Title: Almost quantum correlations

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Abstract: Quantum theory is successfully tested in any experimental lab every day. Apart from its experimental validity, quantum theory also constitutes a robust theoretical framework: small variations of its formalism often lead to highly implausible consequences, such as violation of the no-signalling principle or a significant increase of the computational power. In fact, it has been argued that quantum theory may represent an island in theory space. We show that, at the level of correlations, quantum theory may not be as special as initially thought. In order to do so, we define the set of almost quantum correlations and prove that this set is (i) strictly larger than the set of quantum correlations but (ii) satisfies most of the information principles introduced to characterize quantum correlations, such as local orthogonality, macroscopic locality, no advantage for nonlocal computation or non-trivial communication complexity. We also provide numerical evidence that the set is compatible with information causality. Finally, we briefly discuss how the set of almost quantum correlations naturally emerges in the consistent histories approach to quantum physics.

Quantum and almost quantum correlations

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Perimeter Institute, Waterloo, 9 April 2014

Correlations

One of the main goals of a scientific theory is to predict the correlations between events in different space-time locations:

If one performs a given action x at (q,t) getting result r, what's the probability of getting result r' when performing action y at (q',t')?

Correlations

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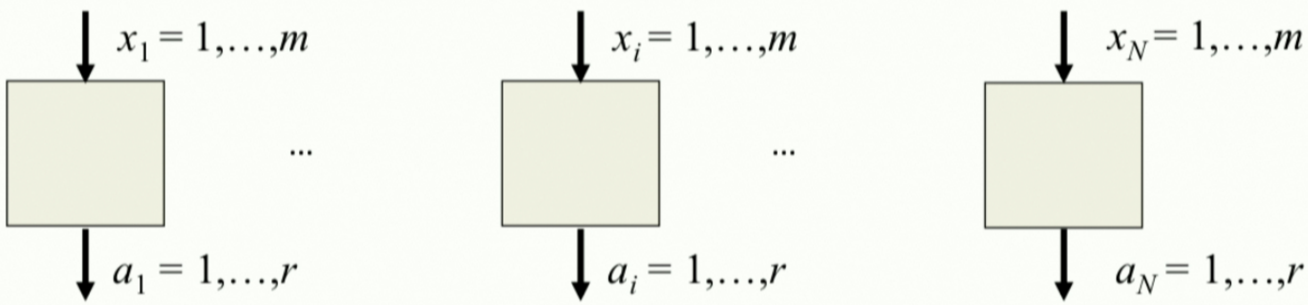
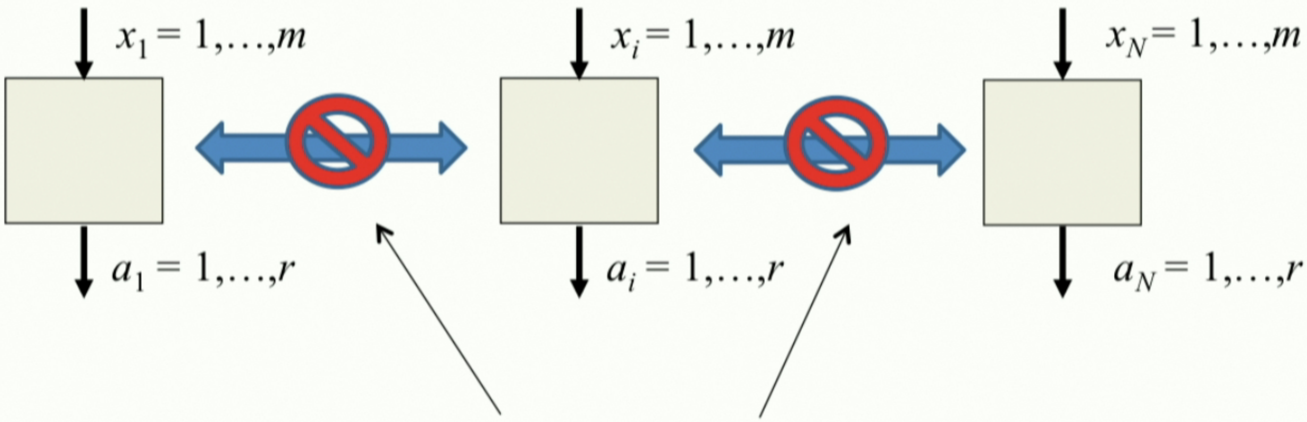
If one performs a given action x at (q,t) getting result r, what's the probability of getting result r' when performing action y at (q',t')?



Correlations: $p(a_1,\ldots,a_N | x_1,\ldots,x_N)$

Correlations

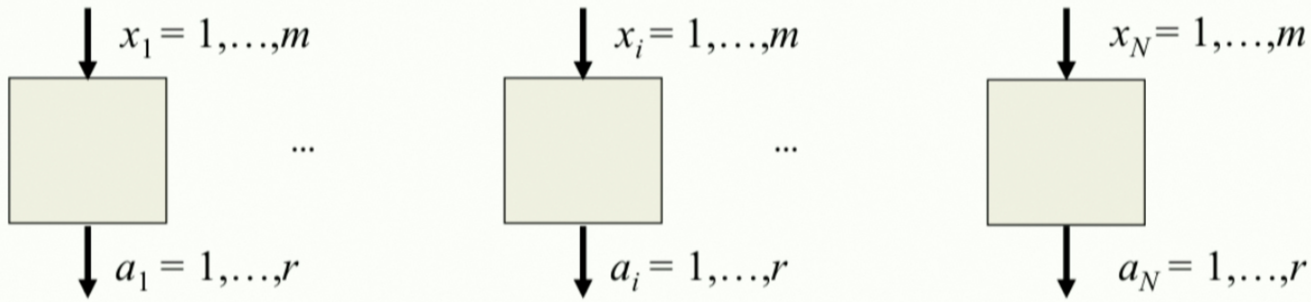
In this talk, we consider only causally disconnected events. That is, the devices do not communicate during the input/output process.



No communication!

Physically, this can be enforced by synchronizing well enough the processes so that they are space-like separated.

Box-world scenario



$$p(a_1,\ldots,a_N | x_1,\ldots,x_N)$$

- Formally, correlations are described by the different conditional probabilities observed in the experiment. They can be collected into a vector of positive numbers satisfying some normalization conditions.

- No assumption or modeling on the devices is made. They are seen as back boxes processing classical information.

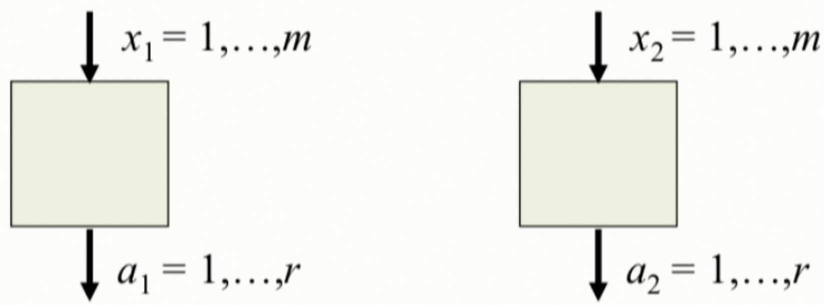
Physical correlations

Physical principles translate into limits on correlations.

