

Title: 13/14 PSI - Explorations in Cosmology - Lecture 5

Date: Apr 11, 2014 10:15 AM

URL: <http://pirsa.org/14040045>

Abstract:

$$ds^2 = \alpha^2(z) \left[- (1 + A) dz^2 + 2B_i dx^i dz + (s_{ij} + \gamma_{ij}) dx^i dx^j \right]$$

$$B_i = \partial$$

$$ds^2 = a^2(\tilde{t}) \left[-(1+2A)d\tilde{t}^2 + 2B_i dx^i d\tilde{t} + (S_{ij} + 2C\delta_{ij}) dx^i dx^j \right]$$

$$B_i = \underbrace{\partial_i B}_{\text{1 scalar}} + \underbrace{\epsilon^{jk} \partial_j b_k}_{\text{2 vector}}$$

$$S_{ij} = 2C\delta_{ij}$$

$$2A) d\tau^2 + 2B_i dx^i d\tau + (S_{ij} + \gamma_{ij}) dx^i dx^j$$

$$B_i = \partial_i B + \sum^{iv} \partial_j b_k$$

$\uparrow$
 1 scalar 2 vector

$$\gamma_{ij} = 2C S_{ij} + E$$

$\uparrow$
 trace scalar

$$h_{ij}^{TT} + \partial_i$$

$$2A) d\tau^2 = 2B_i dx^i d\tau + (\delta_{ij} + \gamma_{ij}) dx^i dx^j$$

$$B_i = \underbrace{\delta}_{\text{Scalar}} + \underbrace{\sum^{iv} \partial_j b_k}_{\text{vector}}$$

$$\gamma_{ij} = 2C \underbrace{\delta_{ij}}_{\substack{\uparrow \\ \text{trace} \\ \text{Scalar}}} + \underbrace{E_{ij}}_{\substack{\uparrow \\ \text{vector}}}$$

$$\partial_i \partial_j E + h_{ij}^{TT} + \underbrace{\partial_i V_j + \partial_j V_i}_{\text{vector}}$$

$$2A) d\tau^2 + 2B_i dx^i d\tau + (S_{ij} + \gamma_{ij}) dx^i dx^j$$

$B_i = \partial_i B$ $\partial_j b_k$
 (Scalar) (vector)

$\gamma_{ij} = 2C \delta_{ij} + E_{ij}$
 (trace scalar)

$\partial_i \partial_j E + h_{ij}^{TT} + \partial_i V_j + \partial_j V_i$
 (tensor) (vector)

Conformal Newtonian Gauge

$$ds^2 = a^2(\tau) \left[-(1+2\psi) d\tau^2 + (1+2\Phi) d\vec{x}^2 \right]$$

Conformal Newtonian Gauge

$$ds^2 = -(1+2\psi)d\hat{t}^2 + (1+2\Phi)d\vec{x}^2$$

Energy-momentum tensor $\bar{P} = \omega \bar{\rho}$

$$\bar{P} = \begin{pmatrix} -1-\delta & (1+\omega)V^i \\ (1+\omega)V_i & \end{pmatrix}$$

Conformal Newtonian Gauge

$$ds^2 = a^2(\hat{t}) \left[-(1+2\psi) d\hat{t}^2 + (1+2\Phi) d\vec{\hat{x}}^2 \right]$$

Energy-Momentum tensor $\bar{P} = w\bar{\rho}$, $\rho = \bar{\rho}(1+S)$ density contrast

$$T^u_v = \bar{\rho} \begin{pmatrix} -1-S & (1+w)V^i \\ -(1+w)V^i & wS\delta^i_j \end{pmatrix}$$

peculiar velocities

Conformal Newtonian Gauge

$$ds^2 = a^2(\hat{t}) \left[-(1+2\psi) d\hat{t}^2 + (1+2\Phi) d\vec{\hat{x}}^2 \right]$$

Energy-Momentum tensor $\bar{P} = w\bar{\rho}$, $\rho = \bar{\rho}(1+S)$

$$T^u_v = \bar{\rho} \begin{pmatrix} -1-S & (1+w)V^i \\ -(1+w)V^i & wS\delta^i_j \end{pmatrix}$$

density contrast

peculiar velocities

Plug into Einstein's Eqns - linearize.

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$$G_{\mu\nu}^{(0)} + \delta G_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{(0)} + 8\pi G_N \delta T_{\mu\nu}$$

$\uparrow$
ISO



Plug into Einstein's Eqns - linearize.

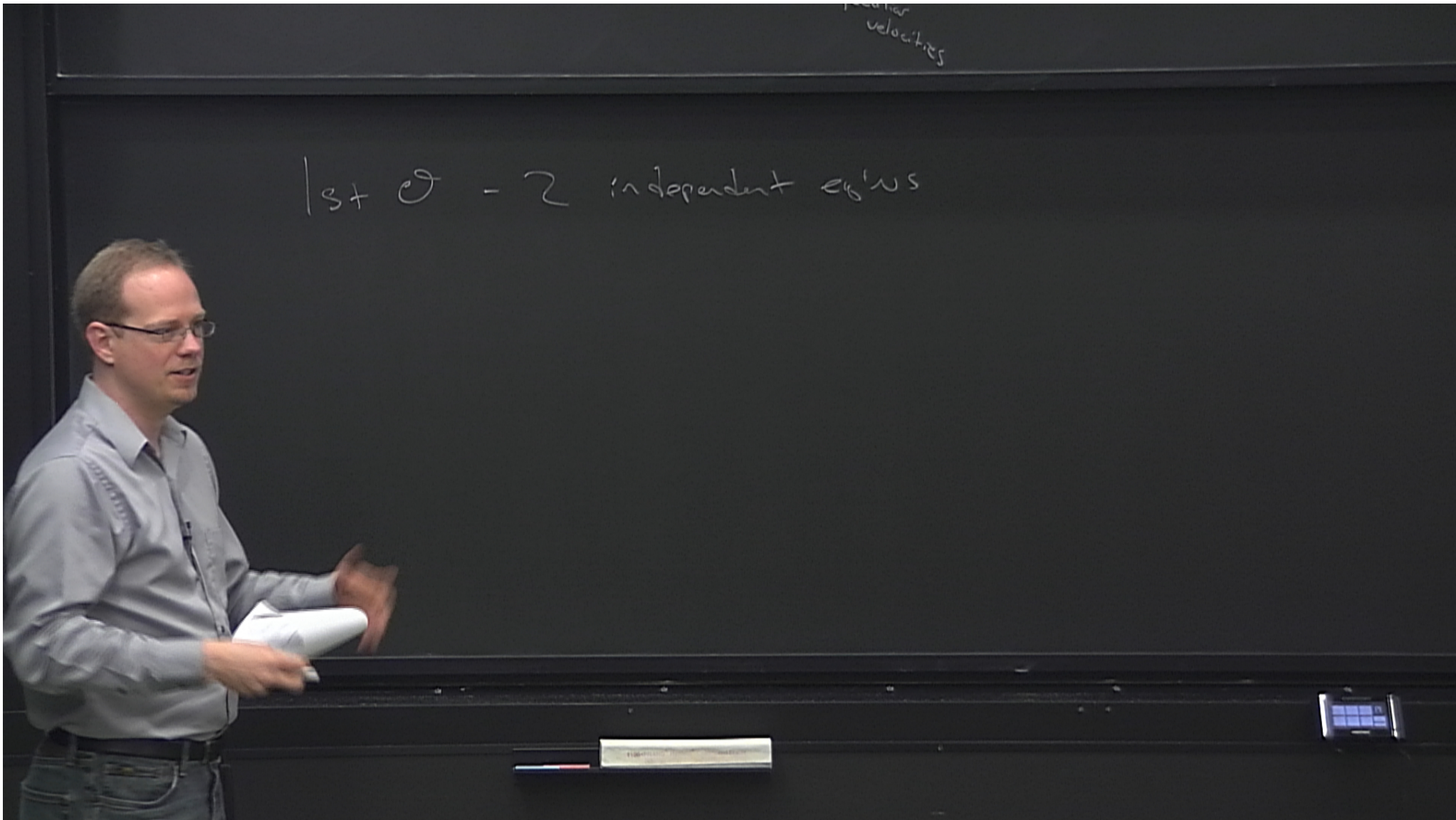
$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$$G_{\mu\nu}^{(0)} + \delta G_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{(0)} + 8\pi G_N \delta T_{\mu\nu}$$

