Title: 13/14 PSI - Explorations in Cosmology - Lecture 2

Date: Apr 08, 2014 10:15 AM

URL: http://pirsa.org/14040042

Abstract:

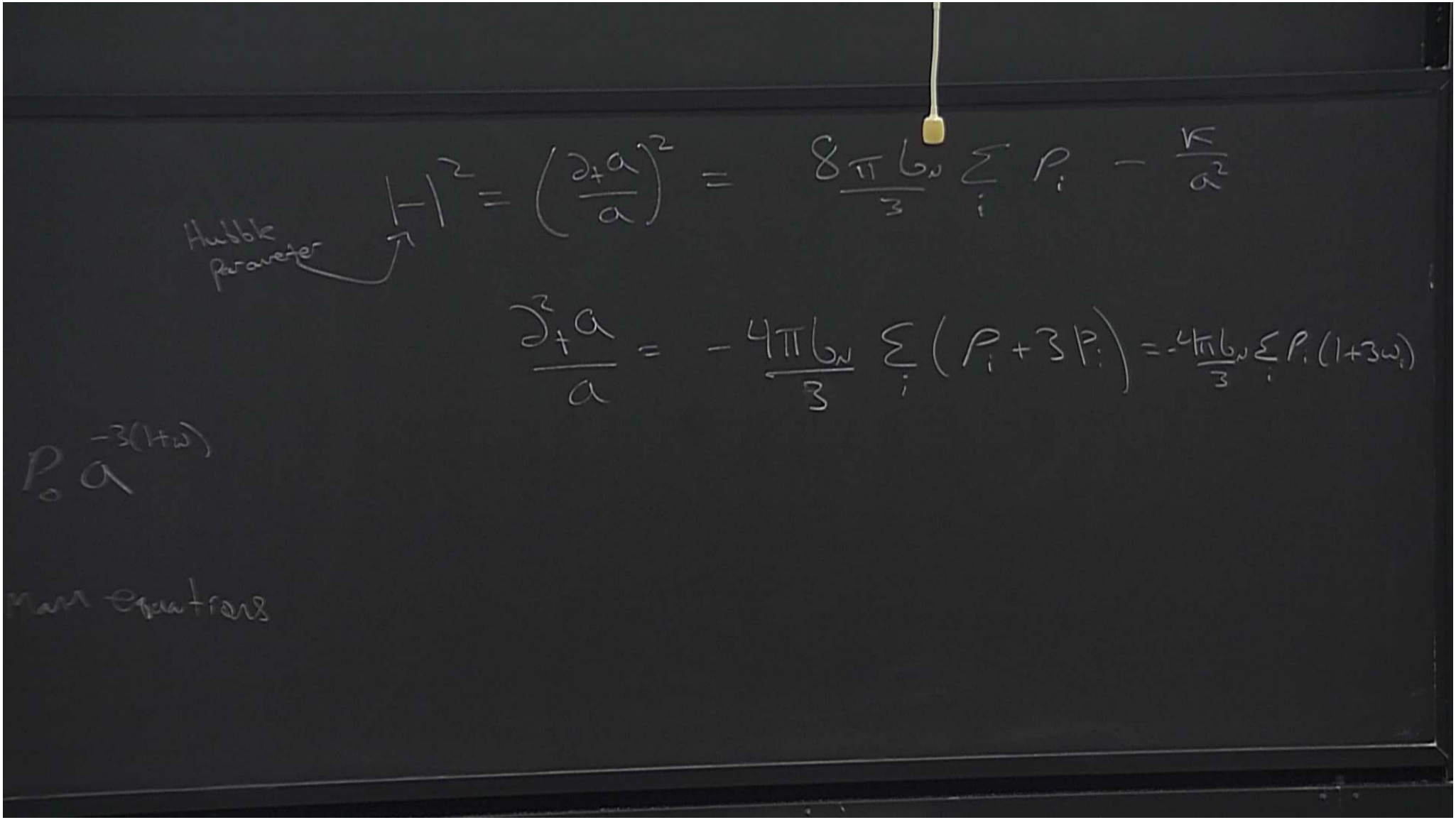
FRW: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

$ds^2 = a(\tau)^2 [-d\tau^2 + d\vec{x}^2]$, $\tau = \int \frac{dt}{a}$

Hubble parameter

$\nabla_\mu T^\mu{}_\nu \Rightarrow \nu = 0 \rightarrow \frac{\partial \rho}{\rho} = -3(1+w)\frac{\partial a}{a} \Rightarrow \rho = \rho_0 a^{-3(1+w)}$

* Stick metric + $T_{\mu\nu}$ for a perfect fluid into Einstein: Freidman equations


$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\sum_i \rho_i - \frac{K}{a^2}$
Hubble parameter
$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\sum_i (\rho_i + 3p_i) = -\frac{4\pi G_N}{3}\sum_i \rho_i (1+3w_i)$
$\rho_0 a^{-3(1+w)}$
main equations