No-signalling correlations: correlations compatible with the no-signalling principle, i.e. the impossibility of instantaneous communication.

$$\sum_{a_{k+1},\ldots,a_N} p(a_1,\ldots,a_N|x_1,\ldots,x_N) = p(a_1,\ldots,a_k|x_1,\ldots,x_k)$$

$p(a_1|x_1)$



Physical correlations

Classical correlations: correlations established by classical means.

$$p\left(a_1,\ldots,a_N \middle| x_1,\ldots,x_N\right) = \sum_{\lambda} p(\lambda) D\left(a_1 \middle| x_1,\lambda\right)\ldots D\left(a_N \middle| x_N,\lambda\right)$$

These are the standard "EPR" correlations. Independently of fundamental issues, these are the correlations achievable by classical resources. Bell inequalities define the limits on these correlations.

Physical correlations

Quantum correlations: correlations established by quantum means.

$$p(a_1,\ldots,a_N|x_1,\ldots,x_N) = \mathrm{tr}\left(\rho M_{a_1}^{x_1} \otimes \cdots \otimes M_{a_N}^{x_N}\right)$$

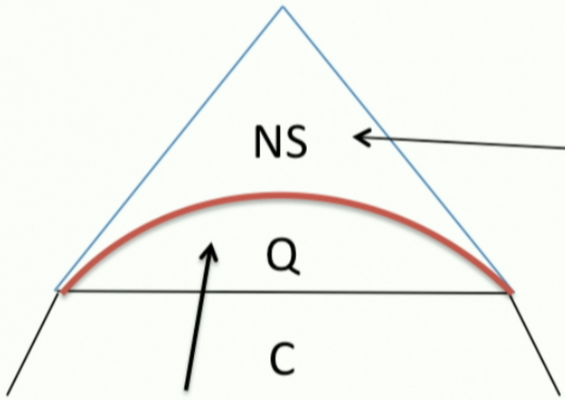
$$\sum_{a_i} M_{a_i}^{x_i} = 1 \qquad M_{a'_i}^{x_i} M_{a_i}^{x_i} = \delta_{a_i a'_i} M_{a_i}^{x_i}$$

Everything is expressed in terms of operators (the quantum state and the measurement projectors) acting on a Hilbert space.

Physical correlations

Bell $C \subset Q \subset NS$ Tsirelson

Popescu-Rohrlich



There exist correlations that are compatible with the no-signalling principle but cannot be obtained by performing local measurements on a quantum (entangled) state.

There exist correlations that cannot be explained by a classical model in which (deterministic) classical instructions specify the outcomes of the devices. These quantum correlations are known as **non-local** and they are detected by the violation of a Bell inequality.

Characterization of Quantum Correlations

Characterizing quantum correlations

Given $p(a,b|x,y)$, does it have a quantum realization?

$$p(a,b|x,y) = tr\left(\rho_{AB} M_a^x \otimes M_b^y\right)$$

$$\sum_a M_a^x = 1$$

$$M_a^x M_{a'}^x = \delta_{a'a} M_a^x$$

Example:

$$p(a,b|0,0) = p(a,b|0,1) = p(a,b|1,0) = \frac{1}{8}\left(2+\sqrt{3}, 2-\sqrt{3}, 2-\sqrt{3}, 2+\sqrt{3}\right)$$

$$p(a,b|1,1) = (0.245, 0.255, 0.255, 0.245)$$

Motivation

- What are the allowed correlations within our current **quantum** description of nature?
- How can we detect the non-quantumness of some observed correlations? Quantum analogues of Bell inequalities.
- What are the limits on correlations associated to the quantum formalism?
- To which extent Quantum Mechanics is useful for information tasks?

Previous work by **Tsirelson**

Device-independent quantum information processing

Quantum hacking

NATURE PHOTONICS | LETTER

Hacking commercial quantum cryptography systems by tailored bright illumination

Lars Lydersen, Carlos Wiechers, Christoffer Wittmann, Dominique Elser, Johannes Skaar & Vadim Makarov

Published online 29 August 2010 | Nature | doi:10.1038/news.2010.436

News

Hackers blind quantum cryptographers

Lasers crack commercial encryption systems, leaving no trace.

Zeeya Merali

How come?!

Quantum hacking

Quantum hacking attacks break the implementation, not the principle.

Theory

- Prepare states in a Hilbert space of dimension two.
- Measure observables in the same space, e.g. spin-1/2 measurements.

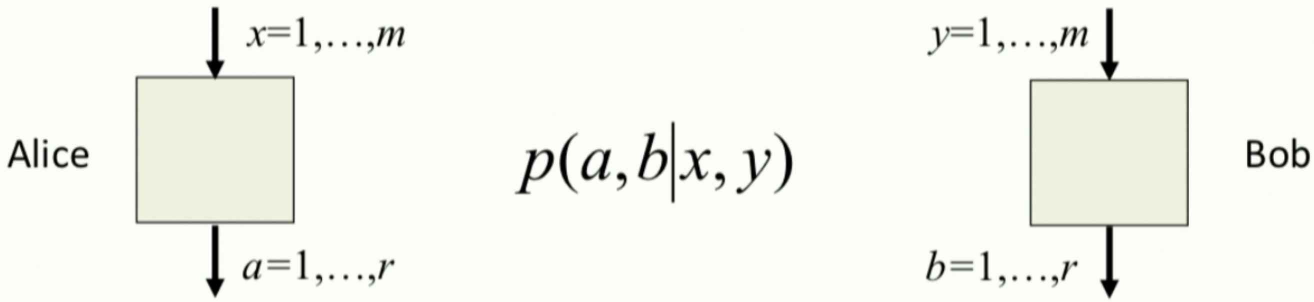
Implementation

- Prepare states using an attenuated laser source.
- Measure polarization of light using single-photon detectors.

Moral: the unavoidable mismatch between theoretical requirements and implementation is an important weakness in quantum information protocols, especially in adversarial scenarios.

A solution to the hacking problem

Device-Independent Quantum Key Distribution



$$p(a,b|x,y)$$

Protocols that establish a secure key only from the observed correlations and without making any assumption about the internal working of the devices used to obtain these correlations.

A. Acín *et al.*, Phys. Rev. Lett. 98, 230501 (2007)

Bell inequality violation

Bell inequality violation is a necessary condition for security.

