

Title: 13/14 PSI - Explorations in Cosmology - Lecture 6

Date: Apr 14, 2014 10:15 AM

URL: <http://pirsa.org/14040048>

Abstract:

Photons

$$\frac{df}{dz} = 0$$

Photons

$$\frac{df}{dz} = C[f]$$

$$= \left[\exp\left\{ \frac{P}{T(1+\Theta)} \right\} - 1 \right]^{-1}$$

$$dN$$

Photons

$$\frac{df}{dz} = C[f]$$

$$F = \left[\exp\left\{ \frac{P}{T(1+\Theta)} \right\} \right]$$

$$dN = f \frac{d^3x}{(2\pi)^3}$$

$$\frac{ST}{T} ($$

Photons

$$= C[f]$$

$$\left[\exp\left\{ \frac{\mu}{T(1+\theta)} \right\} - 1 \right]^{-1}$$

$$\frac{\int d^3p}{(2\pi)^3}$$

$$\frac{S_T}{T}(\hat{p}) = (-1) \left(\vec{x}_{\text{new}}, z_{\text{new}}, \hat{p} \right)$$

Photons

$$\frac{df}{dz} = C[f]$$

$$f \left[\exp\left\{ \frac{p}{T(1+\Theta)} \right\} - 1 \right]^{-1}$$

$$dN = f \frac{d^3x d^3p}{(2\pi)^3}$$

$$\frac{ST}{T}(\hat{p}) = (-1) \left(\vec{x}_{\text{line}}, z_{\text{line}}, \hat{p} \right)$$

Anisotropic
Blackbody

Photons

$$\frac{df}{dz} = C[f]$$

$$f = \left[\exp\left\{ \frac{p}{T(1+\Theta)} \right\} - 1 \right]^{-1}$$

$$dN = f \frac{d^3x d^3p}{(2\pi)^3}$$

$$\frac{S_T}{T}(\hat{p}) = (-1) \left(\vec{x}_{\text{line}}, z_{\text{line}}, \hat{p} \right)$$

Anisotropic
Blackbody

$$+ \hat{p} \leftrightarrow \hat{p}$$

Photons

$$\frac{df}{dz} = C[f]$$

$$f = \left[\exp\left\{ \frac{p}{T(1+\Theta)} \right\} - 1 \right]^{-1}$$

$$dN = f \frac{d^3x d^3p}{(2\pi)^3}$$

$$\frac{ST}{T}(\hat{p}) = (1) (\vec{x}, z, \hat{p})$$

Anisotropic
Blackbody

$$\hat{p} \leftrightarrow \{\theta, \phi\}$$

$$= \cos\theta, \hat{p}$$

Photons

$$\frac{df}{dz} = C[f]$$

$$F = \left[\exp\left\{ \frac{p}{T(1+\Theta)} \right\} - 1 \right]^{-1}$$

$$dN = F \frac{d^3x d^3p}{(2\pi)^3}$$

$$\frac{ST}{T}(\hat{p}) = (1-i) (\vec{x}_{\text{mid}}, z_{\text{mid}}, \hat{p})$$

Anisotropic
Blackbody

convert $\rightarrow \{\theta, \phi\}$

$$\hat{p}_z = \cos\theta$$

$$\hat{p}_y = \sin\theta \sin\phi$$

$$\frac{S_T}{T}(\hat{p}) = \odot(\vec{x}_{\text{here}}, z_{\text{now}}, \hat{p})$$

Anisotropic
Blackbody

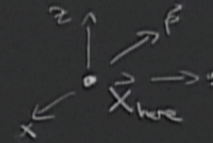
convert $\hat{p} \leftrightarrow \{\theta, \varphi\}$

$$\hat{p}_z = \cos\theta, \quad \hat{p}_x = \sin\theta \cos\varphi$$

$$\hat{p}_y = \sin\theta \sin\varphi$$

$\vec{x}_{\text{here}}$

$$\frac{ST}{T}(\hat{p}) = (-1) \left(\hat{x}_{\text{here}} \hat{z}_{\text{here}} \hat{p} \right)$$



convert $\hat{p} \leftrightarrow$

$$\hat{p}_z = \cos\theta, \quad \hat{p}_x = \sin\theta \cos\phi, \quad \hat{p}_y = \sin\theta \sin\phi$$

