

Title: 13/14 PSI - Explorations in Cosmology - Lecture 11

Date: Apr 22, 2014 10:15 AM

URL: <http://pirsa.org/14040056>

Abstract:

Lecture 4: Non-Gaussianity

Gaussianity

$$p(z) = \frac{1}{\sqrt{2\pi\langle z^2 \rangle}} \exp\left[-\frac{z^2}{2\langle z^2 \rangle}\right]$$

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Properties

- * All odd moments vanish
- * All even moments related to Z

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- * All even moments related to 2nd moment $\langle \xi^2 \rangle$

$$\langle \xi^m \rangle$$

Properties

- * All odd moments vanish
- * All even moments related to 2nd moment $\langle z^2 \rangle$

$$\langle z^m \rangle = \int_{-\infty}^{\infty} dz z^m P(z)$$

Properties

* All odd moments vanish

* All even moments related to 2nd moment $\langle z^2 \rangle$
m odd

$$\langle z^m \rangle = \int_{-\infty}^{\infty} dz z^m P(z) = \begin{cases} 0 & m \text{ odd} \\ \text{related to } \langle z^2 \rangle & m \text{ even} \end{cases}$$

QFT

$$z \rightarrow \hat{z}(z)$$

(consider correlators of free

QFT

$$j \rightarrow \hat{j}(z)$$

(Consider correlators of free QFT
in vacuum state)

$$\hat{j}(q) = \int q \hat{a}(q) + \int q^* \hat{a}^*(-q)$$

QFT

$$z \rightarrow \hat{z}(z)$$

Consider correlators of free QFT
in vacuum state

$$\hat{z}(q) = \int q \hat{a}(q) + \int q^* \hat{a}^+(-q)$$

QFT

$$\mathcal{Z} \rightarrow \hat{\mathcal{Z}}(\mathcal{Z})$$

Consider correlators of free QFT
in vacuum state

Show:

$$\hat{\mathcal{Z}}(q) = \mathcal{Z}_q \hat{a}(q) + \mathcal{Z}_q^* \hat{a}^\dagger(-q)$$

$$[\hat{a}(q), \hat{a}^\dagger(q')] = (2\pi)^3 \delta(q - q')$$

$$\hat{a}(q)|0\rangle = 0$$

$$\langle 0|\hat{a}^\dagger(q) = 0$$

of free QFT

$$\hat{a}^\dagger(-q)$$

$$= (2\pi)^3 \delta(q - q')$$

$$\langle 0 | \hat{a}^\dagger(q) = 0$$

Show: * Correlators of odd # of $\hat{\phi}$'s vanish

$$\langle 0 | \hat{\phi}(q_1) \hat{\phi}(q_2) \hat{\phi}(q_3) | 0 \rangle = 0$$

$\hat{\phi}(x)$

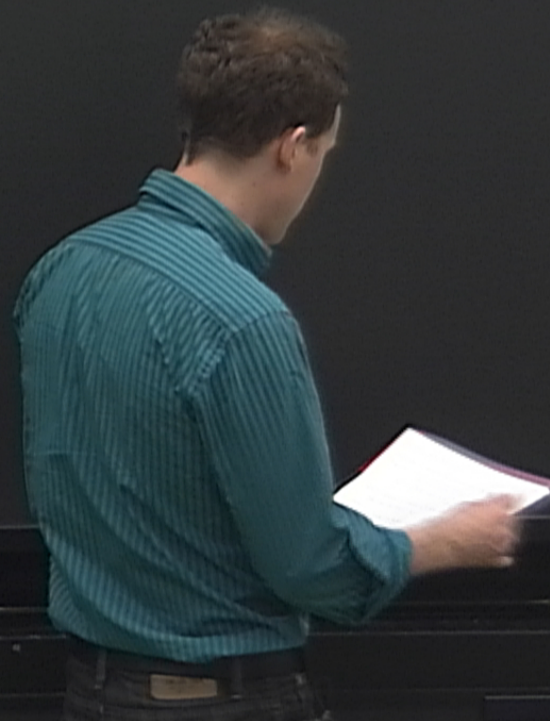
Consider correlators of free QFT
in vacuum state

