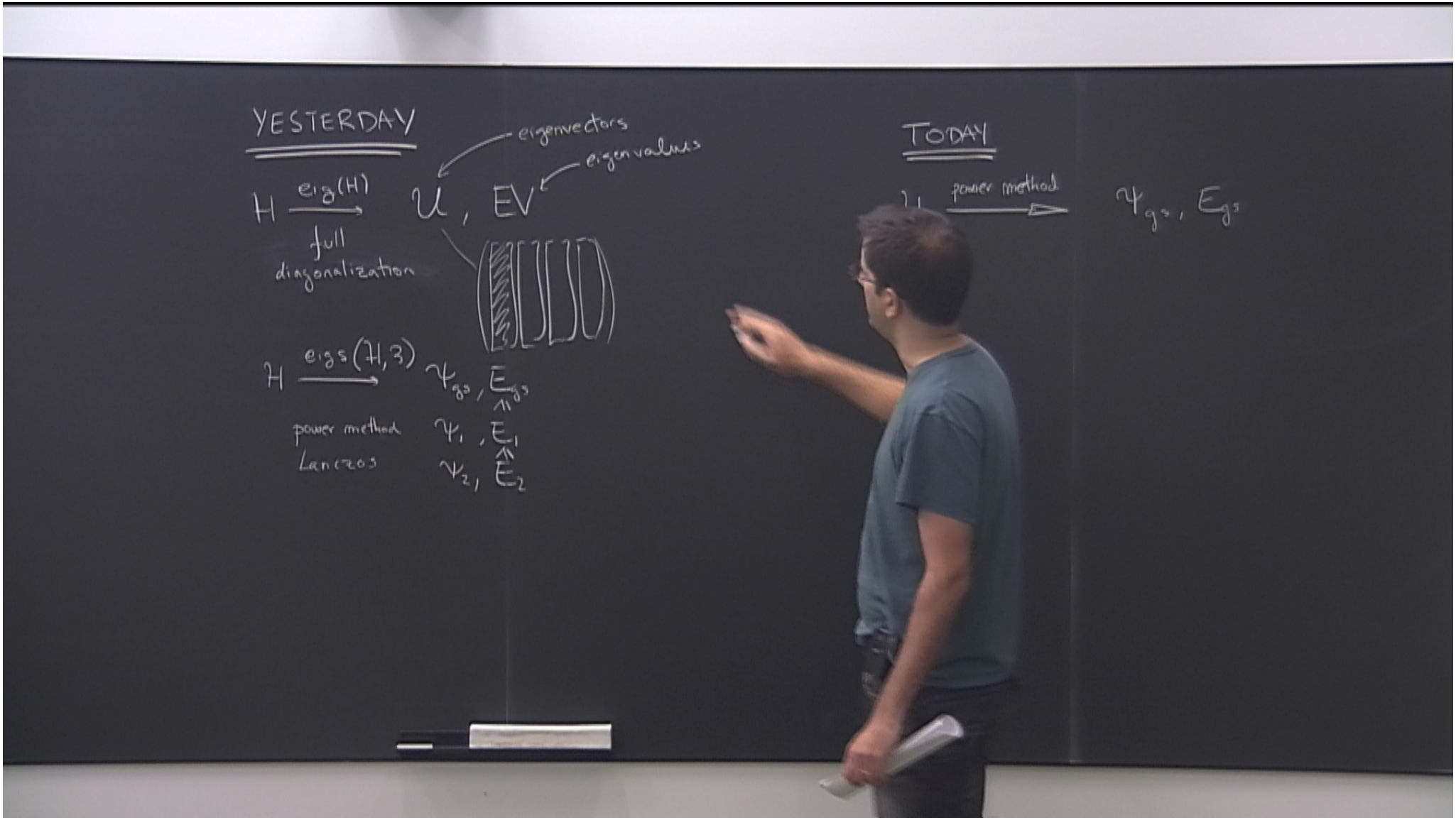
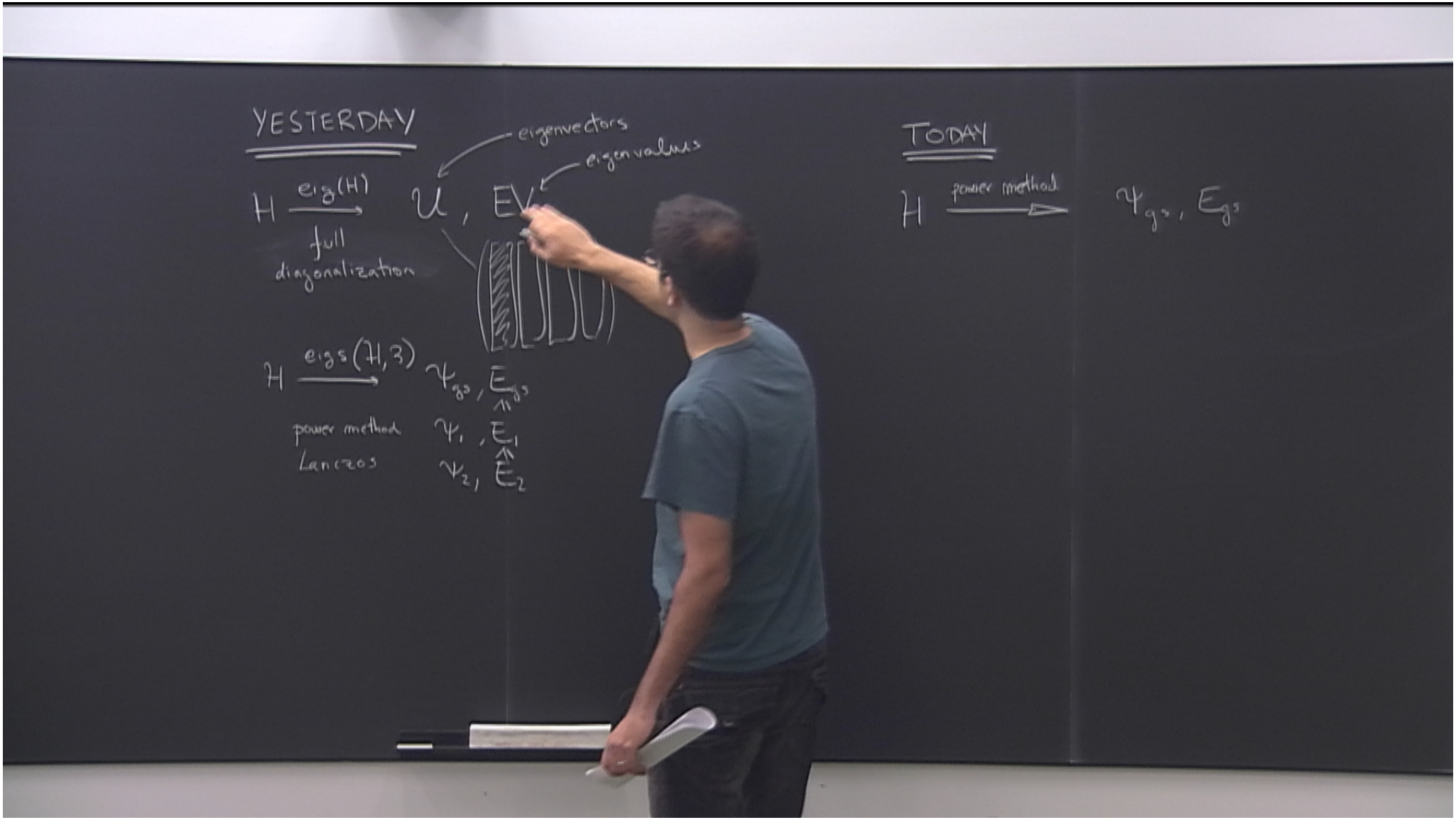
Title: 13/14 PSI - Explorations in Condensed Matter - Lecture 4