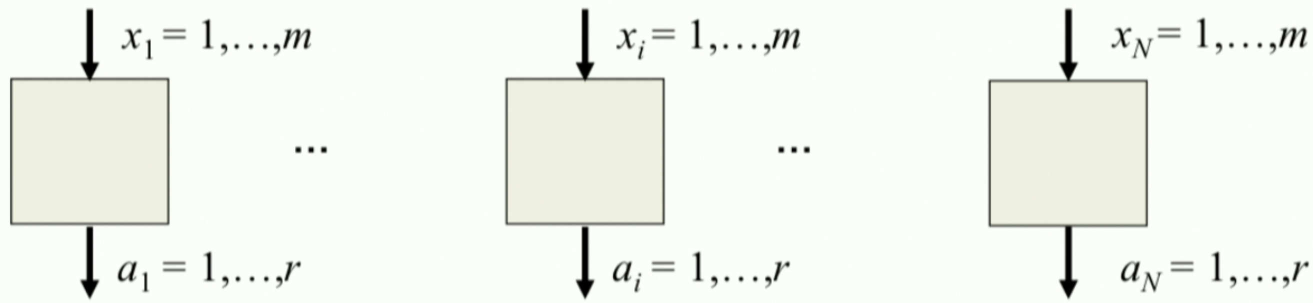
If the correlations are local: $p(a,b|x,y) = \sum_{\lambda} p(\lambda) p(a|x,\lambda) q(b|y,\lambda)$

A perfect copy of the local instructions can go to Eve.

Barrett, Hardy, Kent, PRL 95; Ekert PRL 91

DI quantum information processing

Develop a new form of **quantum information theory** in a scenario where the users' devices are just seen as (quantum) **black boxes** processing classical information. The resulting protocols have **self-certification**.



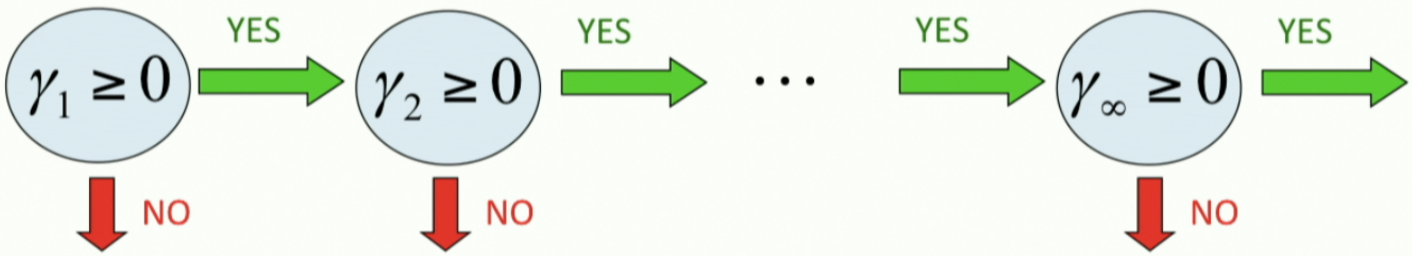
Observed correlations

$$p(a_1,\ldots,a_N | x_1,\ldots,x_N)$$

Characterization of Quantum Correlations

NPA hierarchy

Given a probability distribution $p(a,b|x,y)$, we have defined a hierarchy consisting of a series of tests based on semi-definite programming techniques allowing the detection of supra-quantum correlations.

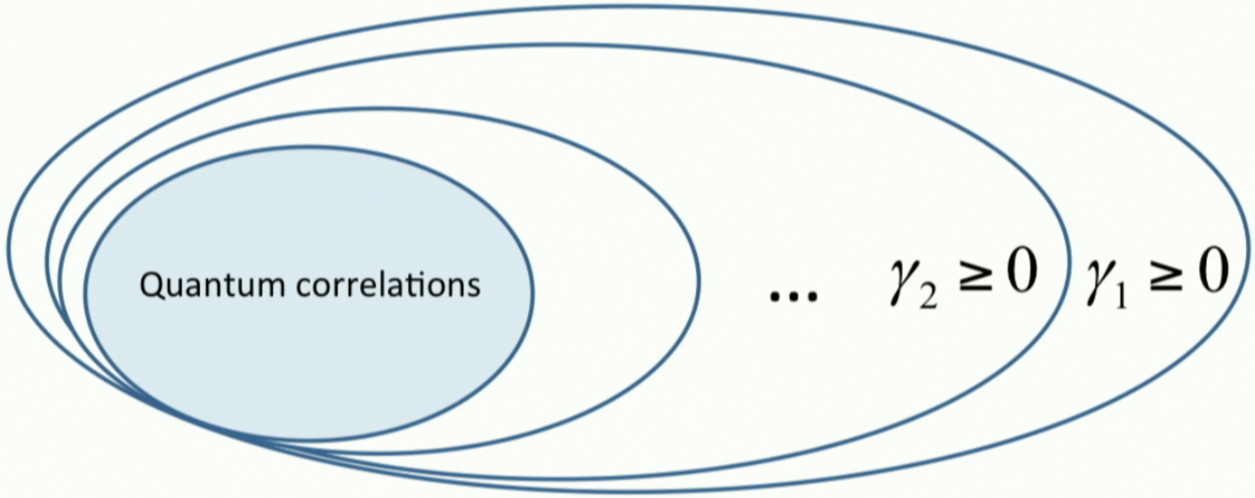


The hierarchy is asymptotically convergent.

Navascués, Pironio, Acin, PRL 2007

Related work by Doherty, Liang, Toner and Wehner

NPA hierarchy



Every step in the hierarchy defines a convex set that is included in the previous step. Convergence is provably attained asymptotically.

In many situations convergence is attained after a few steps. But there is evidence that there may be situations that require an infinite number of steps.

Characterizing quantum correlations

Example:

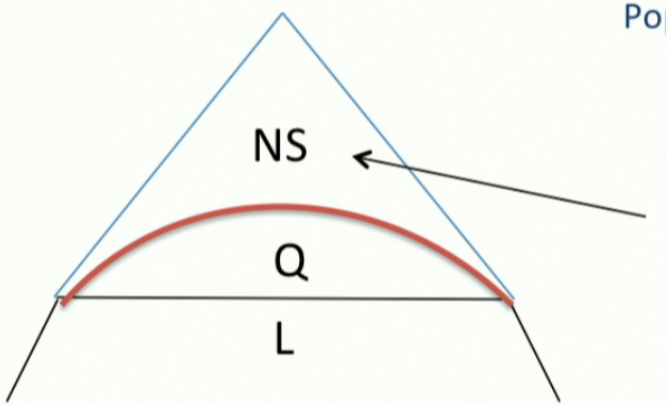
$$p(a,b|0,0) = p(a,b|0,1) = p(a,b|1,0) = \frac{1}{8}\left(2+\sqrt{3},2-\sqrt{3},2-\sqrt{3},2+\sqrt{3}\right)$$

$$p(a,b|1,1) = (0.245,0.255,0.255,0.245)$$

Solution: it is not quantum, that is, there exists no quantum state of two particles and local measurements acting on them that produce these correlations.

The experimental observation of these correlations would imply the failure of quantum physics, as Bell violations did for classical physics.

Why quantum correlations?



Popescu and Rohrlich, Found. Phys. 1994

Q: Why are these correlations not possible in Nature?

A: They are incompatible with quantum laws. That is, there is no quantum state and measurements able to reproduce them.

What would their existence imply operationally?

Information principles have been proposed as the mechanism to bound quantum correlations. Examples: non-trivial communication complexity, information causality, macroscopic locality or local orthogonality.

