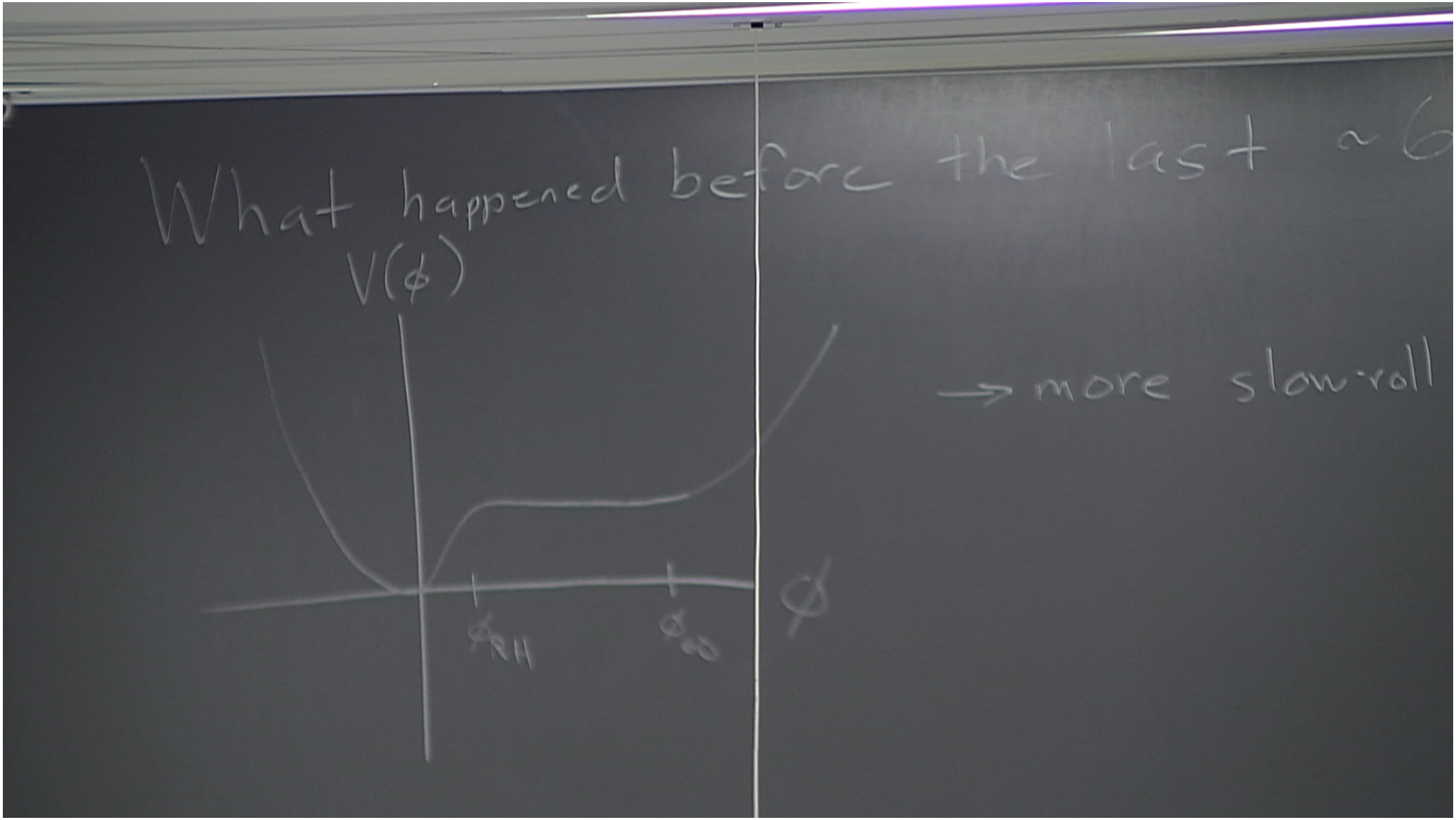
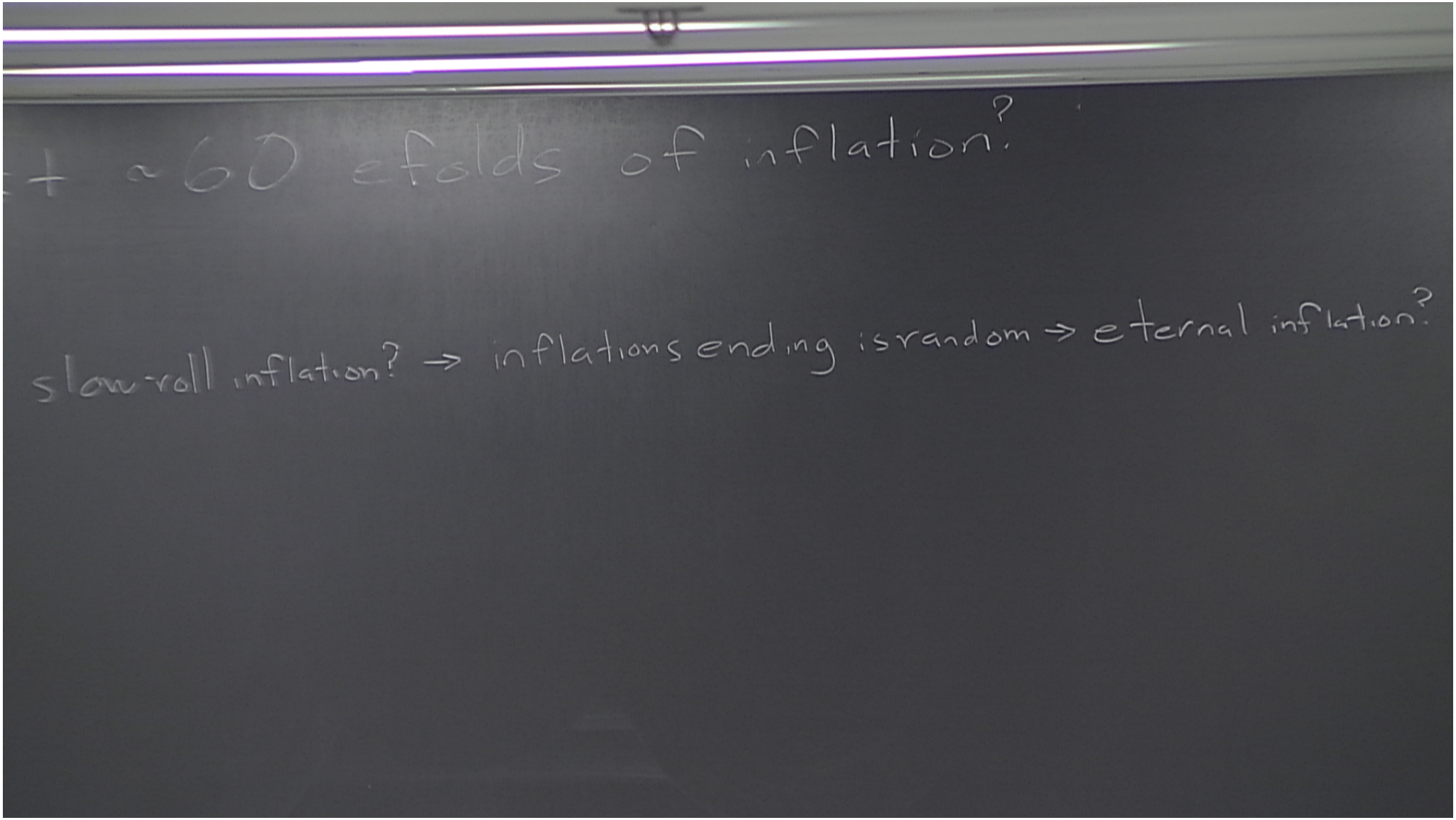
Title: 13/14 PSI - Explorations in Cosmology - Lecture 12

Date: Apr 23, 2014 10:15 AM

URL: http://pirsa.org/14040057

Abstract:

