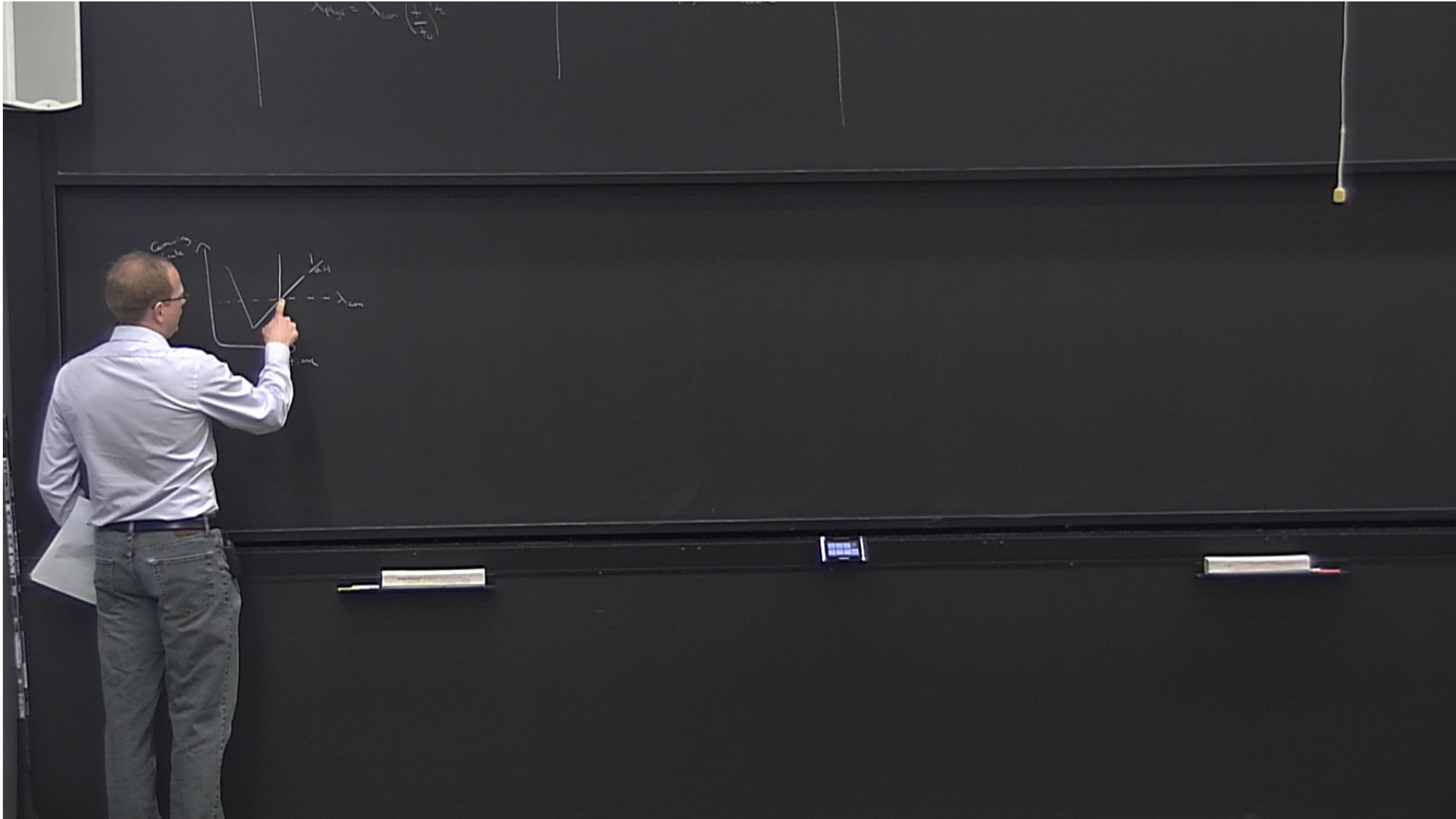


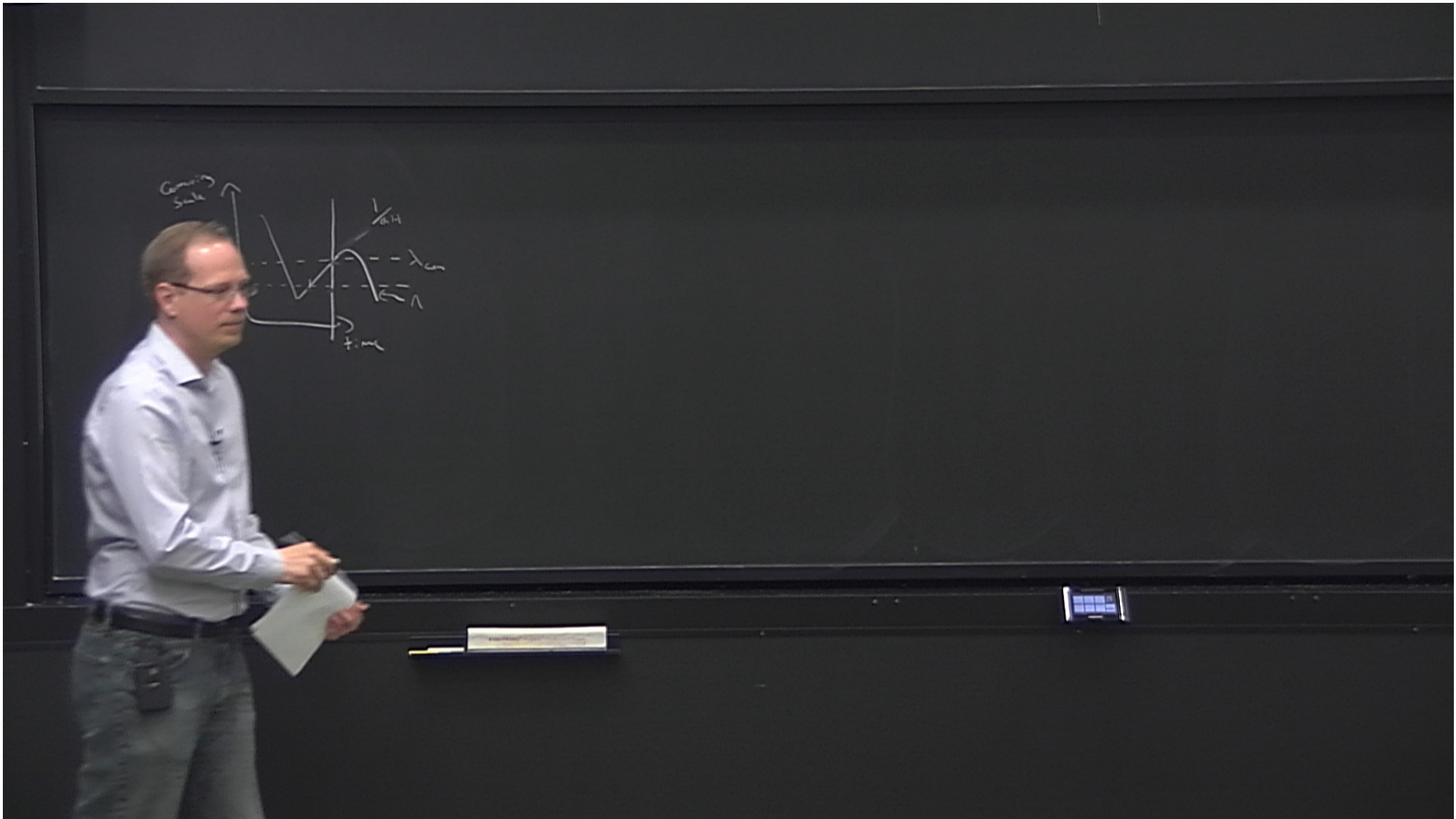
Title: 13/14 PSI - Explorations in Cosmology - Lecture 4

