

Title: 13/14 PSI - Explorations in Condensed Matter - Lecture 10

Date: Mar 28, 2014 10:15 AM

URL: <http://pirsa.org/14040036>

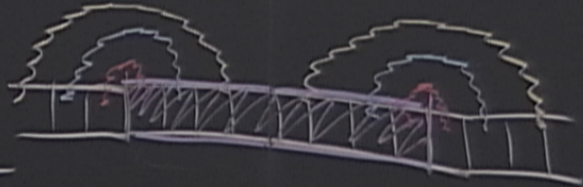
Abstract:



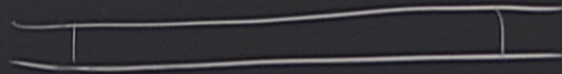
$$L=2 \quad S_{L=2} = 2$$

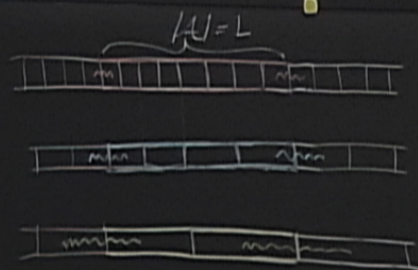


$$L=4 \quad S_{L=4} = 4$$



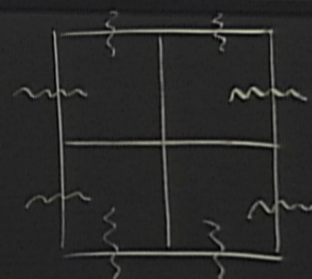
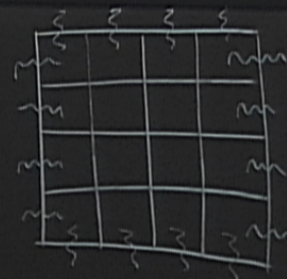
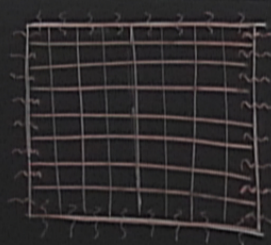
$$L=8 \quad S_{L=8} = 6$$

