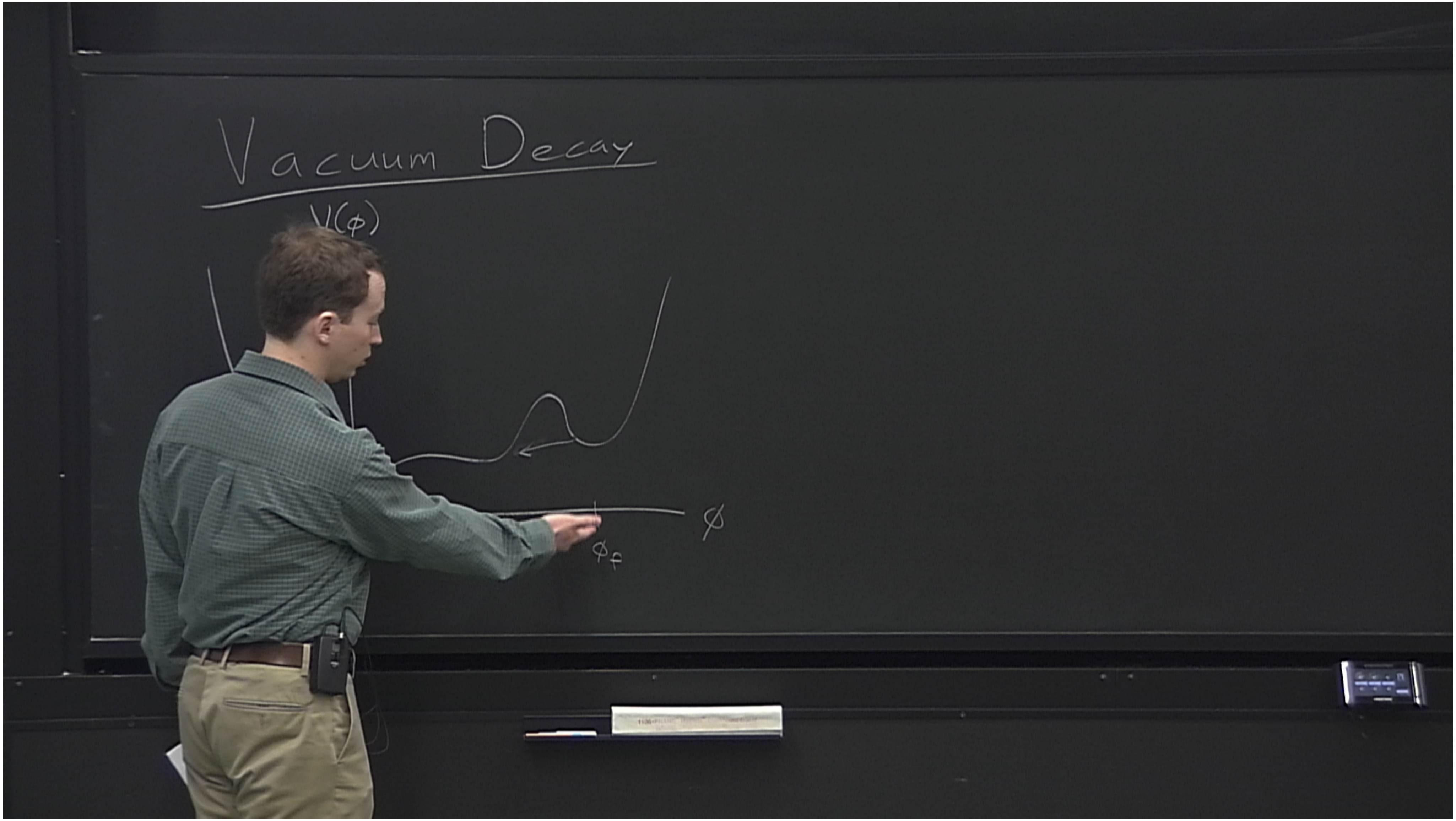
Title: 13/14 PSI - Explorations in Cosmology - Lecture 13

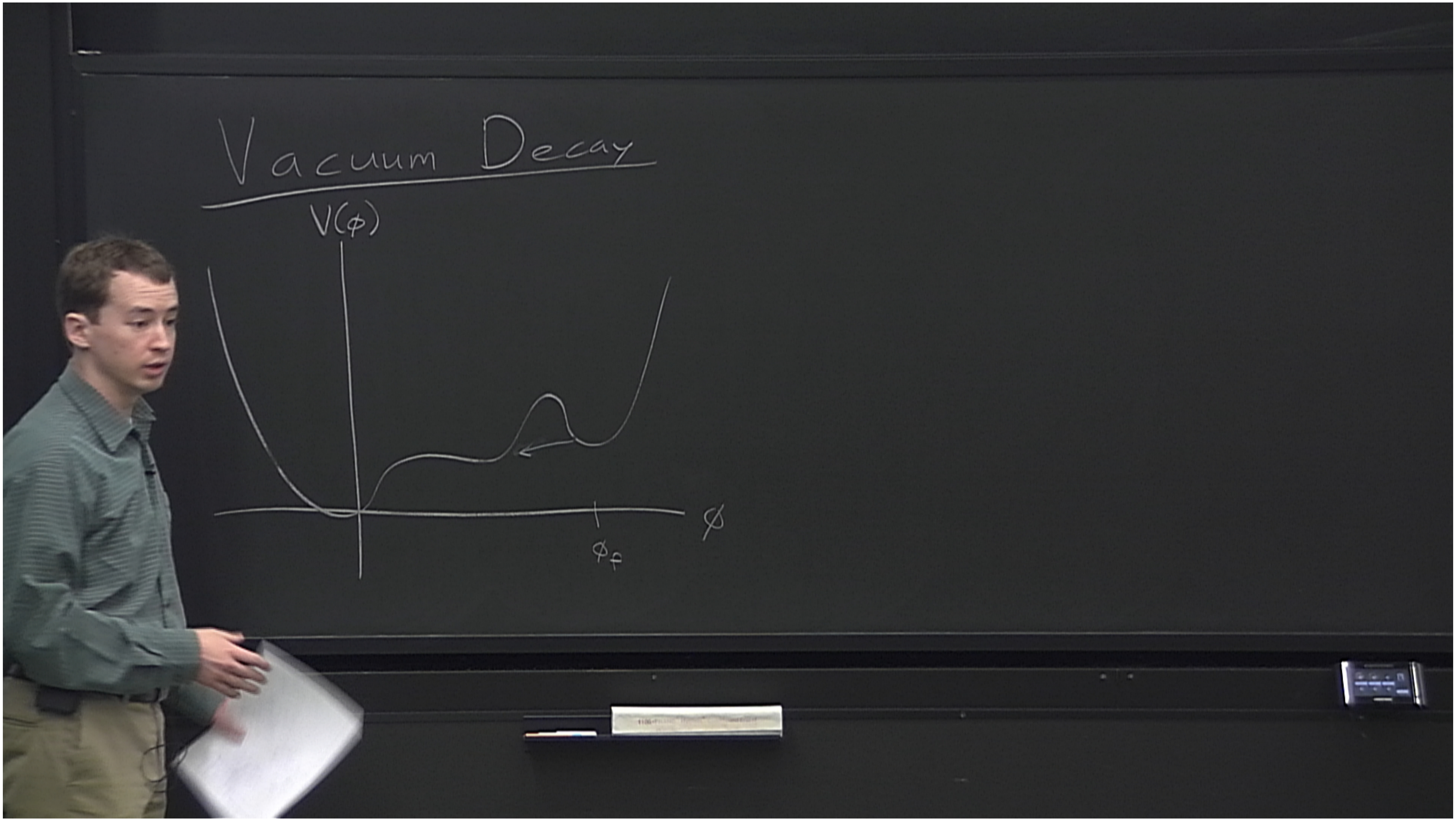
Date: Apr 24, 2014 10:15 AM

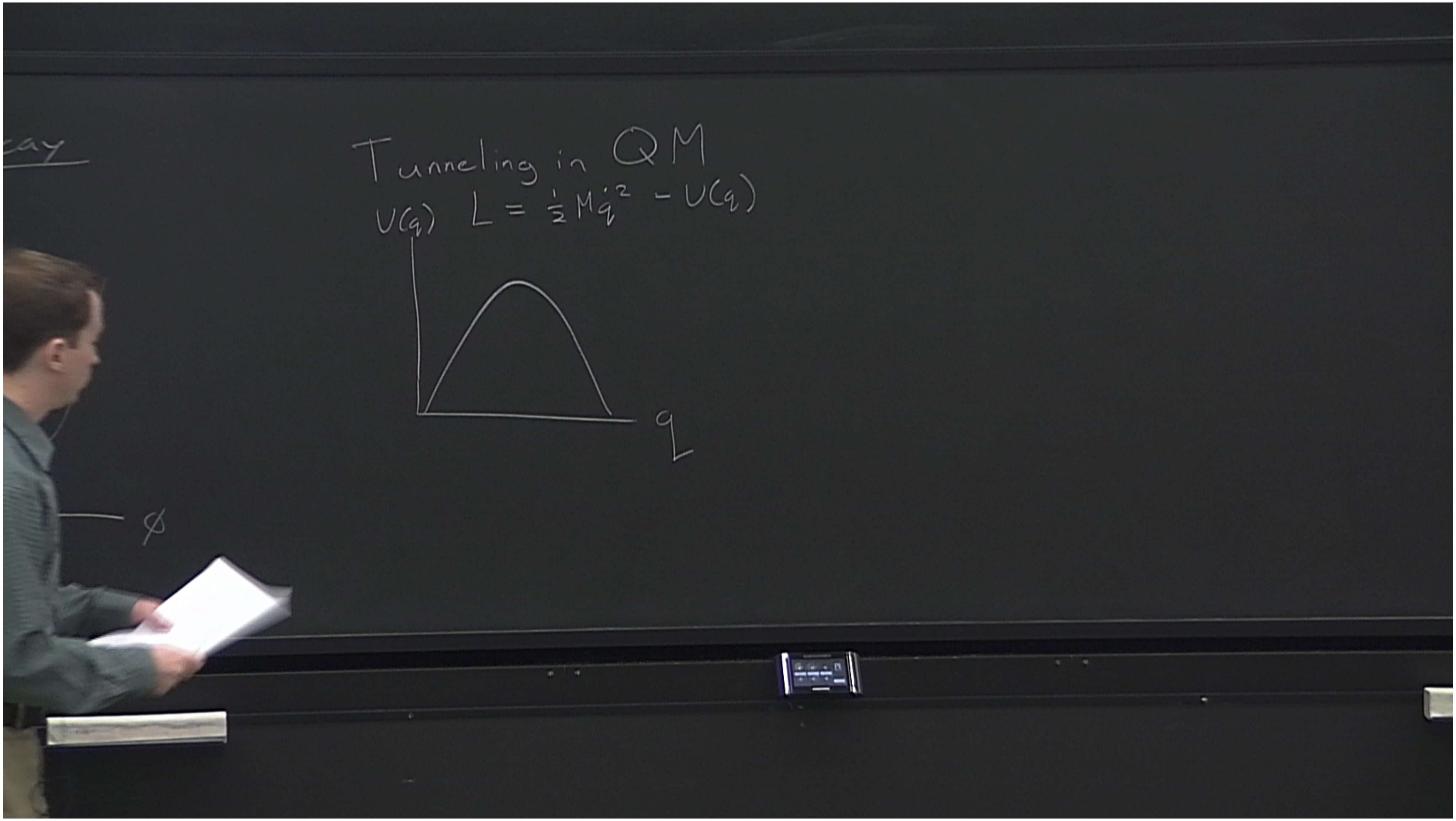
URL: http://pirsa.org/14040058

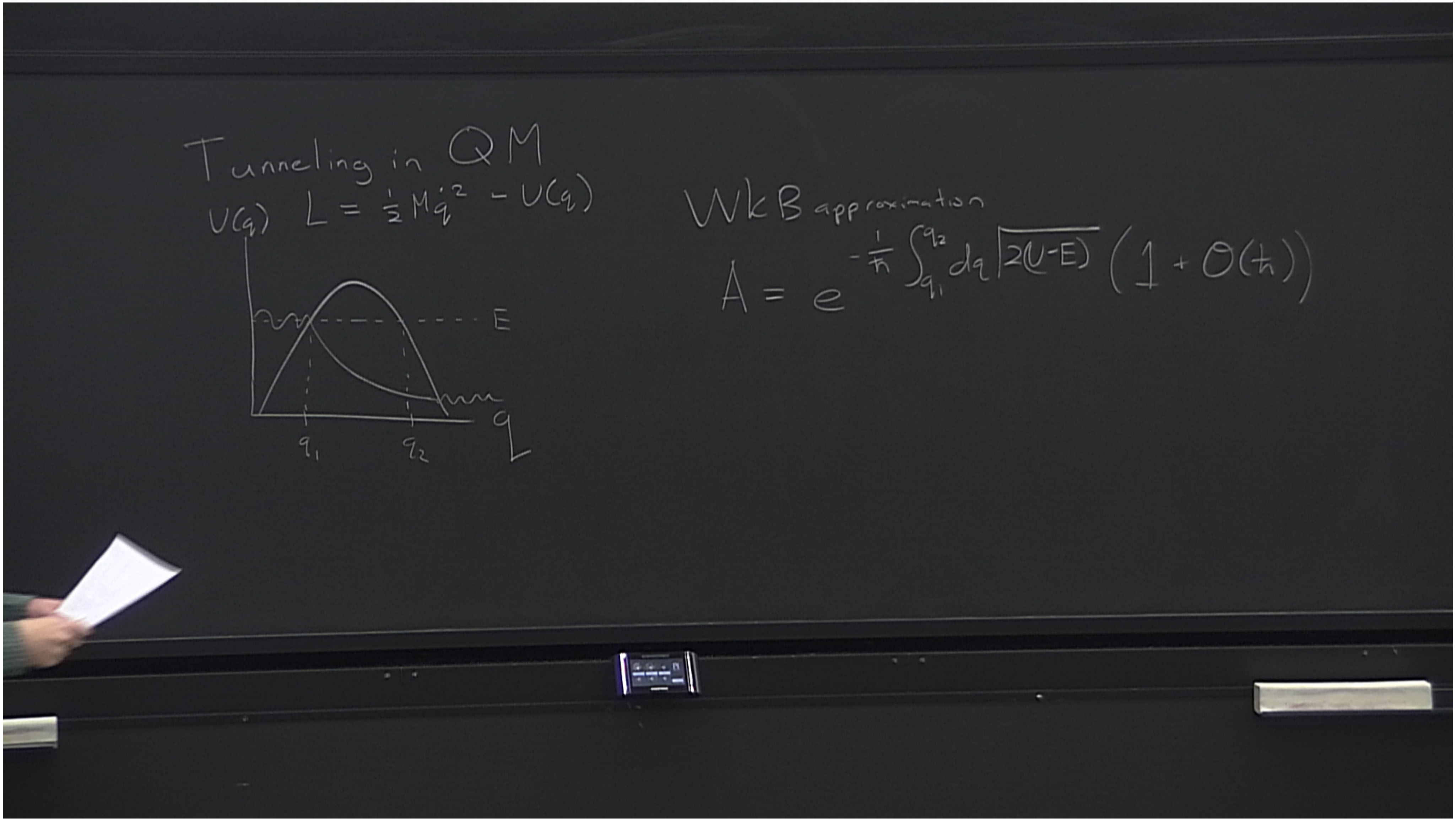
Abstract:


Vacuum Decay


Vacuum Decay
V(ϕ)
ϕ
ϕf

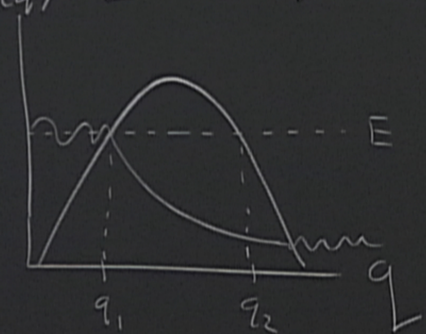

Vacuum Decay
V(ϕ)
ϕ
ϕf


Tunneling in QM
U(q) L = ½ Mq̇² − U(q)
q
ϕ


Tunneling in QM
$U(q)$ $L = \frac{1}{2} M \dot{q}^2 - U(q)$
E
q
q_1
q_2
WKB approximation
$A = e^{-\frac{1}{\hbar}\int_{q_1}^{q_2} dq \sqrt{2(U-E)}}\left(1 + O(\hbar)\right)$