Non-trivial communication complexity

Alice — $x \in \{0,1\}^n$

— 1 bit →

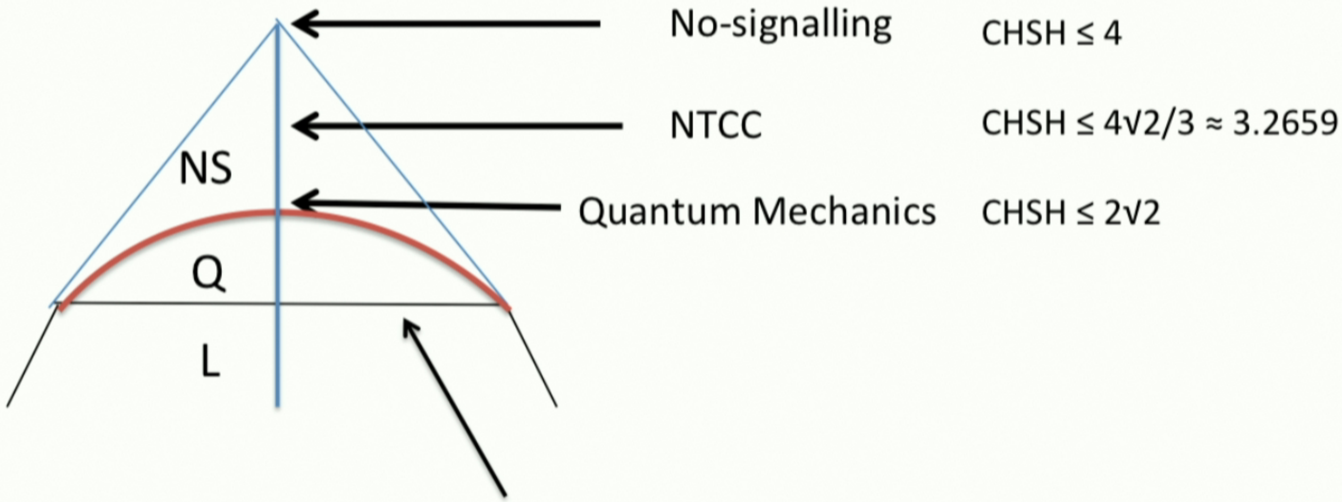
Bob — $y \in \{0,1\}^n$

$$f(x,y) \in \{0,1\}$$

NTCC: Bob cannot guess the value of the function with worst-case probability p>1/2 for all functions and input sizes.

There exist supra-quantum non-signalling correlations that make communication complexity trivial.
Van Dam, PhD Thesis, arXiv:quant-ph/0501159

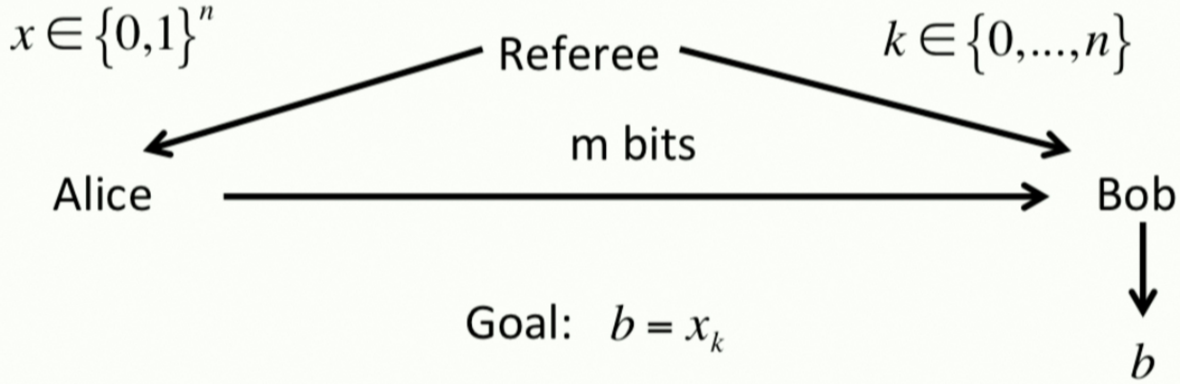
Non-trivial communication complexity



$$\mathrm{CHSH} = A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 \leq 2$$

G. Brassard, H. Buhrman, N. Linden, A. A. Methot, A. Tapp and F. Unger, Phys. Rev. Lett., 96 250401, (2006).

Information causality



The message by Alice should not reveal more than m of her bits: $\sum_k I\left(b : x_k \middle| k\right) \leq m$

M. Pawlowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter, and M. Zukowski, Nature 461, 1101 (2009).

Information causality

No-signalling condition → CHSH ≤ 4

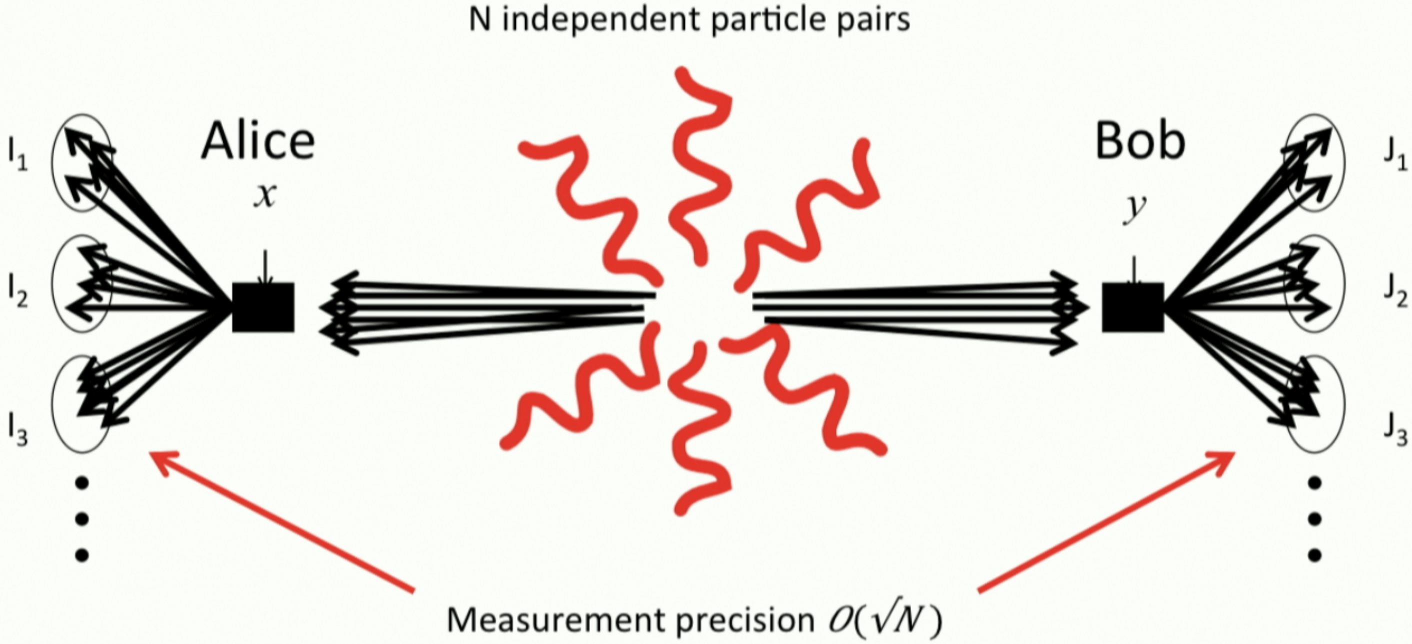
IC → CHSH ≤ 2√2

Quantum Mechanics → CHSH ≤ 2√2

Yet, there are regions in the space of correlations where, at the moment, there is a gap between quantum correlations and information causality.