Hubble parameter

$$H^2 = \left(\frac{\partial_t a}{a}\right)^2 = \frac{8\pi G_N}{3}\sum_i \rho_i - \frac{K}{a^2}$$

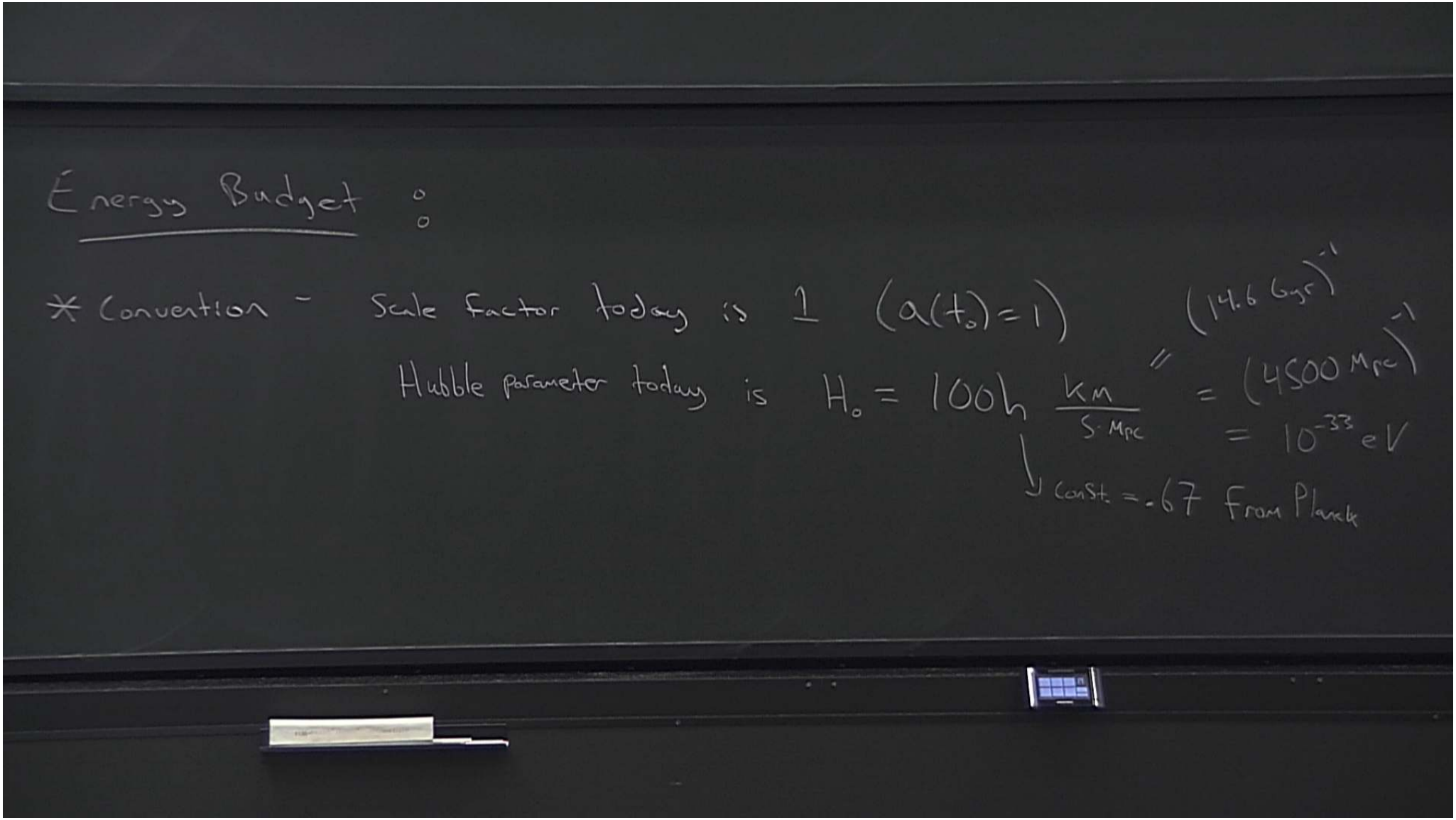
K: $\frac{8\pi G_N}{3}\rho_c$

$$\frac{\partial_t^2 a}{a} = -\frac{4\pi G_N}{3}\sum_i \left(\rho_i + 3P_i\right) = -\frac{4\pi G_N}{3}\sum_i \rho_i (1+3\omega_i)$$

* Accelerated expansion:

for $\partial_t^2 a > 0$, need $\omega_i < -\frac{1}{3}$

(Matter, $\omega = 0$
Radiation, $\omega = \frac{1}{3}$
Curvature, $\omega = -\frac{1}{3}$
Λ, $\omega = -1$)

