Title: 13/14 PSI - Explorations in Condensed Matter - Lecture 2

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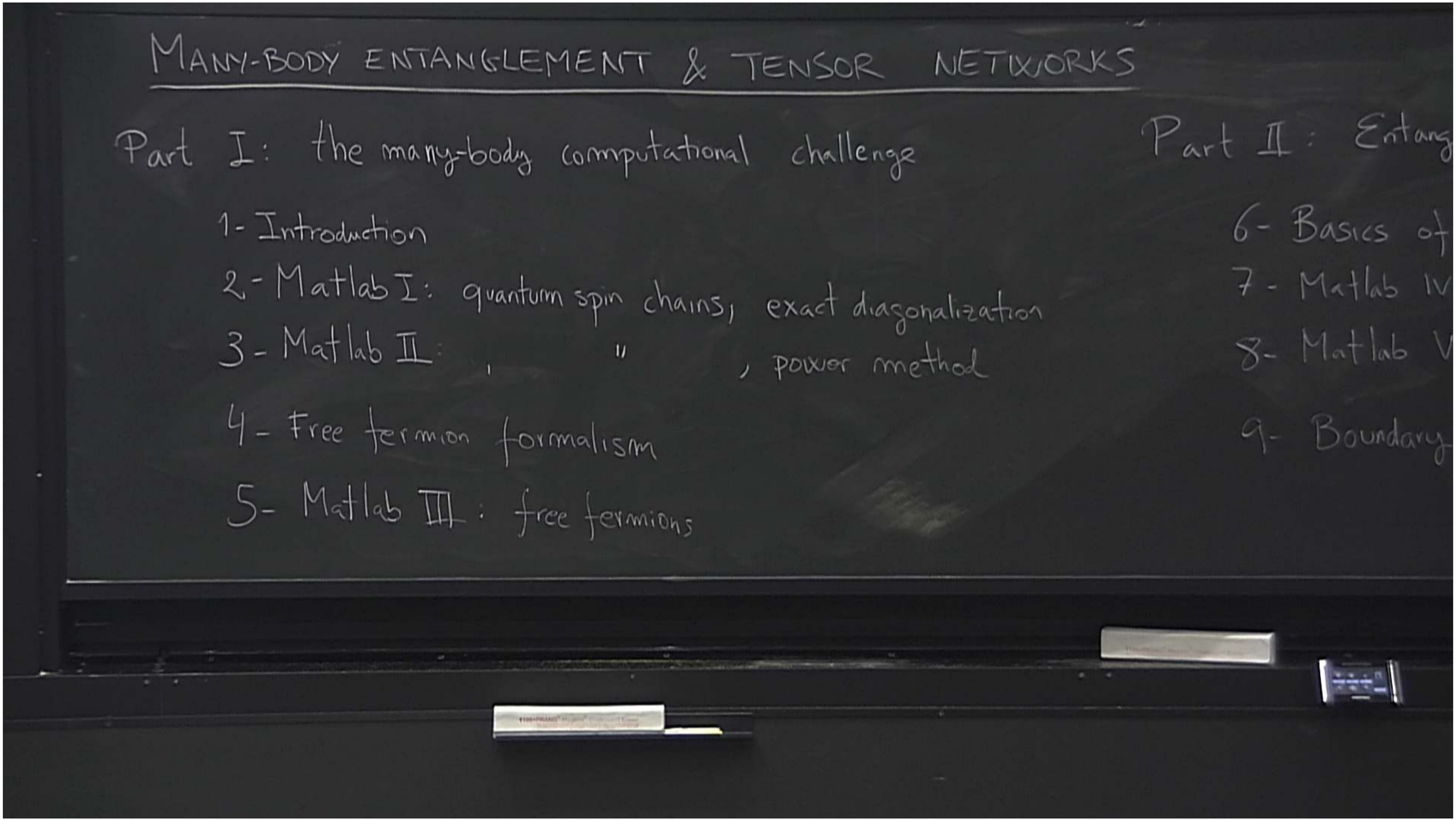
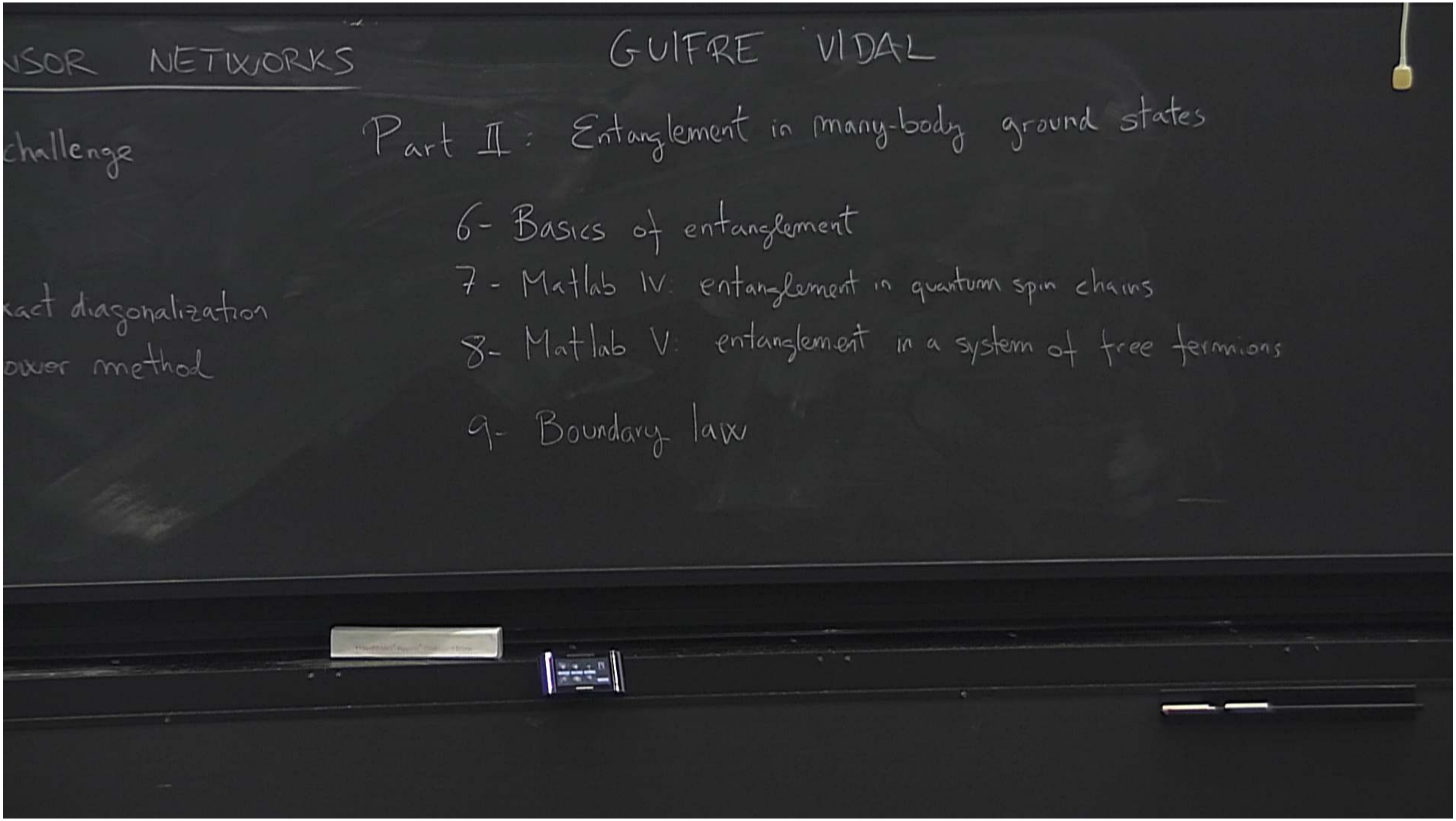
Abstract:


MANY-BODY ENTANGLEMENT & TENSOR NETWORKS
Part I: the many-body computational challenge
1- Introduction
2- Matlab I: quantum spin chains, exact diagonalization
3- Matlab II: " , power method
4- Free fermion formalism
5- Matlab III: free fermions
Part II: Entang
6- Basics of
7- Matlab IV
8- Matlab V
9- Boundary

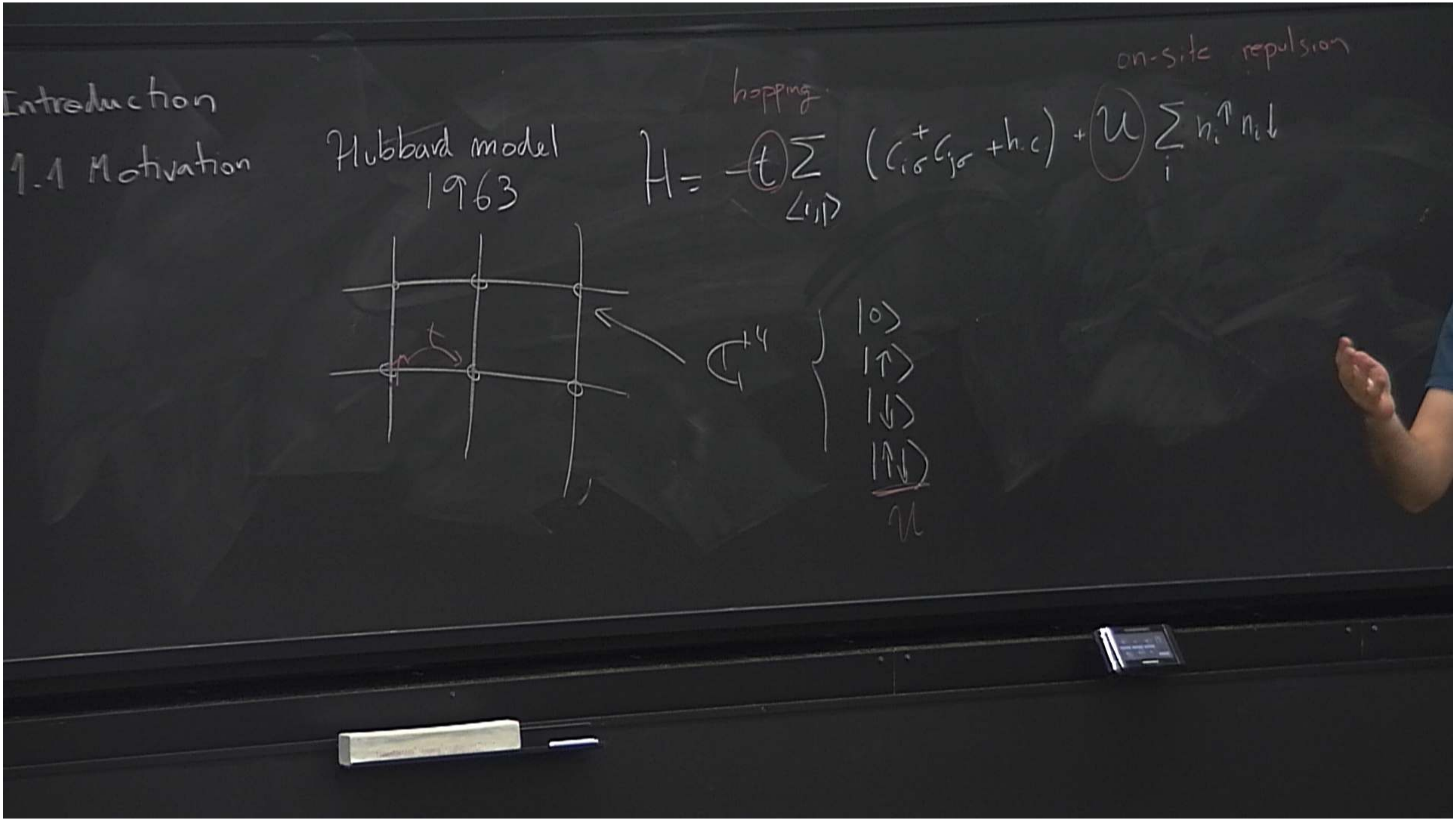

NSOR NETWORKS
GUIFRE VIDAL
challenge
exact diagonalization
power method
Part II: Entanglement in many-body ground states
6- Basics of entanglement
7- Matlab IV: entanglement in quantum spin chains
8- Matlab V: entanglement in a system of free fermions
9- Boundary law