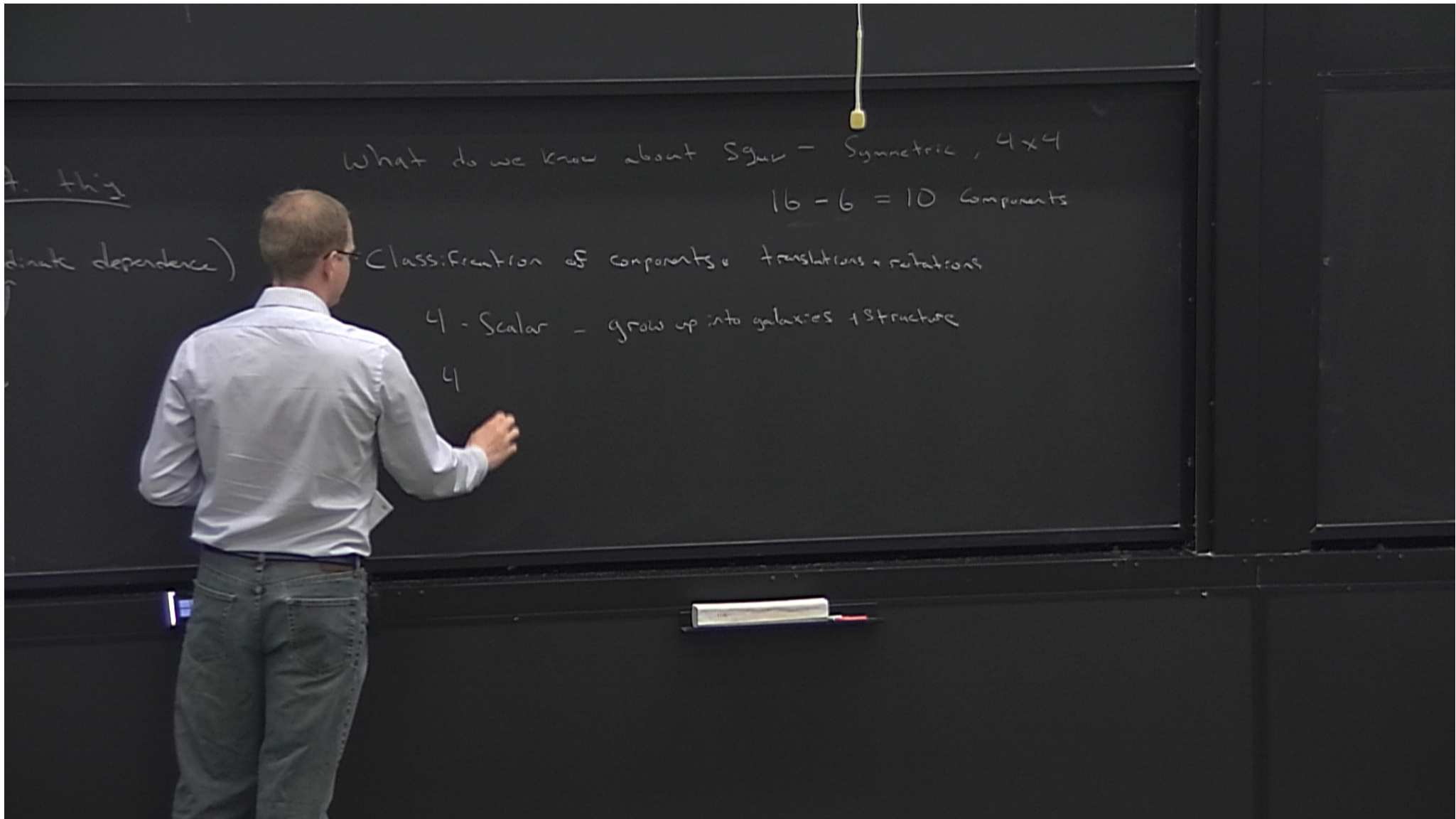
Date: Apr 10, 2014 10:15 AM

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Abstract:







4. this

inate dependence)

What do we know about $S_{\mu\nu}$ - Symmetric, 4×4

$$16 - 6 = 10 \text{ Components}$$

Classification of components: translations + rotations

4 - Scalar - grow up into galaxies + structure

4

4. this

(metric dependence)

What do we know about $S_{\mu\nu}$ - Symmetric, 4×4

$$16 - 6 = 10 \text{ Components}$$

Classification of components: translations + rotations

* 4 - Scalar - grow up into galaxies + structure

4 - Vector - decays in an expanding universe

* 2 - tensor - gravitational waves

But, 4 coordinate freedoms.