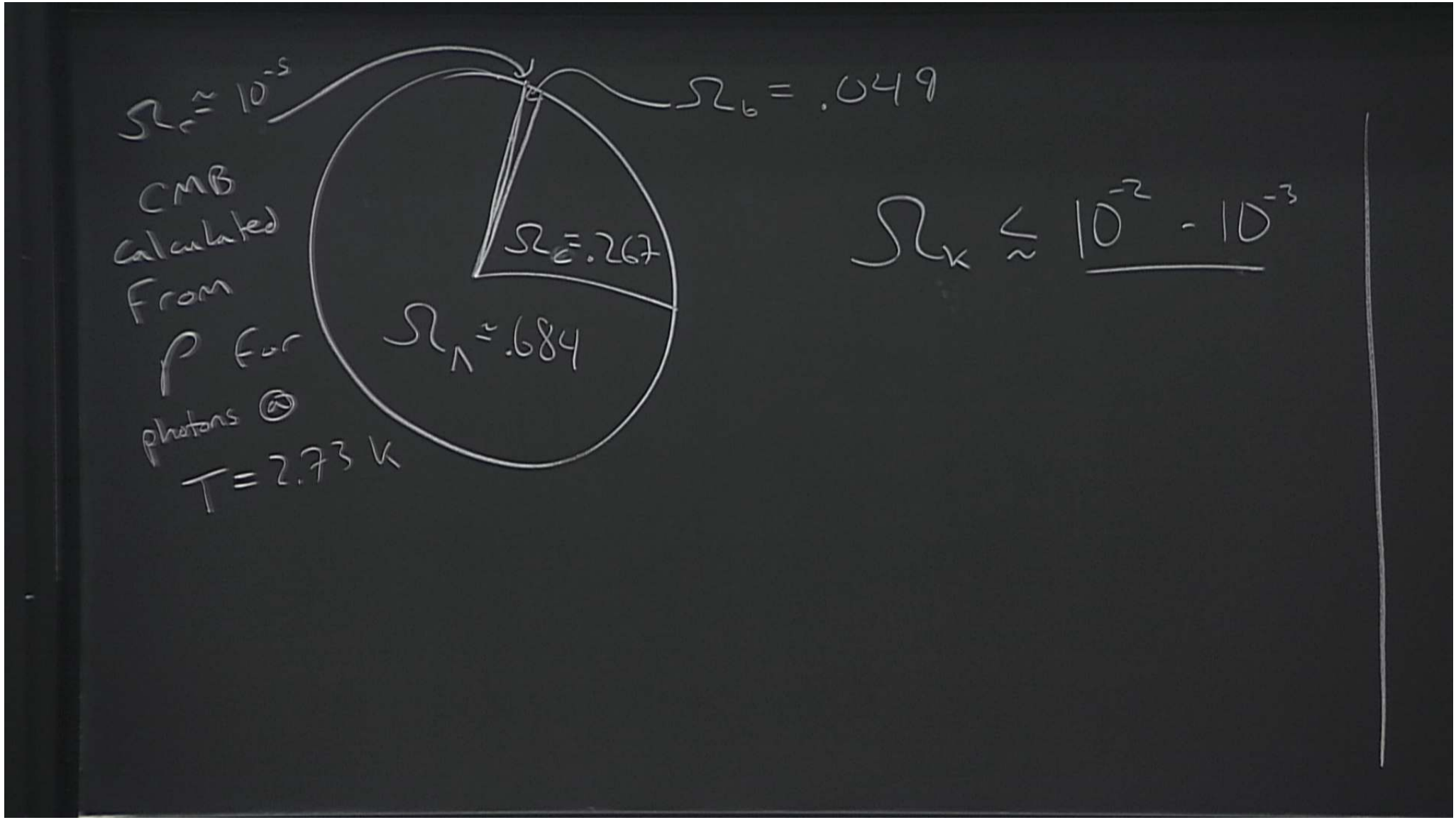

Energy Budget :
* Convention - Scale factor today is 1 $(a(t_0)=1)$
Hubble parameter today is $H_0 = 100h \frac{km}{s \cdot Mpc}$
$= (14.6\ Gyr)^{-1}$
$= (4500\ Mpc)^{-1}$
$= 10^{-33}\ eV$
const. = .67 From Planck

$$\frac{H^2}{H_0^2} = \sum_i \frac{\Omega_i}{a^{3(1+w_i)}} \qquad \Omega_i = \frac{8\pi G_N \rho_{0,i}}{3H_0^2} \quad (= R^{-1})$$

today: $1 = \sum_i \Omega_i$ ~ fraction of energy density in different components