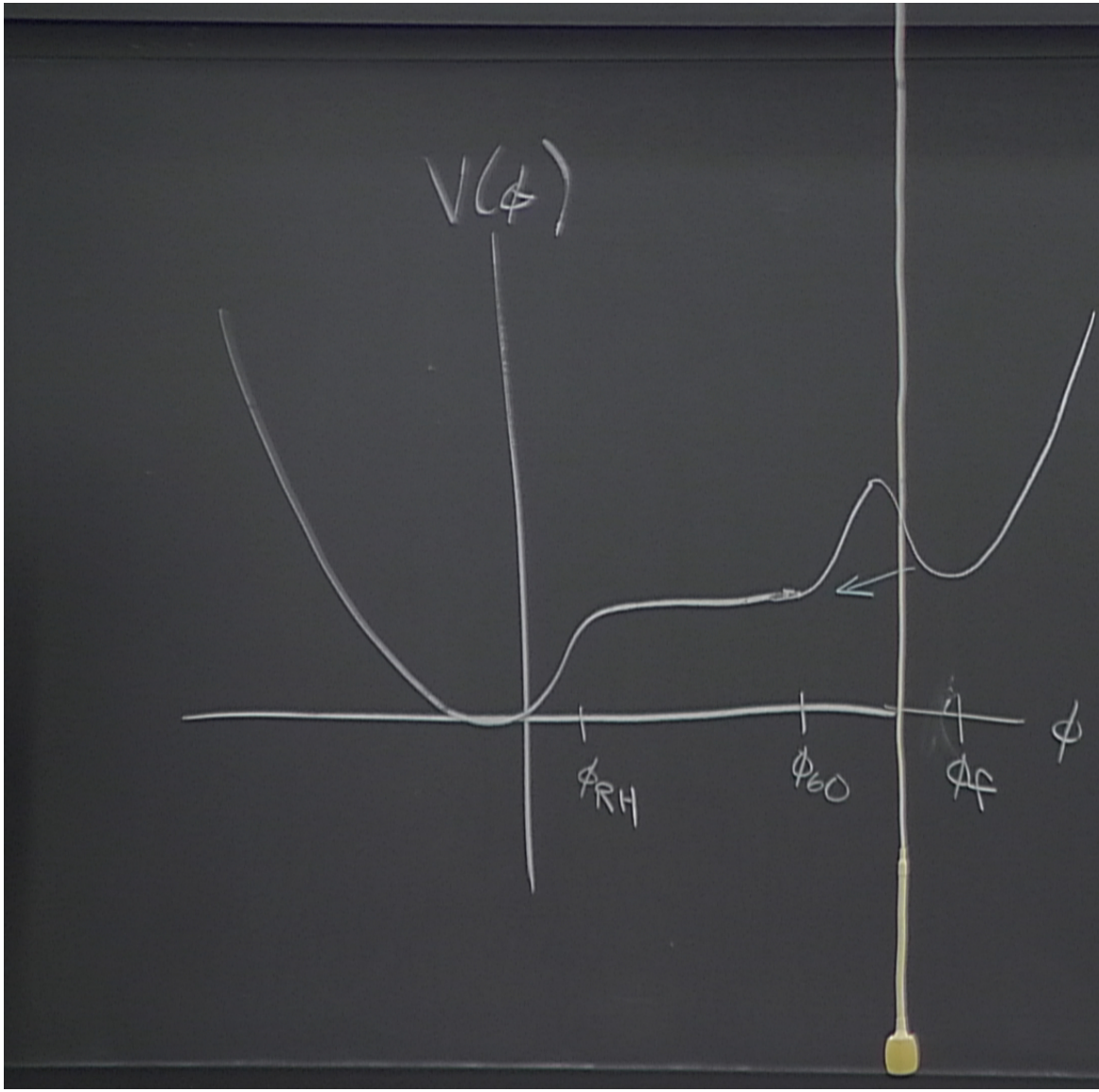

What happened before the last ~6
V(φ)
→ more slow-roll
φ
φ_end


t ~60 efolds of inflation?
slow-roll inflation? → inflations ending is random → eternal inflation?

tunneling from a false vacuum? → inflation ending is random → eternal inflation?

tunneling from a false vacuum? → inflation ending is random → eternal inflation?

1) Stochastic eternal inflation
2) Vacuum Decay
3) False Vacuum Eternal Inflation



tunneling from a false va

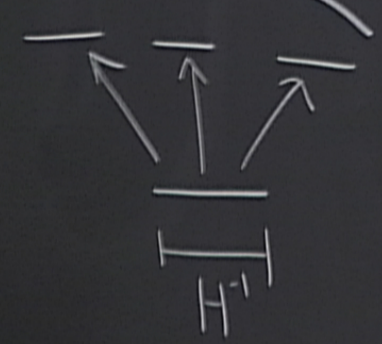
1) Stochastic eter

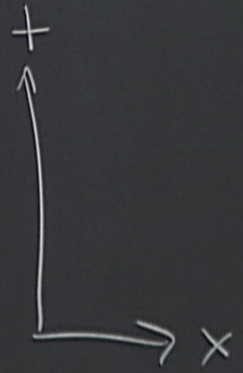
2) Vacuum Dec

3) False Vacuum

3) False Vacuum Eternal Inflati

$e^3 \sim 20$ patches





if $P(\text{inflation ends}) \lesssim \frac{19}{20}$ → eternal inflation

Stochastic Eternal Inflation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\langle \delta\phi^2 \rangle \Longleftrightarrow \langle 0|\hat{\phi}^2|0\rangle$$

For a scalar field in background dS

$$\langle 0|\hat{\phi}^2(t+\Delta t)|0\rangle - \langle 0|\hat{\phi}^2(t)|0\rangle \approx \frac{H^3\Delta t}{4\pi^2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = N(t)$$

