Title: Quantum thermalization and many-body Anderson localization

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Abstract: Progress in physics and quantum information science motivates much recent study of the behavior of strongly-interacting many-body quantum systems fully isolated from their environment, and thus undergoing unitary time evolution. What does it mean for such a system to go to thermal equilibrium? I will explain the Eigenstate Thermalization Hypothesis (ETH), which posits that each individual exact eigenstate of the system's Hamiltonian is at thermal equilibrium, and which appears to be true for most (but not all) quantum many-body systems. Prominent among the systems that do not obey this hypothesis are quantum systems that are many-body Anderson localized and thus do not constitute a reservoir that can thermalize itself. When the ETH is true, one can do standard statistical mechanics using the `single-eigenstate ensembles', which are the limit of the microcanonical ensemble where the `energy window' contains only a single many-body quantum state. These eigenstate ensembles are more powerful than the traditional statistical mechanical ensembles, in that they can also "see" the quantum phase transition in to the localized phase, as well as a rich new world of phases and phase transitions within the localized phase.

Quantum Thermalization and
many-body Anderson localization.

D. H., Rahul Nandkishore, Vadim Oganesyan (CUNY),
Arijeet Pal (Harvard), Shivaji Sondhi.
Princeton.

arXiv: 1305.4915,
Phys. Rev. B 88, 014206 (2013).

Review: arXiv:1404.0686.

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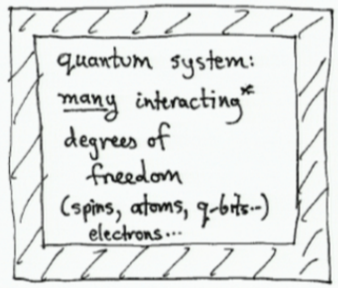
Consider:



fully isolated from its environment.

- as in many thermo./stat. mech. textbooks: of fundamental interest.
- Of <u>current</u> interest due to:
  - many-body A.M.O. physics: is now a reality.
  - some quantum information storage/processing schemes might look like this.

⋮

Today: nonrelativistic, in Euclidean space-time.

(no gravity, black holes...)

* interactions short-range in real space.

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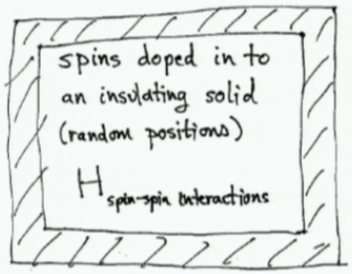
an old
Example. Anderson 1958



neglect spin-phonon interactions.

at energy corresponding to
high temperature for $H_{\text{spin-spin}}$

Not near ground state!

He asked:

→ Does this system constitute a reservoir (or bath) that thermally equilibrates its parts? ←

If yes: Thermalization.

If no: many-body localization.
MBL

Change temperature, or amount of static randomness:

Quantum Phase Transition

Thermal-to-Localized

Basko, Aleiner, Altshuler 2006
Pal, Huse 2010
Even in 1D at high temperatures!

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generally,

initial "state"

$\rho(t=0)$

Hamiltonian H

unitary time-evolution

$\rho(t) = e^{-iHt} \rho(t=0) e^{iHt}$

(Schrödinger picture)

$\hbar = 1$

QM Review:

Pure state $|\Psi_\alpha(t)\rangle$ ("wavefunction" or $\Psi_\alpha(t)$)

obeys Schrödinger eq'n:

$$i \frac{d}{dt} |\Psi_\alpha(t)\rangle = H |\Psi_\alpha(t)\rangle$$

solution: $|\Psi_\alpha(t)\rangle = e^{-iHt} |\Psi_\alpha(0)\rangle$

This pure state gives probability distribution (an operator)

$$\rho_\alpha(t) = |\Psi_\alpha(t)\rangle\langle\Psi_\alpha(t)|$$

(a.k.a. "density matrix")

But, many-body pure states can not be prepared. More realistically, our system is in a mixed state:

$$\rho(t) = \sum_\alpha p_\alpha |\Psi_\alpha(t)\rangle\langle\Psi_\alpha(t)| \quad \text{with} \quad \sum_\alpha p_\alpha = 1$$

(or integrals instead of sums)

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A standard assumption in thermo. + stat. mech.:

System → thermal equilibrium for time $t \to \infty$.

What does this mean for a closed quantum many-body system?

Full system does not thermalize. $\rho(t) \not\to e^{-H/T}/Z$ (Boltzmann-Gibbs)

All detailed information about initial state $\rho(0)$ remains in $\rho(t)$ for all times. But it gets "hidden", becoming impossible to measure as $t \to \infty$. This is "decoherence".

What thermalizes are subsystems.

Thermalization: Full system acts as "reservoir" and thermalizes its subsystems.

→ "Reservoir" of what? Entanglement. ←

Subsystem S defined by "local" operators.

Probability operator for subsystem S:

$$\rho_S(t) = \underset{\bar{S}}{\text{Trace}}\ \rho(t)$$

a.k.a. "reduced density matrix".

Thermal equilibrium for S at temperature T: $\quad \rho_S^{(eq.)}(T) = \underset{\bar{S}}{\text{Trace}} \dfrac{e^{-H/T}}{Z(T)}$
($k_B = 1$)

Thermalization: for all* initial states at that energy density, and all* "accessible" ("local") subsystems:

$$\lim_{\substack{t\to\infty \\ \bar{S}\to\infty}} \rho_S(t) = \lim_{\bar{S}\to\infty} \rho_S^{(eq.)}(T) .$$

* is it "all" or "almost all"?