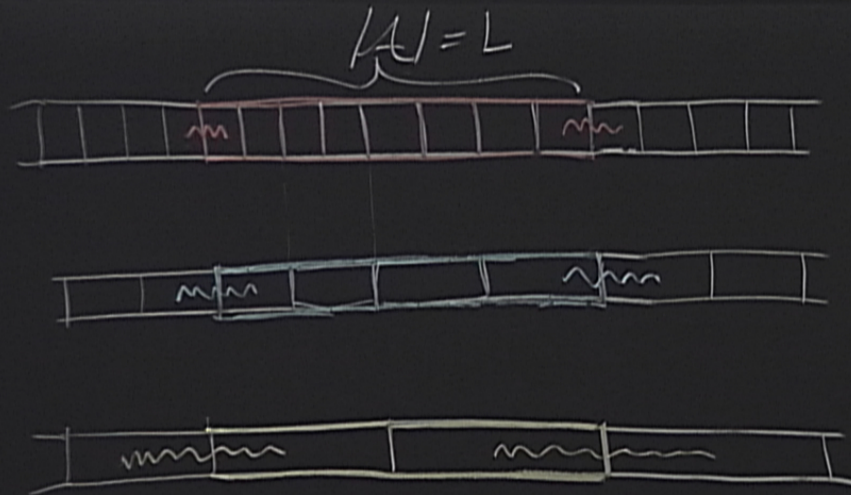




$\left. \begin{matrix} 2 \\ 2 \\ 2 \end{matrix} \right\} \log_2 L$

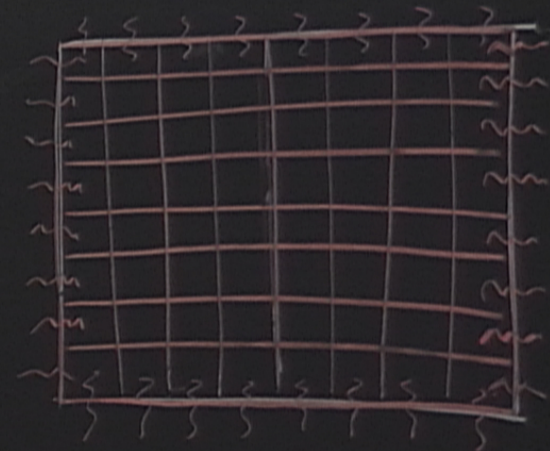
$$S_L = 2k_B \log_2 L$$

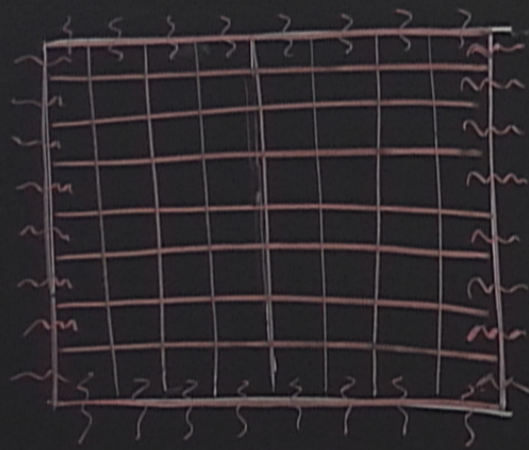




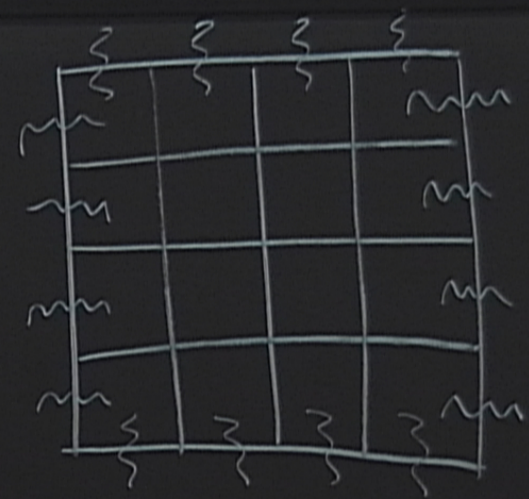
$$\left. \begin{matrix} 2 \\ 2 \\ 2 \end{matrix} \right\} \log_2 L$$

$$S_L = 2 \log_2 L$$



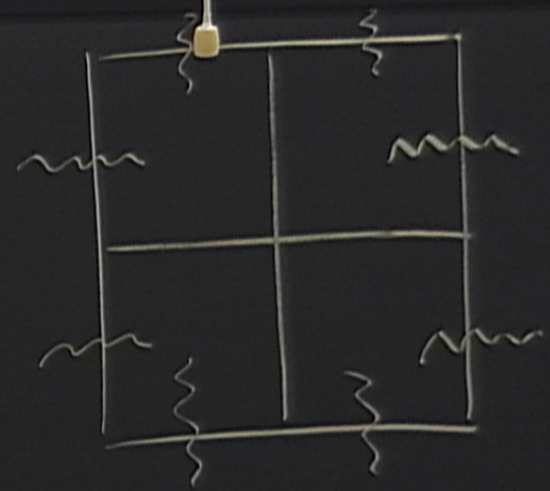


L



$$L_{\text{eff}} = \frac{L}{2}$$

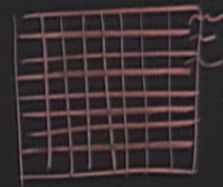
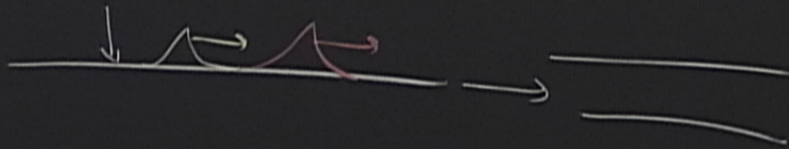
$$L \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right) < 2L$$