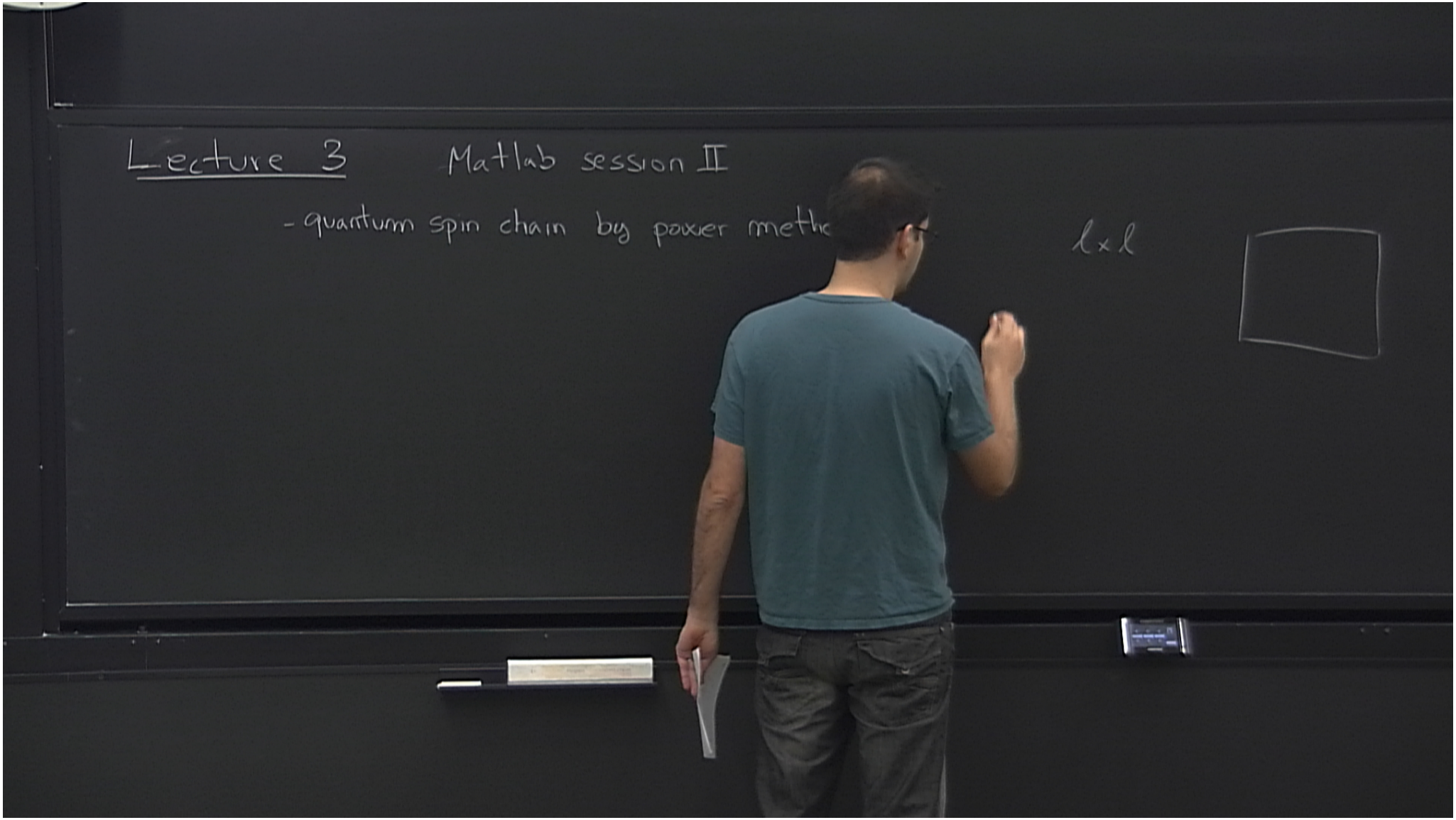
Date: Mar 20, 2014 10:15 AM

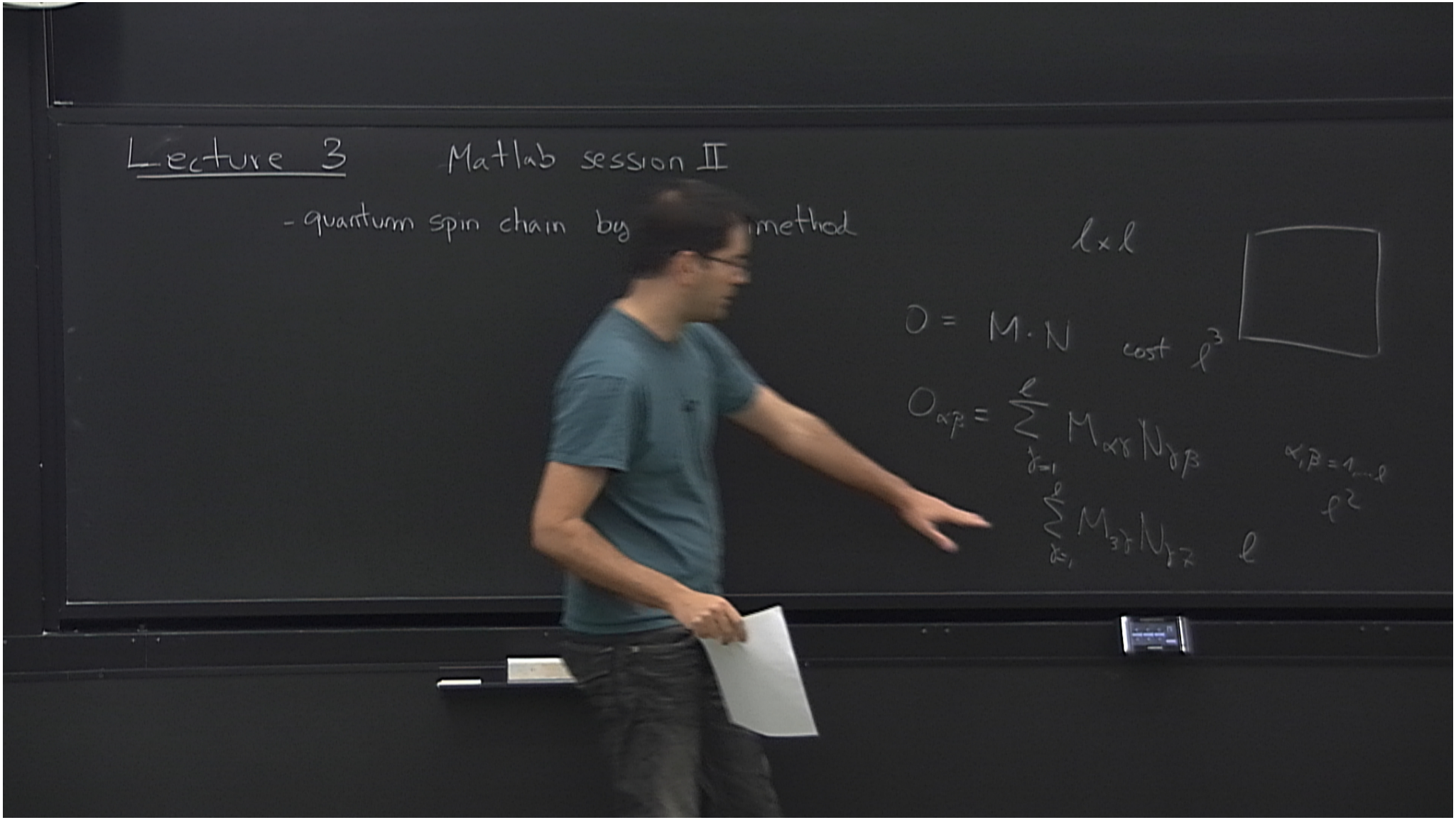
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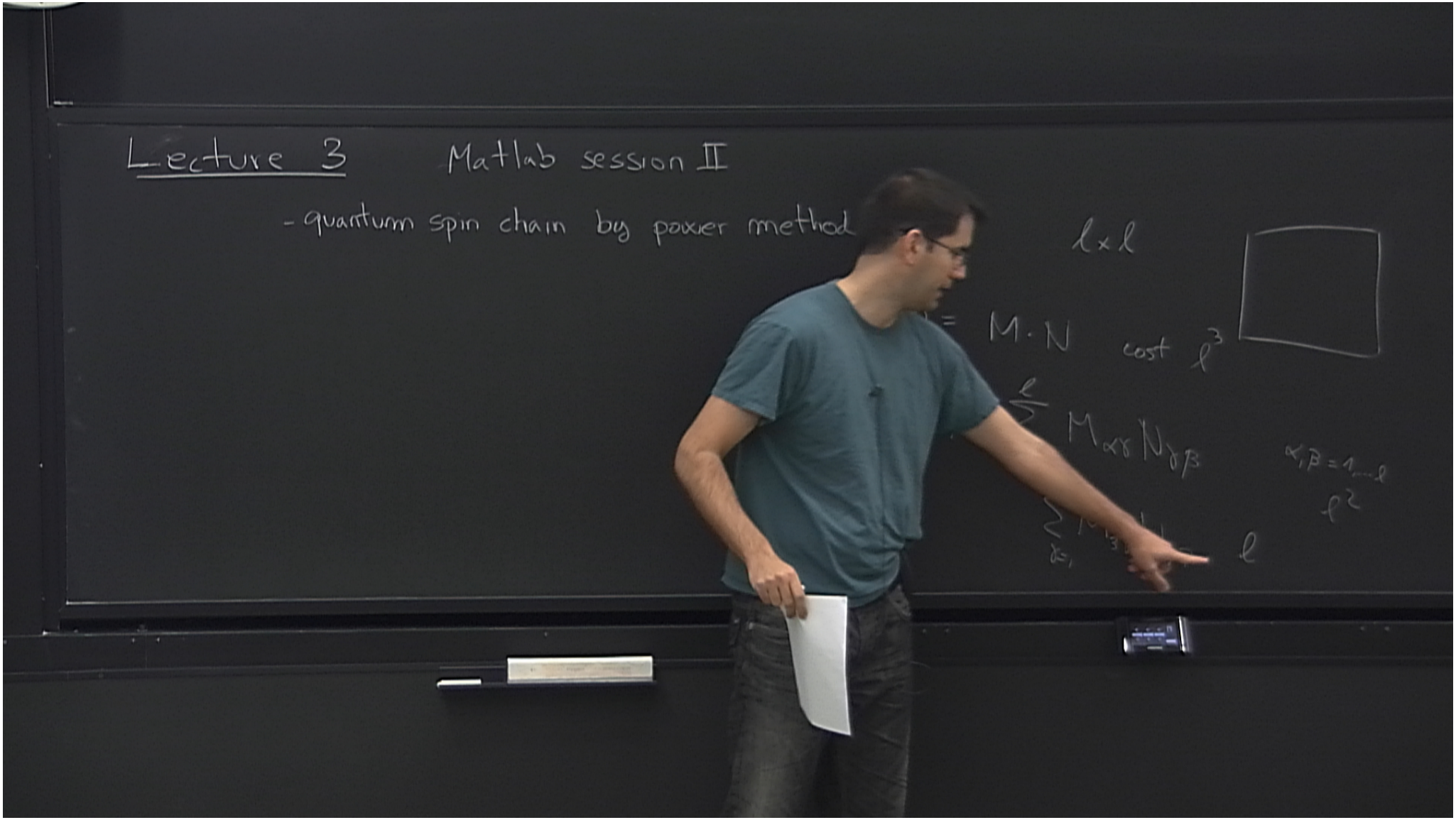
Abstract:

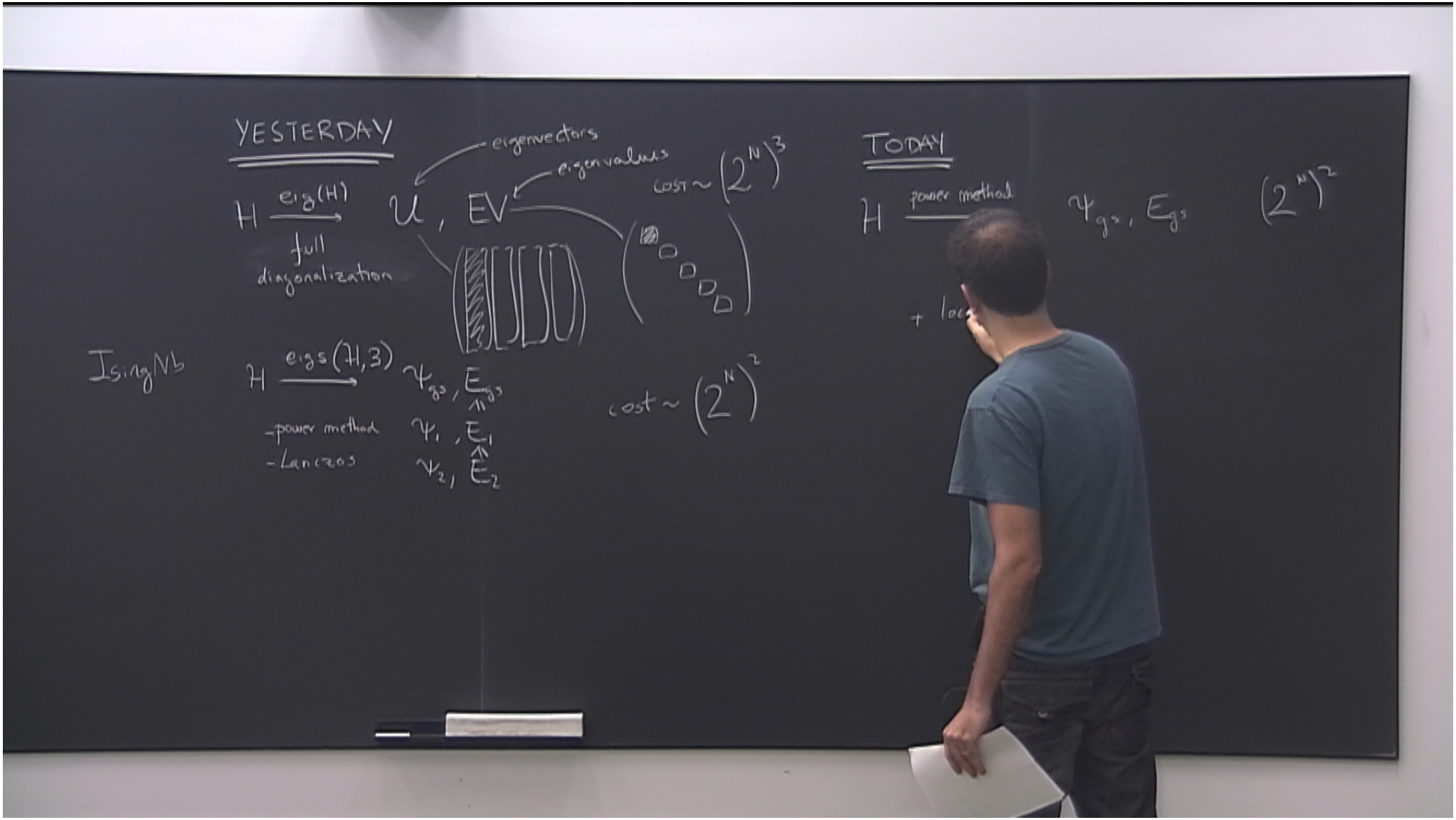

YESTERDAY
eigenvectors
eigenvalues
H
eig(H)
full
diagonalization
U, EV
eigs(H,3)
power method
Lanczos
TODAY
power method

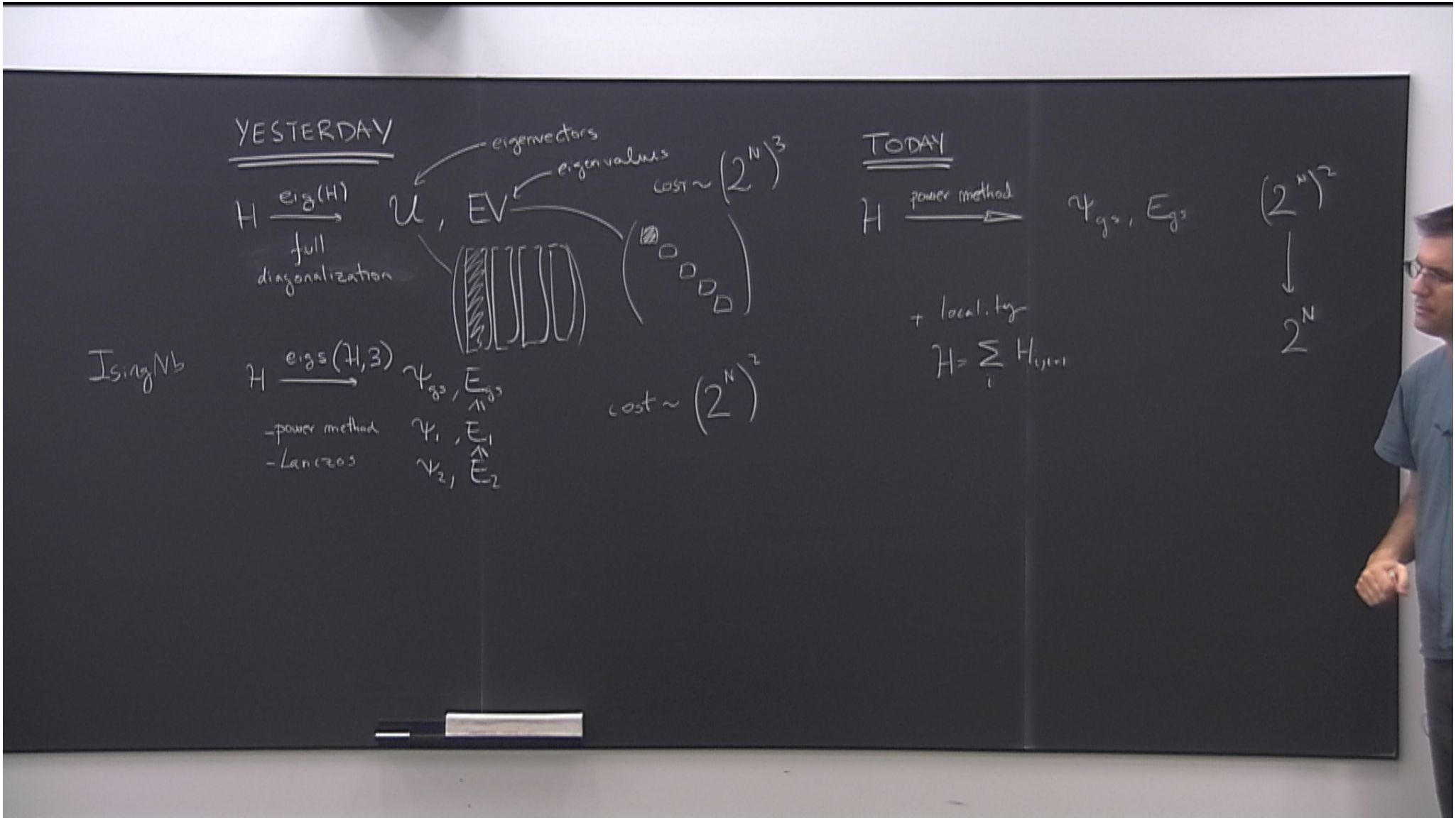

YESTERDAY
eigenvectors
eigenvalues
H
eig(H)
full
diagonalization
U, EV
H
eigs(H,3)
power method
Lanczos
TODAY
H
power method