$$\hat{\phi}(q) = \int d^3x \hat{a}(q) + \int d^3x \hat{a}^\dagger(-q)$$

$$[\hat{a}(q), \hat{a}^\dagger(q')] = (2\pi)^3 \delta(q - q')$$

$$\hat{a}(q)|0\rangle = 0 \quad \langle 0|\hat{a}^\dagger(q) = 0$$

Show: k correlators of odd # of
 $\langle 0|\hat{\phi}(q_1)\hat{\phi}(q_2)\dots\hat{\phi}(q_k)|0\rangle$



of free QFT

$$\hat{a}^+(-q)$$

$$(2\pi)^3 \delta(q-q')$$

$$\langle 0 | \hat{a}^+(q) = 0$$

Show: * Correlators of odd # of ζ 's vanish

$$\langle 0 | \zeta(q_1) \zeta(q_2) \zeta(q_3) | 0 \rangle = 0$$

* Correlators of even # ζ 's given by Wick's theorem

$$\text{eg. } \langle \zeta(q_1) \zeta(q_2) \zeta(q_3) \zeta(q_4) \rangle$$

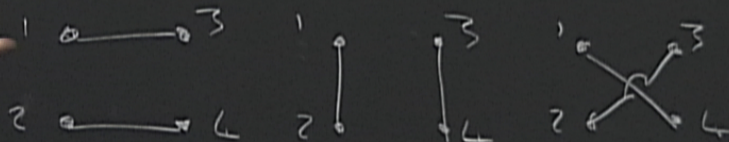
of free QFT

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eg. $\langle \hat{\phi}(q_1) \hat{\phi}(q_2) \hat{\phi}(q_3) \hat{\phi}(q_4) \rangle = \langle \hat{\phi}(q_1) \hat{\phi}(q_2) \rangle \langle \hat{\phi}(q_3) \hat{\phi}(q_4) \rangle$
 $+ (2 \leftrightarrow 3) + (2 \leftrightarrow 4)$



$$\langle 0 | \hat{a}^\dagger(q) = 0$$

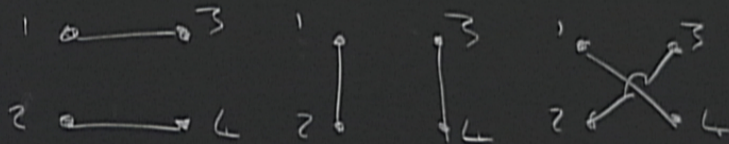
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Moments

back to position space

$$\langle \hat{z}^4(z) \rangle = \int \left(\prod_{i=1}^4 \frac{d^3 q_i}{(2\pi)^3} \right) \langle \hat{z}(q_1) \hat{z}(q_2) \hat{z}(q_3) \hat{z}(q_4) \rangle e^{i \left(\sum_{i=1}^4 q_i \right) \cdot \underline{z}}$$

Moments

back to position space

$$\langle \hat{z}^4(x) \rangle = \int \left(\prod_{i=1}^4 \frac{d^3 q_i}{(2\pi)^3} \right) \langle \hat{z}(q_1) \hat{z}(q_2) \hat{z}(q_3) \hat{z}(q_4) \rangle e^{i \left(\sum_{i=1}^4 q_i \right) \cdot x}$$

$$= \int \frac{d^3 q_1 d^3 q_2}{(2\pi)^6} \langle \hat{z}(q_1) \hat{z}(q_2) \rangle e^{i(q_1 + q_2) \cdot x} \int \frac{d^3 q_3 d^3 q_4}{(2\pi)^6} \langle \hat{z}(q_3) \hat{z}(q_4) \rangle$$

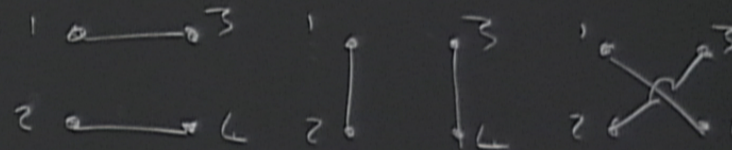
Moments

back to position space

$$\langle \hat{z}^4(z) \rangle = \int \left(\prod_{i=1}^4 \frac{d^3 q_i}{(2\pi)^3} \right) \langle \hat{z}(q_1) \hat{z}(q_2) \hat{z}(q_3) \hat{z}(q_4) \rangle e^{i \left(\sum_{i=1}^4 q_i \right) \cdot z}$$