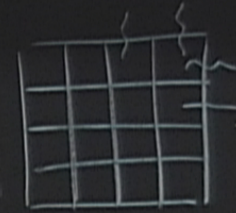
$$L_{\text{eff}} = \frac{L}{4}$$

RG

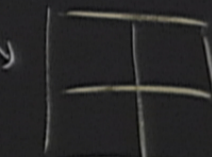
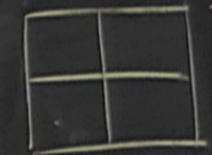
Spin-charge separation



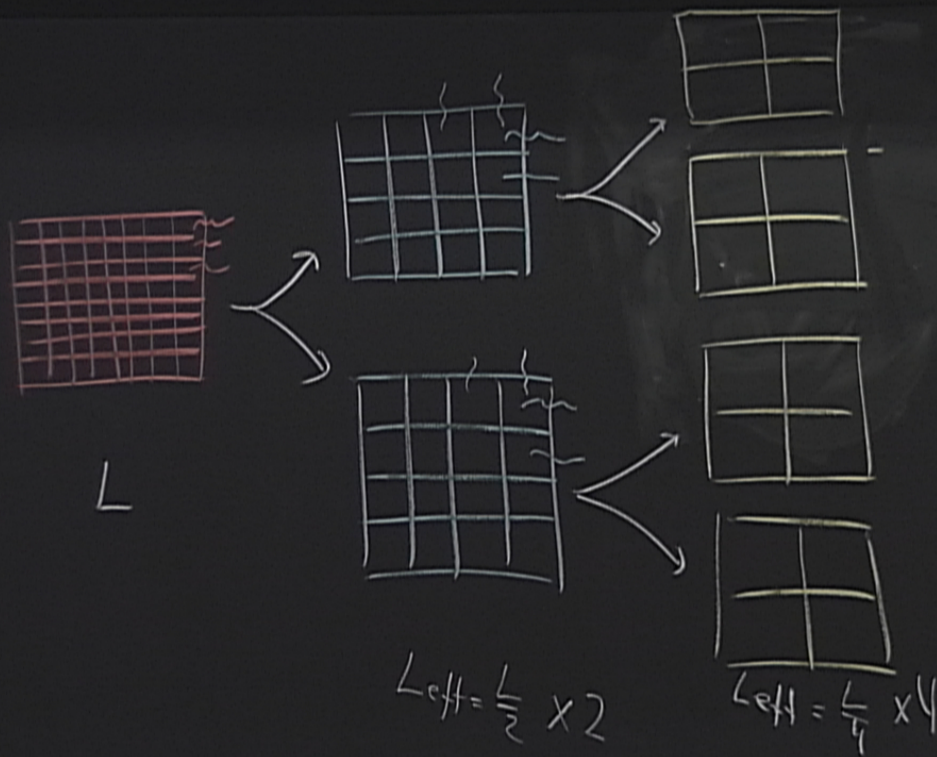
L



$$L_{\text{eff}} = \frac{L}{2} \times 2$$



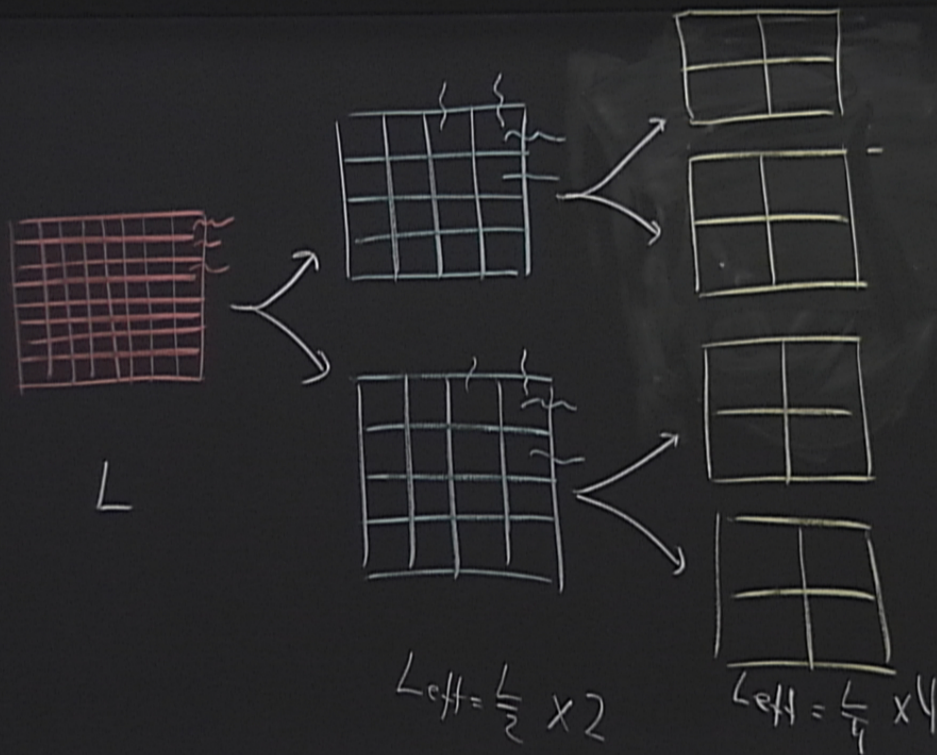
$$L_{\text{eff}} = \frac{L}{4} \times 4$$