Anisotropic
Blackbody

(2π)

Expanding (1) :

① For space : (1) $\ll 1$

$$\psi(\vec{r}, \hat{p}, \hat{z}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{r}} \psi(\vec{k}, \hat{p}, \hat{z})$$

space

Expanding (1) :

① Fourier space : (1) $\ll 1$

$$\hat{\psi}(\vec{x}, \hat{p}, z) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \hat{\psi}(\vec{k}, z)$$

② Harmonic space :

$$\hat{\psi}(\vec{x}, \hat{p}, z) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \hat{\psi}(\vec{k}, z)$$

(1)

Solve for a_{em} From $\Theta(\vec{k}, \hat{p}, \hat{z})$

$$a_{\text{em}}(\vec{x}, \hat{z}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \int d\Omega_z Y_{\text{em}}^*(\hat{p}) \Theta(\vec{k}, \hat{p}, \hat{z})$$

$$a_{\text{em}}(\vec{k}, \hat{z}) Y_{\text{em}}(\hat{p})$$

Solve for $a_{\ell m}$ from $\Theta(\vec{k}, \hat{p}, \hat{z})$

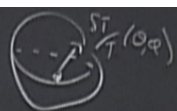
$$a_{\ell m} \left(\int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \right) d\Omega_2 Y_{\ell m}^*(\hat{p}) \Theta(\vec{k}, \hat{p}, \hat{z})$$

③ Moments

$$\mu = \cos(\theta_{np})$$

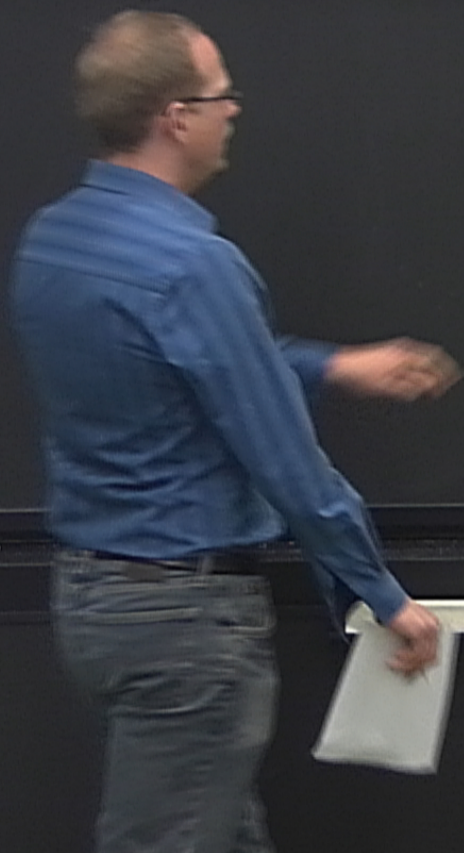
$$(-i)^l (2l+1) P_l^{\vec{\mu}}(\hat{k} \cdot \hat{p})$$

$$a_{\ell m}(\vec{k}, \hat{z}) Y_{\ell m}(\hat{p})$$



$$\psi(\vec{r}, t) = \sum_{\ell} \sum_{m=-\ell}^{\ell} a_{\ell m}(\vec{r}, t) Y_{\ell m}(\hat{r})$$

$$a_{\ell m}(\vec{r}, t) = 4\pi (-i)^{\ell} \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{r}} \psi_{\vec{k}}(\vec{r}, t) Y_{\ell m}(\hat{r})$$



$$= \sum_{\ell} \sum_{m=-\ell}^{\ell} a_{\ell m}(\vec{x}, \vec{z}) Y_{\ell m}(\hat{\vec{r}})$$

