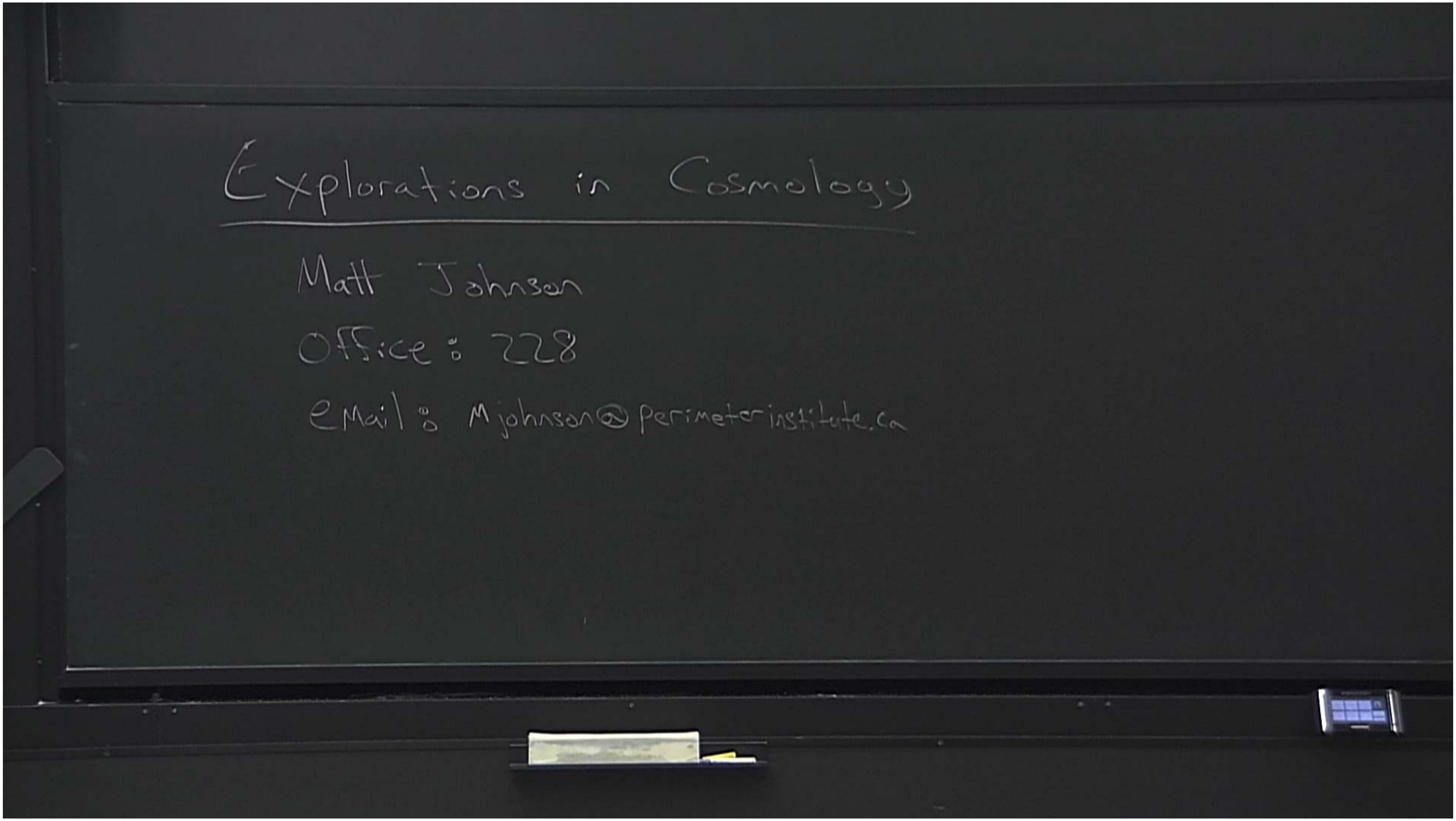
Title: 13/14 PSI - Explorations in Cosmology - Lecture 1

Date: Apr 07, 2014 10:15 AM

URL: http://pirsa.org/14040041

Abstract:


Explorations in Cosmology
Matt Johnson
Office: 228
email: mjohnson@perimeterinstitute.ca

1) Linear Universe ~ 1.5 weeks

2) Eternal Inflation ~3-lectures (Dan Wohns)

3) Calculating Inflationary perterbations ~3 lectures (Paul McFadden)

Cosmology

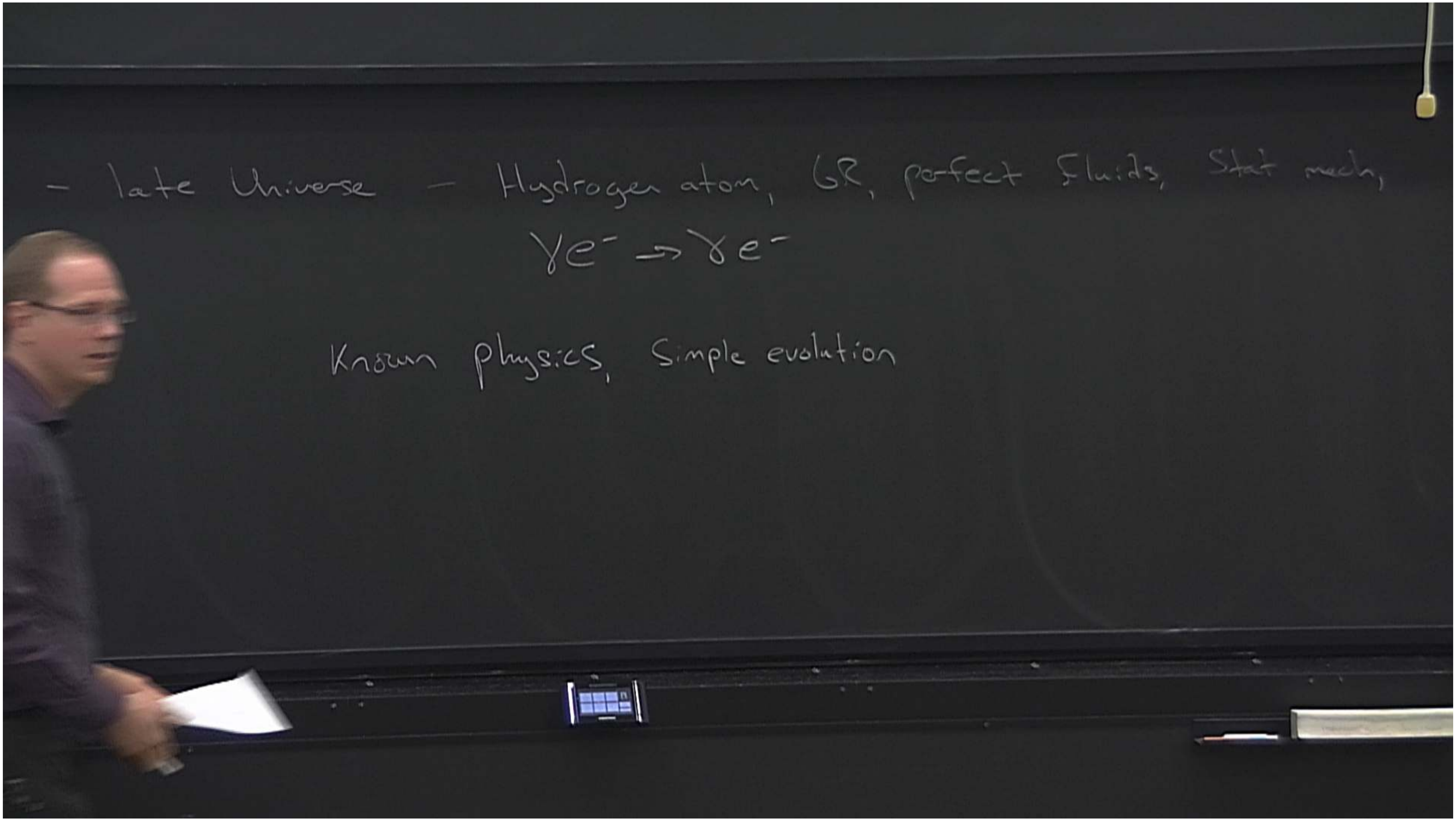
$t <$ seconds — Early Universe ~ Particle Physics, Quatum Gravity, etc.