$$= \int \frac{d^3 q_1 d^3 q_2}{(2\pi)^6} \langle \hat{z}(q_1) \hat{z}(q_2) \rangle e^{i(q_1+q_2) \cdot z} \int \frac{d^3 q_3 d^3 q_4}{(2\pi)^6} \langle \hat{z}(q_3) \hat{z}(q_4) \rangle e^{i(q_3+q_4) \cdot z} +$$

$$\hat{a}(q)|0\rangle=0 \quad \langle 0|\hat{a}^\dagger(q)=0$$



back to position space

$$= \int \left(\prod_{i=1}^4 \frac{d^3 q_i}{(2\pi)^3} \right) \langle \hat{\gamma}(q_1) \hat{\gamma}(q_2) \hat{\gamma}(q_3) \hat{\gamma}(q_4) \rangle e^{i \left(\sum_{i=1}^4 q_i \right) \cdot x}$$

$$= \int \frac{d^3 q_1 d^3 q_2}{(2\pi)^6} \langle \hat{\gamma}(q_1) \hat{\gamma}(q_2) \rangle e^{i(q_1+q_2) \cdot x} \left[\int \frac{d^3 q_3 d^3 q_4}{(2\pi)^6} \langle \hat{\gamma}(q_3) \hat{\gamma}(q_4) \rangle e^{i(q_3+q_4) \cdot x} + (2 \leftrightarrow 3) + (2 \leftrightarrow 4) \right]$$

$$= \langle \hat{\gamma}^2(x) \rangle \langle \hat{\gamma}^2(x) \rangle + \dots = 3 \langle \hat{\gamma}^2(x) \rangle^2$$

In general, find

$$\langle \hat{\mathcal{O}}^n(x) \rangle = \langle \hat{\mathcal{O}}^2(x) \rangle^n$$

In general, find

$$\langle \hat{\mathcal{O}}^n(\underline{x}) \rangle = \langle \hat{\mathcal{O}}(\underline{x}) \rangle^n \times (\# \text{ contractions})$$

In general, find

$$\langle \hat{\mathcal{O}}^{2n}(x) \rangle = \langle \hat{\mathcal{O}}^2(x) \rangle^n \times (\# \text{ contractions})$$

distinct Wick contractions
not counting ordering of pairs
or order within pairs

$(2n)!$

$$e^{i \left(\sum_{i=1}^4 q_i \right) \cdot z}$$

$$\int \frac{d^3 q_3 d^3 q_4}{(2\pi)^6} \langle \hat{\gamma}(q_3) \hat{\gamma}(q_4) \rangle e^{i(q_3 + q_4) \cdot z} + (2 \leftrightarrow 3) + (2 \leftrightarrow 4)$$

$$= 3 \langle \hat{\gamma}^2(z) \rangle$$

$$\begin{aligned} \overline{(3, 3_2)} \overline{(3_3 3_4)} &= (3_2 3_1) (3_3 3_4) \\ (3, 3_2) (3_3 3_4) \\ (3, 3_1) (3_3 3_4) \end{aligned}$$

In general, find

$$\langle \hat{\phi}^{2n}(x) \rangle = \langle \hat{\phi}^2(x) \rangle^n \times (\# \text{ contractions})$$

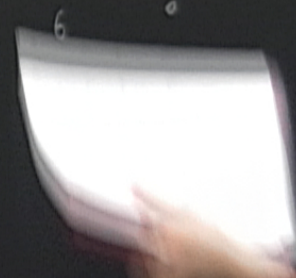
distinct Wick contractions
not counting ordering of pairs
or order within pairs

