

Title: 13/14 PSI - Explorations in Cosmology - Lecture 2

Date: Apr 08, 2014 10:15 AM

URL: <http://pirsa.org/14040042>

Abstract:

FRW: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

$$ds^2 = a(z)^2 [-dz^2 + d\vec{x}^2], \quad z = \int \frac{dt}{a}$$

Hubble
parameter

$$\nabla_\mu T^\mu_\nu \Rightarrow \nu=0 \rightarrow \frac{\partial \rho}{\partial t} = -3(1+w)\frac{\dot{a}}{a} \Rightarrow \rho = \rho_0 a^{-3(1+w)}$$

* Stick metric + $T_{\mu\nu}$ for a perfect fluid into Einstein: Friedmann equations

Hubble parameter $\rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \sum_i \rho_i - \frac{K}{a^2}$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3} \sum_i (\rho_i + 3p_i) = -\frac{4\pi G_N}{3} \sum_i \rho_i (1 + 3w_i)$$

$$\rho_0 a^{-3(1+w)}$$

more equations

Hubble parameter $\rightarrow$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{\kappa}{a^2}$$

$\uparrow$
 $\frac{8\pi G}{3} \rho_c$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i) = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i)$$

* Accelerated expansion:

for $\ddot{a} > 0$, need $w_i < -\frac{1}{3}$

(Matter, $w=0$
radiation, $w=1/3$
Curvature, $w=-1/3$
 Λ , $w=-1$)

Energy Budget :

* Convention - Scale factor today is 1 ($a(t_0)=1$)

Hubble parameter today is $H_0 = 100h \frac{\text{km}}{\text{s} \cdot \text{Mpc}} = (14.6 \text{ Gyr})^{-1} = (4500 \text{ Mpc})^{-1} = 10^{-33} \text{ eV}$

$\downarrow$ Const. = -67 From Planck

$$\frac{H^2}{H_0^2} = \sum_i \frac{\Omega_i}{a^{3(1+w_i)}}$$

$$\Omega_i = \frac{8\pi G \rho_{0,i}}{3 H_0^2}$$

"P"

today: $1 = \sum_i \Omega_i$ ← fraction of energy density in different components

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_c + \overset{=\Omega_m}{\Omega_b}}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

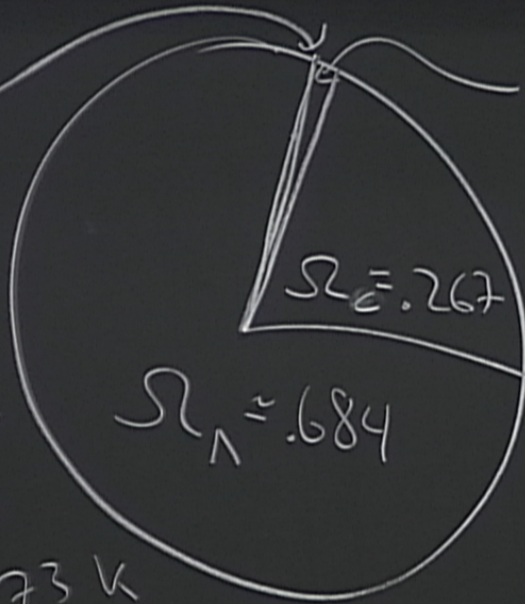
*Note: Ω_i are fractional density today!