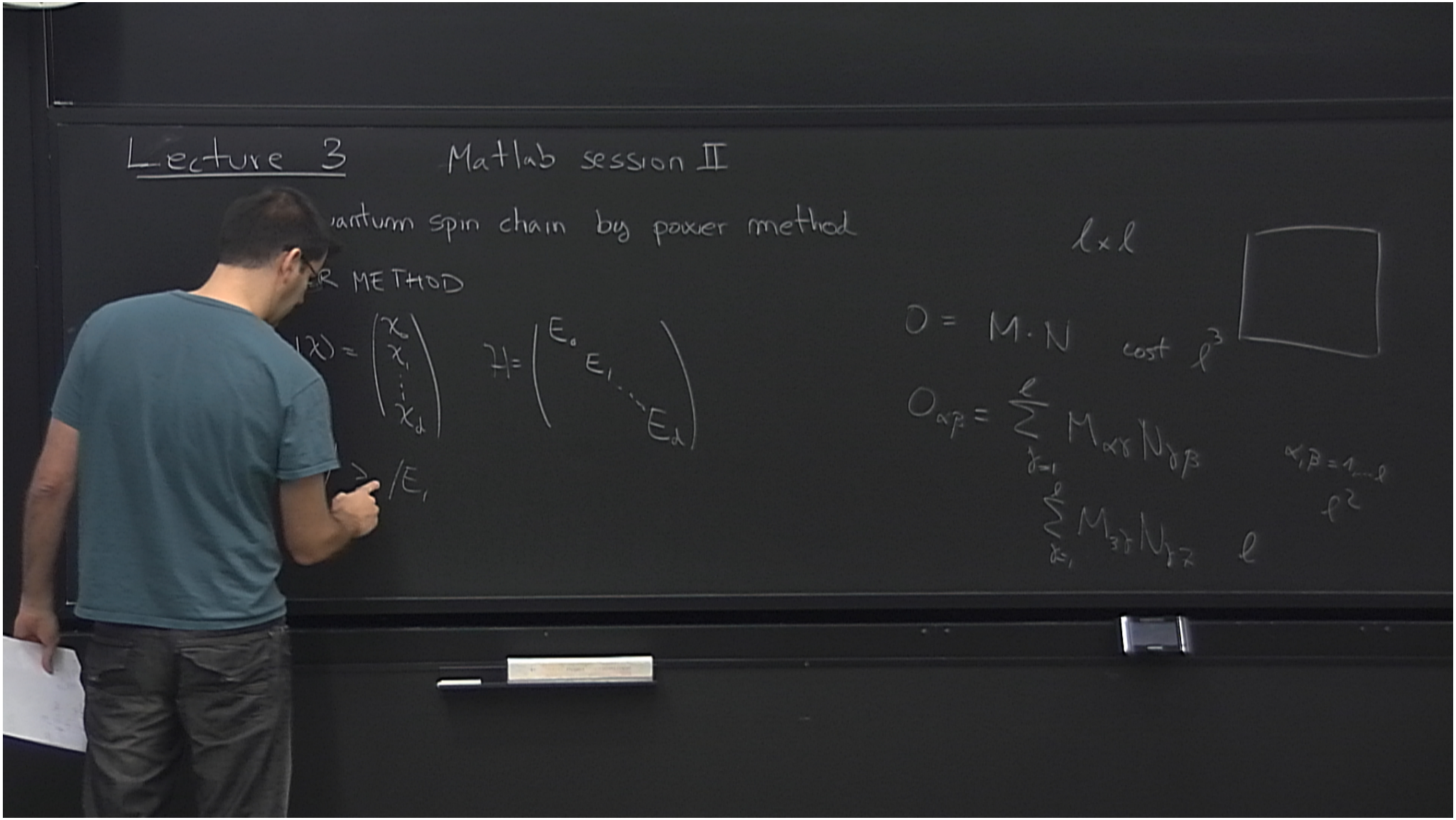

Lecture 3
Matlab session II
- quantum spin chain by power meth
l × l

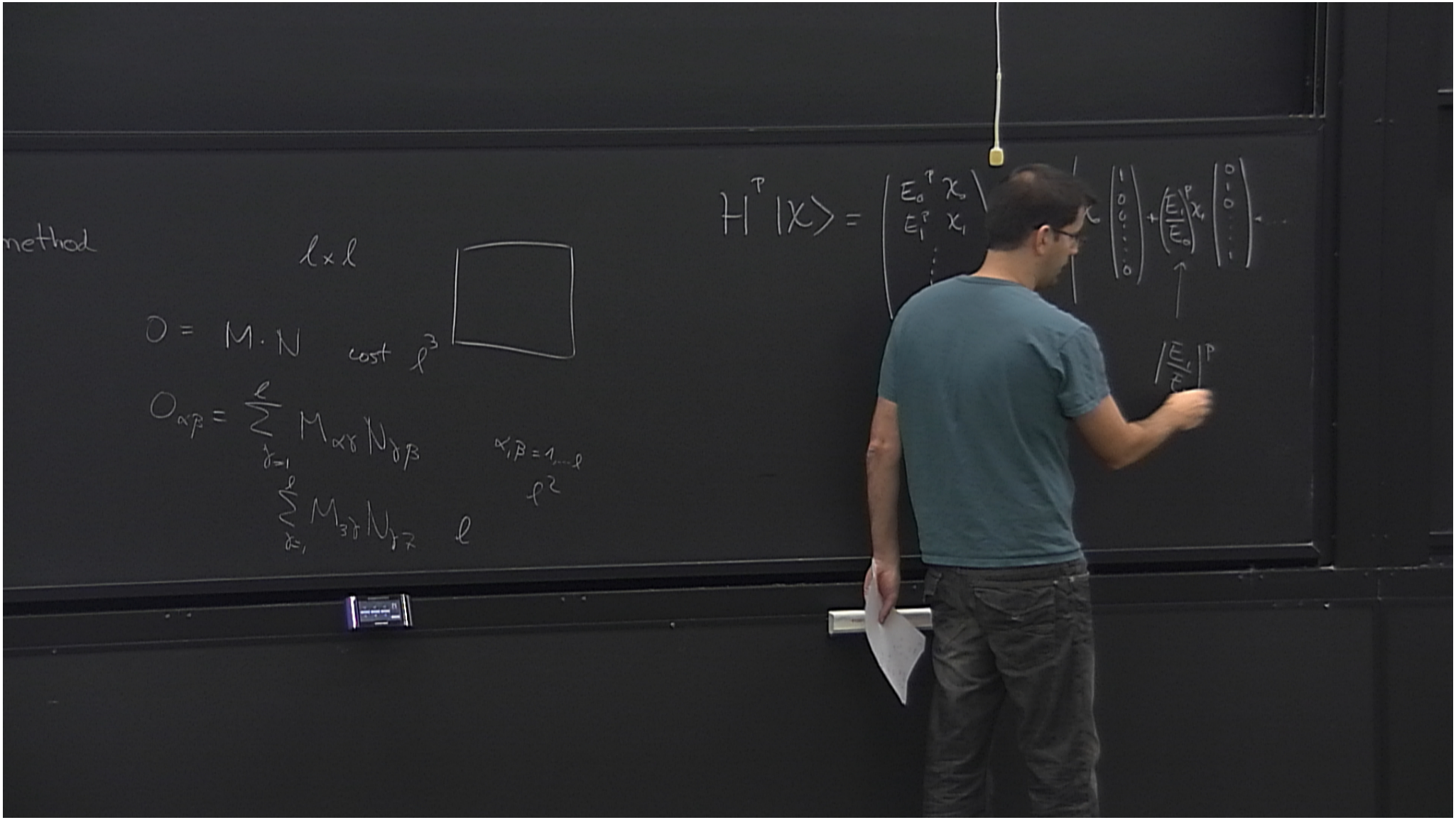

Lecture 3
Matlab session II
- quantum spin chain by
method
$l \times l$
$O = M \cdot N$
cost l^3
$O_{\alpha\beta} = \sum_{\gamma=1}^{l} M_{\alpha\gamma} N_{\gamma\beta}$
$\alpha, \beta = 1, \ldots l$
l^2
$\sum_{\gamma=1}^{l} M_{3\gamma} N_{\gamma 2}$
l

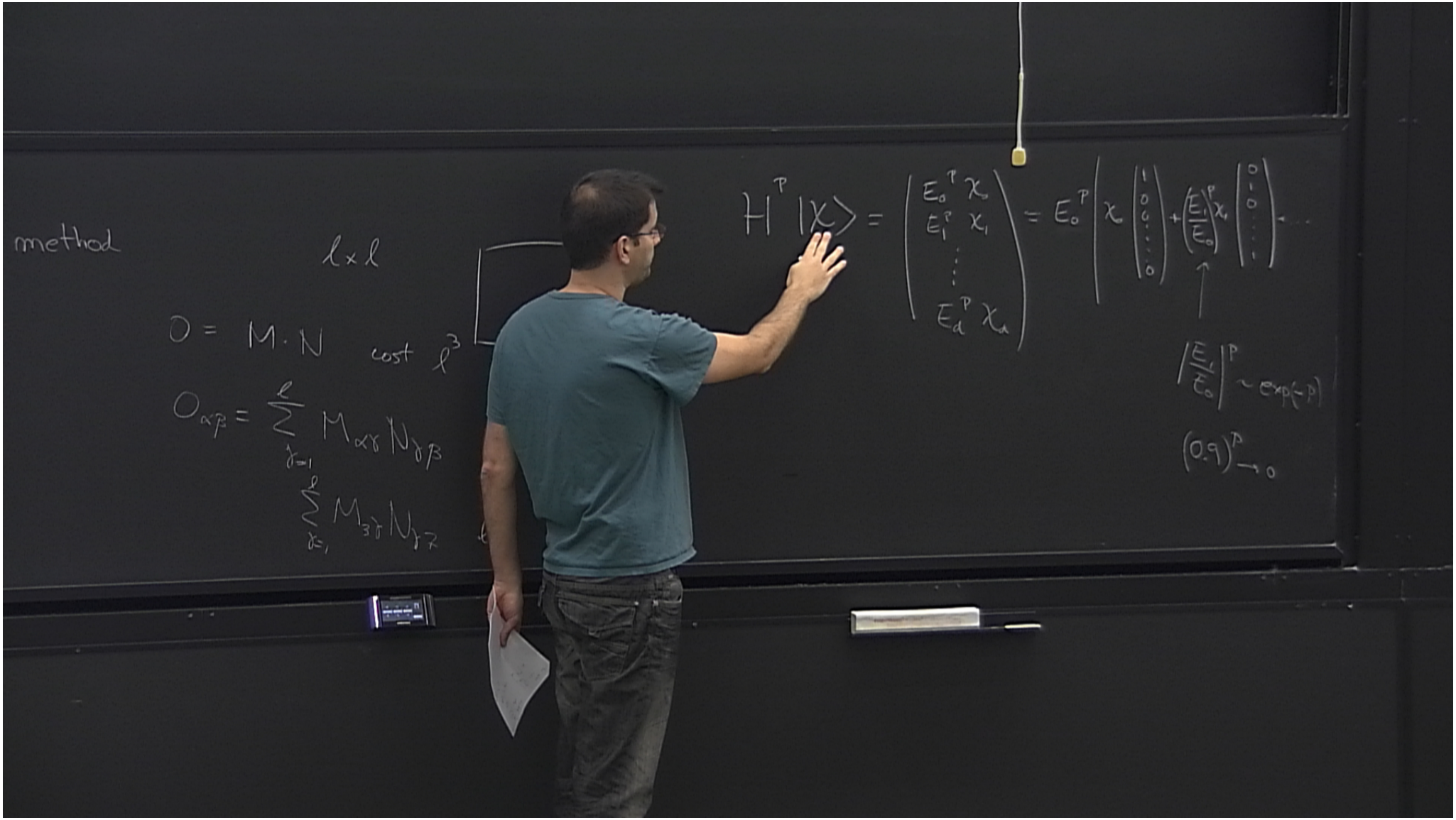

Lecture 3
Matlab session II
- quantum spin chain by power method
$l \times l$
$= M \cdot N$
cost l^3
$\sum_{\gamma=1}^{l} M_{\alpha\gamma} N_{\gamma\beta}$
$\alpha, \beta = 1, \ldots, l$
l^2
l