↑
BBN

Speculative, poorly understood, Unknown initial conditions

<u>But</u> concrete predictions → e.g. Inflation →

Hard but not impossible


- late Universe - Hydrogen atom, GR, perfect fluids, Stat mech,
$\gamma e^- \to \gamma e^-$
Known physics, Simple evolution

- late Universe - Hydrogen atom, GR, perfect fluids, Stat mech,

$\gamma e^- \to \gamma e^-$

Known physics, Simple evolution, Concrete predictions

- late Universe – Hydrogen atom, GR, perfect fluids, Stat mech,

$$\gamma e^- \to \gamma e^-$$

Known physics, simple evolution, concrete predictions

Difficult but do-able

$\boxed{\text{Linear Universe}}$

- late Universe — Hydrogen atom, GR, perfect fluids, Stat mech,

$$\gamma e^- \to \gamma e^-$$

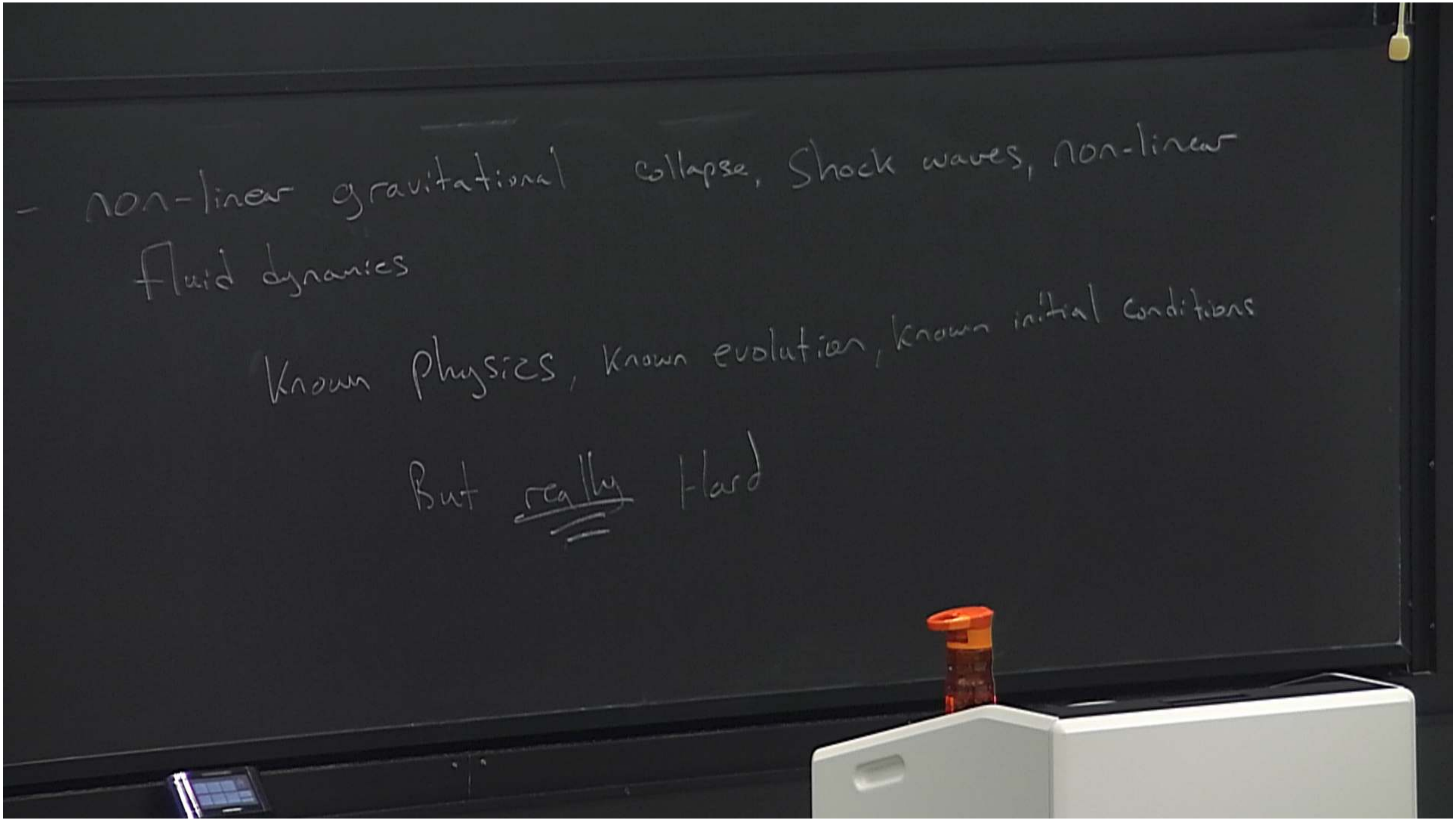
Known physics, Simple evolution, concrete predictions