Moments

$$e^{i\vec{k} \cdot \vec{x}} \Theta(\vec{k}, \vec{z}) Y_{\ell m}(\hat{\vec{r}})$$

$\hat{\vec{r}}$

$a_{\ell m}(\vec{z}_{\text{now}}, \vec{x}_{\text{here}}) \rightarrow$ Not statistical info

random field (Statistically homogeneous)

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell' \ell} \delta_{m' m} C_{\ell}$$

Angular Power Spectrum

- 1) different realizations about fixed point $(\vec{x}_{\text{here}}, \vec{z}_{\text{now}})$
- 2) fixed realization, move point $(\vec{x}, \vec{z})$

$$\langle \Theta(\vec{k}) \Theta(\vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') \langle \Theta(\vec{k})^2 \rangle$$

$\Rightarrow C_{\ell}$ - all info that there is

Boltzmann eqns

$$F = f(z, \vec{x}, p, \hat{p})$$

$$\frac{dF}{dz} = C\{F\}$$

→ LHS first.

$$\frac{dF}{dz} = \frac{\partial F}{\partial z} + \frac{\partial F}{\partial x^i} \frac{dx^i}{dz} + \frac{\partial F}{\partial p} \frac{dp}{dz} + \frac{\partial F}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dz}$$

Expand to linear order: $\mathcal{O}(c)$

Taylor Series
in $\mathcal{O}_T$

$$F = f^{(0)} - \frac{1}{e^{\beta_{\text{reg}}}-1}$$

Boltzmann eqns

$$\frac{dF}{dz} = C[F]$$

→ LHS first.

$$F = f(z, \vec{x}, p, \hat{p})$$

$$\frac{dF}{dz} = \frac{\partial f}{\partial z} + \frac{\partial f}{\partial x^i} \frac{dx^i}{dz} + \frac{\partial f}{\partial p} \frac{dp}{dz} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dz}$$

Expand to linear order: $\omega \ll 1$

Taylor series
in ω

$$F = f^{(0)} - p \frac{\partial f^{(0)}}{\partial p} \left(\frac{\omega}{e^{\frac{p_{\text{max}}}{T}} - 1} \right)$$

Boltzmann eqns

$$\frac{dF}{dz} = C[F]$$

→ LHS first.

$$F = f(z, \vec{x}, p, \hat{p})$$

$$\frac{dF}{dz} = \frac{\partial f}{\partial z} + \frac{\partial f}{\partial x^i} \frac{dx^i}{dz} + \frac{\partial f}{\partial p} \frac{dp}{dz} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dz}$$

Expand to linear order

Taylor series in $\mathcal{O}(T)$

$$F = f^{(0)}(z, p) - p \frac{\partial f^{(0)}}{\partial p}$$

$\left\{ \begin{array}{l} 1 \\ e^{p/T} - 1 \end{array} \right.$

Boltzmann eqns

$$\frac{dF}{d\tau} = C\{F\}$$

→ LHS first.

$$F = f(\tau, \vec{x}, p, \hat{p})$$

$$\frac{dF}{d\tau} = \overset{(1)}{\frac{\partial f}{\partial \tau}} + \overset{(2)}{\frac{\partial f}{\partial x^i} \frac{dx^i}{d\tau}} + \overset{(3)}{\frac{\partial f}{\partial p} \frac{dp}{d\tau}} + \overset{(4)}{\frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{d\tau}}$$

(4) -

Expand to linear order: (4) (4)

Taylor series in (4)

$$F = f^{(0)}(\tau, \vec{x}, p) - p \frac{\partial f^{(0)}}{\partial p}(\tau, \vec{x}, p) + \frac{1}{e^{p/T} - 1} \frac{\partial f^{(0)}}{\partial p}(\tau, \vec{x}, p)$$

Boltzmann eqns

$$\frac{dF}{d\tau} = C[F]$$

→ LHS first.