random variable

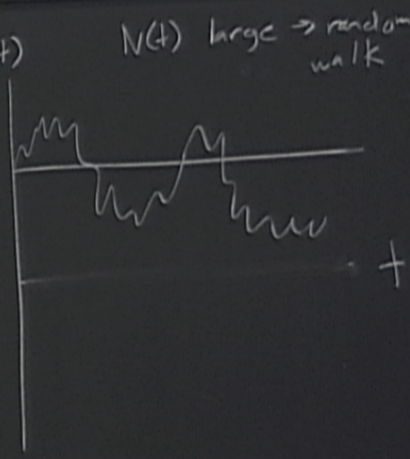
averaged a few H^{-1}

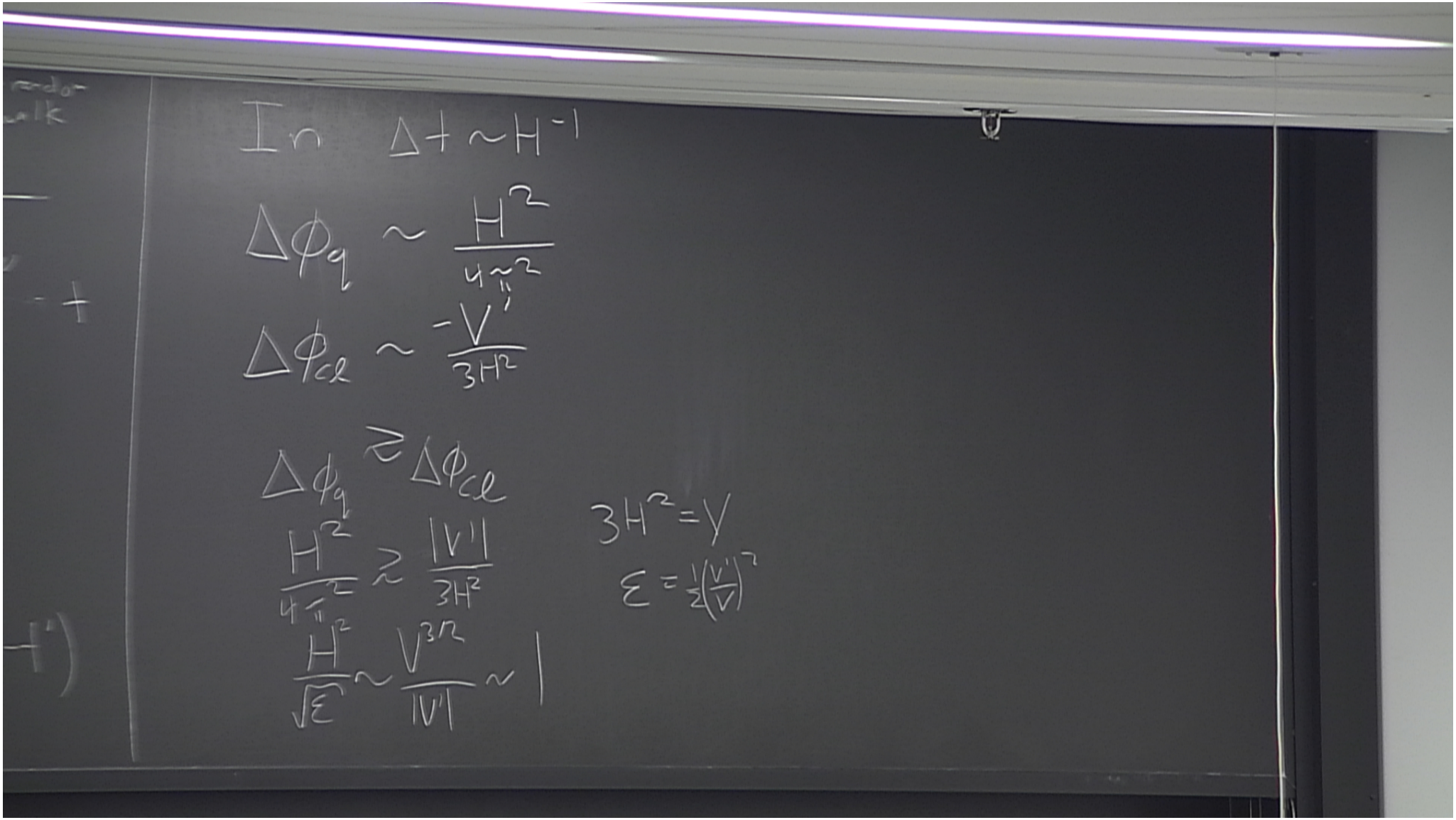
SR limit $V'(\phi)$ small

$$3H\dot{\phi} = N(t)$$

$$\langle \phi(t) \rangle = \int_0^{t'} \frac{\langle N(t) \rangle}{3H} dt' = 0 \Rightarrow \langle N(t) \rangle = 0$$

$$\langle \phi(t)\phi(t') \rangle = \int_0^{t} \int_0^{t'} \frac{\langle N(\tilde{t}) N(\tilde{t}') \rangle}{(3H)^2} d\tilde{t} d\tilde{t}' = \frac{H^3 t}{4\pi^2} \Rightarrow \langle N(t) N(t') \rangle = \frac{9H^5}{4\pi^2} \delta(t - t')$$




In $\Delta t \sim H^{-1}$
$\Delta\phi_q \sim \frac{H^2}{4\pi^2}$
$\Delta\phi_{cl} \sim \frac{-V'}{3H^2}$
$\Delta\phi_q \gtrsim \Delta\phi_{cl}$
$\frac{H^2}{4\pi^2} \gtrsim \frac{|V'|}{3H^2}$
$\frac{H^2}{\sqrt{\epsilon}} \sim \frac{V^{3/2}}{|V'|} \sim 1$
$3H^2 = V$
$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2$

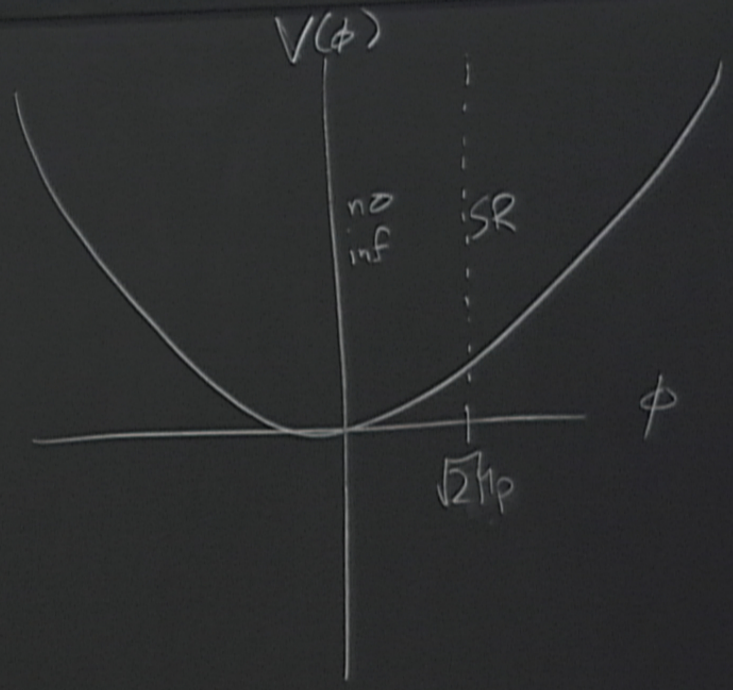
$$\Delta_s^2 \sim \frac{V^3}{|V'|^2}$$

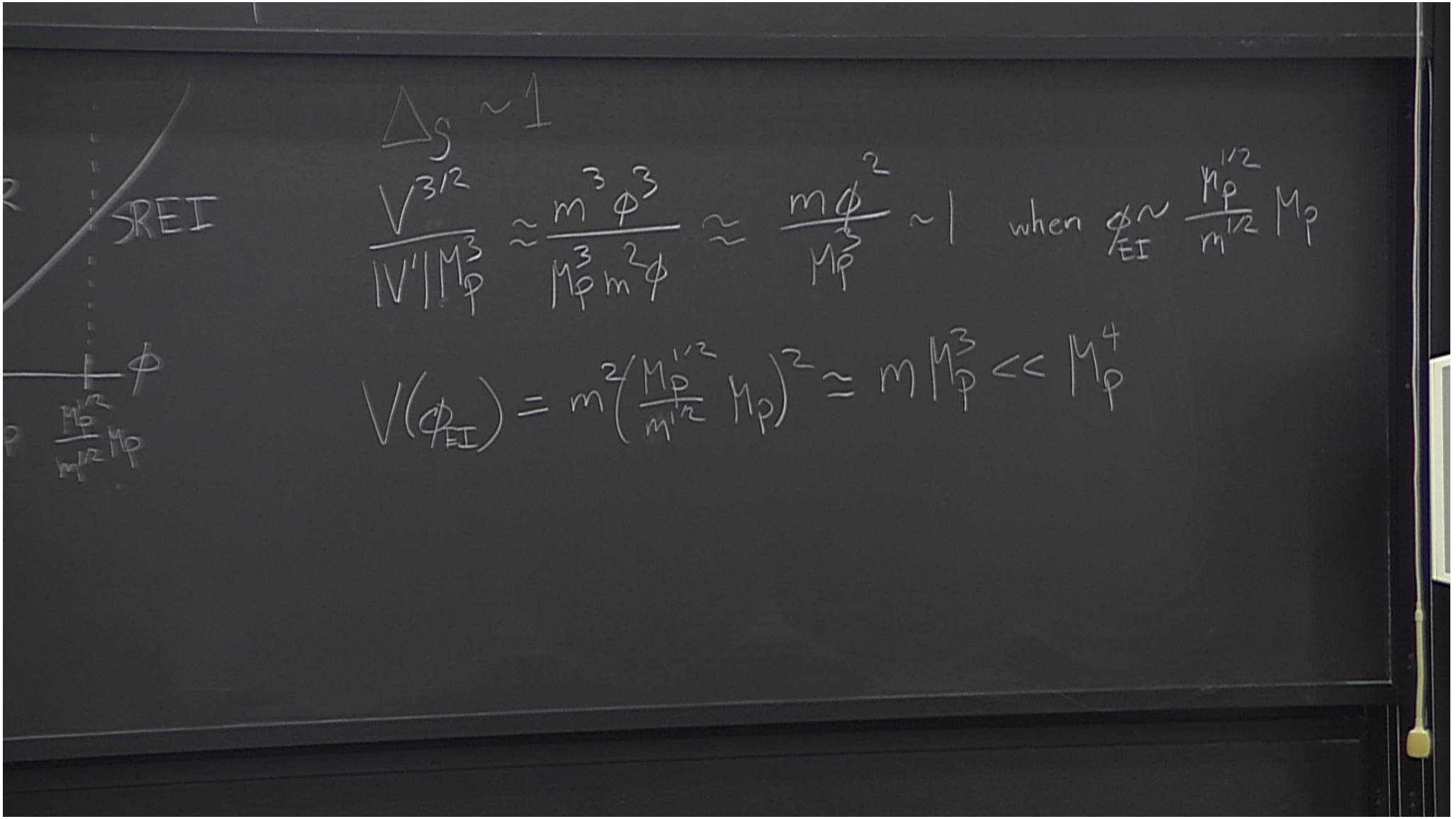
$\Delta_s \gtrsim 1 \Rightarrow$ Eternal Inflation