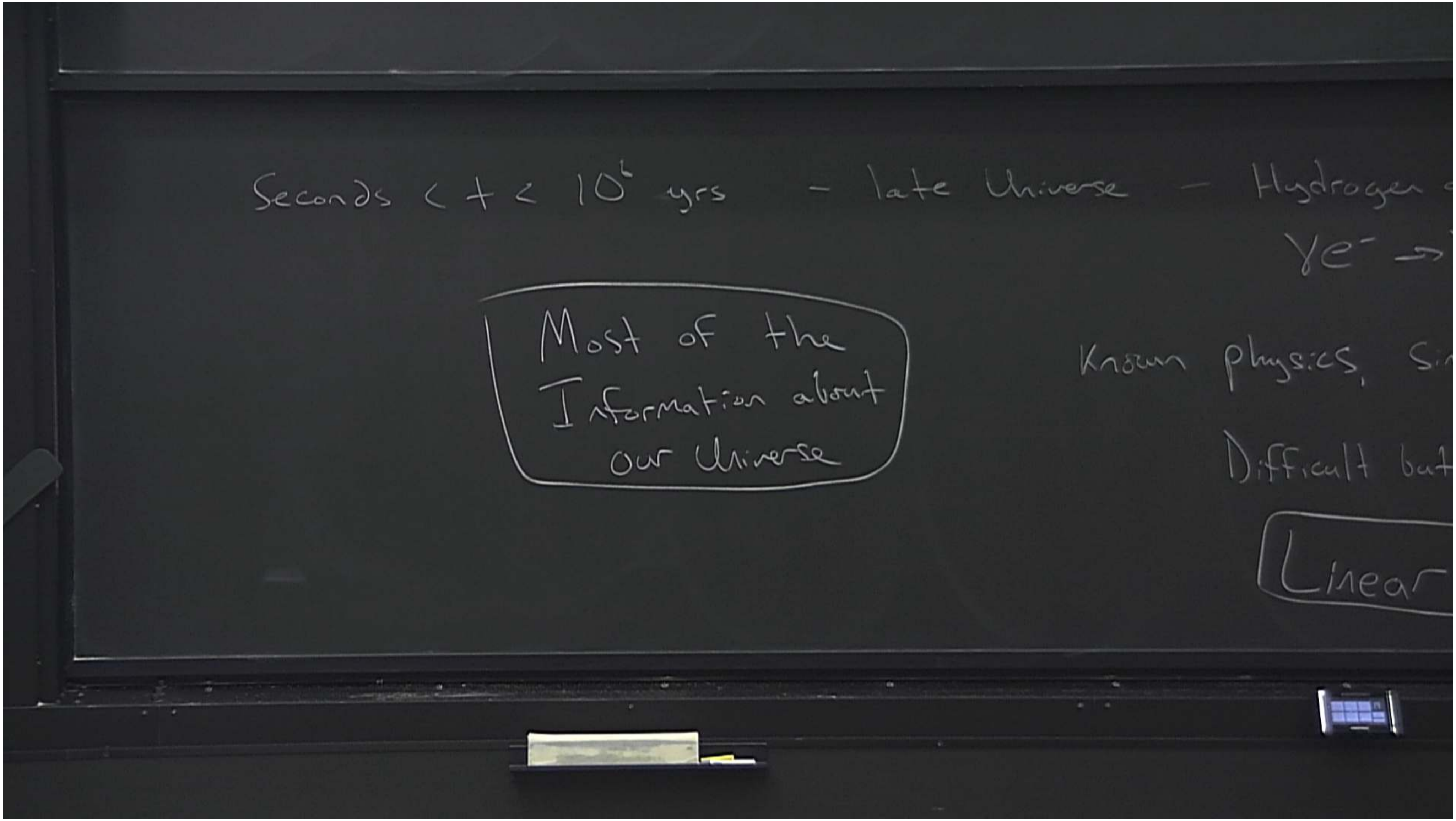
Difficult but do-able

$\boxed{\text{Linear Universe}}$ → Use Perturbation theory

$10^6 < t < 10^9$ yrs $\rightarrow$ Present Universe – non-linear gravit

fluid dynamics


- non-linear gravitational collapse, Shock waves, non-linear
fluid dynamics
Known physics, known evolution, known initial conditions
But really Hard


Seconds < t < 10^6 yrs - late Universe - Hydrogen
γe⁻ →
Most of the Information about our Universe
Known physics,
Difficult but
Linear

- Hydrogen atom, GR, perfect fluids, Stat mech,

$\gamma e^- \to \gamma e^-$

Makes Precision Cosmology Possible

...n physics, Simple evolution, Concrete predictions

Difficult but do-able

Linear Universe → Use Perturbation theory

What is the "theory" of the late Universe?

Relativistic Hydrodynamics

General Relativity

Governing equations:

1) metric – $ds^2 = a(\tau)^2\left[-(1+2\Psi)d\tau^2 + (1+2\Phi)d\vec{x}^2\right]$

FRW + small deviations $(\Psi, \Phi \ll 1)$

2) Einstein's Eq'ns: $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$

density, pressure, metric

3) Sources: $T_{\mu\nu} = \sum_i (\rho_i + P_i) U^i_\mu U^i_\nu + P_i g_{\mu\nu} + \cdots$

4-velocities

Perfect Fluid

Multiple Fluids → photons, visible matter (baryons), dark matter, dark energy, ...

Linear Universe

4) Conservation laws : $\nabla_\mu T^\mu{}_\nu = 0$

covariant derivative

5) Equation of state : $P_i = w_i \rho_i$

e.g. Matter : $w=0$

radiation : $w=1/3$

Λ : $w=-1$

6) Distribution functions:

$$\rho_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k)\, E_i(k)$$

degeneracy

distribution function

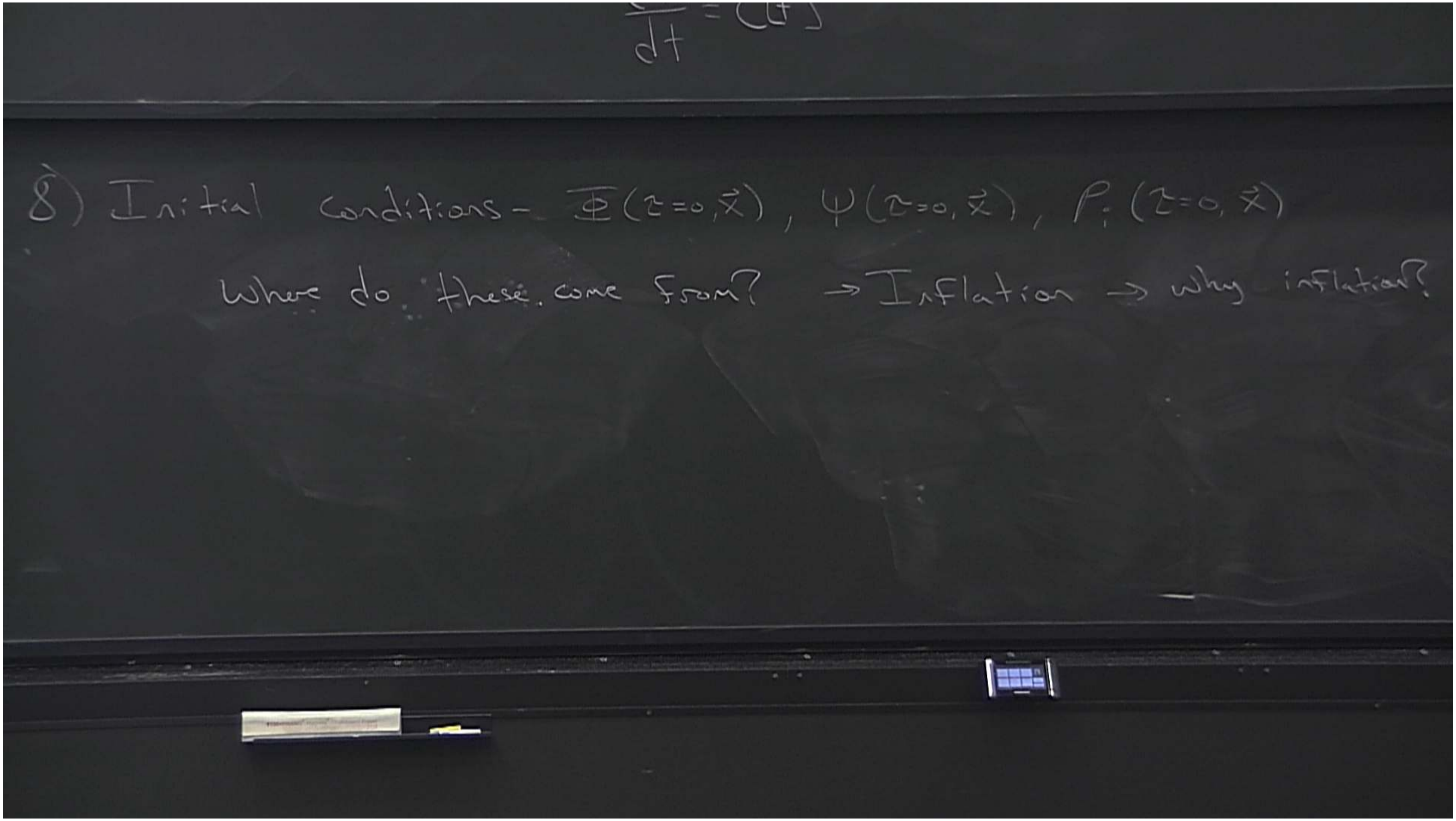
energy

7). Boltzmann equations:

$$\frac{dF}{dt} = 0 \quad \text{(no interaction)} \qquad f_{BE} = \frac{1}{e^{(E-\mu)/T} - 1}$$

$$\frac{df}{dt} = C[f]$$

Collisions


8) Initial conditions - $\Phi(\tau=0,\vec{x})$, $\Psi(\tau=0,\vec{x})$, $\rho_i(\tau=0,\vec{x})$
Where do these come from? $\rightarrow$ Inflation $\rightarrow$ why inflation?