$\uparrow$
ISO



Zeroth $\partial \rightarrow$ Friedmann eqns



medium
velocities

1st $\theta - z$ independent eq'ns

$$\vec{\nabla} \Phi + 3 \frac{\partial \alpha}{\partial z} \left[\frac{\partial \alpha}{\partial a} \psi - \partial_z \Phi \right] = -4\pi G_N a^2 \bar{\rho} \delta$$

$\partial_i \partial_i$

position
velocities

ψ, Φ, z independent eqns

$$\vec{\nabla} \Phi + 3 \frac{\partial \alpha}{\partial z} \left[\frac{\partial \alpha}{\partial z} \psi - \partial_z \Phi \right] = -4\pi G_N \alpha^2 \bar{\rho} \delta$$

$$\partial_i \partial_j (\psi + \Phi) = 0$$

linear
velocities

1st ψ - χ independent eq'ns

$$\vec{\nabla} \Phi + 3 \frac{\partial \alpha}{\partial a} \left[\frac{\partial \alpha}{\partial a} \psi - \partial_c \Phi \right] = -4\pi G_N a^2 \bar{\rho} \delta$$

$$\partial_i \partial_j (\psi + \Phi) = 0$$

just for perfect fluids

For non-zero - $ST^i_j \neq 0$ $i \neq j$

Anisotropic Stress

peculiar
velocities

$\theta - z$ independent eq'ns

$$\vec{\nabla} \Phi + 3 \frac{\partial a}{\partial z} \left[\frac{\partial a}{\partial z} \psi - \partial_z \Phi \right] = -4\pi G a^2 \bar{\rho} \delta$$

$$\partial_i \partial_j (\psi + \Phi) = 0$$

just for perfect fluids

For non-zero - $ST^i_j \neq 0 \ i \neq j$

Anisotropic Stress

- Universe not perfect:

γe^-
↳

$\theta - \tau$ independent eq'ns

$$\vec{\nabla} \bar{\Phi} + 3 \frac{\partial a}{\partial \tau} \left[\frac{\partial a}{\partial \tau} \psi - \partial_{\tau} \bar{\Phi} \right] = -4\pi G_N a^2 \bar{\rho} \delta$$

$$\partial_i \partial_j (\psi + \bar{\Phi}) = 0$$

just for perfect fluids

For non-zero - $ST^i_j \neq 0 \ i \neq j$

Anisotropic Stress

- Universe not perfect:

$\gamma e^- \rightarrow$ } cause anisotropic stress

2 perfect fluids in our Universe -
dark matter
baryons (after decoupling)

$$G_{\mu\nu} = \bar{\rho} g_{\mu\nu}$$

- Universe not perfect:

γe^- } cause anisotropic
 ∇ } stress

$\rho_{ij} \neq 0$
pic Stress

Conservation of T^{μ}_{ν}

2 perfect fluids in our Universe -
dark Matter
baryons (after decoupling)

$\sim \bar{\rho} \delta$

- Universe not perfect:

$\gamma e^- \}$ cause anisotropic
 $\vee$ stress

$i \neq j$

c Stress

Conservation of T^μ_ν

$$\nabla_\mu T^\mu_\nu = 0$$

Give EDM for $S + V^i$

Conservation of T^μ_ν

$$\nabla_\mu T^\mu_\nu = 0$$

Give EOM for $\delta + V^i$

$$\Rightarrow \partial_t \delta + \vec{\nabla} \cdot \vec{V} = -3 \partial_t \Phi$$

$$\partial_t V^i + \frac{\partial a}{a} V^i = -\vec{\nabla} \psi$$

Conservation of T^μ_ν

$$\nabla_\mu T^\mu_\nu = 0$$

Give EOM for $\delta + V^i$

$$\Rightarrow \partial_t \delta + \vec{\nabla} \cdot \vec{V} = -3 \partial_t \Phi$$

$$\Rightarrow \partial_t V^i + \frac{\partial a}{a} V^i = -\vec{\nabla} \psi$$

Conservation of T^μ_ν

$$\nabla_\mu T^\mu_\nu = 0$$

Give EOM for $\delta + V^i$

$$\Rightarrow \partial_t \delta + \vec{\nabla} \cdot \vec{V} = -3 \partial_t \Phi$$

$$\Rightarrow \partial_t V^i + \frac{\partial a}{a} V^i = -\vec{\nabla} \psi$$

1) Go to Fourier Space:

Conservation of T^μ_ν

$$\nabla_\mu T^\mu_\nu = 0$$

Give EOM for $\phi + V^i$

$$\Rightarrow \partial_\mu \phi + \vec{\nabla} \cdot \vec{V} = -3 \partial_\mu \Phi$$

$$\Rightarrow \partial_\mu V^i + \frac{\partial \phi}{\partial V^i} V^i = -\vec{\nabla} \psi$$

1) Go to Fourier Space:
 $\vec{\nabla} \rightarrow i\mathbf{k}$

Conservation of T^μ_ν

$$\nabla_\mu T^\mu_\nu = 0$$

Give EOM for $\phi + V^i$

$$\Rightarrow \partial_\mu \phi + \vec{\nabla} \cdot \vec{V} = -3 \partial_\mu \Phi$$