Example: $V(\phi) = \frac{1}{2} m^2 \phi^2$

$$\varepsilon_V = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{M_p^2}{2\phi^2}$$

$\varepsilon_V = 1$ when $\phi = \sqrt{2} M_p$




SREI
ϕ
$\frac{M_p^{1/2}}{m^{1/2}} M_p$
$\Delta_S \sim 1$
$\frac{V^{3/2}}{|V'| M_p^3} \simeq \frac{m^3 \phi^3}{M_p^3 m^2 \phi} \simeq \frac{m\phi^2}{M_p^3} \sim 1$ when $\phi_{EI} \sim \frac{M_p^{1/2}}{m^{1/2}} M_p$
$V(\phi_{EI}) = m^2 \left(\frac{M_p^{1/2}}{m^{1/2}} M_p\right)^2 \simeq m M_p^3 \ll M_p^4$

$\rho(\phi_f, t_f; \phi_i, t_i)$ = prob dist. of ϕ having a value ϕ_f a

a randomly chosen comoving worldline $x = \text{const}$

$\rho(\phi_f, t_f; \phi_i, t_i)$ = prob dist. of ϕ having a value ϕ_f a

a randomly chosen comoving worldline $x = \text{const}$

$$\rho(\phi_f, t_f; \phi_i, t_i - \Delta t_i) = \int \rho(\phi_f, t_f; \tilde{\phi}, t_i)\, \text{Prob}(\Delta\phi = \tilde{\phi} - \phi_i)\, d\tilde{\phi}$$

$\rho(\phi_f, t_f; \phi_i, t_i)$ = prob dist. of ϕ having a value ϕ_f a

a randomly chosen comoving worldline $x = \text{const}$

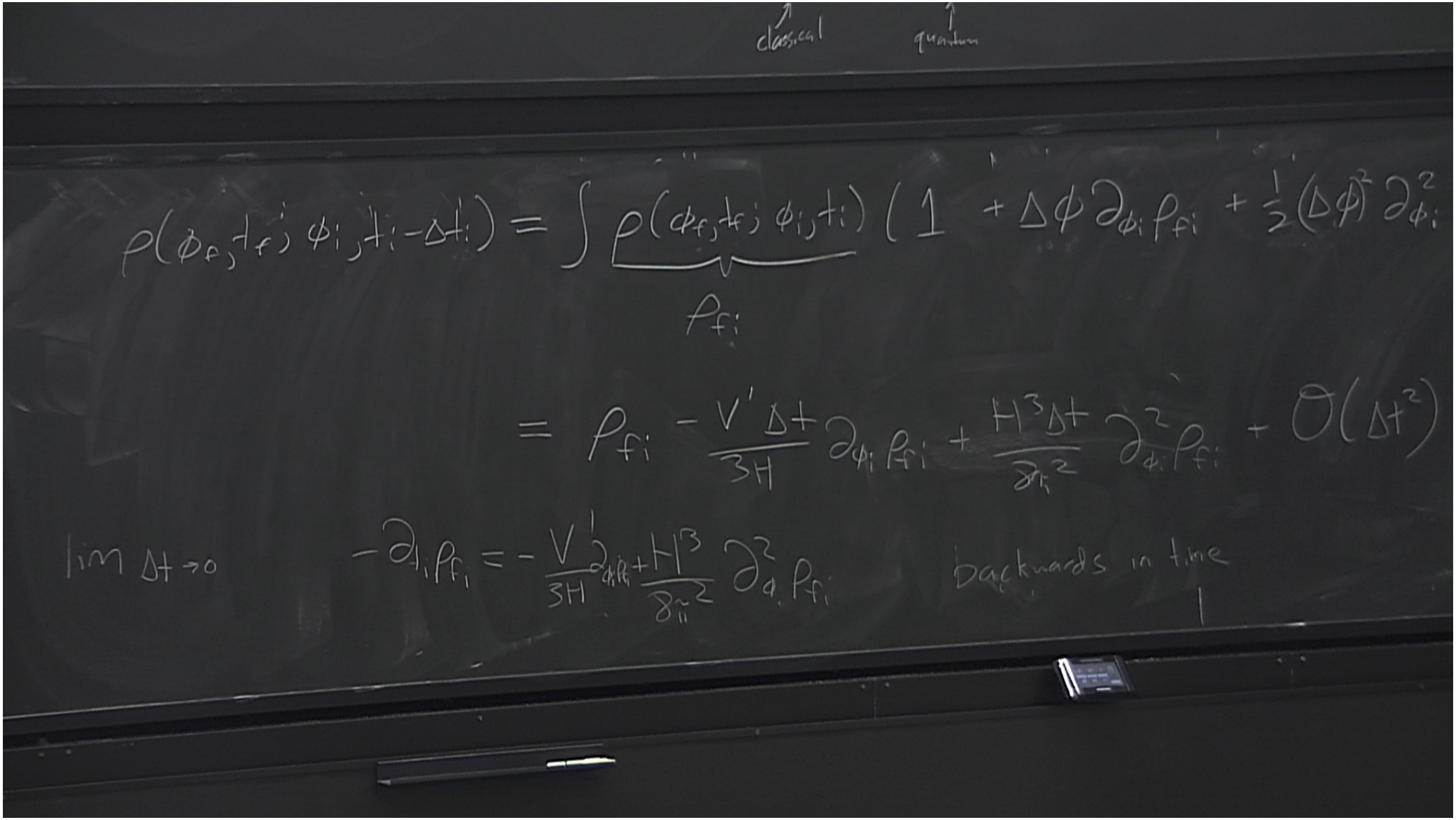
$$\rho(\phi_f, t_f; \phi_i, t_i - \Delta t_i) = \int \rho(\phi_f, t_f; \tilde{\phi}, t_i)\, \text{Prob}(\Delta\phi = \tilde{\phi} - \phi_i)\, d\tilde{\phi}$$

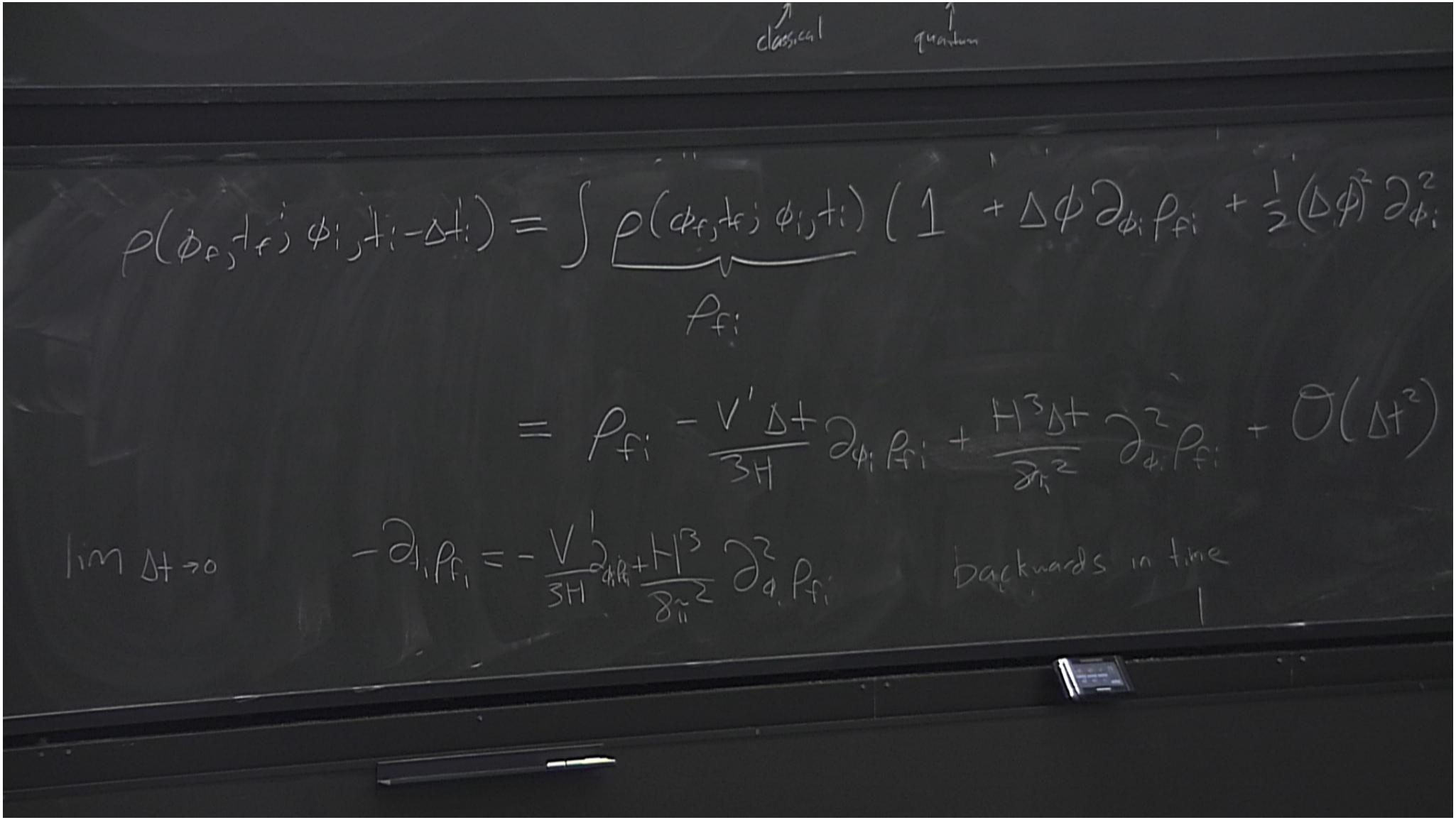
$$= \int \rho(\phi_f, t_f; \phi_i + \Delta\phi(\xi), t_i)\, \rho_G(\xi)\, d\xi$$

