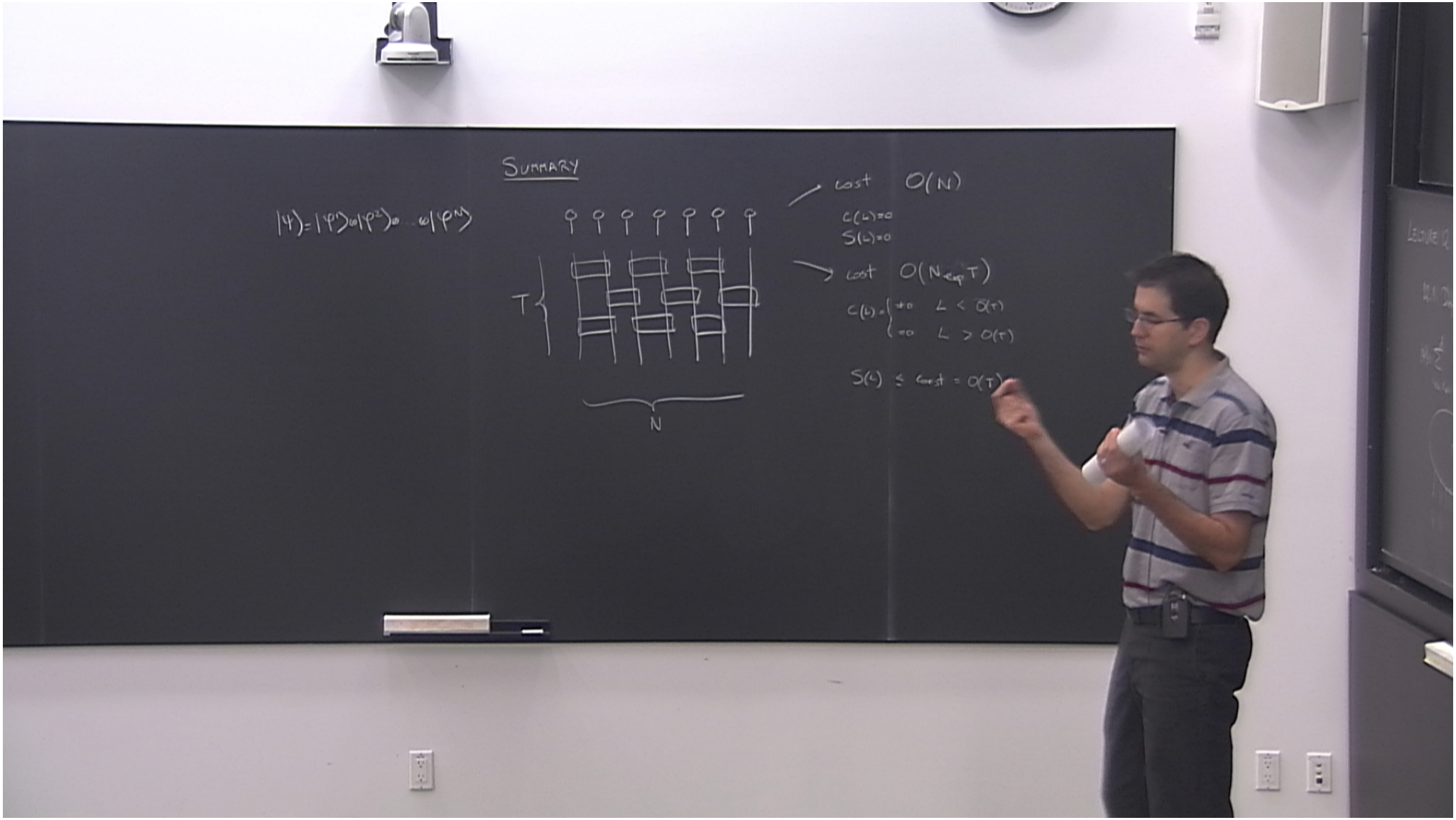
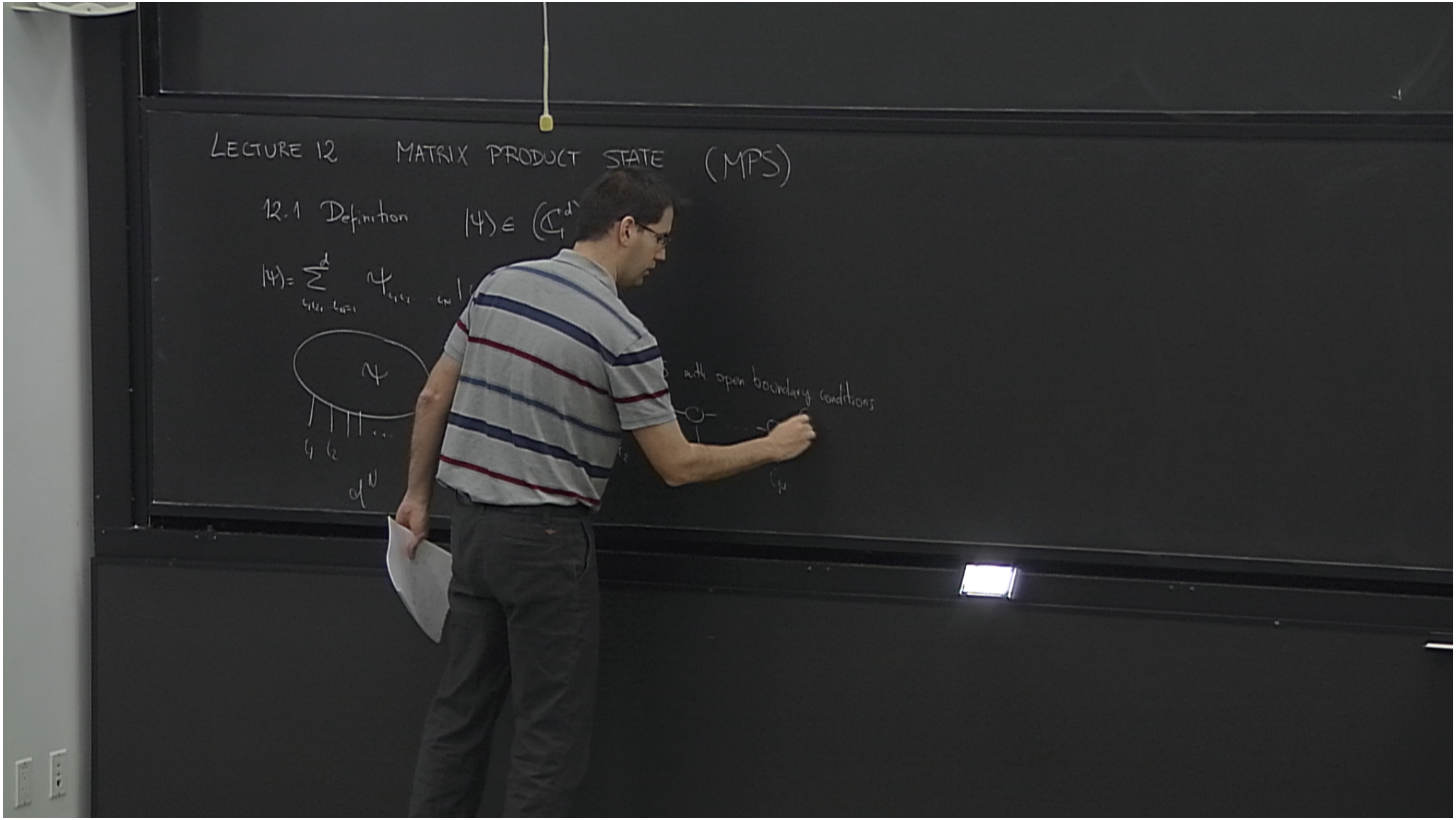
Title: 13/14 PSI - Explorations in Condensed Matter - Lecture 12

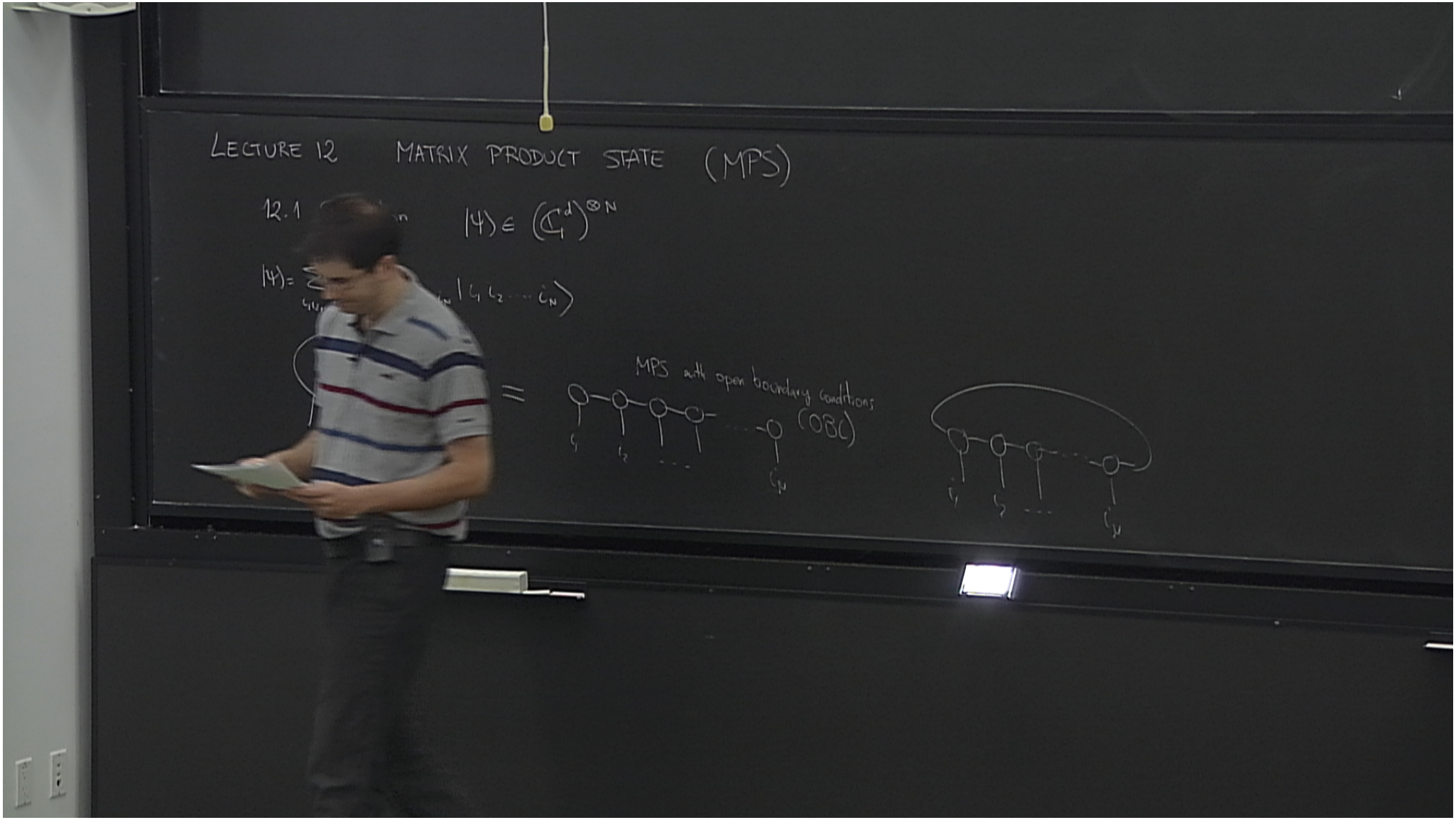
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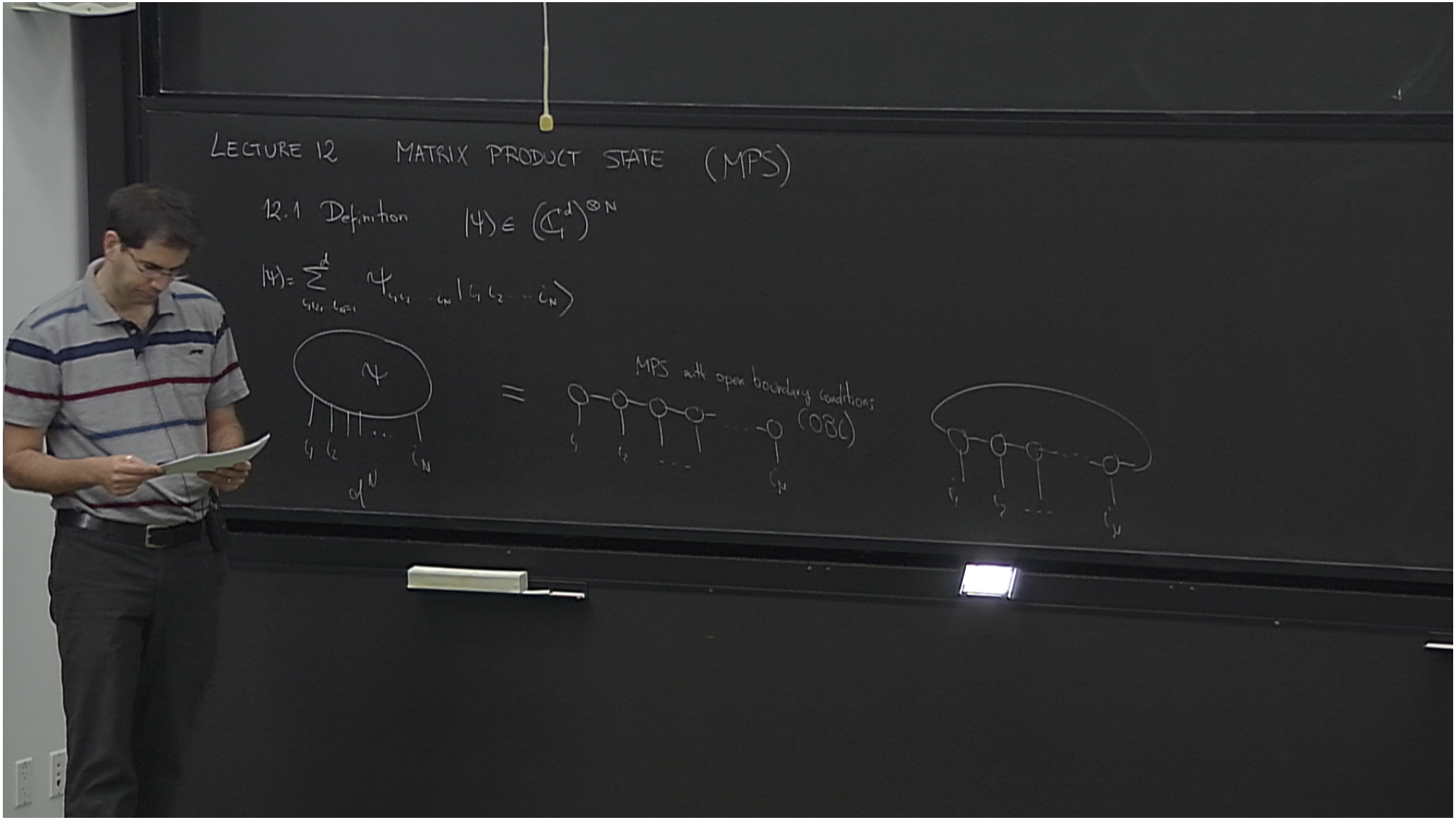
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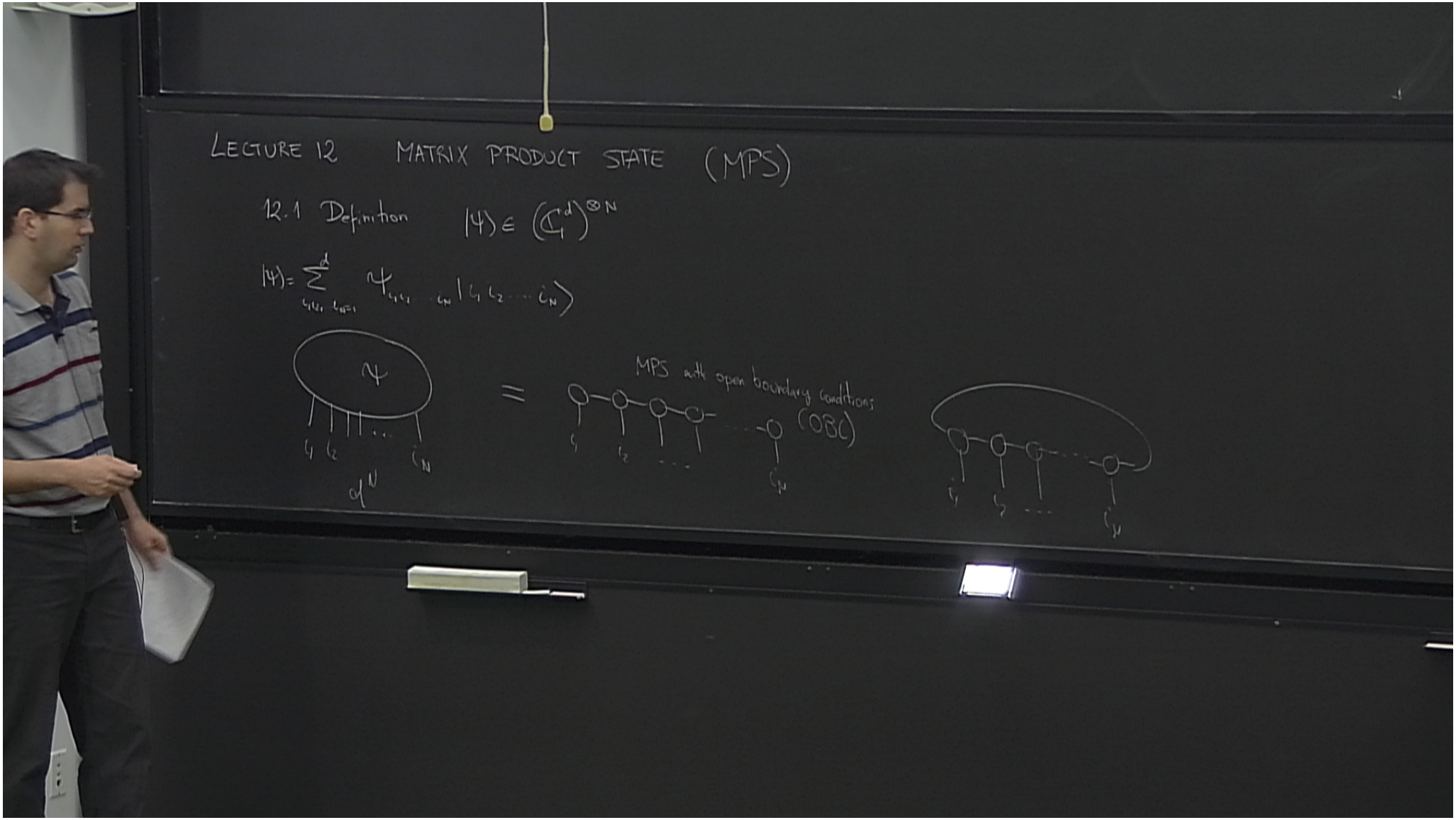
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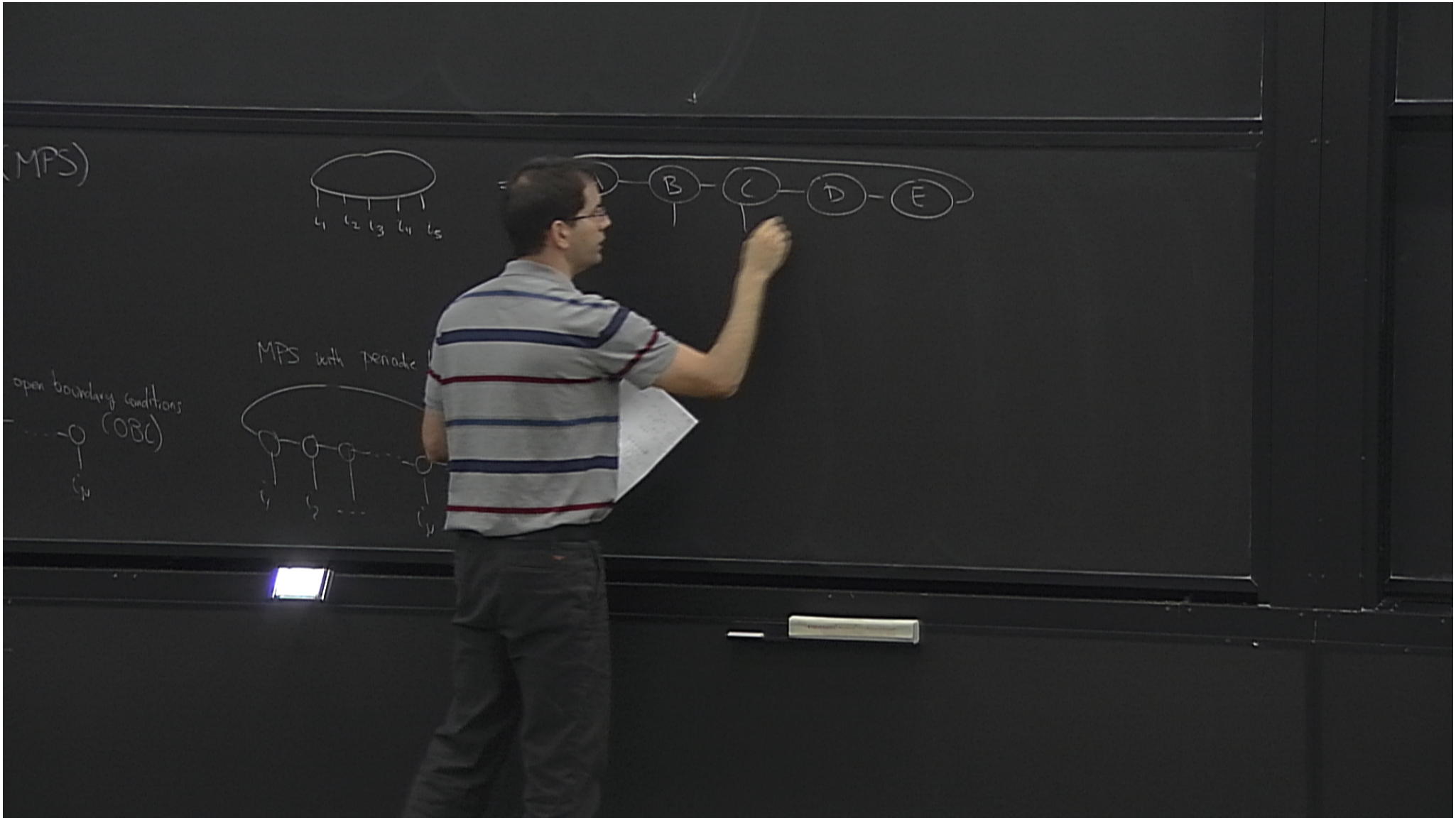