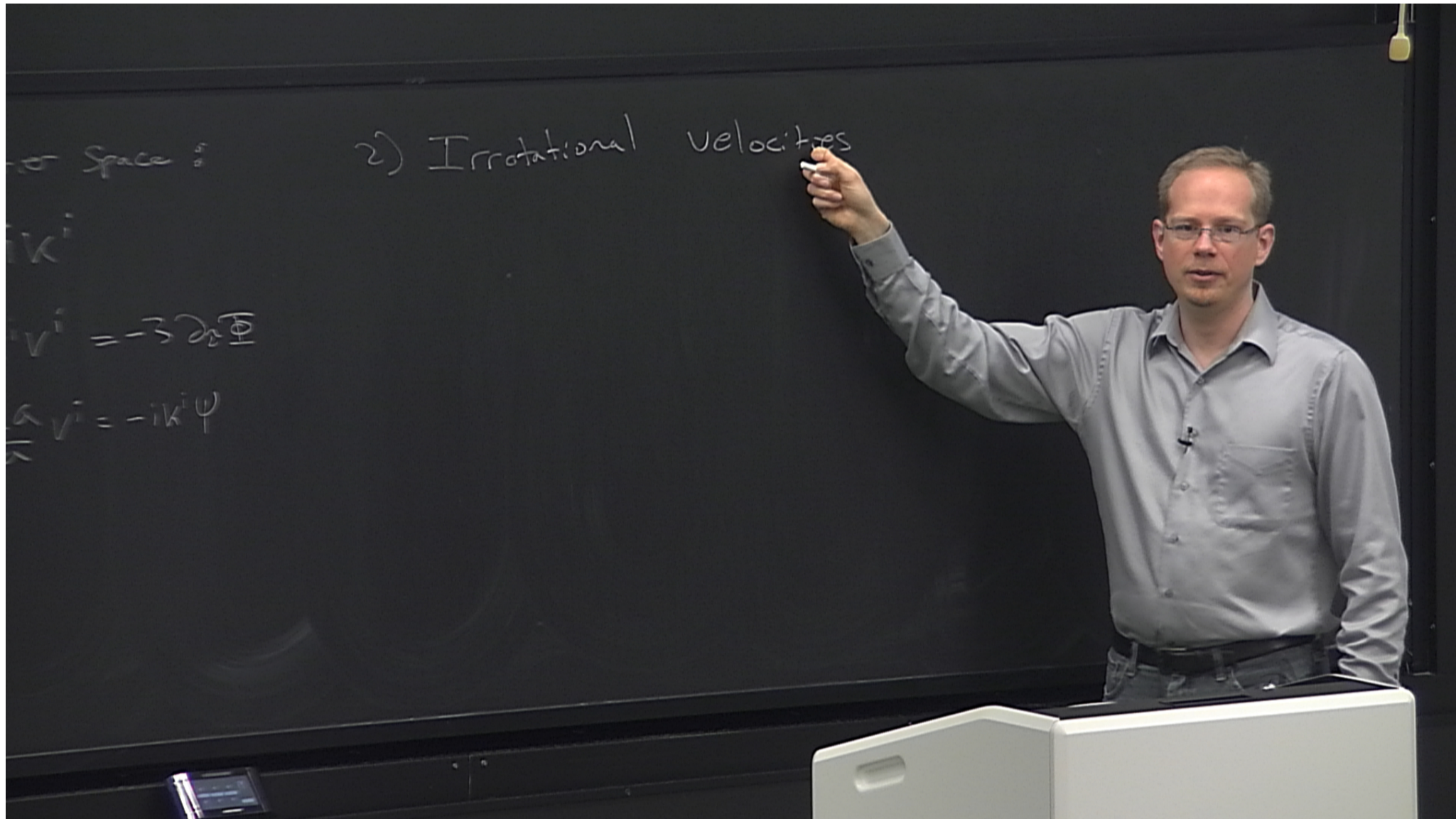
$$\Rightarrow \partial_\mu V^i + \frac{\partial a}{\partial \phi} V^i = -\vec{\nabla} \psi$$

1) Go to Fourier Space:

$$\vec{\nabla} \rightarrow i\mathbf{k}$$

$$\partial_\mu \phi + i\mathbf{k}^i V^i = -3 \partial_\mu \Phi$$

$$\partial_\mu V^i + \frac{\partial a}{\partial \phi} V^i = -i\mathbf{k}^i \psi$$



Space :

$$k^i$$

$$v^i = -3 \partial_i \Phi$$

$$\nabla_i v^i = -i k^i \psi$$

2) Irrotational velocities

$$v^i = \frac{k^i}{k} v \iff \vec{\nabla} \times \vec{v} = 0$$

Space :

k^i

$$v^i = -\partial^i \Phi$$

$$\nabla v^i = -ik^i \psi$$

2) Irrotational velocities

$$v^i = \frac{k^i}{k} v \iff \vec{\nabla} \times \vec{v} = 0$$

Vorticity

Space :

2) Irrotational velocities

$$V^i = \frac{k^i}{k} V \iff \vec{\nabla} \times \vec{V} = 0$$

↑ Vorticity

k^i

$$V^i = -\frac{1}{k} \partial_i \Phi$$

→

$$\partial_t S + \frac{i k^i k^i}{k} V = -\frac{1}{k} \partial_t \Phi$$

$$\frac{1}{k} V^i = -\frac{i k^i}{k} \psi$$

$$\partial_t \left(\frac{k^i}{k} V \right) + \frac{\partial_t a}{a} \frac{k^i}{k} V = -\frac{i k^i}{k} \psi$$

Space :

k^i

$$v^i = -\partial^i \Phi$$

$$v^i = -ik^i \psi$$

2) Irrotational velocities

$$v^i = \frac{k^i}{k} v \iff \vec{\nabla} \times \vec{v} = 0$$

↑ Vorticity

$$\rightarrow \partial_\alpha \delta + \frac{k^i k^j}{k} v = -\partial_\alpha \Phi$$

$$\partial_\alpha \left(\frac{k^i}{k} v \right) + \frac{\partial_\alpha k^i}{k} v = -ik^i \psi$$

$$3) \psi = -\Phi$$

$$\partial_z S + ikV = -3\partial_z \Phi$$

$$\partial_z V + \partial_z \frac{a}{\alpha} V = ik\Phi$$

$$\left. \begin{aligned} \partial_z \delta + ikV &= -3\partial_z \Phi \\ \partial_z V + \partial_z \frac{a}{a} V &= ik\Phi \end{aligned} \right\} \begin{array}{l} \text{dark matter +} \\ \text{baryons (post dec)} \end{array}$$

$$\begin{aligned}
 \partial_z \delta + i k V &= -3 \partial_z \Phi \\
 \partial_z V + \partial_z \frac{a}{a} V &= i k \Phi
 \end{aligned}
 \left. \vphantom{\begin{aligned} \partial_z \delta + i k V &= -3 \partial_z \Phi \\ \partial_z V + \partial_z \frac{a}{a} V &= i k \Phi \end{aligned}} \right\} \begin{array}{l} \text{dark matter +} \\ \text{baryons (post dec)} \end{array}$$

$$-k^2 \Phi + 3 \partial_z \frac{a}{a} \left[-\frac{\partial_z a}{a} \Phi - \partial_z \Phi \right] = -4\pi G_N a^2 \bar{\rho} \delta$$

Newtonian Approximation:

$$\Phi_N = -\Phi$$

Neglect $\partial_c \Phi$ and $\frac{\partial^2 \Phi}{\partial c^2}$

$$\rightarrow k^2 \Phi_N = -4\pi G_N a^2 \bar{\rho} \delta$$

Newtonian Approximation:

$$\Phi_N = -\Phi$$

Neglect $\partial_t \Phi$ and $\frac{\partial a}{a} \Phi$ for $k \gg aH$

$$\rightarrow k^2 \Phi_N = -4\pi G_N a^2 \bar{\rho} \delta$$

Newtonian Approximation:

$$\overline{\Phi}_N = -\overline{\Phi}$$

Neglect $\partial_t \overline{\Phi}$ and $\frac{\partial a}{a} \overline{\Phi}$ for $k \gg aH$

$$\rightarrow k^2 \overline{\Phi}_N = -4\pi G_N a^2 \bar{\rho} \delta$$

$\delta \propto a$ in Matter dom.

$$\overline{\Phi}_N \sim \text{const.}$$

Super-horizon Evolution: $k \rightarrow 0$

Super-horizon Evolution: $K \rightarrow 0$

$$\partial_\tau S = -3\partial_\tau \bar{\Phi} \rightarrow S = -3\bar{\Phi} + C$$



Super-horizon Evolution: $k \rightarrow 0$

$$\partial_\tau S = -3 \partial_\tau \bar{\Phi} \rightarrow S = -3\bar{\Phi} + C$$

$$\partial_\tau V = -\frac{2\alpha}{a} V$$

Super-horizon Evolution: $k \rightarrow 0$

$$\partial_\tau S = -3 \partial_\tau \bar{\Phi} \rightarrow S = -3\bar{\Phi} + C$$

$$\partial_\tau V = -\frac{2\dot{a}}{a} V \rightarrow V \text{ decays}$$

Super-horizon Evolution: $k \rightarrow 0$

$$\partial_\tau S = -3 \partial_\tau \Phi \rightarrow S = -3\Phi + C$$

$$\partial_\tau V = -\frac{\partial_\tau a}{a} V \rightarrow V \text{ decays}$$

$$3 \frac{\partial_\tau a}{a} \left[-\frac{\partial_\tau a}{a} \Phi - \partial_\tau^2 \Phi \right] = -4\pi G_N \bar{\rho} S$$