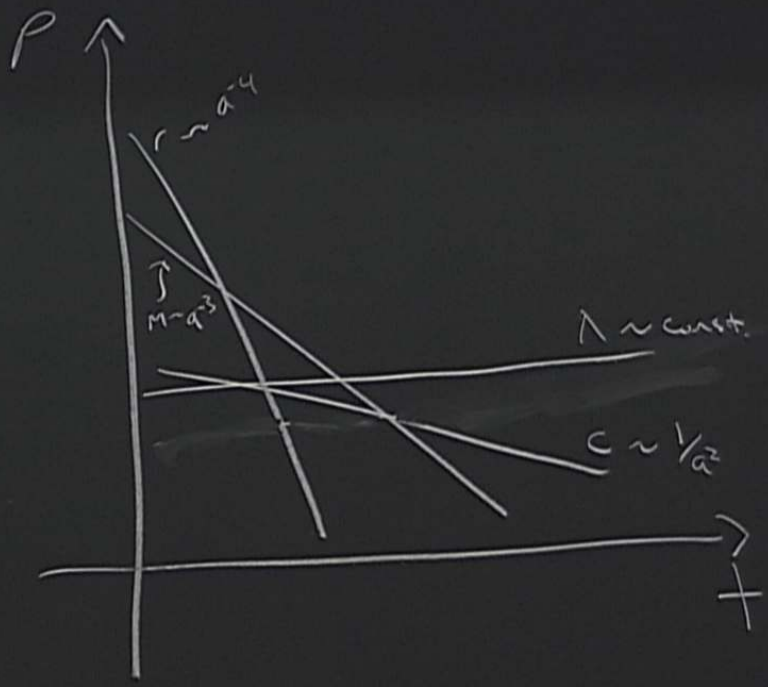
$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_c + \Omega_b}{a^3} + \frac{\Omega_K}{a^2} + \Omega_\Lambda \qquad (\Omega_c + \Omega_b = \Omega_M)$$

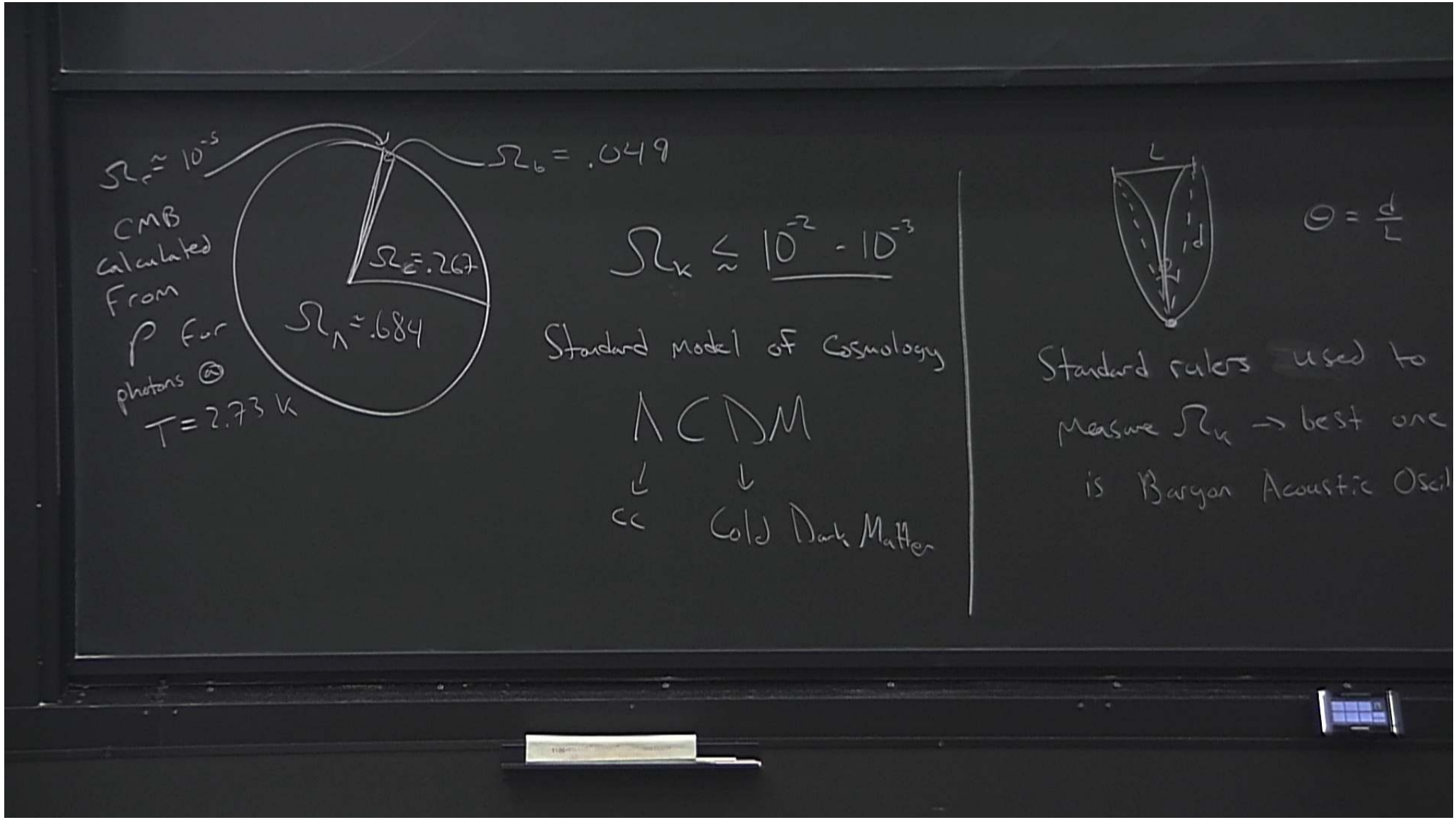
*Note: Ω_i are fractional density today!