SUMMARY
$|\psi\rangle = |\psi^1\rangle \otimes |\psi^2\rangle \otimes \cdots \otimes |\psi^N\rangle$
T
N
cost $O(N)$
$C(L)=0$
$S(L)=0$
cost $O(N \cdot \exp T)$
$C(L) = \begin{cases} \neq 0 & L < O(T) \\ = 0 & L > O(T) \end{cases}$
$S(L) \leq \text{cost} = O(T)$

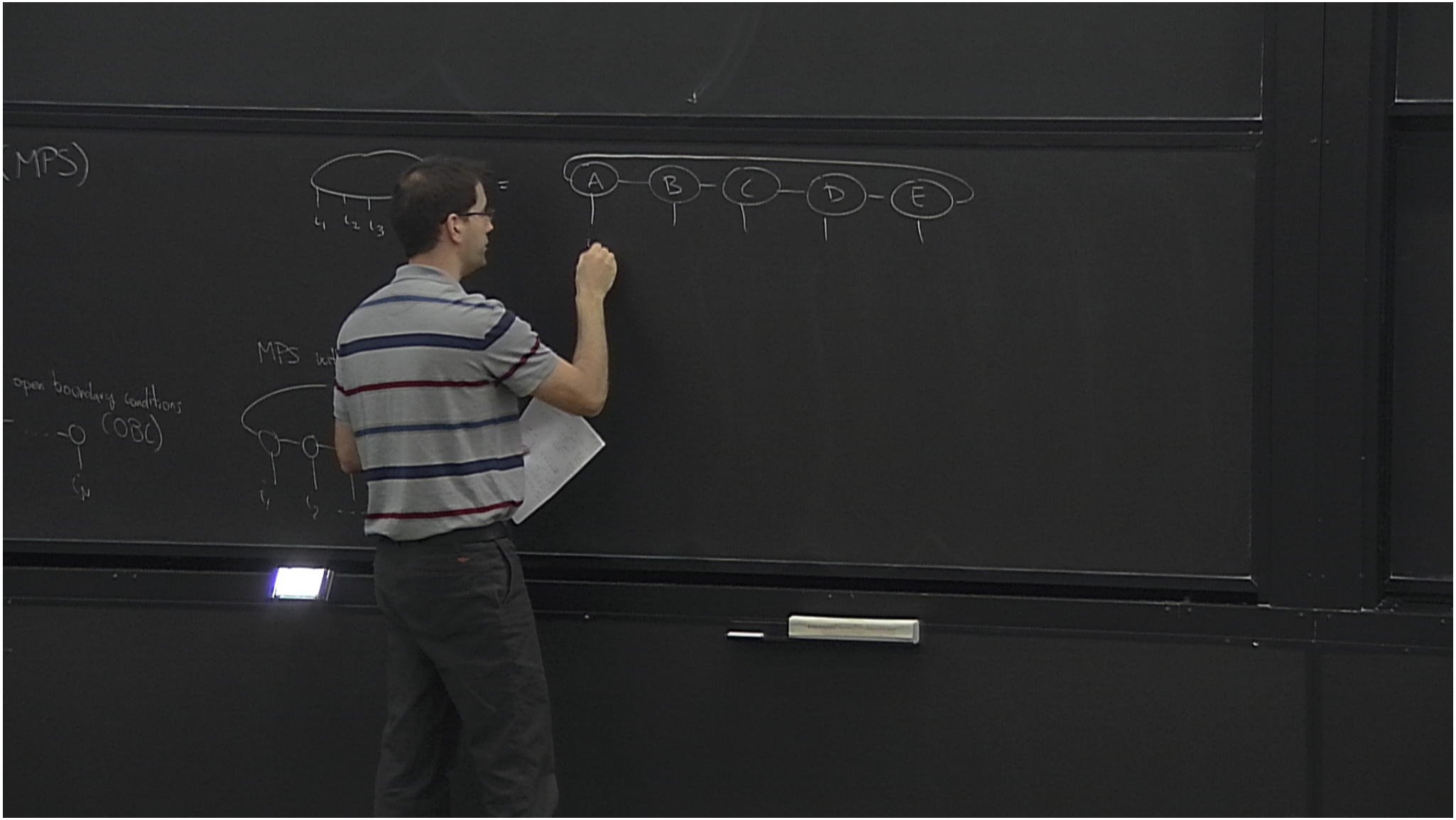

LECTURE 12 MATRIX PRODUCT STATE (MPS)
12.1 Definition
with open boundary conditions

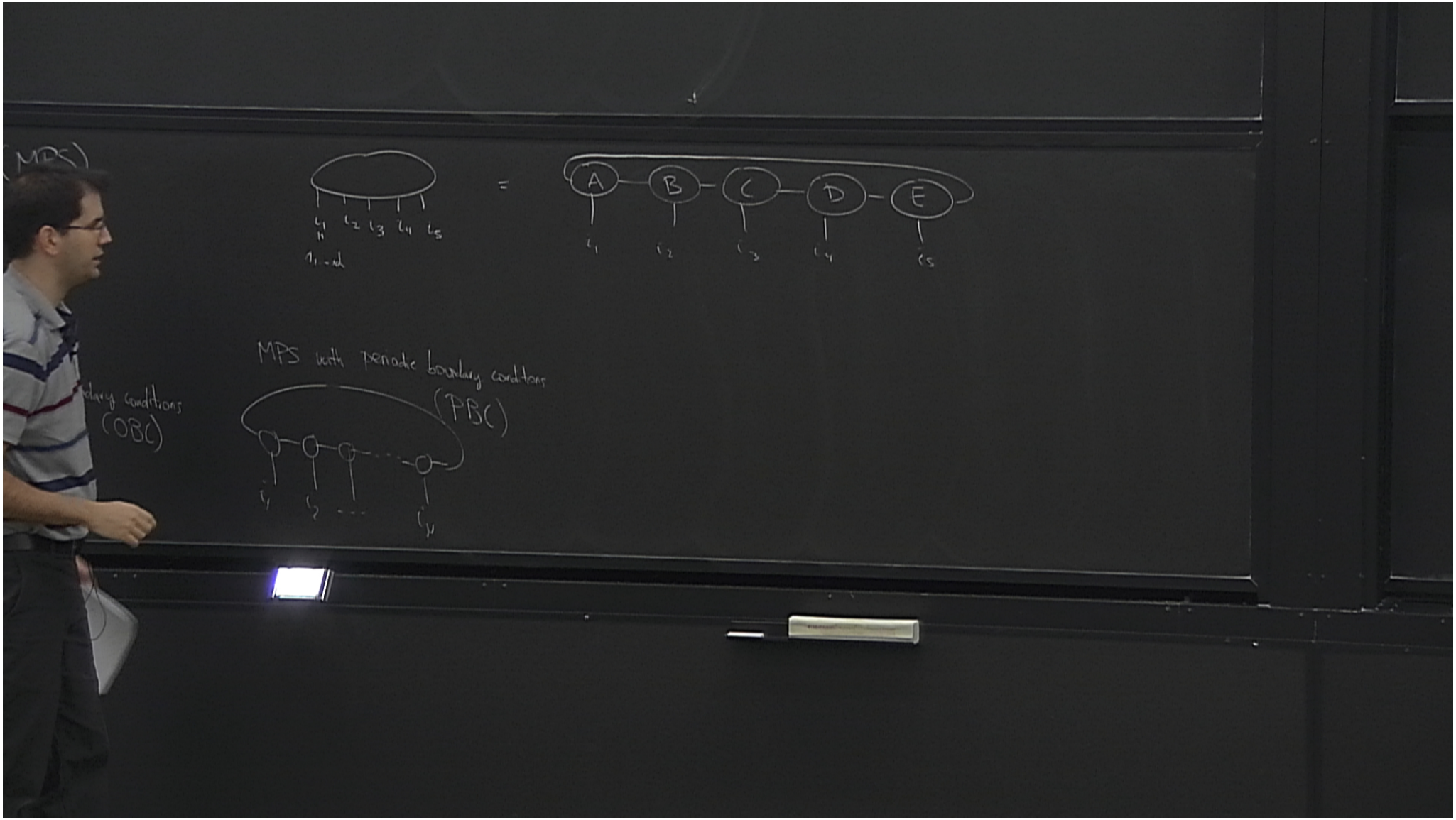

LECTURE 12 MATRIX PRODUCT STATE (MPS)
12.1
$|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
$|\psi\rangle = \sum$
$|i_1 i_2 \cdots i_N\rangle$
=
MPS with open boundary conditions
(OBC)

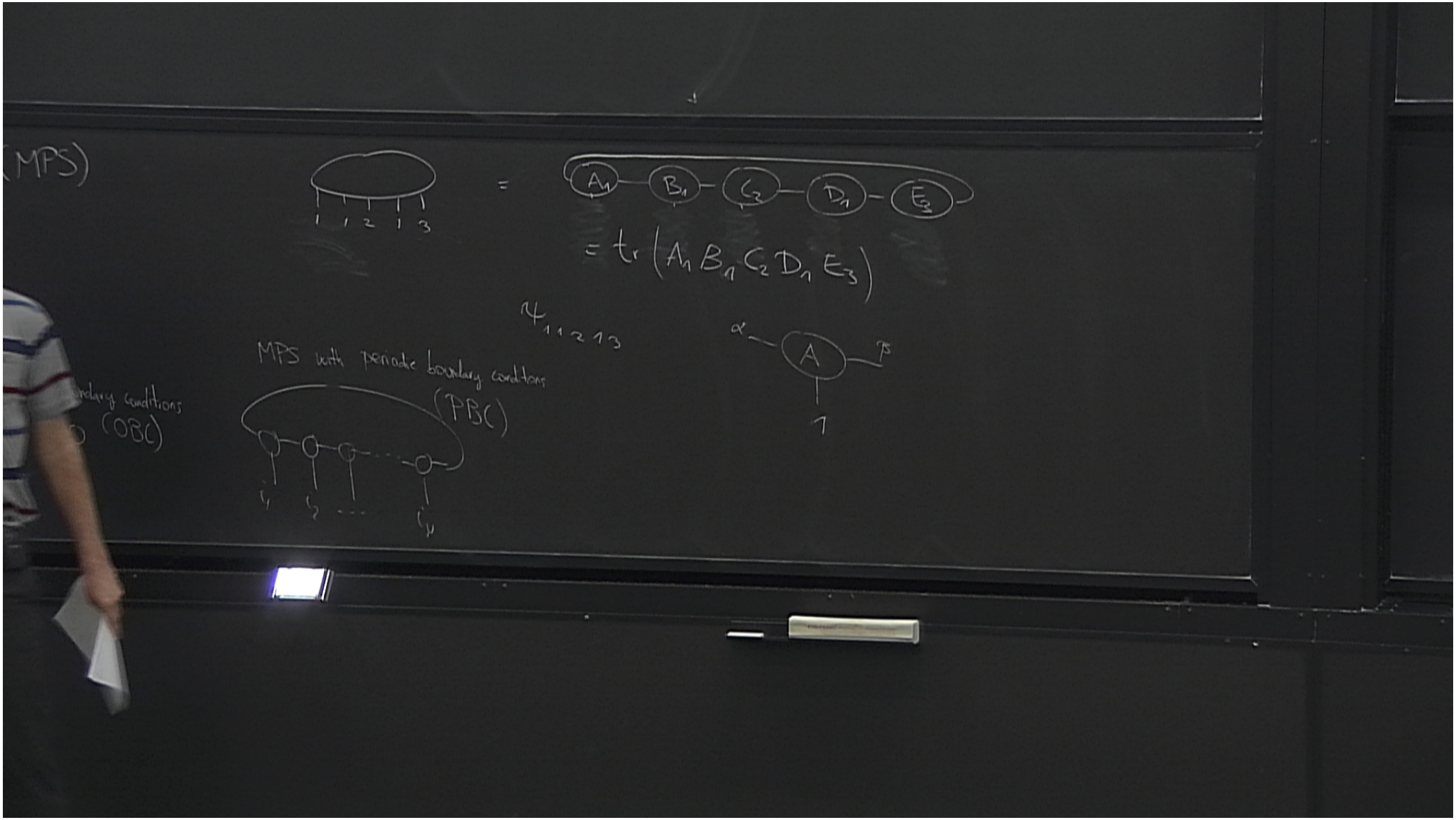

LECTURE 12 MATRIX PRODUCT STATE (MPS)
12.1 Definition
MPS with open boundary conditions (OBC)

