar{\psi}\mu\sigma} = e^{i\pi/4} / \sqrt{2}$$

fusion rules

$$\varepsilon \times \varepsilon = I$$

$$\sigma \times \sigma = I + \varepsilon$$

$$\sigma \times \varepsilon = \sigma$$

$$\mu \times \mu = I + \varepsilon$$

$$\mu \times \varepsilon = \mu$$

$$\psi \times \psi = I$$

$$\bar{\psi} \times \bar{\psi} = I$$

$$\psi \times \bar{\psi} = \varepsilon$$

$$\psi \times \varepsilon = \bar{\psi}$$

$$\bar{\psi} \times \varepsilon = \psi$$

...

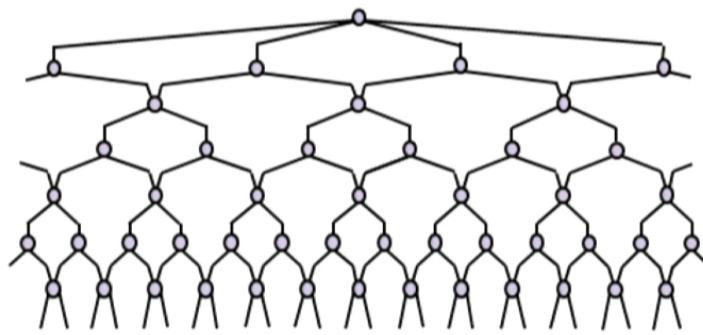
local and
semi-local
subalgebras

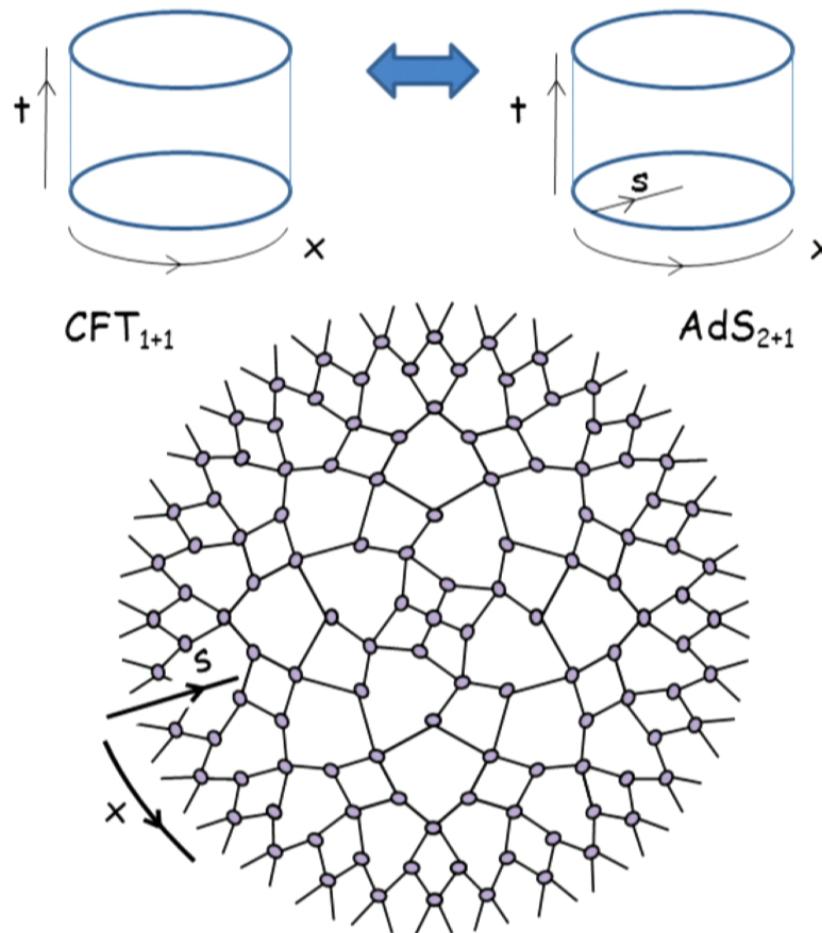
$$\{I, \varepsilon, \sigma, \mu, \psi, \bar{\psi}\}$$