$\Omega_\gamma \simeq 10^{-5}$
CMB calculated From ρ For photons @ $T = 2.73\ K$
$\Omega_b = .049$
$\Omega_c = .267$
$\Omega_\Lambda \simeq .684$
$\Omega_K \lesssim 10^{-2} - 10^{-3}$

$\theta = \frac{d}{L}$

Standard rulers used to measure Ω_k → best one is Baryon Acoustic Oscillations

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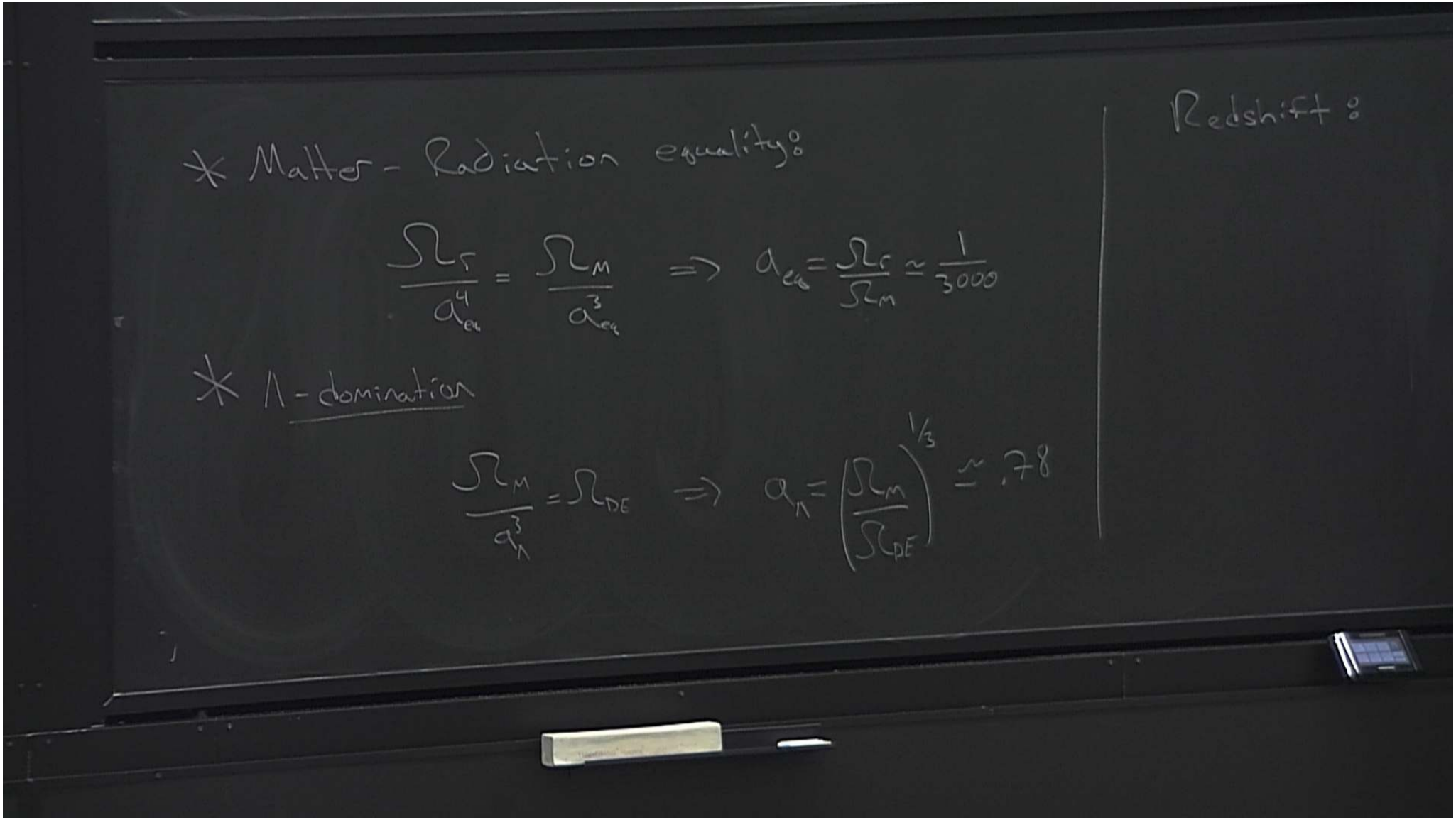
Standard Model of Cosmology
ΛCDM
cc
Cold Dark Matter
L
d
$\theta = \frac{d}{L}$
Standard rulers used to measure Ω_k → best one is Baryon Acoustic Oscil