$$\Omega_r \approx 10^{-5}$$

CMB
calculated
from

ρ for
photons @

$$T = 2.73 \text{ K}$$



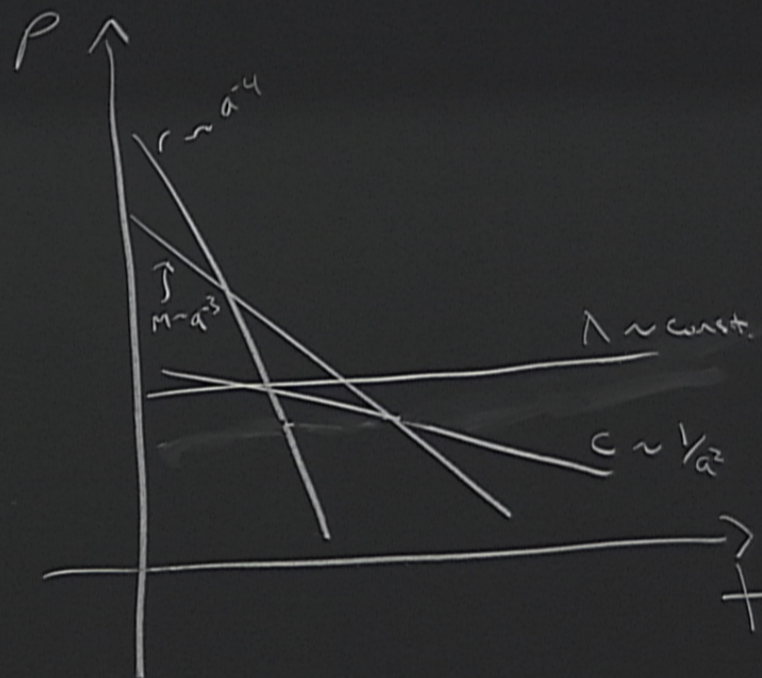
$$\Omega_b = .049$$

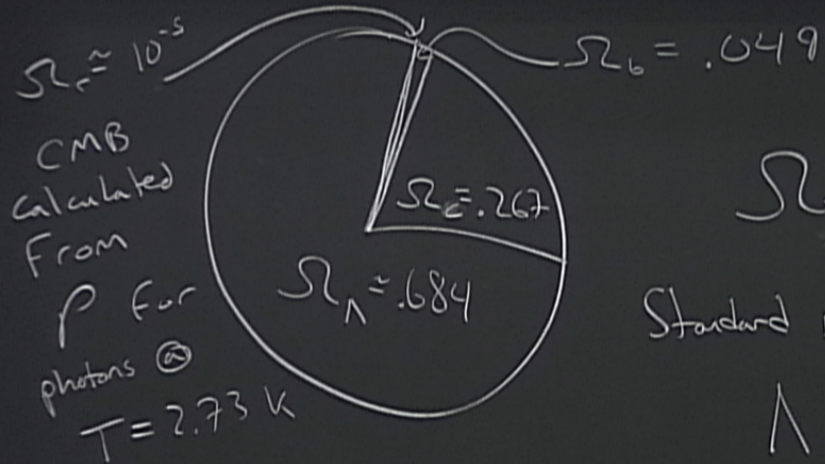
$$\Omega_k \lesssim \underline{10^{-2} - 10^{-3}}$$



$$\Theta = \frac{d}{L}$$

Standard rulers used to
measure $\Omega_k \rightarrow$ best one
is Baryon Acoustic Oscillations





$$\Omega_k \lesssim 10^{-2} - 10^{-3}$$

Standard model of Cosmology

Λ CDM

↓
CC

↓
Cold Dark Matter



$$\theta = \frac{d}{L}$$

Standard rulers used to
measure $\Omega_k \rightarrow$ best one
is Baryon Acoustic Oscil

* Matter - Radiation equality:

$$\frac{\Omega_r}{a_{eq}^4} = \frac{\Omega_m}{a_{eq}^3} \Rightarrow a_{eq} = \frac{\Omega_r}{\Omega_m} \approx \frac{1}{3000}$$

* Λ -domination

$$\frac{\Omega_m}{a_{\Lambda}^3} = \Omega_{DE} \Rightarrow a_{\Lambda} = \left(\frac{\Omega_m}{\Omega_{DE}} \right)^{1/3} \approx .78$$

Redshift:

Redshift :

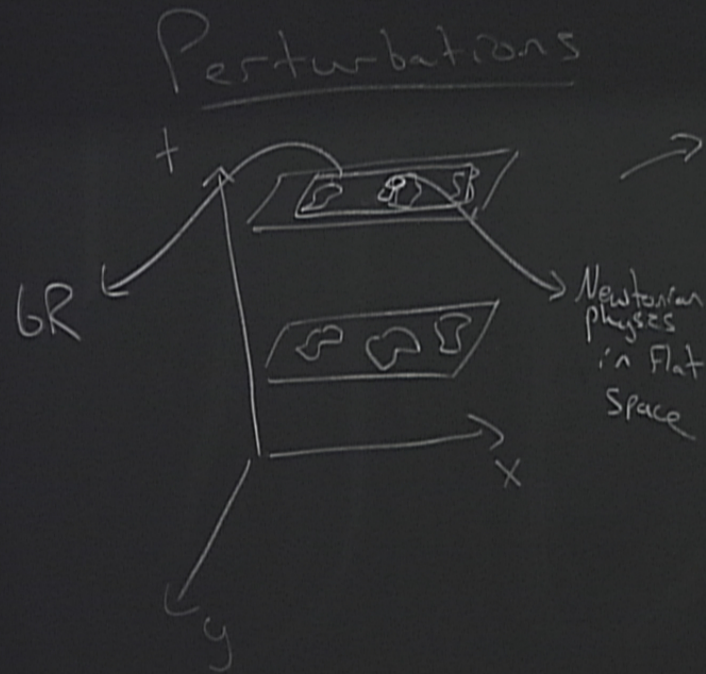
$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} \quad \lambda_{\text{obs}} = \frac{\lambda_{\text{em}}}{a} \quad \rightarrow \quad a = \frac{1}{1+z} \quad \rightarrow \quad 1+z = \frac{1}{a} \quad z = \frac{1}{a} - 1$$



$$z_{\text{eq}} = 3402, \quad z_* = 1090$$

↑ Formation of CMB

$$z_1 = .28,$$
$$z_0 = 0,$$



Small fluctuations
that simply "Punt"
the background.

To extent that you can
neglect back-reaction,
Can do linear theory.