$$F = f(\tau, \vec{x}, p, \hat{p})$$

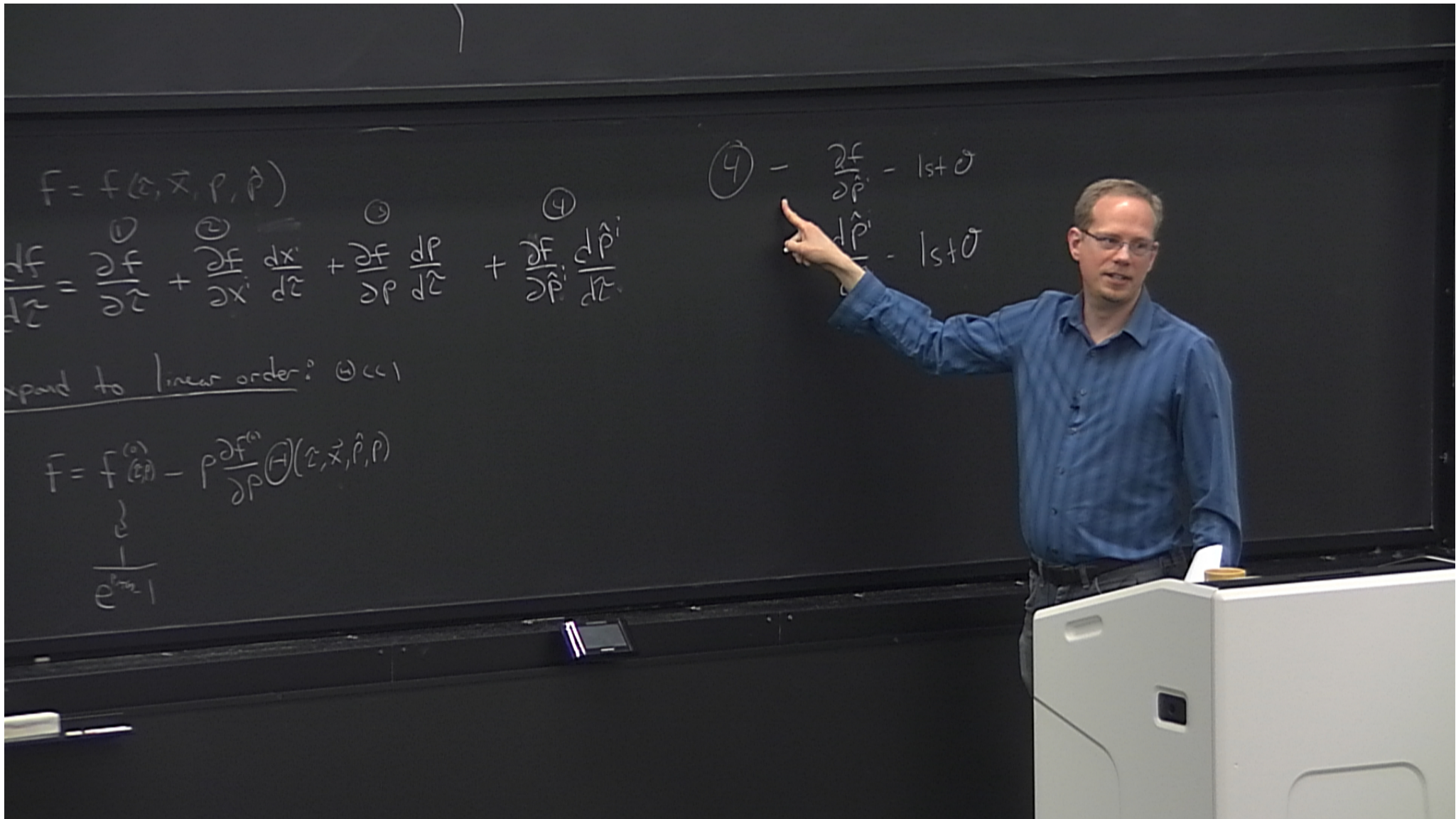
$$\frac{dF}{d\tau} = \overset{(1)}{\frac{\partial F}{\partial \tau}} + \overset{(2)}{\frac{\partial F}{\partial x^i} \frac{dx^i}{d\tau}} + \overset{(3)}{\frac{\partial F}{\partial p} \frac{dp}{d\tau}} + \overset{(4)}{\frac{\partial F}{\partial \hat{p}^i} \frac{d\hat{p}^i}{d\tau}}$$