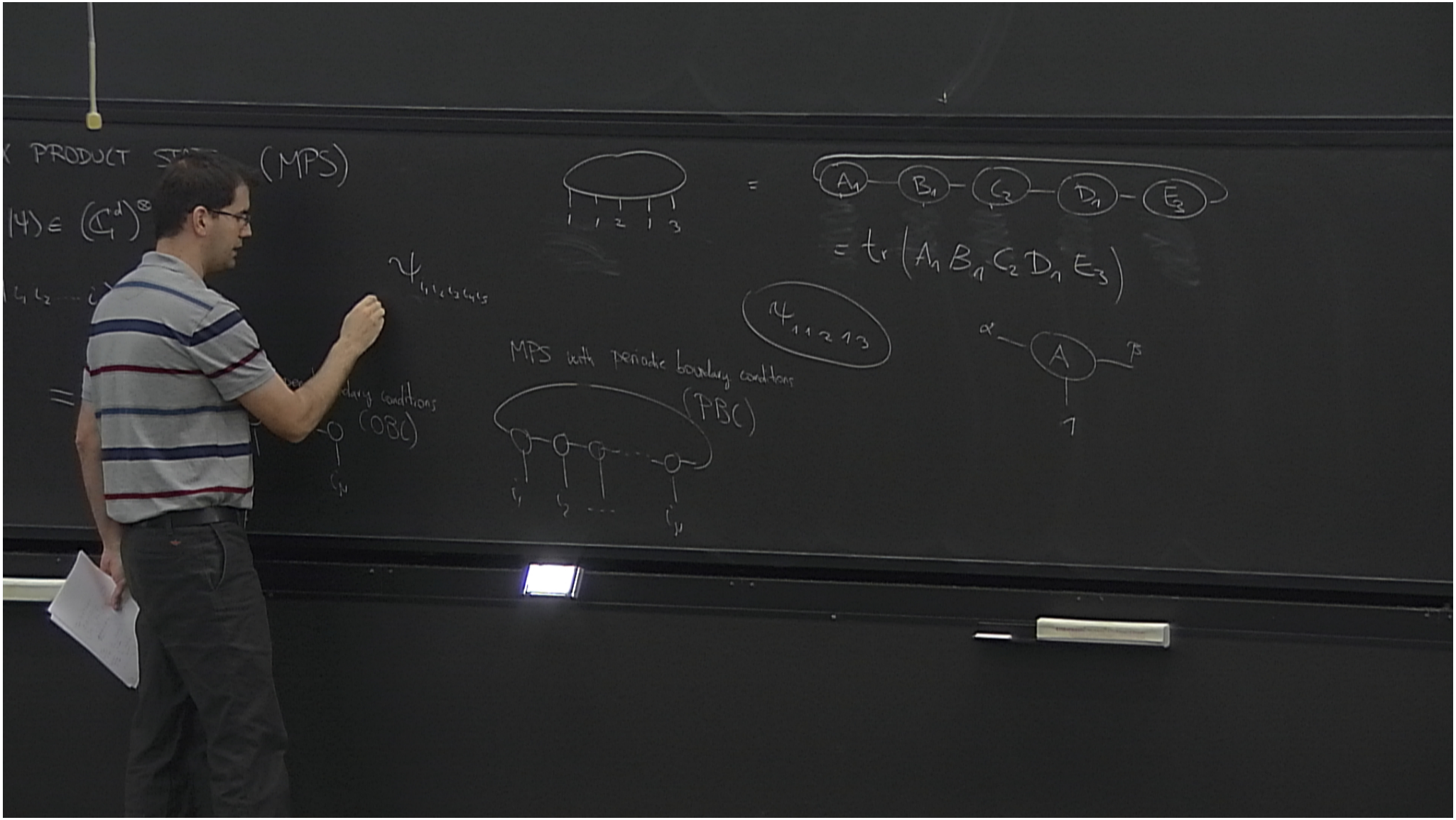

LECTURE 12 MATRIX PRODUCT STATE (MPS)
12.1 Definition
MPS with open boundary conditions (OBC)

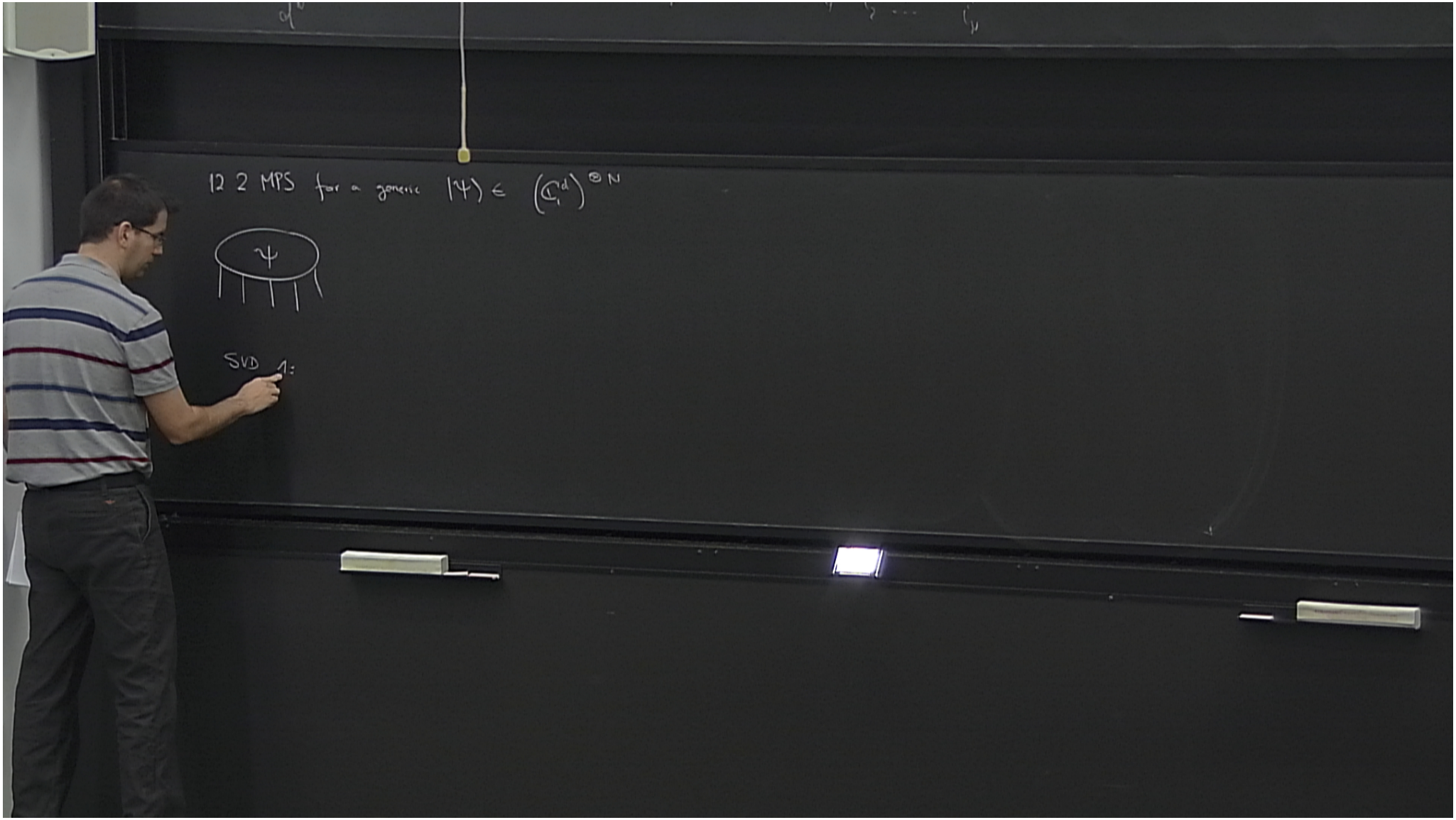

(MPS)
B
C
D
E
open boundary conditions
(OBC)
MPS with periodic

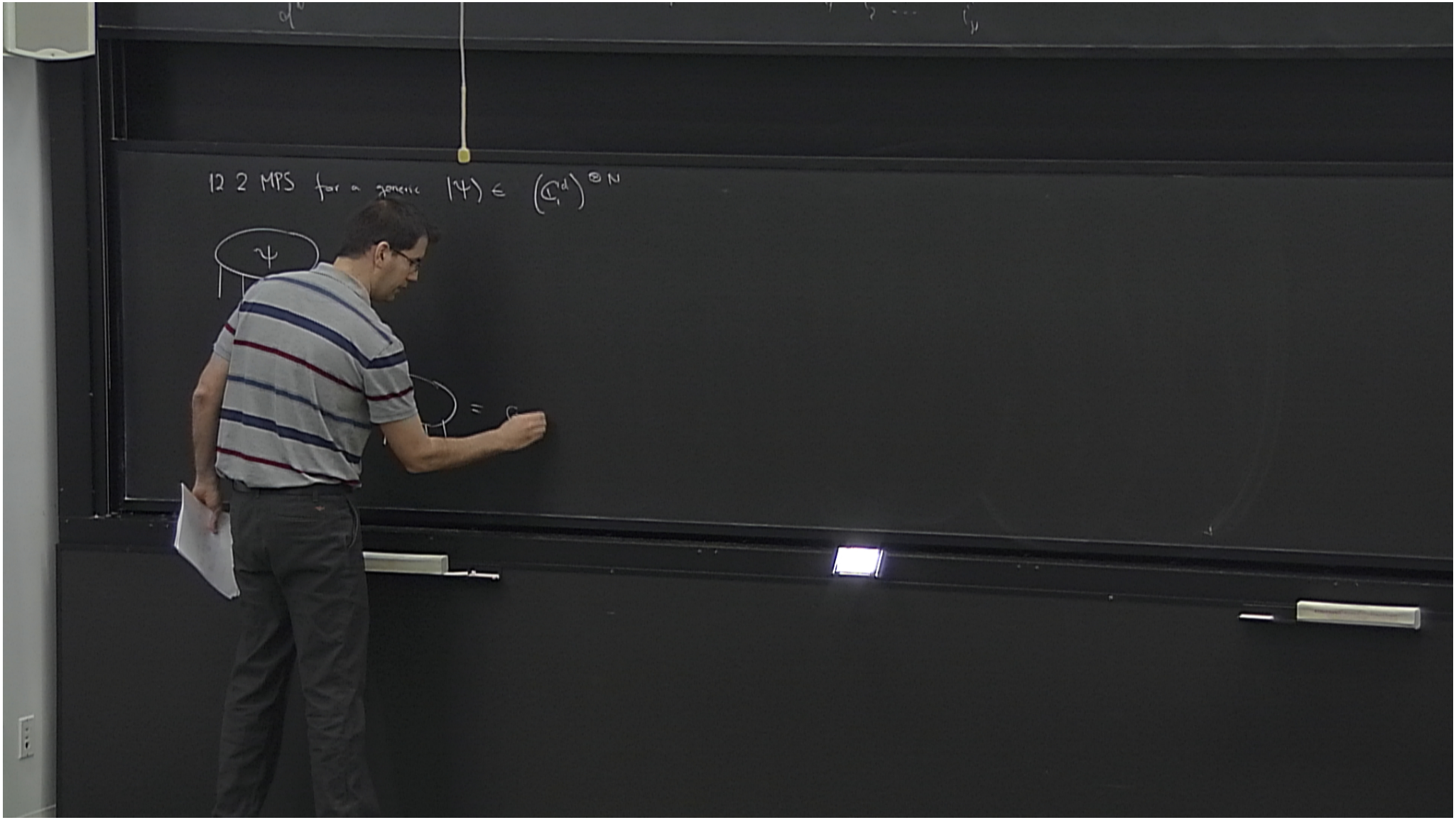

(MPS)
=
A
B
C
D
E
MPS
open boundary conditions
(OBC)

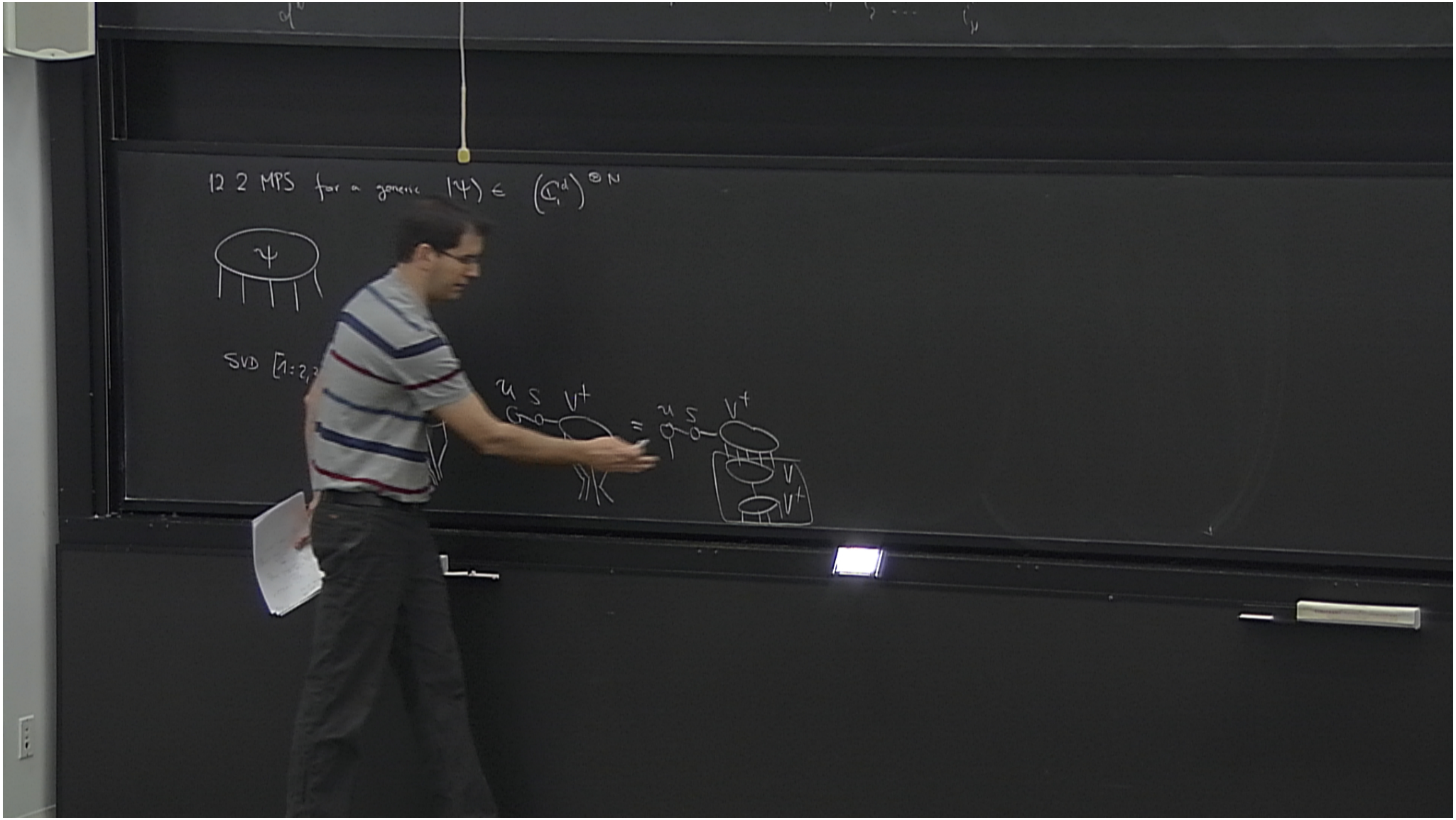

=
A
B
C
D
E
MPS with periodic boundary conditions
(PBC)
(OBC)

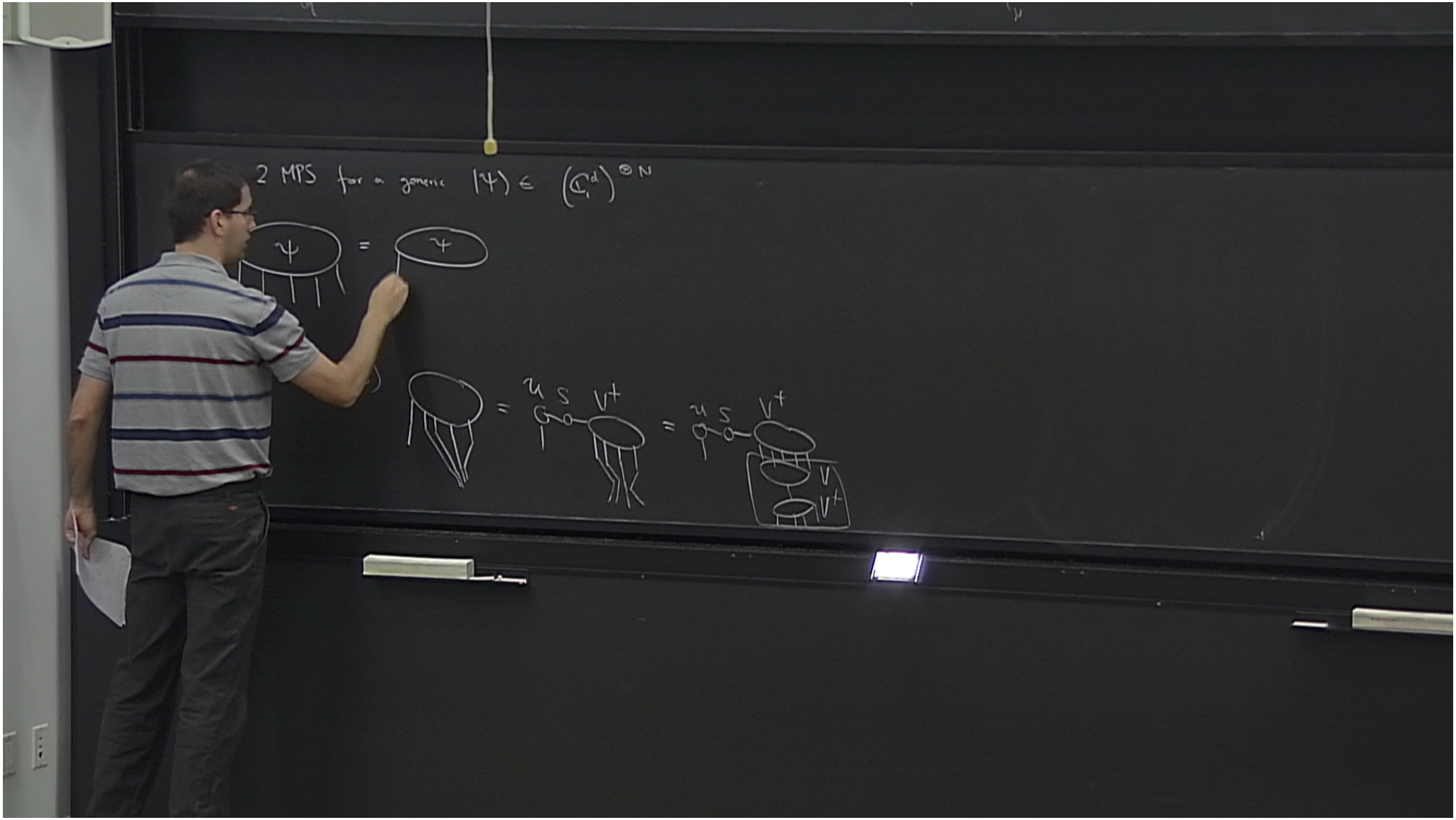

(MPS)
1 1 2 1 3
=
A_1 B_1 C_2 D_1 E_3
$= tr(A_1 B_1 C_2 D_1 E_3)$
Ψ_{11213}
MPS with periodic boundary conditions
(PBC)
boundary conditions
(OBC)
α
A
β
1