Super-horizon Evolution: $k \rightarrow 0$

$$\partial_\tau S = -3 \partial_\tau \bar{\Phi} \rightarrow S = -3\bar{\Phi} + C$$

$$\partial_\tau V = -\frac{\partial_\tau a}{a} V \rightarrow V \text{ decays}$$

$$3 \frac{\partial_\tau a}{a} \left[-\frac{\partial_\tau a}{a} \bar{\Phi} - \partial_\tau \bar{\Phi} \right] = -4\pi G_N \bar{\rho} S$$

$$\hookrightarrow \bar{\Phi} = \text{const.} + \text{decaying}$$

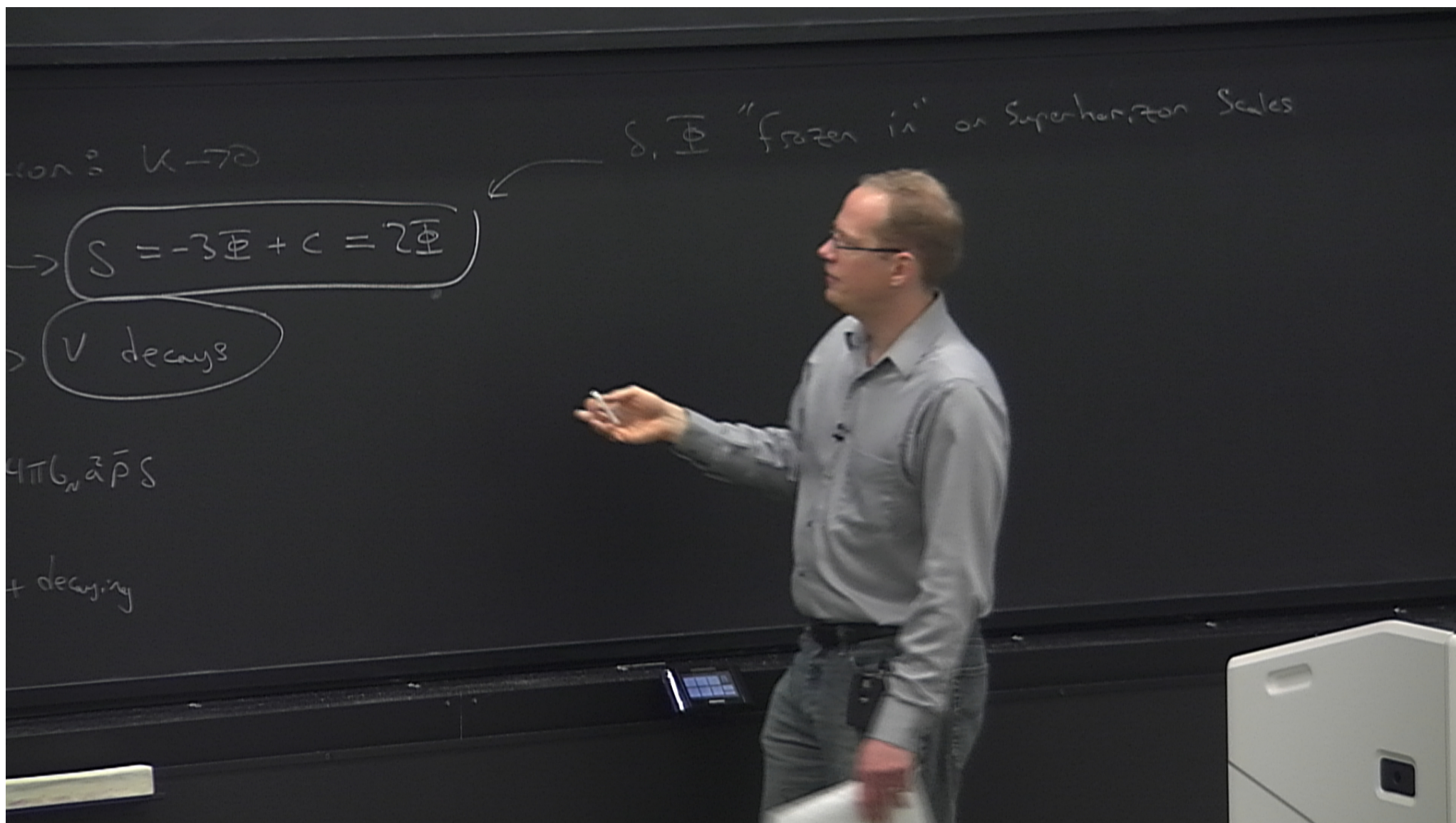
Super-horizon Evolution: $k \rightarrow 0$

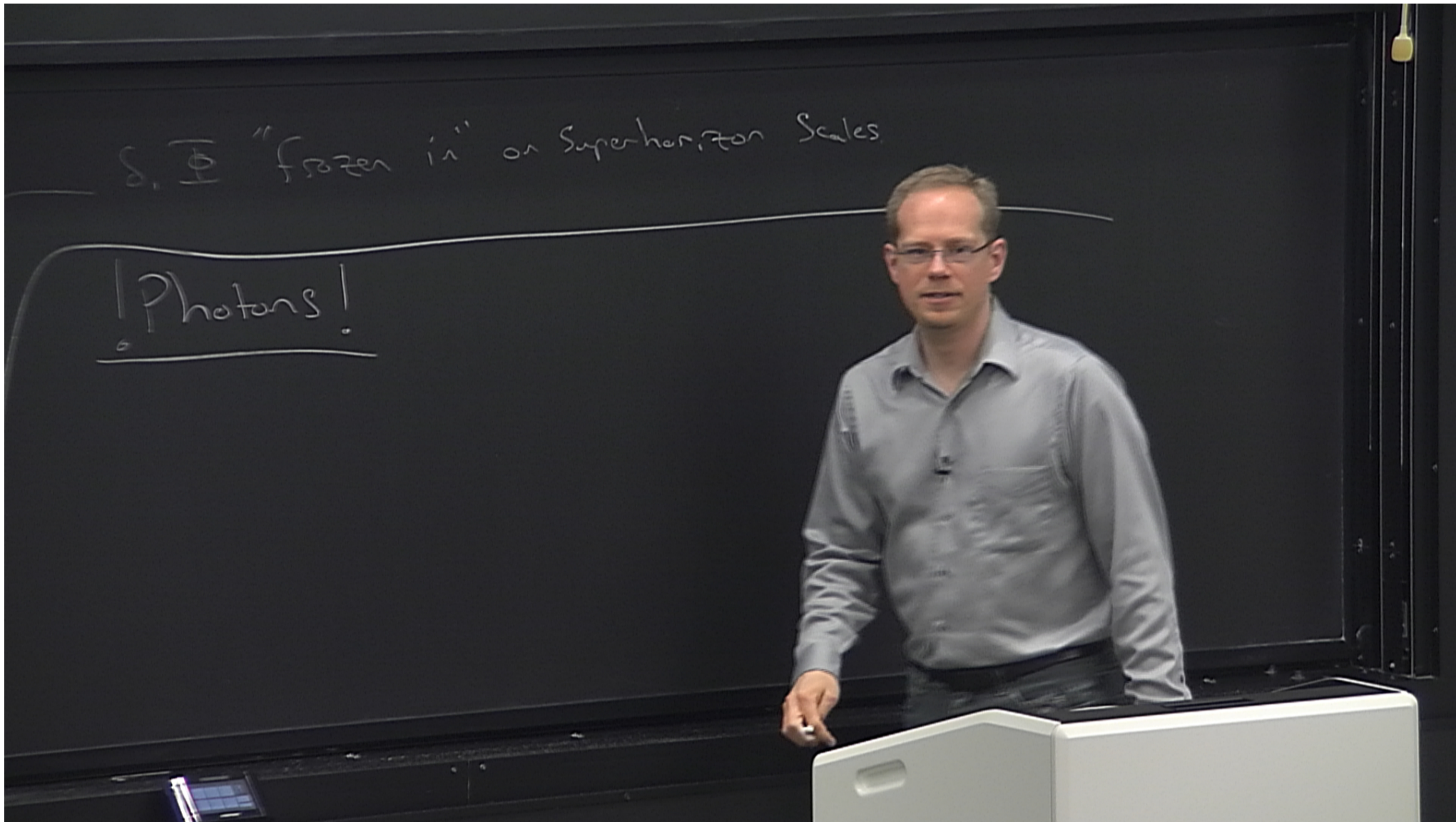
$$\partial_\tau S = -3 \partial_\tau \bar{\Phi} \rightarrow S = -3\bar{\Phi} + C = 2\bar{\Phi}$$