Expand to linear order: (4) CC1

Taylor series in (4)

$$F = \underbrace{f^{(0)}(\tau, \vec{x}, p)}_{\frac{1}{e^{p/T} - 1}} - p \frac{\partial f^{(0)}}{\partial p}(\tau, \vec{x}, p)$$





$$F = f(\tau, \vec{x}, p, \hat{p})$$

$$\frac{dF}{d\tau} = \overset{(1)}{\frac{\partial f}{\partial \tau}} + \overset{(2)}{\frac{\partial f}{\partial x^i} \frac{dx^i}{d\tau}} + \overset{(3)}{\frac{\partial f}{\partial p} \frac{dp}{d\tau}} + \overset{(4)}{\frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{d\tau}}$$

expand to linear order: $\omega \ll 1$

$$\bar{F} = \underbrace{f^{(0)}(\tau)}_{\frac{1}{e^{\tau/\tau_0} - 1}} - p \frac{\partial f^{(0)}}{\partial p}(\tau, \vec{x}, \hat{p}, p)$$

$$\begin{aligned} (4) - \frac{\partial f}{\partial \hat{p}^i} &= 1st\ 0 \\ \frac{d\hat{p}^i}{d\tau} &= 1st\ 0 \end{aligned}$$

$$F = f(\tau, \vec{x}, p, \hat{p})$$

$$\frac{dF}{d\tau} = \frac{\partial F}{\partial \tau} + \frac{\partial F}{\partial x^i} \frac{dx^i}{d\tau} + \frac{\partial F}{\partial p} \frac{dp}{d\tau} + \frac{\partial F}{\partial \hat{p}^i} \frac{d\hat{p}^i}{d\tau}$$