$$\{I, \varepsilon\} \quad \{I, \varepsilon, \sigma\}$$

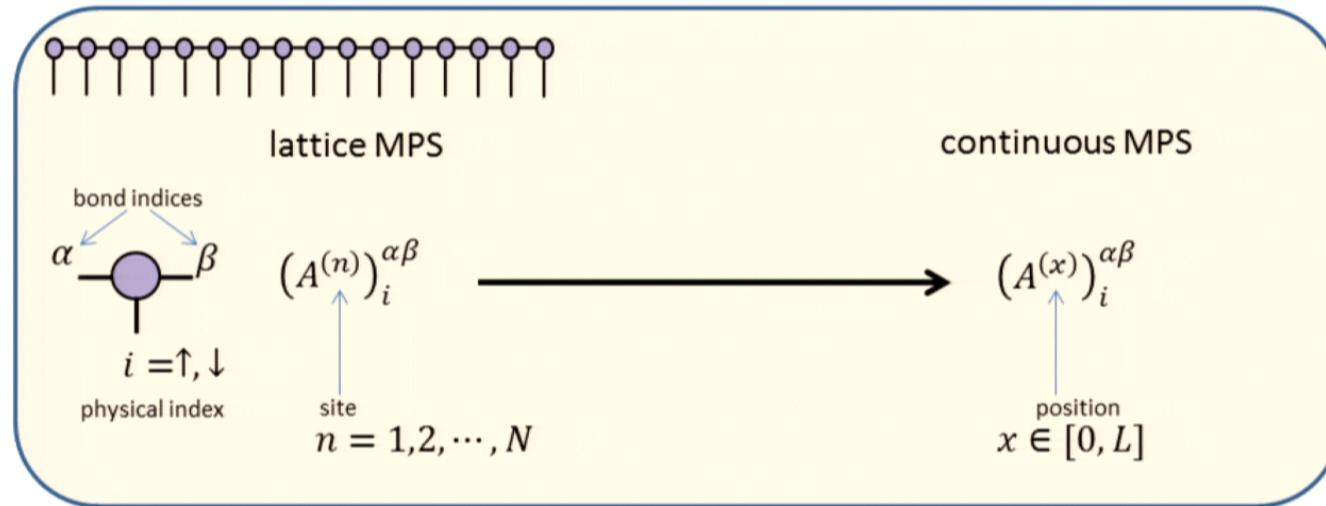
$$\{I, \varepsilon, \mu\} \quad \{I, \varepsilon, \psi, \bar{\psi}\}$$