* Matter - Radiation equality:
$\frac{\Omega_r}{a_{eq}^4} = \frac{\Omega_M}{a_{eq}^3} \Rightarrow a_{eq} = \frac{\Omega_r}{\Omega_M} \simeq \frac{1}{3000}$
* Λ - domination
$\frac{\Omega_M}{a_\Lambda^3} = \Omega_{DE} \Rightarrow a_\Lambda = \left(\frac{\Omega_M}{\Omega_{DE}}\right)^{1/3} \simeq .78$
Redshift:

Redshift:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \rightarrow a = \frac{1}{1+z} \Rightarrow 1+z = \frac{1}{a}$$

$$\lambda_{obs} = \frac{\lambda_{em}}{a}$$

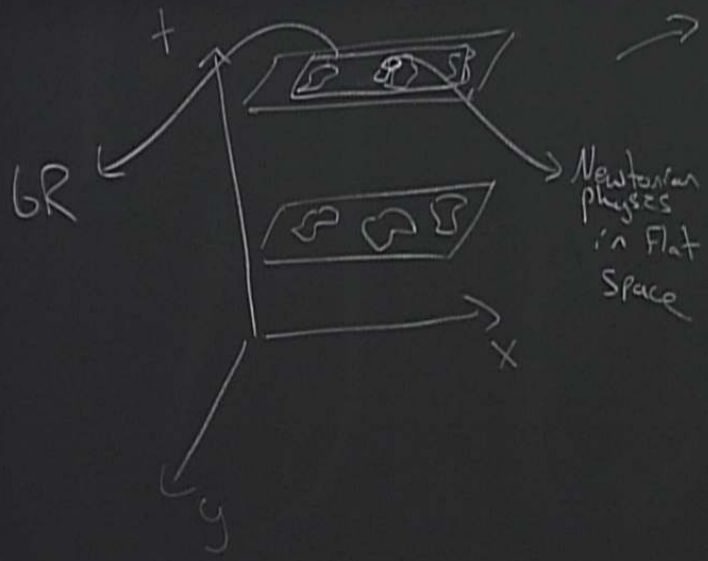
$$z = \frac{1}{a} - 1$$

$z_{eq} = 3402$, $z_* = 1090$ ← Formation of CMB

$z_\Lambda = .28$,

$z_0 = 0$,

Perturbations



→ Small fluctuations that simply "paint" the background.

To extent that you can neglect back-reaction, can do linear theory.

* On small length + time Scales – Newton in flat Space

laws: $\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$ continuity eqn

$\partial_t \vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} + \frac{1}{\rho}\vec{\nabla}p + \vec{\nabla}\Phi = 0$ Euler eqn

$\vec{\nabla}^2\Phi = 4\pi G_N \rho$ Poisson eqn

Variables: $\rho, p, u^\mu = (u^0, \vec{v})$

$p = w\rho$ EOS

* $ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1+2\Phi)d\vec{x}^2\right]$

What to do? Linearize!