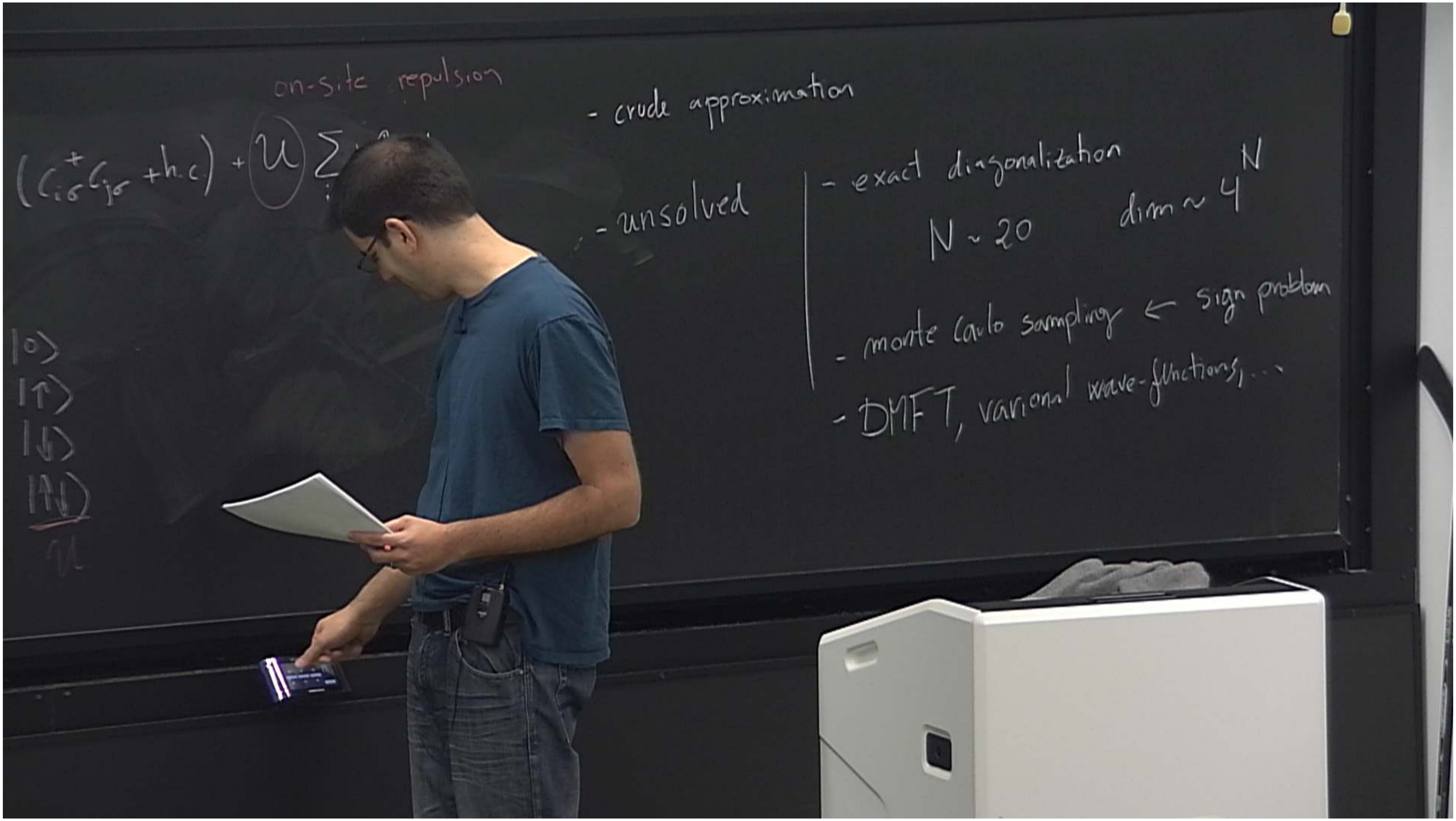
1- Introduction

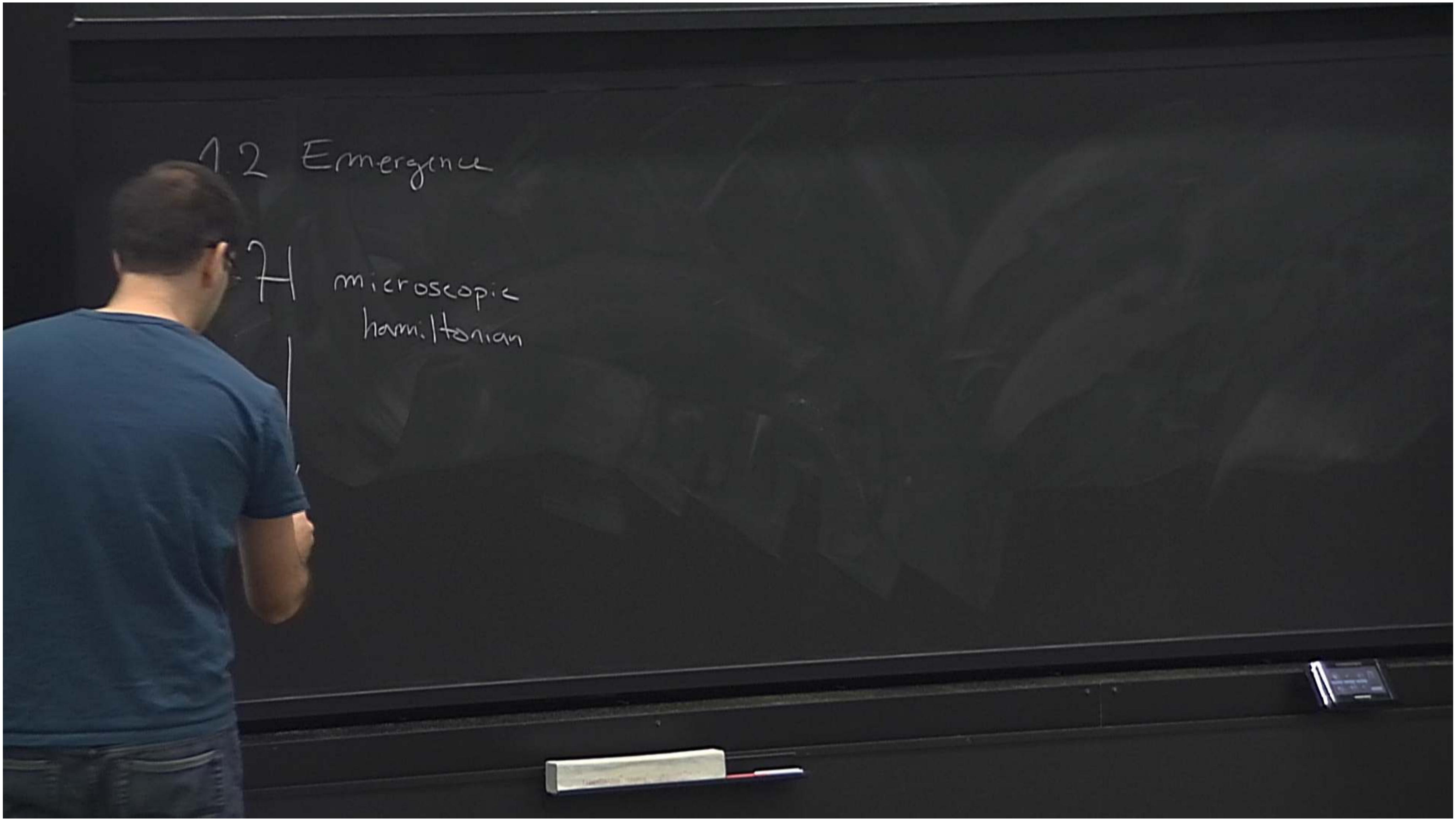
1.1 Motivation

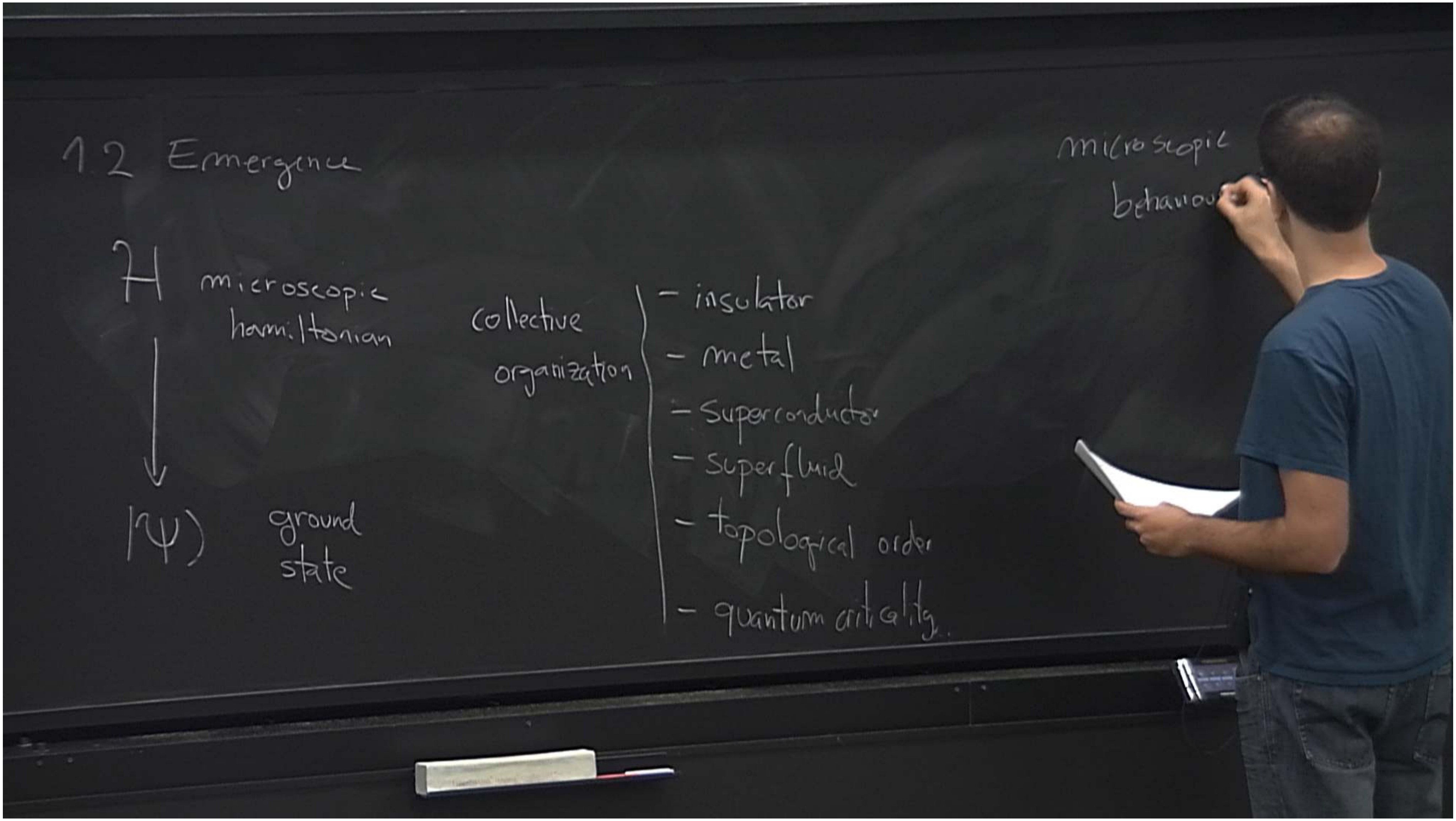
Hubbard model

$$H = -t \sum_{\langle i,j \rangle} \left(c_{i\sigma}^{+} c_{j\sigma} + h.c \right) + U \sum_{i} n$$


Introduction
1.1 Motivation
Hubbard model
1963
hopping
on-site repulsion
$H = -t \sum_{\langle i,j \rangle} (c_{i\sigma}^{+} c_{j\sigma} + h.c) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$
t
$\mathbb{C}^4$
$|0\rangle$
$|\uparrow\rangle$
$|\downarrow\rangle$
$|\uparrow\downarrow\rangle$
U


on-site repulsion
- crude approximation
- unsolved
- exact diagonalization
N ~ 20
dim ~ 4^N
- monte carlo sampling ← sign problem
- DMFT, varional wave-functions, ...


1.2 Emergence
H microscopic
hamiltonian


1.2 Emergence
H
microscopic hamiltonian
|Ψ)
ground state
collective organization
- insulator
- metal
- superconductor
- superfluid
- topological order
- quantum criticality
microscopic
behaviou

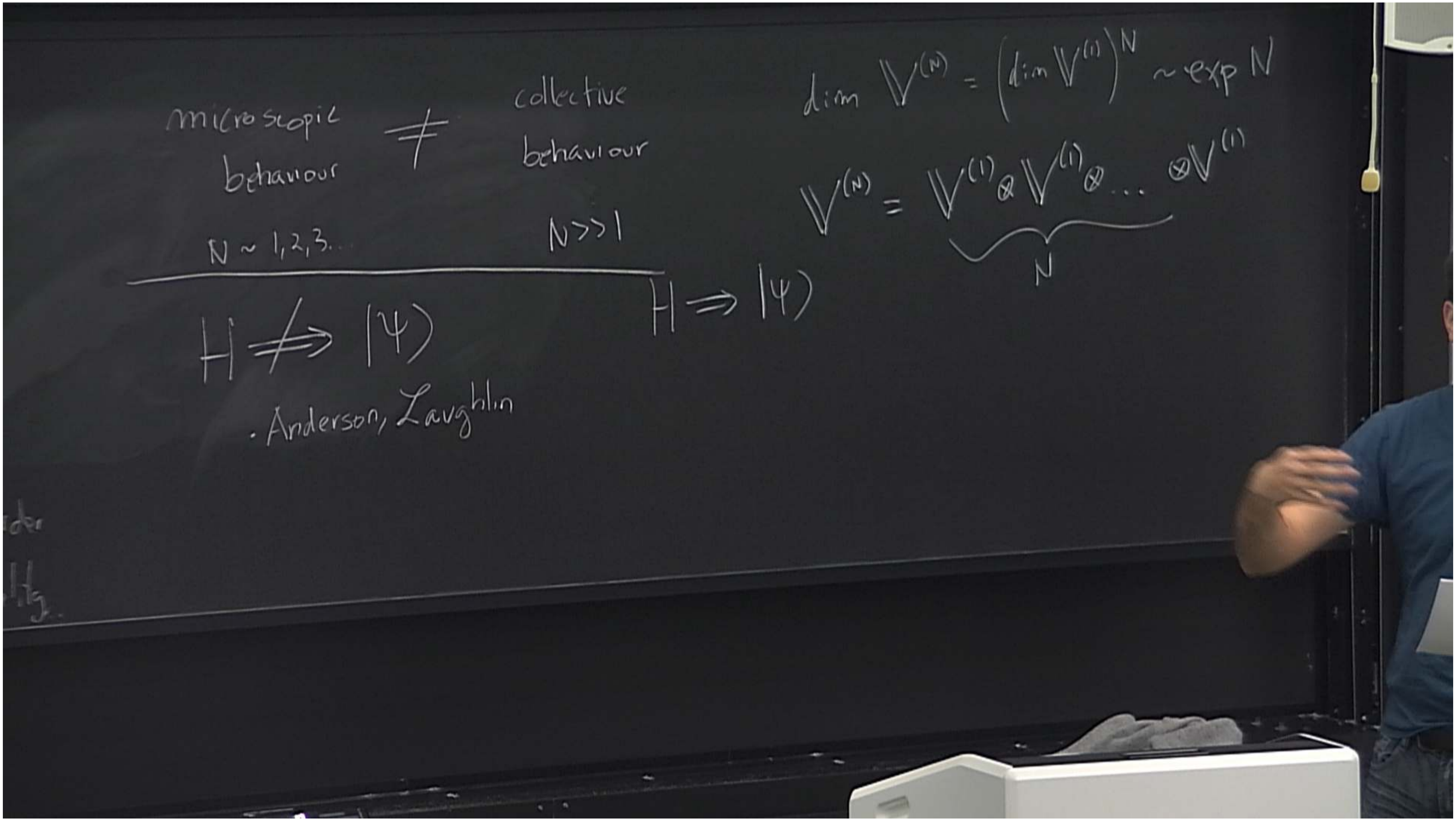
microscopic behaviour $\neq$ collective behaviour