Tunneling in QM

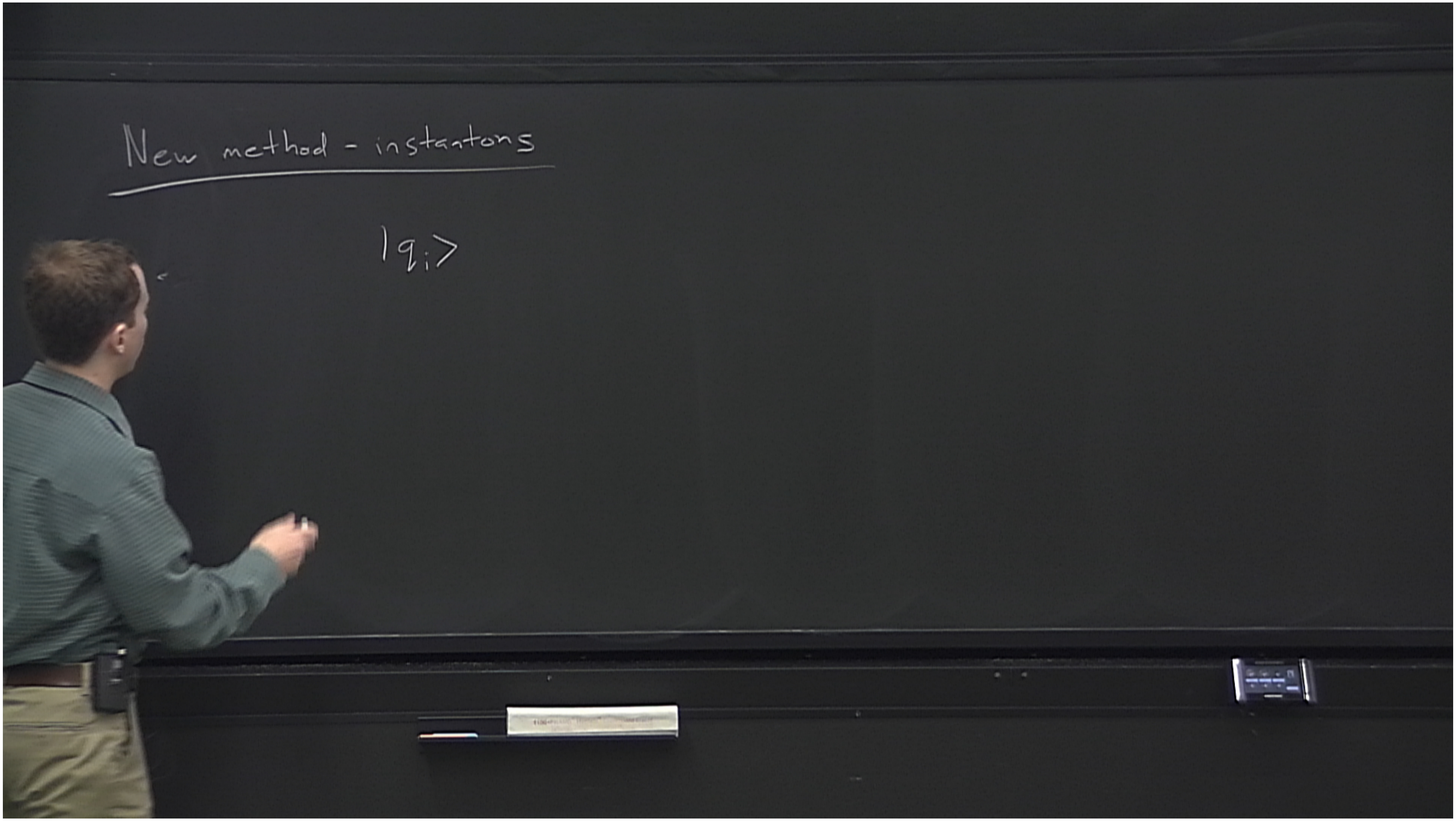
$U(q)$ $\quad L = \frac{1}{2} M \dot{q}^2 - U(q)$



WKB approximation

$$A = e^{-\frac{1}{\hbar}\int_{q_1}^{q_2} dq \sqrt{2(U-E)}}\left(1 + O(\hbar)\right)$$

$A \to 0$ as $\hbar \to 0$ faster than any power of $\hbar$


New method - instantons
$|q_i\rangle$

New method - instantons

$$\langle q_f | e^{-iH\tilde{T}/\hbar} | q_i \rangle = \int [dq]\, e^{iS[q(t)]/\hbar}$$

$$H = \frac{p^2}{2} + U(q)$$

$$q(\tilde{T}/2) = q_f$$

$$q(-\tilde{T}/2) = q_i$$

dom contributions to path integral are sol to EOM

$$M\ddot{q} + V' = 0$$

New method - instantons

$$\langle q_f | e^{-iH\tilde{T}/\hbar} | q_i \rangle = \int [dq]\, e^{iS[q(t)]/\hbar}$$

$$H = \frac{p^2}{2} + U(q)$$

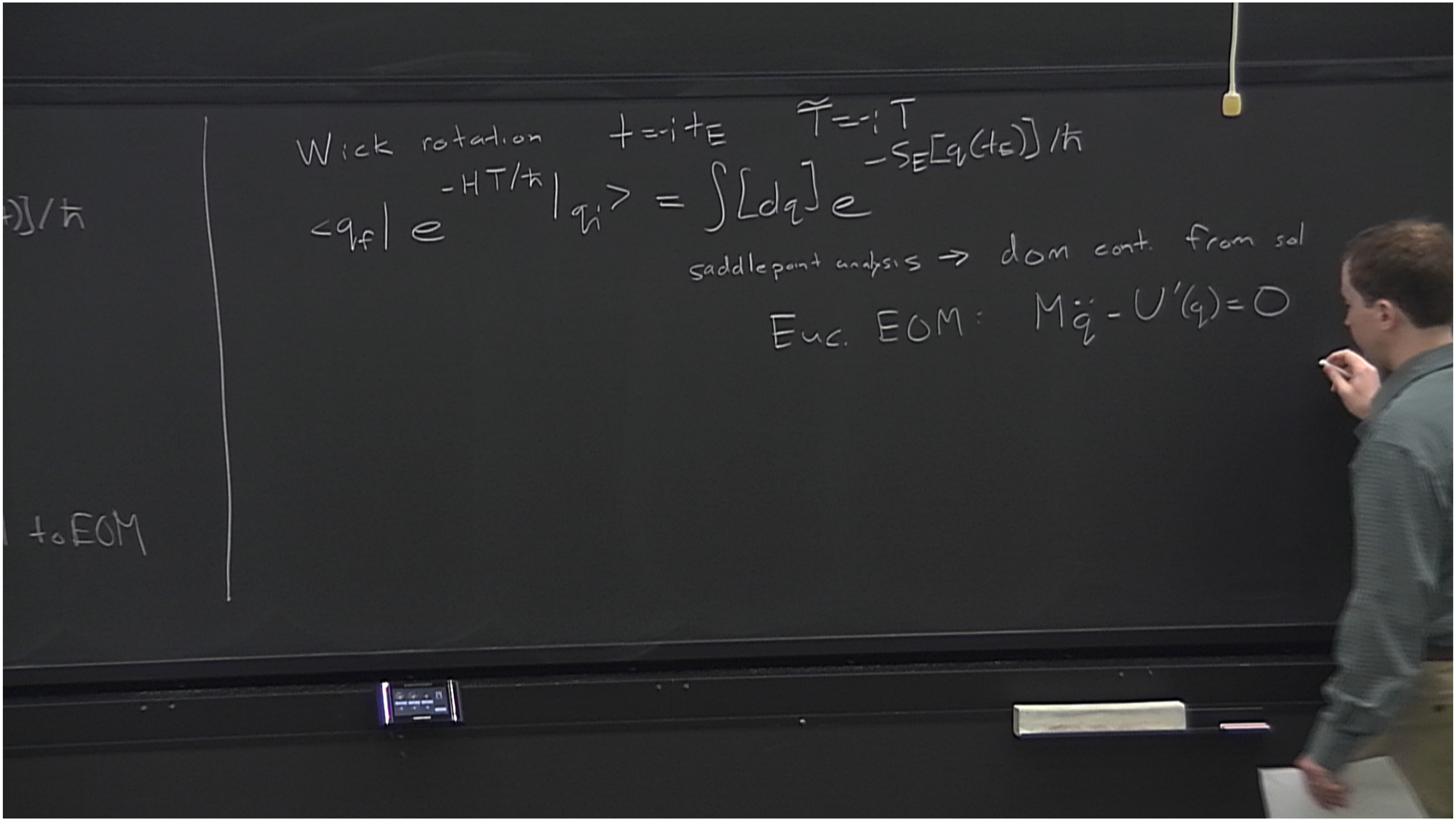
$$q(\tilde{T}/2) = q_f$$

$$q(-\tilde{T}/2) = q_i$$

dom contributions to path integral are sol to EOM

$$M\ddot{q} + V' = 0$$

Wick rotation


Wick rotation t = -i t_E   T̃ = -i T
<q_f| e^{-HT/ħ} |q_i> = ∫[dq] e^{-S_E[q(t_E)]/ħ}
saddlepoint analysis → dom cont. from sol
Euc. EOM: Mq̈ - U'(q) = 0
t)]/ħ
to EOM

Wick rotation $t=-it_E$ $\tilde{T}=-iT$

$$\langle q_f | e^{-HT/\hbar} | q_i \rangle = \int [dq]\, e^{-S_E[q(t_E)]/\hbar}$$

saddlepoint analysis $\rightarrow$ dom cont. from sol

Euc. EOM: $M\ddot{q} - U'(q) = 0$

inverted potential

OM

Wick rotation $t = -i t_E$ $\quad \tilde{T} = -iT$

$$\langle q_f | e^{-HT/\hbar} | q_i \rangle = \int [dq]\, e^{-S_E[q(t_E)]/\hbar}$$

saddlepoint analysis $\rightarrow$ dom cont. from sol

Euc. EOM: $M\ddot{q} - U'(q) = 0$

inverted potential

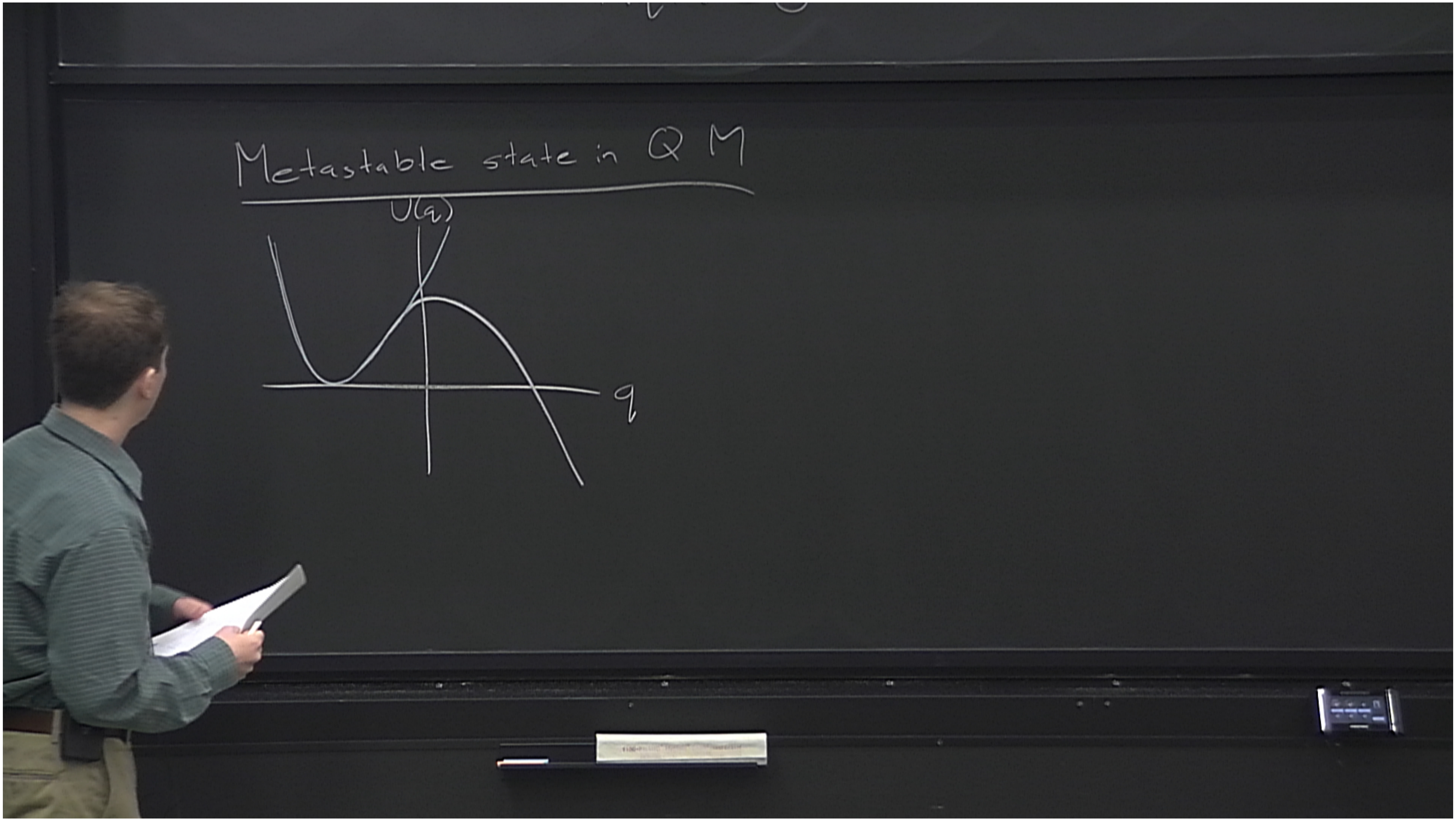
energy eigenstates

$$\langle q_i | e^{-HT/\hbar} | q_i \rangle = \sum_m \langle q_i | e^{-HT/\hbar} | m \rangle \langle m | q_i \rangle$$

$$= \sum_m e^{-E_m T/\hbar} \langle q_i | m \rangle \langle m | q_i \rangle$$

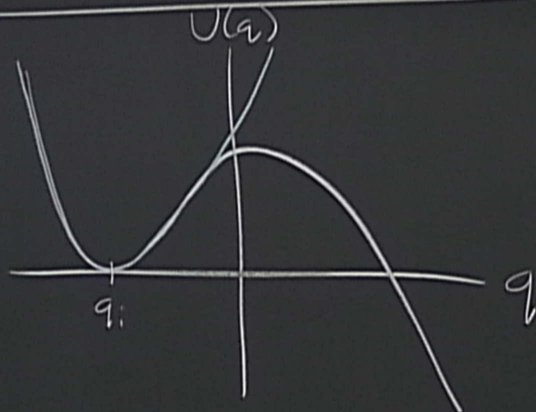
$$\underset{T \to \infty}{\approx} C e^{-E_0 T/\hbar}$$

real #



Metastable state in QM
U(q)
q

Metastable state in QM



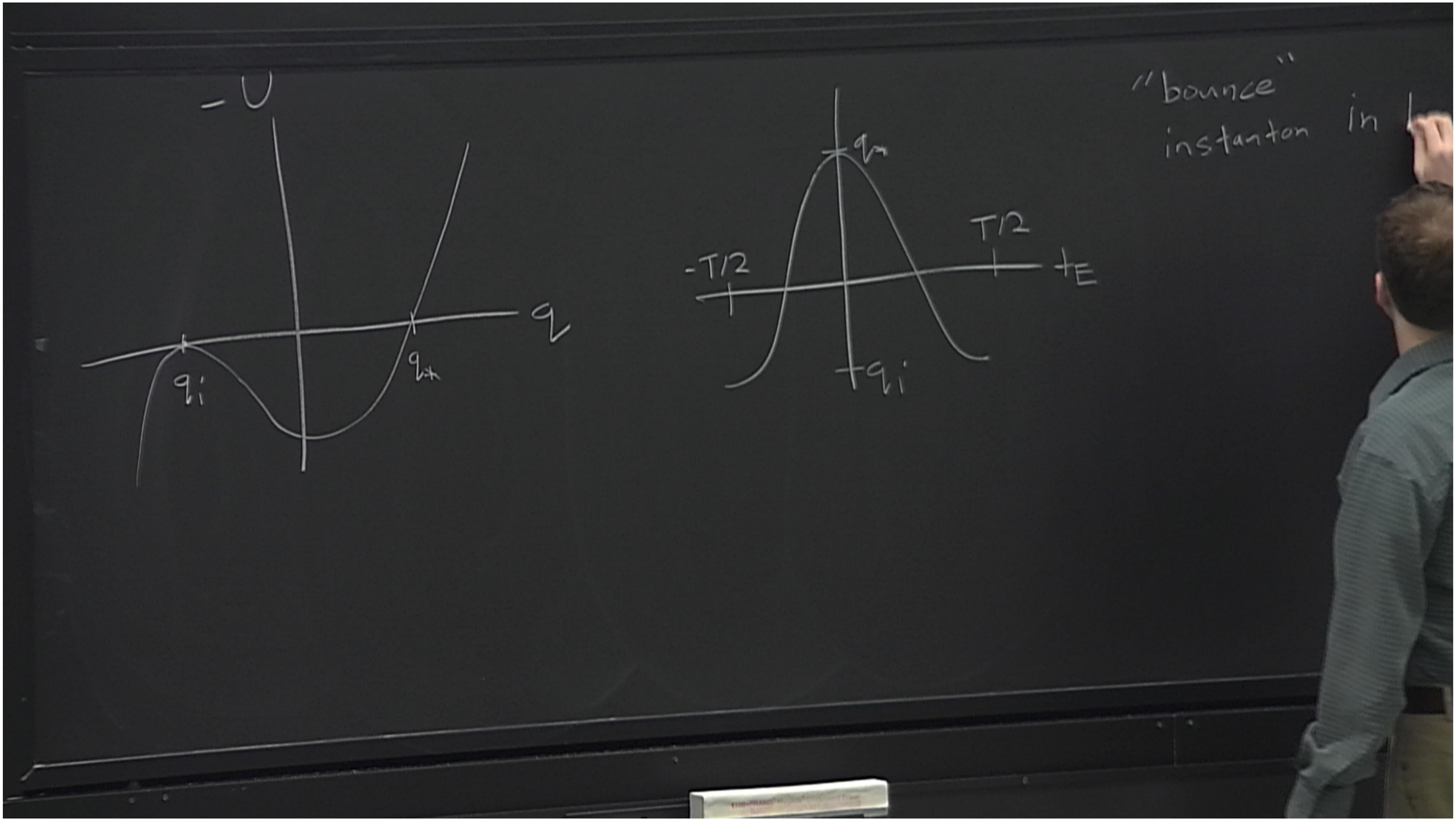
$$e^{-iEt/\hbar} = e^{-i\,\mathrm{Re}(E)t/\hbar + \mathrm{Im}(E)t/\hbar}$$