Equivalent to 4 B-identities.

$\Rightarrow$ 6 independent in $g_{\mu\nu}$

$\rightarrow$ 2 constraints on Scalars $\rightarrow$ 2 dof in Scalars

on vectors $\rightarrow$ 2 dof in vectors

But, 4 coordinate freedoms.

Equivalent to 4 Bianchi Identities

$\Rightarrow$ 6 independent dof in $S_{\mu\nu}$

$\rightarrow$ 2 conditions on $S_{\mu\nu}$ dof in $S_{\mu\nu}$

2 conditions on vectors in vectors

But, 4 coordinate freedoms.

Equivalent to 4 Bianchi Identities

$\Rightarrow$ 6 independent dof in $g_{\mu\nu}$

$\rightarrow$ 2 conditions on Scalars $\rightarrow$ 2 dof in Scalars

2 Conditions on vectors $\rightarrow$ 2 dof in vectors

Focus on Scalars

16

Focus on Scalars

$$ds^2 = \alpha^2(z) \left[-(1+2A) dz^2 - 2\omega_i dz dx^i + \left[(1+2C) S_{ij} + 2\omega_i \omega_j \right] dx^i dx^j \right]$$

2 dof in Scalar

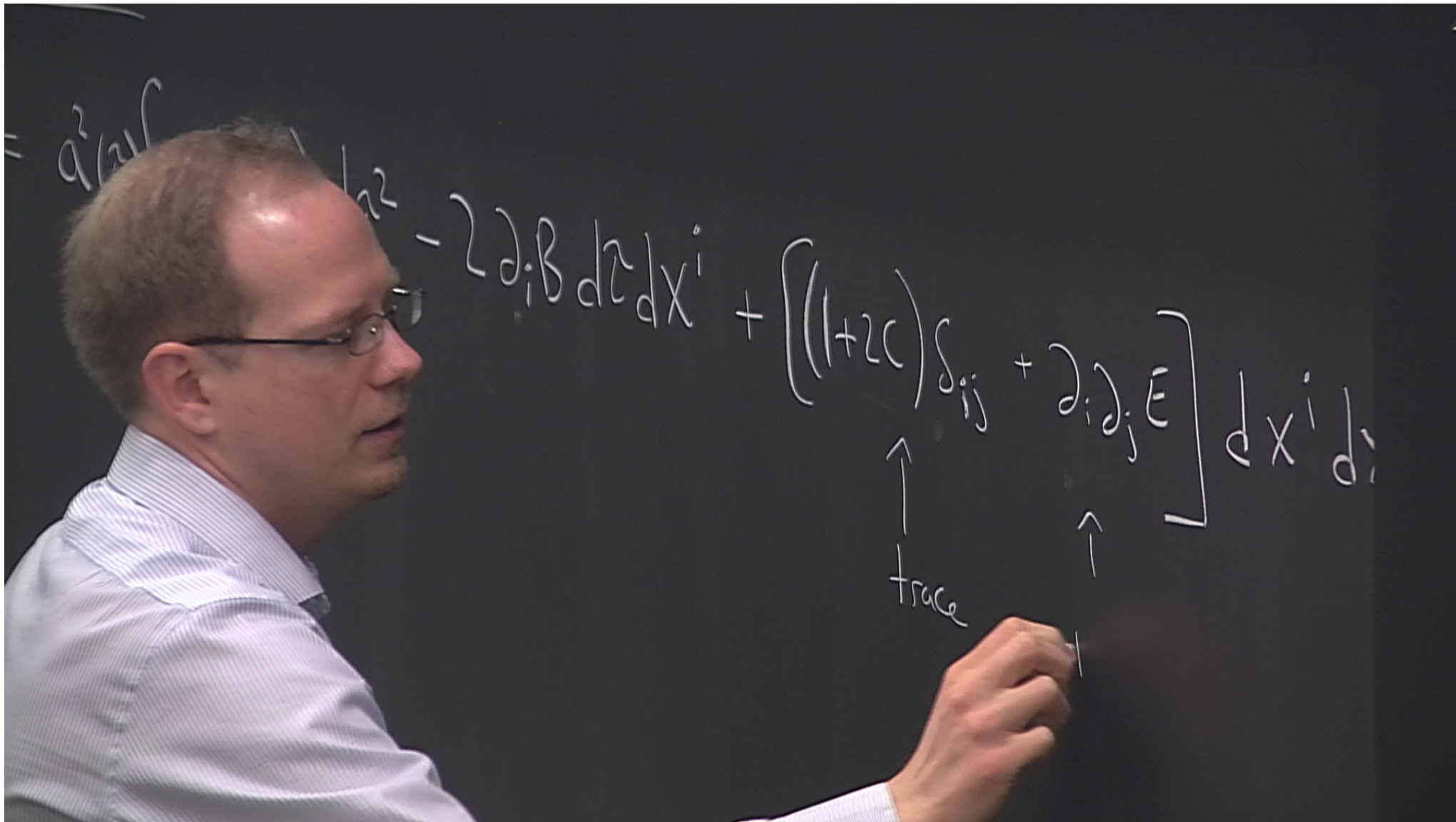
2 dof in vectors

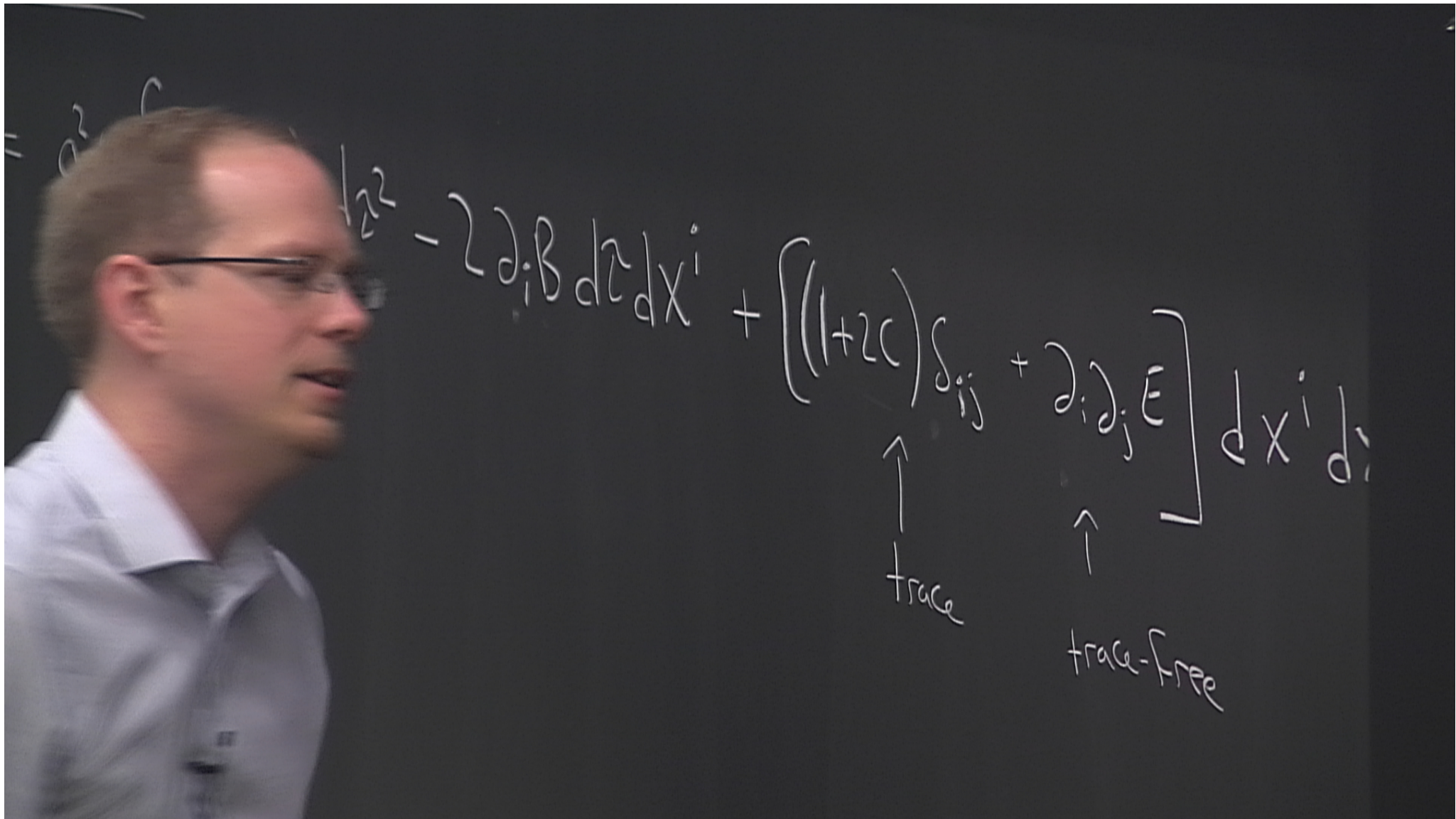
Focus on Scalars

$$ds^2 = a^2(z) \left[-(1+2A) dz^2 - 2\partial_i \phi dz dx^i + \left[(1+2C) \delta_{ij} + 2\partial_i \partial_j E \right] dx^i dx^j \right]$$

2 dof in Scalar

2 dof in vectors





10

Focus on Scalars

$$S^2 = \alpha^2(\tau) \left[-(1+2A) d\tau^2 - 2\partial_i B d\tau dx^i + \left[\underset{\substack{\uparrow \\ \text{trace}}}{(1+2C)S_{ij}} + \underset{\substack{\uparrow \\ \text{trace-free}}}{\partial_i \partial_j E} \right] dx^i dx^j \right]$$

2 dof in Scalar

2 dof in vector

10

Focus on Scalars

$$dS^2 = \alpha^2(\tau) \left[-(1+2A) d\tau^2 - 2\partial_i B d\tau dx^i + \left[\underset{\substack{\uparrow \\ \text{trace}}}{(1+2C)S_{ij}} + \underset{\substack{\uparrow \\ \text{trace-free}}}{\partial_i \partial_j E} \right] dx^i dx^j \right]$$