* on small length + time scales - Newton in Flat Space

laws: $\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0$ continuity eqn

$\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} + \frac{1}{\rho} \vec{\nabla} p + \vec{\nabla} \Phi = 0$ Euler eqn

$\vec{\nabla}^2 \Phi = 4\pi G \rho$ Poisson eqn

Variables: $\rho, p, u^\mu = (u^0, \vec{v})$

$p = w\rho$ EOS

* $ds^2 = a^2(t) [-(1+2\Phi)dt^2 + (1-2\Phi)d\vec{x}^2]$

What to do? Linearize!

$$P = \overset{\text{2nd } \sigma}{\bar{P}} + \overset{\text{1st } \sigma}{\delta P(x,t)}, \quad \delta P \ll \bar{P} \quad (\text{linear})$$

$\uparrow$
const.

$$\vec{V} = \delta \vec{V}(x,t)$$

$$\underline{\Phi} = \delta \underline{\Phi}(x,t)$$

e.g. continuity eqn

$$\partial_t(\bar{P} + SP) + \vec{\nabla} \cdot ((\bar{P} + SP) \delta \vec{V}) = 0$$

$$\underbrace{\partial_t SP}_{1st\ \mathcal{O}} + \underbrace{\bar{P} \vec{\nabla} \cdot \delta \vec{V}}_{1st\ \mathcal{O}} + \underbrace{\vec{\nabla} \cdot (SP \delta \vec{V})}_{2nd\ \mathcal{O}} = 0 \quad \text{linearized EOM}$$

Repeat for other EOM

Define $\circ \quad C_s^2 = \frac{\partial \bar{P}}{\partial \rho} = \omega$

$$\Rightarrow \partial_t^2 \delta P - C_s^2 \vec{\nabla}^2 \delta P - 4\pi G_N \bar{\rho} \delta P = 0$$

Go to Fourier space:

$$\delta P(x,t) = \int \frac{d^3 k}{(2\pi)^{3/2}} \delta P(k,t) e^{i\vec{k} \cdot \vec{x}}$$

*Note: Ω_i are fractional density today!

$$\nabla^2 \Rightarrow -k^2$$

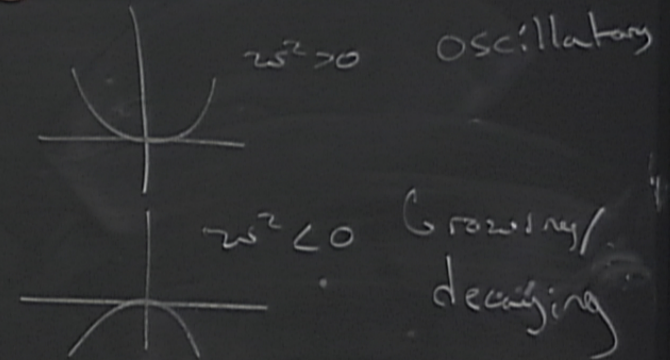
$$\Rightarrow \partial_t^2 SP(k,t) + \underbrace{\left[k^2 c_s^2 - 4\pi G \bar{\rho} \right]}_{w^2} SP = 0$$

Pressure
gravity

$$SP \propto e^{\pm i w t}$$

$w^2 = 0$ defines bndry $\rightarrow$ defines a scale, the Jean's Scale.

$$K_J = \sqrt{\frac{4\pi G \bar{\rho}}{c_s^2}}, \quad \lambda_J = \frac{2\pi}{K_J}$$



$$K_J = \sqrt{\frac{4\pi G \rho}{c_s^2}}$$

$\lambda > \lambda_J$, Growth / decay
 $\lambda < \lambda_J$, Oscillations $\rightarrow$ Sound waves

Matter: $w=0 \rightarrow c_s=0$
 $\Rightarrow \lambda_J=0$

Collapse happens on all scales
 because no pressure

Radiation: $w=1/3$, $c_s = \frac{1}{\sqrt{3}}$

$$\lambda_J \approx \frac{H^{-1}}{\sqrt{3}}$$

Collapse of relativistic matter
 Can't be described by Newton

Can this be applied to cosmology? Not if $t \sim H^{-1}$
 $\ell \sim H^{-1}$

$$H^2 = \frac{8\pi G}{3} \rho \Rightarrow \lambda_J \approx \sqrt{\frac{c_s^2}{H^2}} \approx c_s H^{-1} \quad c_s^2 = w$$

Matter: $w=0 \rightarrow c_s=0$

$$\Rightarrow \lambda_J = 0$$

Collapse happens on all scales
because no pressure

Radiation: $w=1/3, c_s=1/\sqrt{3}$

$$\lambda_J \approx \frac{H^{-1}}{\sqrt{3}}$$

Collapse of relativistic matter
can't be described by Newton

What about time scales?

$$w \approx \frac{1}{3} \Rightarrow \rho \approx \frac{3}{2} H^2$$

$$S^3 \propto e^{3Ht} \propto e^{+4t}$$

$$T \sim \frac{1}{w} \sim H^{-1}$$

Bad, can't use Newtonian thing on all times

Can fix by adding expansion for Matter