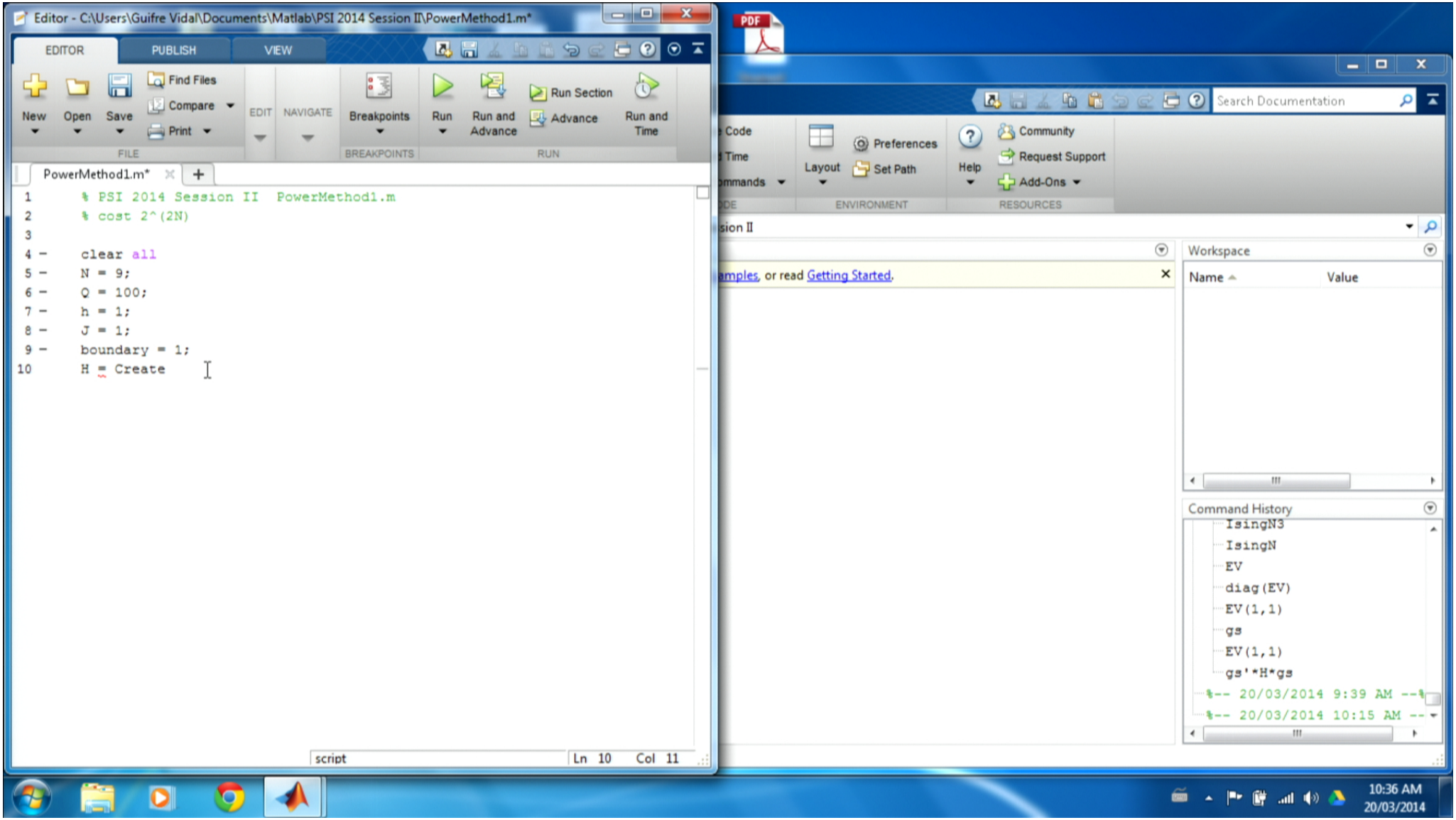

YESTERDAY
eigenvectors
eigenvalues
H
eig(H)
full diagonalization
U , EV
cost ~ (2^N)^3
Ising/Nb
H
eigs(H,3)
-power method
-Lanczos
ψ_gs , E_gs
ψ_1 , E_1
ψ_2 , E_2
cost ~ (2^N)^2
TODAY
H
power method
ψ_gs , E_gs
(2^N)^2

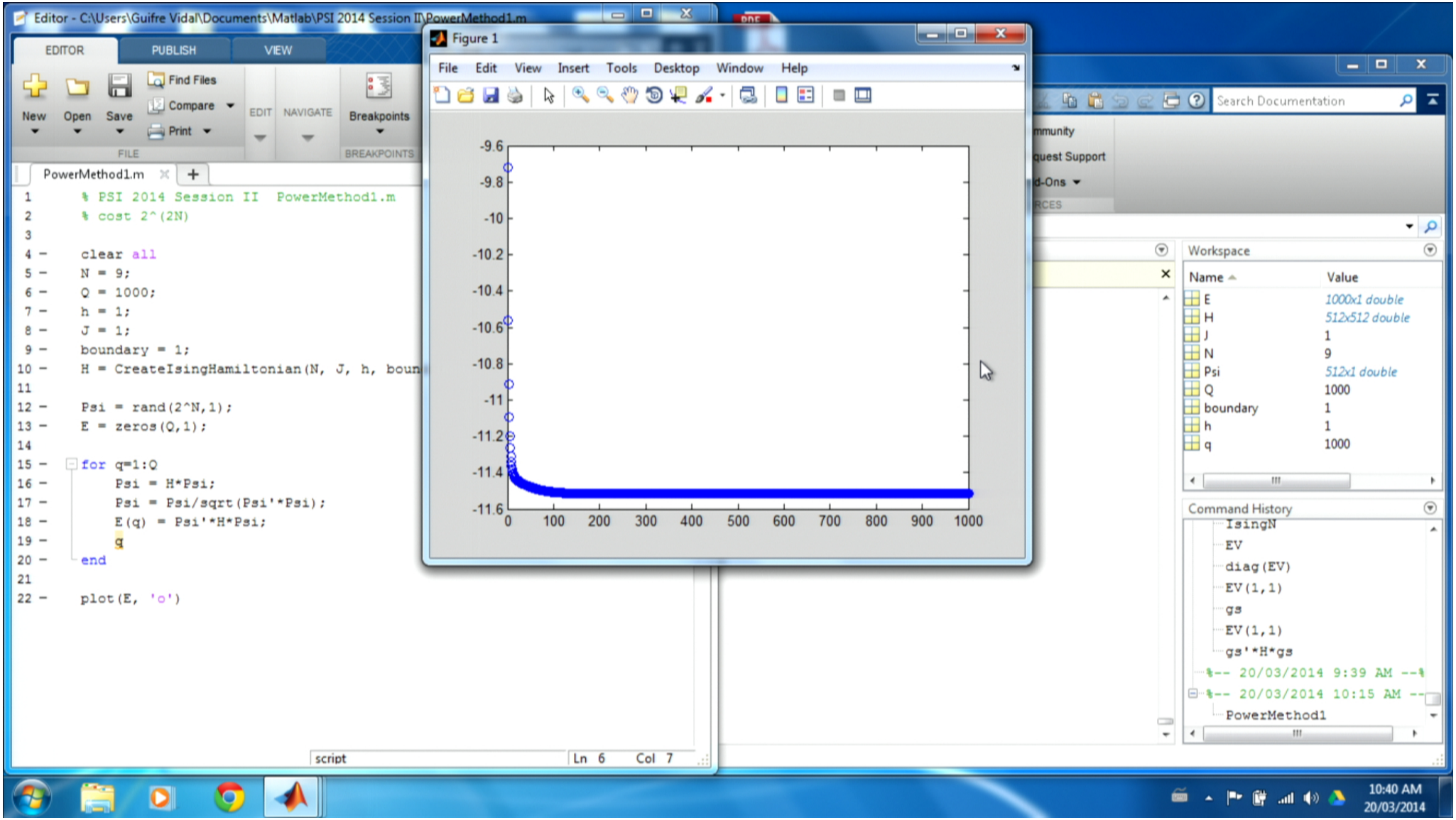

YESTERDAY
eigenvectors
eigenvalues
H
eig(H)
full diagonalization
U , EV
cost ~ (2^N)^3
TODAY
H
power method
ψ_gs , E_gs
(2^N)^2
2^N
+ locality
H = Σ_i H_{i,i+1}
Ising/Nb
H
eigs(H,3)
ψ_gs , E_gs
ψ_1 , E_1
ψ_2 , E_2
-power method
-Lanczos
cost ~ (2^N)^2

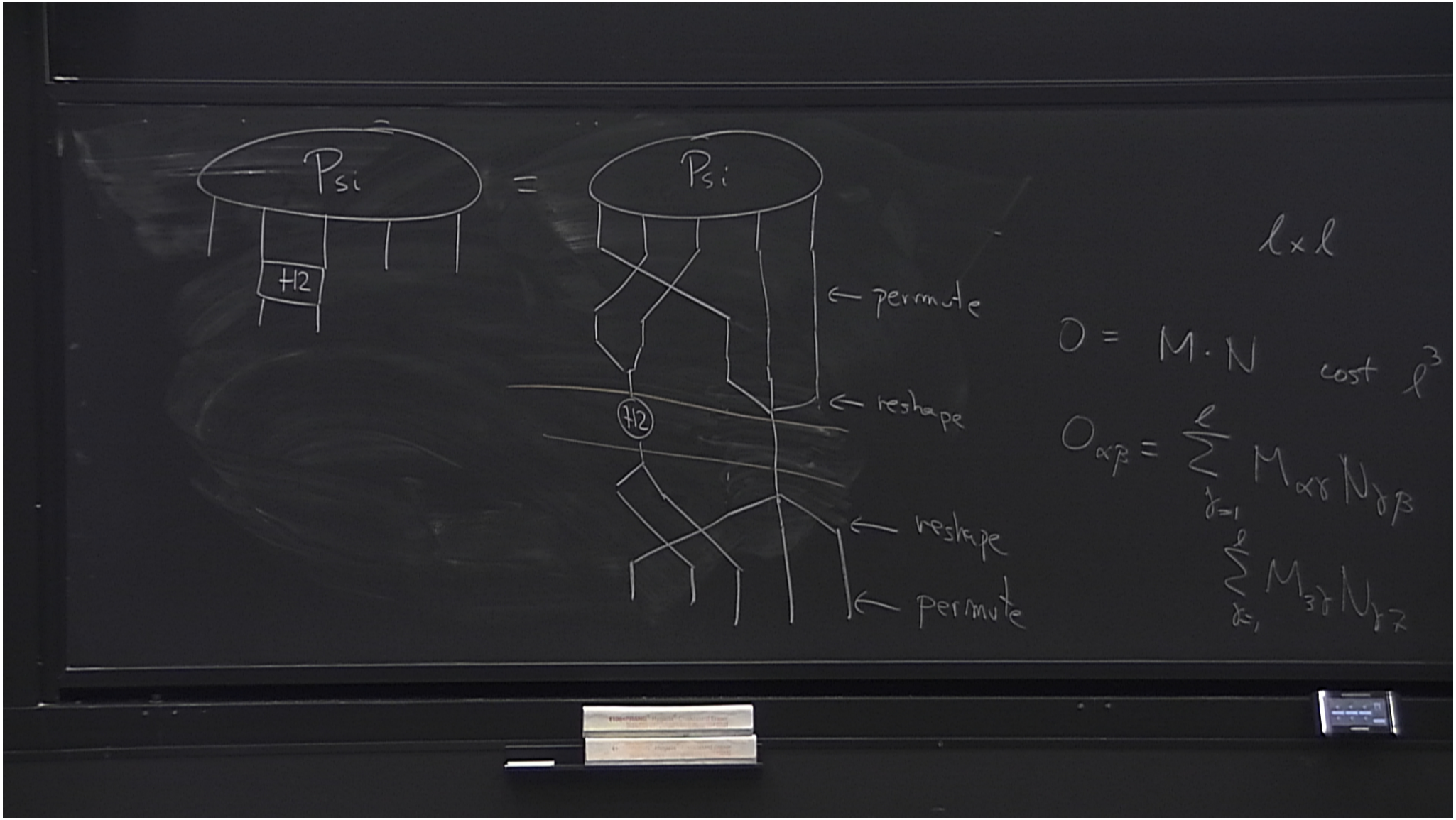

Lecture 3
Matlab session II
antum spin chain by power method
R METHOD
$|\chi\rangle = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \chi_d \end{pmatrix}$
$\mathcal{H} = \begin{pmatrix} E_0 & & & \\ & E_1 & & \\ & & \ddots & \\ & & & E_d \end{pmatrix}$
$/E_1$
$l \times l$
$O = M \cdot N$ cost l^3
$O_{\alpha\beta} = \sum_{\gamma=1}^{l} M_{\alpha\gamma} N_{\gamma\beta}$
$\alpha, \beta = 1 \dots l$
l^2
$\sum_{\alpha=1}^{l} M_{3\alpha} N_{\alpha 2}$ l