Focus on Scalars

$$ds^2 = \alpha^2(\tau) \left[-(1+2A) d\tau^2 - 2\partial_i B d\tau dx^i + \left[\underset{\substack{\uparrow \\ \text{trace}}}{(1+2C)S_{ij}} + \underset{\substack{\uparrow \\ \text{trace-free}}}{\partial_i \partial_j E} \right] dx^i dx^j \right]$$

4-free fws = $A(\tau, \vec{x}), B(\tau, \vec{x}), C(\tau, \vec{x}), E(\tau, \vec{x})$

2 dof in Scal

2 dof in ve

Focus on Scalars

$$dS^2 = \alpha^2(\tau) \left[-(1+2A) d\tau^2 - 2\partial_i B d\tau dx^i + \left[\underset{\substack{\uparrow \\ \text{trace}}}{(1+2C)S_{ij}} + \underset{\substack{\uparrow \\ \text{trace-free}}}{\partial_i \partial_j E} \right] dx^i dx^j \right]$$

4-free fws = $A(\tau, \vec{x}), B(\tau, \vec{x}), C(\tau, \vec{x}), E(\tau, \vec{x})$

Put our coordinate freedom in a gauge Fix $\circ \{^{\mu}(\tau, \vec{x})$

$$\tilde{X}^{\mu} = X^{\mu} + \xi^{\mu} \leftarrow 4 \text{ components} = 4 \text{ coordinate choices}$$

2 dof in Scalar

2 dof in vectors

$$\xi^\mu = (\xi^0, \underbrace{\sum^{ij} \partial_j \xi + \sum^{ijk} \partial_j \xi^k}_{\text{our choices}})$$

$$\xi^\mu = \left(\xi^0, \underbrace{\delta^{ij} \partial_j \xi}_{\substack{\uparrow \\ 2 \text{ scalar choices}}} + \underbrace{\epsilon^{ijk} \partial_j \xi^k}_{\uparrow} \right)$$



$$\xi^{\mu} = \left(\xi^0, \underbrace{\delta^{ij} \partial_j \xi}_{\substack{\uparrow \\ 2 \text{ scalar choices}}} + \underbrace{\delta^{ijk} \partial_j \xi^k}_{\substack{\uparrow \\ 2 \text{ vector choices}}} \right)$$