$$L \left(1 + \frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \dots \right)$$

$$L \left(\underbrace{1 + 1 + 1 + \dots}_{\log L} \right)$$

$$= L \log L$$



$$L \left(1 + \frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \dots \right)$$

$$L \left(\underbrace{1 + 1 + 1 + \dots}_{\log L} \right)$$

$$= \underline{L \log L}$$

$$S_L \sim L^2 \text{ bulk law}$$

LECTURE 10

(B)

ENTANGLEMENT AS A THEORETICAL TOOL

10.1 Universality

A) $\hat{H}$ local $\rightarrow$ (4)
boundary law

Quantum criticality / CFT $1+1$
in 1D

CFT
 $1+1$ →

conformal data

- central charge

- primary fields

ϕ_α { - conformal dimension ($h_{\alpha}, \bar{h}_{\alpha}$)

- Operator product expansion
OPE

③ Quantum criticality / CFT 1+1
in 1D

CFT $1+1 \rightarrow$ conformal data

- central charge
- primary fields

$\phi_\alpha \left\{ \begin{array}{l} \text{- conformal dimension } (h_\alpha, \bar{h}_\alpha) \\ \text{- Operator product expansion OPE} \end{array} \right.$

Scaling dimensions

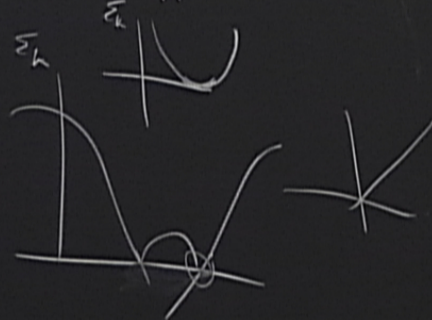
$$\Delta_\alpha = h_\alpha + \bar{h}_\alpha$$

conformal spin

$$S_\alpha = h_\alpha - \bar{h}_\alpha$$

③ Quantum criticality / CFT 1+1
in 1D

CFT 1+1



conformal data

- central charge

- primary fields

③

ϕ_α { - conformal dimension ($h_\alpha, \bar{h}_\alpha$)

- Operator product expansion
OPE

$$S_L = \frac{c}{3} \log L$$

Scaling dimensions

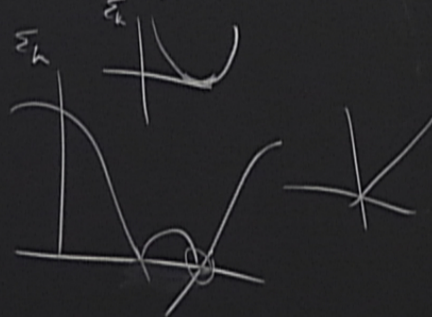
$$\Delta_\alpha = h_\alpha + \bar{h}_\alpha$$