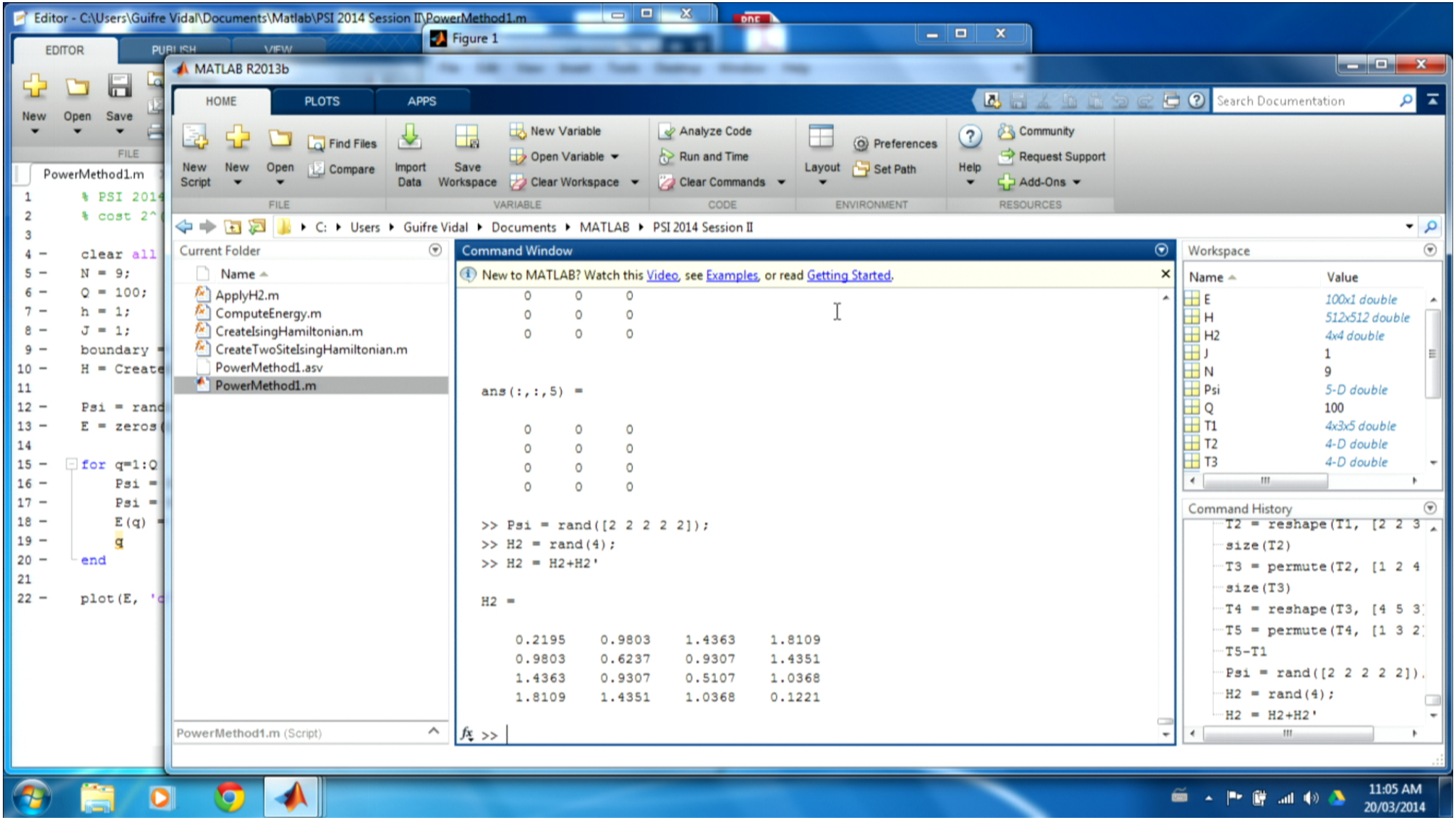

method
$l \times l$
$O = M \cdot N$ cost l^3
$O_{\alpha\beta} = \sum_{\gamma=1}^{l} M_{\alpha\gamma} N_{\gamma\beta}$
$\alpha, \beta = 1, \ldots, l$
l^2
$\sum_{\alpha=1}^{l} M_{3\gamma} N_{\gamma 7}$ l
$H^P |X\rangle =$

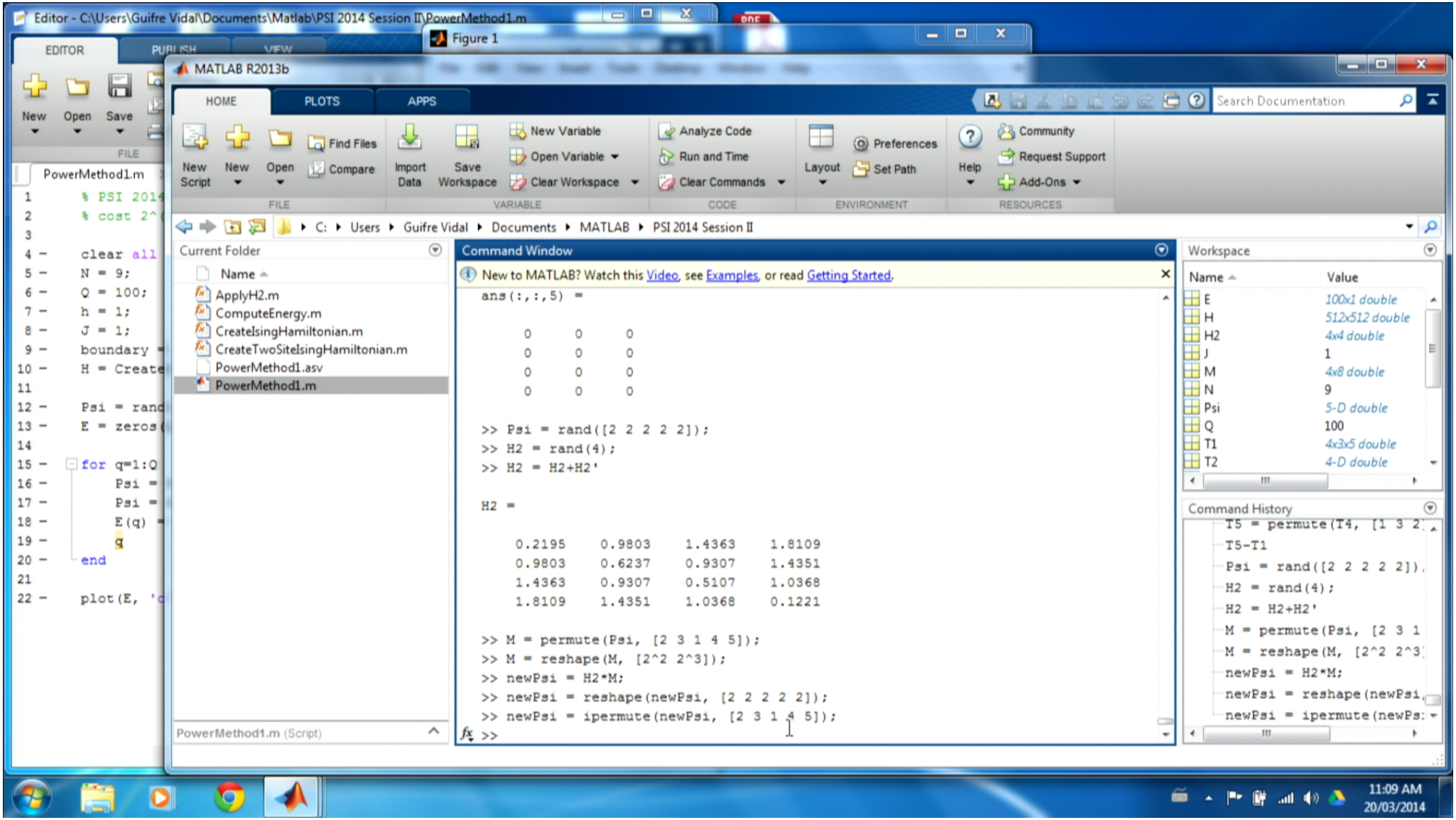

method
$l \times l$
$O = M \cdot N$ cost l^3
$O_{\alpha\beta} = \sum_{\gamma=1}^{l} M_{\alpha\gamma} N_{\gamma\beta}$
$\sum_{\gamma=1}^{l} M_{3\gamma} N_{\gamma 7}$
$H^p |\chi\rangle = \begin{pmatrix} E_0^p \chi_0 \\ E_1^p \chi_1 \\ \vdots \\ E_d^p \chi_d \end{pmatrix} = E_0^p \left(\chi_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \left(\frac{E_1}{E_0}\right)^p \chi_1 \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \cdots \right)$
$\left|\frac{E_1}{E_0}\right|^p \sim \exp(-p)$
$(0.9)^p \to 0$

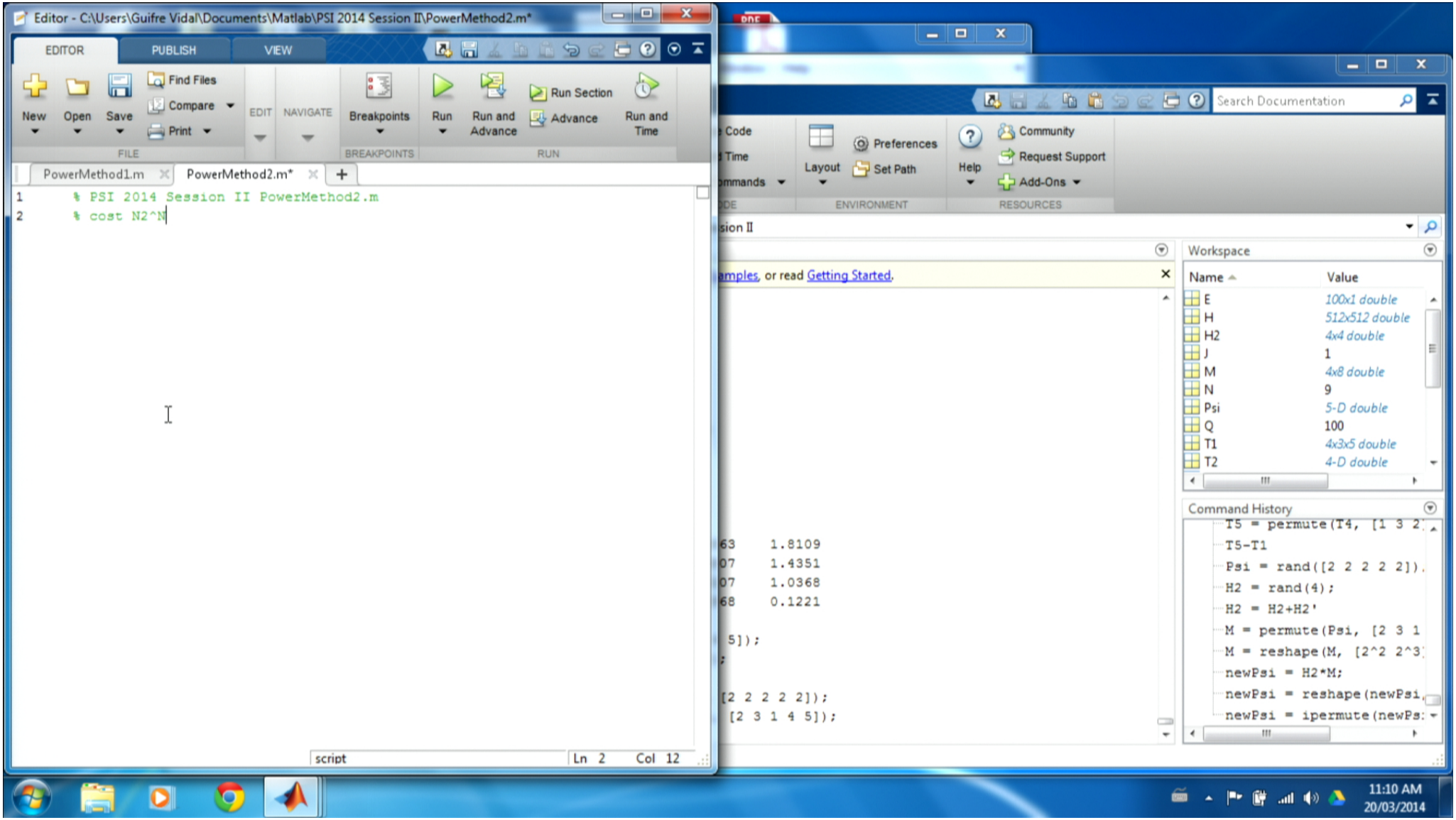

Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod1.m*
EDITOR
PUBLISH
VIEW
New
Open
Save
Find Files
Compare
Print
FILE
EDIT
NAVIGATE
Breakpoints
Run
Run and Advance
Run Section
Advance
Run and Time
BREAKPOINTS
RUN
PowerMethod1.m*
1 % PSI 2014 Session II PowerMethod1.m
2 % cost 2^(2N)
3
4 - clear all
5 - N = 9;
6 - Q = 100;
7 - h = 1;
8 - J = 1;
9 - boundary = 1;
10 H = Create
script
Ln 10 Col 11
Search Documentation
Layout
Preferences
Set Path
Help
Community
Request Support
Add-Ons
ENVIRONMENT
RESOURCES
Workspace
Name
Value
Command History
IsingN3
IsingN
EV
diag(EV)
EV(1,1)
gs
EV(1,1)
gs'*H*gs
%-- 20/03/2014 9:39 AM --%
%-- 20/03/2014 10:15 AM --
10:36 AM
20/03/2014