$N \sim 1, 2, 3 \ldots$

$N \gg 1$

$$\dim \mathbb{V}^{(N)} = \left(\dim \mathbb{V}^{(1)}\right)^N \sim \exp N$$

$$\mathbb{V}^{(N)} = \underbrace{\mathbb{V}^{(1)} \otimes \mathbb{V}^{(1)} \otimes \ldots \otimes \mathbb{V}^{(1)}}_{N}$$

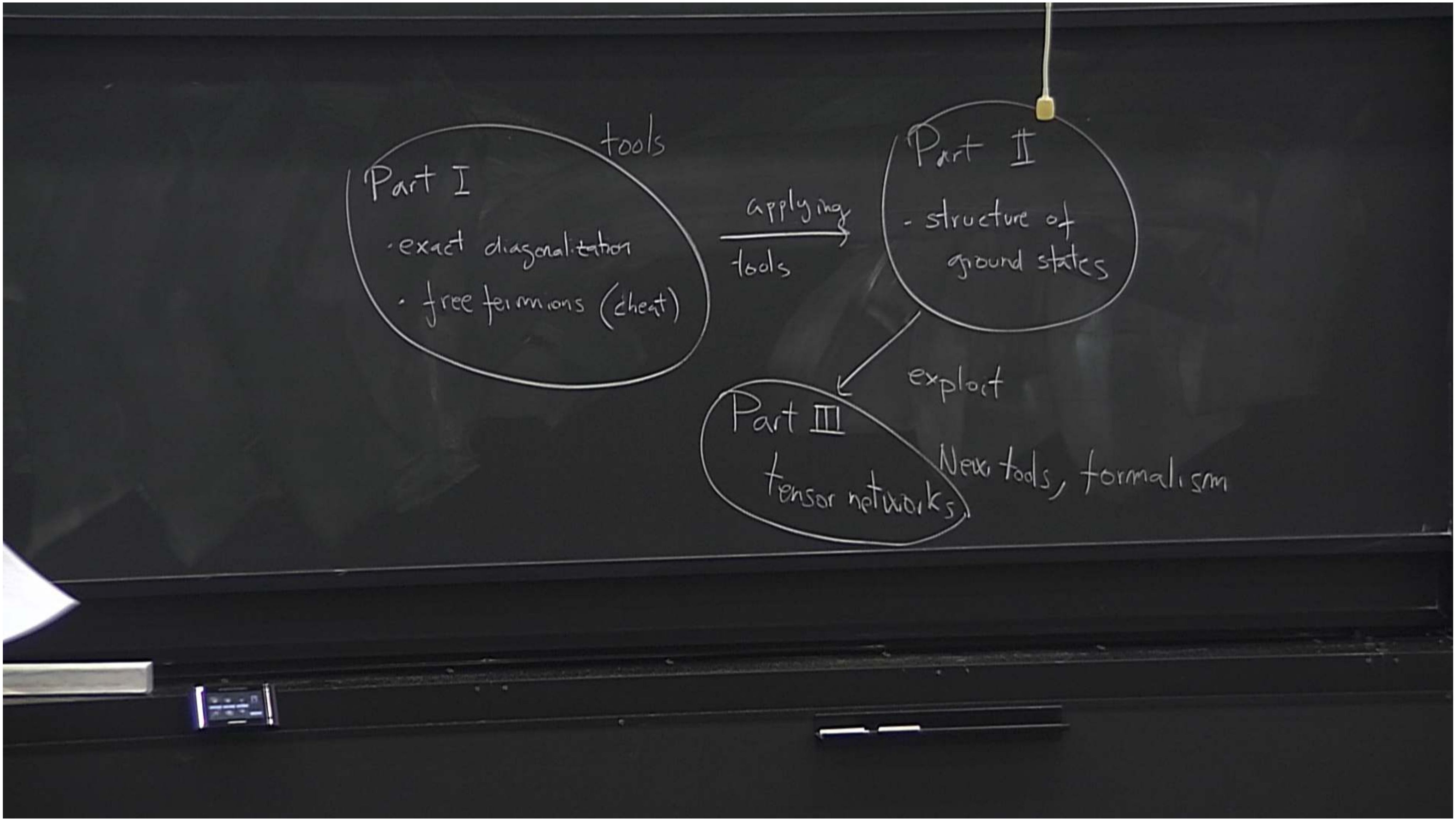

microscopic behaviour ≠ collective behaviour
N ~ 1,2,3...
N >> 1
H ⇏ |ψ⟩
H ⇒ |ψ⟩
. Anderson, Laughlin
dim V^(N) = (dim V^(1))^N ~ exp N
V^(N) = V^(1) ⊗ V^(1) ⊗ ... ⊗ V^(1)
N

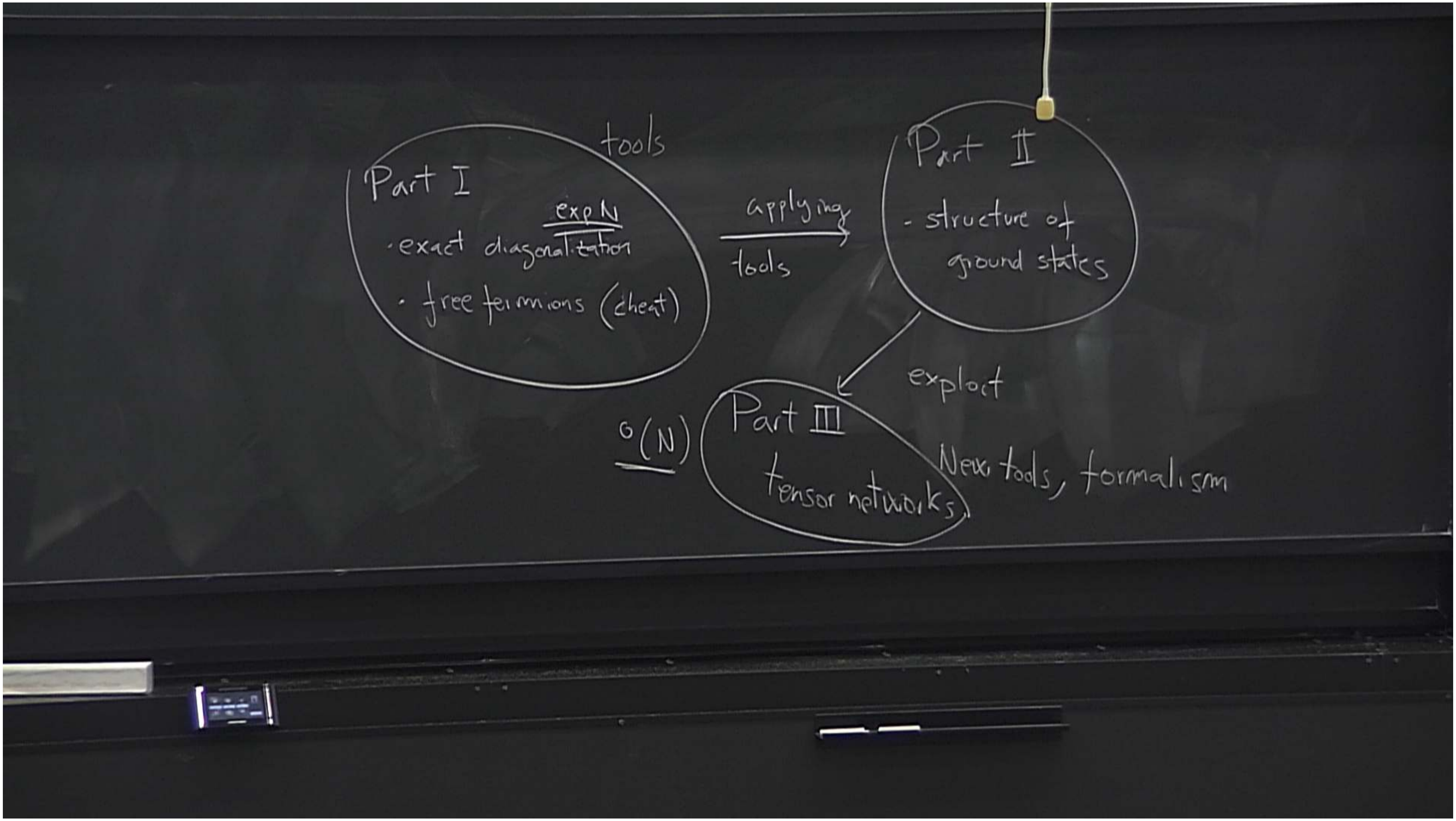
microscopic behaviour	$\neq$	collective behaviour
$N \sim 1, 2, 3 \ldots$		$N \gg 1$
$H \not\Rightarrow \vert\psi\rangle$		$H \Rightarrow \vert\psi\rangle$