FRW + Small deviations ($\Psi, \Phi \ll 1$)

Homogeneous Universe: Perfect fluids, FRW metric, Cons of $T^{\mu}{}_{\nu}$ + Fri

FRW metric: $$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{\left(1+\frac{kr^2}{4}\right)^2} + r^2\, d\Omega_2^2\right]$$

scale factor

metric on S^2

$$d\Omega_2^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

$k<0$ - open

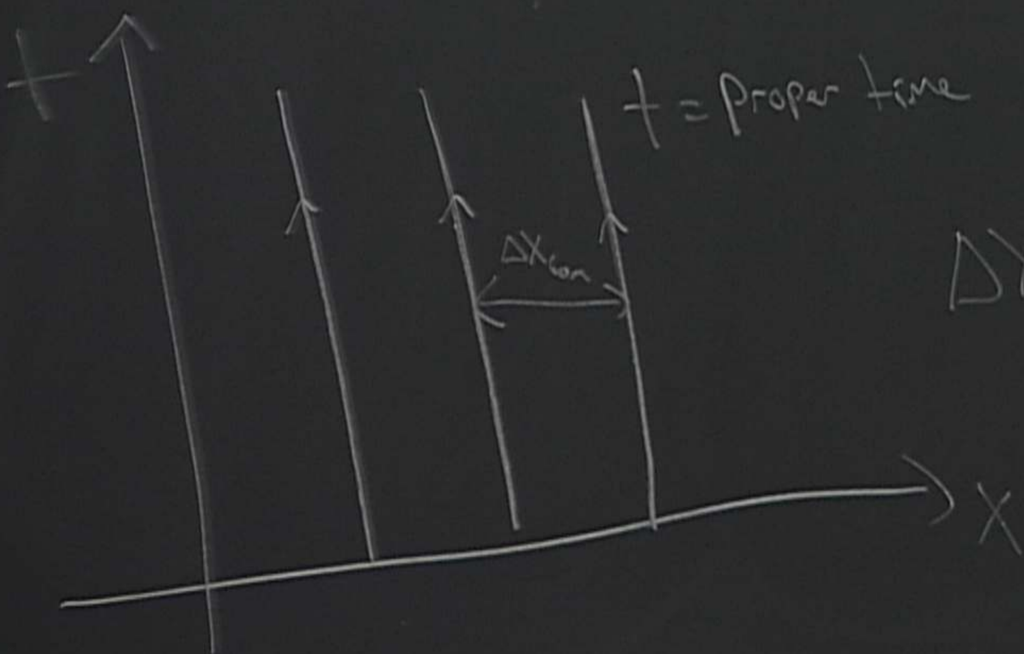
$\boxed{k=0 \text{ - flat}}$

$k>0$, closed

Homogeneous Universe: Perfect fluids, ...

FRW metric: $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{\left(1+\frac{kr^2}{4}\right.}\right.$