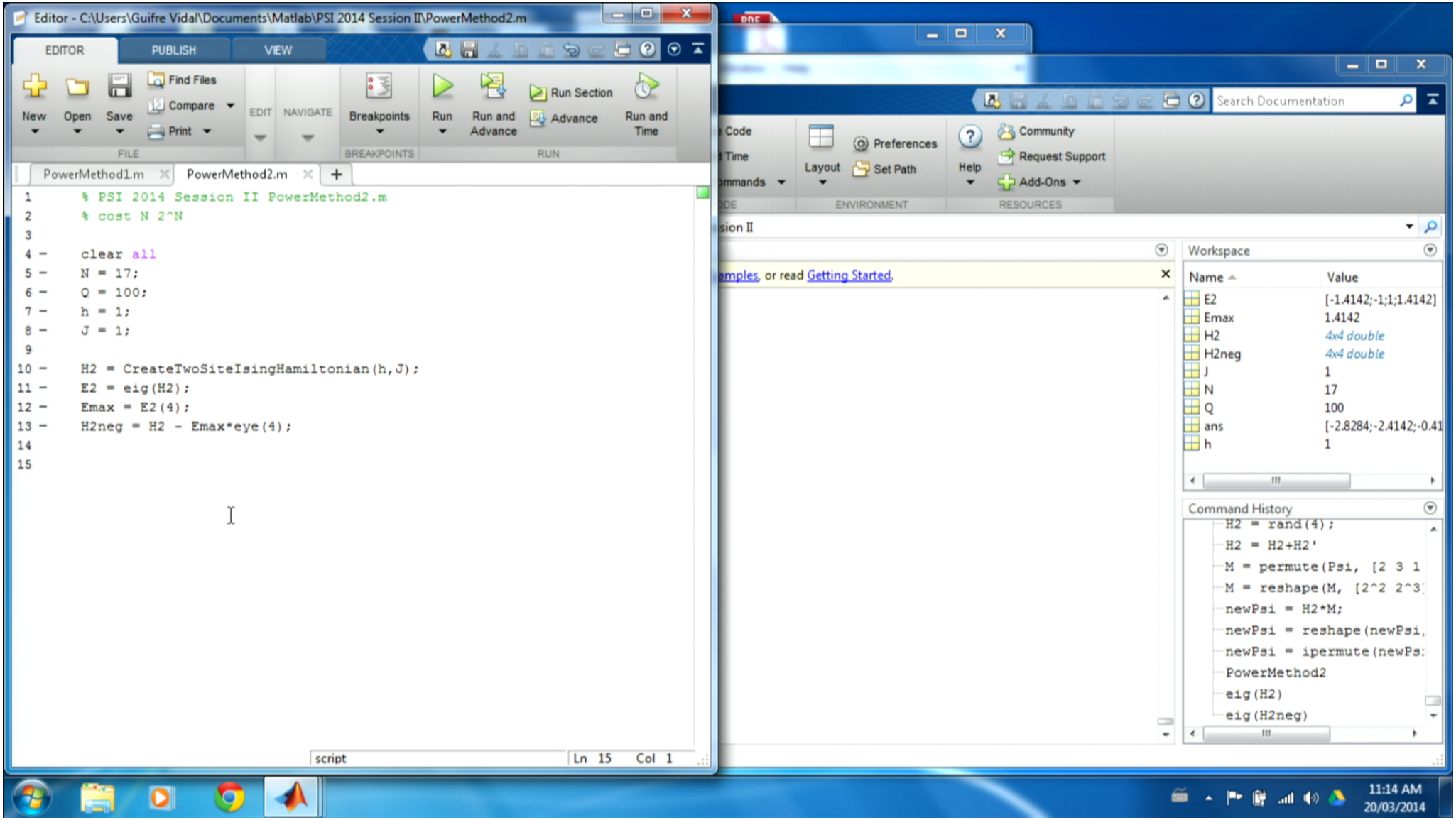

Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod1.m
EDITOR
PUBLISH
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Find Files
Compare
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FILE
EDIT
NAVIGATE
Breakpoints
BREAKPOINTS
PowerMethod1.m
1 % PSI 2014 Session II PowerMethod1.m
2 % cost 2^(2N)
3
4 clear all
5 N = 9;
6 Q = 1000;
7 h = 1;
8 J = 1;
9 boundary = 1;
10 H = CreateIsingHamiltonian(N, J, h, boun
11
12 Psi = rand(2^N,1);
13 E = zeros(Q,1);
14
15 for q=1:Q
16 Psi = H*Psi;
17 Psi = Psi/sqrt(Psi'*Psi);
18 E(q) = Psi'*H*Psi;
19 q
20 end
21
22 plot(E, 'o')
script
Ln 6 Col 7
Figure 1
File Edit View Insert Tools Desktop Window Help
-9.6
-9.8
-10
-10.2
-10.4
-10.6
-10.8
-11
-11.2
-11.4
-11.6
0 100 200 300 400 500 600 700 800 900 1000
Search Documentation
Workspace
Name Value
E 1000x1 double
H 512x512 double
J 1
N 9
Psi 512x1 double
Q 1000
boundary 1
h 1
q 1000
Command History
IsingN
EV
diag(EV)
EV(1,1)
gs
EV(1,1)
gs'*H*gs
%-- 20/03/2014 9:39 AM --%
%-- 20/03/2014 10:15 AM --
PowerMethod1
10:40 AM
20/03/2014

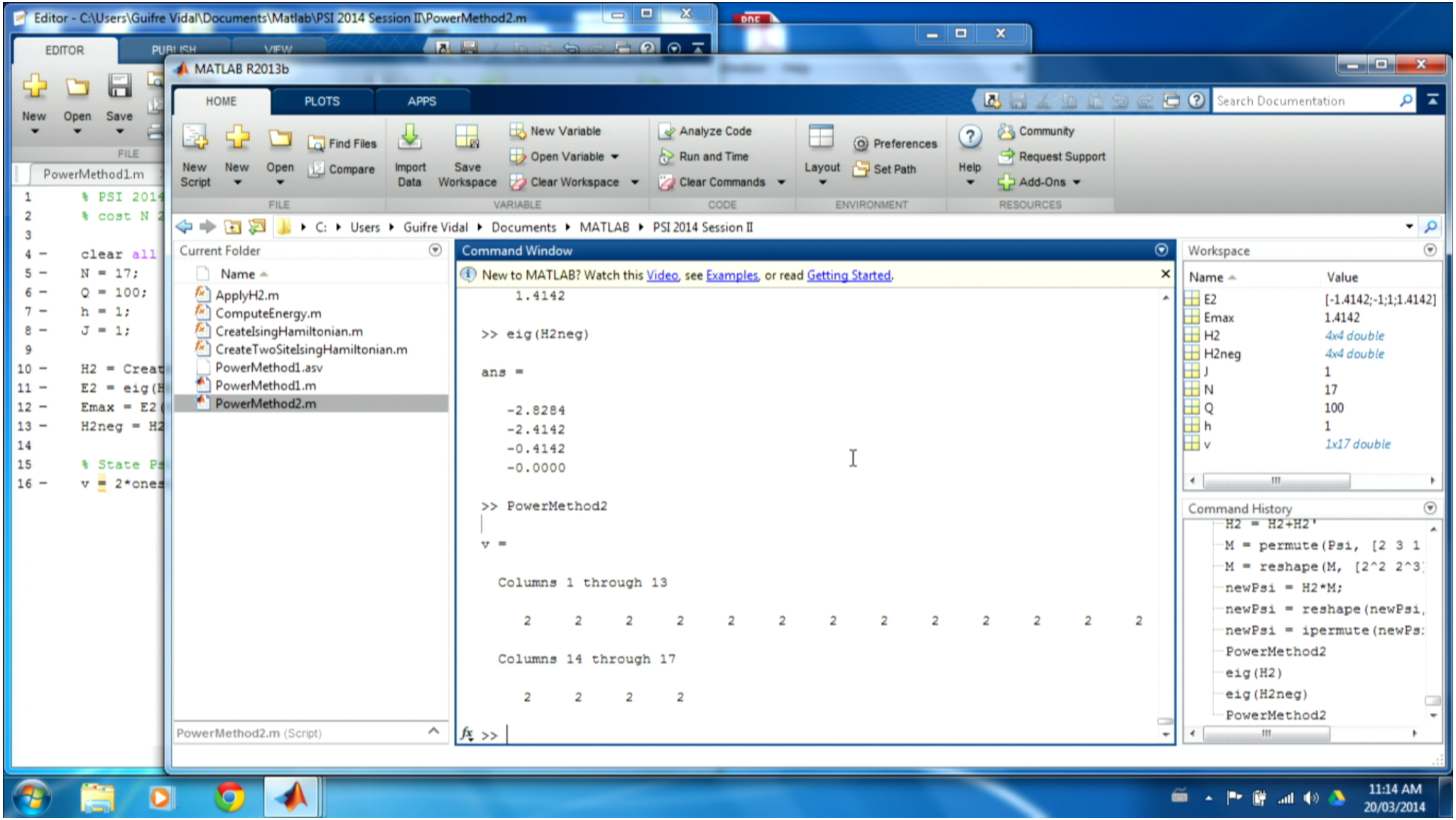

Psi
=
Psi
H2
H2
← permute
← reshape
← reshape
← permute
l × l
O = M · N
cost l^3
O_αβ = Σ_{γ=1}^{l} M_αγ N_γβ
Σ_{γ=1}^{l} M_3γ N_γ7