conformal spin

$$S_\alpha = h_\alpha - \bar{h}_\alpha$$

③ Quantum criticality / CFT 1+1
in 1D

CFT 1+1



conformal data

- central charge
- primary fields

③

- conformal dimension ($h, \bar{h}$)
- Operator product expansion
OPE

$$S_L = \frac{c}{3} \log L$$

Scaling dimensions

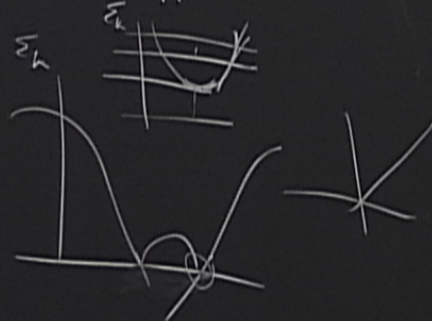
$$\Delta_\alpha = h_\alpha + \bar{h}_\alpha$$

conformal spin

$$S_\alpha = h_\alpha - \bar{h}_\alpha$$

② Quantum criticality / CFT 1+1
in 1D

CFT 1+1



conformal data

- central charge
- primary fields

③

- conformal dimension ($h, \bar{h}$)
- Operator product expansion OPE

$$S_L = \frac{c}{3} \log L$$

Scaling dimensions

$$\Delta_\alpha = h_\alpha + \bar{h}_\alpha$$

conformal spin

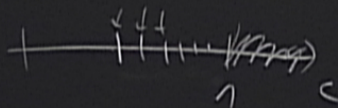
$$S_\alpha = h_\alpha - \bar{h}_\alpha$$

unitary minimal models

$$c = 1 - \frac{6}{m(m+1)}$$

$$m = 3, 4, 5, 6, \dots$$

Ⓢ Topo

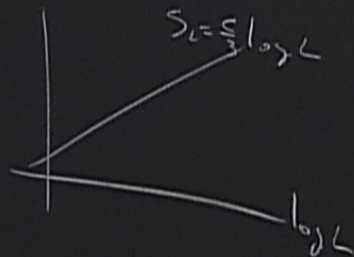


$$c = \frac{1}{2} \quad \text{Ising CFT}$$

$$c = 7/10 \quad \text{tricritical Ising}$$

$$c = 4/5 \quad \text{3-state Potts model}$$

$$c = 6/7 \quad \text{tricritical "}$$



③ Topological order / TQFT 2+1

anyon model

- list of anyon types i

- quantum dimension d_i

$$D \equiv \sqrt{\sum_i d_i^2} \quad \text{total quantum dimension}$$

- fusion rules

Toric code

$$i = 1, e, m, \epsilon$$

$$d_1 = d_e = d_m = d_\epsilon = 1$$

$$D = 2$$

③ Topological order / TQFT 2+1

anyon model

- list of anyon types i

- quantum dimension d_i

$$D \equiv \sqrt{\sum_i d_i^2} \quad \text{total quantum dimension}$$

- fusion rules

$$N_{ij}^k$$

$$i \times j \rightarrow k$$

Toric code

$$i = 1, e, m, \epsilon$$

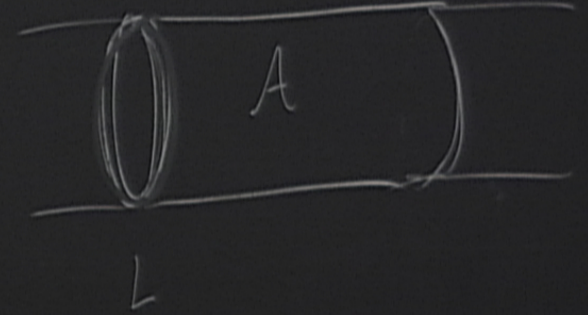
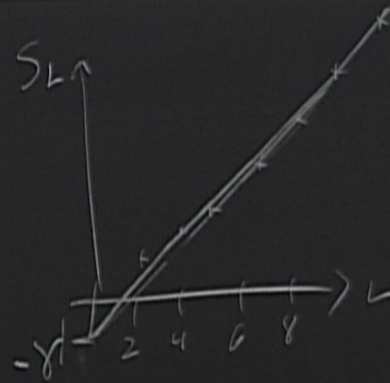
$$d_1 = d_e = d_m = d_\epsilon = 1$$

$$D = 2$$

$$S_L = \alpha L - \gamma$$


 L

α non-universal
 γ universal



10.2 ENTANGLEMENT SPECTRUM

$$S(A) = -\sum p_k \log p_k \longrightarrow \{p_k\}$$

A) Quantum criticality / CFT 1+1

$$p_k = e^{-\lambda_k}$$

entanglement
(energy)