Macroscopic Locality

Non-local correlations cannot be observed in macroscopic experiments.
M. Navascués and H. Wunderlich, Proc. Royal Soc. A 466, 881-890 (2009).



Macroscopic Locality

No-signalling condition → CHSH ≤ 4

ML → CHSH ≤ 2√2

Quantum Mechanics → CHSH ≤ 2√2

The principle is known to be larger than the quantum set.

Local orthogonality: a multipartite principle

Local orthogonality: different outcomes of the same measurement by one of the observers define orthogonal events, independently of the rest of measurements.

Event	Input	Output
e_1	x_1	$a_1 \ldots \boldsymbol{a_i} \ldots a_N$
e_2	$x'_1 \ldots \boldsymbol{x_i} \ldots x_N$	$a'_1 \ldots \boldsymbol{a'_i} \ldots a'_N$

1. N events are orthogonal if they are pairwise orthogonal.
2. The sum of probabilities of orthogonal events is bounded by 1. $\sum_i p(e_i) \leq 1$

T. Fritz, A. B. Sainz, R. Augusiak, J. B. Brask, R. Chaves, A. Leverrier and A. Acín, Nature Communications 4, 2263 (2013) .

Local Orthogonality

No-signalling condition → CHSH ≤ 4

LO → CHSH ≤ 2.88

Quantum Mechanics → CHSH ≤ 2√2

We know examples of supra-quantum correlations not detectable by any bipartite principle that violate LO.

Will this ever stop?

- On the one hand, it seems very difficult to extract the Hilbert space formalism only from information principles acting on the correlations.

- On the other hand, it appears very difficult to prove that an information principle is not good, as one has to consider all possible applications of the principle.

Almost quantum correlations

We have identified the set of almost quantum correlations, a set of non-signalling correlations with the following properties:

- It is provably supra-quantum.
- It can be proven to satisfy all known information principles proposed for quantum correlations, except information causality.
- The existing numerical evidence suggests that it also satisfies information causality.

Almost quantum correlations

The set $\tilde{Q}$ of almost quantum correlations is defined as:

Definition 1. *Let $P(a_1, ..., a_n|x_1, ..., x_n)$ be an n-partite non-signalling distribution. We say that $P(a_1, ..., a_n|x_1, ..., x_n)$ is* almost quantum *iff there exist a Hilbert space $\mathcal{H}$, a normalized state $|\phi\rangle \in \mathcal{H}$ and projector operators $\{E_k^{a,x}\} \subset B(\mathcal{H})$ with the properties:*

(i) $\sum_a E_k^{a,x} = \mathbb{I}$, *for all* x, k.

(ii) $E_1^{a_1,x_1} ... E_n^{a_n,x_n}|\phi\rangle = E_{\pi(1)}^{a_{\pi(1)},x_{\pi(1)}} ... E_{\pi(n)}^{a_{\pi(n)},x_{\pi(n)}}|\phi\rangle$, *where $\pi \in S_n$ is an arbitrary permutation of the parties* $\{1, ..., n\}$.

(iii) $P(a_1, ..., a_n|x_1, ..., x_n) = \langle\phi|\prod_{k=1}^n E_k^{a_k,x_k}|\phi\rangle$.

Property (ii) implies that the projectors "commute only when acting on the state".

The set of almost quantum correlations contains the set of quantum correlations Q.

Almost quantum correlations

The set $\tilde{Q}$ of almost quantum correlations can be alternatively defined as:

Lemma 3. *Let $P(a_1, ..., a_n|x_1, ..., x_n)$ be a non-signalling n-partite distribution. $P(a_1, ..., a_n|x_1, ..., x_n) \in \tilde{Q}$ iff, for any event $(\bar{a}|\bar{x})$ with $a_k \neq 0$ for all parties k involved, there exists a vector $|\bar{a}, \bar{x}\rangle \in \mathcal{H}$ with the properties*

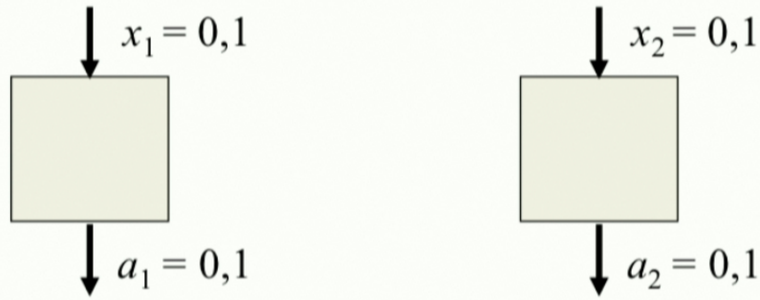
(a) $\langle \bar{a}', \bar{x}'|\bar{a}, \bar{x}\rangle = 0$, *if* $(\bar{a}'|\bar{x}') \perp (\bar{a}|\bar{x})$.

(b) $P(\bar{a}|\bar{x}) = \langle \phi|\bar{a}, \bar{x}\rangle$, *where* $|\phi\rangle$ *is the (normalized) vector corresponding to the null event, i.e., none of the parties measures.*

(c) $\langle \bar{a}, \bar{a}', \bar{x}, \bar{x}'|\bar{a}, \bar{a}'', \bar{x}, \bar{x}''\rangle = \langle \bar{a}', \bar{x}'|\bar{a}, \bar{a}'', \bar{x}, \bar{x}''\rangle = \langle \bar{a}, \bar{a}', \bar{x}, \bar{x}'|\bar{a}'', \bar{x}''\rangle$, *where* $(\bar{a}|\bar{x})$ *is any event not involving the measuring parties in the events* $(\bar{a}'|\bar{x}')$ *and* $(\bar{a}''|\bar{x}'')$.

The set admits an efficient SDP characterization: it corresponds to the level 1+AB of the NPA hierarchy. M. Navascués, S. Pironio and A. Acín, New J. Phys. (2008).