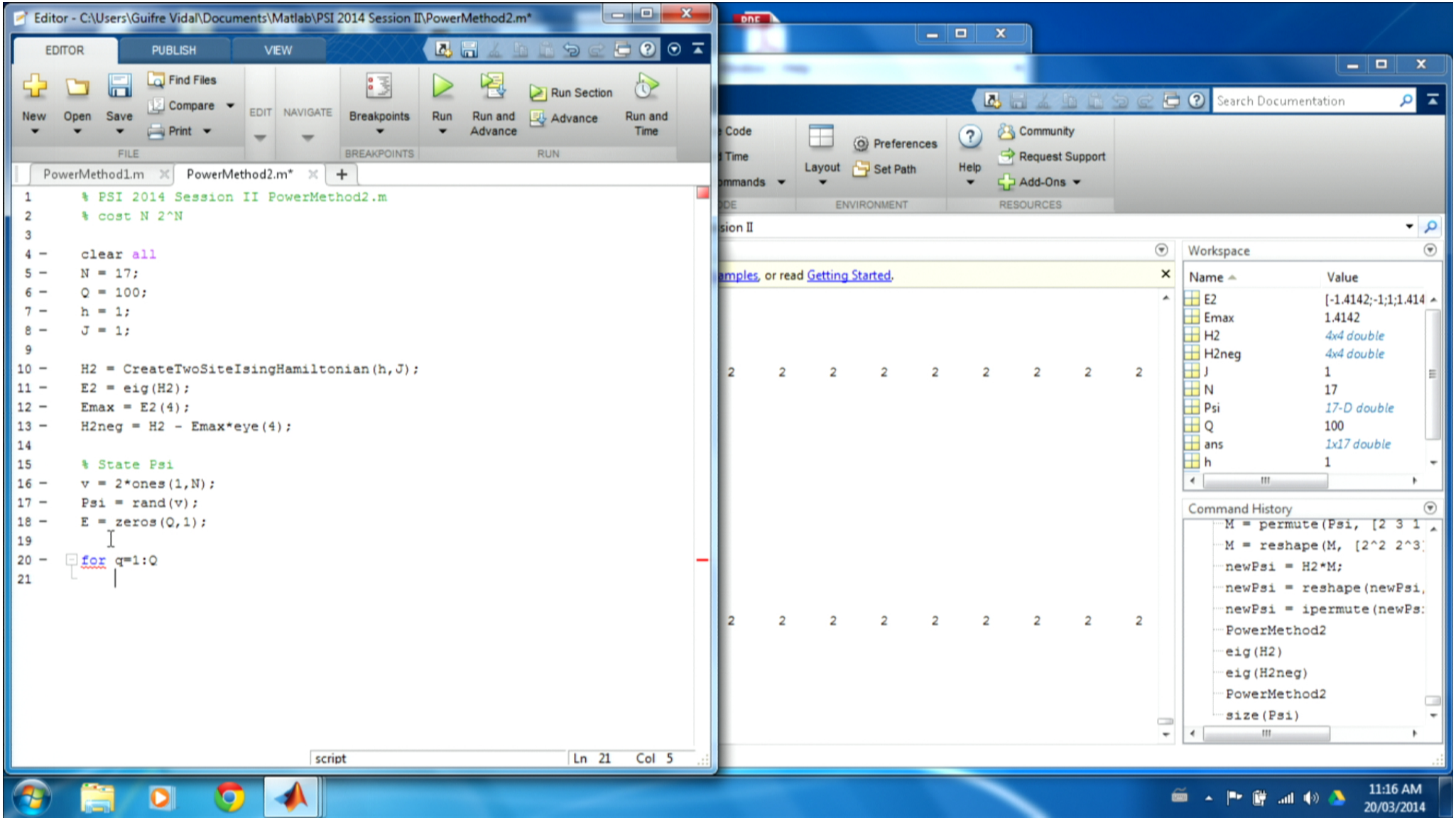

Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod1.m
Figure 1
MATLAB R2013b
HOME
PLOTS
APPS
Search Documentation
New Script
New
Open
Find Files
Compare
Import Data
Save Workspace
New Variable
Open Variable
Clear Workspace
Analyze Code
Run and Time
Clear Commands
Layout
Preferences
Set Path
Help
Community
Request Support
Add-Ons
FILE
VARIABLE
CODE
ENVIRONMENT
RESOURCES
C: ▸ Users ▸ Guifre Vidal ▸ Documents ▸ MATLAB ▸ PSI 2014 Session II
Current Folder
Name
ApplyH2.m
ComputeEnergy.m
CreateIsingHamiltonian.m
CreateTwoSiteIsingHamiltonian.m
PowerMethod1.asv
PowerMethod1.m
PowerMethod1.m (Script)
Command Window
New to MATLAB? Watch this Video, see Examples, or read Getting Started.
ans(:,:,5) =
>> Psi = rand([2 2 2 2 2]);
>> H2 = rand(4);
>> H2 = H2+H2'
H2 =
0.2195 0.9803 1.4363 1.8109
0.9803 0.6237 0.9307 1.4351
1.4363 0.9307 0.5107 1.0368
1.8109 1.4351 1.0368 0.1221
Workspace
Name
Value
E 100x1 double
H 512x512 double
H2 4x4 double
J 1
N 9
Psi 5-D double
Q 100
T1 4x3x5 double
T2 4-D double
T3 4-D double
Command History
T2 = reshape(T1, [2 2 3
size(T2)
T3 = permute(T2, [1 2 4
size(T3)
T4 = reshape(T3, [4 5 3
T5 = permute(T4, [1 3 2
T5-T1
Psi = rand([2 2 2 2 2]).
H2 = rand(4);
H2 = H2+H2'
11:05 AM
20/03/2014

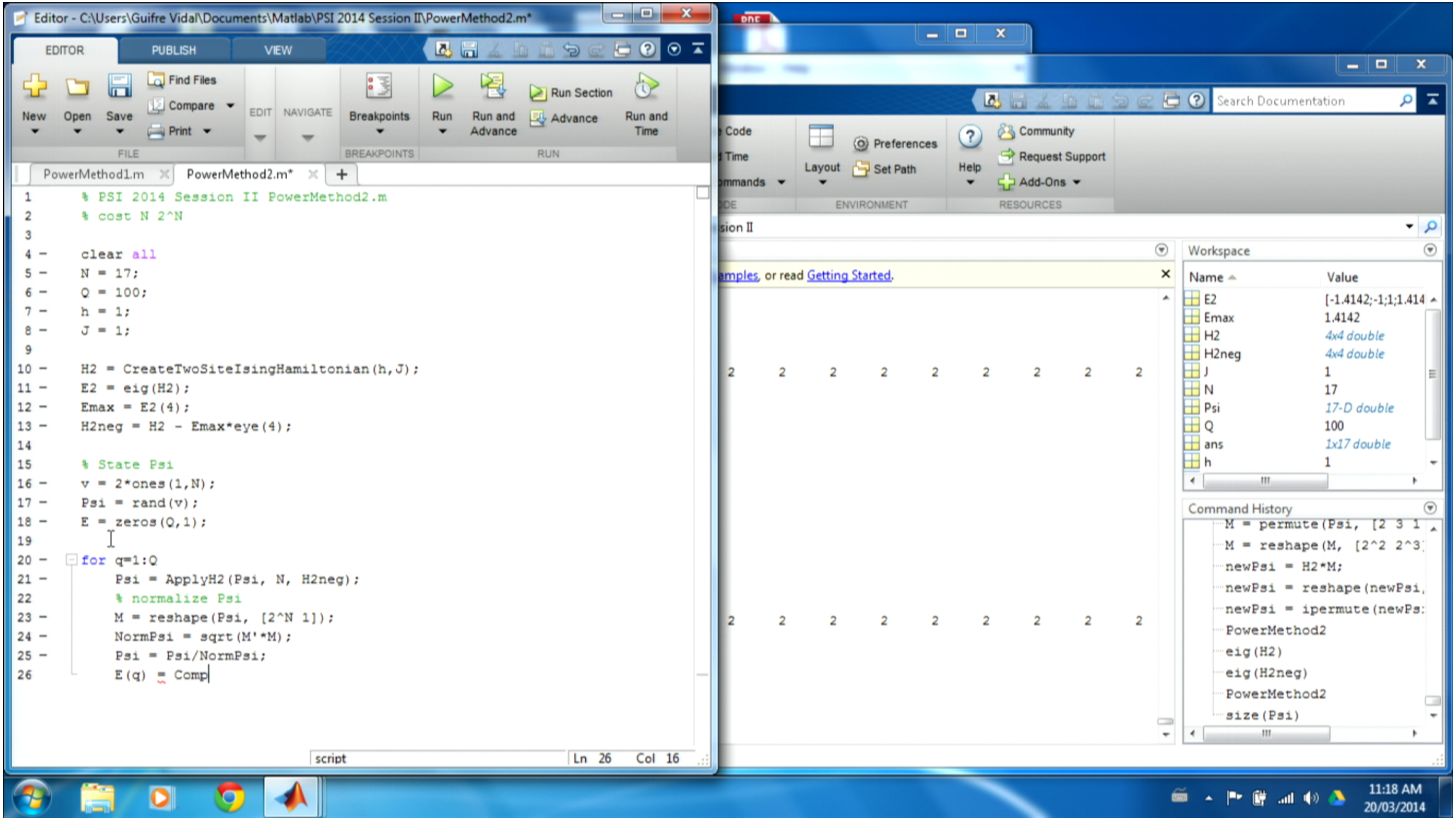

Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod1.m
Figure 1
MATLAB R2013b
HOME
PLOTS
APPS
Search Documentation
New Script
New
Open
Find Files
Compare
Import Data
Save Workspace
New Variable
Open Variable
Clear Workspace
Analyze Code
Run and Time
Clear Commands
Layout
Preferences
Set Path
Help
Community
Request Support
Add-Ons
FILE
VARIABLE
CODE
ENVIRONMENT
RESOURCES
C: ▸ Users ▸ Guifre Vidal ▸ Documents ▸ MATLAB ▸ PSI 2014 Session II
Current Folder
Name
ApplyH2.m
ComputeEnergy.m
CreateIsingHamiltonian.m
CreateTwoSiteIsingHamiltonian.m
PowerMethod1.asv
PowerMethod1.m
PowerMethod1.m (Script)
Command Window
New to MATLAB? Watch this Video, see Examples, or read Getting Started.
ans(:,:,5) =
0 0 0
0 0 0
0 0 0
0 0 0
>> Psi = rand([2 2 2 2 2]);
>> H2 = rand(4);
>> H2 = H2+H2'
H2 =
0.2195 0.9803 1.4363 1.8109
0.9803 0.6237 0.9307 1.4351
1.4363 0.9307 0.5107 1.0368
1.8109 1.4351 1.0368 0.1221
>> M = permute(Psi, [2 3 1 4 5]);
>> M = reshape(M, [2^2 2^3]);
>> newPsi = H2*M;
>> newPsi = reshape(newPsi, [2 2 2 2 2]);
>> newPsi = ipermute(newPsi, [2 3 1 4 5]);
>>
Workspace
Name
Value
E 100x1 double
H 512x512 double
H2 4x4 double
J 1
M 4x8 double
N 9
Psi 5-D double
Q 100
T1 4x3x5 double
T2 4-D double
Command History
T5 = permute(T4, [1 3 2
T5-T1
Psi = rand([2 2 2 2 2]).
H2 = rand(4);
H2 = H2+H2'
M = permute(Psi, [2 3 1
M = reshape(M, [2^2 2^3
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs:
11:09 AM
20/03/2014

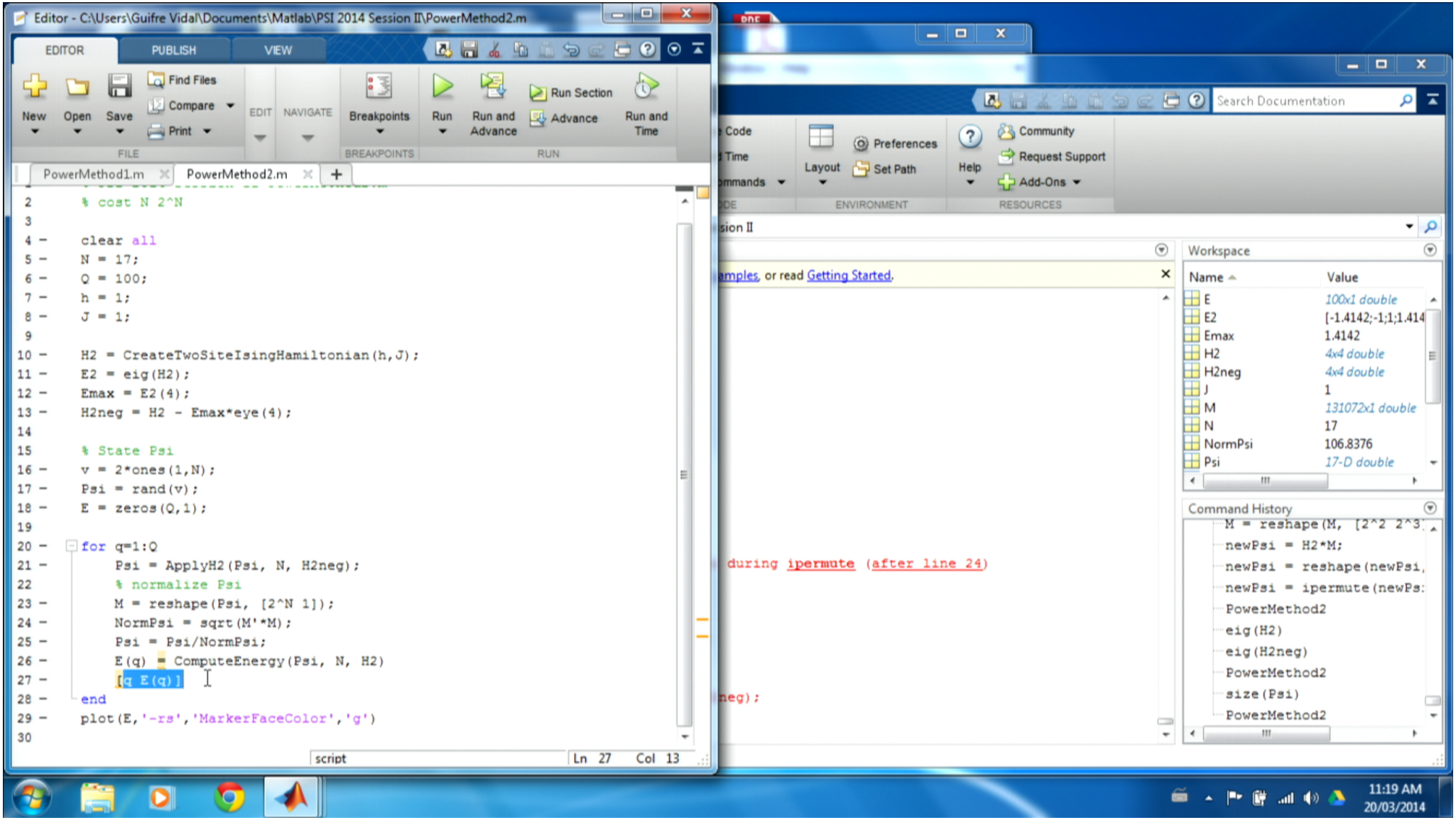

Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod2.m*
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New
Open
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Find Files
Compare
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FILE
EDIT
NAVIGATE
Breakpoints
BREAKPOINTS
Run
Run and Advance
Run Section
Advance
Run and Time
RUN
PowerMethod1.m
PowerMethod2.m*
1 % PSI 2014 Session II PowerMethod2.m
2 % cost N2^N
script
Ln 2 Col 12
Search Documentation
Layout
Preferences
Set Path
ENVIRONMENT
Help
Community
Request Support
Add-Ons
RESOURCES
Workspace
Name
Value
E 100x1 double
H 512x512 double
H2 4x4 double
J 1
M 4x8 double
N 9
Psi 5-D double
Q 100
T1 4x3x5 double
T2 4-D double
Command History
T5 = permute(T4, [1 3 2
T5-T1
Psi = rand([2 2 2 2 2]).
H2 = rand(4);
H2 = H2+H2'
M = permute(Psi, [2 3 1
M = reshape(M, [2^2 2^3
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs:
63 1.8109
07 1.4351
07 1.0368
68 0.1221
5]);
[2 2 2 2 2]);
[2 3 1 4 5]);
11:10 AM
20/03/2014