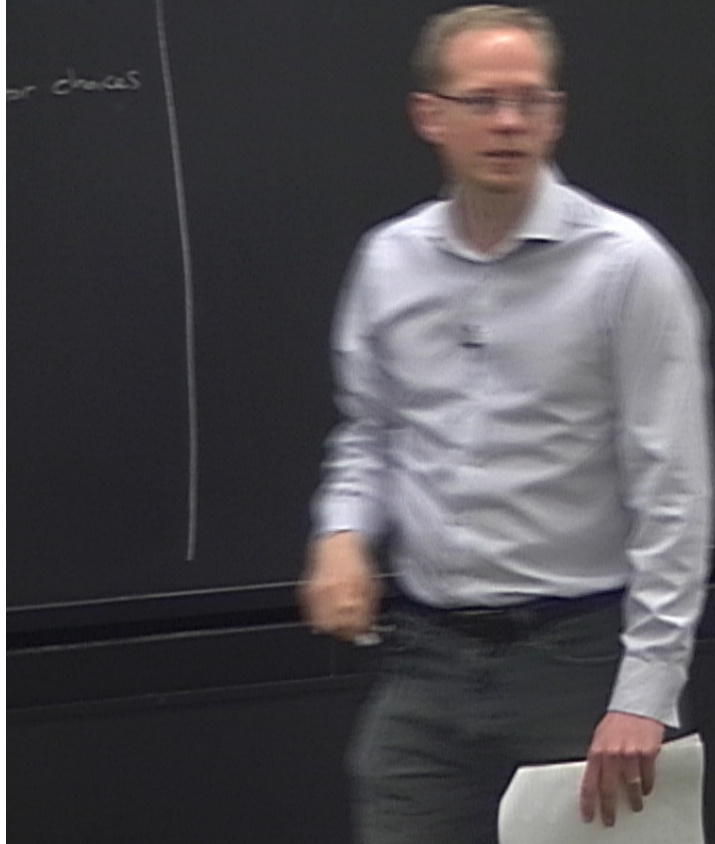
See what happens to metric

$$\Rightarrow \tilde{\tau} = \tau + \xi^0$$

$$\tilde{x}^i = x^i + \delta^{ij} \partial_j \xi$$

See what happens to metric under a change in coordinates (change in ξ^{μ})

ξ^{μ}
or changes



See what happens to metric under a change in coordinates (change in $\tilde{g}^{\mu\nu}$)

$$g_{\mu\nu}(x^\alpha) = \frac{\partial \tilde{X}^\alpha}{\partial X^\mu} \frac{\partial \tilde{X}^\beta}{\partial X^\nu} \tilde{g}_{\alpha\beta}(\tilde{x}^\gamma)$$

See what happens to metric under a change in coordinates (change in $\tilde{x}^\mu$)

$$g_{\mu\nu}(x^\alpha) = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial \tilde{x}^\beta}{\partial x^\nu} \tilde{g}_{\alpha\beta}(\tilde{x}^\gamma)$$

linear coordinate shifts, $\tilde{x}^\mu \ll 1 \rightarrow$ choice of x vs $\tilde{x}$

step 1)

See what happens to metric under a change in coordinates (change in $\tilde{g}^{\mu\nu}$)

$$g_{\mu\nu}(x^0) = \frac{\partial \tilde{X}^\alpha}{\partial X^\mu} \frac{\partial \tilde{X}^\beta}{\partial X^\nu} \tilde{g}_{\alpha\beta}(\tilde{x})$$

linear coordinate shifts, $\tilde{x}^\mu = x^\mu + \epsilon^\mu$ $\rightarrow$ choice of X vs $\tilde{X}$ in ξ arbitrary @ 1st $\mathcal{O}$
choose X

step 1) Taylor expand $\tilde{g}_{\alpha\beta}(\tilde{x})$

$$\tilde{g}_{\alpha\beta}(\tilde{x}) = g_{\alpha\beta}^{(0)}(\tilde{x}) + \delta g_{\alpha\beta}$$

See what happens to metric under a change in coordinates (change in $\tilde{g}^{\mu\nu}$)

$$g_{\mu\nu}(x^a) = \frac{\partial \tilde{X}^\alpha}{\partial X^\mu} \frac{\partial \tilde{X}^\beta}{\partial X^\nu} \tilde{g}_{\alpha\beta}(\tilde{x}^r)$$

linear coordinate shifts, $\tilde{x}^\mu \ll 1 \rightarrow$ choice of X vs $\tilde{X}$ arbitrary @ 1st $\mathcal{O}$

step 1) Taylor expand $\tilde{g}_{\alpha\beta}(\tilde{x}^r)$

$$\tilde{g}_{\alpha\beta}(\tilde{x}^r) = g_{\alpha\beta}^{(0)}(\tilde{x}^r) + \delta g_{\alpha\beta}(\tilde{x}^r) = g_{\alpha\beta}^{(0)}(x^r) + \left(\partial_r g_{\alpha\beta}^{(0)}(x^r) \right) \tilde{x}^r +$$

Step 2) evaluate partials:

$$\frac{\partial \tilde{X}^\alpha}{\partial X^\mu} = \frac{\partial}{\partial X^\mu} (X^\alpha + \xi^\alpha) = \delta^\alpha_\mu + \partial_\mu \xi^\alpha$$

Step 3)

Step 2) evaluate partials: $\frac{\partial \tilde{X}^\alpha}{\partial X^\mu} = \frac{\partial}{\partial X^\mu} (X^\alpha + \xi^\alpha) = \delta^\alpha_\mu + \partial_\mu \xi^\alpha$

Step 3) Substitute into $\bar{T}_\mu$ law, and keep 1st order only:

$$g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} = (\delta^\alpha_\mu + \partial_\mu \xi^\alpha) (\delta^\beta_\nu + \partial_\nu \xi^\beta) \left(g_{\alpha\beta}^{(0)} + (\partial_\gamma g_{\alpha\beta}^{(0)}) \xi^\gamma + \tilde{g}_{\alpha\beta} \right)$$