$$\partial_\tau V = -\frac{\partial_\tau a}{a} V \rightarrow V \text{ decays}$$

$$3 \frac{\partial_\tau a}{a} \left[-\frac{\partial_\tau a}{a} \bar{\Phi} - \partial_\tau \bar{\Phi} \right] = -4\pi G_N \bar{\rho} S$$

$$\hookrightarrow \bar{\Phi} = \text{const.} + \text{decaying}$$





δ, Φ "frozen in" on Super-horizon Scales

! Photons ! $\rightarrow$ Radiation dominated early on
 $\hookrightarrow$ CMB

δ, Φ "frozen in" on Super-horizon Scales

Photons! $\rightarrow$ Radiation dominated early on
 $\rightarrow$ CMB

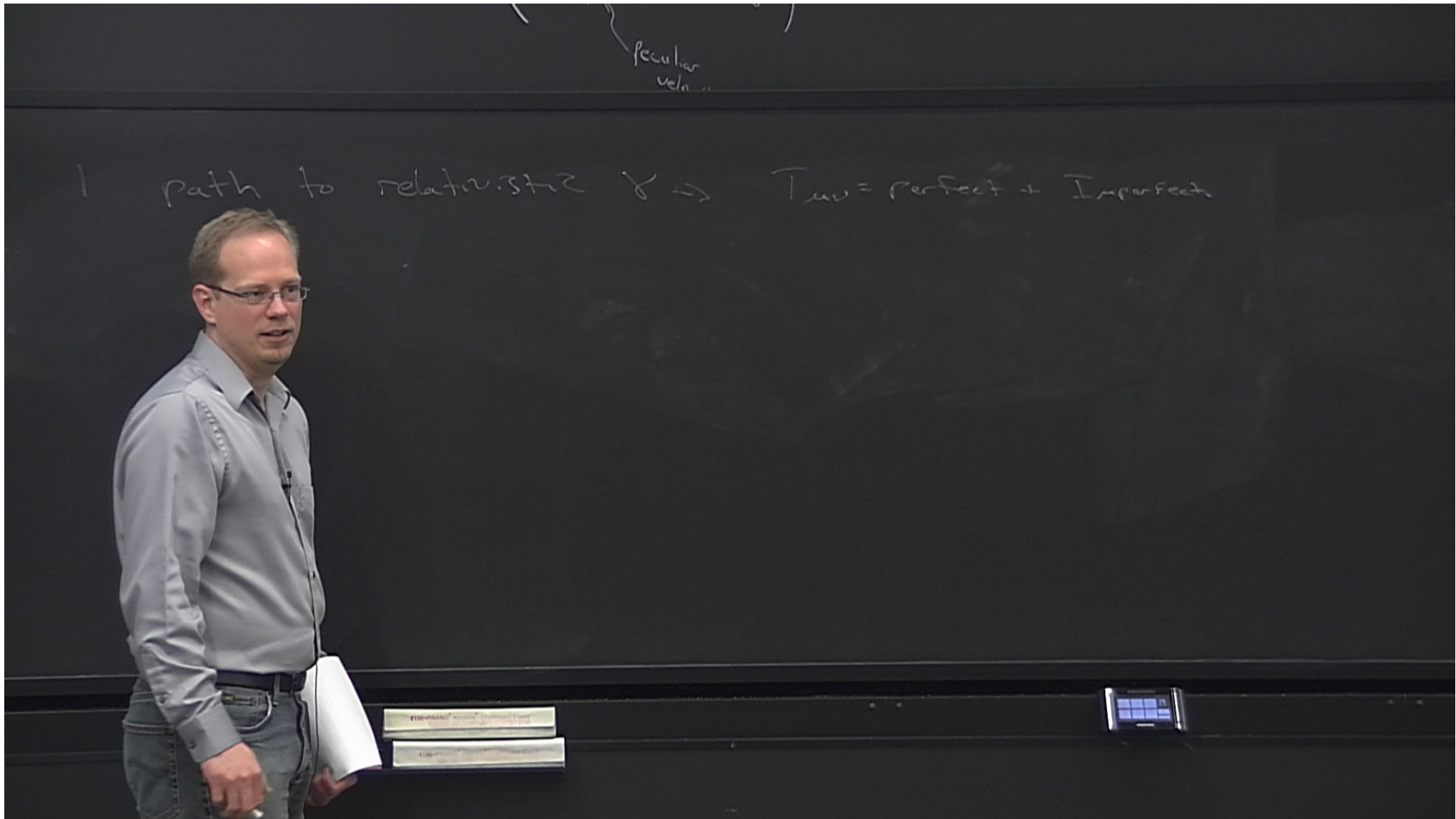
$\downarrow$
Newtonian $\rightarrow \delta \sim$ damped oscillations on scales $k \gg aH$
 $\lambda_J \sim H^{-1}$