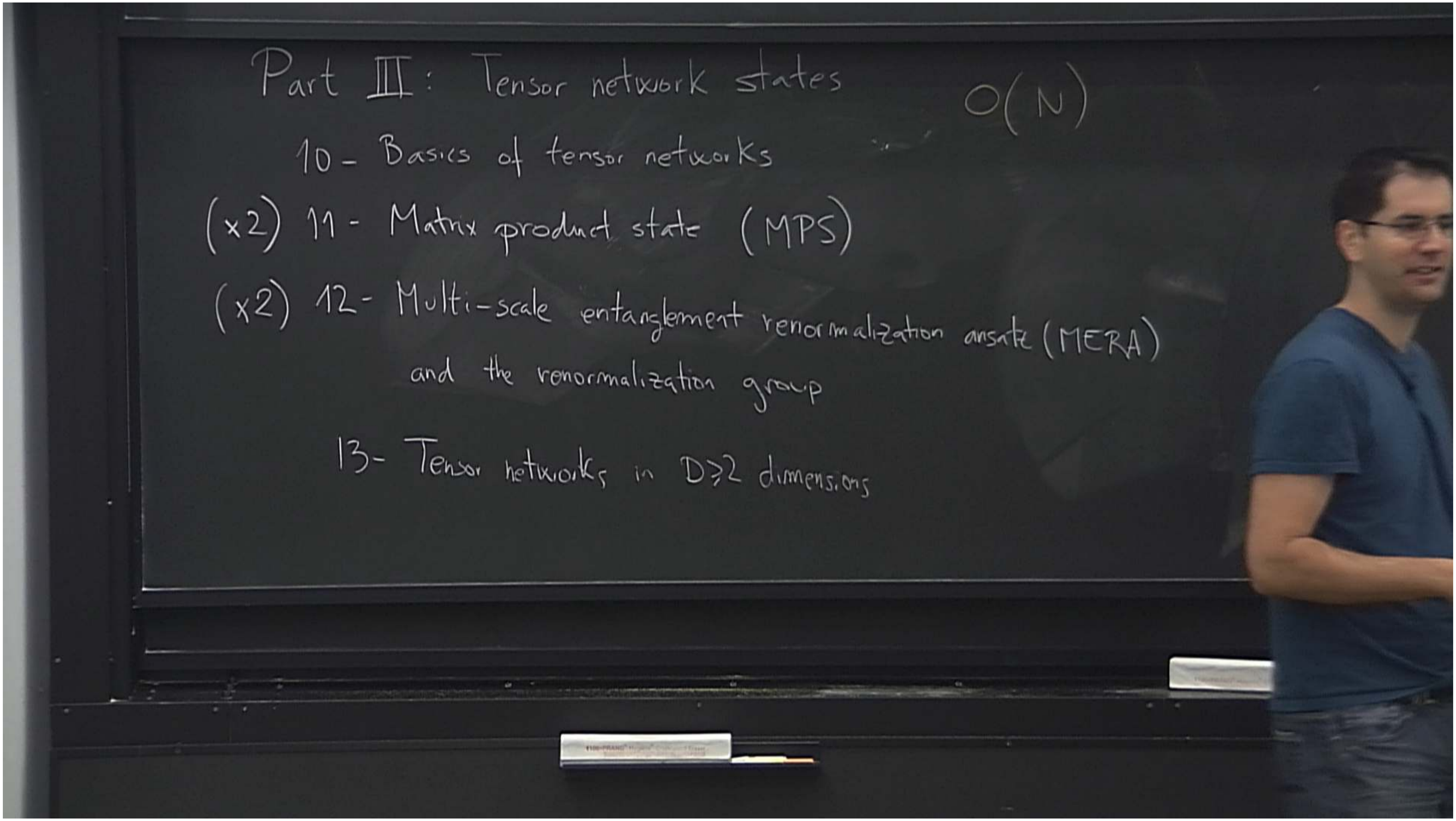
· Anderson, Laughlin

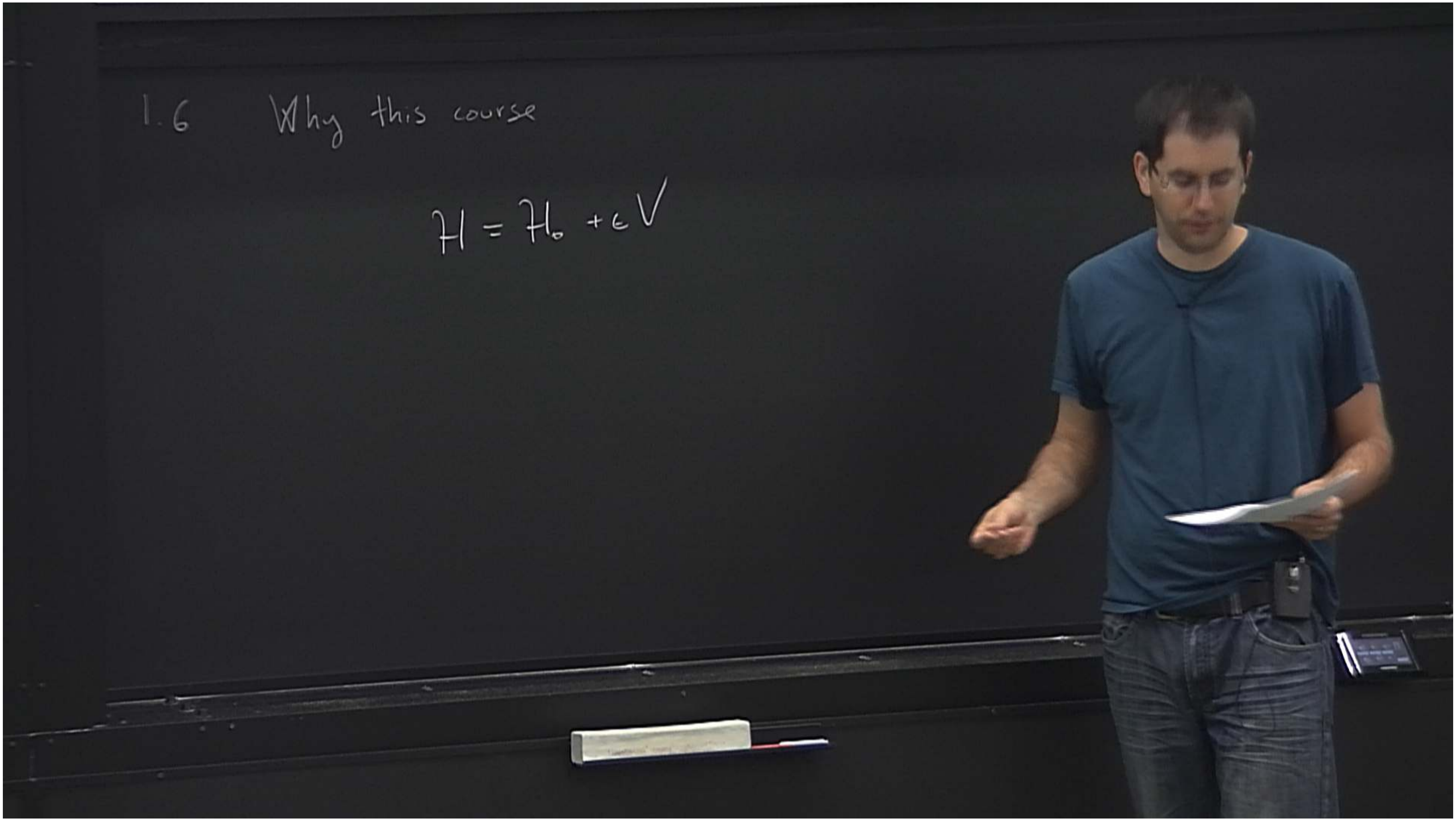
$$\dim \mathbb{V}^{(N)} = \left(\dim \mathbb{V}^{(1)}\right)^N \sim \exp N$$

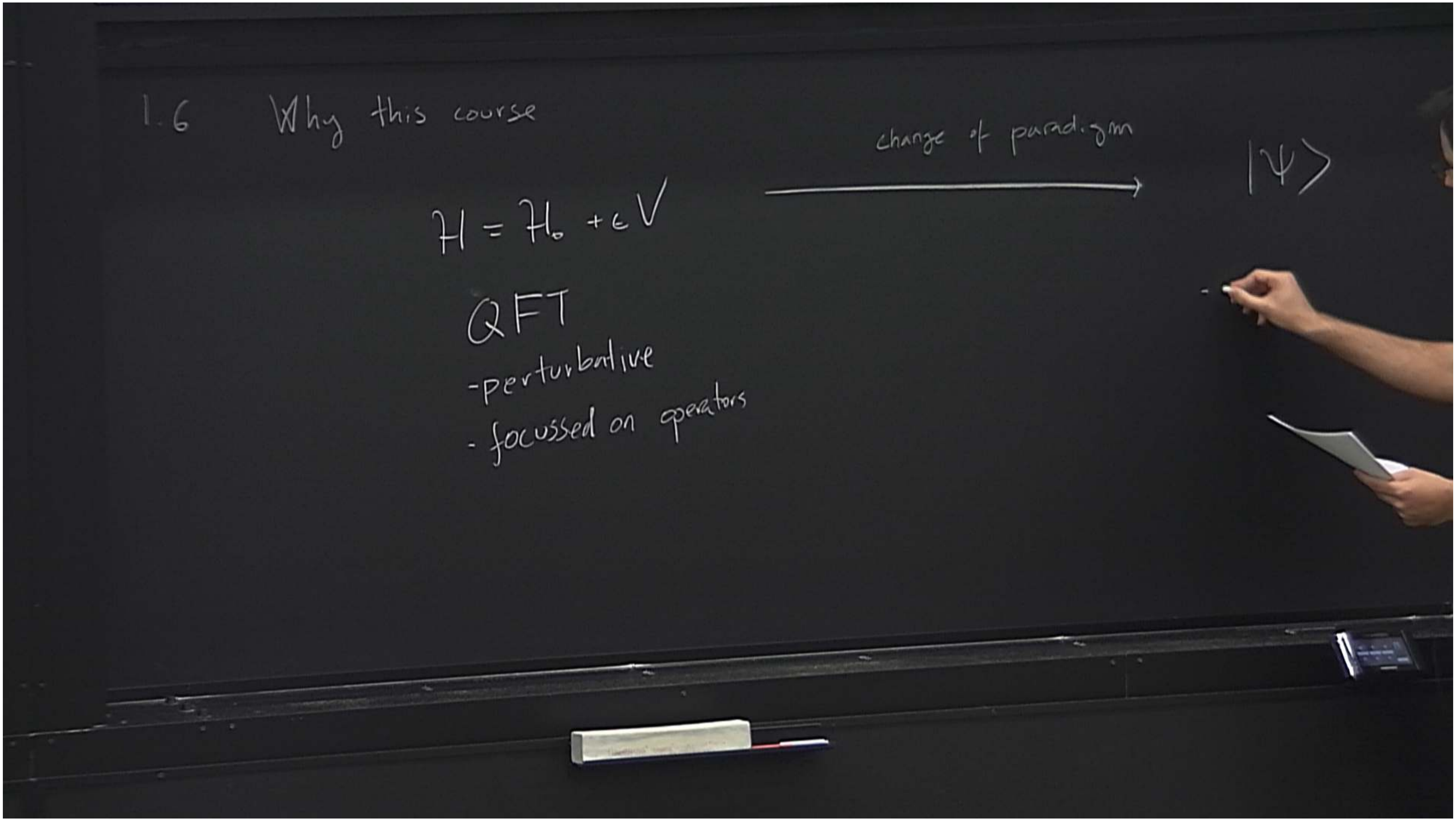
$$\mathbb{V}^{(N)} = \underbrace{\mathbb{V}^{(1)} \otimes \mathbb{V}^{(1)} \otimes \ldots \otimes \mathbb{V}^{(1)}}_{N}$$

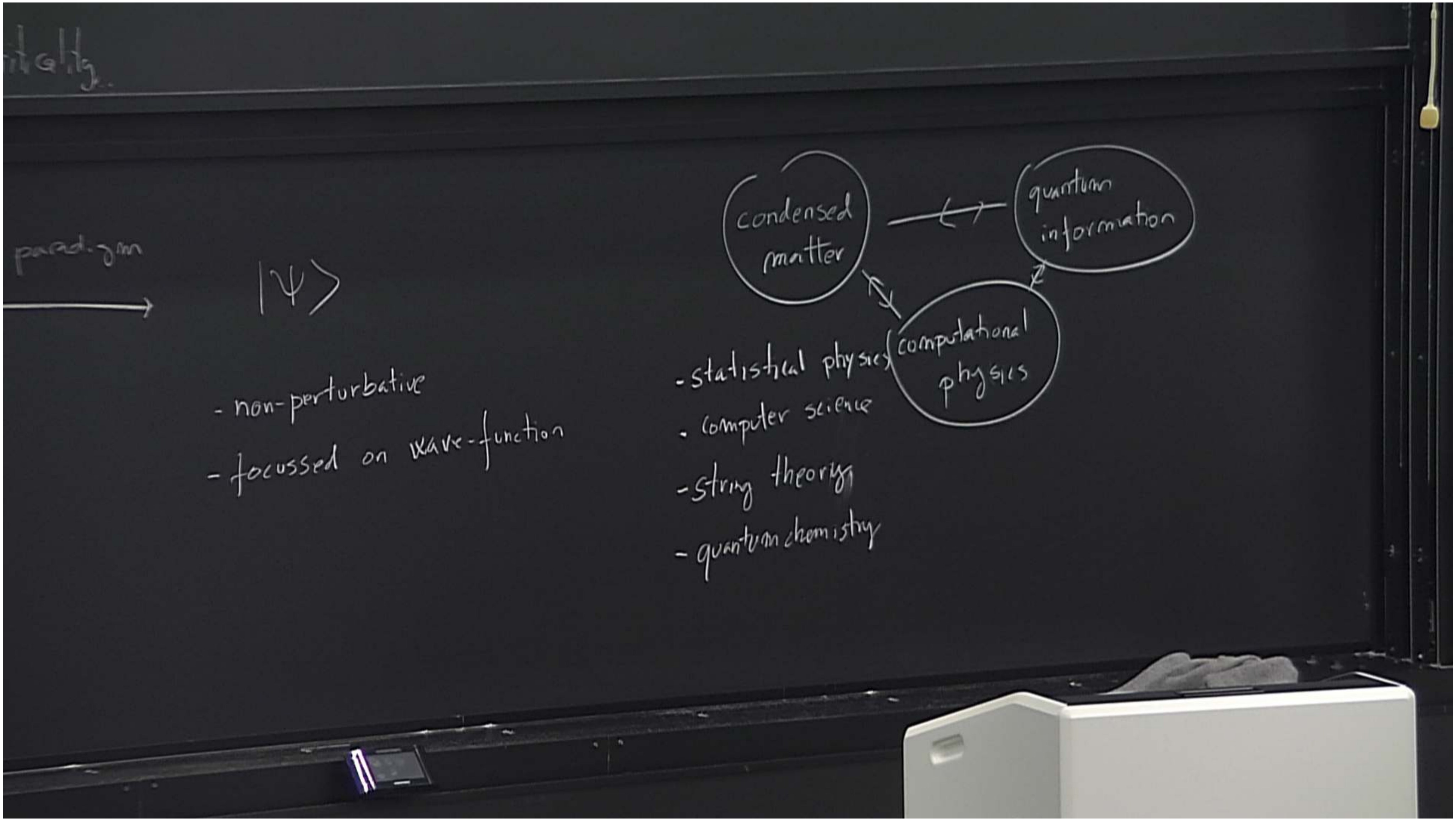

Part I
· exact diagonalization
· free fermions (cheat)
tools
applying
tools
Part II
· structure of ground states
exploit
Part III
tensor networks
New tools, formalism


tools
Part I
exp N
· exact diagonalization
· free fermions (cheat)
applying
tools
Part II
· structure of ground states
exploit
O(N)
Part III
tensor networks
New tools, formalism


Part III: Tensor network states
O(N)
10 - Basics of tensor networks
(x2) 11 - Matrix product state (MPS)
(x2) 12 - Multi-scale entanglement renormalization ansatz (MERA)
and the renormalization group
13 - Tensor networks in D≥2 dimensions


1.6 Why this course
$\mathcal{H} = \mathcal{H}_0 + \epsilon V$


1.6 Why this course
change of paradigm
$|\psi\rangle$
$\mathcal{H} = \mathcal{H}_0 + \epsilon V$
QFT
-perturbative
- focussed on operators