$$\Gamma = -\frac{\mathrm{Im}(E)}{2\hbar}$$

Find Γ by evaluating

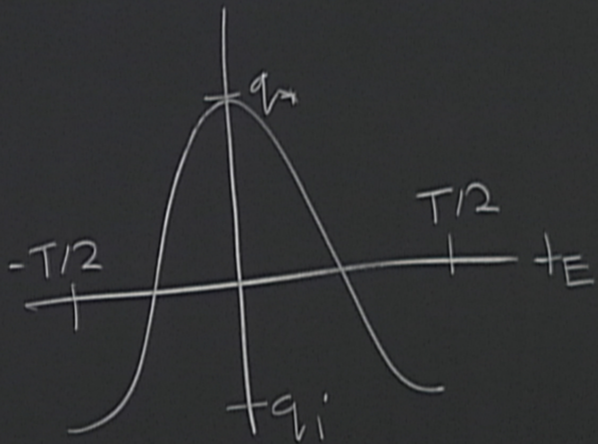
$$\int [dq]\, e^{-S_E[q(t)]}$$

$$q[T/2] = q_i = q(-T/2) \quad \text{in } T \to \infty$$

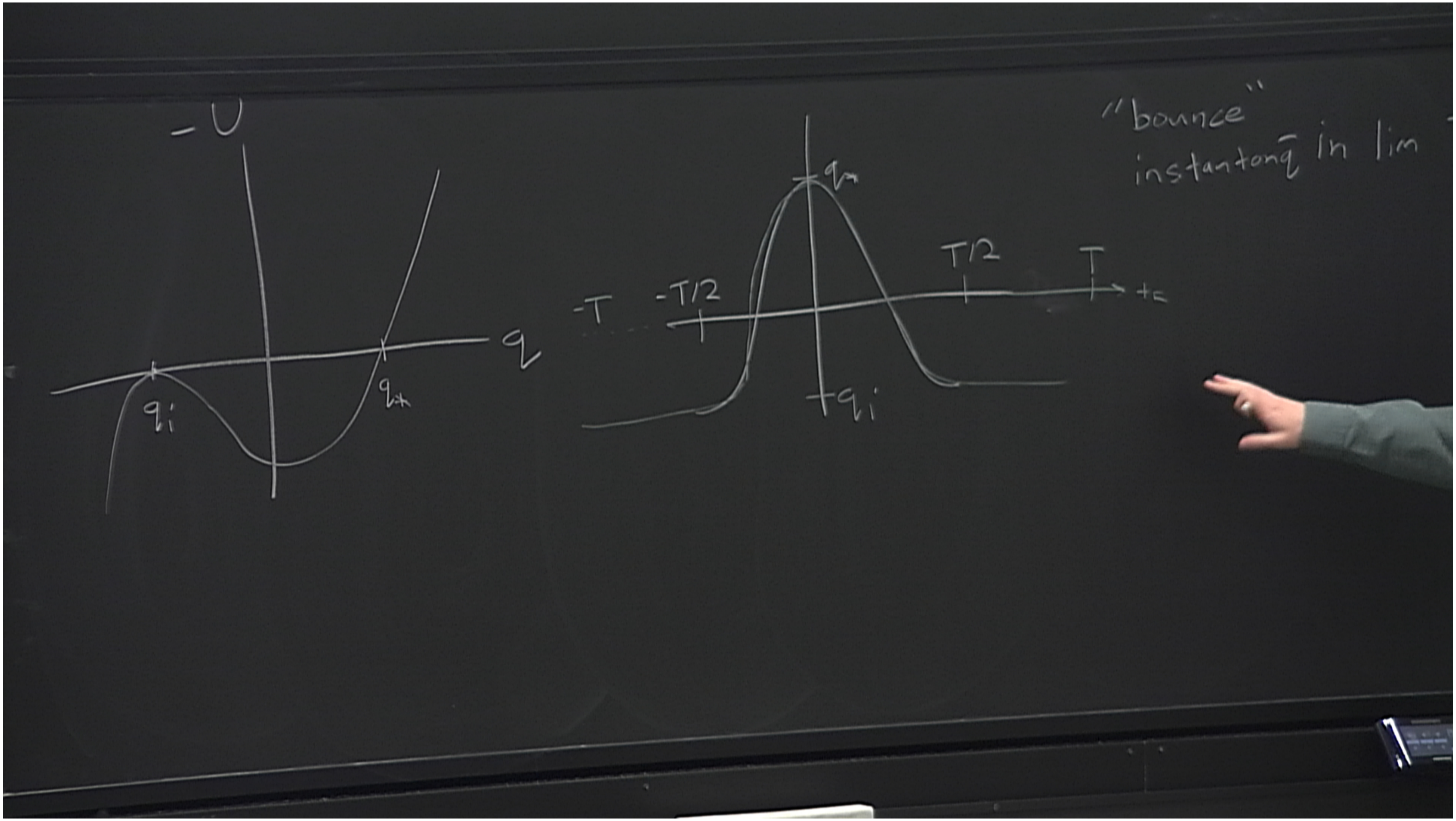


−U
q
q_i
q_*
−T/2
T/2
t_E
q_*
q_i
"bounce"
instanton in

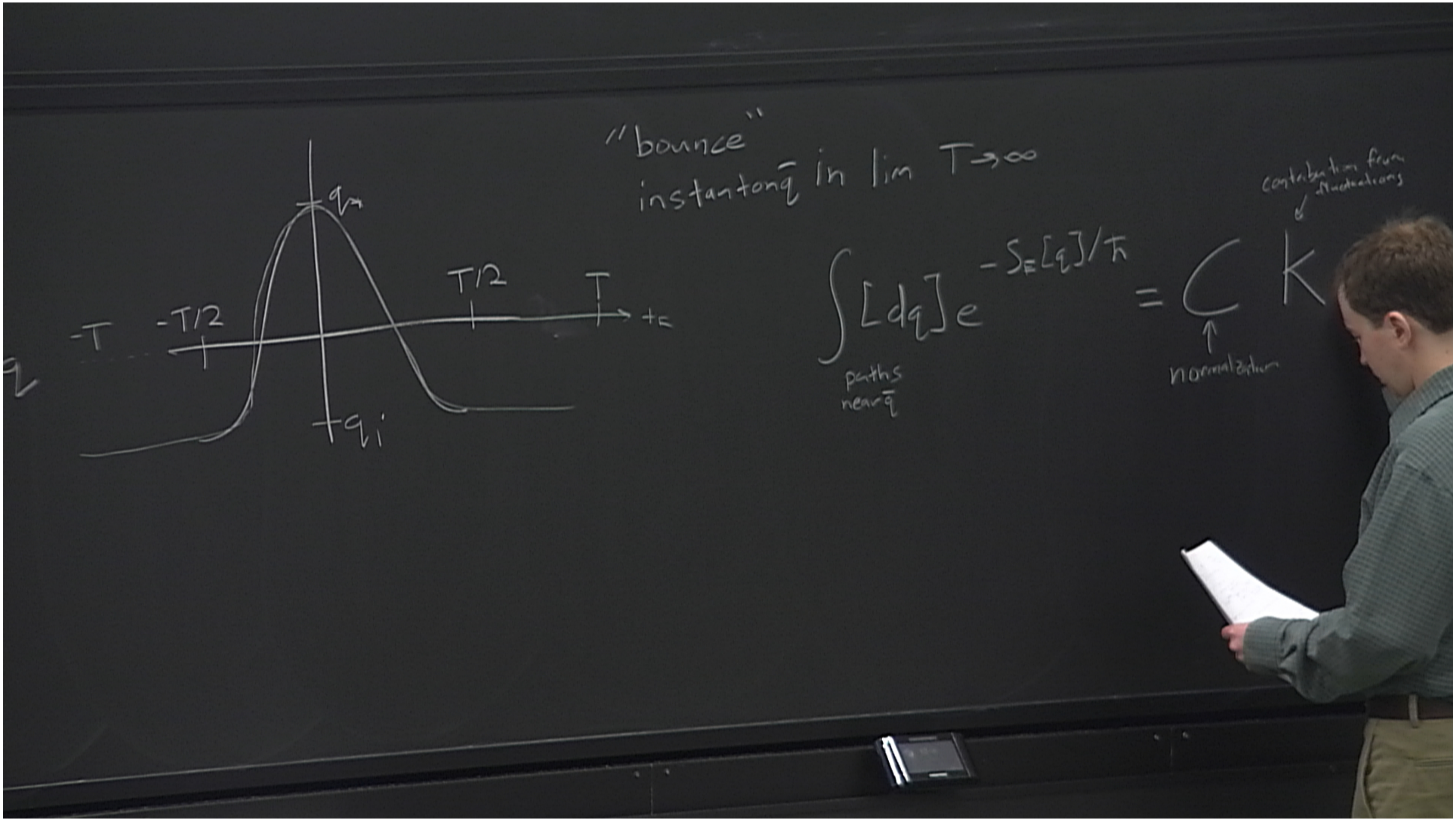
q

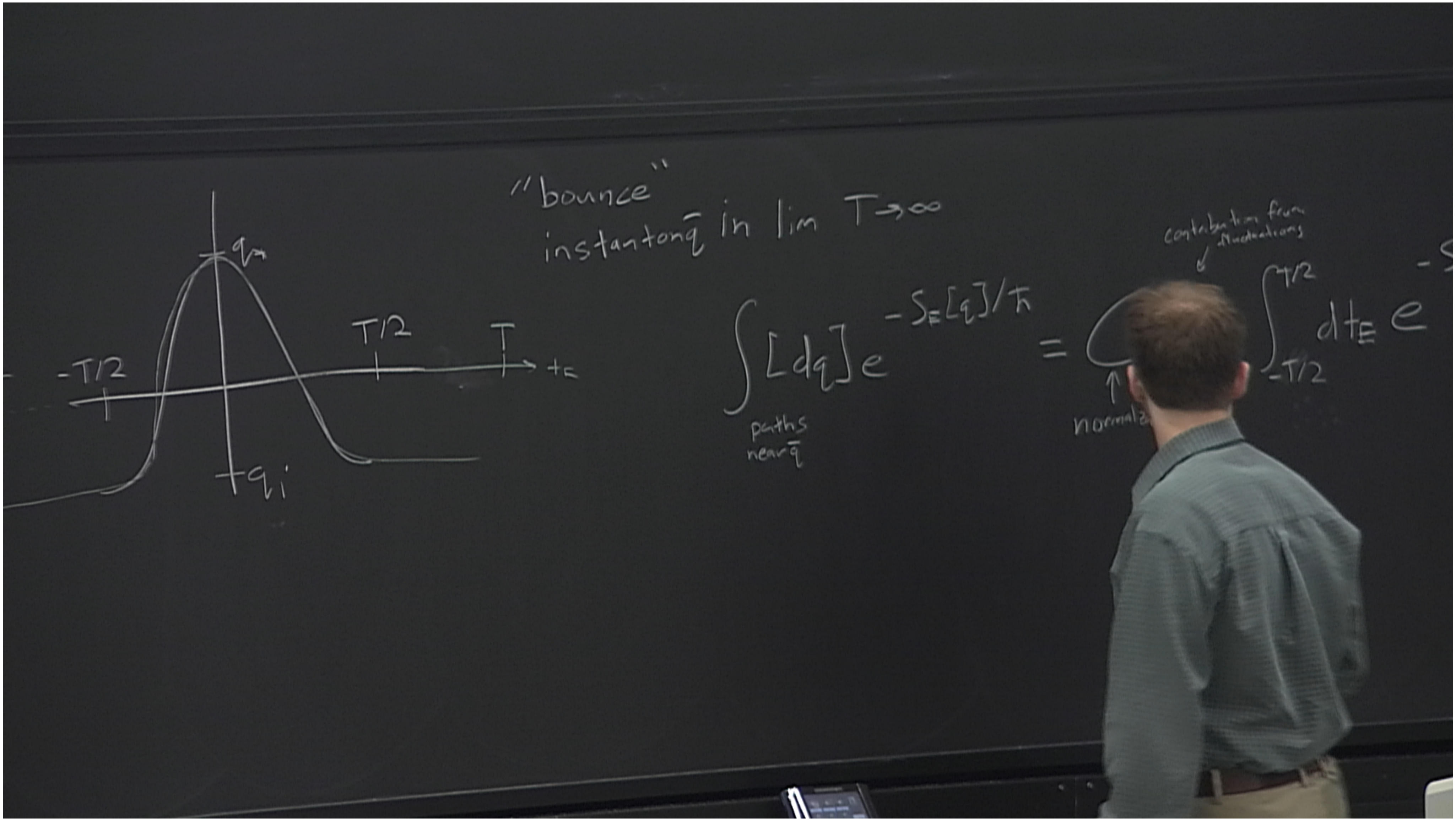


"bounce"
instanton $\bar{q}$ in $\lim T\to\infty$



-U
q
q_i
q_*
-T
-T/2
T/2
T
q_*
q_i
"bounce"
instanton in lim


"bounce"
instanton q̄ in lim T→∞
contribution from fluctuations
normalization
paths near q̄
-T
-T/2
T/2
T


"bounce"
instanton $\bar{q}$ in lim $T\to\infty$
q_*
$-T/2$
$T/2$
T
t_E
q_i
$\int [dq]\, e^{-S_E[q]/\hbar} =$
paths near $\bar{q}$
normaliz
contribution from fluctuations
$\int_{-T/2}^{T/2} dt_E\, e$