expand to linear order: $\omega \ll 1$

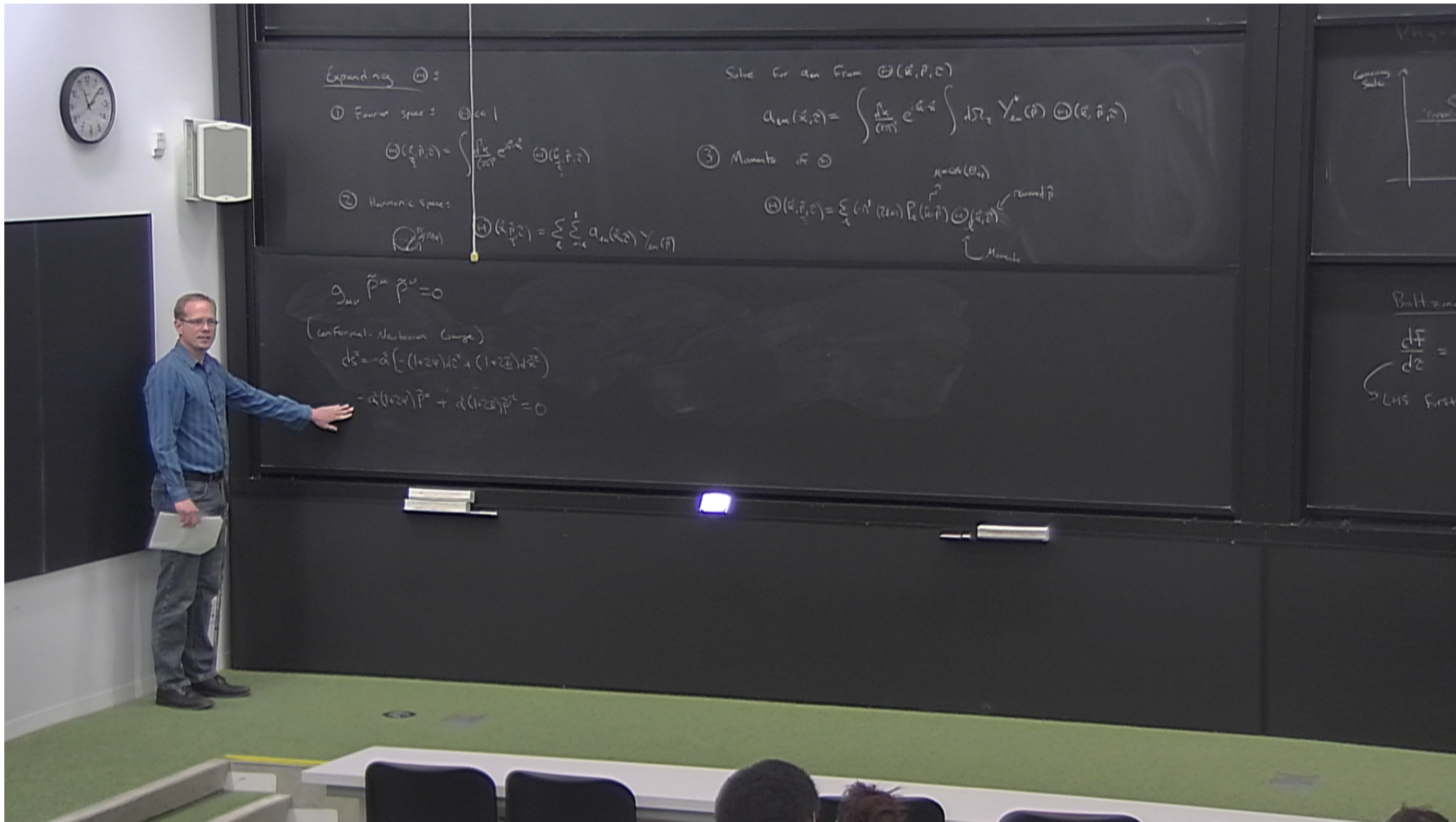
$$\bar{F} = \underbrace{f^{(0)}(\tau, p)}_{\frac{1}{e^{\beta p} - 1}} - p \frac{\partial f^{(0)}}{\partial p}(\tau, \vec{x}, \hat{p}, p)$$

$$\textcircled{4} - \frac{\partial F}{\partial \hat{p}^i} - 1st \text{ order} \quad 2nd \text{ order} \rightarrow \text{discarded}$$

$$\frac{d\hat{p}^i}{d\tau} - 1st \text{ order}$$

$$\textcircled{2} \text{ Need } \frac{dx^i}{d\tau} = \frac{dx^i}{d\lambda} \frac{d\lambda}{d\tau}$$

$\vec{p}^u$



$$(1) (\vec{x}, \hat{p}, \hat{z}) = \sum_l \sum_{m=-l}^l a_{lm}(\vec{x}, \hat{z}) Y_{lm}(\hat{p})$$

↑ Moment

$$\text{define } \vec{p}^2 = \alpha^2 (1 + 2\Phi) \tilde{p}^2$$

$\Rightarrow$

$$d\vec{z}^2 + (1 + 2\Phi) d\vec{x}^2$$

$$\vec{a}(1 + 2\Phi) \tilde{p}^2 = \epsilon$$

$$(1) (\vec{x}, \hat{p}, \hat{z}) = \sum_{\ell} \sum_{m=-\ell}^{\ell} a_{\ell m}(\vec{x}, \hat{z}) Y_{\ell m}(\hat{p})$$