10.2 ENTANGLEMENT SPECTRUM

$$S(A) = - \sum_{p_k} \log p_k \quad \xrightarrow{\substack{\text{eigenvalues of } \hat{\rho}^A}} \{p_k\}$$

$$p_k = e^{-\lambda_k}$$

A) Quantum criticality / CFT 1+1

$$\boxed{C=1}$$

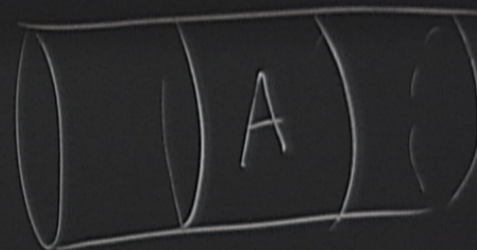
free boson with compactification radius R

$$S_L \sim \frac{c}{3} \log L \quad \{ \lambda_k \}$$

$$S_L \sim \frac{c}{3} \log L \quad \left\{ \downarrow_k \right\}$$

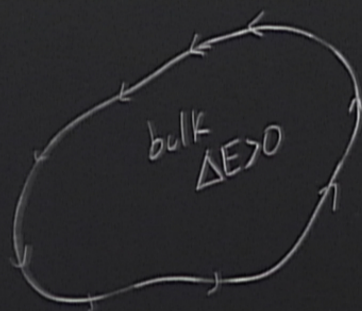
③ Topological order / TQFT in $2+1$

$D=2$ { Toric code
doubled semion
Ising anyon model
⋮

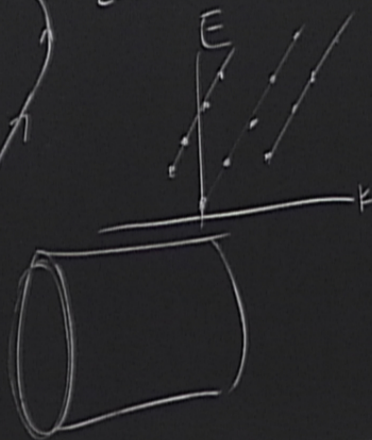


chiral topological order

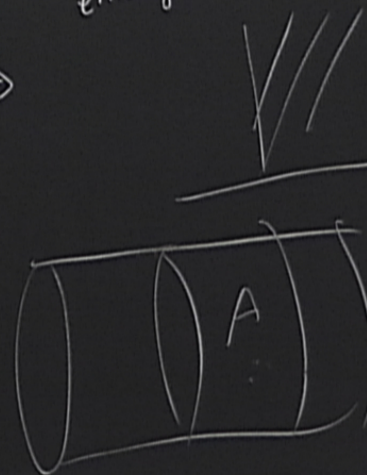
FQH, chiral spin liquid



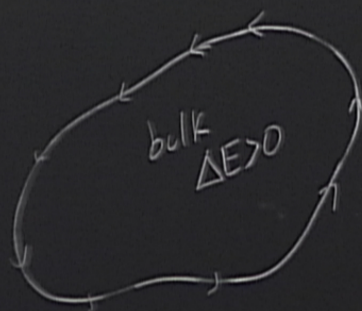
chiral CFT



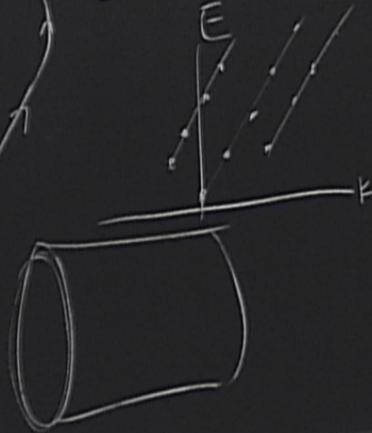
entanglement spectrum



chiral topological order
FQH, chiral spin liquid



chiral CFT



entanglement spectrum