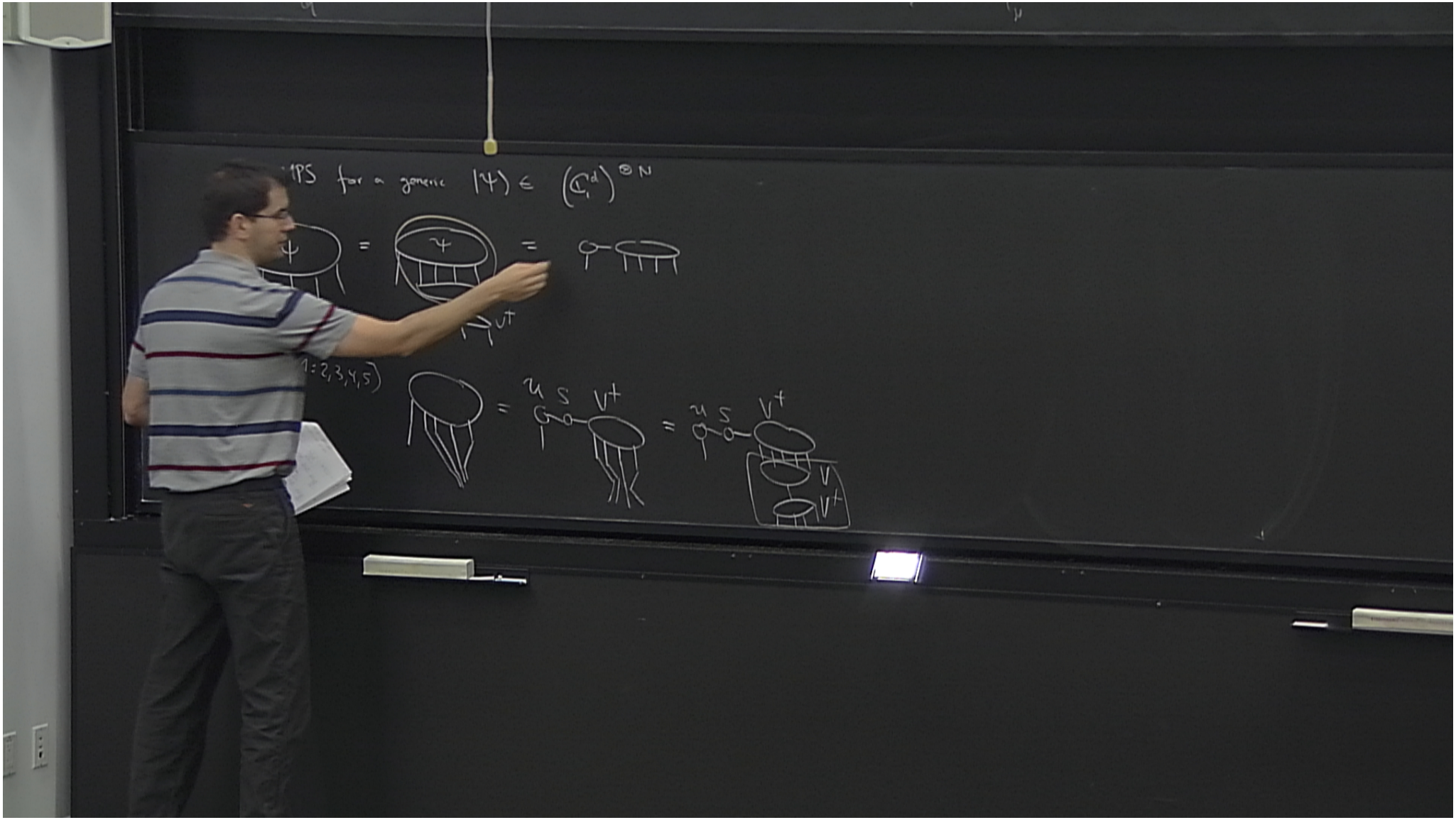

PRODUCT
(MPS)
(OBC)
MPS with periodic boundary conditions
(PBC)
= tr(A1 B1 C2 D1 E3)

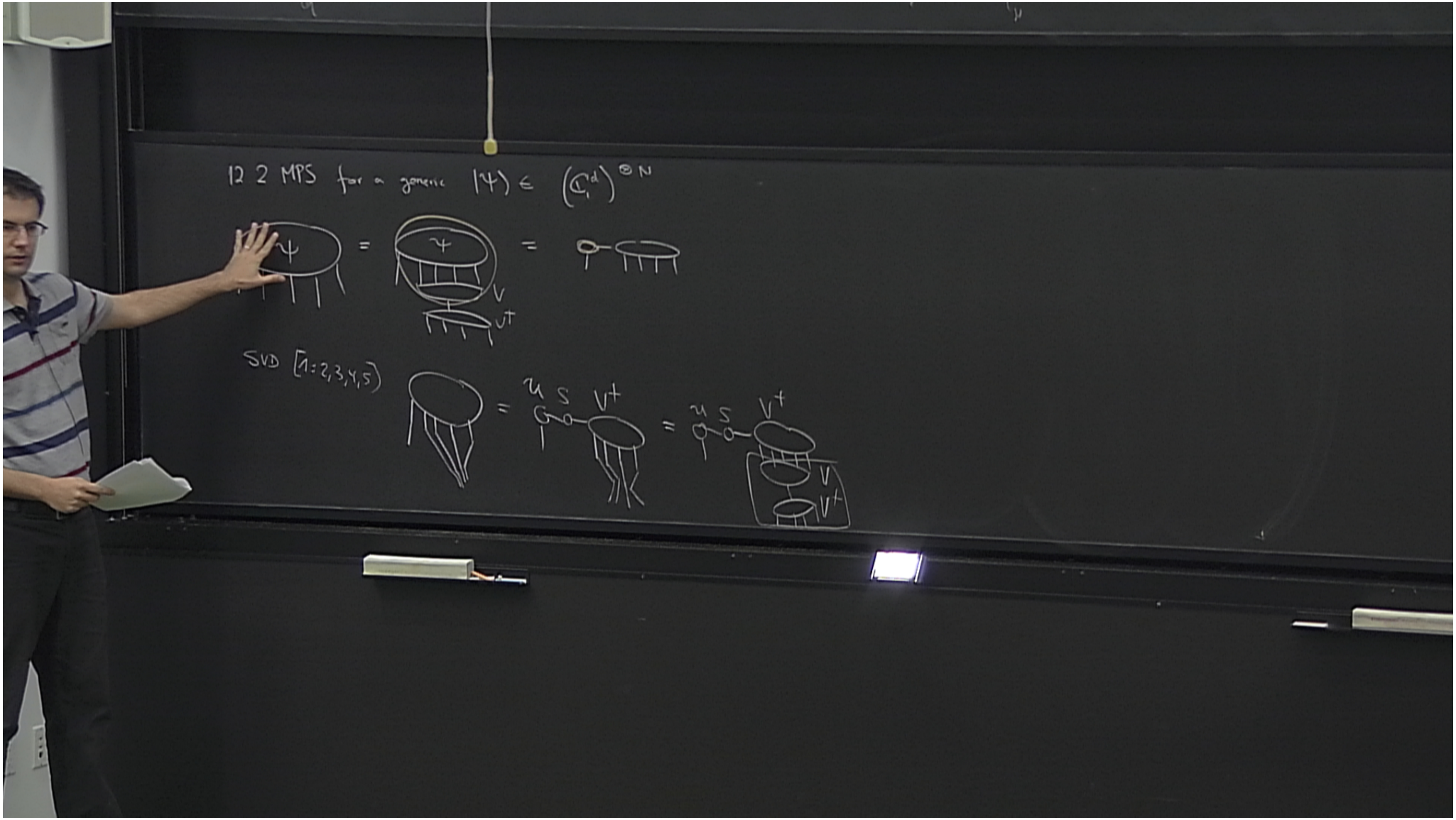

12 2 MPS for a generic $|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
Ψ
SVD 1:

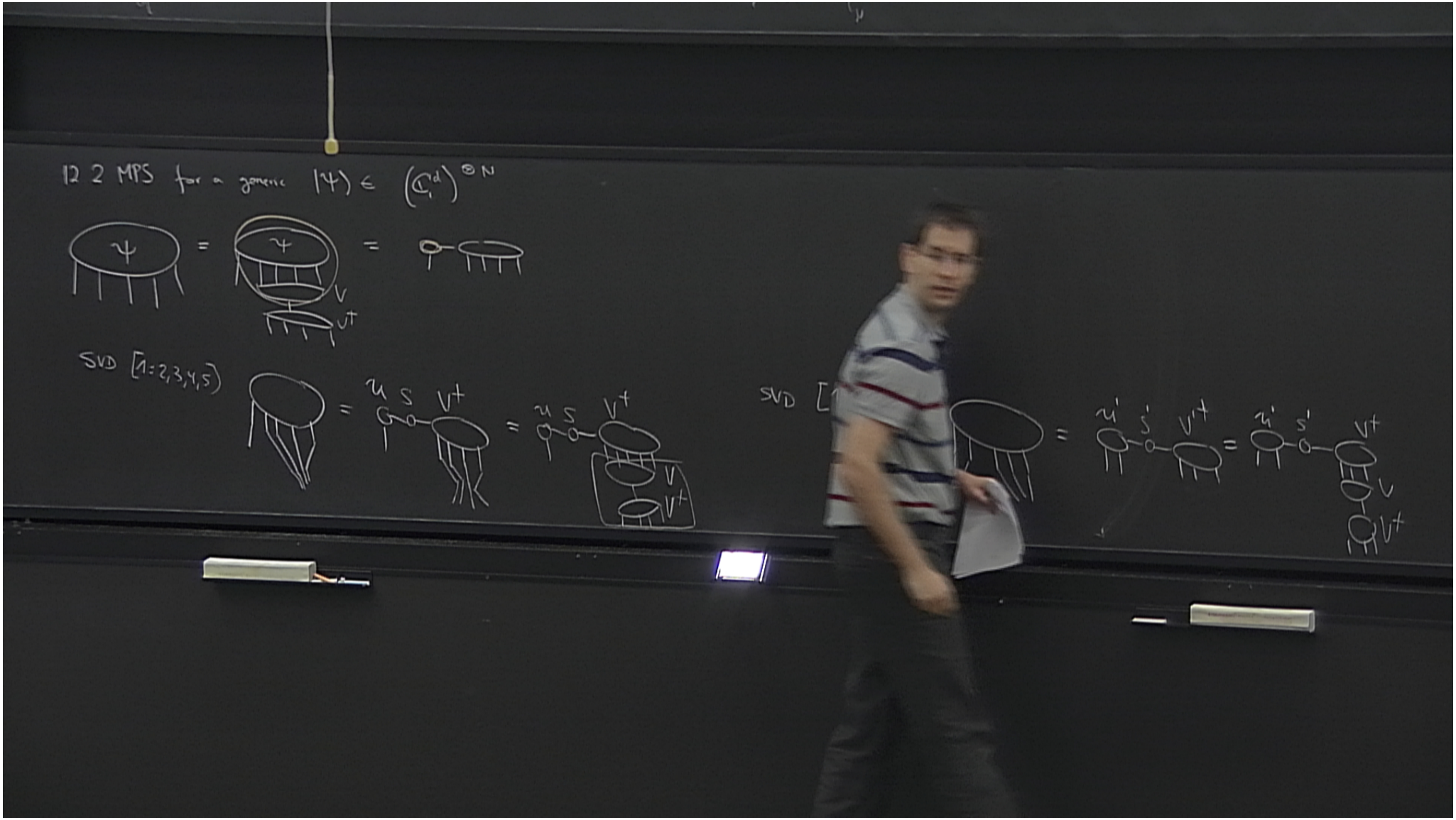

12 2 MPS for a generic $|\Psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
Ψ

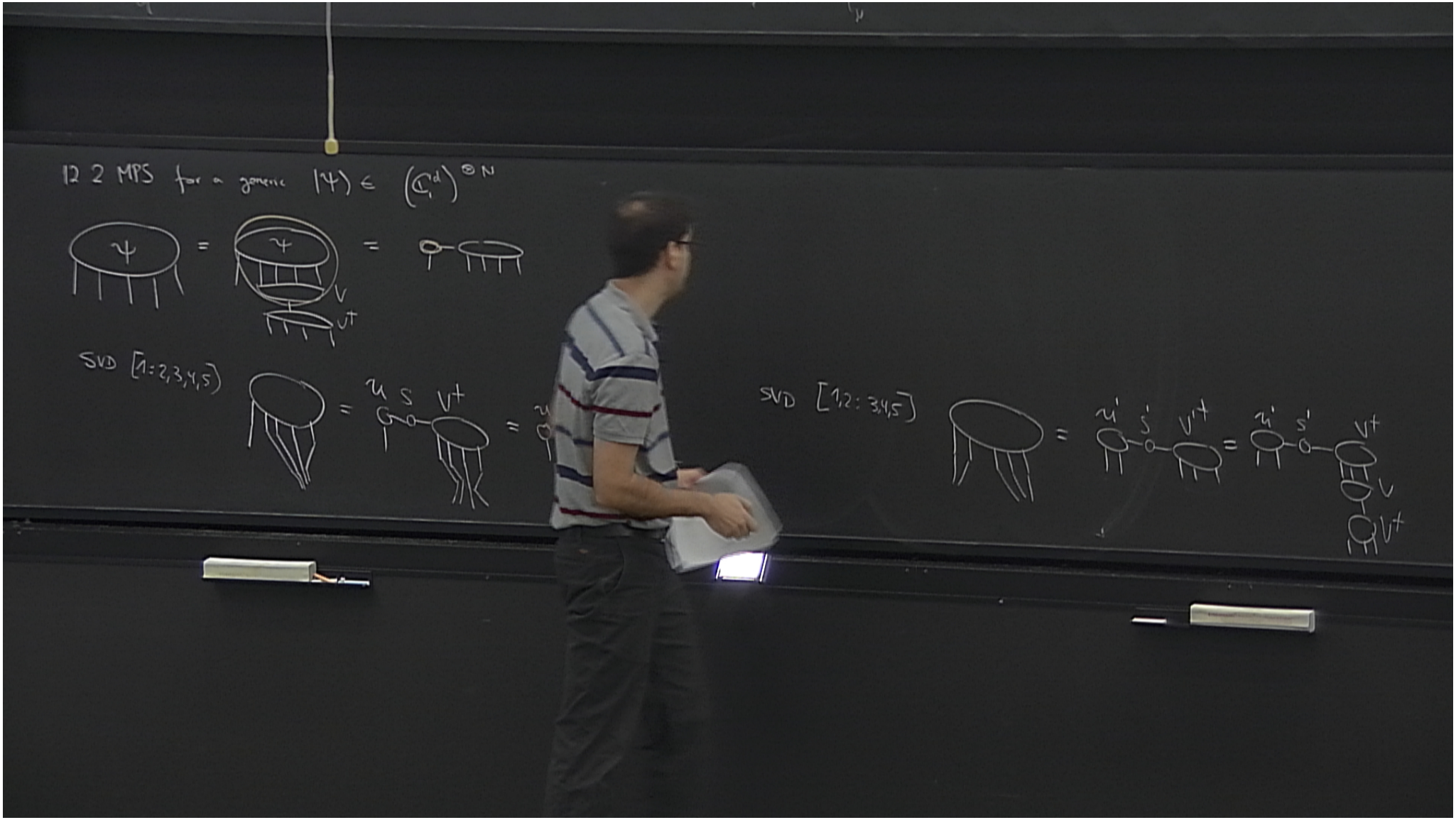

12 2 MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
SVD
U S V†
=
U S V†
V
V†