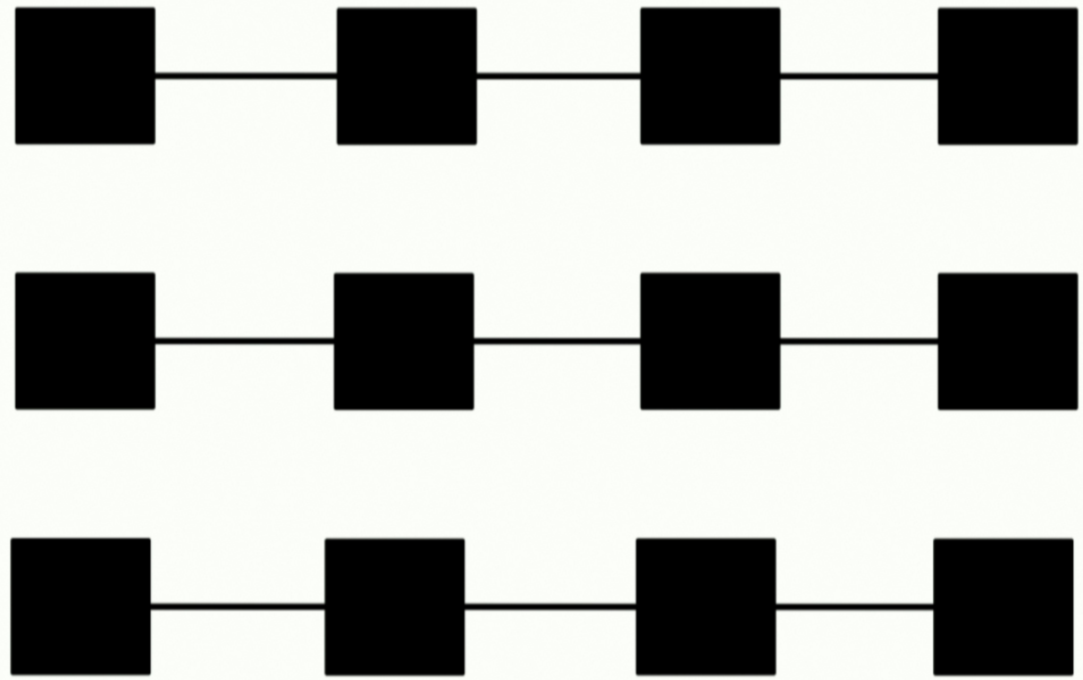
$\tilde{Q}$ is larger than the quantum set



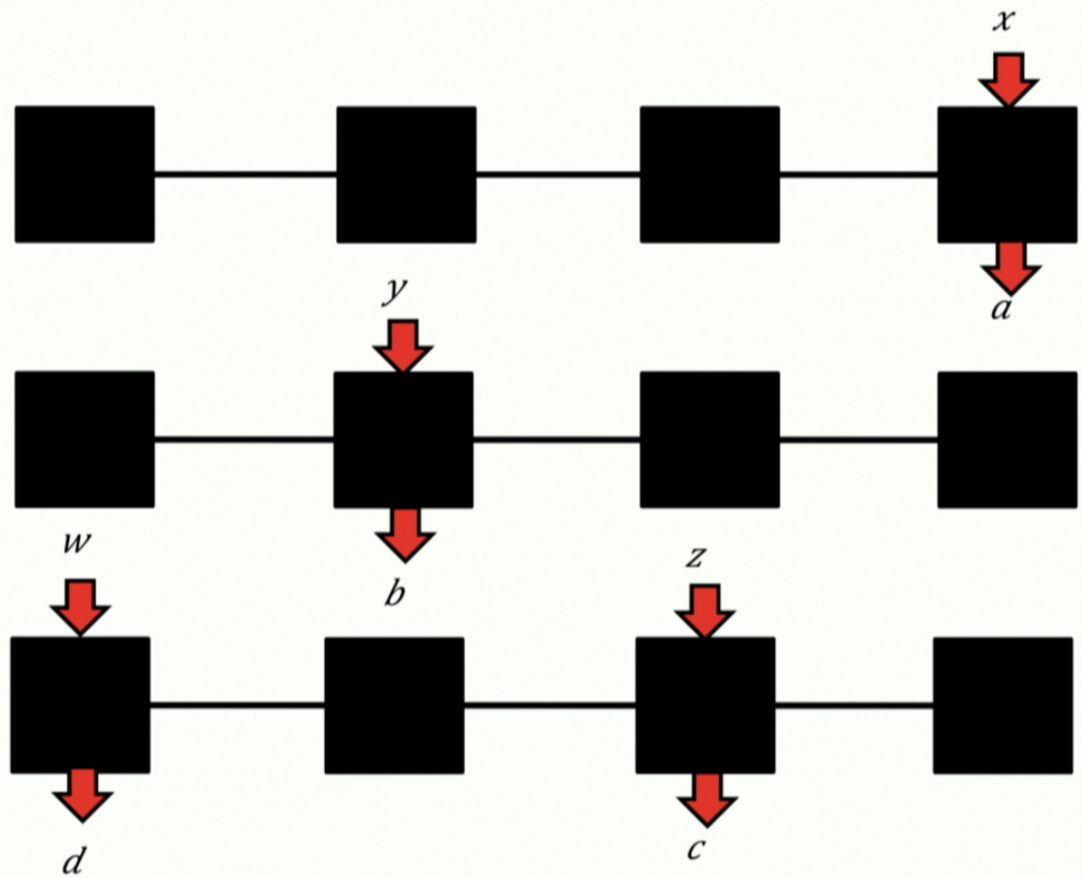
$$I \equiv -\frac{30}{31}P_1(1|0) + \frac{167}{9}P_1(1|1) + \frac{167}{9}P_2(1|0) - \frac{30}{31}P_2(1|1) - \frac{174}{11}P(1,1|0,0) - \frac{244}{23}P(1,1|1,0) + \frac{74}{11}P(1,1|0,1) - \frac{174}{11}P(1,1|1,1)$$

For quantum correlations this quantity is lower bounded by -1.
There exist almost quantum correlations that give $I \approx$ -1.052.