"bounce"
instanton $\bar{q}$ in $\lim T \to \infty$

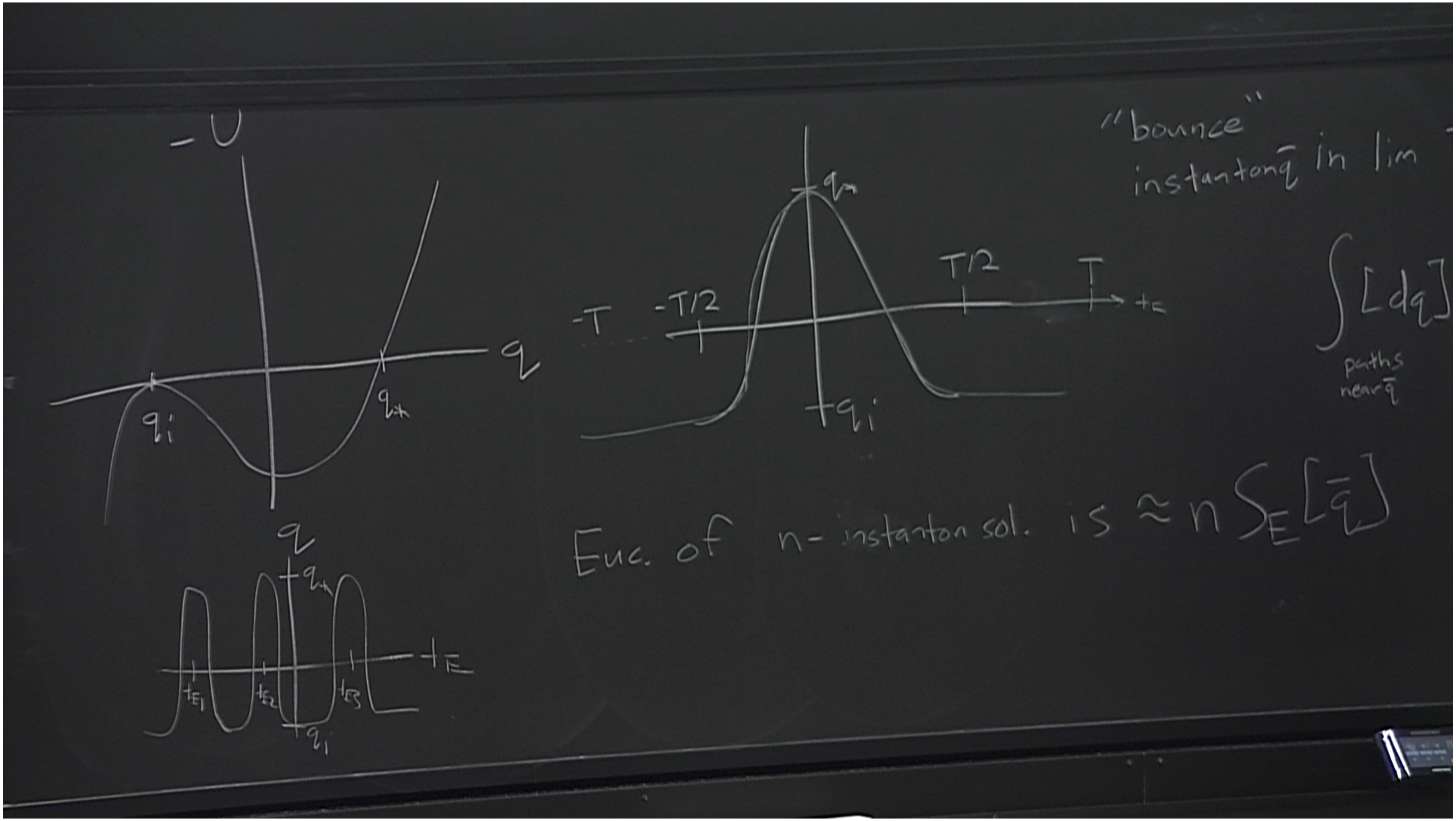
$\to t_E$

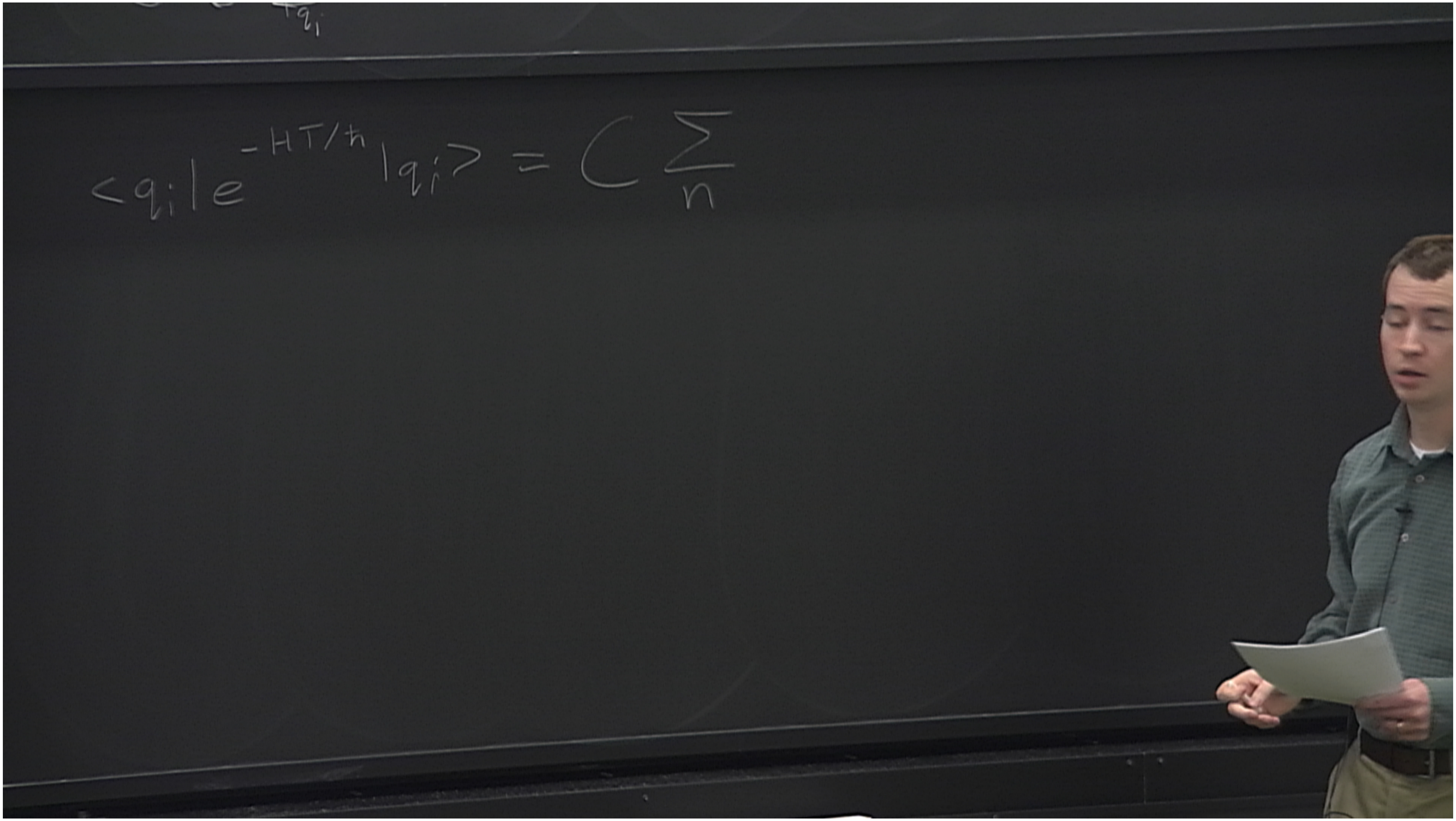
contribution from fluctuations

$$\int_{\substack{\text{paths}\\ \text{near } \bar{q}}} [dq]\, e^{-S_E[q]/\hbar} = \lim_{T\to\infty} C\, K \int_{-T/2}^{T/2} dt_E\, e^{-S_E[\bar{q}]/\hbar}$$

normalization

center instanton anywhere


-U
q
q_i
q_*
-T
-T/2
T/2
T
t_E
"bounce"
instanton q̄ in lim
∫[dq]
paths near q̄
Euc. of n-instanton sol. is ≈ n S_E[q̄]
t_E1
t_E2
t_E3

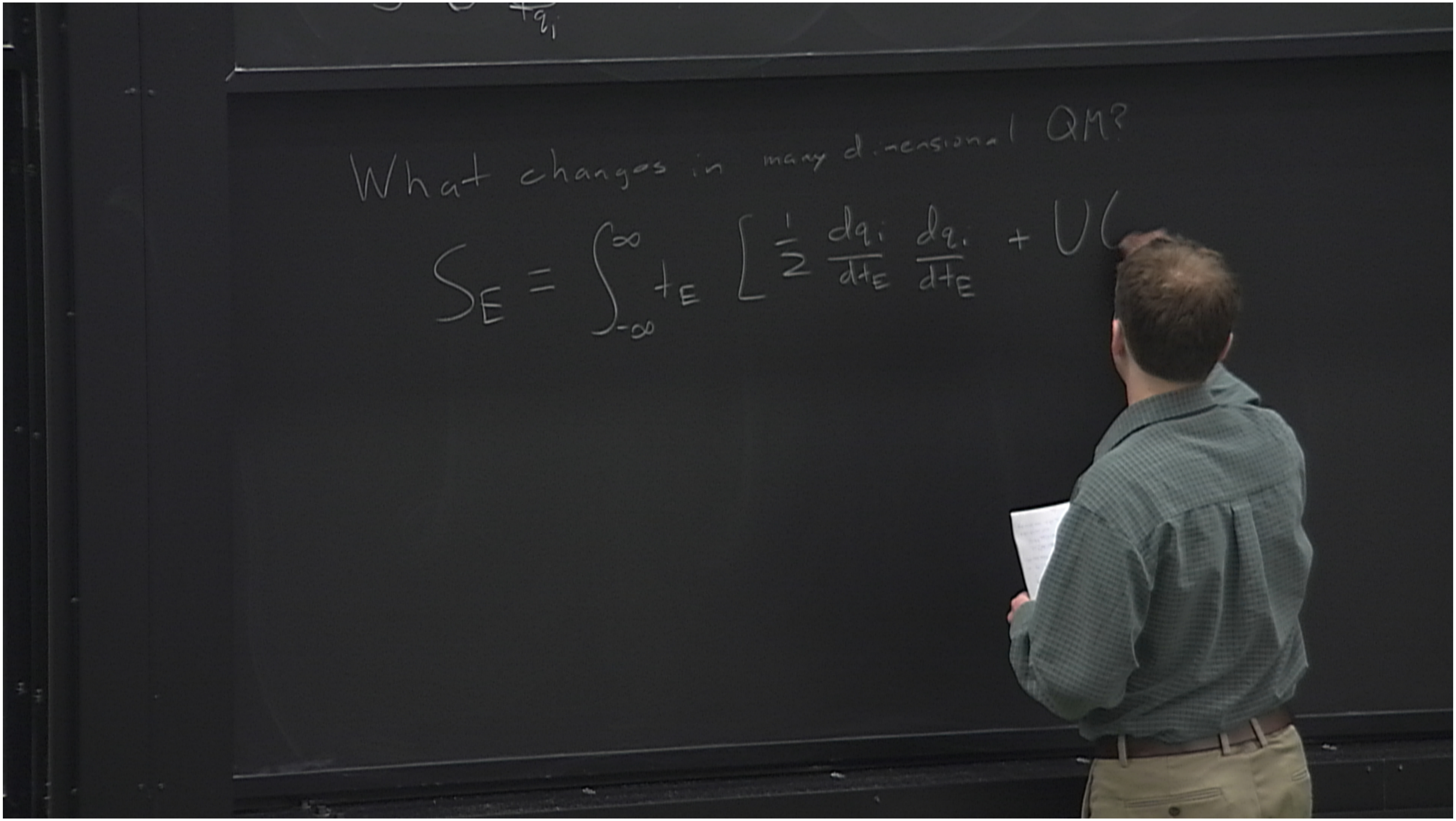

$\langle q_i | e^{-HT/\hbar} | q_i \rangle = C \sum_n$

$$\langle q_i | e^{-HT/\hbar} | q_i \rangle = C \sum_n K^n \int_{-\frac{T}{2}}^{\frac{T}{2}} dt_{E_n} \int_{-\frac{T}{2}}^{t_{E_n}} dt_{E_{n-1}} \cdots \int$$

$${}^{n}|q_i\rangle = C\sum_n k^n \int_{-\frac{T}{2}}^{\frac{T}{2}} dt_{E_n}\int_{-\frac{T}{2}}^{t_{E_n}} dt_{E_{n-1}} \cdots \int_{-\frac{T}{2}}^{t_{E_2}} dt_{E_1}\, e^{-nS_E[\bar{q}]/\hbar}$$

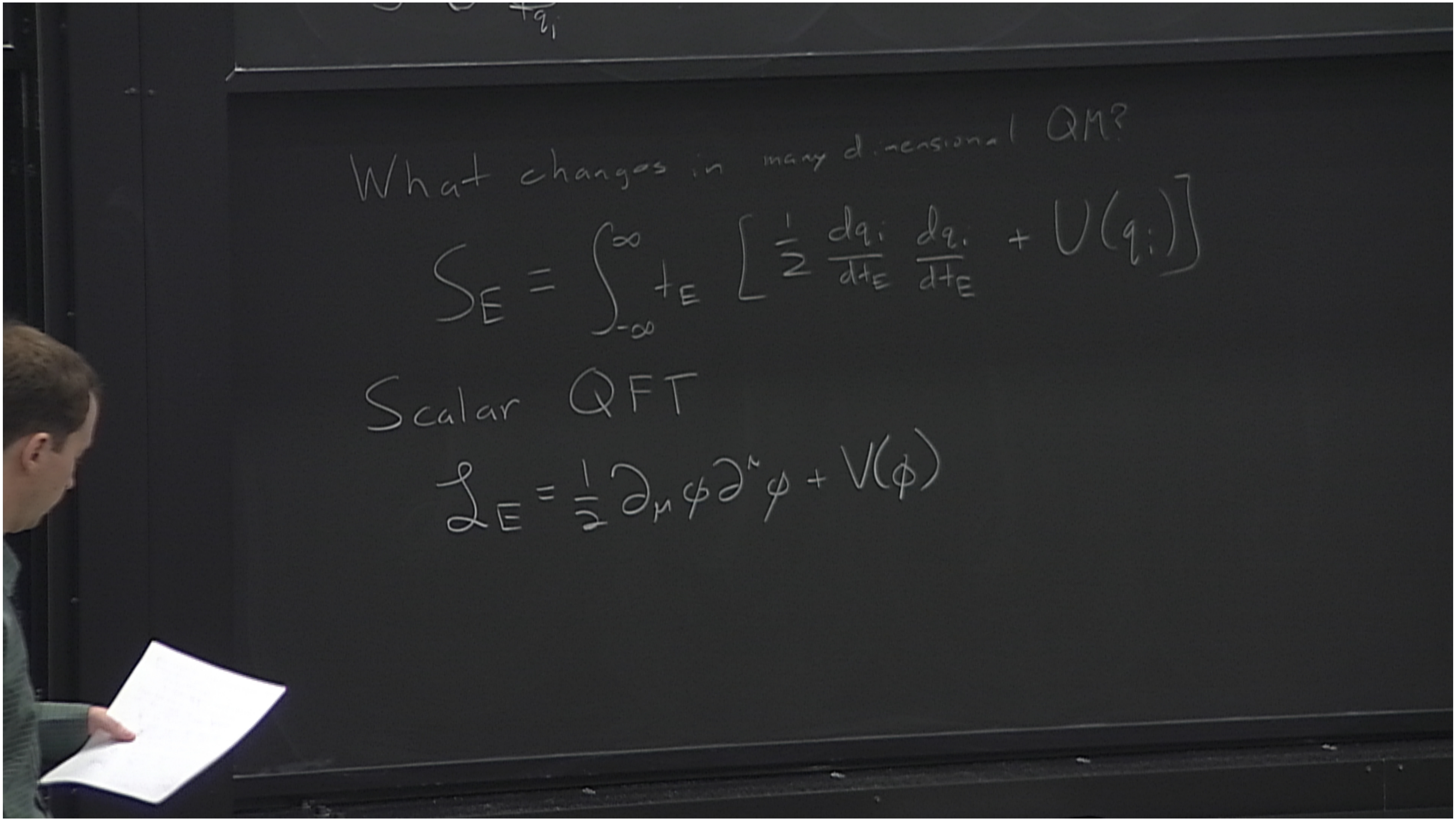
$$= C\sum_n k^n \frac{T^n}{n!}\, e^{-nS_E[\bar{q}]/\hbar}$$

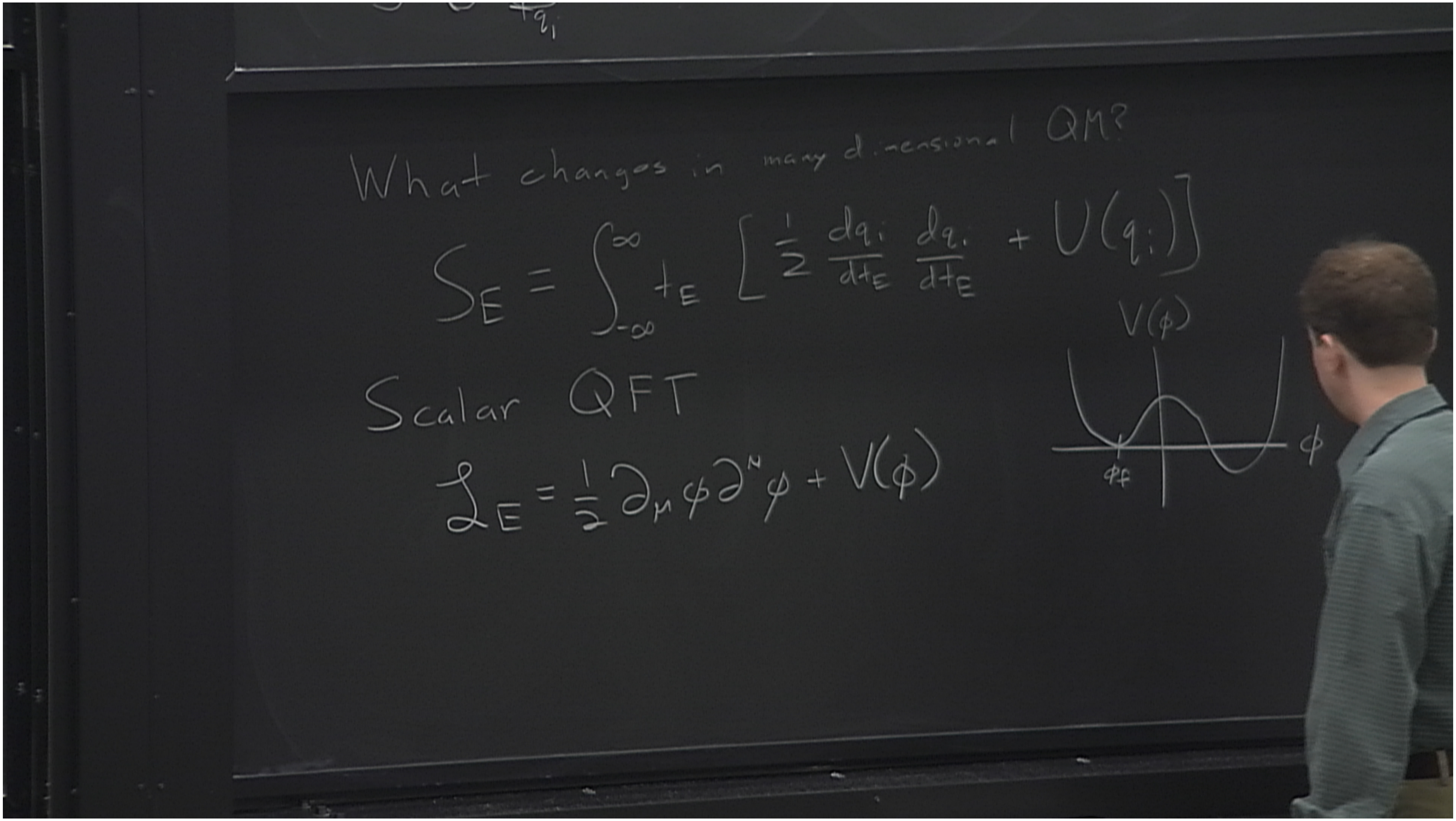
$$= C\exp\left[kT e^{-S_E[\bar{q}]/\hbar}\right]$$