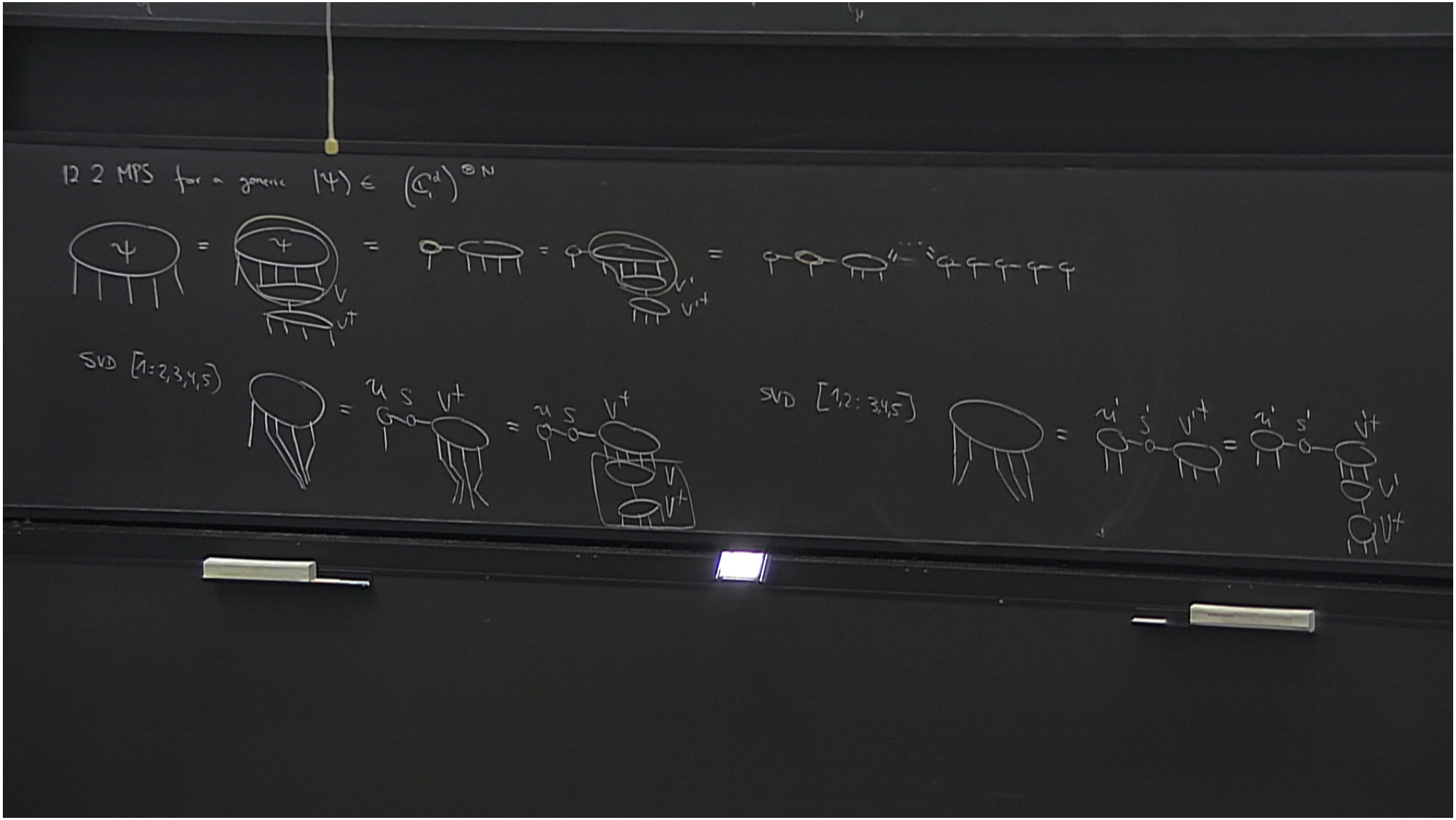

2 MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
ψ
=
ψ
=
U
S
V†
=
U
S
V†
V
V†

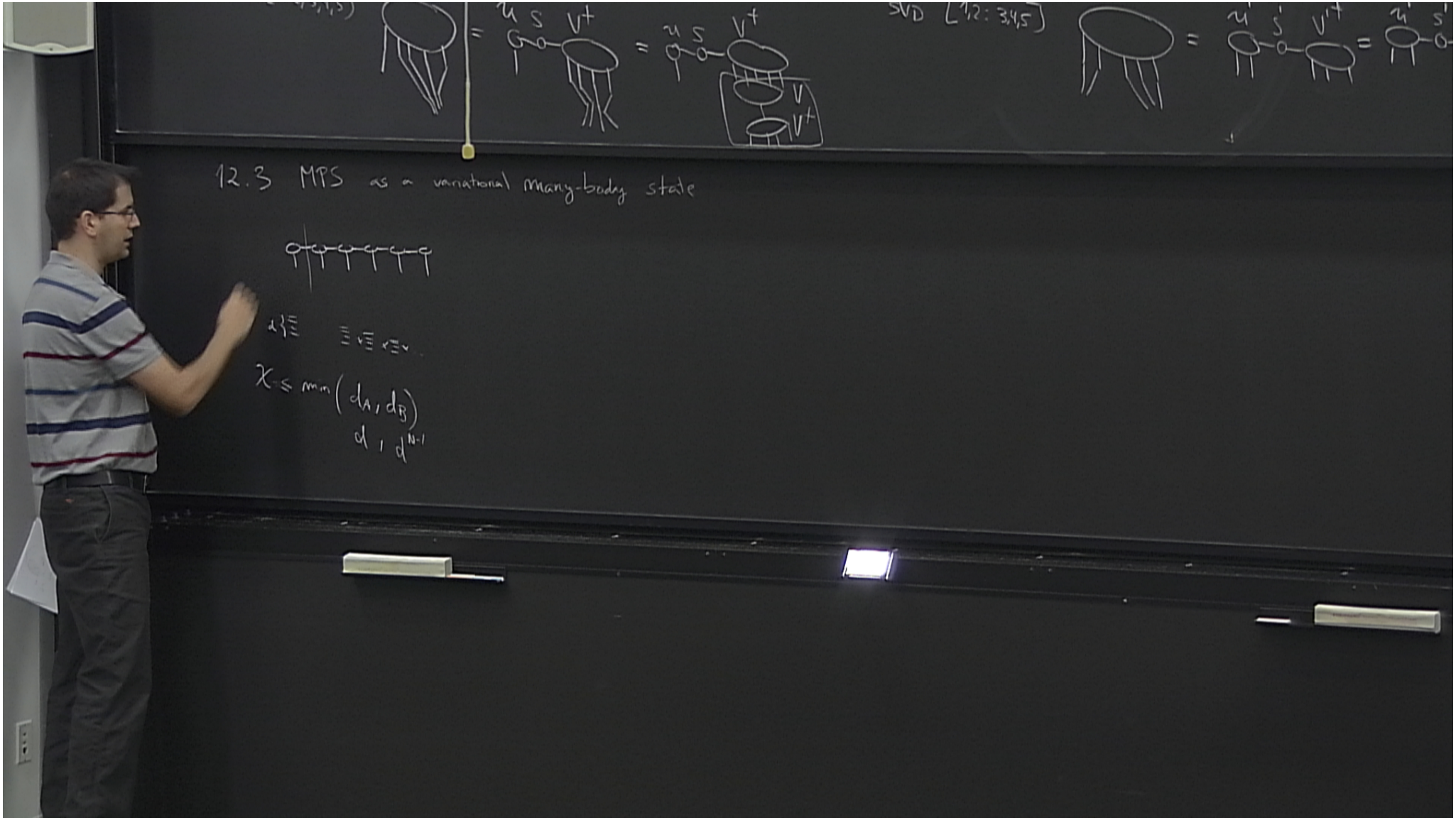

MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
ψ
Ψ
$\Psi^{U^\dagger}$
$1:2,3,4,5)$
Ψ
S
$V^\dagger$
V
$V^\dagger$

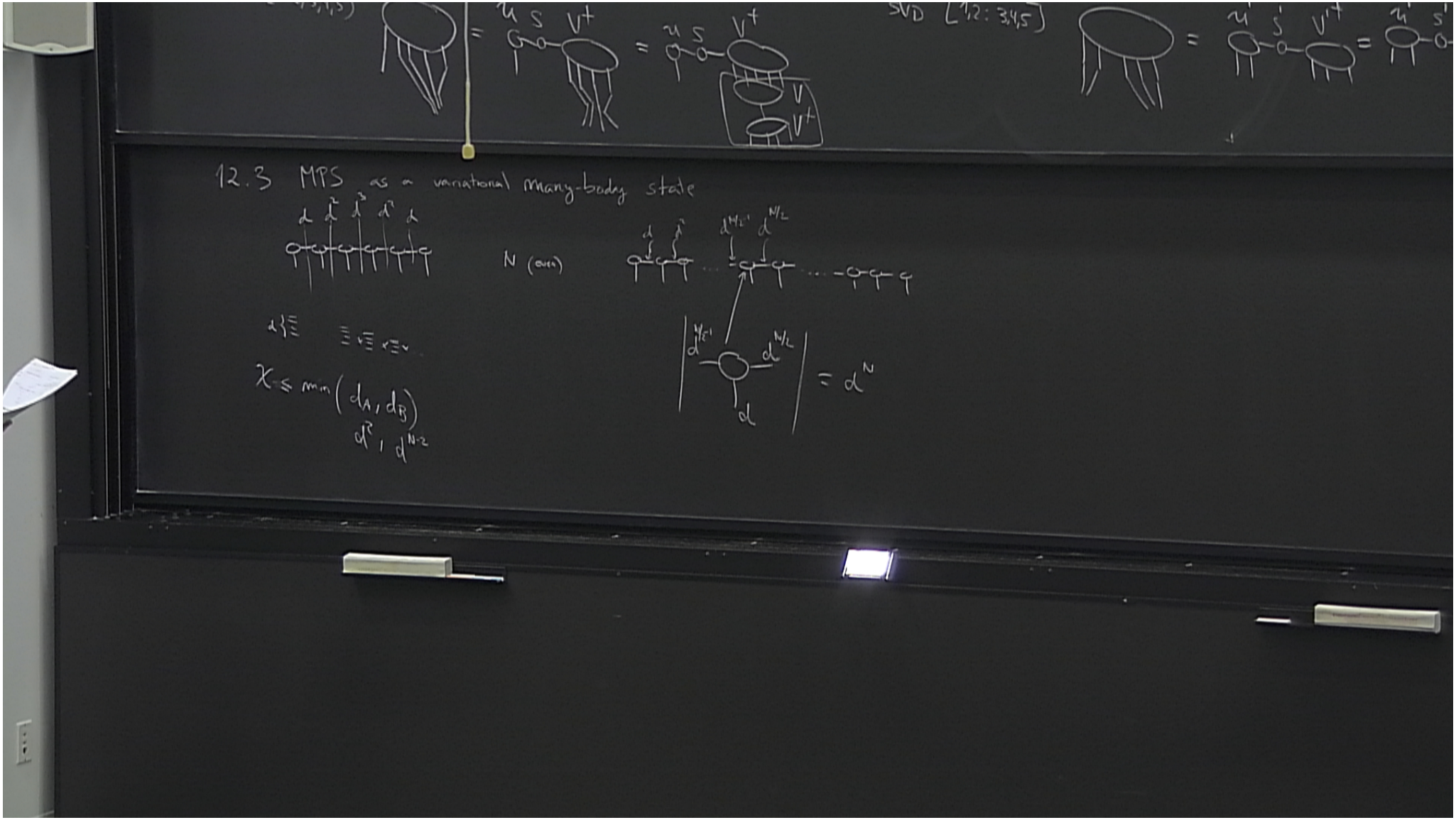

12 2 MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
ψ = ψ = V V†
SVD [1:2,3,4,5]
= U S V† = U S V† V V†

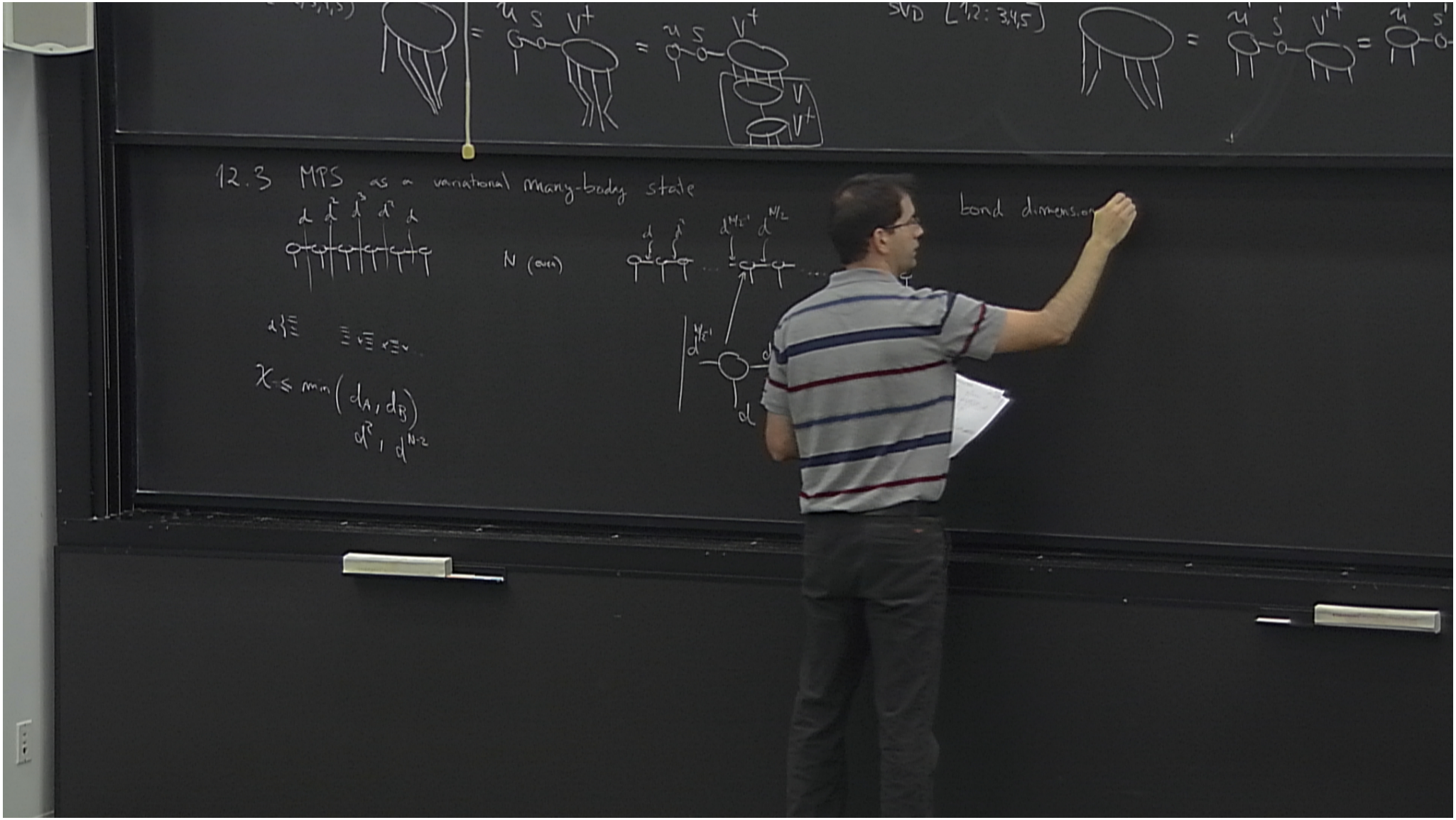

12 2 MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
SVD [1=2,3,4,5]
SVD [


12 2 MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
SVD [1:2,3,4,5]
SVD [1,2:3,4,5]

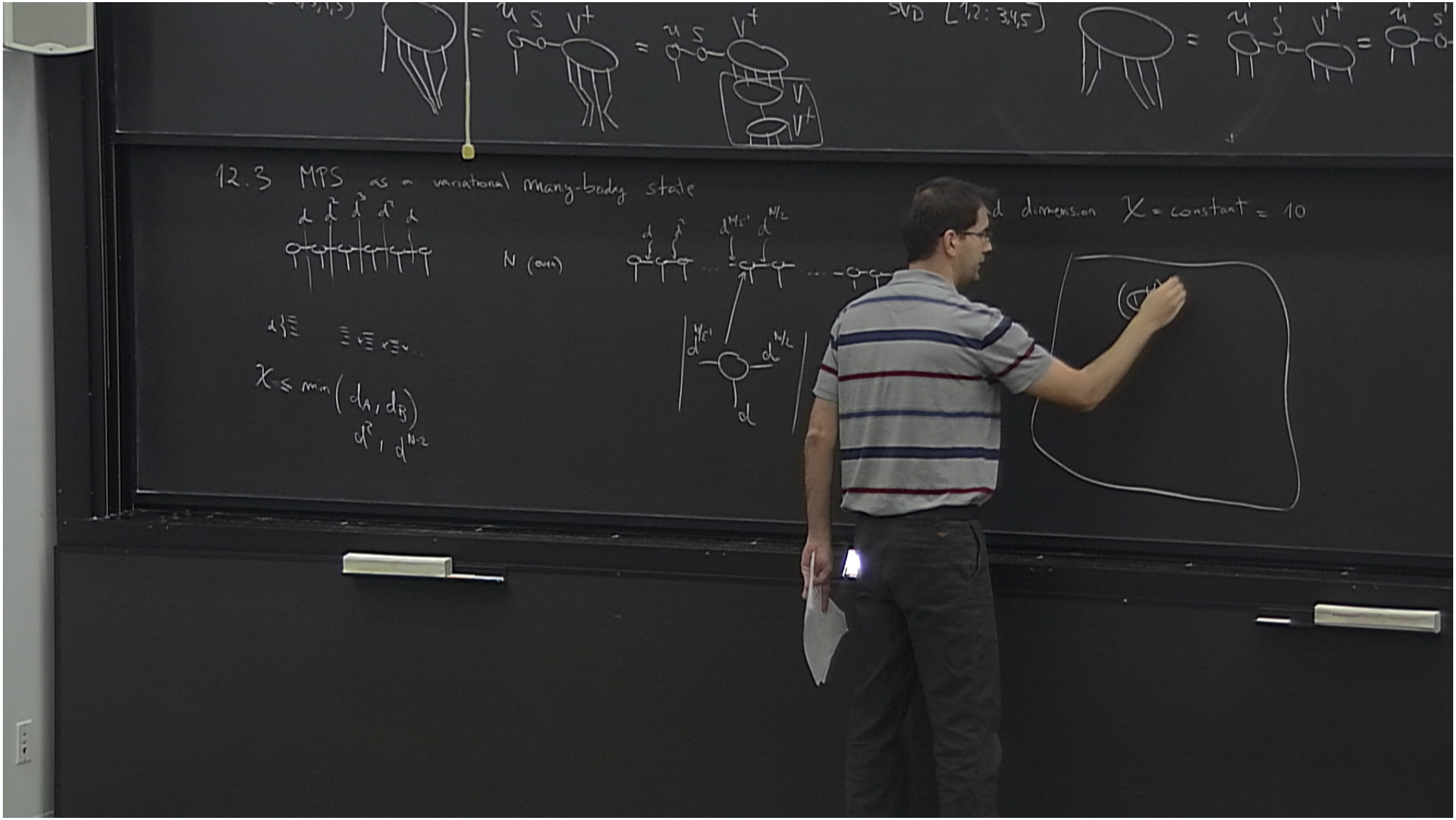

12 2 MPS for a generic $|\psi\rangle \in (\mathbb{C}^d)^{\otimes N}$
SVD [1:2,3,4,5]
SVD [1,2:3,4,5]