Step 2) evaluate partials: $\frac{\partial \tilde{X}^\alpha}{\partial X^\mu} = \frac{\partial}{\partial X^\mu} (X^\alpha + \xi^\alpha) = \delta^\alpha_\mu + \partial_\mu \xi^\alpha$

Step 3) Substitute into $\overline{T}I$ law, and keep 1st order only:

$$g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} = (\delta^\alpha_\mu + \partial_\mu \xi^\alpha) (\delta^\beta_\nu + \partial_\nu \xi^\beta) \left(g_{\alpha\beta}^{(0)} + (\partial_\gamma g_{\alpha\beta}^{(0)}) \xi^\gamma + \tilde{g}_{\alpha\beta} \right)$$

Step 2) evaluate partials: $\frac{\partial \tilde{X}^\alpha}{\partial X^\mu} = \frac{\partial}{\partial X^\mu} (X^\alpha + \xi^\alpha) = \delta^\alpha_\mu + \partial_\mu \xi^\alpha$

Step 3) Substitute into $\overline{II}$ law, and keep 1st order only:

$$g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} = (\delta^\alpha_\mu + \partial_\mu \xi^\alpha) (\delta^\beta_\nu + \partial_\nu \xi^\beta) \left(g_{\alpha\beta}^{(0)} + (\partial_\gamma g_{\alpha\beta}^{(0)}) \xi^\gamma + \tilde{g}_{\alpha\beta} \right)$$

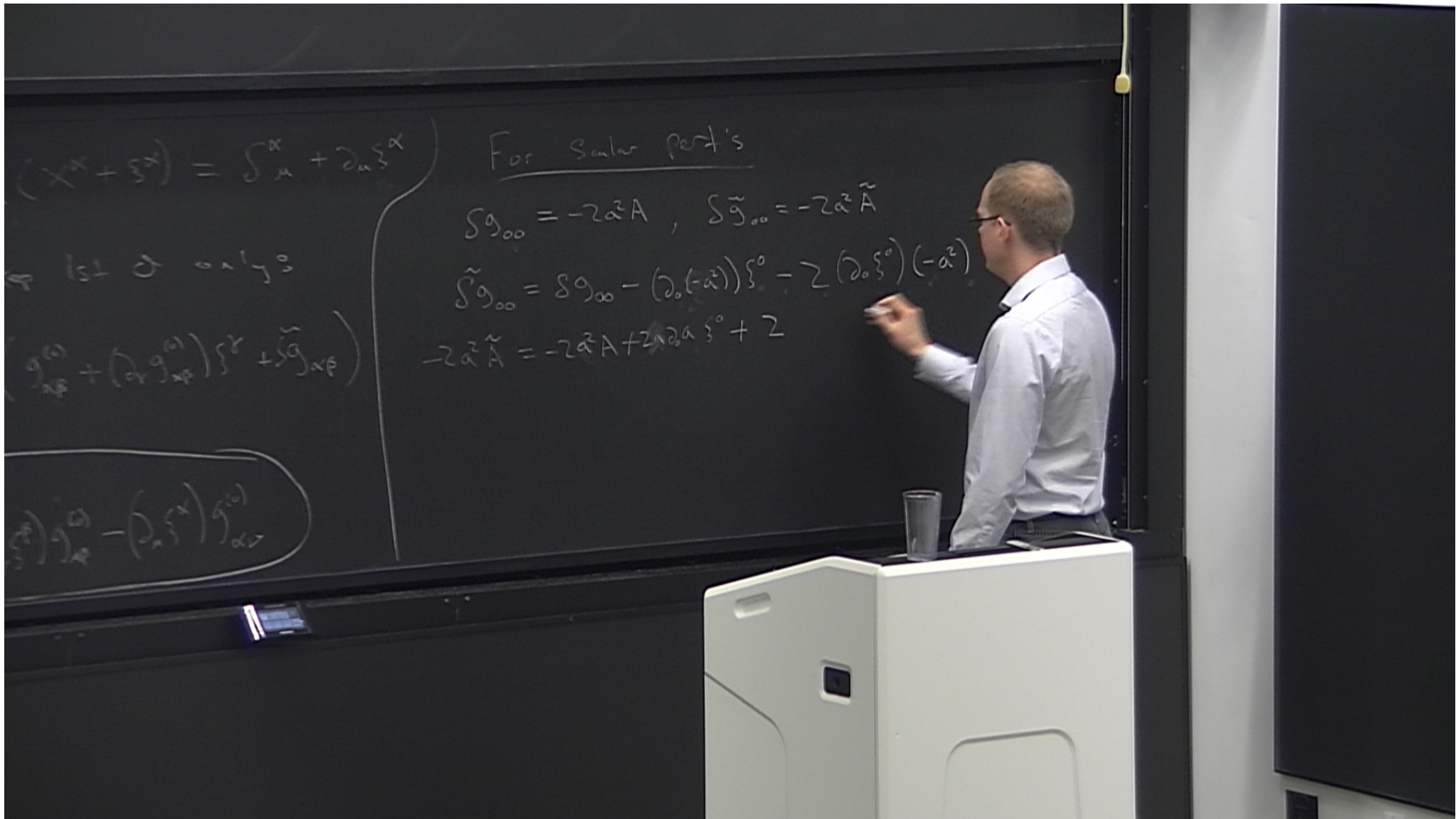
Step 2) evaluate partials: $\frac{\partial \tilde{X}^\alpha}{\partial X^\mu} = \frac{\partial}{\partial X^\mu} (X^\alpha + \xi^\alpha) = \delta^\alpha_\mu + \partial_\mu \xi^\alpha$

Step 3) Substitute into TT law, and keep 1st order terms

$$g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} = (\delta^\alpha_\mu + \partial_\mu \xi^\alpha) (\delta^\beta_\nu + \partial_\nu \xi^\beta) (g_{\alpha\beta}^{(0)} + (\partial_\gamma g_{\alpha\beta}^{(0)}) \xi^\gamma)$$