↑ Moments

$$\text{define } \tilde{p}^z = \alpha^2 (1 + 2\Phi) \tilde{p}^{iz}$$

$$\Rightarrow \tilde{p}^0 = \frac{p}{\alpha} (1 - \Psi)$$

$$\tilde{p}^i = \frac{p \hat{p}^i}{\alpha} (1 - \Phi)$$

change)

$$(\vec{x}, \hat{p}, \hat{z}) \rightarrow (\vec{x}, \hat{p}, \hat{z})$$

$$\vec{a} (1 + 2\Phi) \tilde{p}^{iz} = 0$$

$$\tilde{p}^0 = \frac{p}{\alpha (1 + 2\Phi)^{1/2}} \approx p \alpha (1 - \Psi)$$

$$(14) (\vec{x}, \hat{p}, z) = \sum_l \sum_{m=-l}^l a_{lm}(\vec{x}, z) Y_{lm}(\hat{p})$$

↑ Momenta

define $\tilde{p}^z = \alpha^2 (1+2\Phi) \tilde{p}^{iz}$

$$\Rightarrow \tilde{p}^0 = \frac{p}{a} (1-\psi)$$

$$\tilde{p}^i = \frac{p \hat{p}^i}{a} (1-\psi)$$

$$\frac{dx^i}{dz}$$

range)

$$d\vec{x}^2 + (1+2\Phi) d\vec{x}^2$$

$$\vec{a}(1+2\Phi) \tilde{p}^{iz} = 0$$

$$\tilde{p}^0 = \frac{p}{a(1+\psi)^{1/2}} \approx p/a(1-\psi)$$

$\gamma_{em}(\hat{p})$

↑ Moments

$$p^z = \alpha^2 (1 + 2\Phi) \tilde{p}^{i,z}$$

$$\Rightarrow \tilde{p}^0 = \frac{p}{\alpha} (1 - \psi)$$

$$\tilde{p}^i = \frac{p \hat{p}^i}{\alpha} (1 - \Phi)$$

$$\Rightarrow \frac{dx^i}{dz} = \frac{\tilde{p}^i}{\tilde{p}^0} = \frac{\frac{p \hat{p}^i}{\alpha} (1 - \Phi)}{\frac{p}{\alpha} (1 - \psi)} = \hat{p}^i (1 - \Phi + \psi)$$

③ Need $\frac{dP}{d\tau} \rightarrow$ From geodesic eqn

$$\frac{d\tilde{p}^\mu}{d\lambda} = -\Gamma_{\alpha\beta}^\mu \tilde{p}^\alpha \tilde{p}^\beta$$