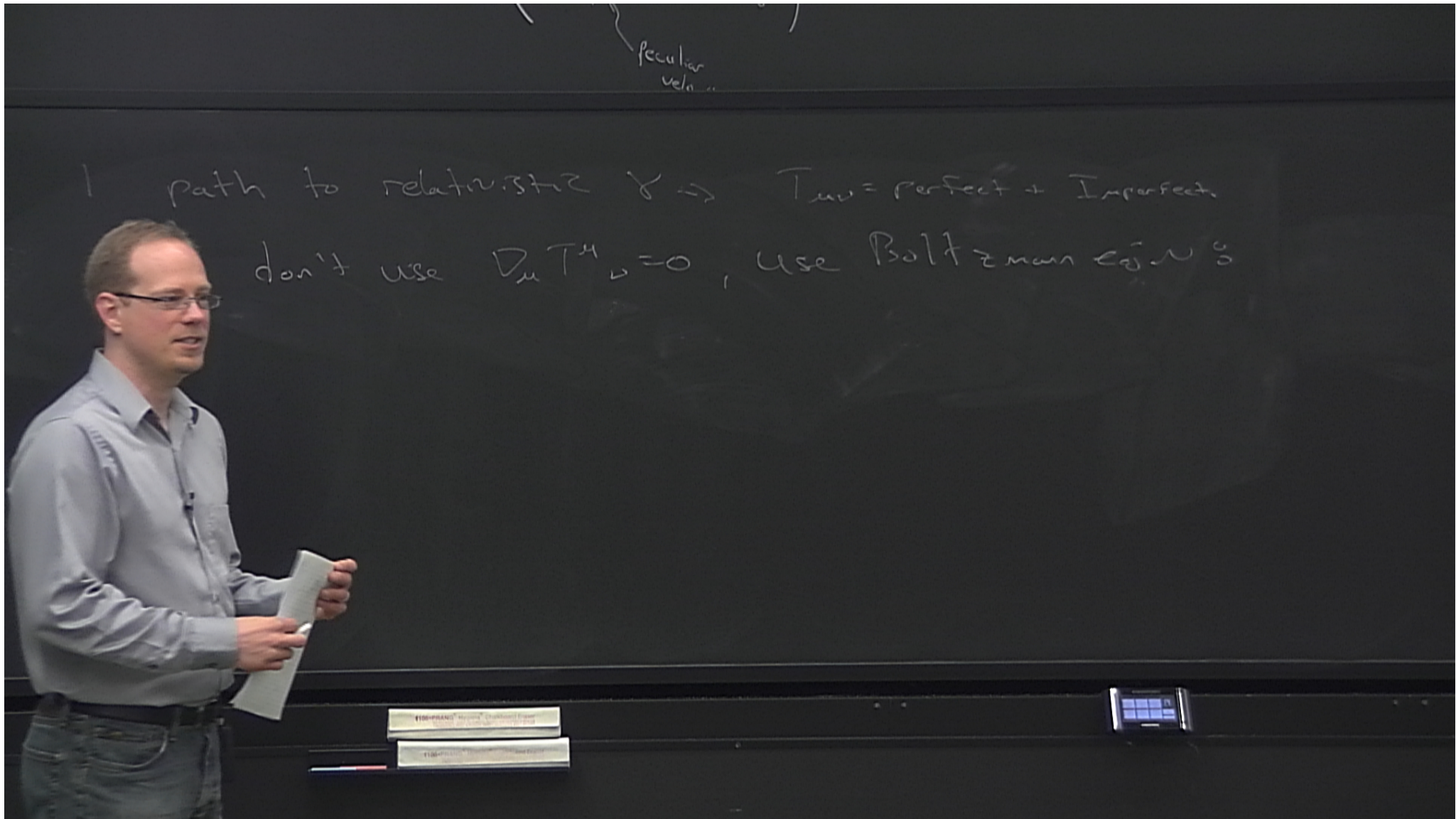
δ, Φ "frozen in" on Super-horizon Scales

! Photons ! $\rightarrow$ Radiation dominated early on
 $\hookrightarrow$ CMB

$\downarrow$
Newtonian $\rightarrow \delta \sim$ damped oscillations on scales $k \gg aH$
 $\lambda_J \sim H^{-1}$

$\Rightarrow$ Relativistic treatment!

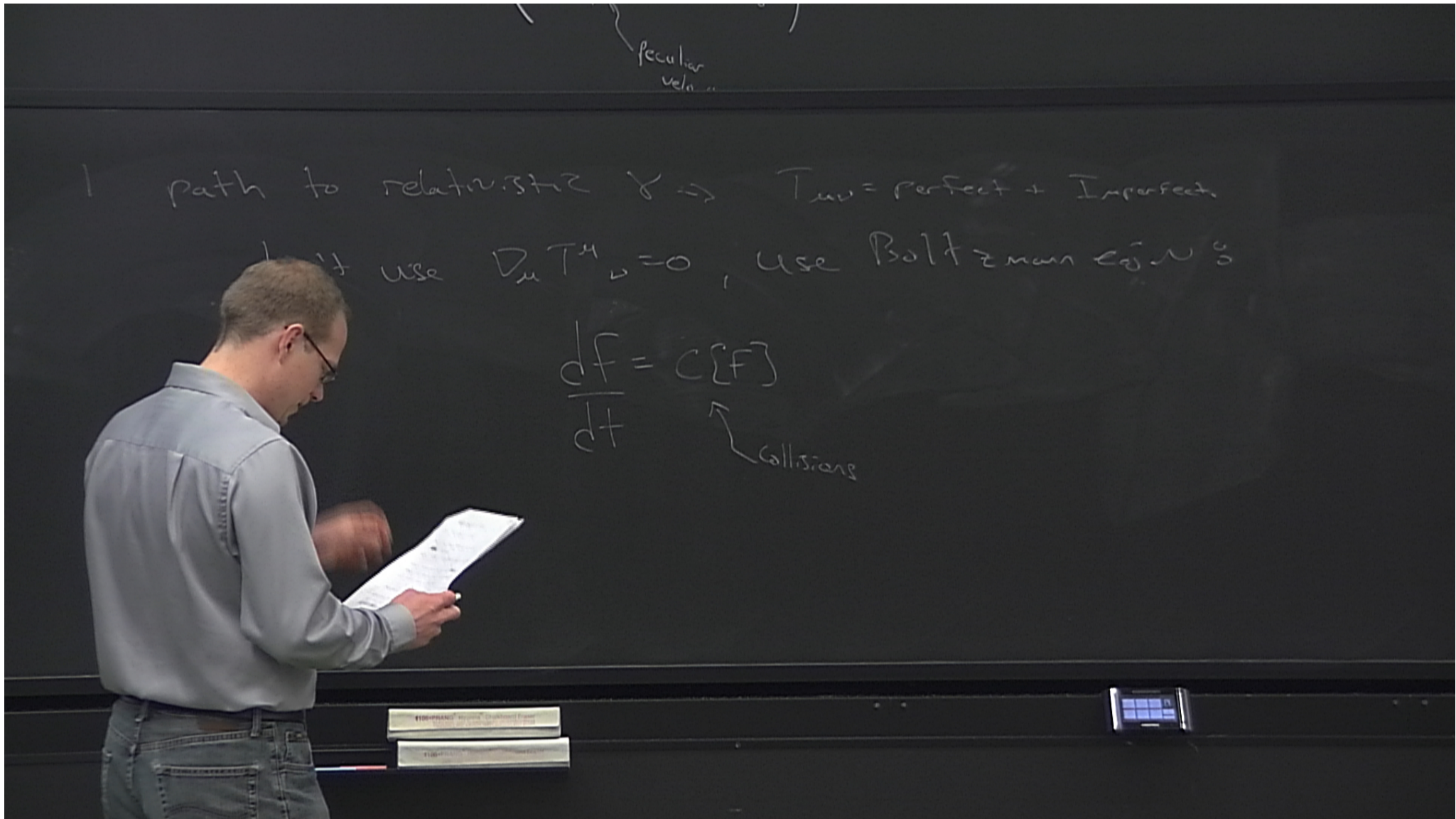




peculiar
velocity

1 path to relativistic $\gamma \rightarrow T_{\mu\nu} = \text{perfect} + \text{Imperfect}$

don't use $\nabla_\mu T^{\mu\nu} = 0$, use Boltzmann eqns



peculiar
vel.

1 path to relativistic $\gamma \rightarrow T_{\mu\nu} = \text{perfect} + \text{Imperfect}$ | Distr.

don't use $\nabla_\mu T^\mu_\nu = 0$, use Boltzmann eqn

$$\frac{df}{dt} = C[F]$$

↑
Collisions

ect + Imperfect.

$z_{\text{max}} \approx N$

Distribution Function :

$$dN = f(\vec{x}, \vec{p}, t) \frac{d^3x d^3p}{(2\pi)^3}$$

↑
Momentum,
not pressure

ect + Imperfect.