$\tilde{Q}$ is closed under post-processing

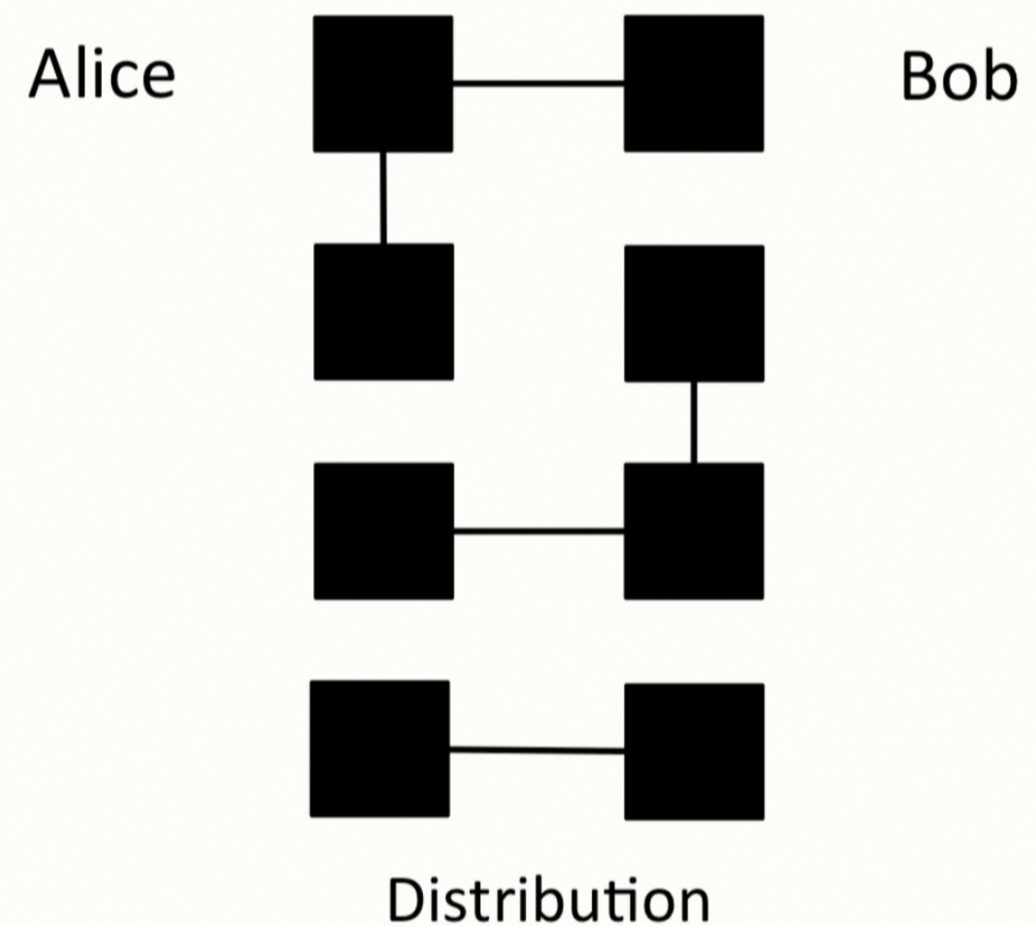


$\tilde{Q}$ is closed under post-processing

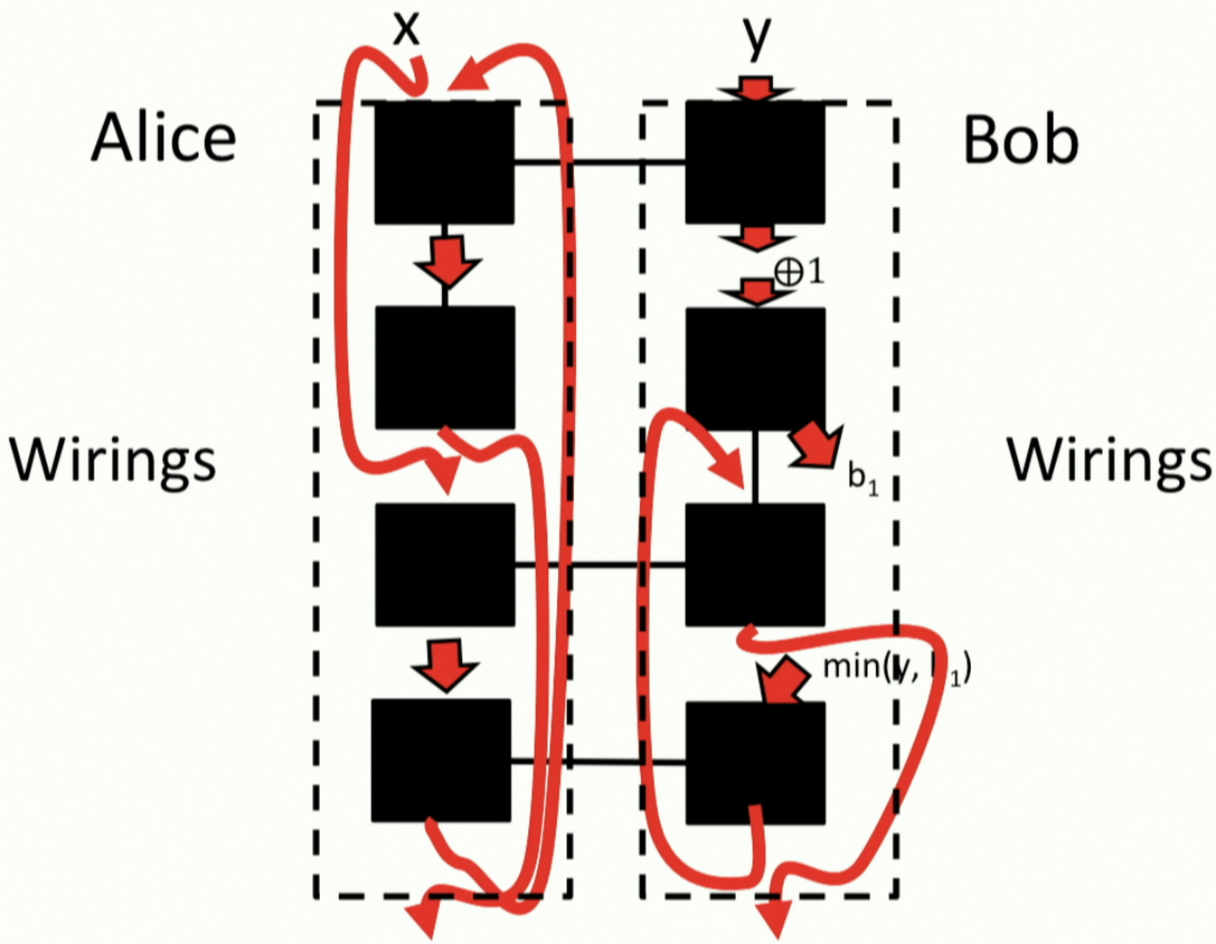


Post-selection

$\tilde{Q}$ is closed under post-processing



$\tilde{Q}$ is closed under post-processing



$\tilde{Q}$ has non-trivial communication complexity

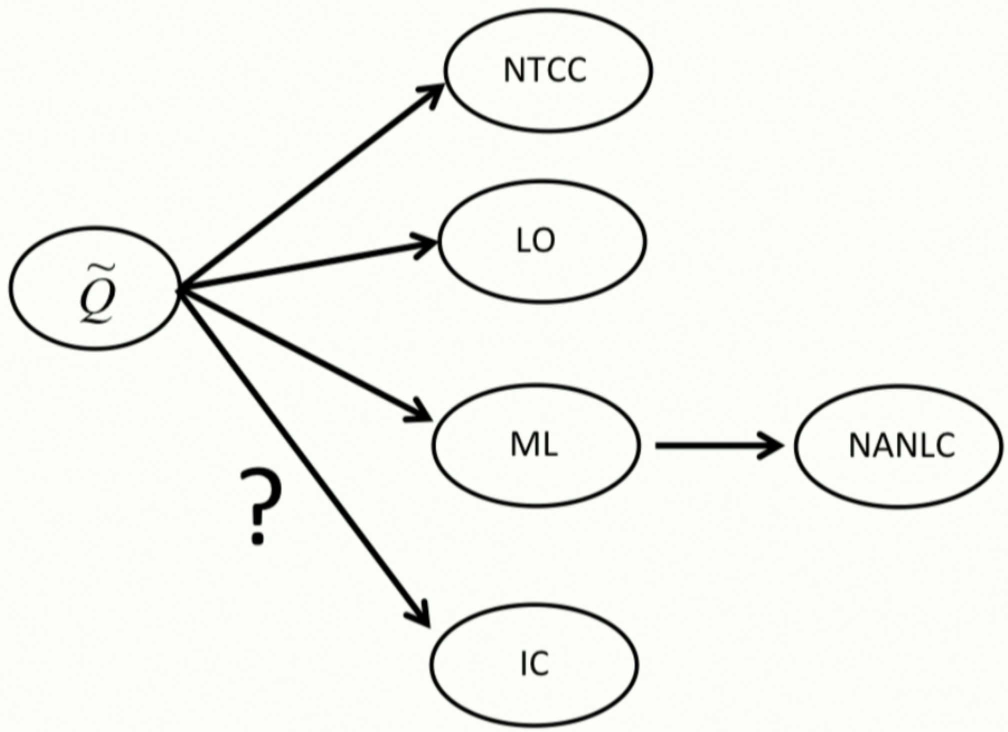
Alice | Bob

$x \in \{0,1\}^n$ —— m bits ——→ $y \in \{0,1\}^n$

↓

Alice and Bob have to compute the inner product: $x \cdot y \in \{0,1\}$

$\tilde{Q}$ and information principles



$\tilde{Q}$ and information causality

Known bounds on nonlocality from IC

J. Allcock, N. Brunner, M. Pawlowski and V. Scarani, Phys. Rev. A 80, 040103(R) (2009).

A. Ahanj, S. Kunkri, A. Rai, R. Rahaman and P. S. Joag, Physical Review A 81, 032103 (2010).

D. Cavalcanti, A. Salles and V. Scarani, Nat. Commun. 1, 136 (2010).

Y. Xiang and W. Ren, QIC, Vol. 11, No. 11&12 0948--0956 (2011).

T. H. Yang, D. Cavalcanti, M. Almeida, C. Teo and V. Scarani, New J. Phys. 14, 013061 (2012).

For all the known cases, the set of almost quantum correlations provide the same or tighter bounds on non-signalling correlations than information causality.

Almost quantum correlations

- Quantum physics has been and is tested experimentally in many different situations.
- From a theoretical point of view, it is a robust formalism. Attempts to modify the theory have led to implausible consequences, such as signalling or more computational power. It has sometimes been said that quantum physics may be special and represent “an island in theory space”. S. Aaronson, arXiv:quant-ph/0401062
- Our work proves that, at least at the level of correlations, it is possible to modify the structure of quantum theory apparently without running into any operational problem.

Speculations…

- The set of almost quantum correlations has a simple characterization in terms of SDP.

Speculations…

- The set of almost quantum correlations has a simple characterization in terms of SDP.
- The best known characterization of the quantum set is by means of a hierarchy of SDP programs, the NPA hierarchy.
- It is even unknown whether the problem of deciding if some correlations belong to the quantum set is decidable or not.
- A related question: are systems of finite dimension enough to generate all quantum correlations?
- Do you expect correlations in nature to be decidable?

A theory beyond $\mathcal{Q}$?