$$\frac{(2n)!}{2^n n!}$$

ordering pairs

Non-Gaussianity (ie nonzero higher connected correlators)
requires interactions

→ nontrivial vertices



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requires interactions

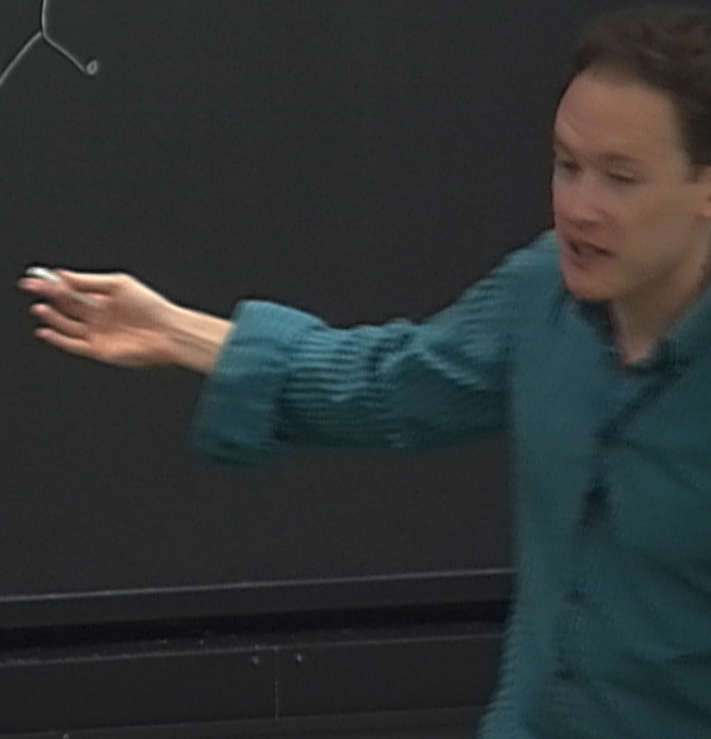


→ nontrivial vertices

Non-Gaussianity (ie nonzero higher connected correlators)
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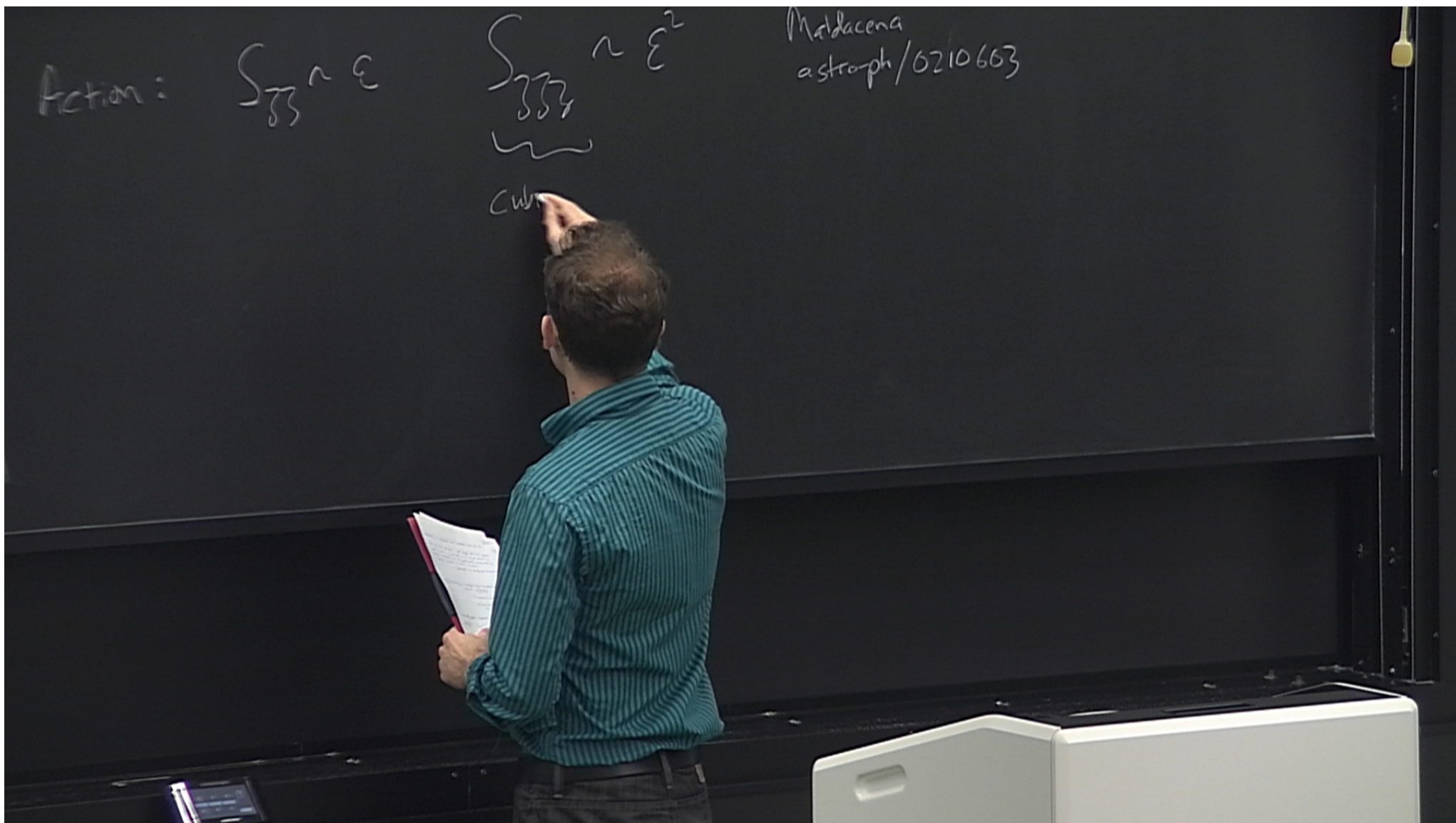