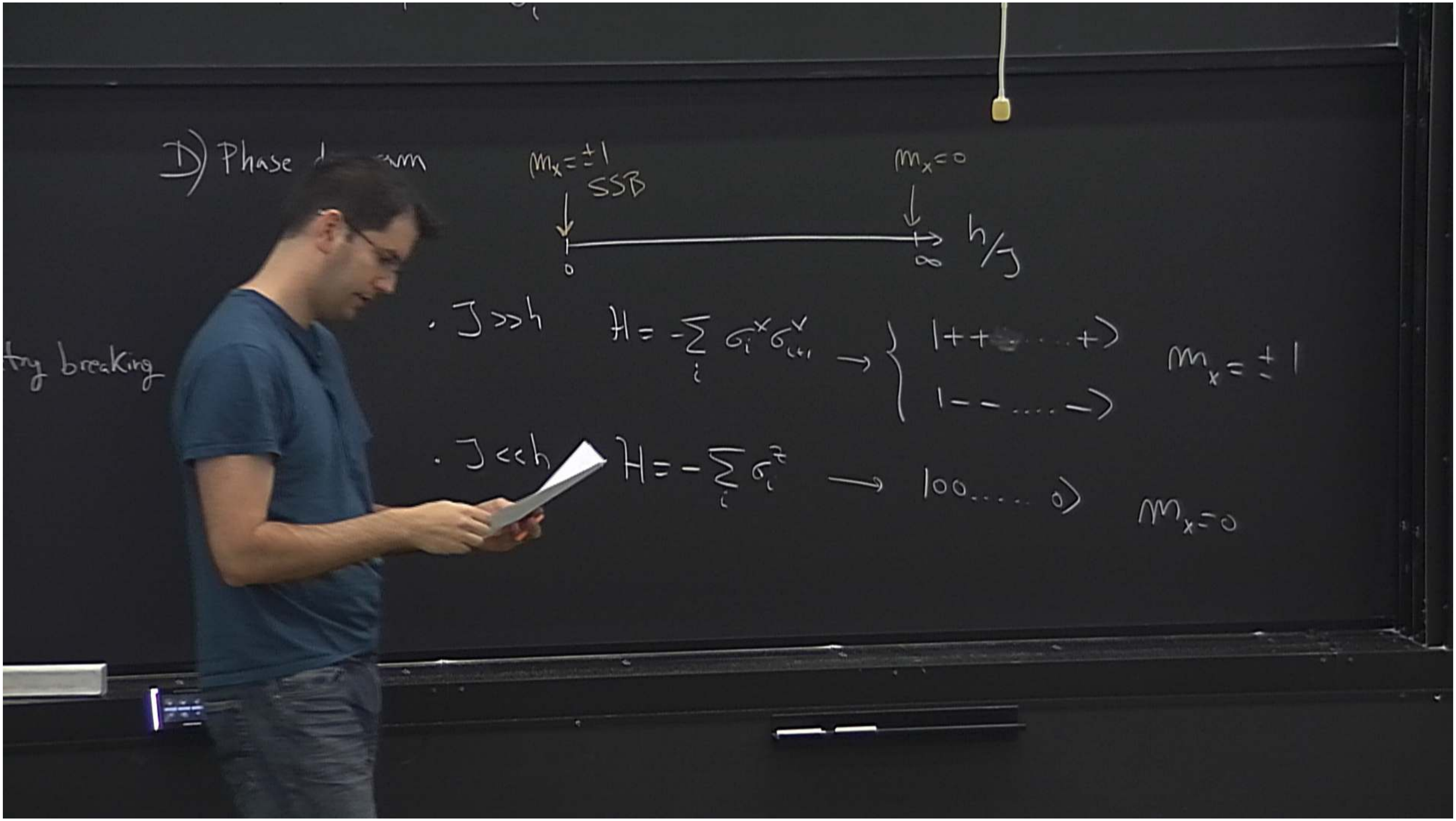

paradigm
$|\psi\rangle$
- non-perturbative
- focussed on wave-function
condensed matter
quantum information
computational physics
- statistical physics
- computer science
- string theory
- quantum chemistry

C) order parameter

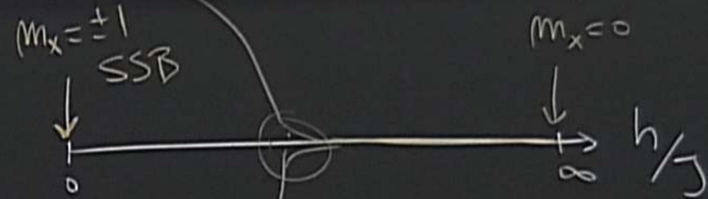
$$m_x \equiv \frac{1}{N}\sum_i \langle \sigma_i^x \rangle = \begin{cases} 0 & \text{symmetric phase} \\ \neq 0 & \text{spontaneous symmetry breaking} \end{cases}$$

ground state

D) Phase diagram


D) Phase
$m_x = \pm 1$
SSB
$m_x = 0$
0
∞
h/J
$\cdot\ J \gg h$
$H = -\sum_i \sigma_i^x \sigma_{i+1}^x \rightarrow \begin{cases} |{+}{+}\ldots{+}\rangle \\ |{-}{-}\ldots{-}\rangle \end{cases}$
$m_x = \pm 1$
$\cdot\ J \ll h$
$H = -\sum_i \sigma_i^z \rightarrow |00\ldots0\rangle$
$m_x = 0$
try breaking