Scale factor

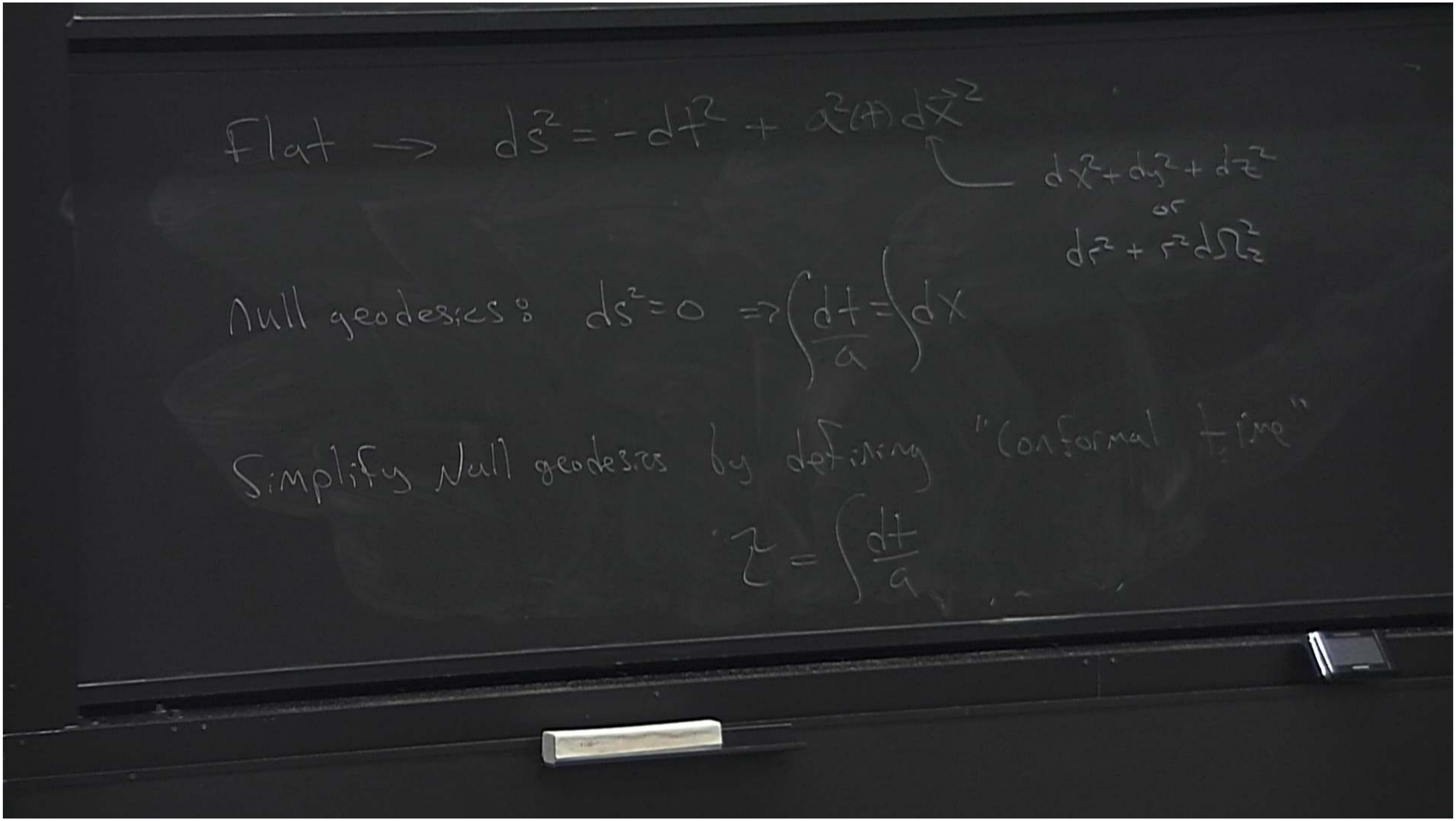


$\Delta X_{physical} = a\,\Delta X_{comoving}$

$k<0$

$\boxed{k=0}$

$k>$

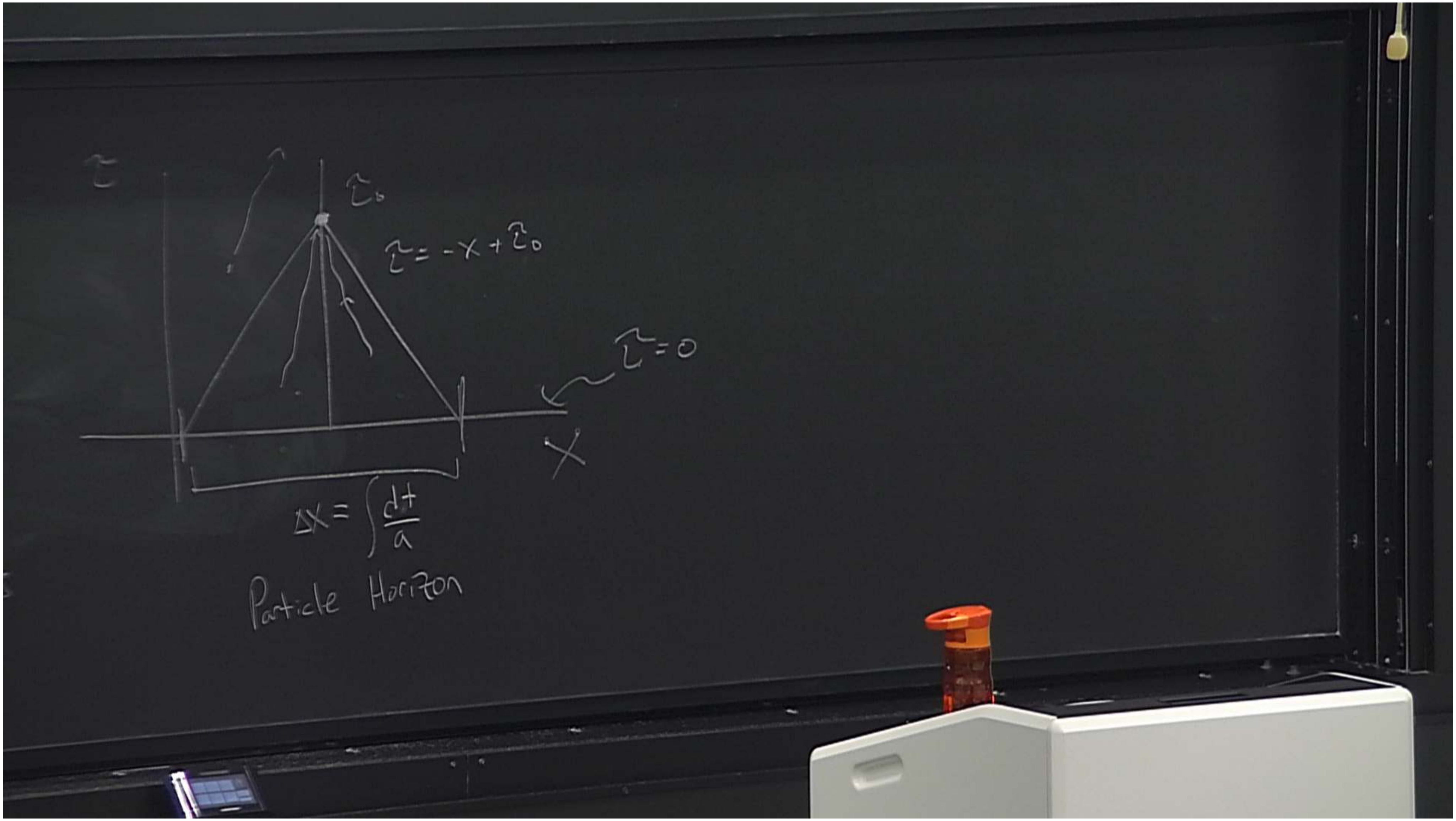

Flat $\rightarrow$ $ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
$dx^2 + dy^2 + dz^2$
or
$dr^2 + r^2 d\Omega_2^2$
Null geodesics: $ds^2 = 0 \Rightarrow \int \frac{dt}{a} = \int dx$
Simplify Null geodesics by defining "Conformal time"
$\tau = \int \frac{dt}{a}$

$$dt^2 = a^2 d\tau^2$$

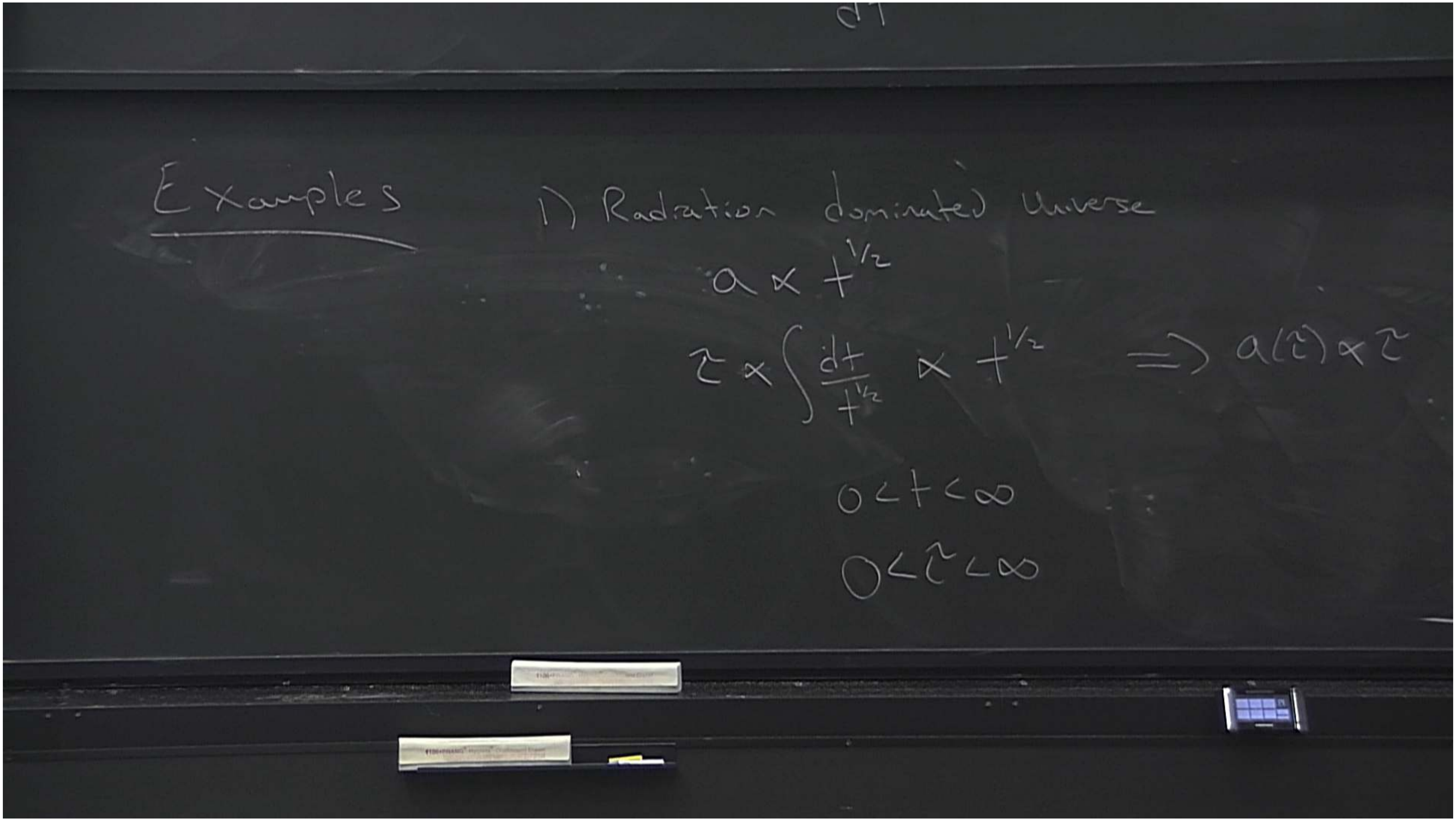
$$\Rightarrow ds^2 = \underbrace{a^2(\tau)}_{\text{Conformal Factor}}\left[-d\tau^2 + d\vec{x}^2\right]$$