$z_{\text{max}} \propto \sqrt{N}$

Distribution Function:

$$dN = f(\vec{x}, \vec{p}, t) \frac{d^3x d^3p}{(2\pi)^3}$$

↑
Momentum,
not pressure

Bose-Einstein's

$$f = \frac{1}{e^{(E_{\text{kin}})/T} - 1}$$

Fermi-Dirac - $f = \frac{1}{e^{(E_{\text{kin}})/T} + 1}$

Perfect + Imperfect
 $z_{\text{max}} \propto N^0$

Distribution Function

$$d f(\vec{x}, \vec{p}, t) \frac{d^3x d^3p}{(2\pi)^3}$$

↑
Momentum,
not pressure

$$f = \frac{1}{e^{(\epsilon_{\text{min}})/T} - 1}$$

Bosons

$$\text{Fermi-Dirac} - f = \frac{1}{e^{(\epsilon_{\text{min}})/T} + 1}$$

Fermions

ect + Imperfect.
Z main eqn is

Distribution Function is

$$d f(\vec{x}, \vec{p}, t) \frac{d^3x d^3p}{(2\pi)^3}$$

↑
Momentum,
not pressure

$$f = \frac{1}{e^{(\epsilon - \mu)/T} - 1}$$

Bosons

Fermi-Dirac - $f = \frac{1}{e^{(\epsilon - \mu)/T} + 1}$

Fermions

Homogeneous: for χ , $E = |\vec{P}|$

$$P = g \int F(\vec{P}) \cdot \vec{P} \frac{d^3P}{(2\pi)^3}$$

↑
of
polarization
states

Homogeneous: for γ , $E = |\vec{P}|$

$$P = g \int F(\vec{P}) \vec{P} \frac{d^3P}{(2\pi)^3}$$

↑
of
polarization
states

$$= \frac{\pi^2}{15} T^4$$

Inhomogeneous case:

$$1) \mu = 0$$

Inhomogeneous case:

1) $\mu = 0 \rightarrow$ No energy injection

Inhomogeneous case:

1) $\mu = 0 \rightarrow$ No energy injection
Spectral distortions

Inhomogeneous case:

1) $\mu = 0 \rightarrow$ No energy injection
Spectral distortions

2)

$$f = \frac{1}{\exp\left[\frac{P}{T(1+\Theta(t, \vec{x}, \vec{p}))}\right] - 1}$$

Inhomogeneous Case:

1) $\mu = 0 \rightarrow$ No energy injection
Spectral distortions

2)

$$f = \frac{1}{\exp\left[\frac{P}{T}(1 + \Theta(t, \vec{x}, \vec{p}))\right] - 1}$$

$$|\Theta| \ll 1$$

2 perfect fluids in our Universe -
dark matter
baryons (after decoupling)

What are we after?

What are we after?

$$dN_{\text{here+now}} = \left[\int F(\vec{x}, \vec{p}, t_{\text{now}}) \delta^{(3)}(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

What are we after?

$$\rho(\vec{x}, \vec{p}, t_{\text{now}}) = \left[\int F(\vec{x}, \vec{p}, t_{\text{now}}) \delta^{(3)}(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

$$= f(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{d^3p}{(2\pi)^3}$$

What are we after?

$$dN_{\text{here+now}} = \left[\int F(\vec{x}, \vec{p}, t_{\text{now}}) \delta^{(3)}(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right]$$

$$= f(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{d^3p}{(2\pi)^3}$$

$$= f(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{p^2 d\Omega_2 dp}{(2\pi)^3}$$

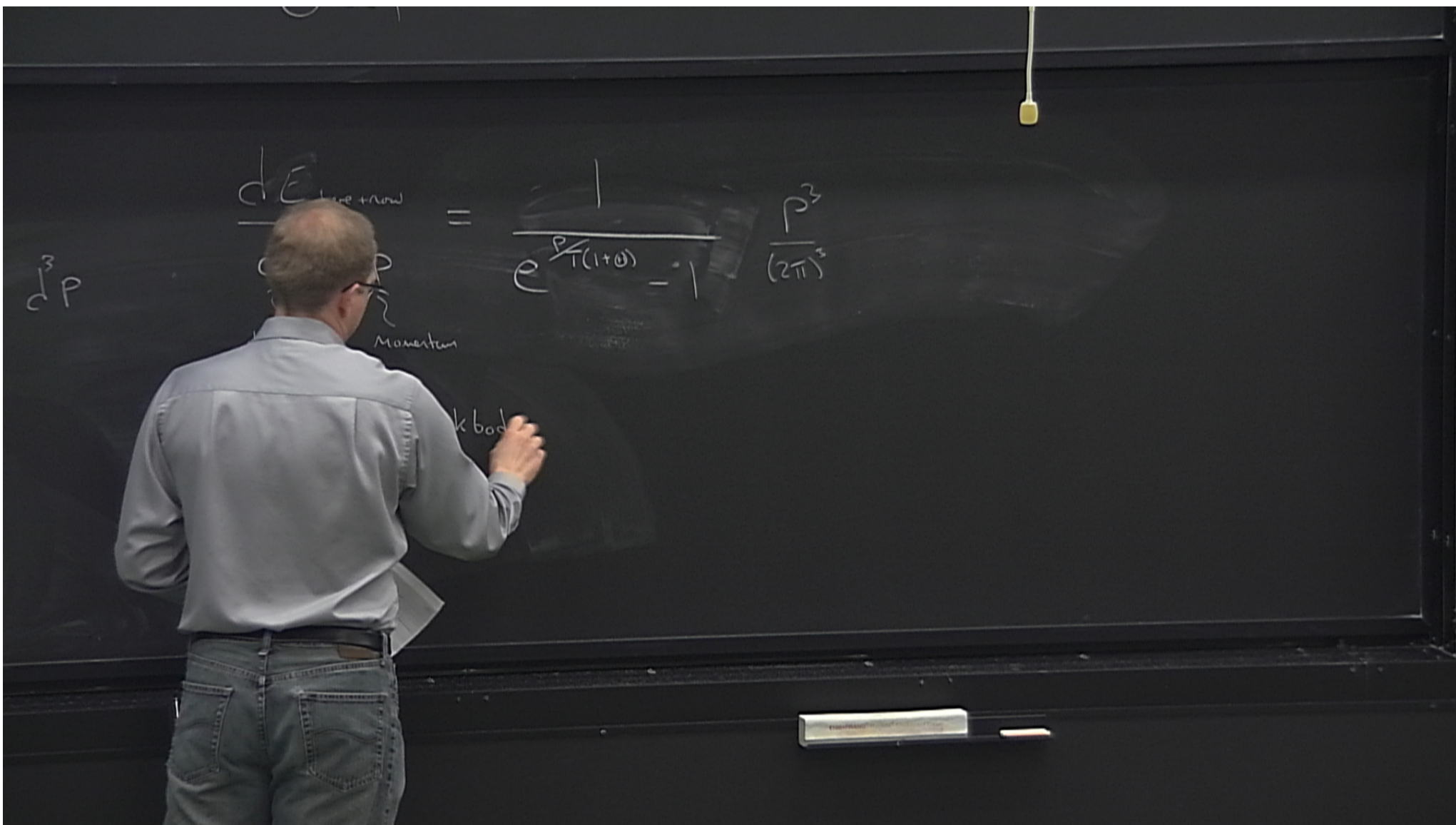
$$d^3p \quad \frac{dN_{\text{here+now}}}{d\Omega dp} = f(\vec{x}_{\text{here+now}}, \vec{p}) \frac{p^2}{(2\pi)^3}$$

$$\frac{dE_{\text{here+now}}}{d\Omega dp} = f(\vec{x}_{\text{here+now}}, \vec{p}) \frac{p^3}{(2\pi)^3}$$

3P

$$\frac{dE_{\text{here+now}}}{d\Omega \, dp} = f(\vec{x}_{\text{here+now}}, \vec{p}) \frac{p^3}{(2\pi)^3}$$