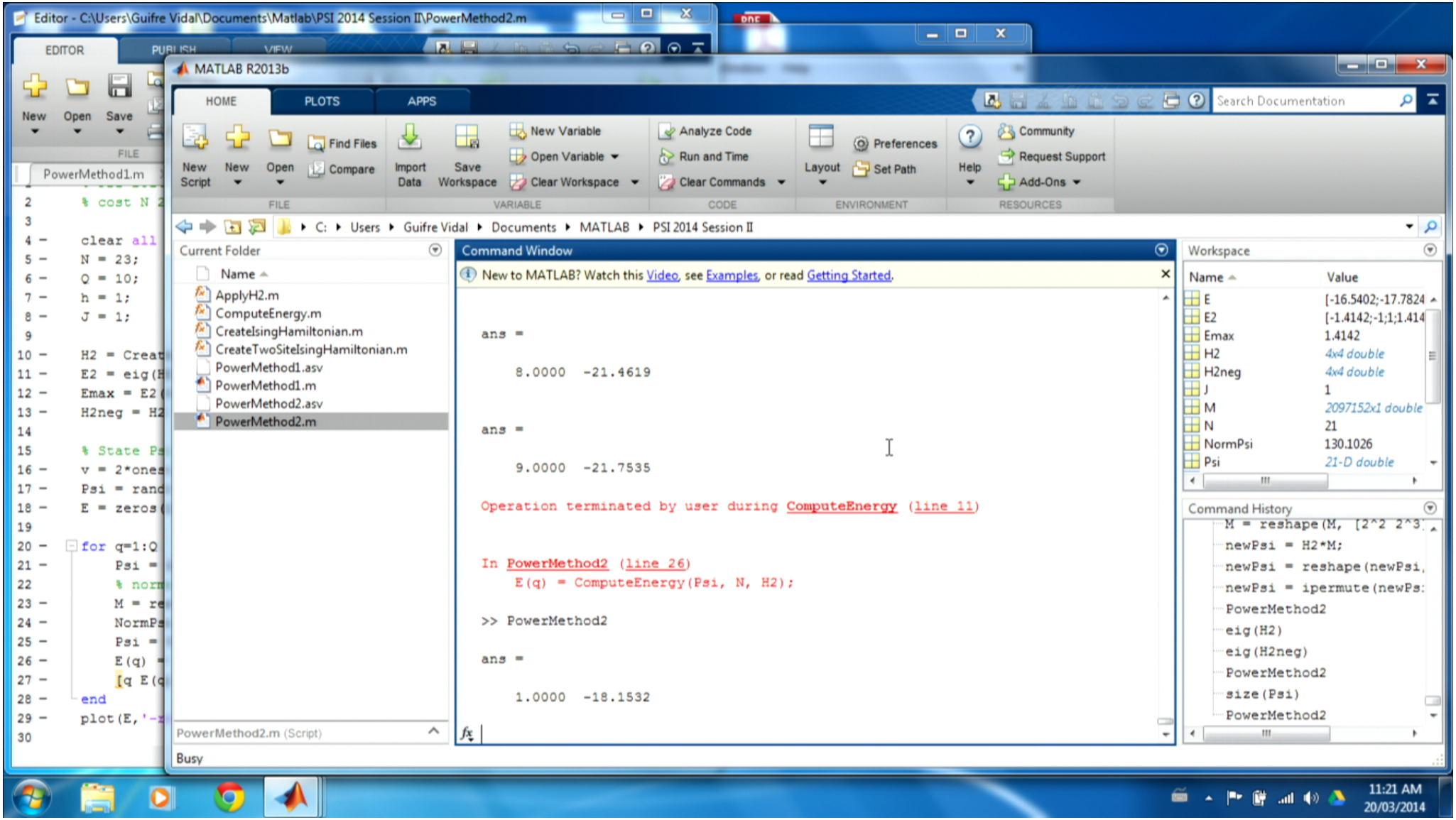

Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod2.m
EDITOR
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Open
Save
Find Files
Compare
Print
FILE
EDIT
NAVIGATE
Breakpoints
BREAKPOINTS
Run
Run and Advance
Run Section
Advance
Run and Time
RUN
PowerMethod1.m
PowerMethod2.m
1 % PSI 2014 Session II PowerMethod2.m
2 % cost N 2^N
3
4 - clear all
5 - N = 17;
6 - Q = 100;
7 - h = 1;
8 - J = 1;
9
10 - H2 = CreateTwoSiteIsingHamiltonian(h,J);
11 - E2 = eig(H2);
12 - Emax = E2(4);
13 - H2neg = H2 - Emax*eye(4);
14
15
script
Ln 15 Col 1
Search Documentation
Preferences
Set Path
Layout
Help
Community
Request Support
Add-Ons
ENVIRONMENT
RESOURCES
Workspace
Name
Value
E2 [-1.4142;-1;1;1.4142]
Emax 1.4142
H2 4x4 double
H2neg 4x4 double
J 1
N 17
Q 100
ans [-2.8284;-2.4142;-0.41
h 1
Command History
H2 = rand(4);
H2 = H2+H2'
M = permute(Psi, [2 3 1
M = reshape(M, [2^2 2^3
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs
PowerMethod2
eig(H2)
eig(H2neg)
11:14 AM
20/03/2014


Editor - C:\Users\Guifre Vidal\Documents\Matlab\PSI 2014 Session II\PowerMethod2.m
EDITOR
PUBLISH
VIEW
New
Open
Save
FILE
PowerMethod1.m
1 % PSI 2014
2 % cost N 2
3
4 - clear all
5 - N = 17;
6 - Q = 100;
7 - h = 1;
8 - J = 1;
9
10 - H2 = Creat
11 - E2 = eig(H
12 - Emax = E2(
13 - H2neg = H2
14
15 % State Ps
16 - v = 2*ones
MATLAB R2013b
HOME
PLOTS
APPS
Search Documentation
New Script
New
Open
Find Files
Compare
Import Data
Save Workspace
New Variable
Open Variable
Clear Workspace
Analyze Code
Run and Time
Clear Commands
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Community
Request Support
Add-Ons
FILE
VARIABLE
CODE
ENVIRONMENT
RESOURCES
C: ▸ Users ▸ Guifre Vidal ▸ Documents ▸ MATLAB ▸ PSI 2014 Session II
Current Folder
Name
ApplyH2.m
ComputeEnergy.m
CreateIsingHamiltonian.m
CreateTwoSiteIsingHamiltonian.m
PowerMethod1.asv
PowerMethod1.m
PowerMethod2.m
PowerMethod2.m (Script)
Command Window
New to MATLAB? Watch this Video, see Examples, or read Getting Started.
1.4142
>> eig(H2neg)
ans =
-2.8284
-2.4142
-0.4142
-0.0000
>> PowerMethod2
v =
Columns 1 through 13
2 2 2 2 2 2 2 2 2 2 2 2 2
Columns 14 through 17
2 2 2 2
>>
Workspace
Name
Value
E2 [-1.4142;-1;1;1.4142]
Emax 1.4142
H2 4x4 double
H2neg 4x4 double
J 1
N 17
Q 100
h 1
v 1x17 double
Command History
H2 = H2+H2'
M = permute(Psi, [2 3 1
M = reshape(M, [2^2 2^3
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs
PowerMethod2
eig(H2)
eig(H2neg)
PowerMethod2
11:14 AM
20/03/2014


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NAVIGATE
Breakpoints
BREAKPOINTS
Run
Run and Advance
Run Section
Advance
Run and Time
RUN
PowerMethod1.m
PowerMethod2.m*
1 % PSI 2014 Session II PowerMethod2.m
2 % cost N 2^N
3
4 clear all
5 N = 17;
6 Q = 100;
7 h = 1;
8 J = 1;
9
10 H2 = CreateTwoSiteIsingHamiltonian(h,J);
11 E2 = eig(H2);
12 Emax = E2(4);
13 H2neg = H2 - Emax*eye(4);
14
15 % State Psi
16 v = 2*ones(1,N);
17 Psi = rand(v);
18 E = zeros(Q,1);
19
20 for q=1:Q
21
script
Ln 21 Col 5
Search Documentation
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Community
Request Support
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ENVIRONMENT
RESOURCES
Workspace
Name
Value
E2 [-1.4142;-1;1;1.414
Emax 1.4142
H2 4x4 double
H2neg 4x4 double
J 1
N 17
Psi 17-D double
Q 100
ans 1x17 double
h 1
Command History
M = permute(Psi, [2 3 1
M = reshape(M, [2^2 2^3
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs
PowerMethod2
eig(H2)
eig(H2neg)
PowerMethod2
size(Psi)
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Breakpoints
BREAKPOINTS
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Run and Advance
Run Section
Advance
Run and Time
RUN
PowerMethod1.m
PowerMethod2.m*
1 % PSI 2014 Session II PowerMethod2.m
2 % cost N 2^N
3
4 clear all
5 N = 17;
6 Q = 100;
7 h = 1;
8 J = 1;
9
10 H2 = CreateTwoSiteIsingHamiltonian(h,J);
11 E2 = eig(H2);
12 Emax = E2(4);
13 H2neg = H2 - Emax*eye(4);
14
15 % State Psi
16 v = 2*ones(1,N);
17 Psi = rand(v);
18 E = zeros(Q,1);
19
20 for q=1:Q
21 Psi = ApplyH2(Psi, N, H2neg);
22 % normalize Psi
23 M = reshape(Psi, [2^N 1]);
24 NormPsi = sqrt(M'*M);
25 Psi = Psi/NormPsi;
26 E(q) = Comp
script
Ln 26 Col 16
Search Documentation
Layout
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Add-Ons
RESOURCES
Workspace
Name
Value
E2 [-1.4142;-1;1;1.414
Emax 1.4142
H2 4x4 double
H2neg 4x4 double
J 1
N 17
Psi 17-D double
Q 100
ans 1x17 double
h 1
Command History
M = permute(Psi, [2 3 1
M = reshape(M, [2^2 2^3
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs
PowerMethod2
eig(H2)
eig(H2neg)
PowerMethod2
size(Psi)
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Run and Advance
Run Section
Advance
Run and Time
FILE
EDIT
NAVIGATE
BREAKPOINTS
RUN
PowerMethod1.m
PowerMethod2.m
% cost N 2^N
clear all
N = 17;
Q = 100;
h = 1;
J = 1;
H2 = CreateTwoSiteIsingHamiltonian(h,J);
E2 = eig(H2);
Emax = E2(4);
H2neg = H2 - Emax*eye(4);
% State Psi
v = 2*ones(1,N);
Psi = rand(v);
E = zeros(Q,1);
for q=1:Q
Psi = ApplyH2(Psi, N, H2neg);
% normalize Psi
M = reshape(Psi, [2^N 1]);
NormPsi = sqrt(M'*M);
Psi = Psi/NormPsi;
E(q) = ComputeEnergy(Psi, N, H2)
[q E(q)]
end
plot(E,'-rs','MarkerFaceColor','g')
script
Ln 27
Col 13
Search Documentation
Layout
Preferences
Set Path
Help
Community
Request Support
Add-Ons
ENVIRONMENT
RESOURCES
Workspace
Name
Value
E
100x1 double
E2
[-1.4142;-1;1;1.414
Emax
1.4142
H2
4x4 double
H2neg
4x4 double
J
1
M
131072x1 double
N
17
NormPsi
106.8376
Psi
17-D double
Command History
M = reshape(M, [2^2 2^3]
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs:
PowerMethod2
eig(H2)
eig(H2neg)
PowerMethod2
size(Psi)
PowerMethod2
during ipermute (after line 24)
11:19 AM
20/03/2014


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Current Folder
Name
ApplyH2.m
ComputeEnergy.m
CreateIsingHamiltonian.m
CreateTwoSiteIsingHamiltonian.m
PowerMethod1.asv
PowerMethod1.m
PowerMethod2.asv
PowerMethod2.m
PowerMethod2.m (Script)
Command Window
New to MATLAB? Watch this Video, see Examples, or read Getting Started.
ans =
8.0000 -21.4619
ans =
9.0000 -21.7535
Operation terminated by user during ComputeEnergy (line 11)
In PowerMethod2 (line 26)
E(q) = ComputeEnergy(Psi, N, H2);
>> PowerMethod2
ans =
1.0000 -18.1532
Busy
Workspace
Name
Value
E
E2
Emax
H2
H2neg
J
M
N
NormPsi
Psi
Command History
newPsi = H2*M;
newPsi = reshape(newPsi,
newPsi = ipermute(newPs:
PowerMethod2
eig(H2)
eig(H2neg)
PowerMethod2
size(Psi)
PowerMethod2
11:21 AM
20/03/2014