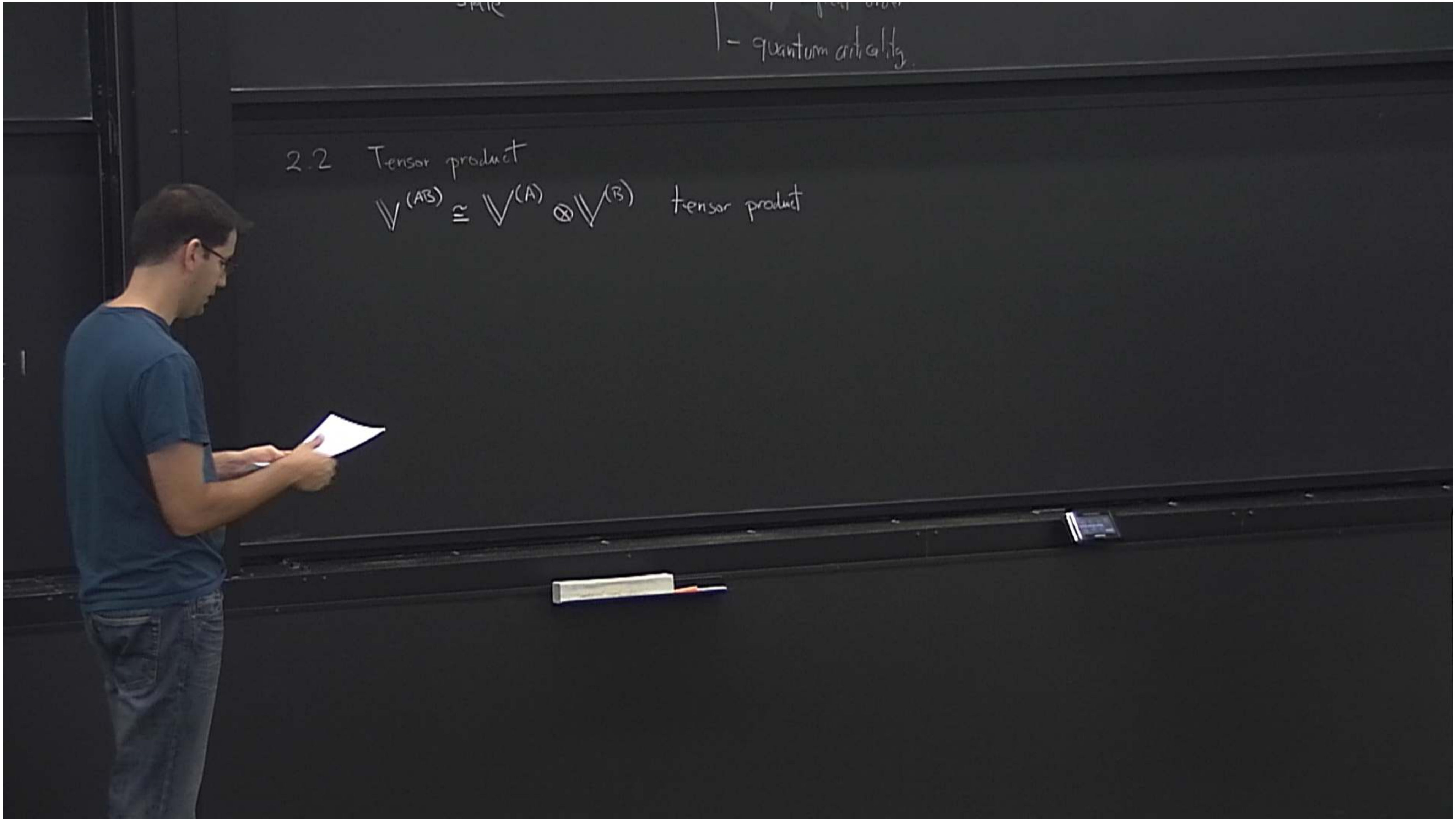
D) Phase diagram

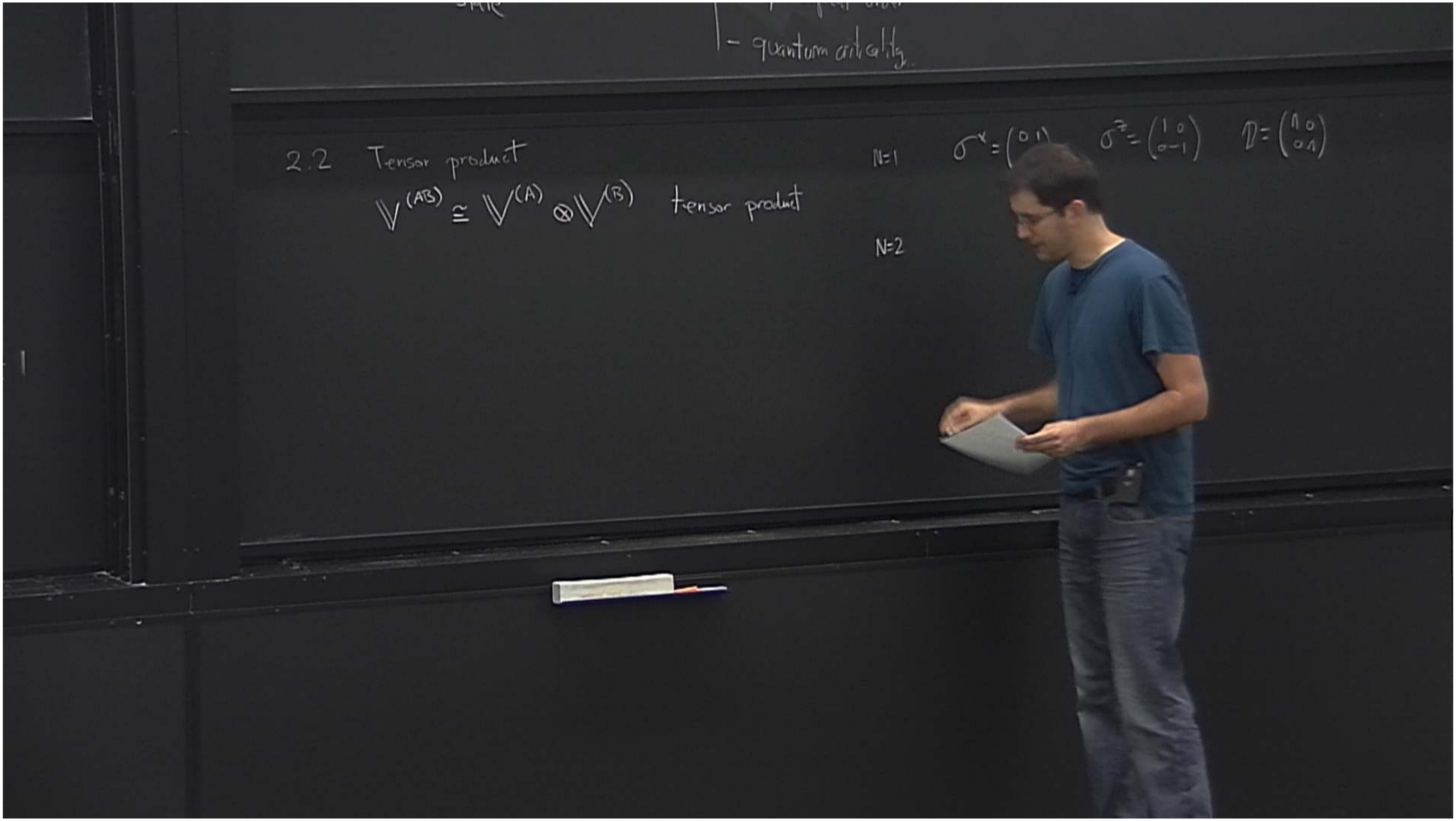


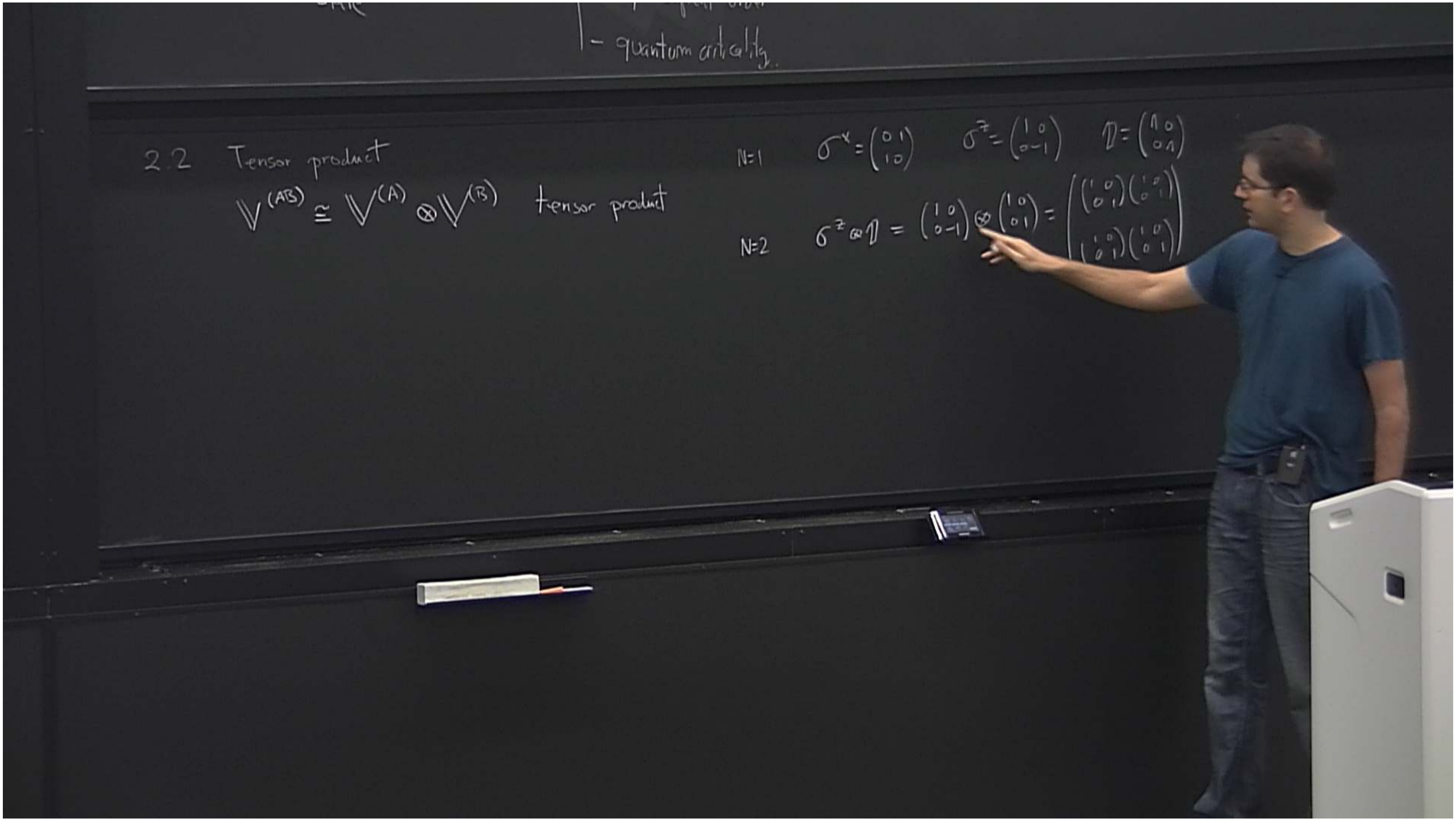
try breaking

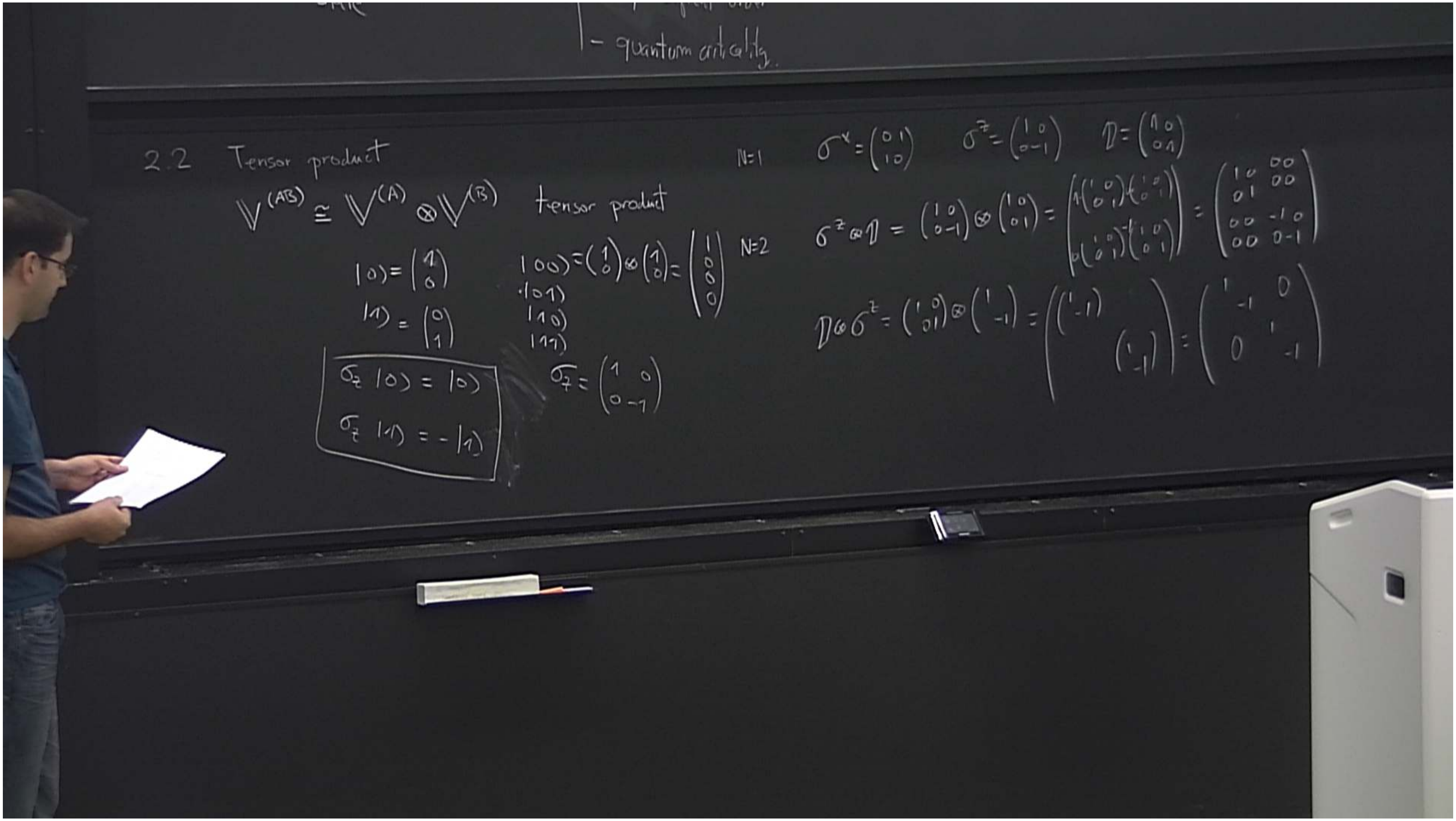
- $J \gg h$ $\quad H = -\sum_i \sigma_i^x \sigma_{i+1}^x \rightarrow \begin{cases} |++\ldots+\rangle \\ |--\ldots-\rangle \end{cases}$ $\quad m_x = \pm 1$

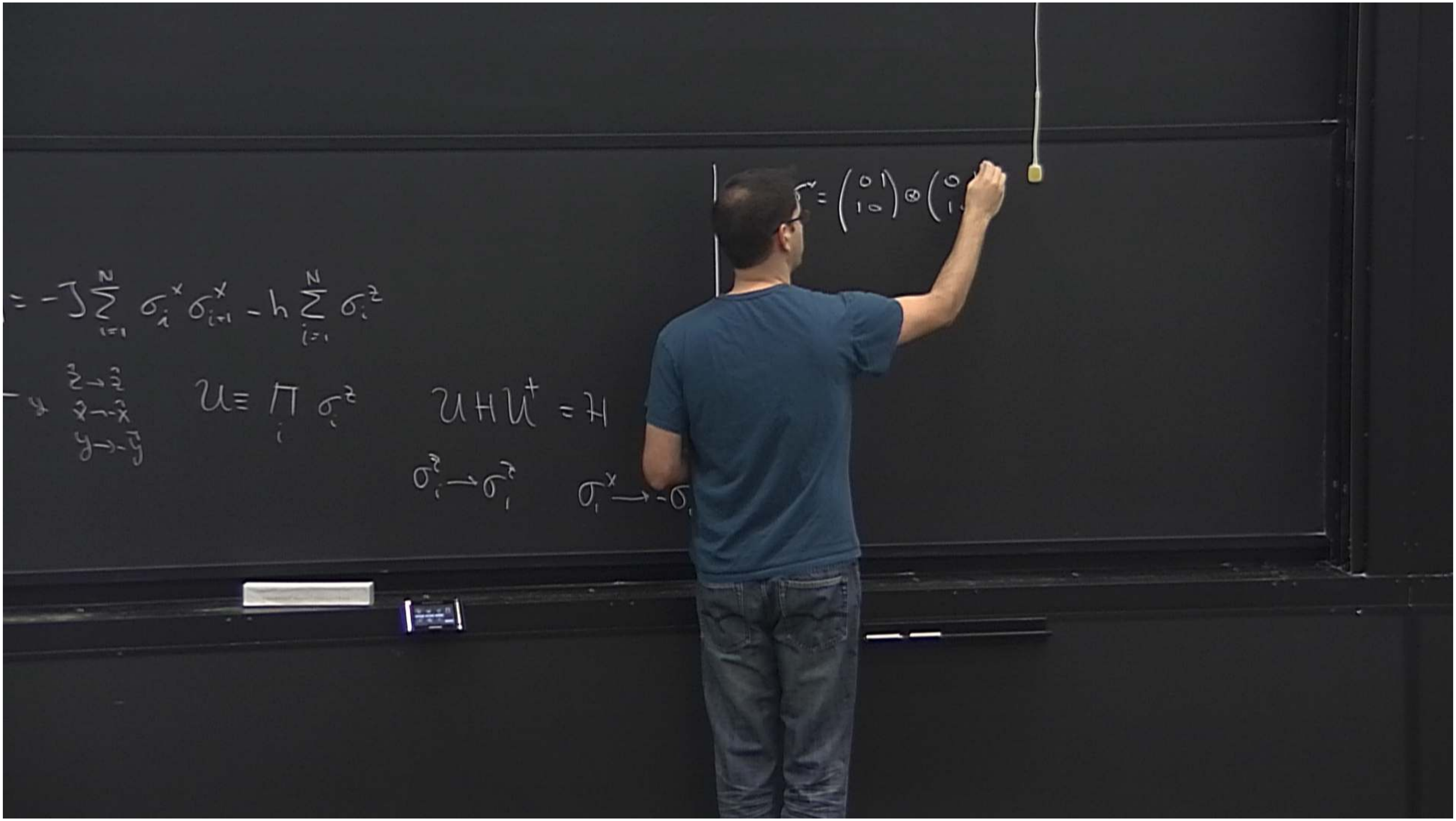
- $J \ll h$ $\quad H = -\sum_i \sigma_i^z \longrightarrow |00\ldots0\rangle$ $\quad m_x = 0$

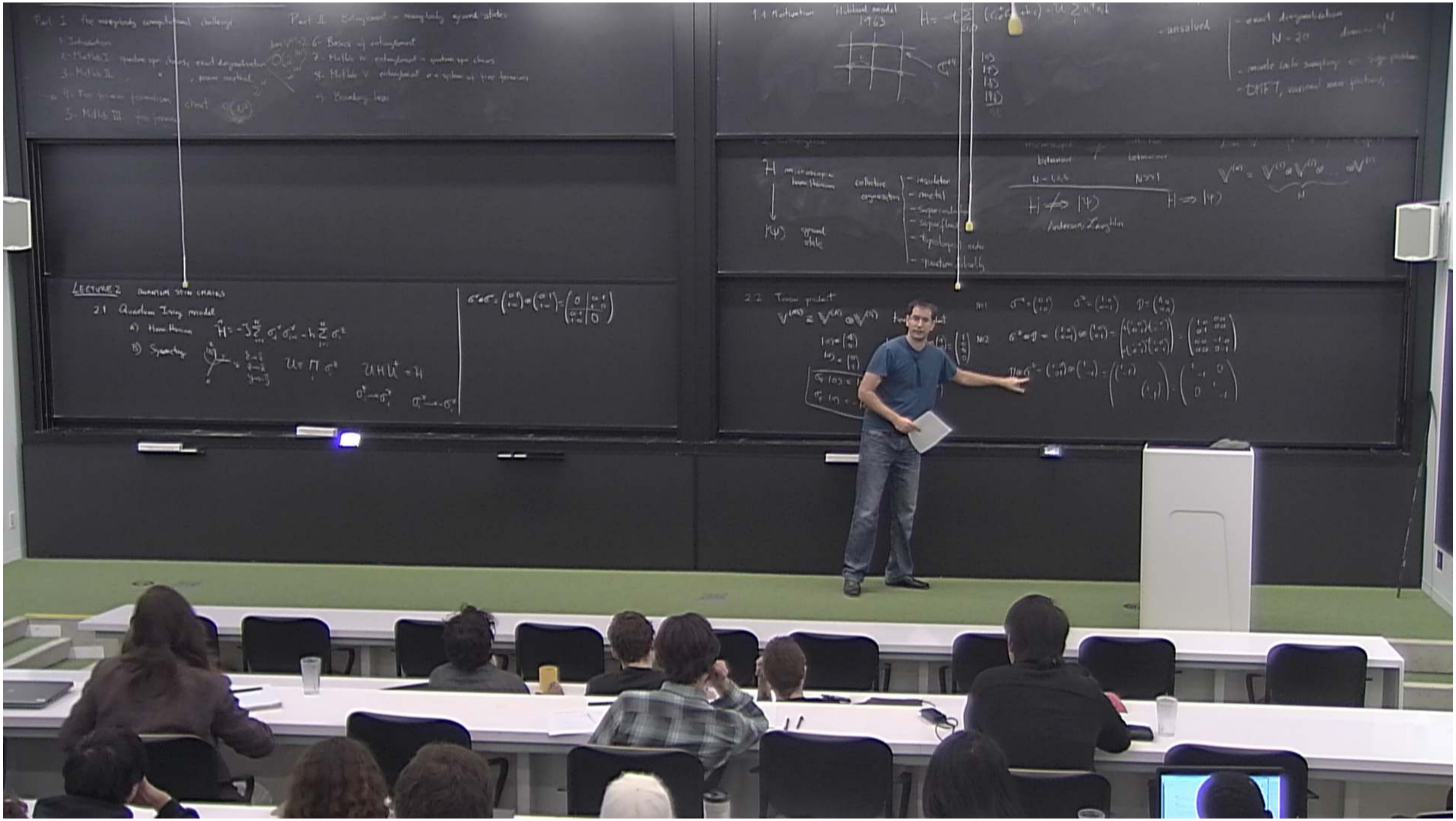

- quantum criticality
2.2 Tensor product
$\mathbb{V}^{(AB)} \cong \mathbb{V}^{(A)} \otimes \mathbb{V}^{(B)}$ tensor product