What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(\right.$

What changes in many dimensional QM?

$$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$$


What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
Scalar QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$

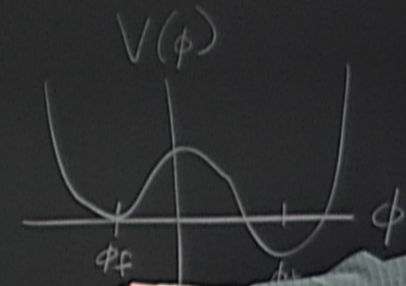

What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} dt_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
Scalar QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$
$V(\phi)$
ϕ
ϕ_f

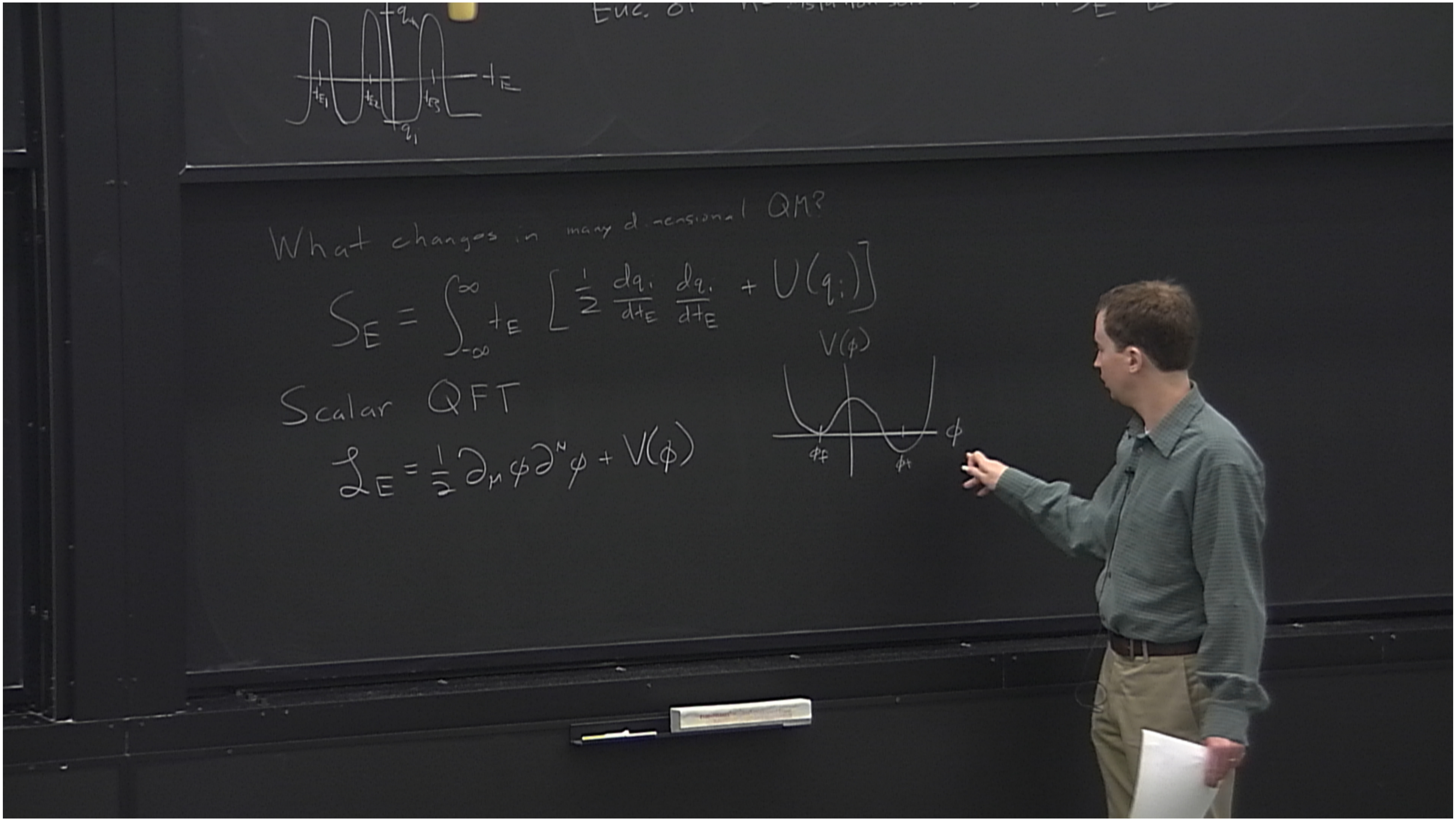
What changes in many dimensional QM?

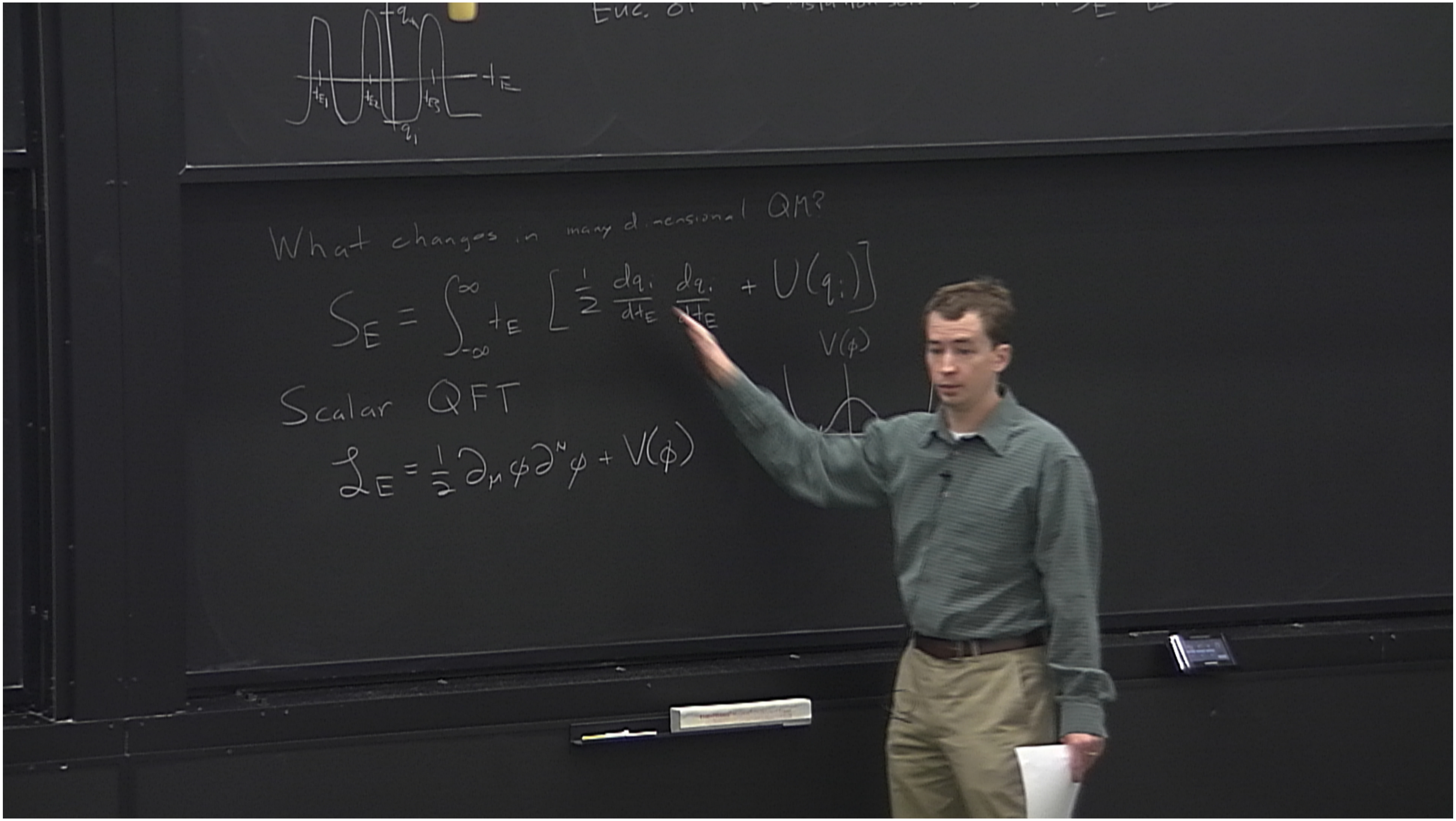
$$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$$

Scalar QFT

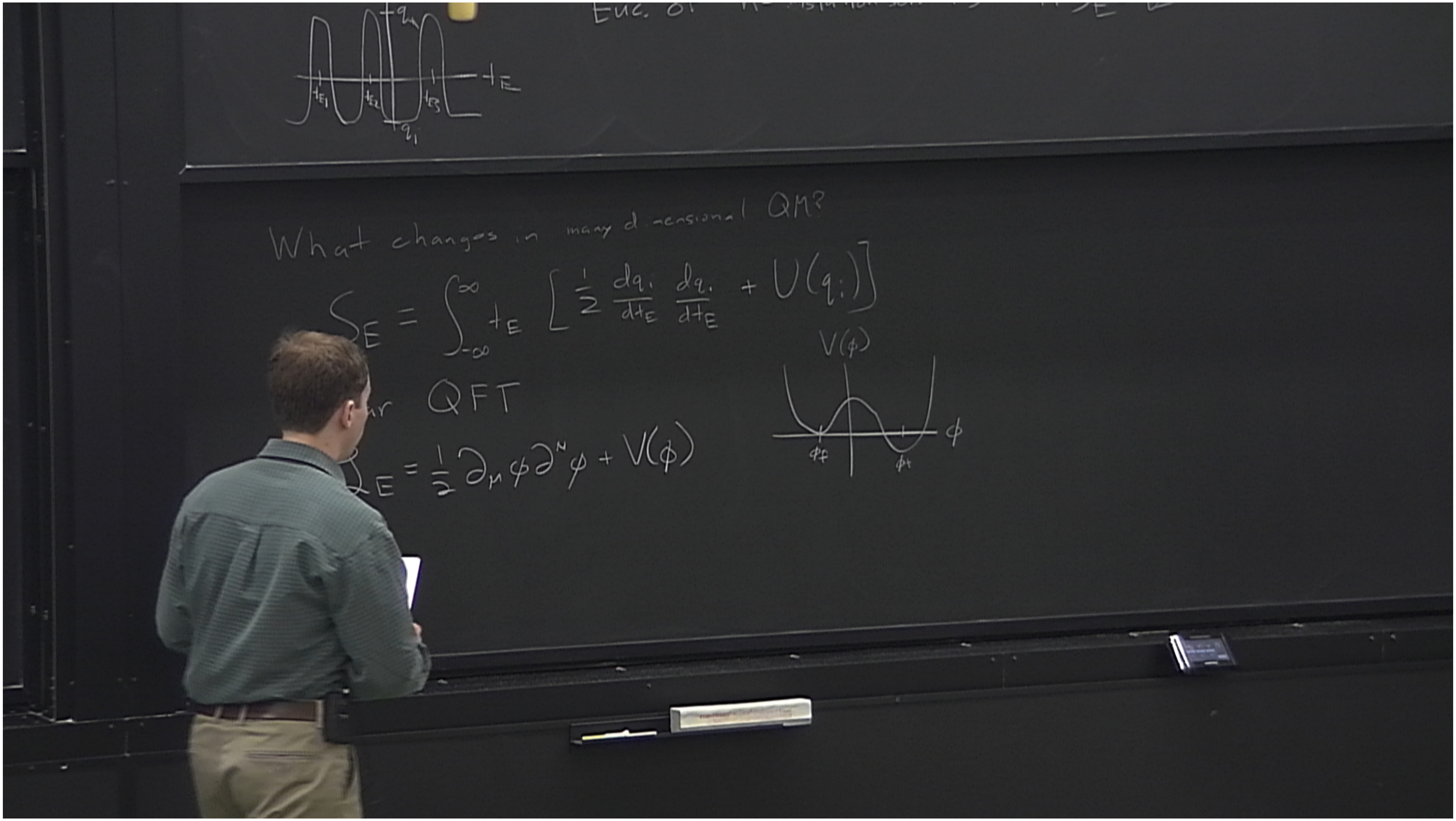
$$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$$

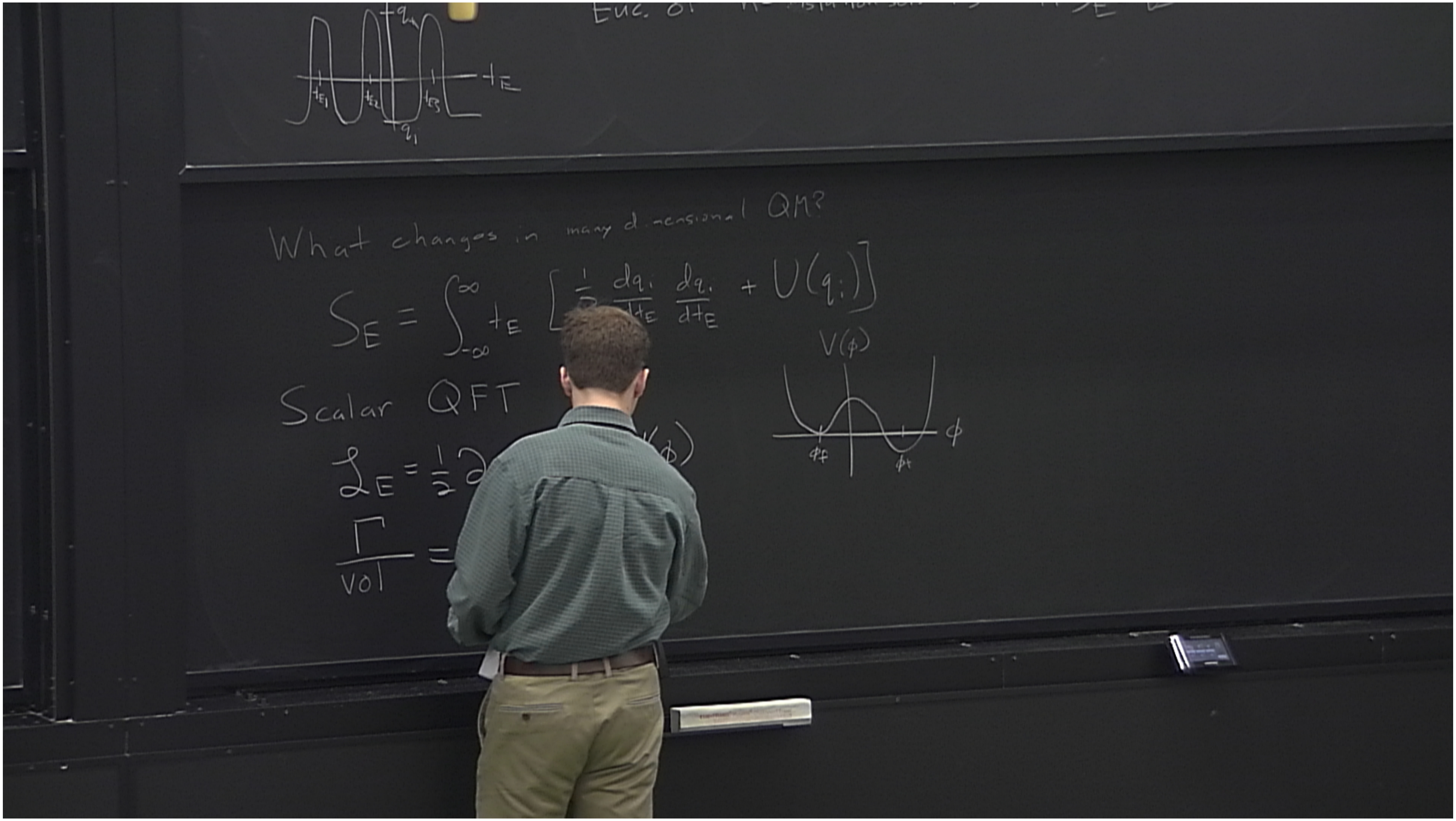



What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
Scalar QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$
$V(\phi)$
ϕ
ϕ_f
ϕ_t
t_E
t_{E1}
t_{E2}
t_{E3}
q_1