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Eigenstate Thermalization Hypothesis (ETH)

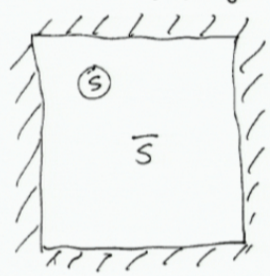
Deutsch 1991, Srednicki 1994, Tasaki 1998, ...

If all initial states thermalize, consider eigenstates of H

$H|n\rangle = E_n|n\rangle$ $\qquad \rho^{(n)} = |n\rangle\langle n|$ is time-independent. so must be thermal.

ETH: in limit $\bar{S} \to \infty$: $\quad \rho_S^{(n)} = \mathrm{Trace}_{\bar{S}}\, |n\rangle\langle n| = \rho_S^{(eq)}(T_n)$

for all eigenstates, subsystems, with $\langle H \rangle_{T_n} = E_n$



ETH is a strong claim. Hard to test.

It is stronger than is necessary for thermo. + stat. mech.

Appears to be true for many* systems. (exact diagonalizations of H's)

Has an appealing "simplicity".

*but not all

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When ETH is true, we have a new stat. mech. ensemble that (like the others) gives correct thermodynamics:

→ <u>single-eigenstate</u> microcanonical ensembles ←

For system satisfying ETH: To be out of thermal equilibrium (e.g., a temperature gradient) requires special <u>coherence</u> between amplitudes of eigenstates in $\langle n|\rho|m\rangle$.

With time, this coherence is lost due to dephasing: $e^{i(E_n-E_m)t}$:

→ Equilibration is "just" dephasing. ←

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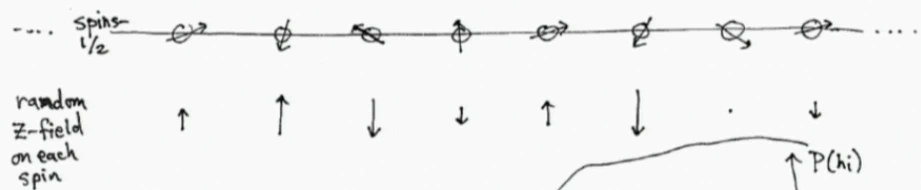
But: ETH is false for systems that are many-body Anderson localized.

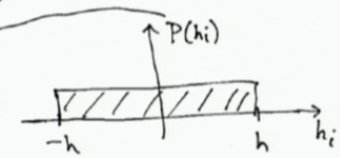
such systems do not thermalize, are not reservoirs.

Example: spin-1/2 (q-bit) chain with random static fields.

Pal + Huse 2010



$$H = \sum_i \left(J\, \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + h_i \sigma_{iz} \right)$$



probability distribution of h_i

or: ↑ = particle, ↓ = no particle

h_i = random potential

$\sigma_{iz}\sigma_{i+1\,z}$ = interaction

$\sigma_i^+ \sigma_{i+1}^-$ = hopping

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$$H = \sum_i \left(J \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + h_i \sigma_{iz} \right)$$

↖ static + random

For $J=0$, $h>0$, this is just independent spins,

is trivially localized: no transport of energy or σ_z

each σ_{iz} is conserved, each spin just Larmor precesses.

eigenstates are simply any spin pattern:

$|n\rangle = \cdots\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\cdots$ not thermal. so ETH false.

Add small J: remains localized: (Basko, et al. 2006, Pal + Huse 2010).

[Proof for a similar model: Imbrie 2014.]

each σ_{iz} is "dressed" to make a less-trivial

localized conserved pseudospin (Oganesyan + Huse 2013 + others 2013)

This is Anderson localization with interactions at high energy.

Larger J: system becomes thermal (Pal + Huse 2010).

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$$H = \sum_i \left(J \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + h_i \sigma_{iz} \right)$$

Eigenstate (or dynamic) phase diagram at $E = 0$ ("$T=\infty$")
(from diagonalization of H's)

0 (Bethe) — thermal phase — $(h/J)_c \simeq 3 \pm 1$ — many-body localized phase → h/J

thermal phase		many-body localized phase
system <u>is</u> a reservoir		system is <u>not</u> a reservoir.
all local info in $\rho(t=0)$ spreads through entire system for $t \to \infty$, thus is "hidden".		<u>some</u> local info in $\rho(t=0)$ remains localized. A quantum memory!
(<u>diffusive</u> transport of energy and σ_z (spin))		(<u>strictly</u> <u>zero</u> thermal and spin conductivity)
ETH is true	eigenstate phase transition(s?)	ETH is false
entanglement spreads <u>ballistically</u> (Kim+Huse 2013) → thermal entropy	scaling? Universality class?	entanglement spreads only as $\sim \log t$, remains subthermal. a quantum "glass".

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Localization protected quantum order: (Huse, et al. 2013)
(Pekker, Refael et al. 2013)
⋮

In many-body localized phase of spin systems,
lattice guage theories, ...

Can have long-range order (including topological order)
that is stabilized by localization, No mobile thermal fluctuations.
even at high temperature + in low dimensions.

Single-eigenstate statistical mechanics reveals a rich
new world of phases and phase transitions that
is invisible to equilibrium stat mech.

SUMMARY

Dynamics of <u>closed</u> many-body quantum systems.

- Thermalization
- Eigenstate Thermalization (ETH)
- Single-eigenstate statistical mechanics
- Many-body localization. (MBL)

- Experiments: thermalization is the norm.

  what will be the best system
  for exploring MBL in the lab?

  (atoms, molecules, condensed matter, ions,
  circuits, NV centers in diamond....?)

- What if H is not random? Can randomness in the state localize itself?

  (analog to structural glass)

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