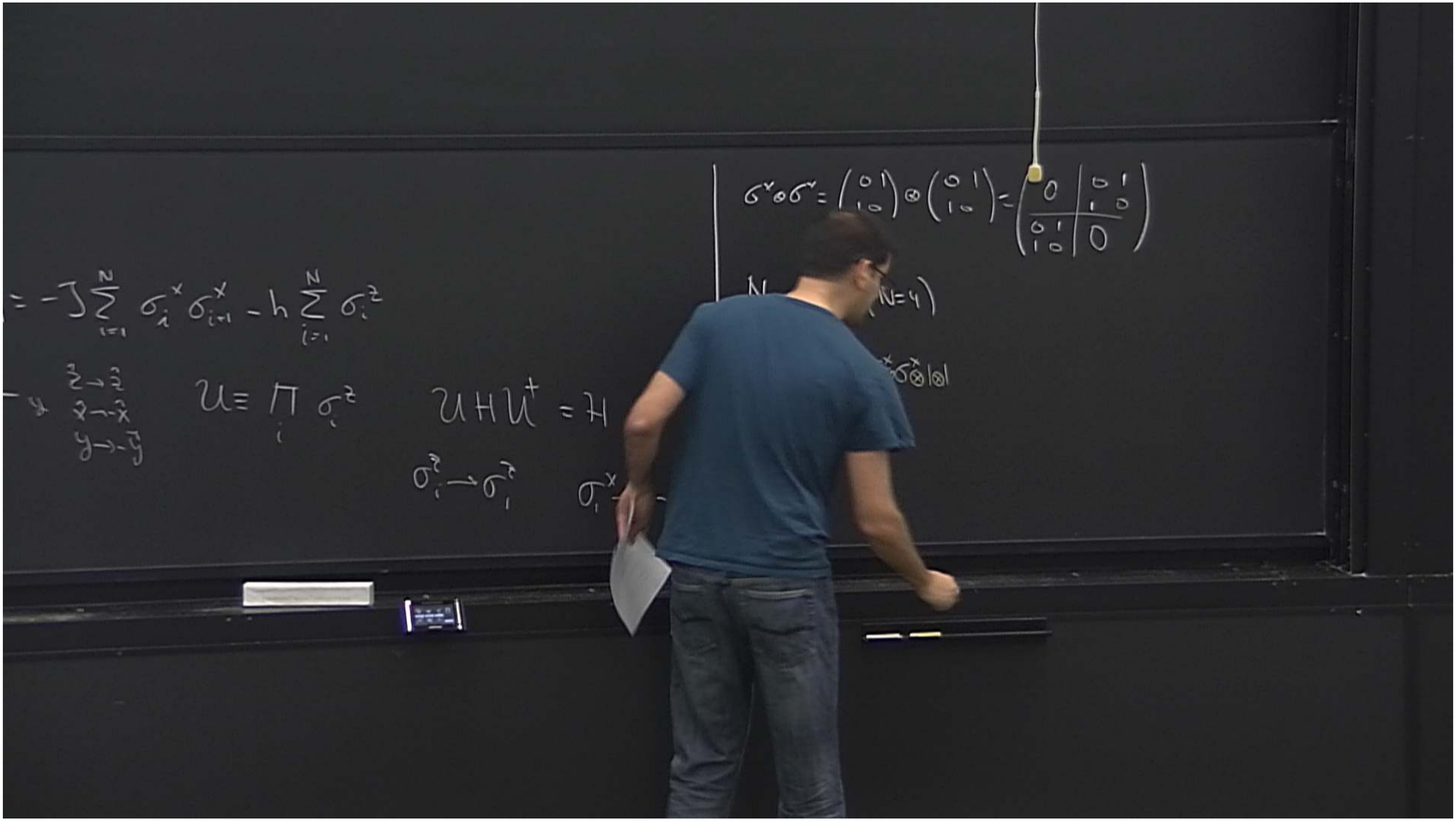

- quantum criticality
2.2 Tensor product
$\mathbb{V}^{(AB)} \cong \mathbb{V}^{(A)} \otimes \mathbb{V}^{(B)}$ tensor product
N=1
$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
N=2

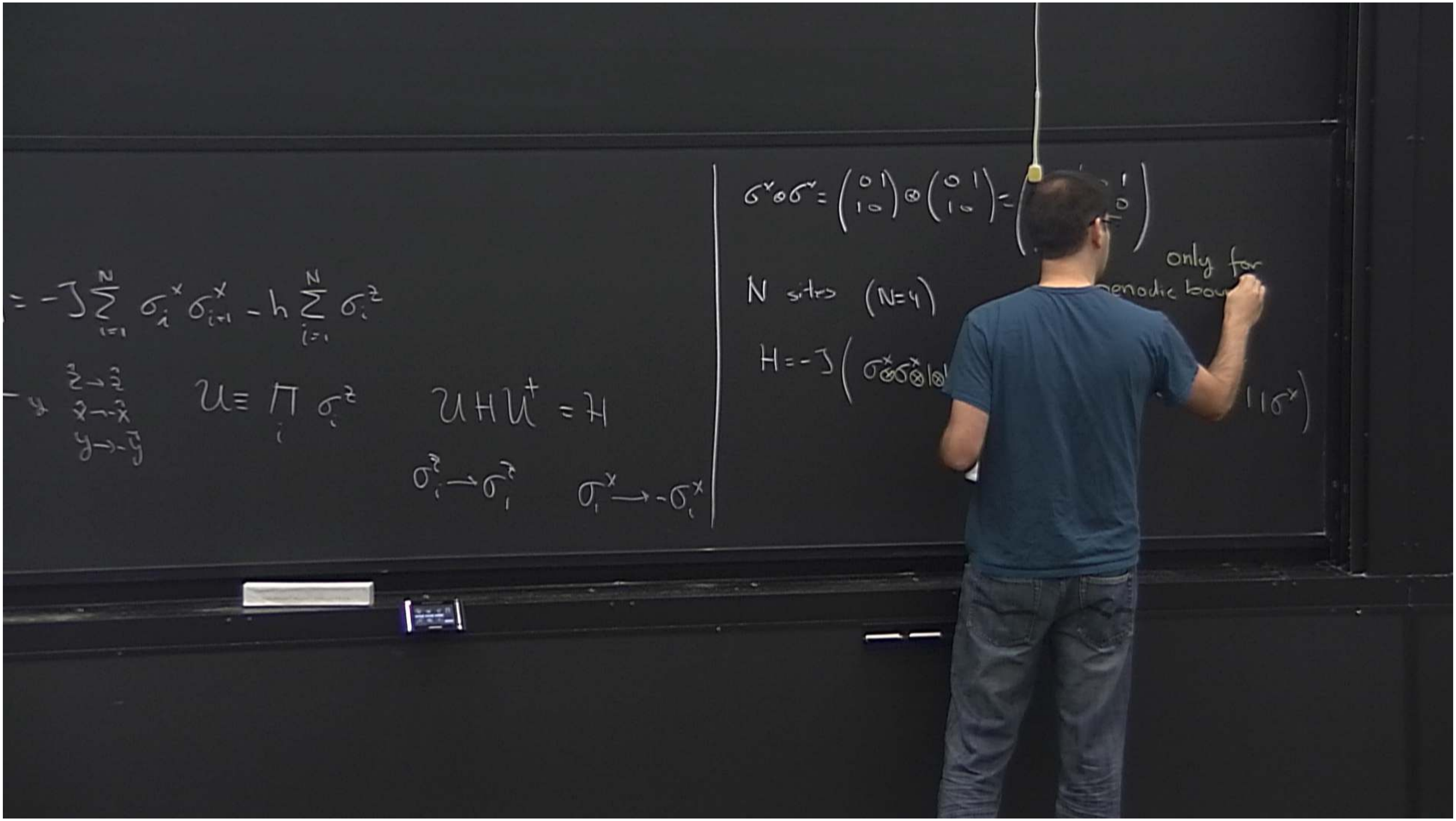

- quantum criticality
2.2 Tensor product
$\mathbb{V}^{(AB)} \cong \mathbb{V}^{(A)} \otimes \mathbb{V}^{(B)}$ tensor product
N=1 $\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
N=2 $\sigma^z \otimes \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}$

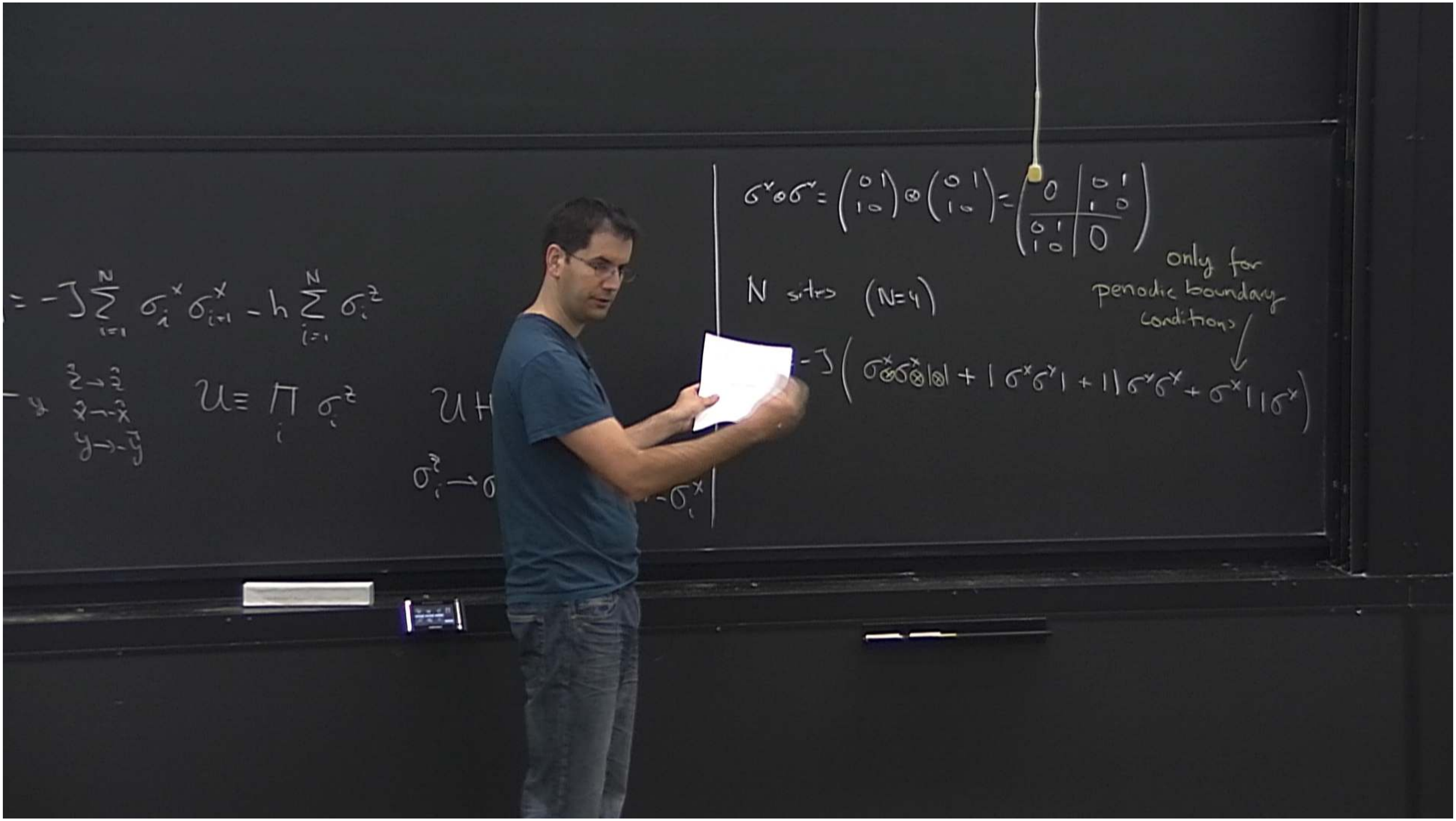

- quantum criticality
2.2 Tensor product
V^(AB) ≅ V^(A) ⊗ V^(B) tensor product
|0⟩ = (1 0)
|1⟩ = (0 1)
σz |0⟩ = |0⟩
σz |1⟩ = -|1⟩
|00⟩ = (1 0) ⊗ (1 0) = (1 0 0 0)
|01⟩
|10⟩
|11⟩
σz = (1 0 0 -1)
N=1
σx = (0 1 1 0)
σz = (1 0 0 -1)
𝟙 = (1 0 0 1)
N=2
σz ⊗ 𝟙 = (1 0 0 -1) ⊗ (1 0 0 1)
𝟙 ⊗ σz = (1 0 0 1) ⊗ (1 -1)