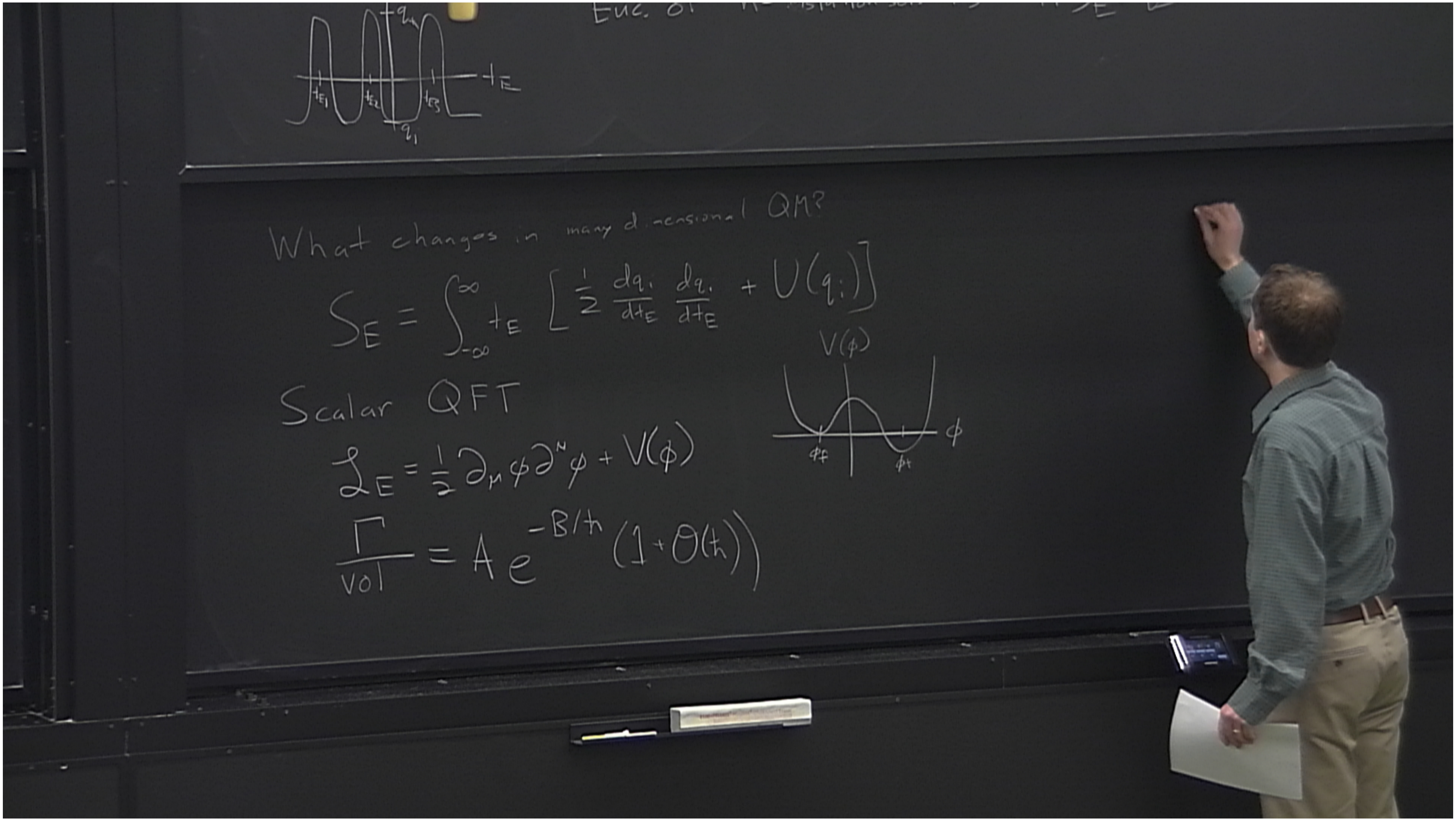


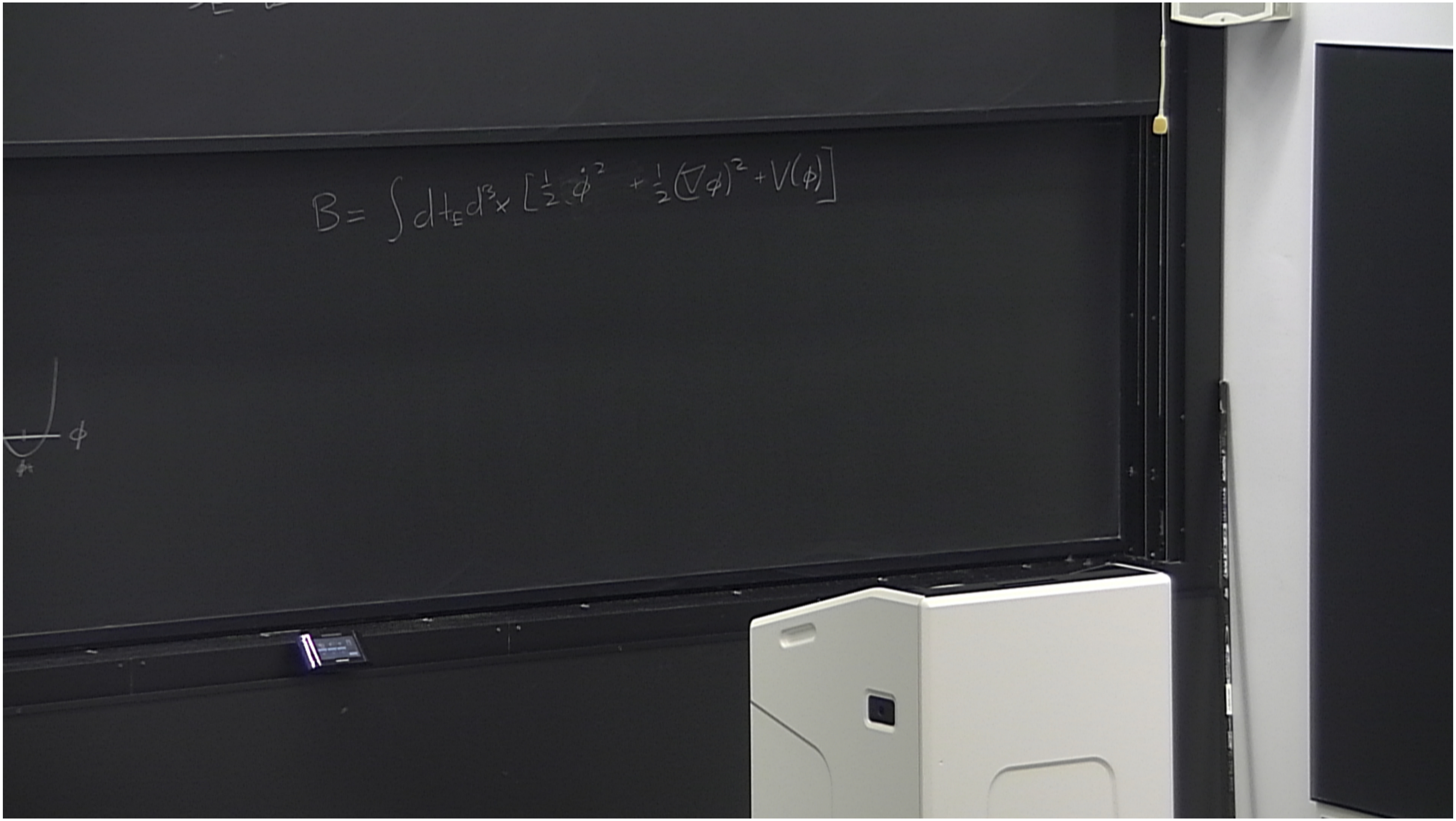
What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} dt_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
$V(\phi)$
Scalar QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$

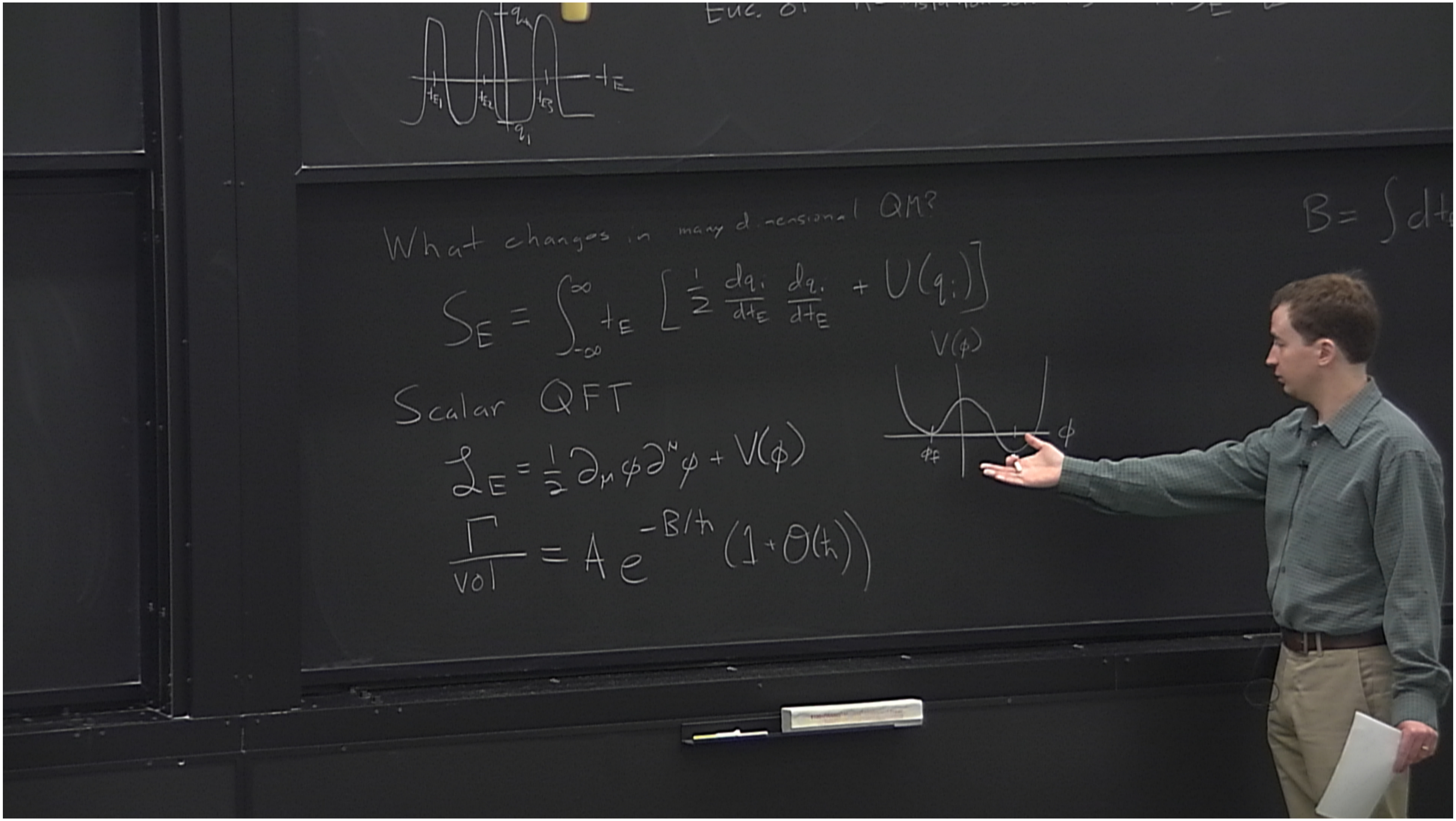

t_E
q_1
What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$
$V(\phi)$
ϕ
ϕ_F
ϕ_T



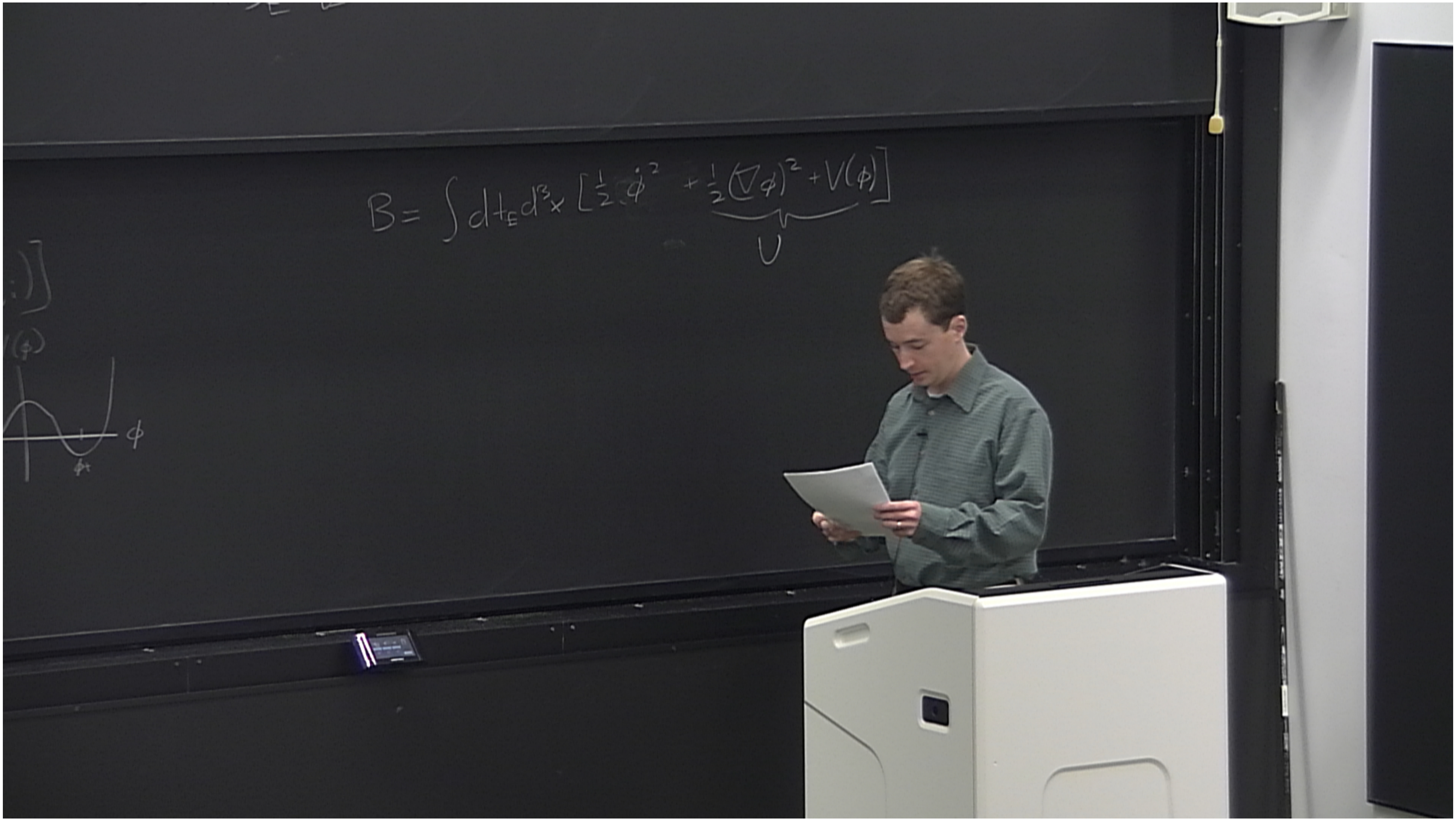
What changes in many dimensional QM?
Scalar QFT
V(φ)
φ
Vol