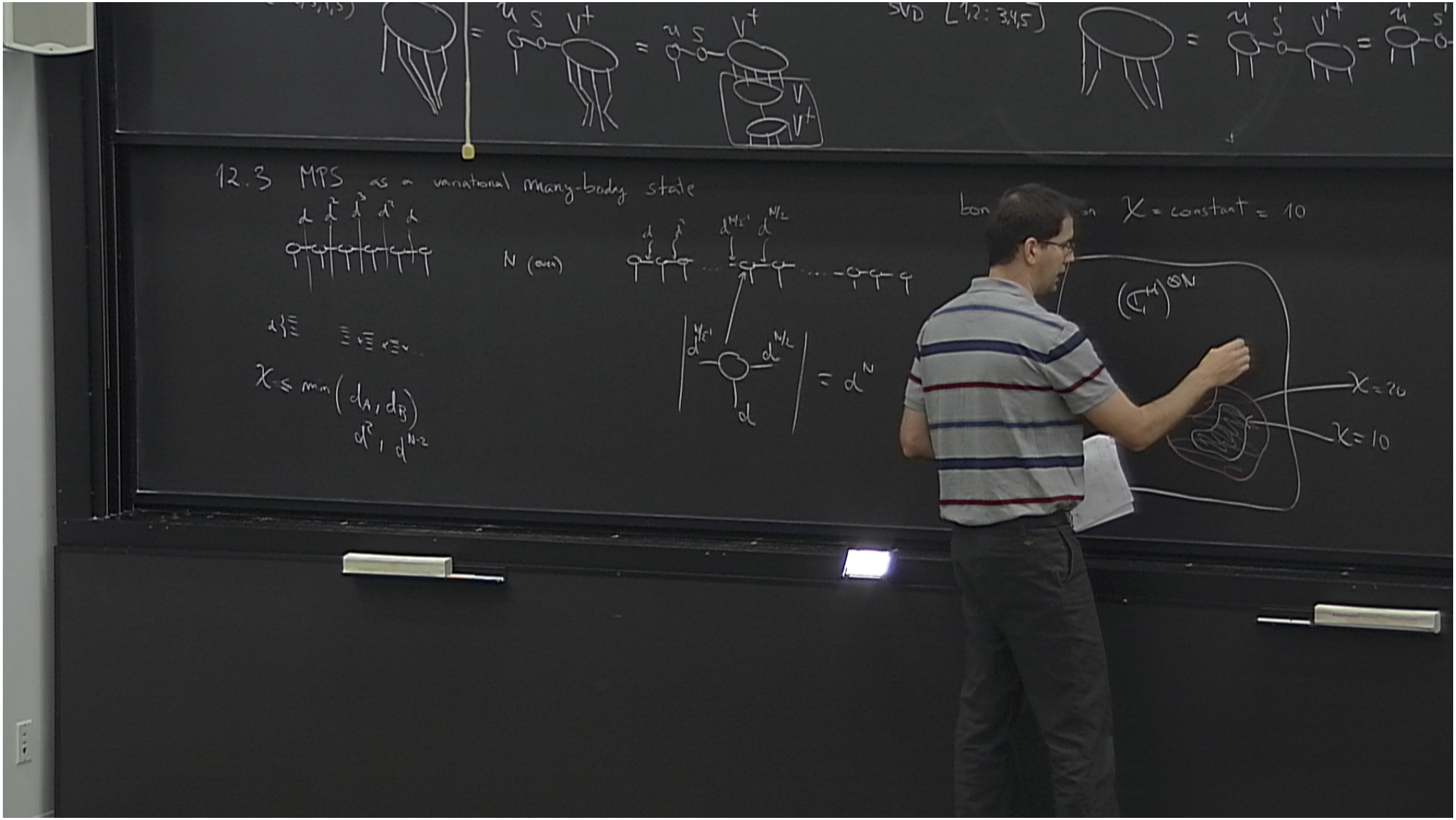

SVD [1,2 : 3,4,5]
12.3 MPS as a variational many-body state
$\chi \leq \min\left(d_A, d_B\right)$
$d \cdot d^{N-1}$

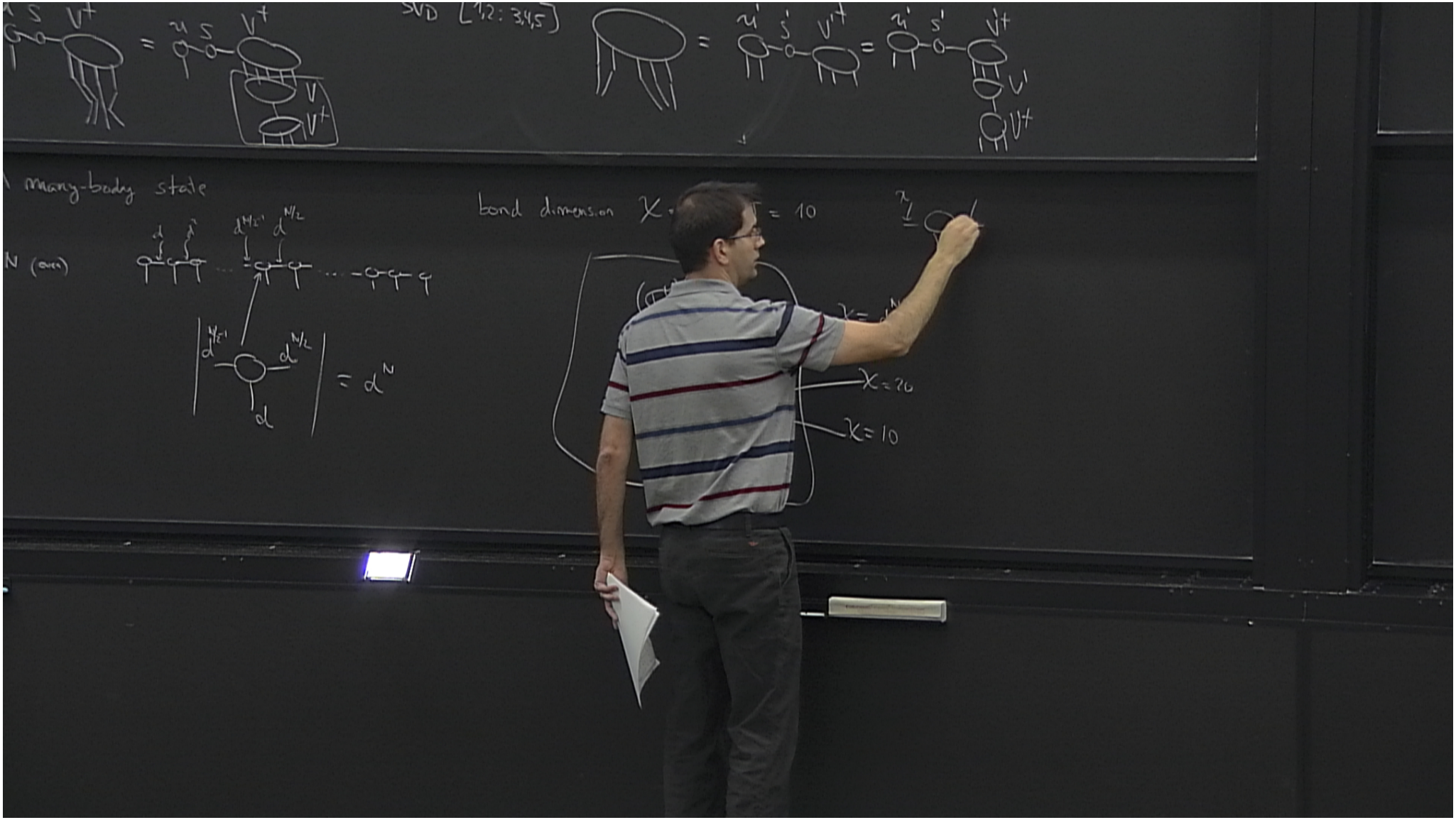

SVD [1,2 : 3,4,5]
12.3 MPS as a variational many-body state
N (even)
$\chi \leq \min(d_A, d_B)$
d^2, d^{N-2}
$= d^N$

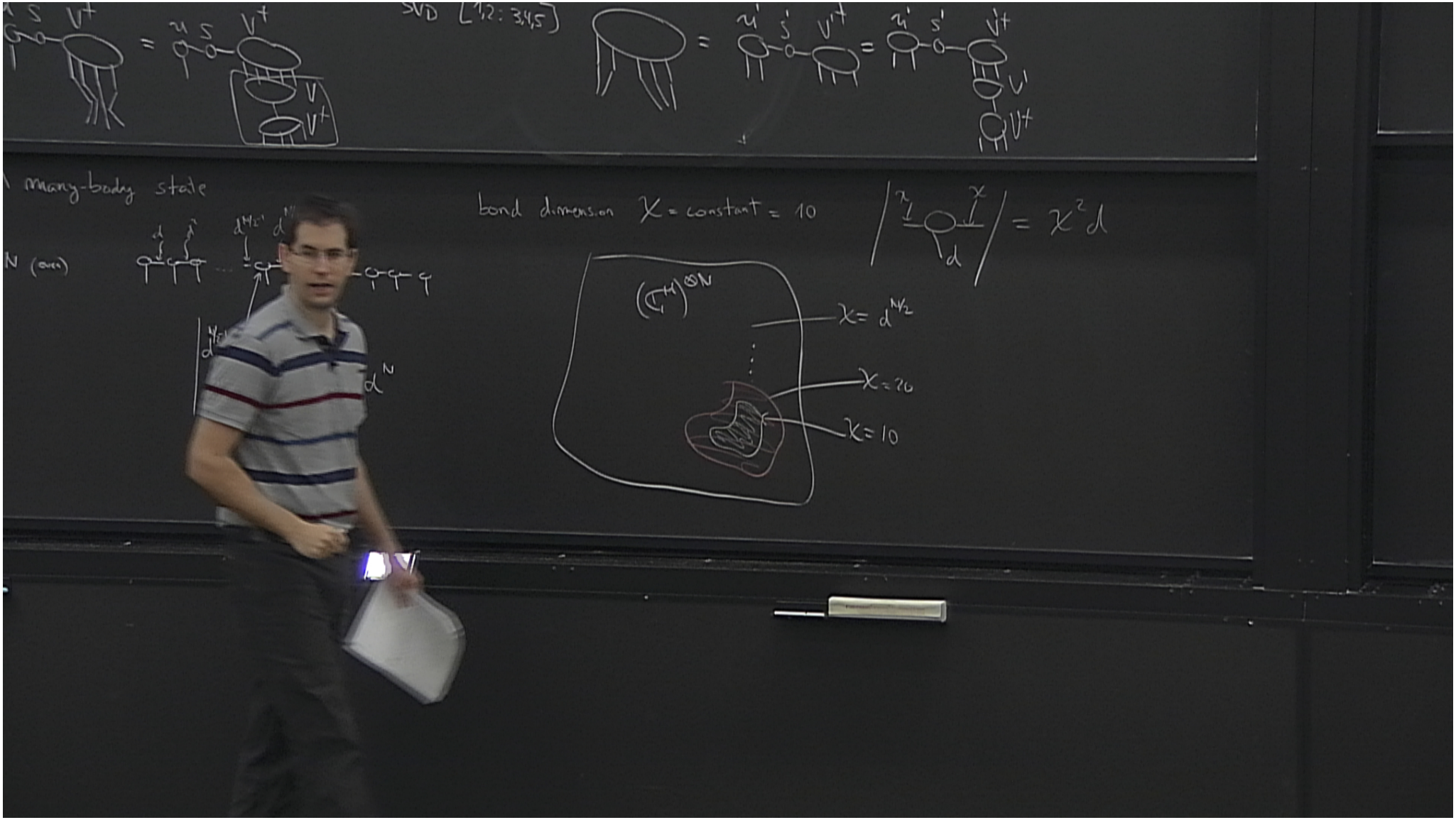

12.3 MPS as a variational many-body state
N (even)
bond dimension
$\chi \leq \min(d_A, d_B)$
d^2 ; d^{N-2}

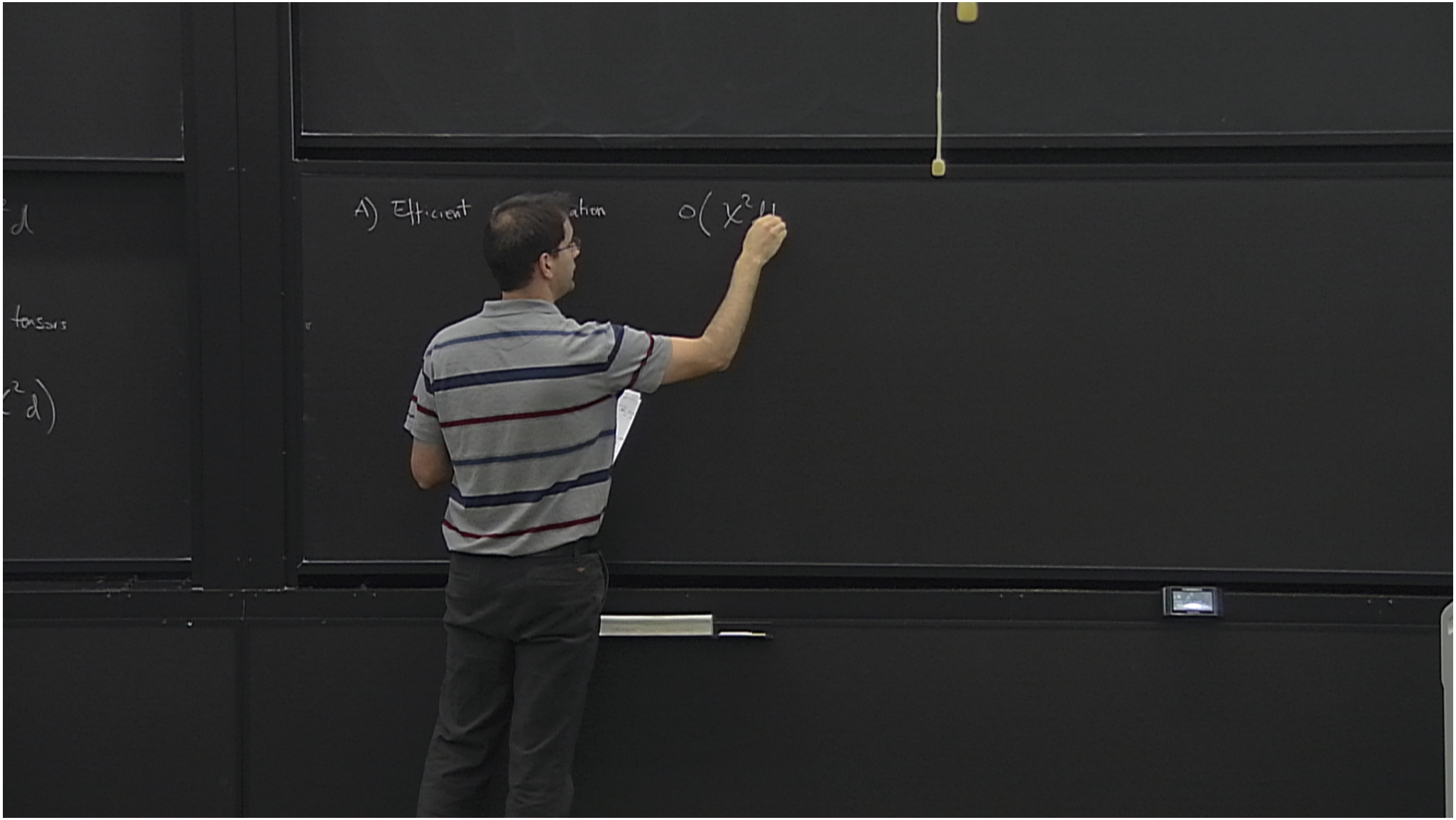

SVD [1,2 : 3,4,5]
12.3 MPS
a variational many-body state
bond dimension χ = constant
N (even)
= d^N

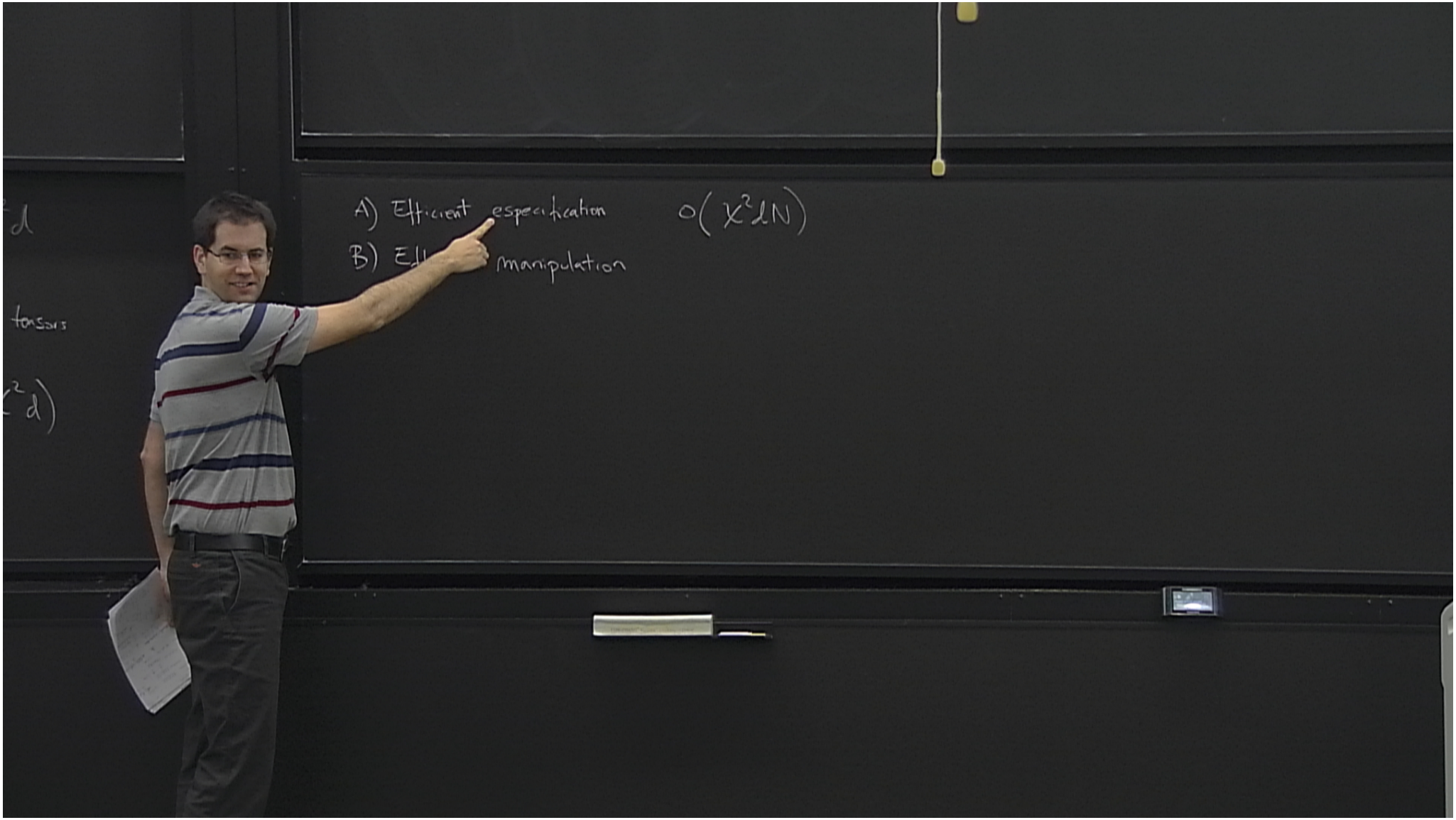

12.3 MPS as a variational many-body state
d dimension χ = constant = 10
N (even)
χ ≤ min(d_A, d_B)
d^2 ; d^{N-2}


12.3 MPS as a variational many-body state
N (even)
$\chi \leq \min(d_A, d_B)$
d^2, d^{N-2}
$(\mathbb{C}^d)^{\otimes N}$
$\chi = 20$
$\chi = 10$
$= d^N$


SVD [1,2 : 3,4,5]
many-body state
bond dimension
= 10
χ = 20
χ = 10
= d^N


SVD [1,2 : 3,4,5]
many-body state
bond dimension χ = constant = 10
= χ² d
(ℂ^H)^⊗N
χ = d^{N/2}
χ = 20
χ = 10