$\underbrace{\quad}_{\text{direction}} \quad \underbrace{\quad}_{\text{momentum}}$



$$d^3p \quad \frac{dE_{\text{here + now}}}{d\Omega \, dp} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$\underbrace{\quad}_{\text{direction}} \quad \underbrace{\quad}_{\text{momentum}}$

$(-1) = 0 \rightarrow \text{Blackbody}$

$$d^3p \quad \frac{E_{\text{here+now}}}{d\Omega \, dp} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$\underbrace{\quad}_{\text{direction}} \quad \underbrace{\quad}_{\text{momentum}}$

$(-1) = 0 \rightarrow \text{Blackbody}$

$$(-1) = (-1)(\hat{p}) = (-1)(\Omega_2)$$

$$\frac{d^3 p}{d\Omega dp} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$\underbrace{\quad}_{\text{direction}} \quad \underbrace{\quad}_{\text{momentum}}$

$$\langle H \rangle = 0 \rightarrow \text{Blackbody}$$

$$\langle H \rangle = \langle H \rangle(\hat{p}) = \langle H \rangle(\Omega_z)$$

$$d^3p \quad \frac{dE_{\text{here+now}}}{d\Omega \, dp} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$\underbrace{\quad}_{\text{direction}} \quad \underbrace{\quad}_{\text{momentum}}$

$$\langle u \rangle = 0 \rightarrow \text{Blackbody}$$

$$\langle u \rangle = \langle u \rangle(\hat{p}) = \langle u \rangle(\Omega_z) \rightarrow \text{Blackbody w/ observation angle dependent } T$$

$$d^3p \quad \frac{dE_{\text{here+now}}}{d\Omega \underbrace{dp}_{\substack{\text{direction} \\ \text{momentum}}}} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$$(H) = 0 \rightarrow \text{Blackbody}$$

$$(H) = (H)(\hat{p}) = (H)(\Omega_2) \rightarrow \text{Blackbody w/ observation angle dependent } T$$

$$(H) = (H)(\vec{p}) \rightarrow \text{Distorted Blackbody, w/ anisotropic distortion}$$

$$d^3p \quad \frac{dE_{\text{here+now}}}{d\Omega \, dp} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$\underbrace{\quad}_{\text{direction}} \quad \underbrace{\quad}_{\text{momentum}}$

$(-1) = 0 \rightarrow \text{Blackbody}$

$(-1) = (-1)(\hat{p}) = (-1)(\Omega_2) \rightarrow \text{Blackbody w/ observation angle dependent } T$

$(-1) = (-1)(\vec{p}) \rightarrow \text{Distorted Blackbody, w/ anisotropic distortion}$

$$d^3p \quad \frac{dE_{\text{here+now}}}{d\Omega \, dp} = \frac{1}{e^{\frac{p}{T(1+u)}} - 1} \frac{p^3}{(2\pi)^3}$$

$\underbrace{d\Omega}_{\text{direction}} \quad \underbrace{dp}_{\text{momentum}}$

why - γe^-
doesn't change $|\vec{p}|$
very much.

$(-1) = 0 \rightarrow \text{Blackbody}$

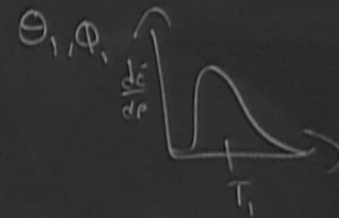
$(-1) = (-1)(\hat{p}) = (-1)(\Omega_2) \rightarrow \text{Blackbody w/ observation angle dependent } T$

$(-1) = (-1)(\vec{p}) \rightarrow \text{Distorted Blackbody, w/ anisotropic distortion}$

What are we after?

$$dN_{\text{here+now}} = \left[\int \bar{F}(\vec{x}, \vec{p}, t_{\text{now}}) \delta^{(3)}(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

$$\begin{aligned} &= f(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{d^3p}{(2\pi)^3} \\ &= f(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{p^2 d\Omega_2 dp}{(2\pi)^3} \end{aligned}$$



$$\langle H \rangle = 0$$

$$\langle H \rangle = \langle H \rangle$$

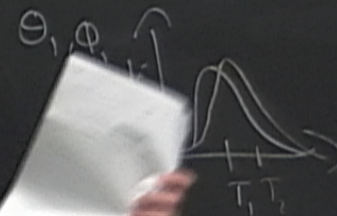
$$\langle H \rangle = \langle H \rangle$$

What are we after?

$$dN_{\text{here+now}} = \left[\int F(\vec{x}, \vec{p}, t) \delta(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

$$= f(\vec{x}_{\text{here}})$$

$$= f(\vec{x}_w)$$



$$\frac{d\sigma}{dS} \int_{\text{direct}}$$

$$\langle H \rangle = 0$$

$$\langle H \rangle = \langle H \rangle$$

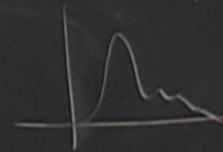
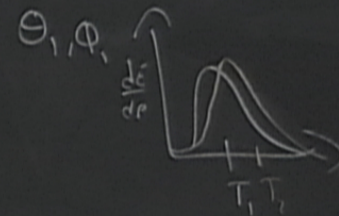
$$\langle H \rangle = \langle H \rangle$$

What are we after?

$$dN_{\text{here+now}} = \left[\int \bar{F}(\vec{x}, \vec{p}, t_{\text{now}}) \delta^{(3)}(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

$$= \bar{f}(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{d^3p}{(2\pi)^3}$$

$$= \bar{f}(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{p^2 d\Omega_2 dp}{(2\pi)^3}$$



$\frac{dN}{dS}$
direct

$$\langle H \rangle = 0$$

$$\langle H \rangle = \langle H \rangle$$

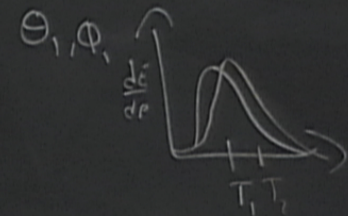
$$\langle H \rangle = \langle H \rangle$$

What are we after?

$$dN_{\text{here+now}} = \left[\int F(\vec{x}, \vec{p}) (\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

$$= f(\vec{x})$$

$$= f(\vec{x})$$



$$\frac{d\sigma}{d\Omega}$$

direct

$$\langle H \rangle = 0$$

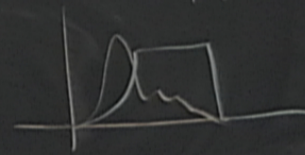
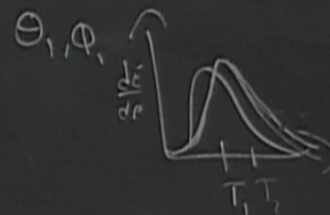
$$\langle H \rangle = \langle H \rangle$$

$$\langle H \rangle = \langle H \rangle$$

What are we after?

$$dN_{\text{here+now}} = \left[\int \bar{F}(\vec{x}, \vec{p}, t_{\text{now}}) \delta^{(3)}(\vec{x} - \vec{x}_{\text{here}}) \frac{d^3x}{(2\pi)^3} \right] d^3p$$

$$\begin{aligned} &= \bar{F}(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{d^3p}{(2\pi)^3} \\ &= \bar{F}(\vec{x}_{\text{here}}, \vec{p}, t_{\text{now}}) \frac{p^2 d\Omega_z dp}{(2\pi)^3} \end{aligned}$$



$$\frac{dE_{\text{here+now}}}{d\Omega \frac{dp}{\text{direction}}}$$

$$\langle H \rangle = 0 \rightarrow$$

$$\langle H \rangle = \langle H \rangle(\vec{p}) =$$

$$\langle H \rangle = \langle H \rangle(\vec{p}) =$$