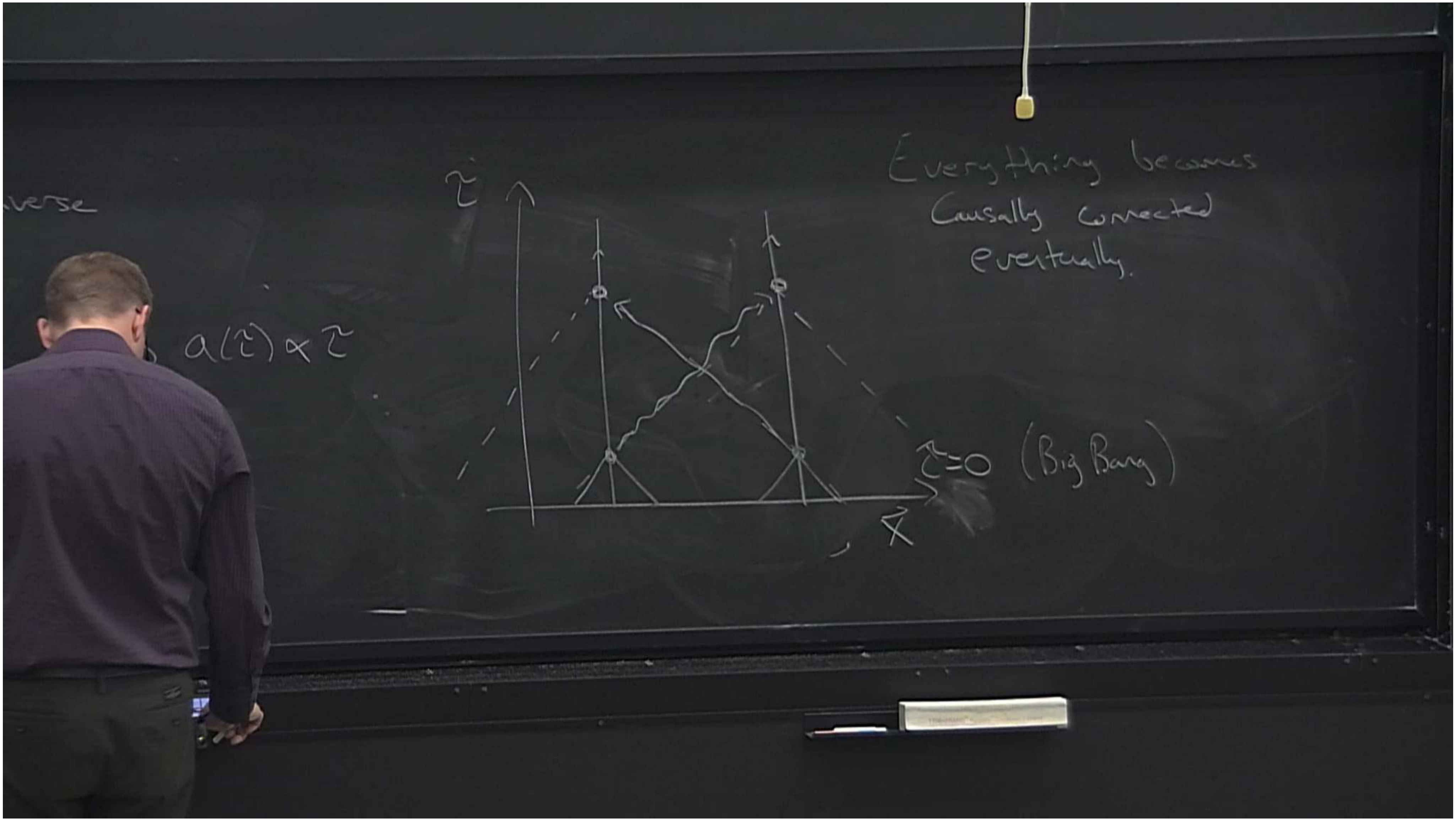
Null geodesics: $ds^2 = 0 \Rightarrow d\tau = d\vec{x}$ travel on 45° lines

$$\tau = \pm x + \text{const.}$$

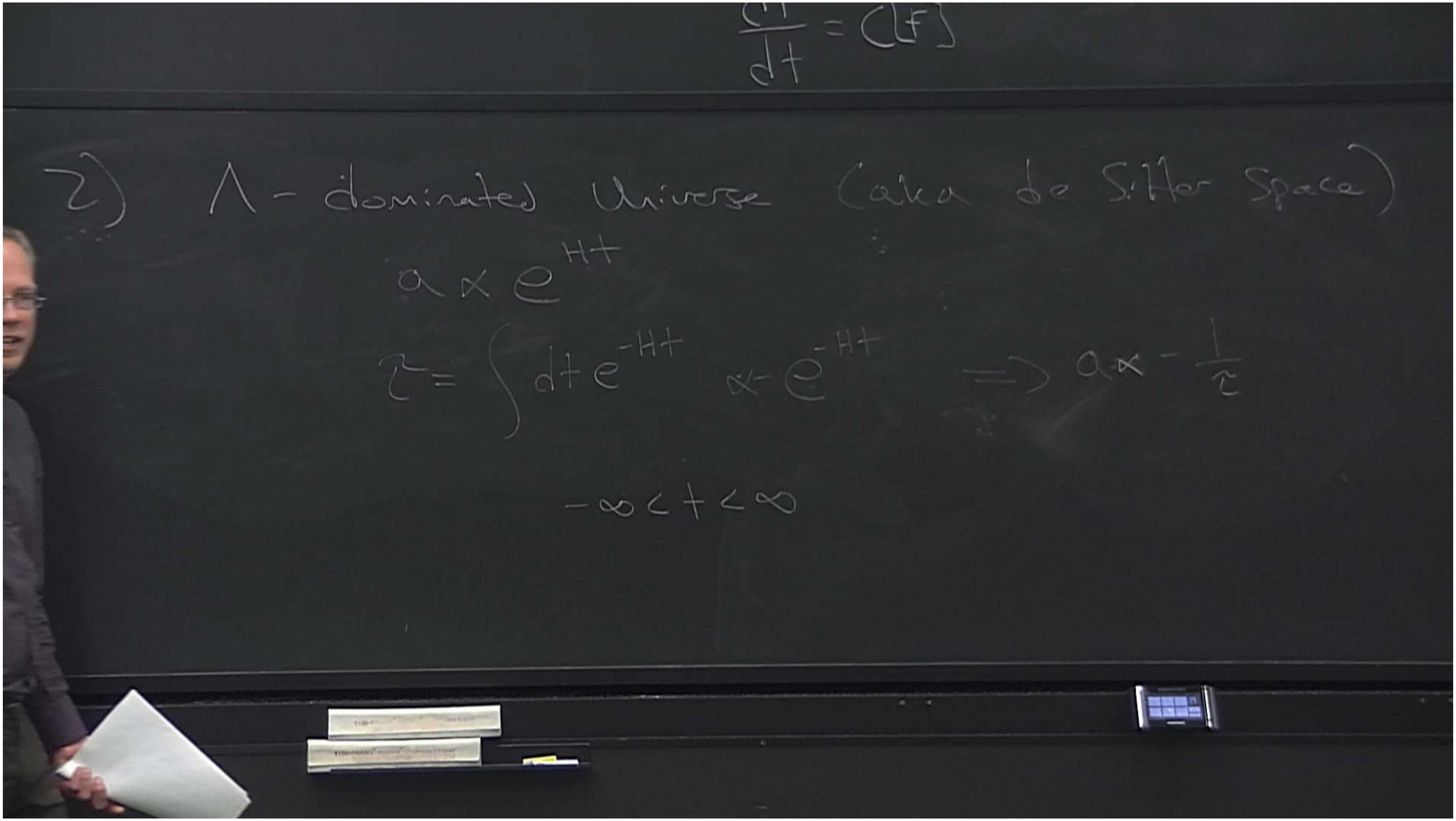


τ
τ_0
$\tau = -x + \tau_0$
$\tau = 0$
x
$\Delta x = \int \frac{dt}{a}$
Particle Horizon