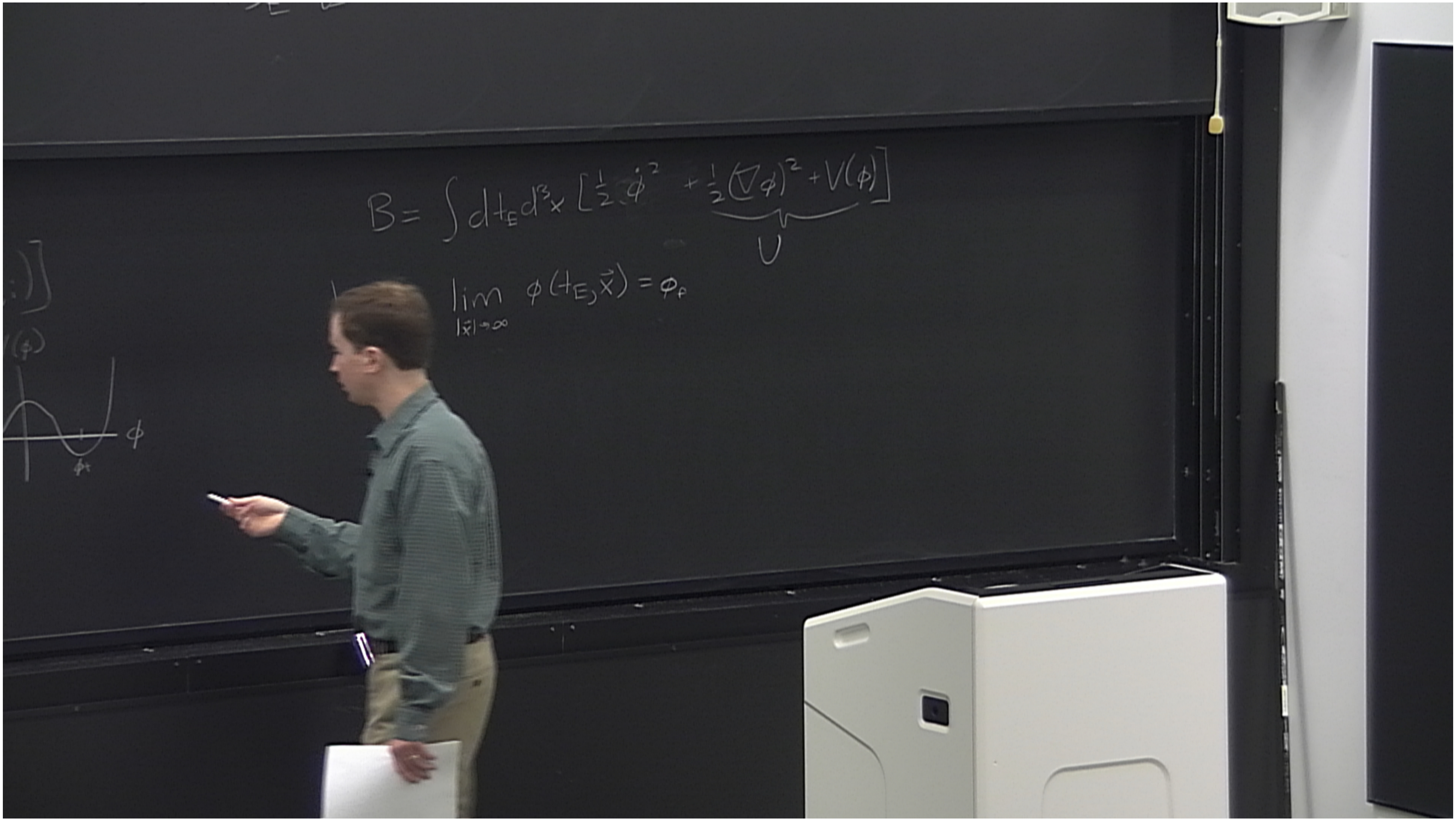

What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
$V(\phi)$
ϕ
ϕ_F
ϕ_T
Scalar QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$
$\frac{\Gamma}{\mathrm{Vol}} = A e^{-B/\hbar} (1 + O(\hbar))$
t_E
q_1

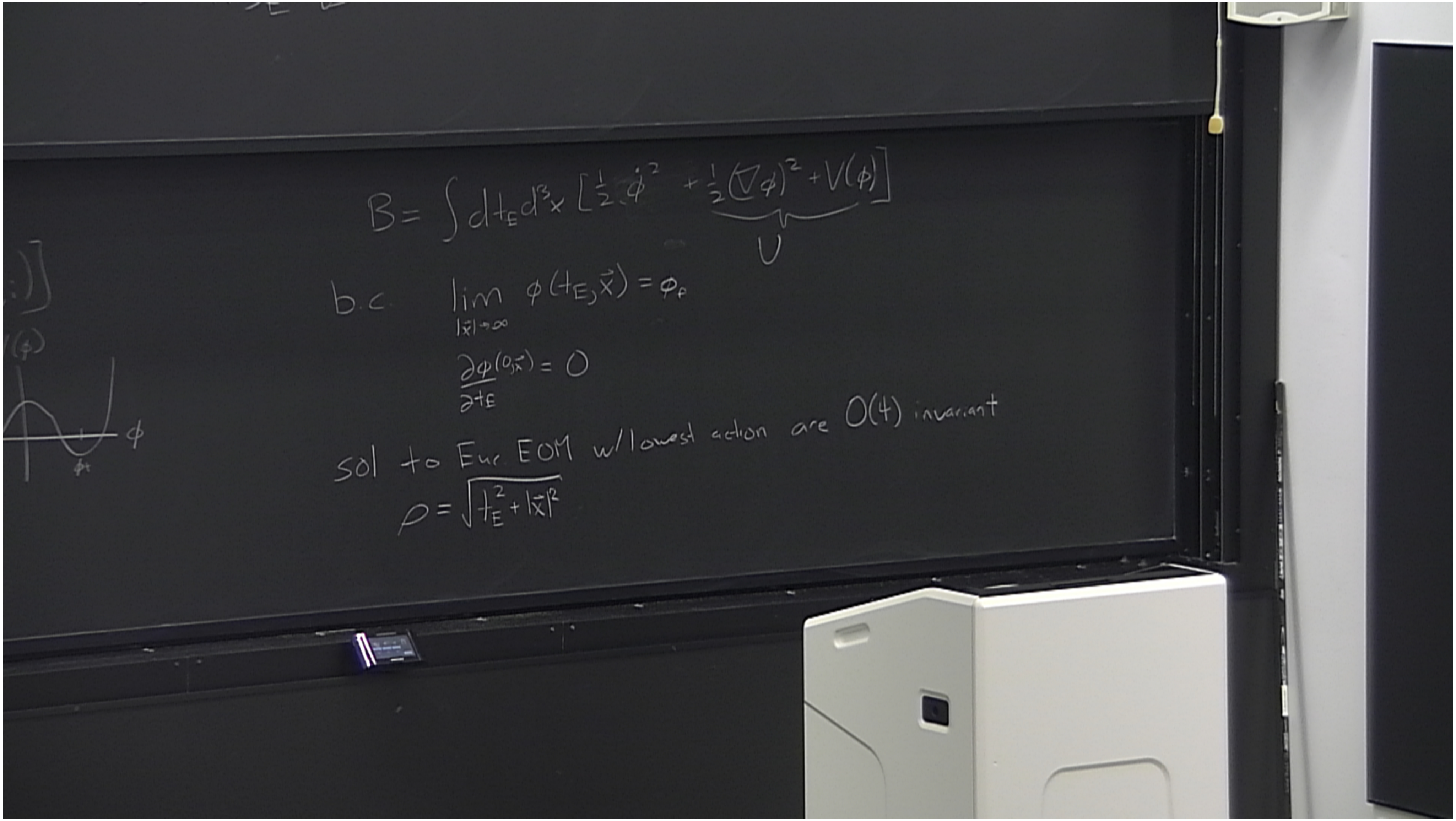

$B = \int dt_E d^3x \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right]$
ϕ

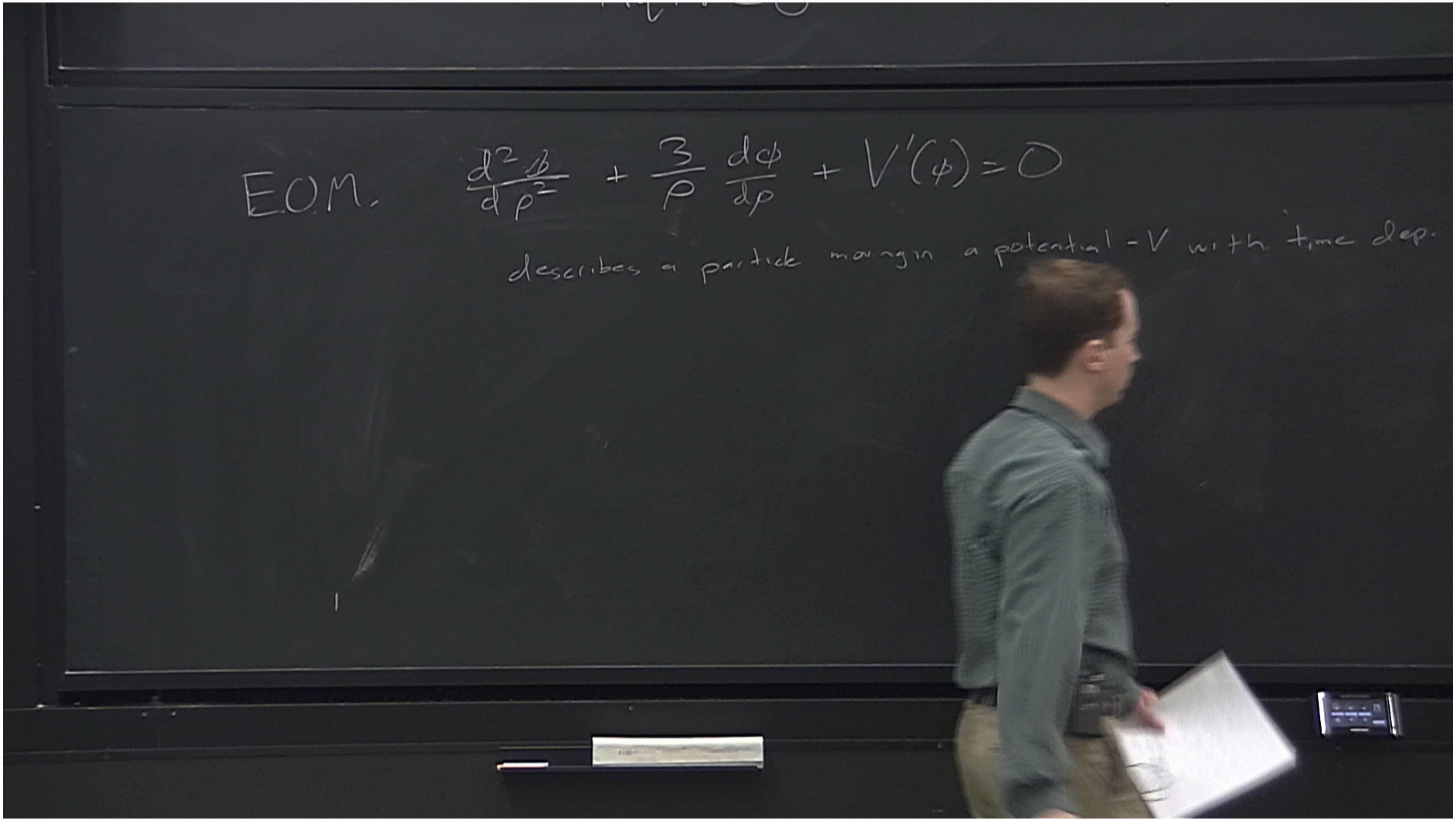


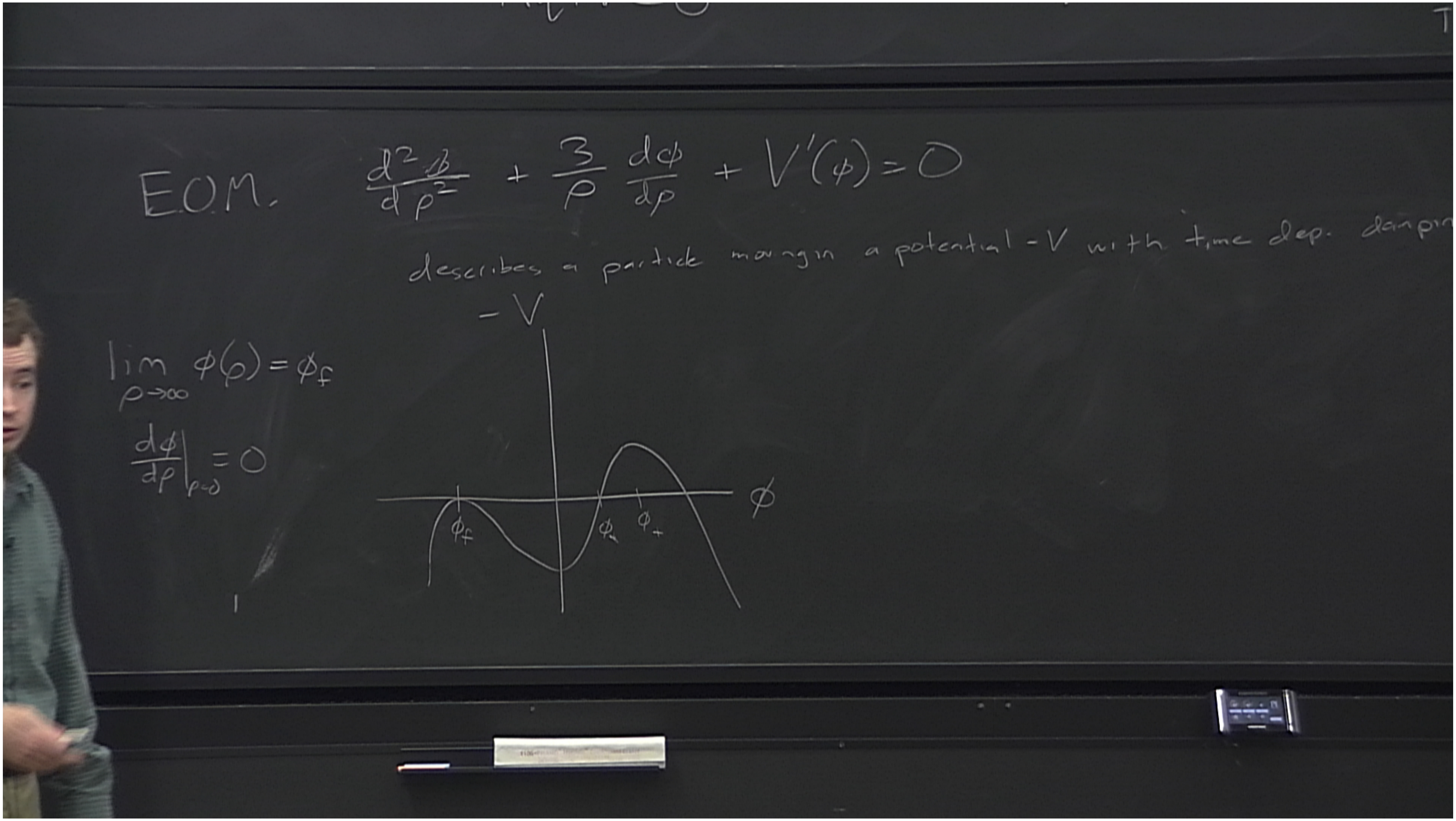
t_E
t_{E1} t_{E2} t_{E3}
q_1
What changes in many dimensional QM?
$S_E = \int_{-\infty}^{\infty} t_E \left[\frac{1}{2} \frac{dq_i}{dt_E} \frac{dq_i}{dt_E} + U(q_i) \right]$
Scalar QFT
$\mathcal{L}_E = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$
$\frac{\Gamma}{\mathrm{Vol}} = A e^{-B/\hbar} (1 + O(\hbar))$
$V(\phi)$
ϕ
ϕ_F
$B = \int dt$