Keep linear in ξ , solve for $\delta \tilde{g}$:

$$\delta \tilde{g}_{\mu\nu} = \delta g_{\mu\nu} - (\partial_\mu g_{\nu\alpha}^{(0)}) \xi^\alpha - (\partial_\nu g_{\mu\alpha}^{(0)}) \xi^\alpha$$



$$(X^\alpha + \xi^\alpha) = \delta^\alpha_\mu + \partial_\mu \xi^\alpha$$

1st order only

$$g_{\alpha\beta}^{(1)} + (\partial_\alpha g_{\mu\beta}^{(0)}) \xi^\mu + \tilde{g}_{\alpha\beta}$$

$$\xi^\mu g_{\mu\alpha}^{(0)} - (\partial_\alpha \xi^\mu) g_{\mu\alpha}^{(0)}$$

For scalar pert's

$$\delta g_{00} = -2a^2 A, \quad \delta \tilde{g}_{00} = -2a^2 \tilde{A}$$

$$\tilde{\delta} g_{00} = \delta g_{00} - (\partial_0 (-a^2)) \xi^0 - 2 (\partial_0 \xi^0) (-a^2)$$

$$-2a^2 \tilde{A} = -2a^2 A + 2a \partial_a \xi^0 + 2$$

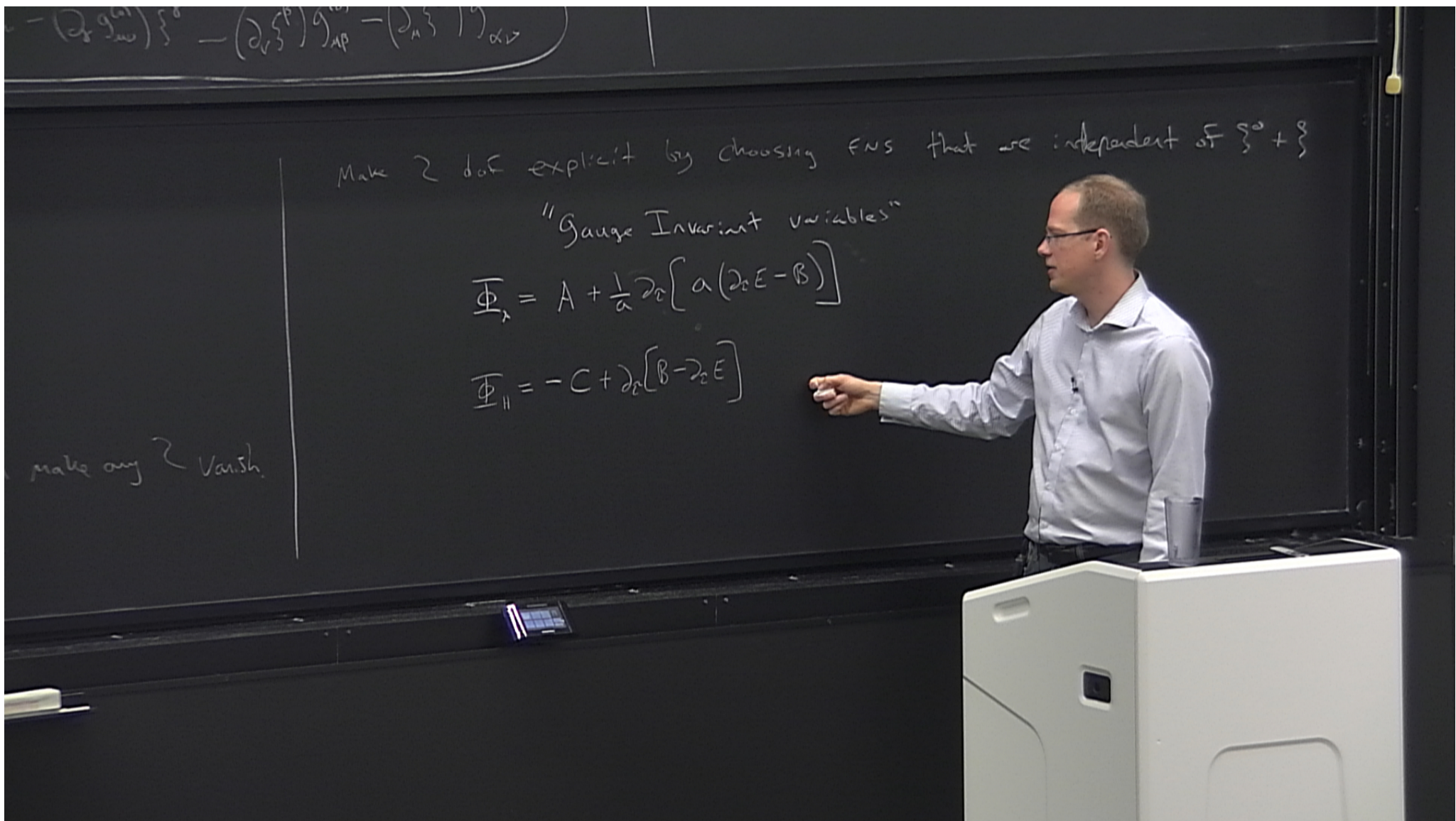
$$\delta g_{\mu\nu} = \delta g_{\mu\nu} - (\partial_\mu g_{\nu\rho}) \xi^\rho - (\partial_\nu g_{\mu\rho}) \xi^\rho - (\partial_\mu \xi^\rho) g_{\nu\rho} - (\partial_\nu \xi^\rho) g_{\mu\rho}$$

For others:

$$\tilde{B} = B + \partial_\mu \xi - \xi^\mu$$

$$\tilde{C} = C + \frac{\partial_\mu \xi^\mu}{2}$$

$$\tilde{E} = E + \xi$$



10

Nice aspect :

$$\overline{\Phi_A} = \Psi$$
$$\overline{\Phi_H} = -\overline{\Phi}$$

$$\Phi_A = \Psi$$

$$\Phi_H = -\Phi$$

metric.

on Scales, Φ, Ψ depend on

→ Motivates another gauge choice

Early Universe → ?