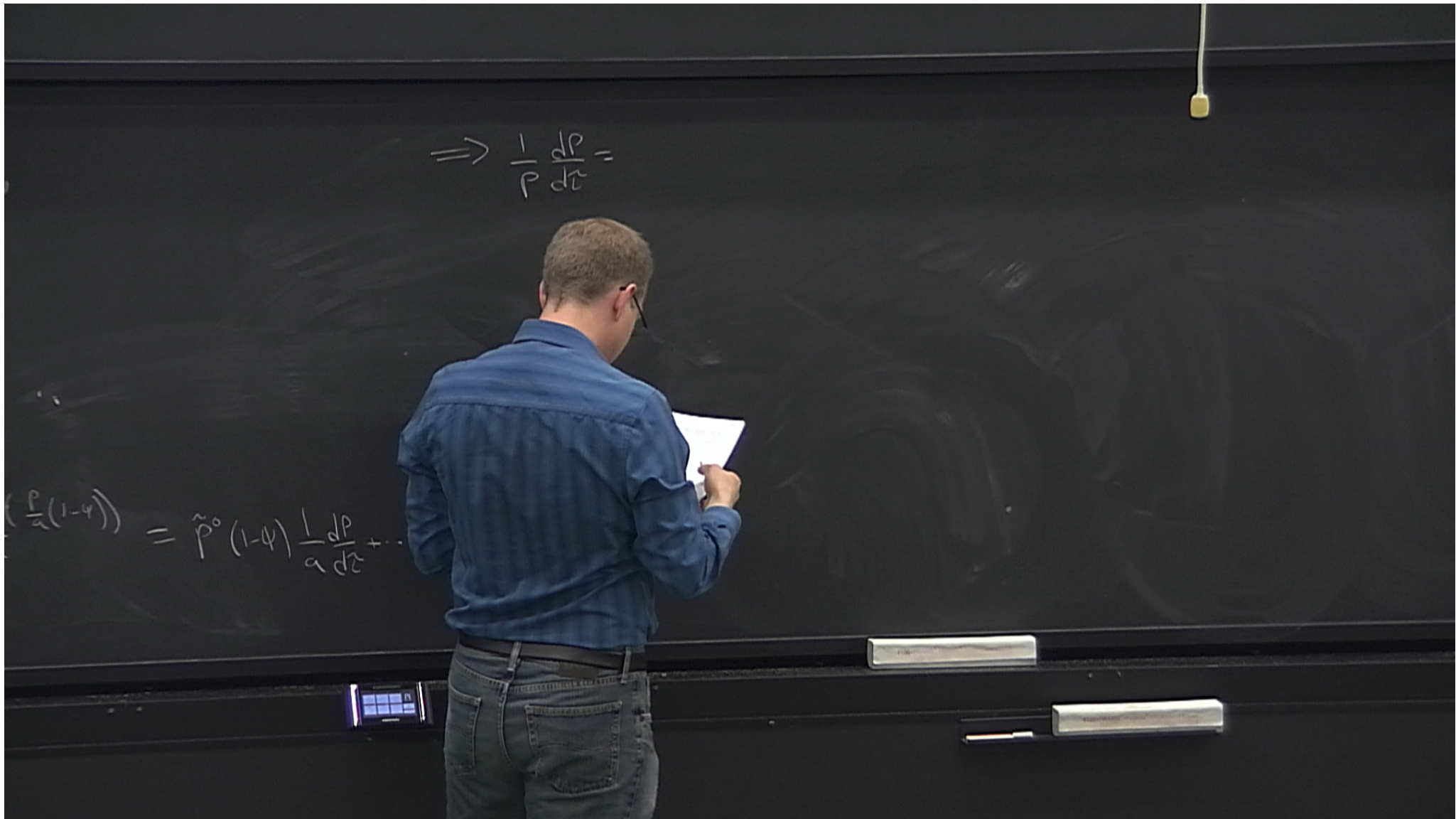
$$\frac{d\tilde{p}^0}{d\lambda} = \frac{d\tau}{d\lambda} \frac{d\tilde{p}^0}{d\tau} = \tilde{p}^0 \frac{d\tilde{p}^0}{d\tau}$$

③ Need $\frac{dP}{d\tau}$ $\rightarrow$ From geodesic eqn

$$\frac{d\tilde{p}^\mu}{d\lambda} = -\Gamma_{\alpha\beta}^\mu \tilde{p}^\alpha \tilde{p}^\beta$$

0th component:

$$\frac{d\tilde{p}^0}{d\lambda} = \frac{d\tau}{d\lambda} \frac{d\tilde{p}^0}{d\tau} = \tilde{p}^0 \frac{d\tilde{p}^0}{d\tau} \rightarrow \tilde{p}^0 \frac{d(\frac{P}{a(1-\psi)})}{d\tau} = \tilde{p}^0 (1-\psi) \frac{1}{a} \frac{dP}{d\tau} + \dots$$



$$\Rightarrow \frac{1}{P} \frac{dP}{dt} = - \underbrace{\frac{\partial \alpha}{\partial t}}_{1)} - \underbrace{\partial_t \Phi}_{3)} - \underbrace{\hat{P}^i \partial_{x^i} \psi}_{2)}$$

What changes momentum in an expanding Universe?

1) expansion

2) Fall into potential wells

3) evolving potential wells

$$\left(\frac{P}{a(1-\psi)} \right) = \tilde{P}^0(1-\psi) \frac{1}{a} \frac{dP}{dt} + \dots$$