→ nontrivial vertices



$$\langle \{q_1\} \{q_2\} \{q_3\} \{q_4\} \rangle_c = \langle \{q_1\} \{q_2\} \{q_3\} \{q_4\} \rangle_{d.s.}$$

$$- \langle \{q_1\} \{q_2\} \rangle \langle \{q_3\} \{q_4\} \rangle - \dots$$



slow-roll approx. sup

Action: $S_{\text{eff}} \propto \epsilon$

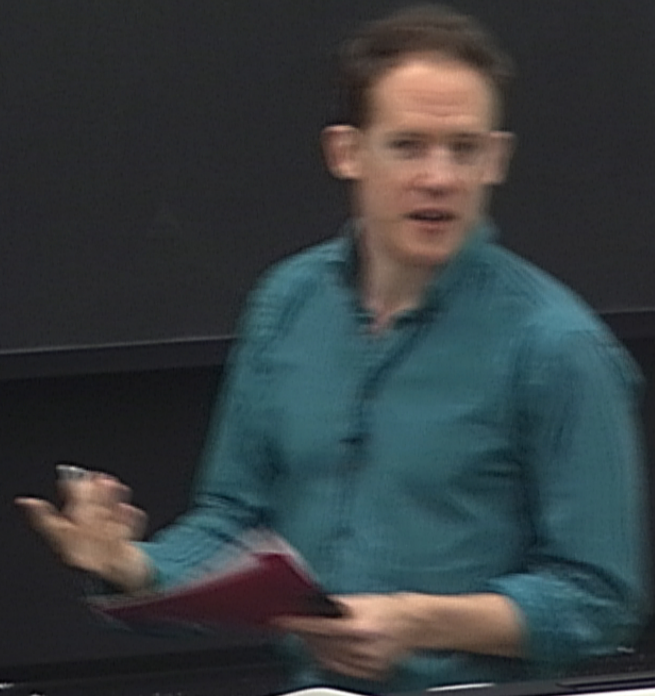
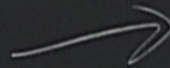
$\epsilon \ll 1$

* single field S12 inflation
nearly Gaussian

$$S_{\text{eff}} \propto \epsilon^2$$

Cubic term gives
leading interactions

Maldacena
astro-ph/0210603



slow-roll approx. sup

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Maldacena
astro-ph/0210663

slow-roll approx. sup

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$$\underbrace{S_{\text{eff}}}_{\text{cubic term}} \propto \epsilon^2$$

cubic term gives
leading interactions

Maldacena
astro-ph/0210603

→ NG tells you about interaction terms in
inflationary Lagrangian, i.e. dynamics

How to quantify NG

lowest order statistic = 3pt correlator

→ zero for Gaussian

$$\delta \sim \frac{\delta T}{T} \sim 10^{-5}$$

$$\langle \delta(q_1) \delta(q_2) \delta(q_3) \rangle$$

→ higher correlators get smaller

How to quantify NG

lowest order statistic = 3pt correlator

→ zero for Gaussian

$$\zeta \sim \frac{\delta T}{T} \sim 10^{-5}$$

$$\langle \zeta(q_1) \zeta(q_2) \zeta(q_3) \rangle = B(q_1, q_2, q_3) \underbrace{(2\pi)^3 \delta(q_1 + q_2 + q_3)}_{\text{momentum conservation}}$$