A) Efficient
tensors

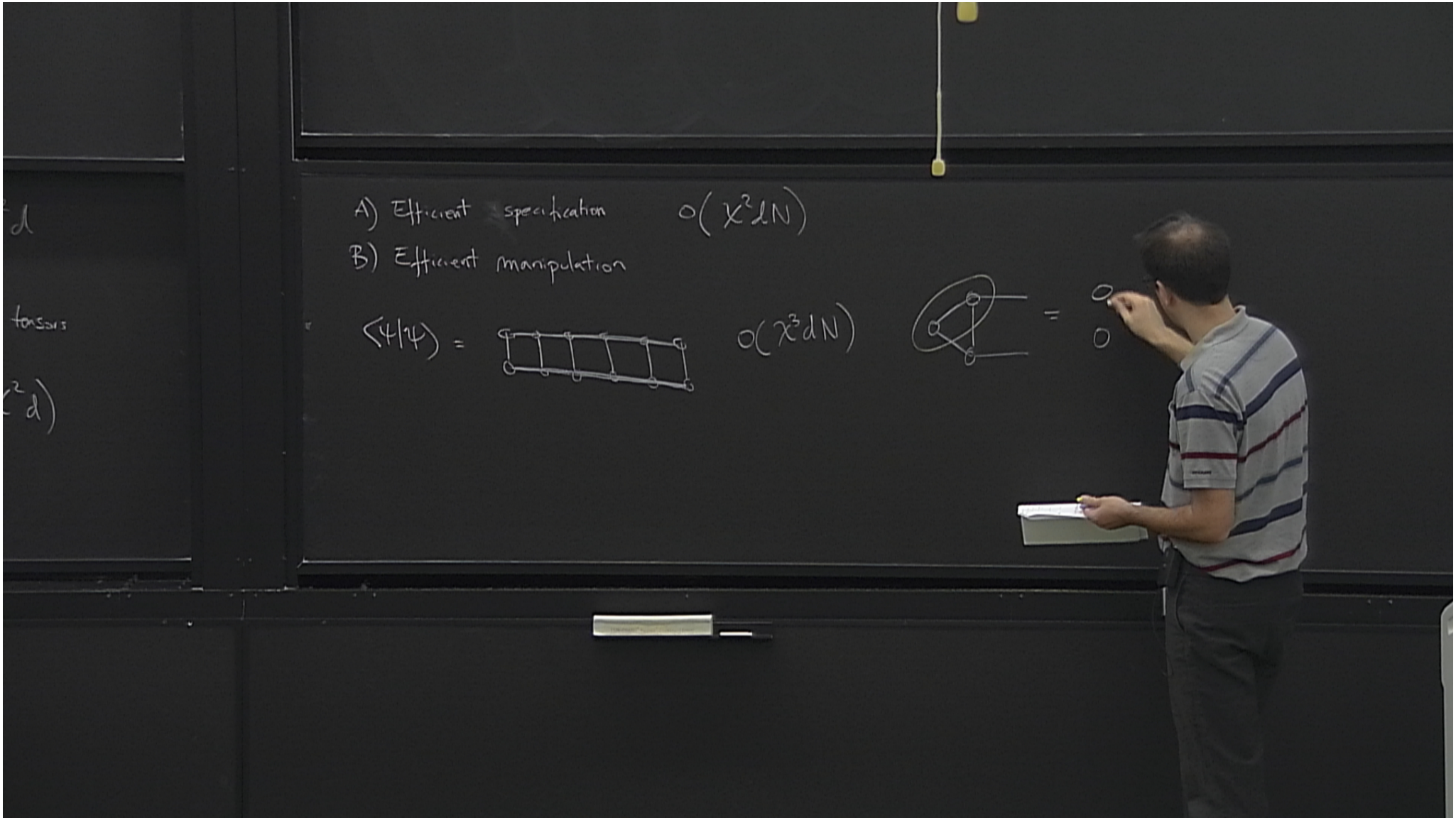

A) Efficient specification
O(χ²dN)
B) manipulation
tensors

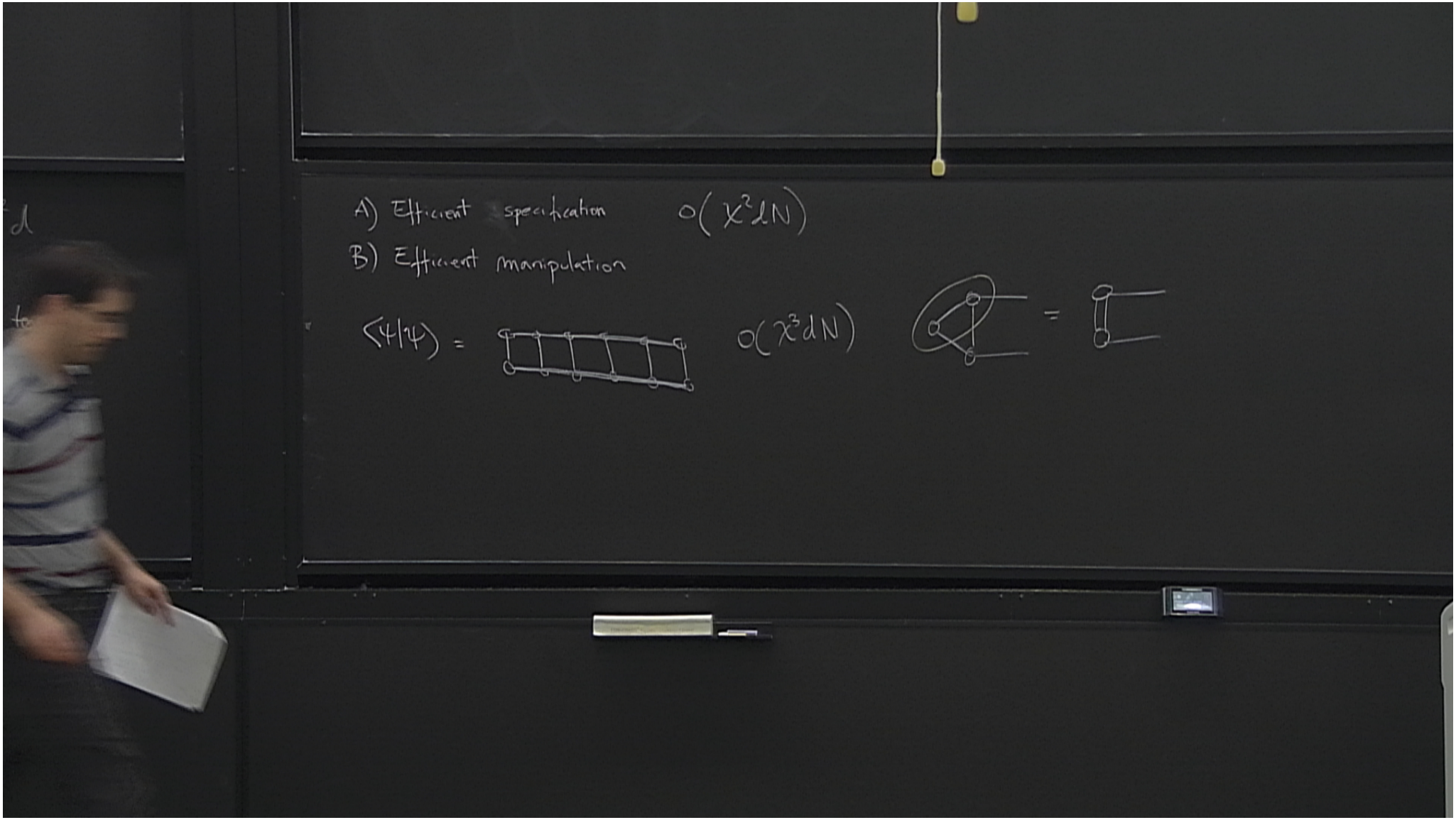
A) Efficient specification $O(\chi^2 dN)$

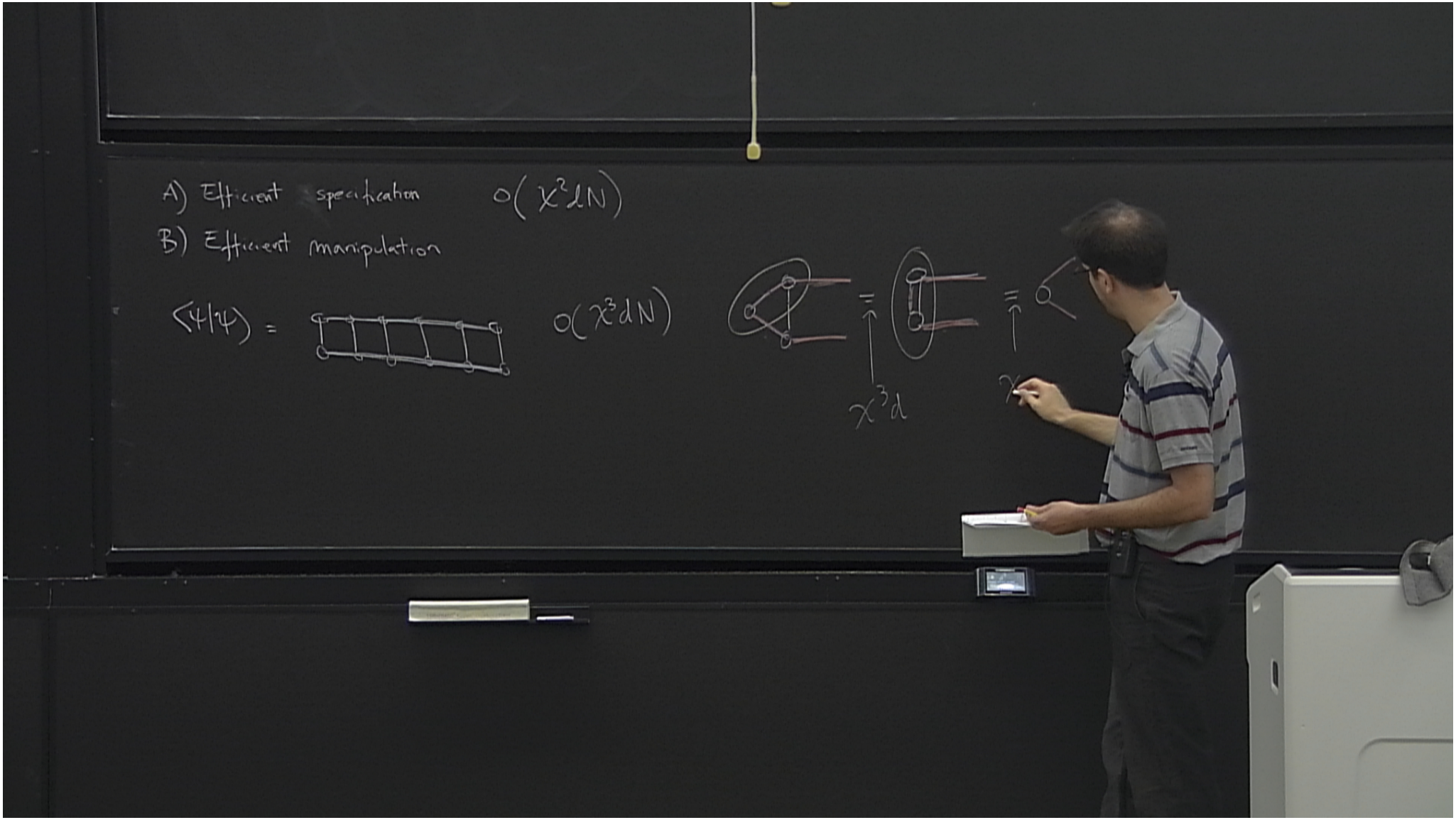
B) Efficient manipulation

$\langle \psi | \psi \rangle =$ 


A) Efficient specification $O(\chi^2 dN)$
B) Efficient manipulation
$\langle\psi|\psi\rangle =$
$O(\chi^3 dN)$

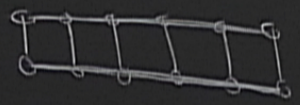

A) Efficient specification $O(\chi^2 dN)$
B) Efficient manipulation
$\langle\psi|\psi\rangle =$
$O(\chi^3 dN)$
$=$
0
0


A) Efficient specification $O(\chi^2 dN)$
B) Efficient manipulation
$\langle \psi | \psi \rangle =$
$O(\chi^3 dN)$
=

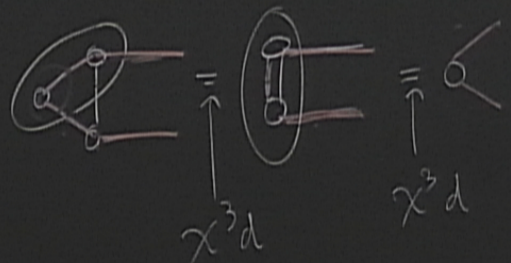

A) Efficient specification $O(\chi^2 dN)$
B) Efficient manipulation
$\langle \psi | \psi \rangle =$
$O(\chi^3 dN)$
$\chi^3 d$

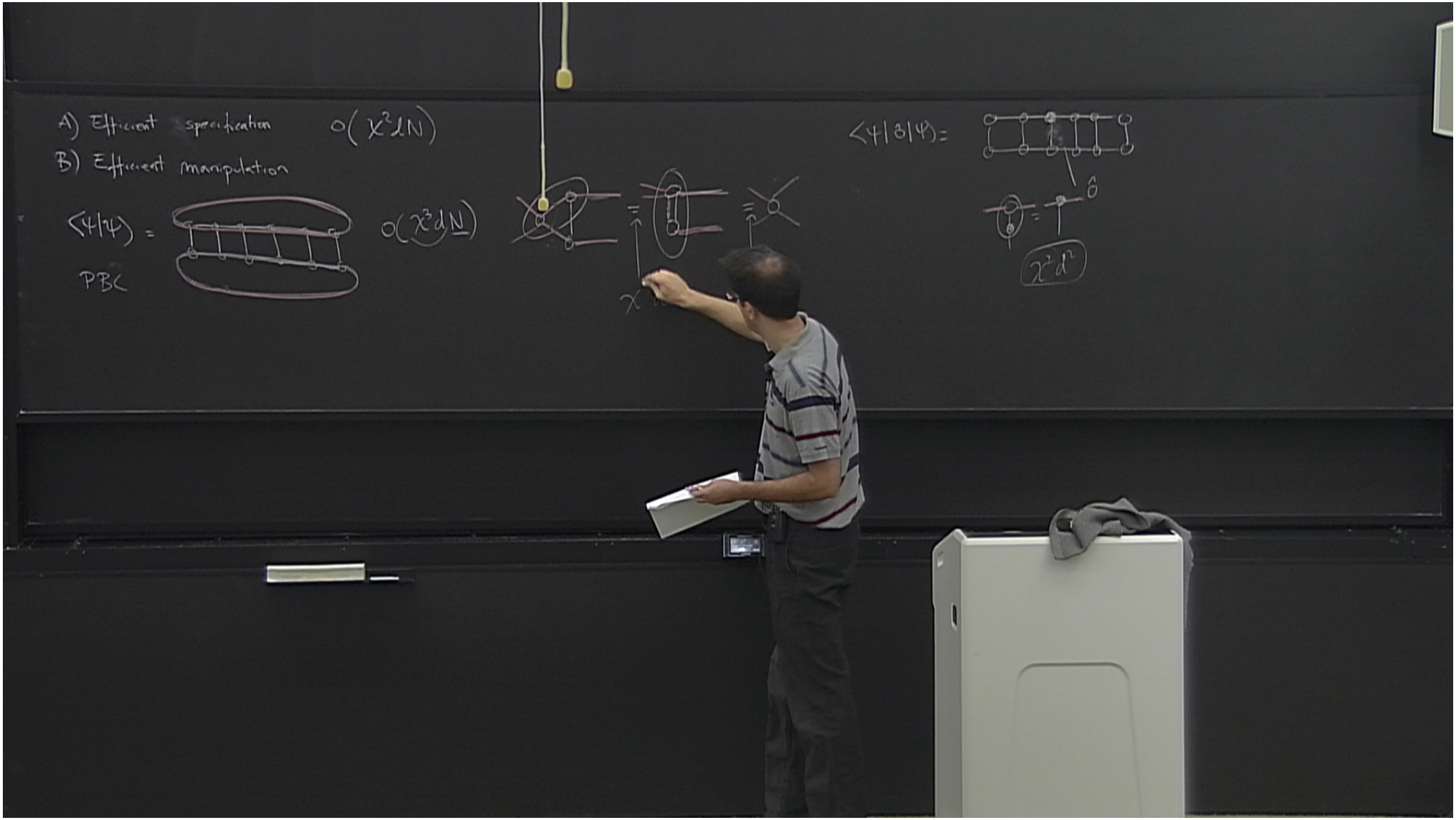
A) Efficient specification $O(X^2 \log N)$