MERA and HOLOGRAPHY

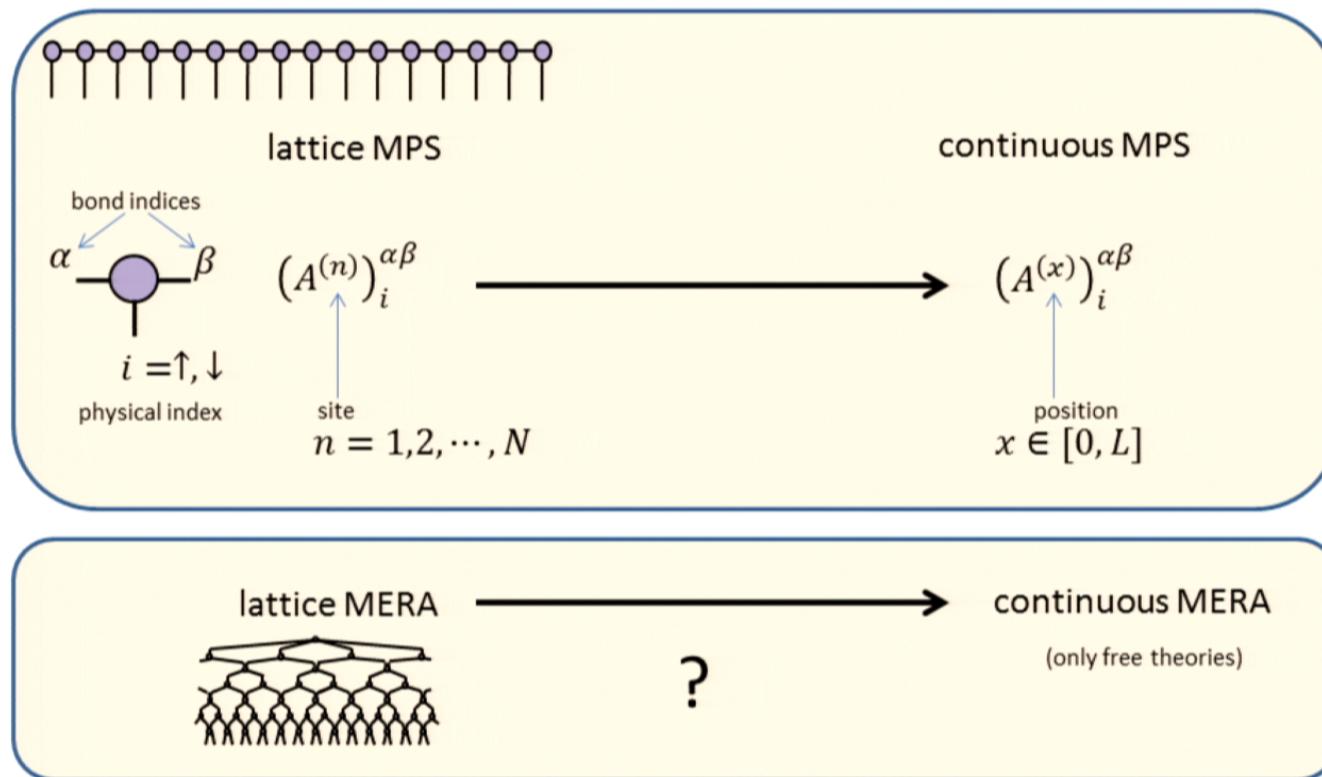




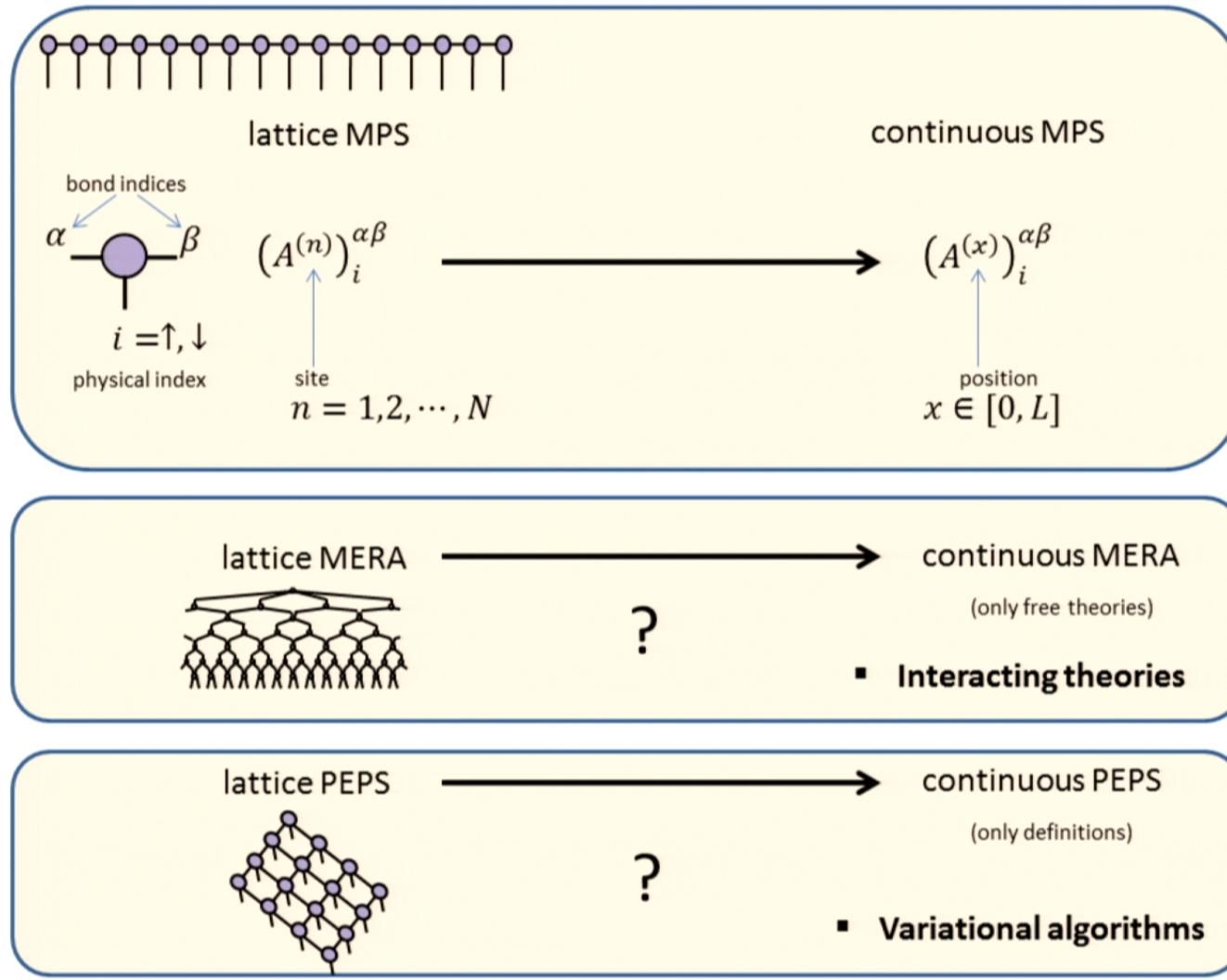
continuous tensor networks



continuous tensor networks



continuous tensor networks





THANKS!