(ρ_G: Gaussian)

$$\Delta\phi = -\frac{V'\,\Delta t}{3H} + \xi \sqrt{\frac{H^3 \Delta t}{4\pi^2}}$$

(first term: classical; ξ term: quantum)


classical
quantum
$\rho(\phi_f, t_f; \phi_i, t_i - \Delta t_i) = \int \underbrace{\rho(\phi_f, t_f; \phi_i, t_i)}_{P_{fi}} \left(1 + \Delta\phi\, \partial_{\phi_i} P_{fi} + \frac{1}{2}(\Delta\phi)^2 \partial_{\phi_i}^2\right.$
$= P_{fi} - \frac{V'\Delta t}{3H} \partial_{\phi_i} P_{fi} + \frac{H^3 \Delta t}{\partial_{\pi_i}^2} \partial_{\phi_i}^2 P_{fi} + O(\Delta t^2)$
$\lim \Delta t \to 0 \quad -\partial_{t_i} P_{fi} = -\frac{V'}{3H} \partial_{\phi_i} P_{fi} + \frac{H^3}{\tilde{g}_{ii}^2} \partial_{\phi_i}^2 P_{fi}$
backwards in time

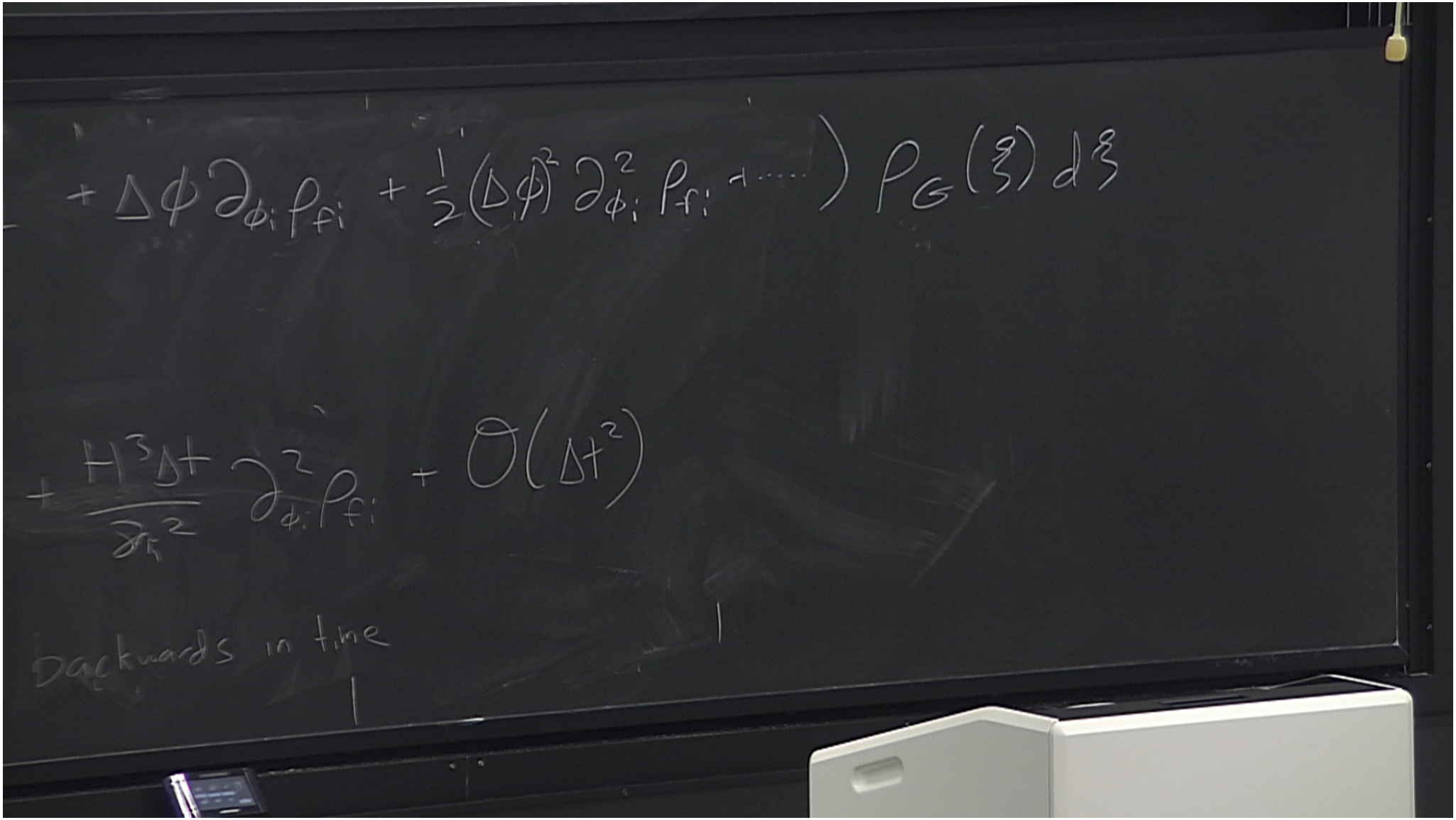

classical
quantum
$\rho(\phi_f, t_f; \phi_i, t_i - \Delta t_i) = \int \underbrace{\rho(\phi_f, t_f; \phi_i, t_i)}_{\rho_{fi}} \left(1 + \Delta\phi\, \partial_{\phi_i} \rho_{fi} + \frac{1}{2}(\Delta\phi)^2 \partial_{\phi_i}^2\right.$
$= \rho_{fi} - \frac{V'\Delta t}{3H} \partial_{\phi_i} \rho_{fi} + \frac{H^3 \Delta t}{\partial_{\pi_i}^2} \partial_{\phi_i}^2 \rho_{fi} + \mathcal{O}(\Delta t^2)$
$\lim \Delta t \to 0 \quad -\partial_{t_i} \rho_{fi} = -\frac{V'}{3H} \partial_{\phi_i} \rho_{fi} + \frac{H^3}{\tilde{g}_{11}^2} \partial_{q_i}^2 \rho_{fi}$
backwards in time