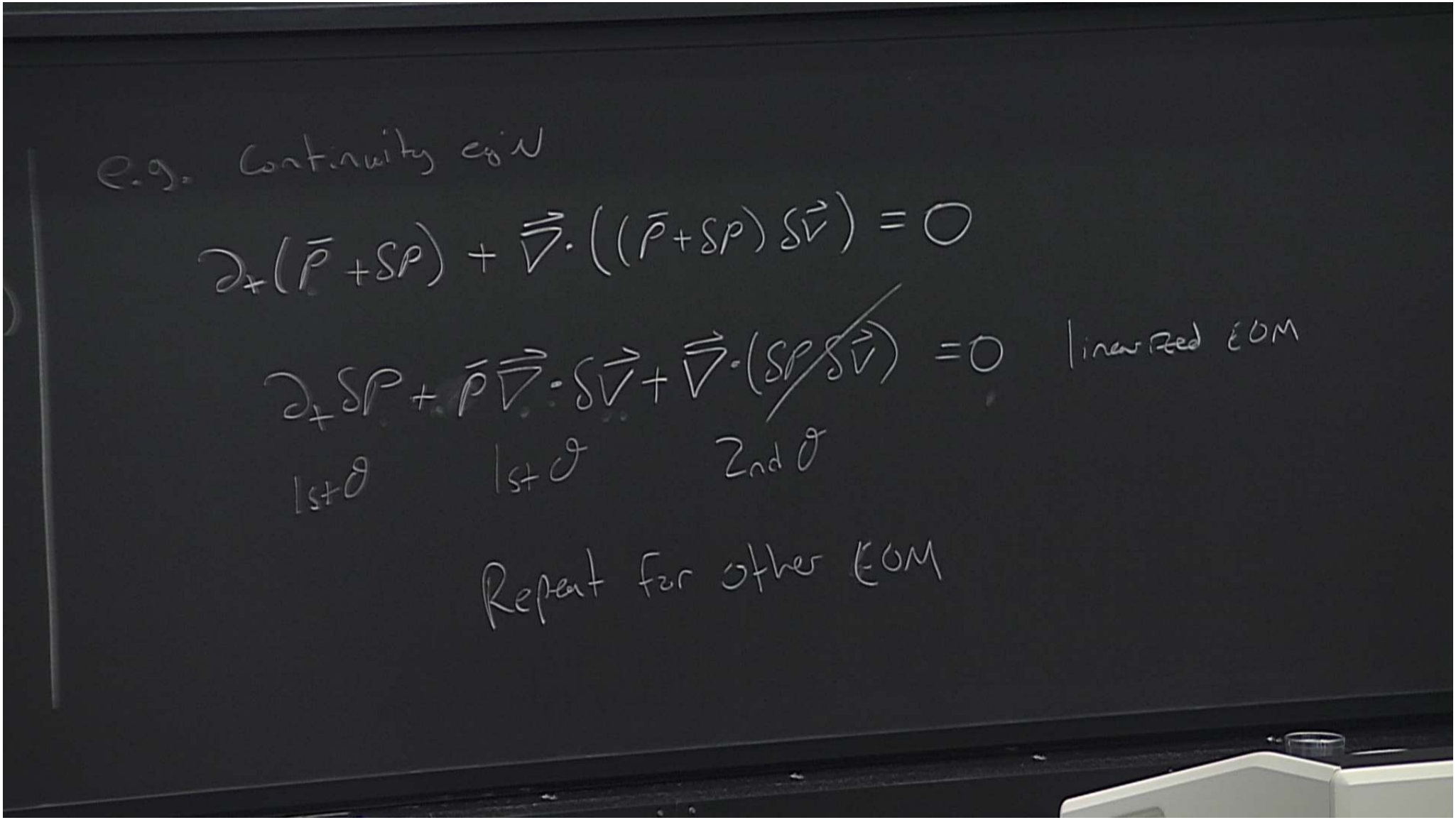
zeroth σ 1st σ

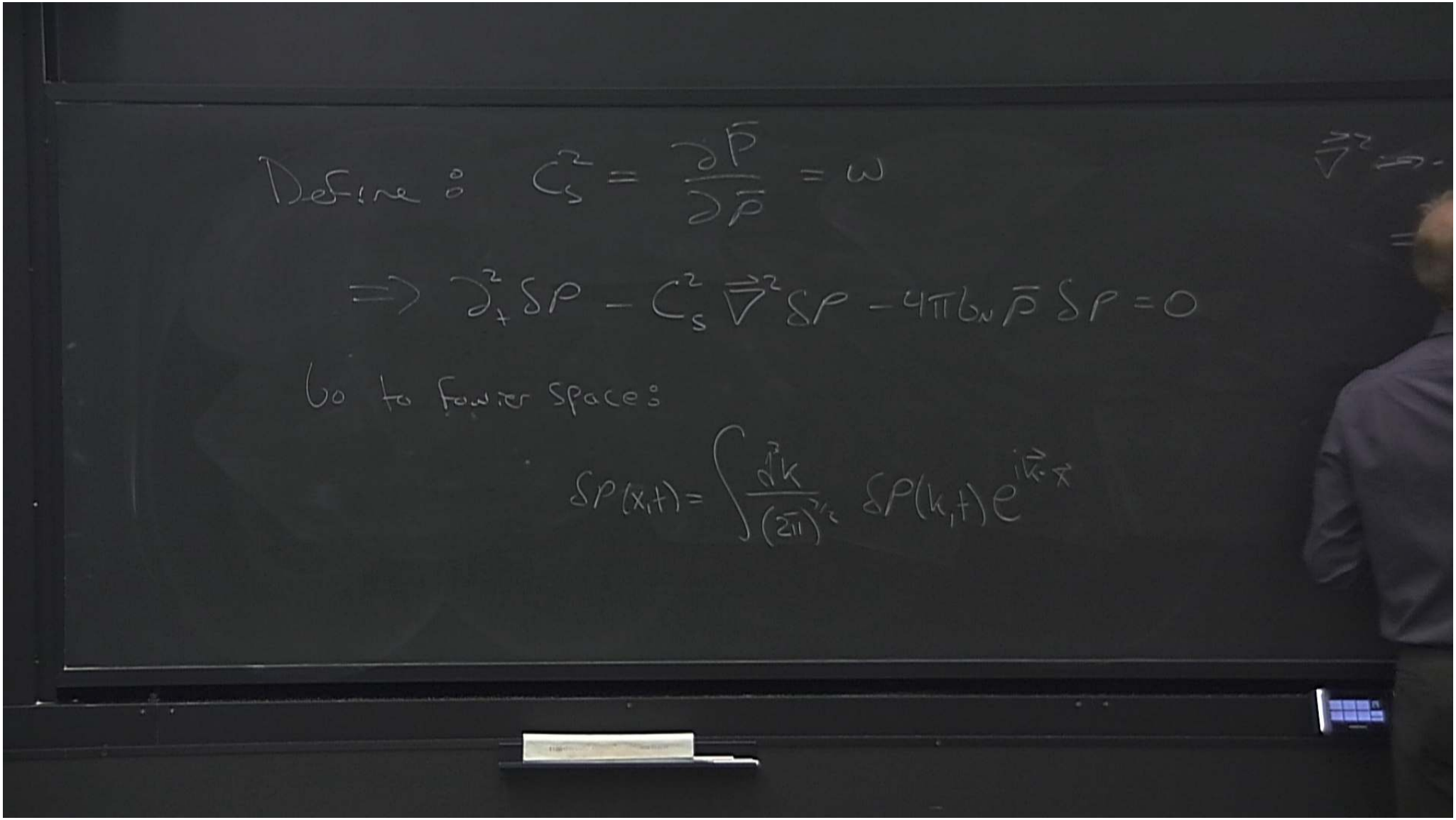
$$\rho = \bar{\rho} + \delta\rho(x,t) \quad , \quad \delta\rho \ll \bar{\rho} \text{ (linear)}$$