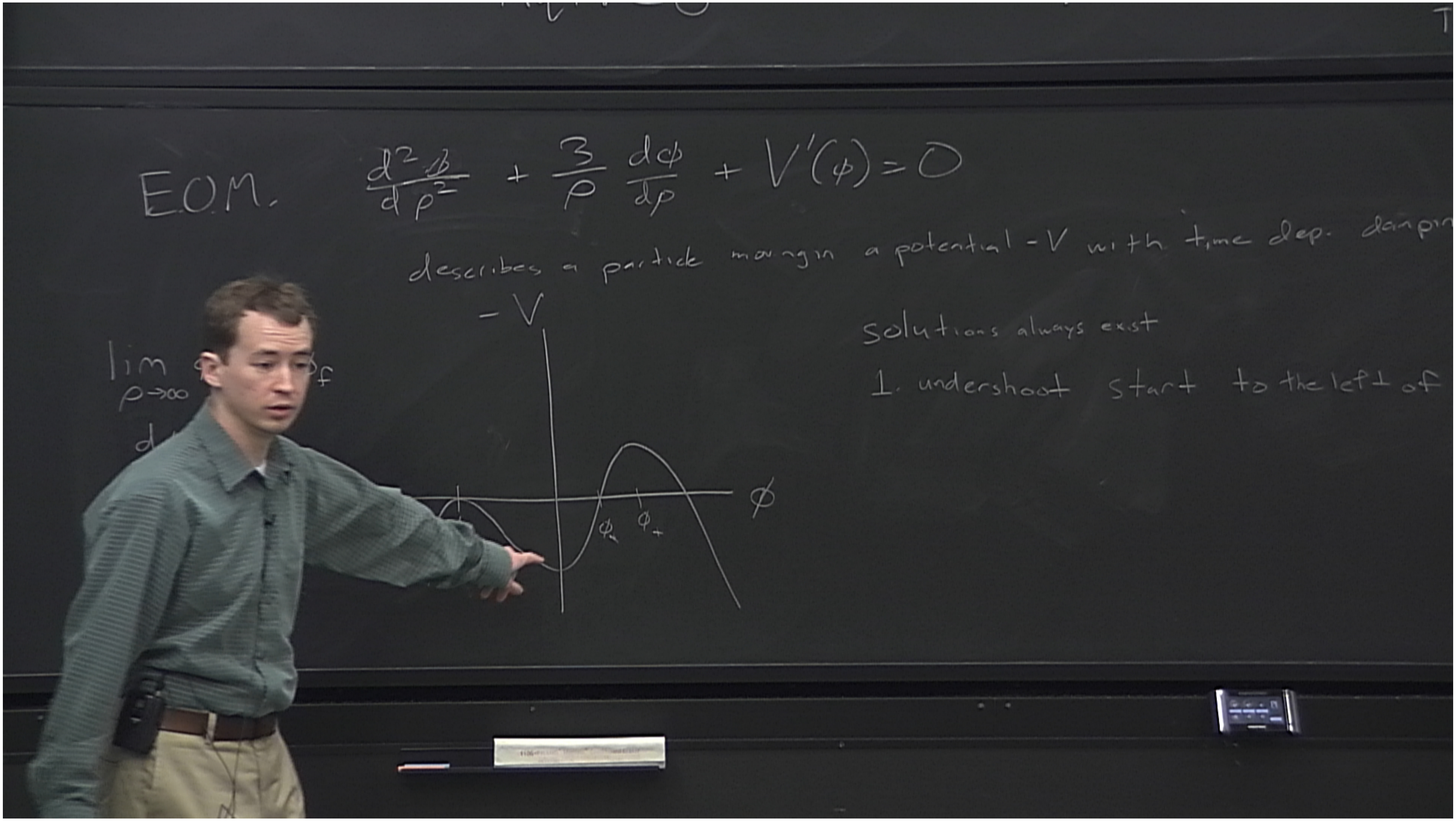

B = ∫ dt_E d³x [½ φ̇² + ½(∇φ)² + V(φ)]
U
V(φ)
φ
φ+

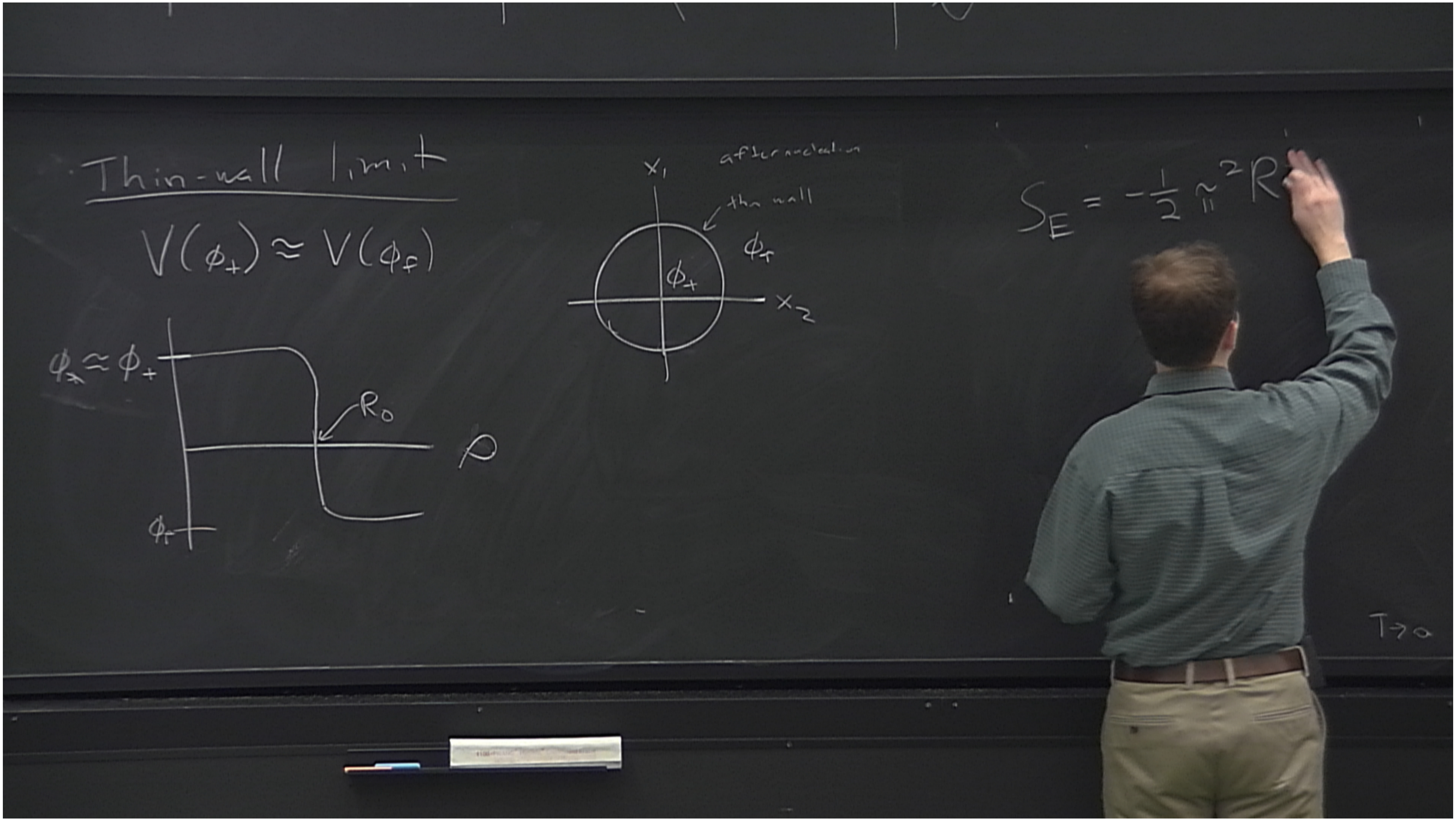

$B = \int dt_E\, d^3x \left[\frac{1}{2}\dot{\phi}^2 + \underbrace{\frac{1}{2}(\nabla\phi)^2 + V(\phi)}_{U} \right]$
$\lim_{|\vec{x}|\to\infty} \phi(t_E, \vec{x}) = \phi_F$
ϕ

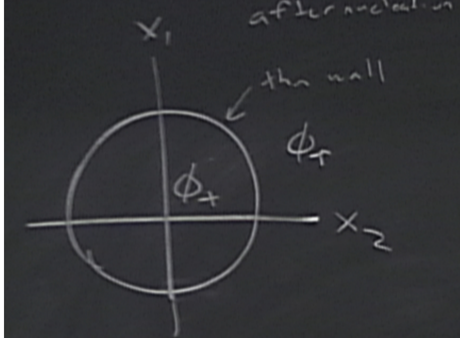

$B = \int dt_E d^3x \left[\frac{1}{2}\dot{\phi}^2 + \underbrace{\frac{1}{2}(\nabla\phi)^2 + V(\phi)}_{U} \right]$
b.c. $\lim_{|\vec{x}|\to\infty} \phi(t_E, \vec{x}) = \phi_F$
$\frac{\partial\phi(0,\vec{x})}{\partial t_E} = 0$
sol to Euc EOM w/lowest action are O(4) invariant
$\rho = \sqrt{t_E^2 + |\vec{x}|^2}$
$V(\phi)$
ϕ


E.O.M.
$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} + V'(\phi) = 0$
describes a particle moving in a potential -V with time dep.


E.O.M. $\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} + V'(\phi) = 0$
describes a particle moving in a potential $-V$ with time dep. dampin
$-V$
$\lim_{\rho\to\infty} \phi(\rho) = \phi_f$
$\left.\frac{d\phi}{d\rho}\right|_{\rho=0} = 0$
ϕ
ϕ_f
ϕ_-
ϕ_+


EOM. $\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} + V'(\phi) = 0$
describes a particle moving in a potential -V with time dep. dampin
-V
ϕ
ϕ_-
ϕ_+
solutions always exist
1. undershoot start to the left of
lim
$\rho \to \infty$


Thin-wall limit
V(φ+) ≈ V(φf)
φ* ≈ φ+
R0
ρ
φf
x1
after nucleation
thin wall
φf
φ+
x2
SE = −1/2 ~² R
T→0



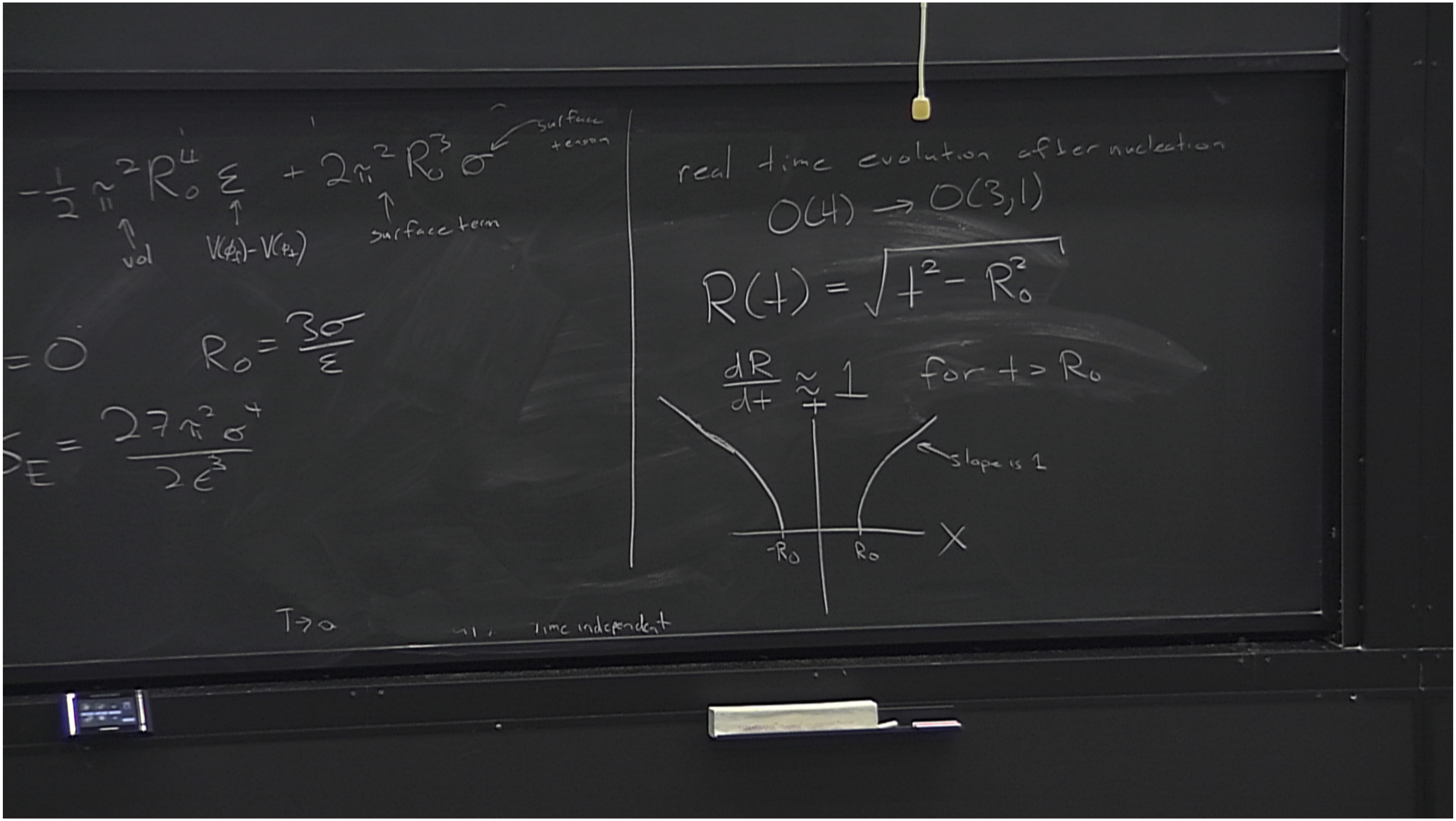
$$S_E = -\frac{1}{2}\pi^2 R_0^4\, \varepsilon + 2\pi^2 R_0^3\, \sigma$$

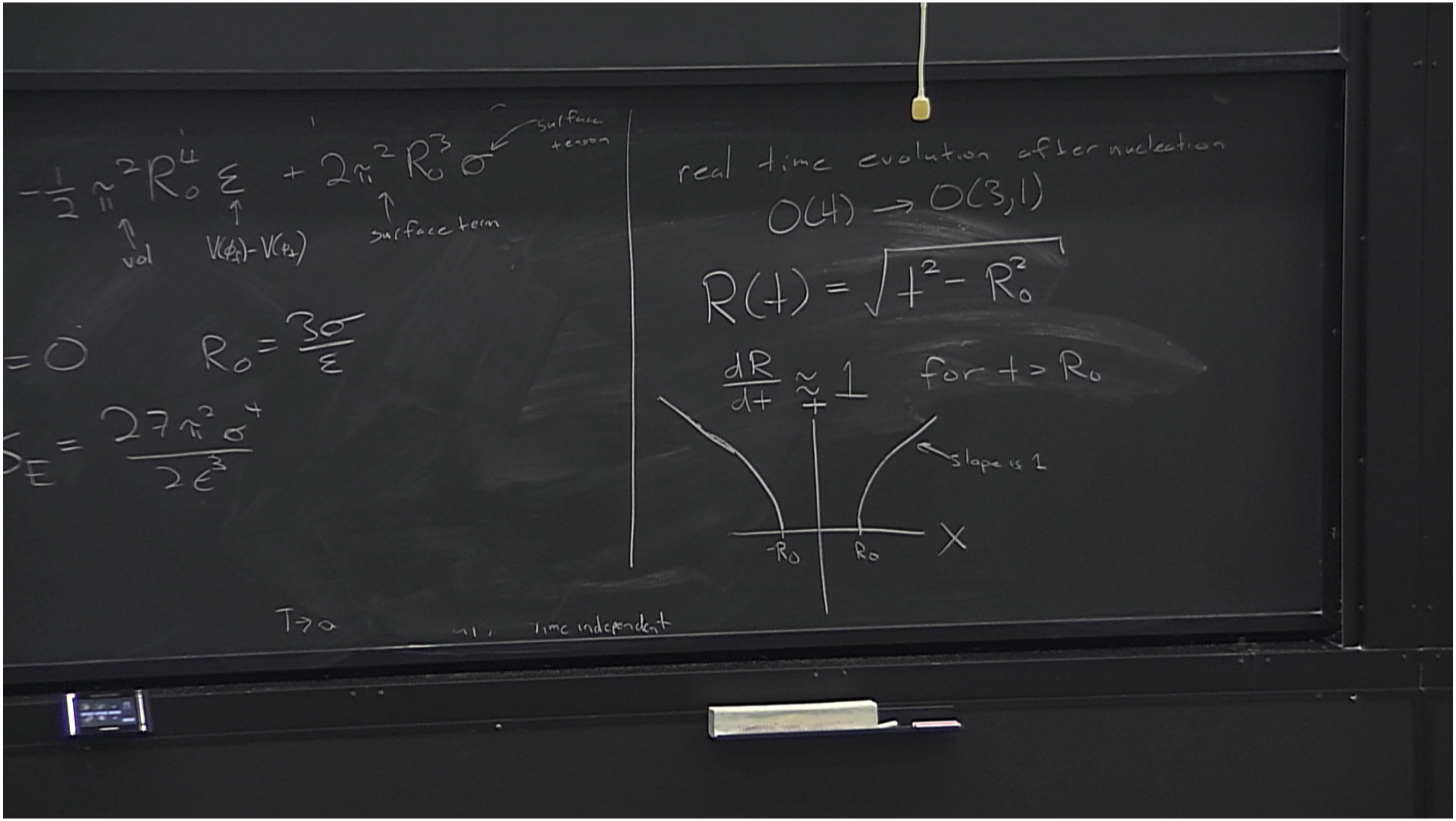
vol $\qquad V(\phi_f) - V(\phi_t) \qquad$ surface term $\qquad$ surface tension

$$\frac{dS_E}{dR_0} = 0 \qquad R_0 = \frac{3\sigma}{\varepsilon}$$

$$S_E = \frac{27\pi^2\sigma^4}{2\epsilon^3}$$

$T \to \infty$ $\qquad$ time independent


$-\frac{1}{2}\pi^2 R_0^4 \varepsilon + 2\pi^2 R_0^3 \sigma$
surface tension
vol
$V(\phi_f)-V(\phi_t)$
surface term
$=0$
$R_0 = \frac{3\sigma}{\varepsilon}$
$S_E = \frac{27\pi^2\sigma^4}{2\epsilon^3}$
$T \to \sigma$
time independent
real time evolution after nucleation
$O(4) \to O(3,1)$
$R(t) = \sqrt{t^2 - R_0^2}$
$\frac{dR}{dt} \approx 1$ for $t > R_0$
slope is 1
$-R_0$
R_0
X
t


surface tension
$-\frac{1}{2}\pi^2 R_0^4 \, \varepsilon + 2\pi^2 R_0^3 \, \sigma$
vol
$V(\phi_f)-V(\phi_t)$
surface term
$= 0$
$R_0 = \frac{3\sigma}{\varepsilon}$
$S_E = \frac{27\pi^2\sigma^4}{2\epsilon^3}$
real time evolution after nucleation
$O(4) \to O(3,1)$
$R(t) = \sqrt{t^2 - R_0^2}$
$\frac{dR}{dt} \approx 1$ for $t > R_0$
slope is 1
$-R_0$
R_0
X
t
time independent