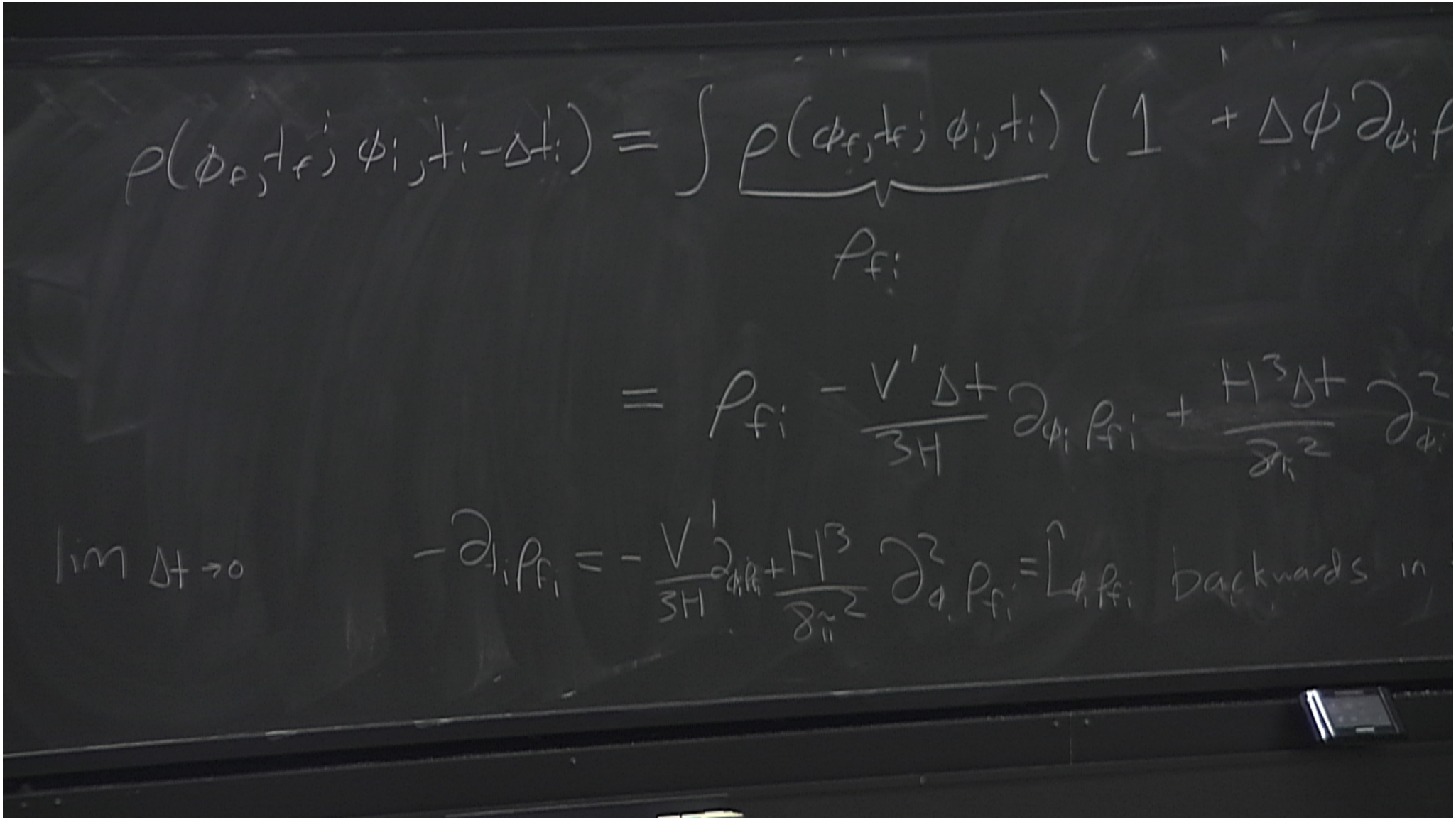
$$\rho(\phi_f, t_f; \phi_i, t_i - \Delta t_i) = \int \underbrace{\rho(\phi_f, t_f; \phi_i, t_i)}_{\rho_{fi}} \left(1 + \Delta\phi \partial\right.$$

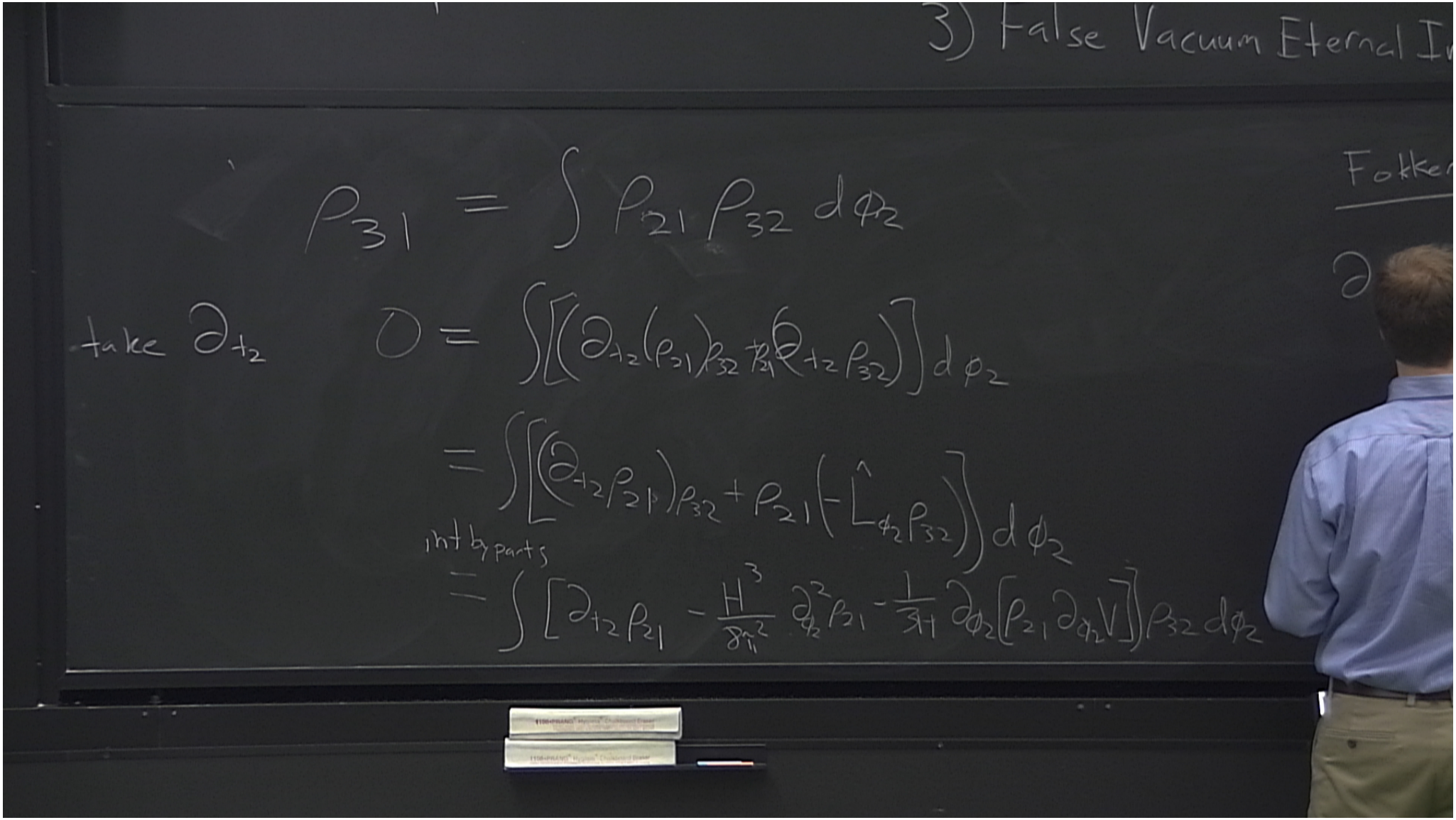
$$= \rho_{fi} - \frac{V' \Delta t}{3H} \partial_{\phi_i} \rho_{fi} + \frac{H^3 \Delta t}{\partial_{\pi_i}^2}$$