Examples
1) Radiation dominated Universe
$a \propto t^{1/2}$
$\tau \propto \int \frac{dt}{t^{1/2}} \propto t^{1/2} \Longrightarrow a(\tau) \propto \tau$
$0 < t < \infty$
$0 < \tau < \infty$



verse
a(τ) ∝ τ
τ
Everything becomes
Causally connected
eventually.
τ=0 (Big Bang)
x


$\frac{d}{dt} = C[F]$
2) Λ - dominated Universe (aka de Sitter Space)
$a \propto e^{Ht}$
$\tau = \int dt\, e^{-Ht} \propto -e^{-Ht} \Rightarrow a \propto -\frac{1}{\tau}$
$-\infty < t < \infty$

$$\frac{d\ }{dt} = C[F]$$

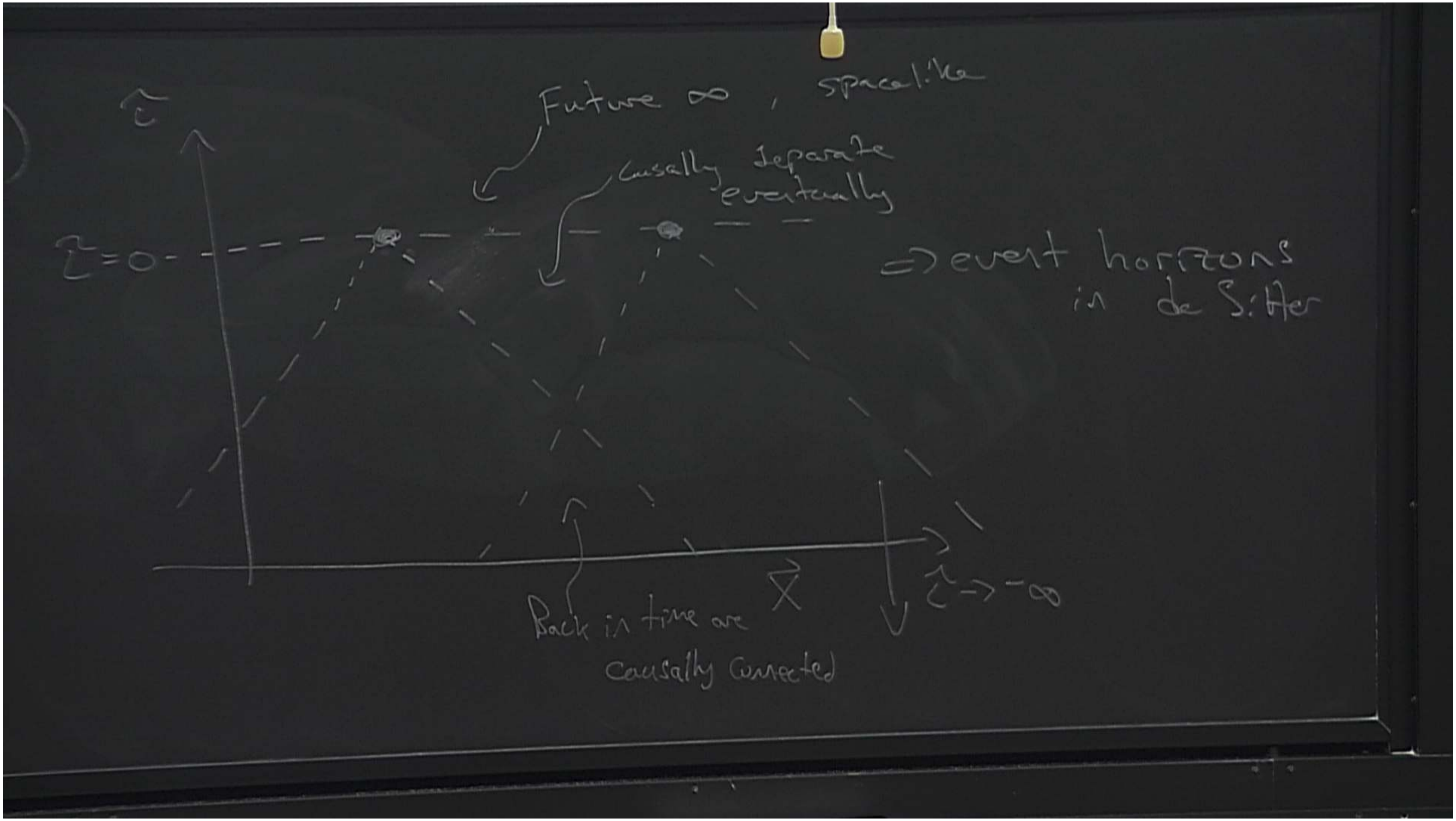
2) Λ - dominated Universe (aka de Sitter Space)

$$a \propto e^{Ht}$$

$$\tau = \int dt\, e^{-Ht} \propto -e^{-Ht} \Longrightarrow a \propto -\frac{1}{\tau}$$

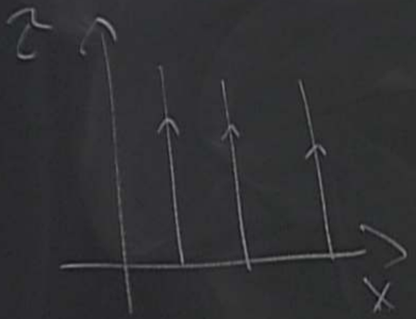
$$-\infty < t < \infty$$

$$-\infty < \tau < 0^-$$


Future ∞ , spacelike
causally separate
eventually
⇒ event horizons
in de Sitter
Back in time are
causally connected