Late Universe

* $T^{\mu\nu}$ transforms too!!

$$T^{\mu}_{\nu} = (-\rho, \rho, \rho, \rho)$$

$$\hookrightarrow \tilde{\rho}(\tilde{x}^{\nu}) = \rho(x^{\nu}) + (\partial_{\mu} \rho(x^{\nu})) \xi^{\mu}$$

$$\tilde{p}(\tilde{x}^{\nu}) = p(x^{\nu}) + (\partial_{\mu} p(x^{\nu})) \xi^{\mu}$$

* $T^{\mu\nu}$ transforms too!!

$$T^{\mu}_{\nu} = (-P, P, P, P), \quad T_{\mu\nu} = (P + P)u_{\alpha}u_{\beta} + P g_{\alpha\beta}$$

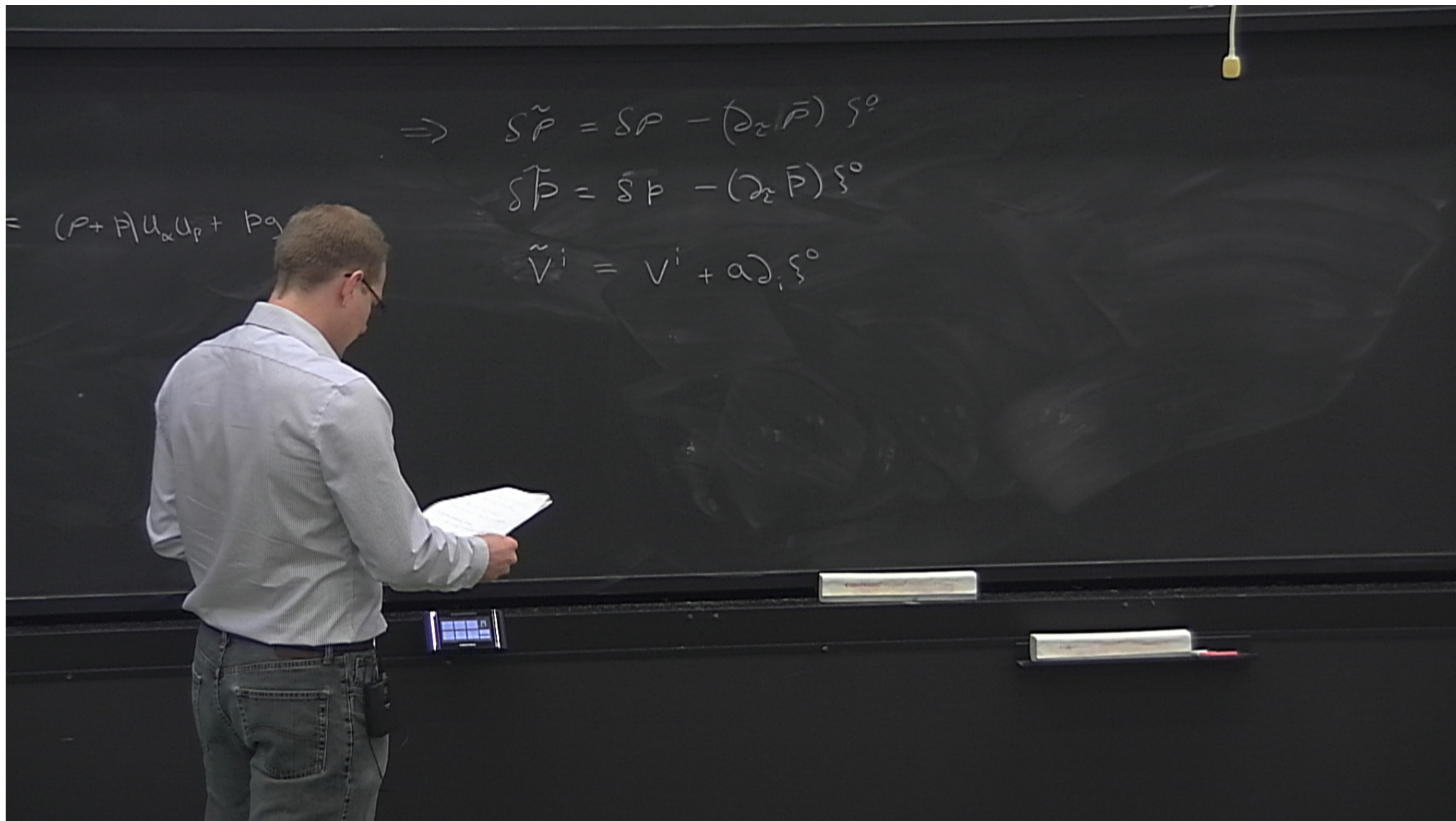
$$(\rightarrow) \tilde{P}(\tilde{x}^{\nu}) = P(x^{\nu}) + (\partial_{\mu} P(x^{\nu})) \xi^{\mu}$$

$$\tilde{P}(\tilde{x}^{\nu}) = P(x^{\nu}) + (\partial_{\mu} P(x^{\nu})) \xi^{\mu}$$

$$\mathcal{L}_{\mu}(\tilde{x}) = \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\mu}} \tilde{\mathcal{L}}_{\alpha}(\tilde{x}) \rightarrow \text{expand}$$

$$\Rightarrow \tilde{S} \tilde{P} = S P$$

$$\tilde{S} \tilde{P} = S P$$



$$\Rightarrow \tilde{S}P = SP - (\partial_z \bar{P}) \xi^0$$

$$\tilde{S}\bar{P} = S\bar{P} - (\partial_z P) \xi^0$$

$$\tilde{V}^i = V^i + \alpha \partial_i \xi^0$$

$$\delta T^\mu_\nu = \begin{pmatrix} -SP & (\bar{P} + P)V^i \\ (\bar{P} + P)V^i & \delta_{ij} SP \end{pmatrix}$$

Comoving Gauge

δT

$$\Rightarrow \tilde{S}P = SP - (\partial_z \bar{P}) \xi^0$$

$$\tilde{S}\bar{P} = \bar{S}P - (\partial_z \bar{P}) \xi^0$$

$$\tilde{V}^i = V^i + \alpha \partial_i \xi^0$$

$$\delta T^\mu_\nu = \begin{pmatrix} -SP & (\bar{P} + P)V^i \\ (\bar{P} + P)V^i & S_{ij}SP \end{pmatrix}$$

Comoving Gauge

$$\delta T^0_i = 0 \Rightarrow V^i = 0$$

$$B = 0$$

$$ds^2 = a^2(\tau) \left[-(1+2A)d\tau^2 + \left[(1+2S)S_{ij} + \partial_i \partial_j E \right] dx^i dx^j \right]$$