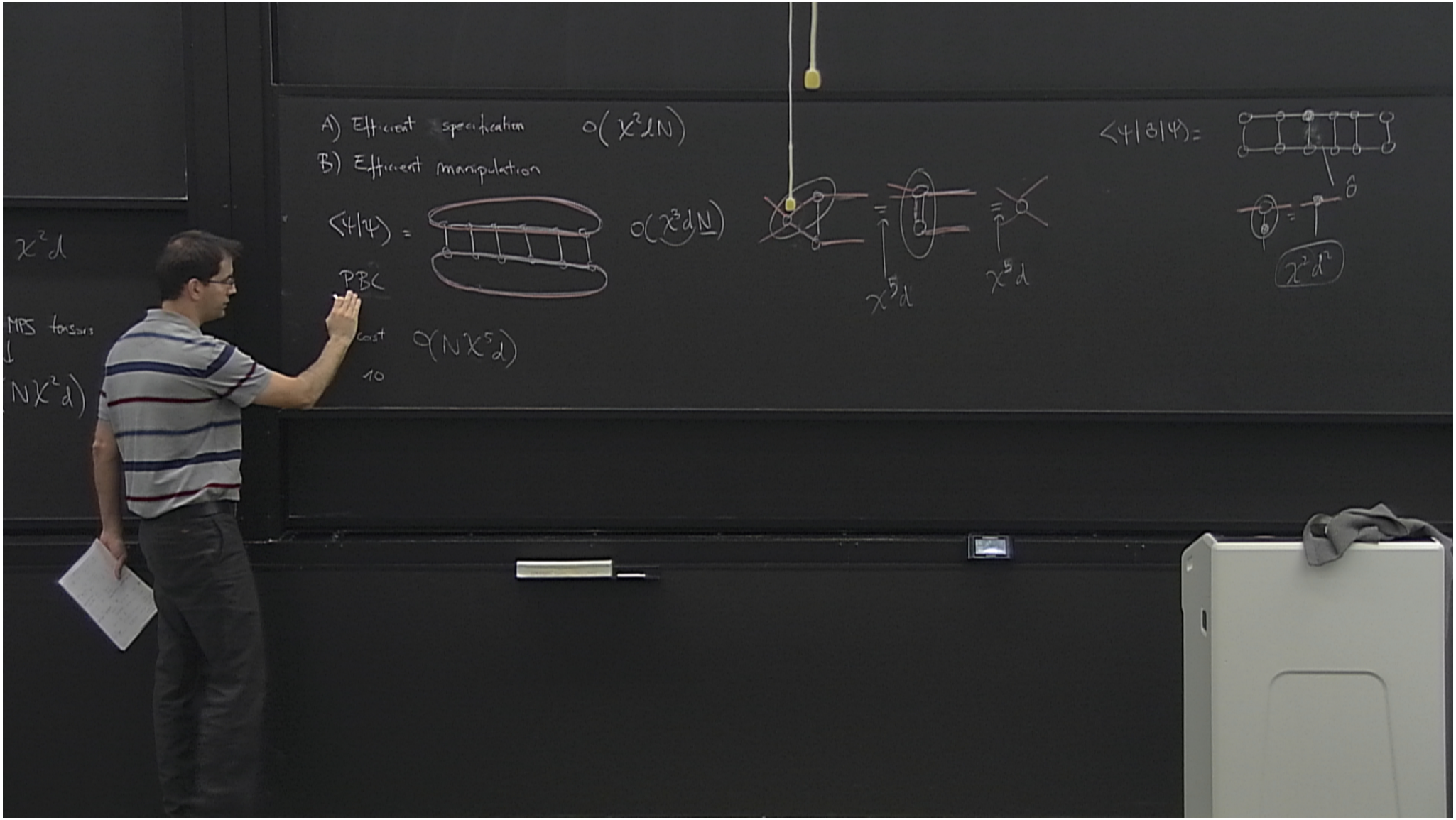
B) Efficient manipulation

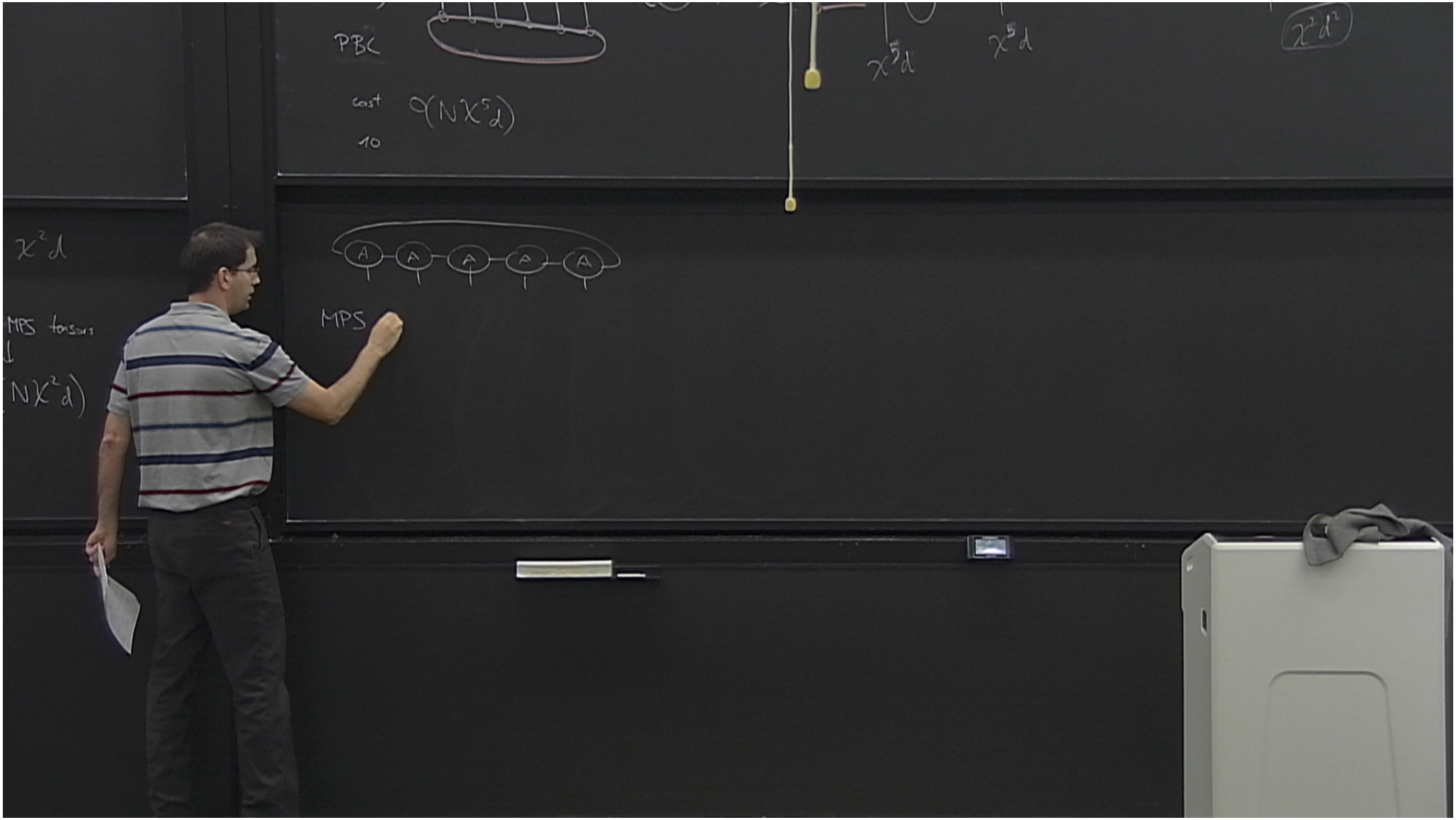
$\langle \psi | \psi \rangle =$ 

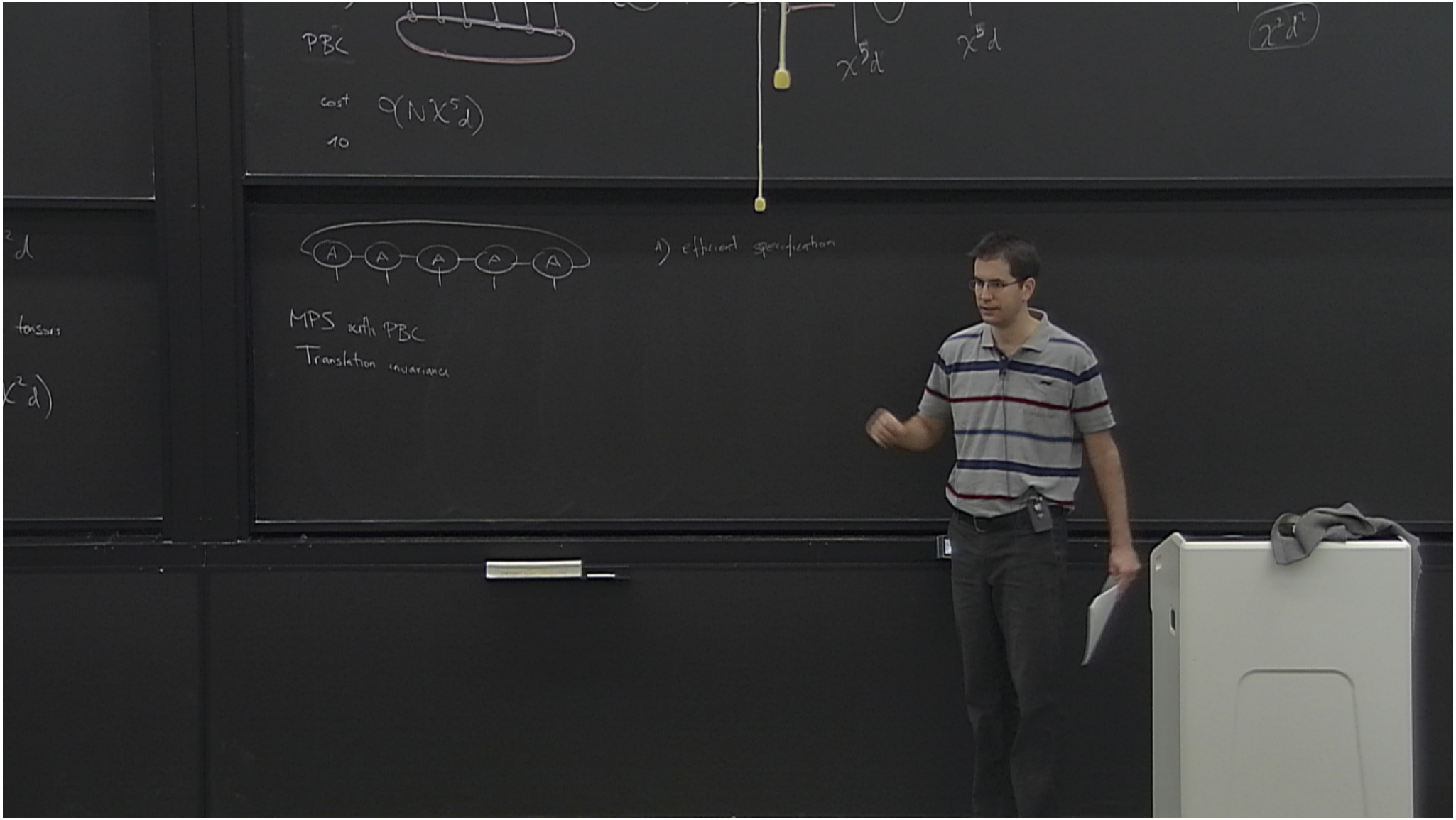
$O(X^3 \log N)$

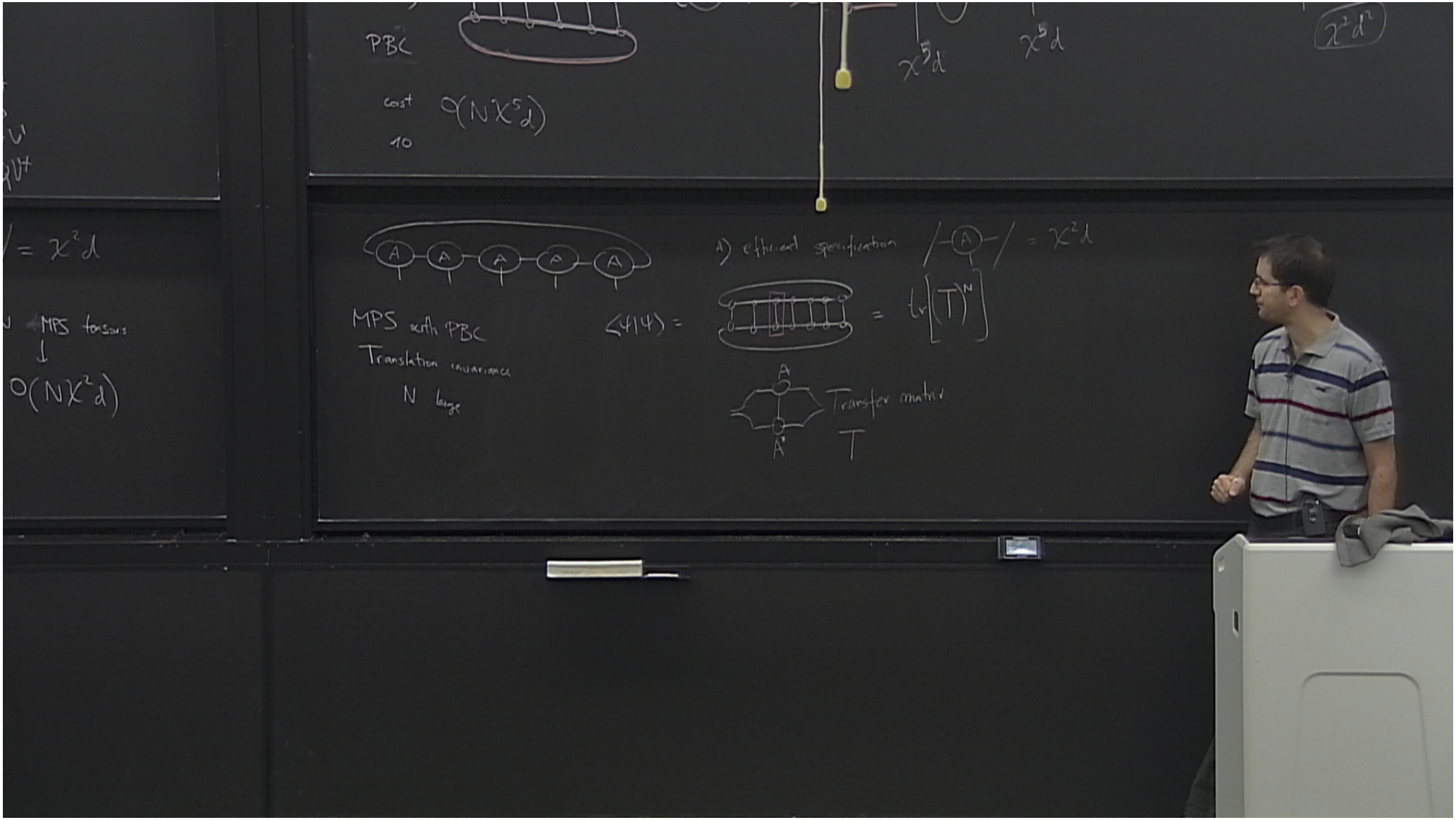


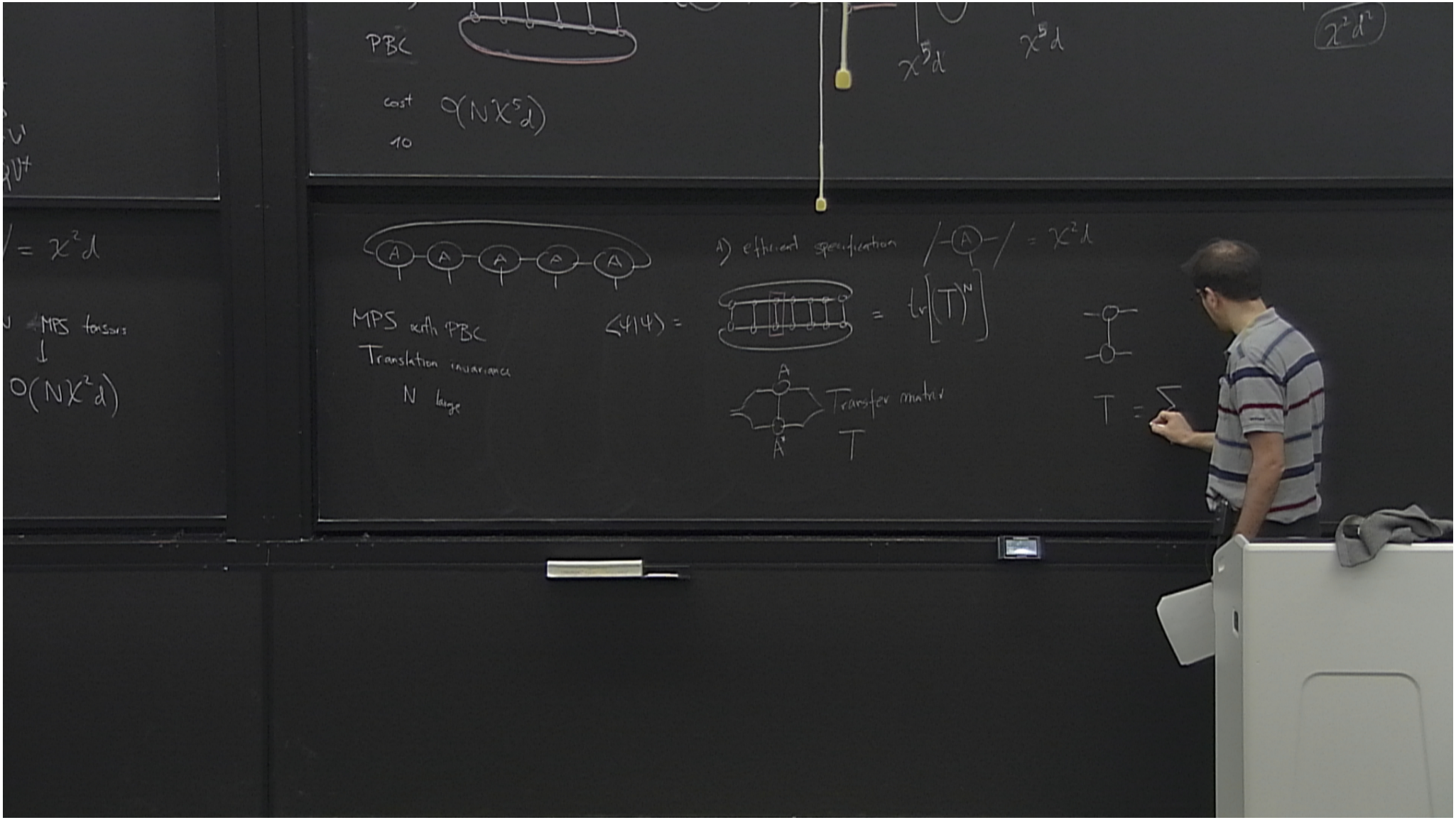

A) Efficient specification
B) Efficient manipulation
PBC


A) Efficient specification
B) Efficient manipulation
PBC
cost
10
MPS tensors


PBC
cost
10
MPS


PBC
cost
10
MPS with PBC
Translation invariance
1) efficient specification
tensors


PBC
cost
10
$O(NX^5d)$
X^5d
X^5d
X^2d^2
$= X^2d$
MPS tensors
$O(NX^2d)$
A A A A A
MPS with PBC
Translation invariance
N large
$\langle \psi | \psi \rangle =$
A) efficient specification
$= X^2d$
$= tr\left[(T)^N\right]$
A
A*
Transfer matrix
T


PBC
cost
$O(NX^5d)$
10
X^5d
X^5d
X^2d^2
$= X^2d$
MPS tensors
$O(NX^2d)$
MPS with PBC
Translation invariance
N large
$\langle \psi | \psi \rangle =$
A) efficient specification
$= X^2d$
$= tr\left[(T)^N\right]$
Transfer matrix
T
$T = \sum$