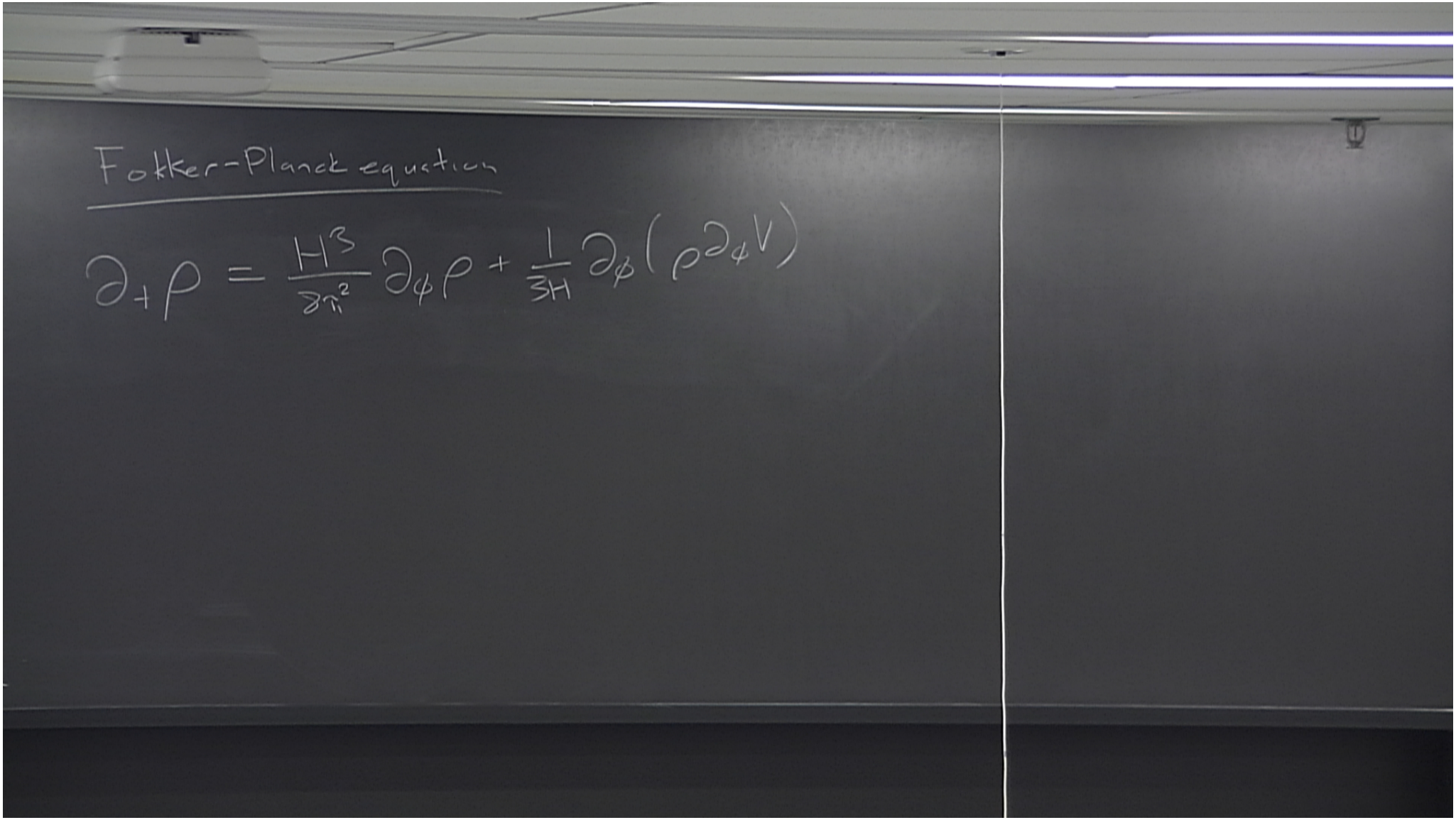
$$\lim \Delta t \to 0 \qquad -\partial_{t_i} \rho_{fi} = -\frac{V'}{3H} \partial_{\phi_i} \rho_{fi} + \frac{H^3}{\delta_{\pi\pi}^2} \partial_{q_i}^2 \rho_{fi}$$

backwards

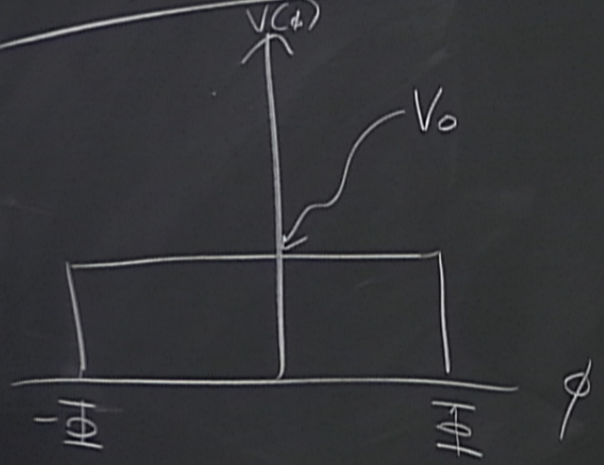

$+\Delta\phi\,\partial_{\phi_i} p_{fi} + \frac{1}{2}(\Delta\phi)^2\,\partial^2_{\phi_i} p_{fi} + \cdots\Big)\,\rho_G(\xi)\,d\xi$
$+\frac{H^3\Delta t}{\partial_{f_i}^2}\,\partial^2_{\phi_i} p_{fi} + O(\Delta t^2)$
backwards in time


$\rho(\phi_f, t_f; \phi_i, t_i - \Delta t_i) = \int \underbrace{\rho(\phi_f, t_f; \phi_i, t_i)}_{P_{fi}} (1 + \Delta\phi \partial_{\phi_i} f$
$= P_{fi} - \frac{V' \Delta t}{3H} \partial_{\phi_i} P_{fi} + \frac{H^3 \Delta t}{\partial_{\phi_i}^2} \partial_{\phi_i}^2$
$\lim \Delta t \to 0 \quad -\partial_{t_i} P_{f_i} = -\frac{V'}{3H} \partial_{\phi_i} P_{f_i} + \frac{H^3}{\partial_{\phi_i}^2} \partial_{\phi_i}^2 P_{fi} = \hat{L}_{\phi_i} P_{fi}$ backwards in


3) False Vacuum Eternal I
Fokke
$\rho_{31} = \int \rho_{21}\rho_{32}\, d\phi_2$
take ∂_{t_2}
$0 = \int [(\partial_{t_2}(\rho_{21})\rho_{32} + \rho_{21}(\partial_{t_2}\rho_{32})]\, d\phi_2$
$= \int [(\partial_{t_2}\rho_{21})\rho_{32} + \rho_{21}(-\hat{L}_{\phi_2}\rho_{32})]\, d\phi_2$
int by parts
$= \int [\partial_{t_2}\rho_{21} - \frac{H^3}{8\pi^2}\partial_{\phi_2}^2\rho_{21} - \frac{1}{3H}\partial_{\phi_2}[\rho_{21}\partial_{\phi_2}V]]\rho_{32}\, d\phi_2$


Fokker-Planck equation
$\partial_t \rho = \frac{H^3}{2\pi^2} \partial_\phi \rho + \frac{1}{3H} \partial_\phi (\rho \partial_\phi V)$

Toy Model of EI



inf ends when $|\phi| > \Phi$

$$\rho = f(t)\, g(\phi)$$

$$\frac{1}{f}\frac{\partial f}{\partial t} = \frac{H^3}{8\pi^2}\frac{1}{g}\frac{\partial^2 g}{\partial \phi^2} = C = -\frac{n^2 H^3}{32\Phi^2}$$