($\bar{\rho}$: const.)

$$\vec{V} = \delta\vec{V}(x,t)$$

$$\underline{\Phi} = \delta\underline{\Phi}(x,t)$$


e.g. Continuity eqN
$\partial_t(\bar{\rho}+\delta\rho) + \vec{\nabla}\cdot((\bar{\rho}+\delta\rho)\delta\vec{v}) = 0$
$\partial_t\delta\rho + \bar{\rho}\vec{\nabla}\cdot\delta\vec{v} + \vec{\nabla}\cdot(\delta\rho\,\delta\vec{v}) = 0$ linearized EOM
1st O
1st O
2nd O
Repeat for other EOM


Define: $c_s^2 = \frac{\partial \bar{p}}{\partial \bar{\rho}} = \omega$
$\Rightarrow \partial_t^2 \delta\rho - c_s^2 \vec{\nabla}^2 \delta\rho - 4\pi G_N \bar{\rho} \delta\rho = 0$
Go to Fourier Space:
$\delta\rho(x,t) = \int \frac{d^3k}{(2\pi)^3} \delta\rho(k,t) e^{i\vec{k}\cdot\vec{x}}$

*Note: Ω_i are Fractional density today

$\vec{\nabla}^2 \Rightarrow -k^2$

Pressure ↓ gravity ↓

$\Rightarrow \partial_t^2 \delta P(k,t) + \underbrace{\left[k^2 c_s^2 - 4\pi G_N \bar{\rho}\right]}_{\omega^2} \delta P = 0$

$\omega^2 > 0$ oscillatory

$\omega^2 < 0$ Growing/decaying

$\delta P \propto e^{\pm i\omega t}$

$\omega^2 = 0$ defines bndry → defines a scale, the Jeans Scale,

$k_J = \sqrt{\frac{4\pi G_N \bar{\rho}}{c_s^2}}$, $\lambda_J = \frac{2\pi}{k_J} =$

$\lambda > \lambda_J$, Growth / decay

$\lambda < \lambda_J$, Oscillations → Sound waves

Can this be applied to cosmology? Not if $t \sim H^{-1}$ $\ell \sim H^{-1}$

$H^2 = \frac{8\pi G_N}{3}\bar{\rho} \Rightarrow \lambda_J \simeq \sqrt{\frac{c_s^2}{H^2}} \simeq c_s H^{-1}$ $c_s^2 = w$

Matter: $w = 0 \Rightarrow c_s = 0$

$\Rightarrow \lambda_J = 0$

Collapse happens on all scales because no pressure

Radiation: $w = 1/3$, $c_s = \frac{1}{\sqrt{3}}$

$\lambda_J \simeq \frac{H^{-1}}{\sqrt{3}}$

Collapse of relativistic matter can't be described by Newton

$K_J = \sqrt{\frac{4\pi G_N \bar{\rho}}{c_s^2}}$

$\frac{H^2}{H_0^2}$

today

$\frac{H^2}{H_0^2} =$

Matter: $w=0 \Rightarrow c_s=0$

$\Rightarrow \lambda_J = 0$

Collapse happens on all scales because no pressure

Radiation: $w=1/3$, $c_s = \frac{1}{\sqrt{3}}$

$\lambda_J \simeq \frac{H^{-1}}{\sqrt{3}}$

Collapse of relativistic Matter can't be described by Newton

What about time scales?

$\omega \simeq i\sqrt{4\pi G_N \bar{\rho}} \simeq \frac{3}{2} H$

$space^{\omega t} \propto e^{t/\tau}$

$T \sim \frac{1}{\omega} \sim H^{-1}$

Bad, can't use Newtonian thy on all times

Can fix by adding expansion, for Matter