Coherent histories approach to quantum physics motivated by quantum gravity.

Quantum measure

1. for all $\alpha \in \mathfrak{A}$, $\mu(\alpha) \geq 0$ (*Positivity*);
2. for all mutually disjoint $\alpha, \beta, \gamma \in \mathfrak{A}$,
$$\mu(\alpha \cup \beta \cup \gamma) - \mu(\alpha \cup \beta) - \mu(\beta \cup \gamma) - \mu(\gamma \cup \alpha) + \mu(\alpha) + \mu(\beta) + \mu(\gamma) = 0$$
(*Quantum Sum Rule*);
3. $\mu(\Omega) = 1$ (*Normalisation*).

The quantum sum rule implies the lack of interference between triple of histories.

A theory beyond Q ?

Decoherence functional: a map from pairs of events to the complex numbers such that:

1. For all $\alpha, \beta \in \mathfrak{A}$, we have $D(\alpha, \beta) = D(\beta, \alpha)^*$ (*Hermiticity*).
2. For all $\alpha, \beta, \gamma \in \mathfrak{A}$ with $\beta \cap \gamma = \emptyset$, we have $D(\alpha, \beta \cup \gamma) = D(\alpha, \beta) + D(\alpha, \gamma)$ (*Bi-additivity*).
3. $D(\Omega, \Omega) = 1$ (*Normalisation*).
4. For all $\alpha \in \mathfrak{A}$, $D(\alpha, \alpha) \geq 0$ (Positivity).

The existence of a quantum measure is equivalent to the existence of a decoherence functional.

Sometimes the stronger condition that D is semi-definite positive is imposed.

A theory beyond $\mathcal{Q}$?

In the case of correlations, one is interested in the existence of a strongly positive joint quantum measure (SPJQM).

Dowker, Henson and Wallden (and independently Navascués) have proven that the existence of a SPJQM singles out the set of almost quantum correlations.

Theorem 21. $SPJQM_{b} = Q^{1+AB}$.

F. Dowker, J. Henson and P. Wallden, NJP 2014

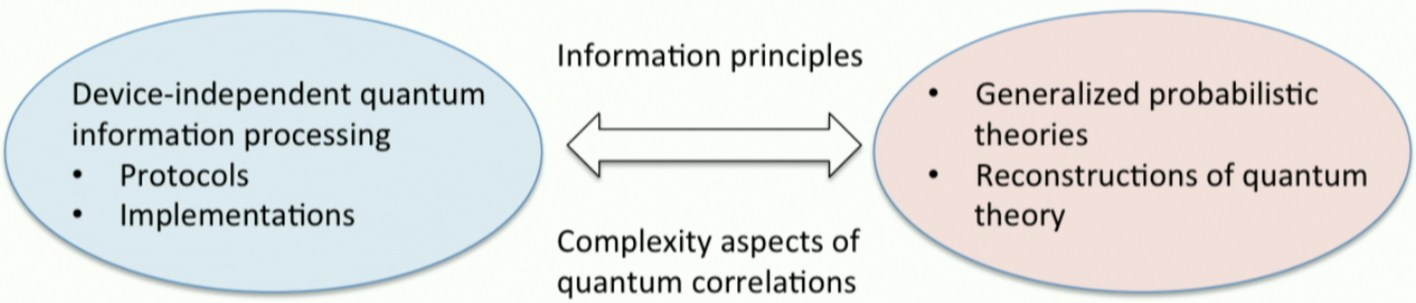
Is this connection accidental?

Conclusions

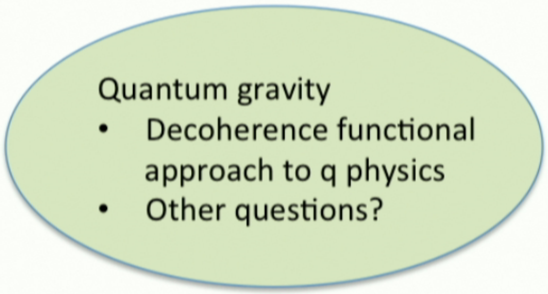
- The set of almost quantum correlations is an intriguing object.
- It has an efficient numerical characterization.
- It is in many senses not more powerful than quantum correlations.
- It proves that, at the level of correlations, quantum physics may not be as "special" as expected.
- It has an intriguing connection with quantum gravity concepts.
- The starting point for a consistent theory beyond quantum physics?

Conclusions

Quantum correlations have proven to be a fruitful arena for questions in quantum information theory and foundations of quantum physics. It allows one to study quantum physics without Hilbert spaces.



Can these techniques be of any use for quantum gravity problems?



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