→ higher correlators get smaller

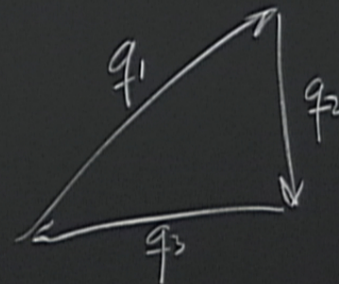
→ zero for Gaussian

1st.c = 3pt correlator

$$\langle \zeta(q_1) \zeta(q_2) \zeta(q_3) \rangle = \underbrace{B(q_1, q_2, q_3)}_{\text{bispectrum}} \underbrace{(2\pi)^3 \delta(q_1 + q_2 + q_3)}_{\text{momentum conservation}}$$

bispectrum

(translation invariance in position space)

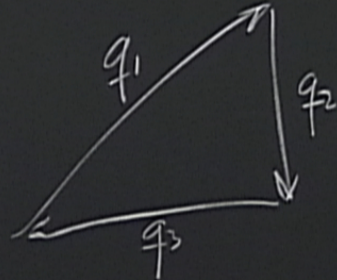


for
region

$$(2\pi)^3 \delta(q_1 + q_2 + q_3)$$

momentum con

(translation invar



* Measure from 3pt ℓ^n of CMB
temperature fluctuations

* extract primordial (ie. inflationary)
part by subtracting all late-time
contributions

Komatsu 1003.6097

$\angle y_1 y_2 y_3 = \angle y_1 y_2 y_3$

$- \angle y_1 y_2 y_3 - \dots$



Parametrisation

$$B(q_1, q_2, q_3) = \text{amplitude} \times \text{shape function}$$

"fNL"

eg. "local-type" NG

$$\mathcal{L}(\pi)$$



Parametrisation

$$B(q_1, q_2, q_3) = \text{amplitude} \times \text{shape function}$$

"f_{NL}"

eg. "local-type" NG

$$\zeta(z) = \zeta_g(z) + \frac{3}{5} f_{NL}^{\text{local}} \left(\ddot{\zeta}_g(z) - \underbrace{\langle \ddot{\zeta}_g(z) \rangle}_{\langle \ddot{\zeta}_g(z) \rangle = 0} \right)$$

$$\langle \ddot{\zeta}_g(z) \rangle = 0$$

Parametrisation

$$B(q_1, q_2, q_3) = \text{amplitude} \times \text{shape function}$$

"f_{NL}"

eg. "local-type" NG

$$\delta(z) = \delta_g(z) + \frac{3}{5} f_{NL}^{local} \left(\delta_g^2(z) - \underbrace{\langle \delta_g^2(z) \rangle} \right)$$

$$\langle \delta(z) \rangle = \langle \delta_g(z) \rangle = 0$$

$$B(q_1, q_2, q_3) = \frac{6}{5} \ln \bar{A} \left(\underbrace{\frac{1}{q_1^3 q_2^3} + \frac{1}{q_2^3 q_3^3} + \frac{1}{q_3^3 q_1^3}}_{\text{"shape } f^n" \text{ for local NG}} \right)$$

$q_1 \rightarrow 0$

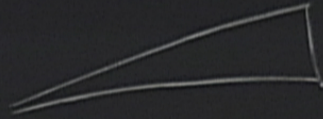
$$= \frac{6}{5} \ln A^2 \left(\underbrace{\frac{1}{q_1^3 q_2^3} + \frac{1}{q_2^3 q_3^3} + \frac{1}{q_3^3 q_1^3}}_{\text{"shape } f^n" \text{ for local NG}} \right)$$

$$q_1 \rightarrow 0$$


$$\left. \begin{array}{r} 1 \\ \hline q_3^3 q_1^3 \end{array} \right\}$$

- local NG

$$q_1 \rightarrow 0$$



SR inflation

$$f_{NL} \sim \epsilon, \eta \ll 1$$

$$\left. \begin{array}{r} 1 \\ \hline q_3^3 q_1^3 \end{array} \right\}$$

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