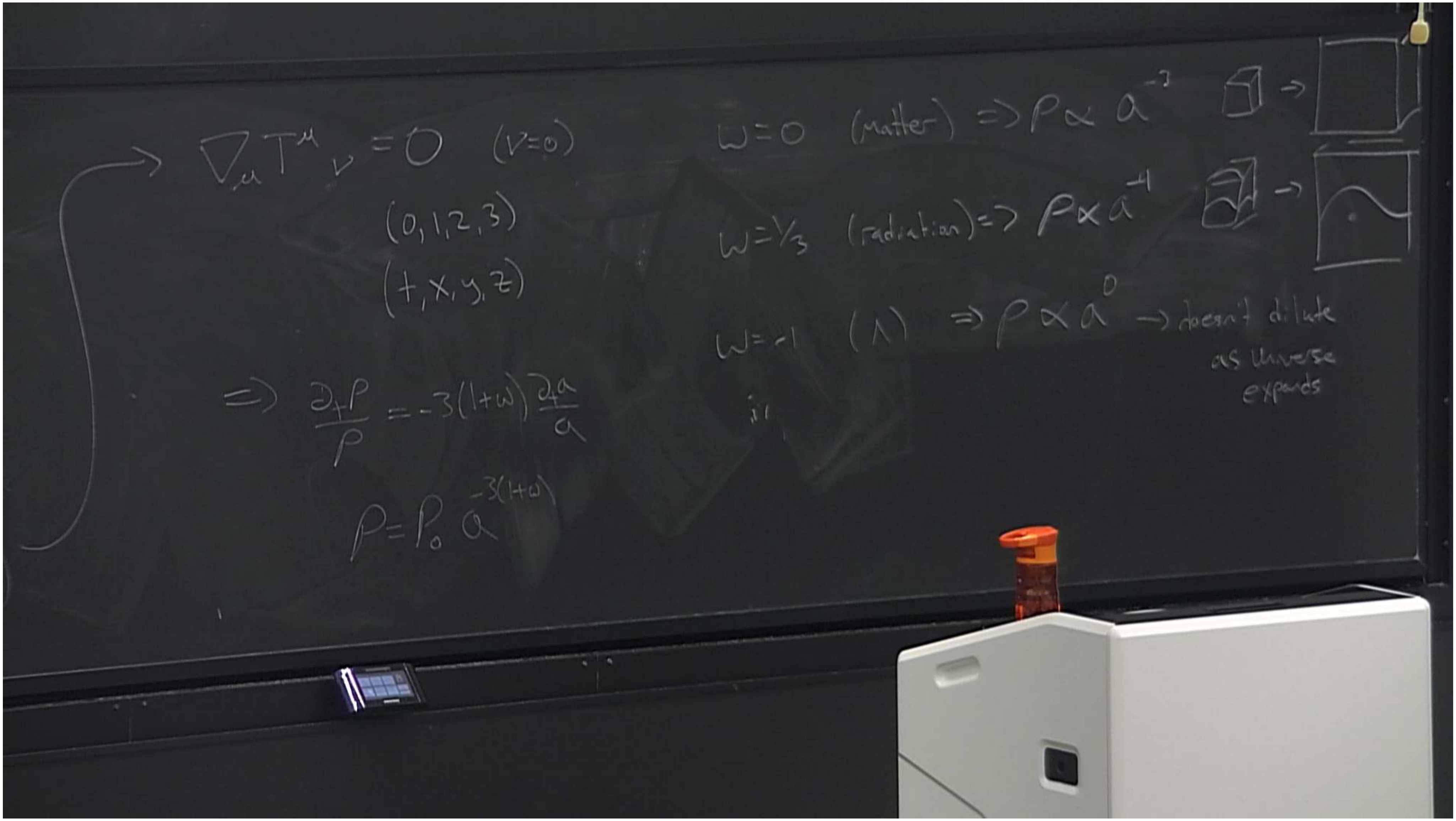
Back to FRW + evolution:

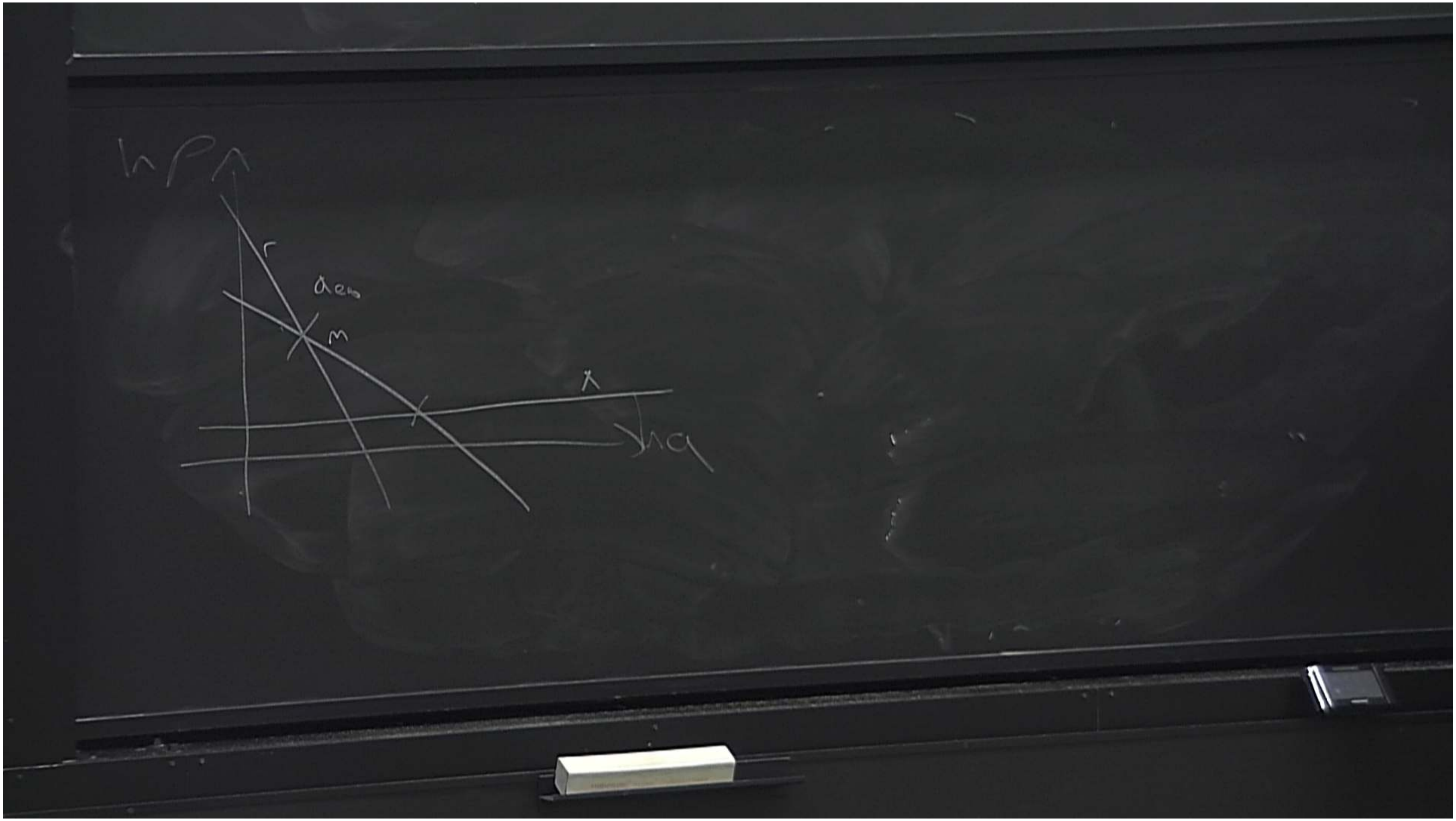
* $T^{\mu}{}_{\nu}$



4-velocity of fluid

$$u^{\mu} = (1,0,0,0)$$


$\nabla_\mu T^\mu{}_\nu = 0 \quad (\nu = 0)$
$(0,1,2,3)$
(t,x,y,z)
$\Rightarrow \frac{\partial_t \rho}{\rho} = -3(1+w)\frac{\partial_t a}{a}$
$\rho = \rho_0 a^{-3(1+w)}$
$w=0$ (matter) $\Rightarrow \rho \propto a^{-3}$
$w=1/3$ (radiation) $\Rightarrow \rho \propto a^{-4}$
$w=-1$ (Λ) $\Rightarrow \rho \propto a^{0}$ → doesn't dilute as Universe expands


ln ρ
r
a_em
m
λ
ln a