BC. $\rho(\pm\Xi) = 0$

$g = \cos\left[\frac{n\pi\phi}{2\Xi}\right] \quad n = 1, 3, 5, \ldots$

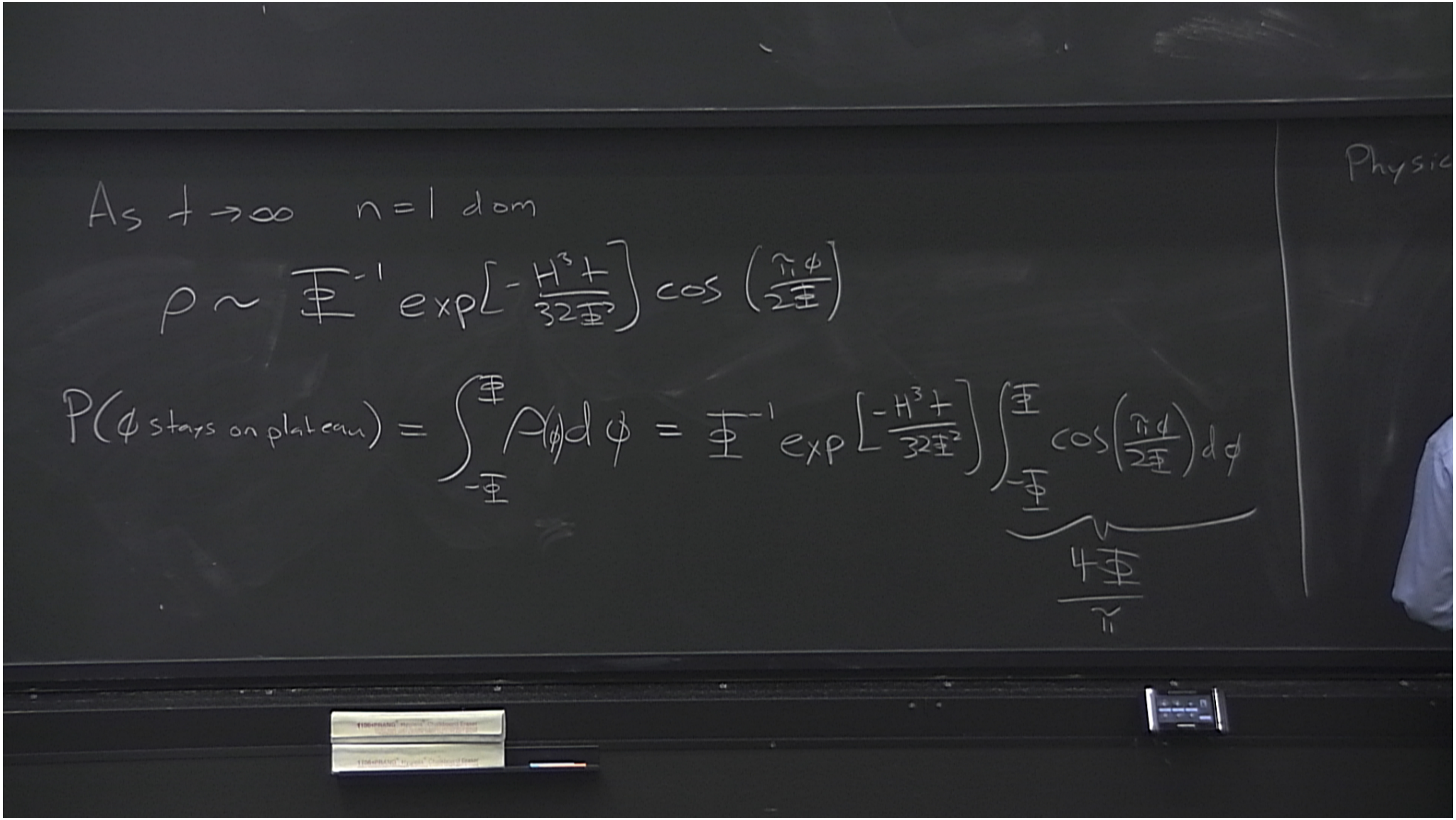
$f = A_n \exp\left[\frac{-n^2H^2}{8\Xi^2}t\right]$

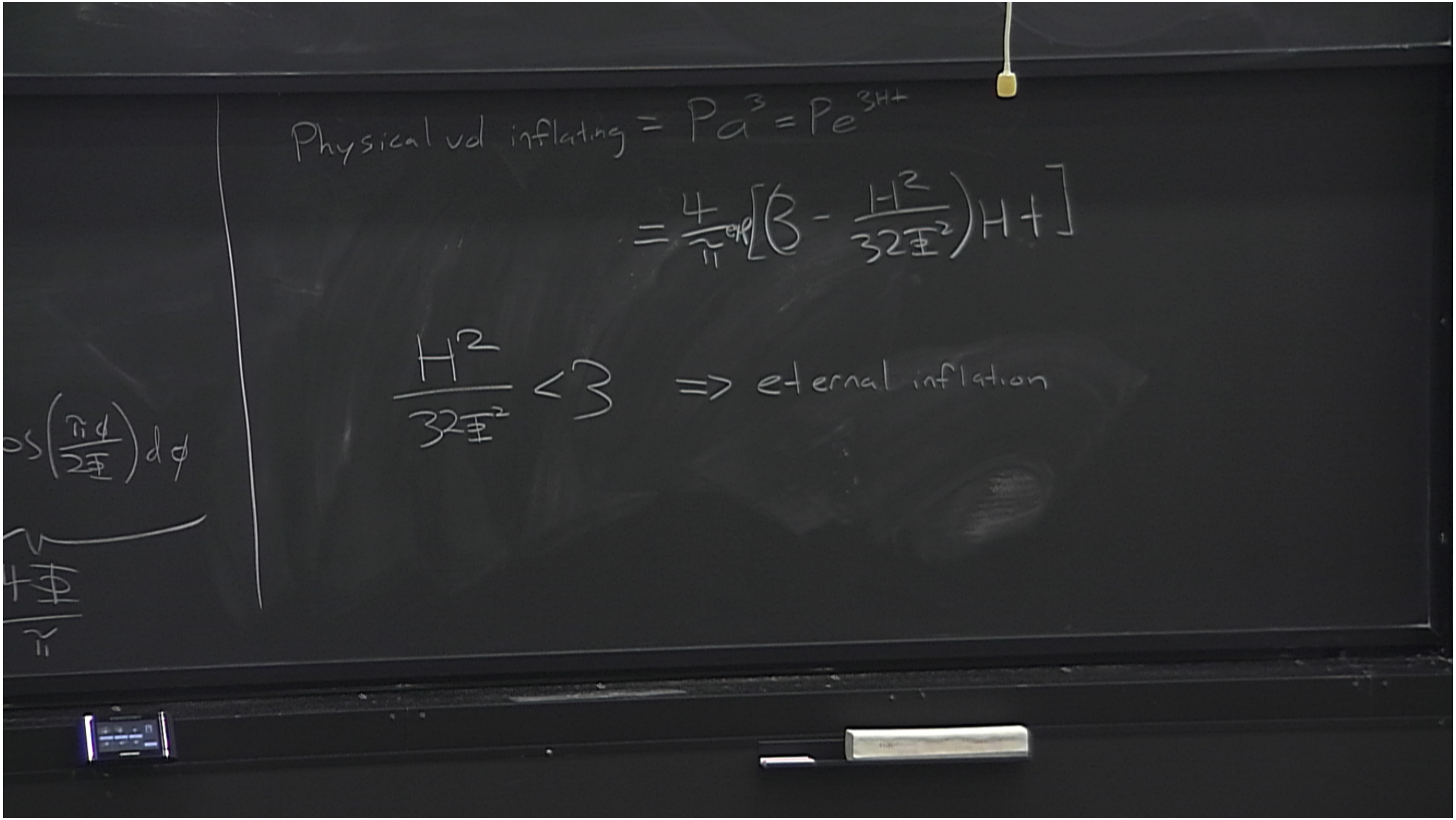
$\rho = \sum_{\text{odd}} A_n \exp\left[\frac{-n^2H^2}{32\Xi^2}t\right]\cos\left[\frac{n\pi\phi}{2\Xi}\right]$

$-\frac{n^2H^3}{32\Xi^2}$

Assume $\rho(0) = \delta(\phi) \Rightarrow A_n = \Xi^{-1}$

Stochastic Eternal Inflation


As $t \to \infty$ n=1 dom
$\rho \sim \Phi^{-1} \exp\left[-\frac{H^3 t}{32\Phi^2}\right] \cos\left(\frac{\pi\phi}{2\Phi}\right)$
$P(\phi \text{ stays on plateau}) = \int_{-\Phi}^{\Phi} P(\phi) d\phi = \Phi^{-1} \exp\left[-\frac{H^3 t}{32\Phi^2}\right] \int_{-\Phi}^{\Phi} \cos\left(\frac{\pi\phi}{2\Phi}\right) d\phi$
$\frac{4\Phi}{\pi}$
Physic


Physical vol inflating = $Pa^3 = Pe^{3Ht}$
$= \frac{4}{3\pi}\left[\left(\beta - \frac{H^2}{32\pi^2}\right)Ht\right]$
$\frac{H^2}{32\pi^2} < 3 \Rightarrow$ eternal inflation