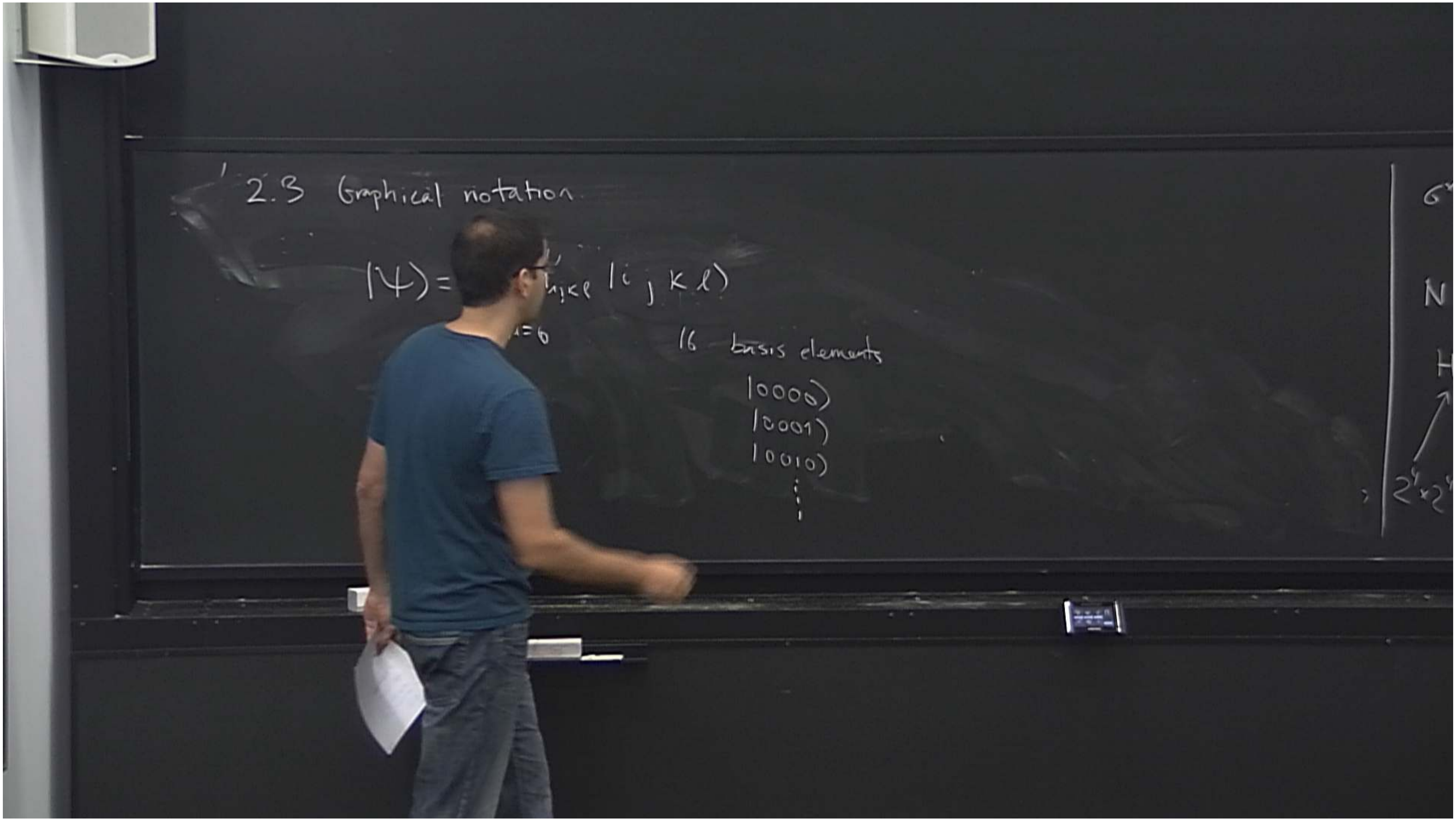

$= -J\sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x - h\sum_{i=1}^{N} \sigma_i^z$
$\hat{z} \to \hat{z}$
$\hat{x} \to -\hat{x}$
$y \to -\hat{y}$
$\mathcal{U} \equiv \prod_i \sigma_i^z$
$\mathcal{U} H \mathcal{U}^+ = H$
$\sigma_i^z \to \sigma_i^z$
$\sigma_i^x \to -\sigma_i$

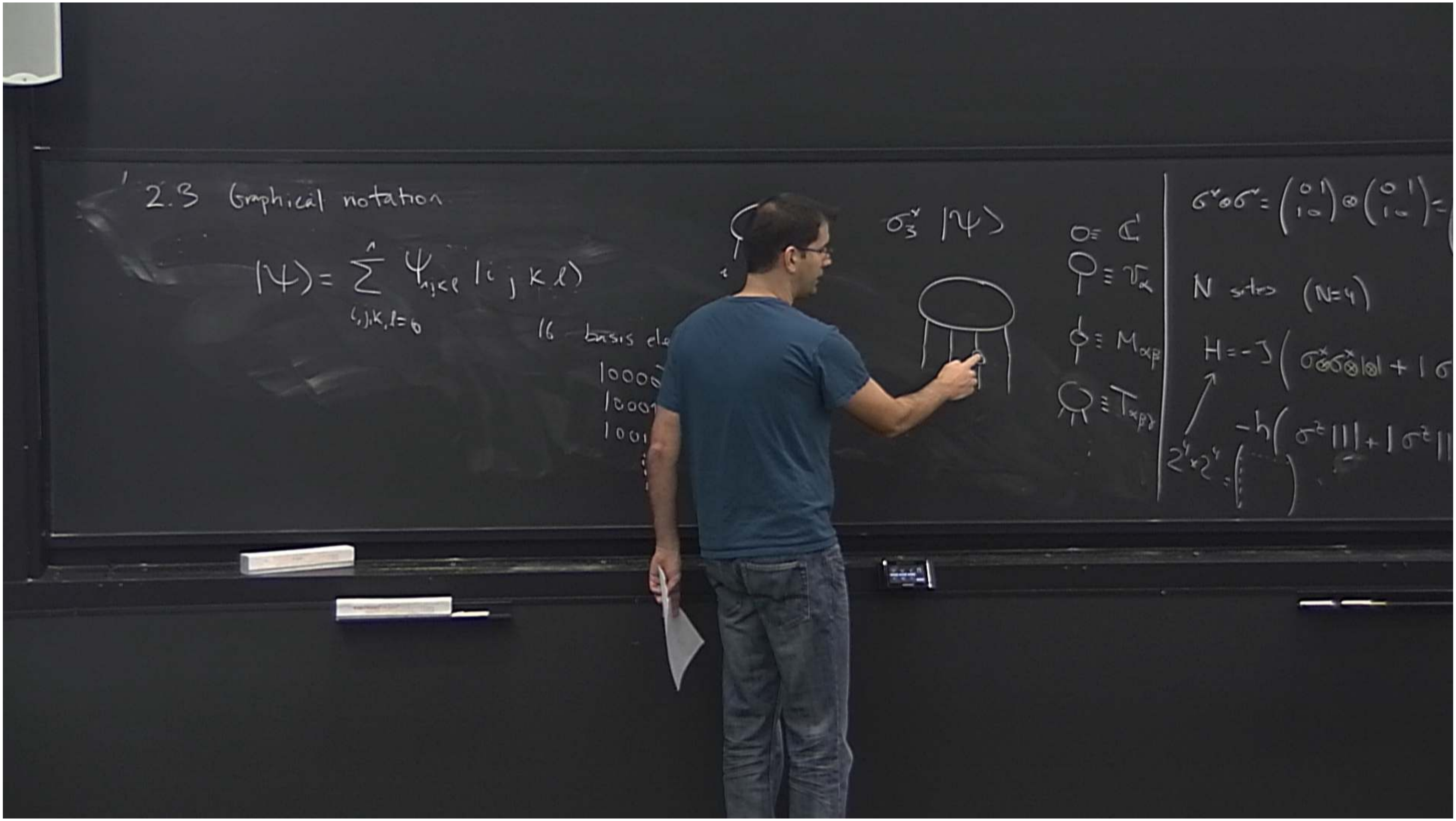


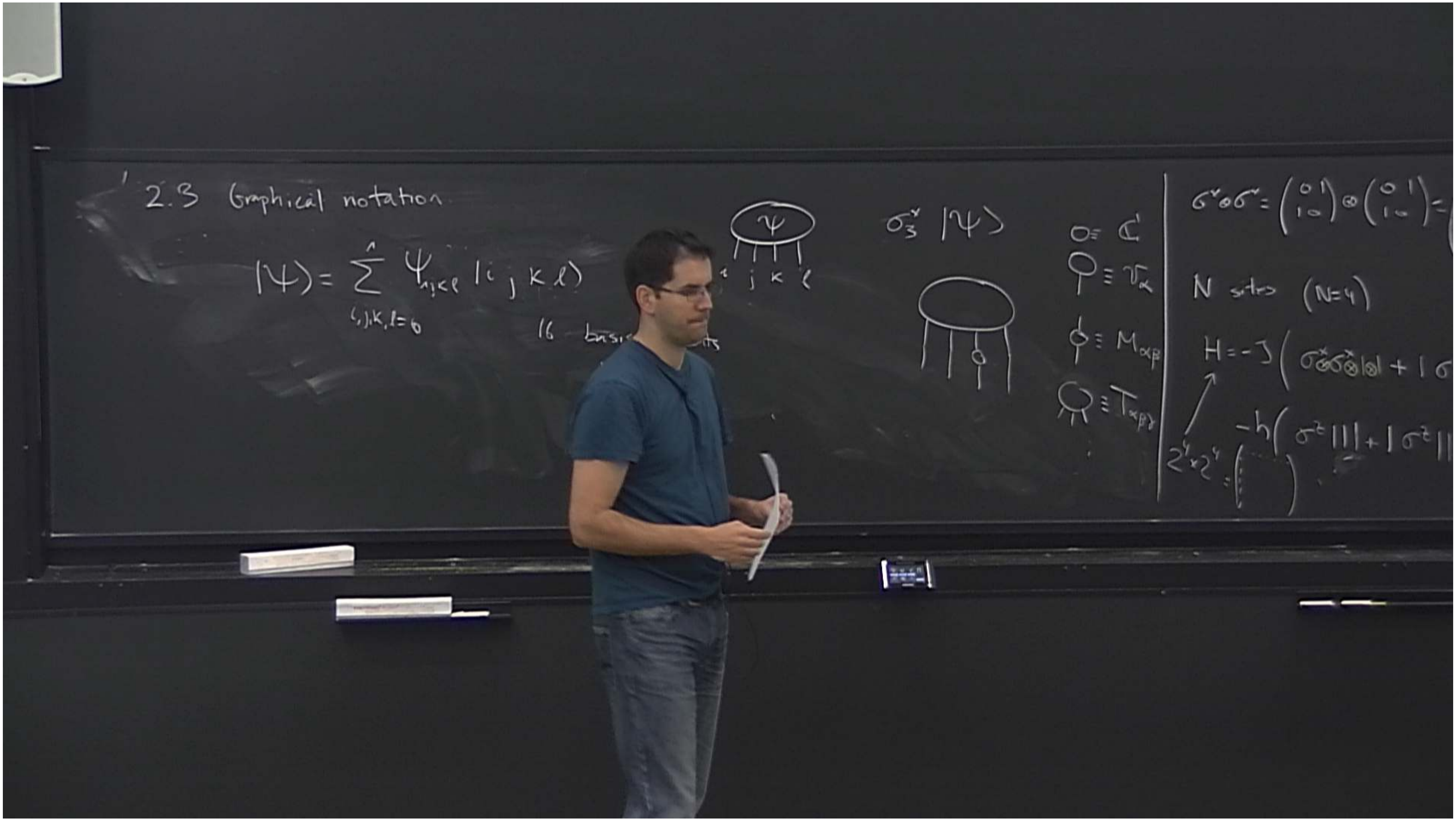

U ≡ Π σ^z
UHU^+ = H


N sites (N=4)
only for
U H U+ = H


N sites (N=4)
only for periodic boundary conditions


2.3 Graphical notation
16 basis elements
|0000)
|0001)
|0010)


2.3 Graphical notation
16 basis ele
N sites (N=4)


2.3 Graphical notation
16 basis
N sites (N=4)