Title: 13/14 PSI - Explorations in Condensed Matter - Lecture 2

Date: Mar 18, 2014 10:15 AM

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Abstract:

MANY-BODY ENTANGLEMENT & TENSOR NETWORKS

Part I: the many-body computational challenge

1- Introduction

2- Matlab I: quantum spin chains, exact diagonalization

3- Matlab II: " , power method

4- Free fermion formalism

5- Matlab III: free fermions

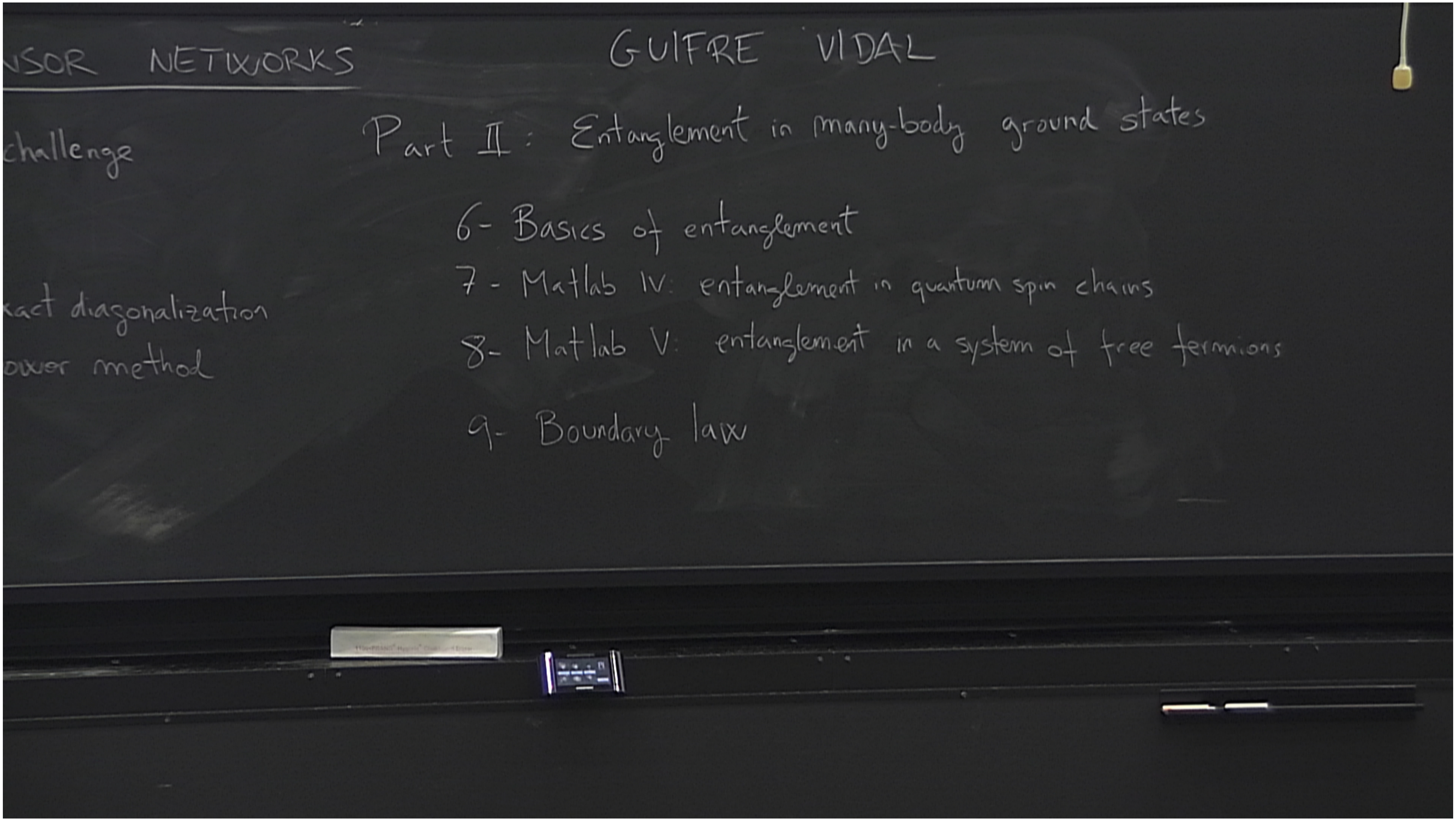
Part II: Entang

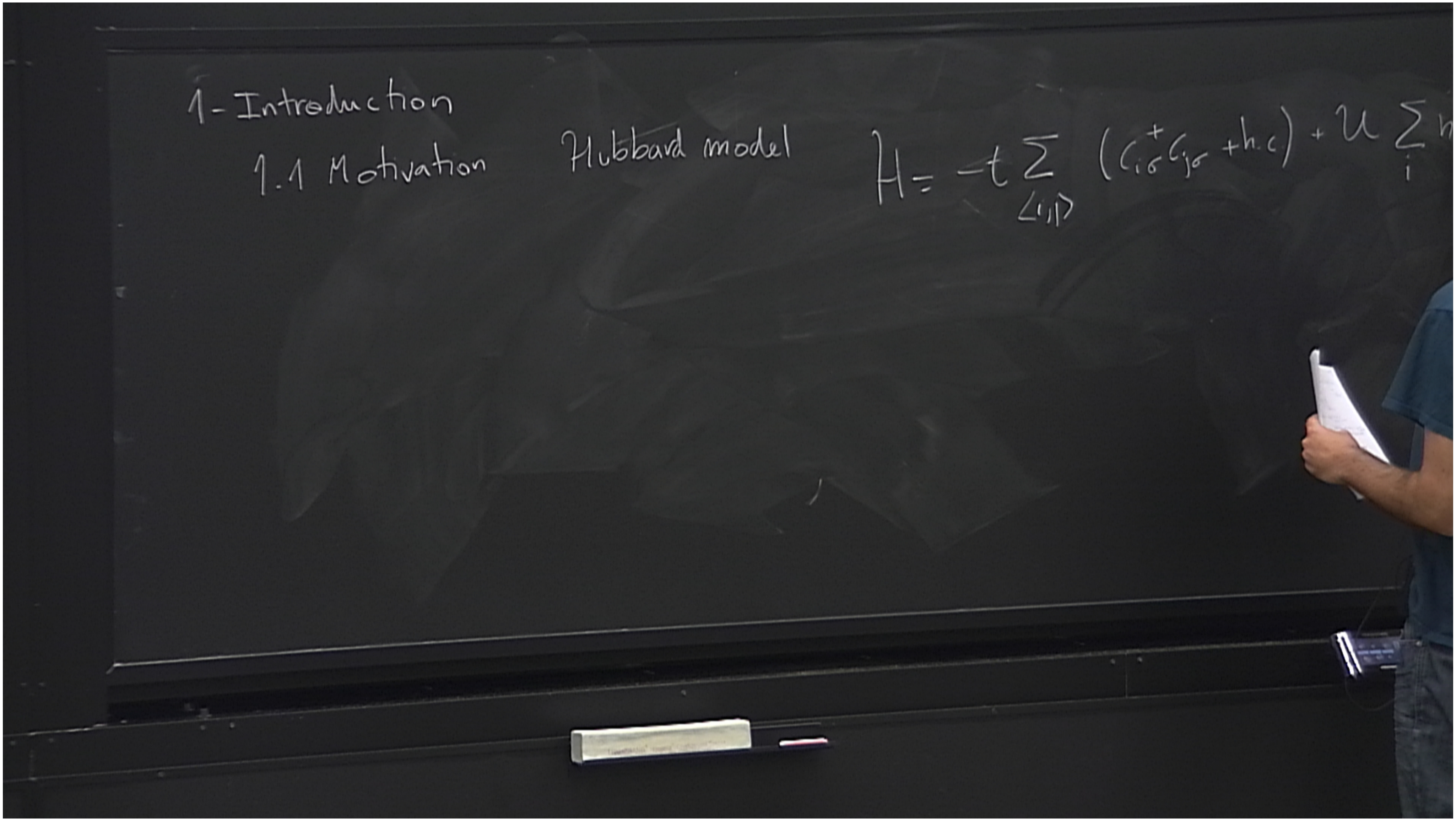
6- Basics of

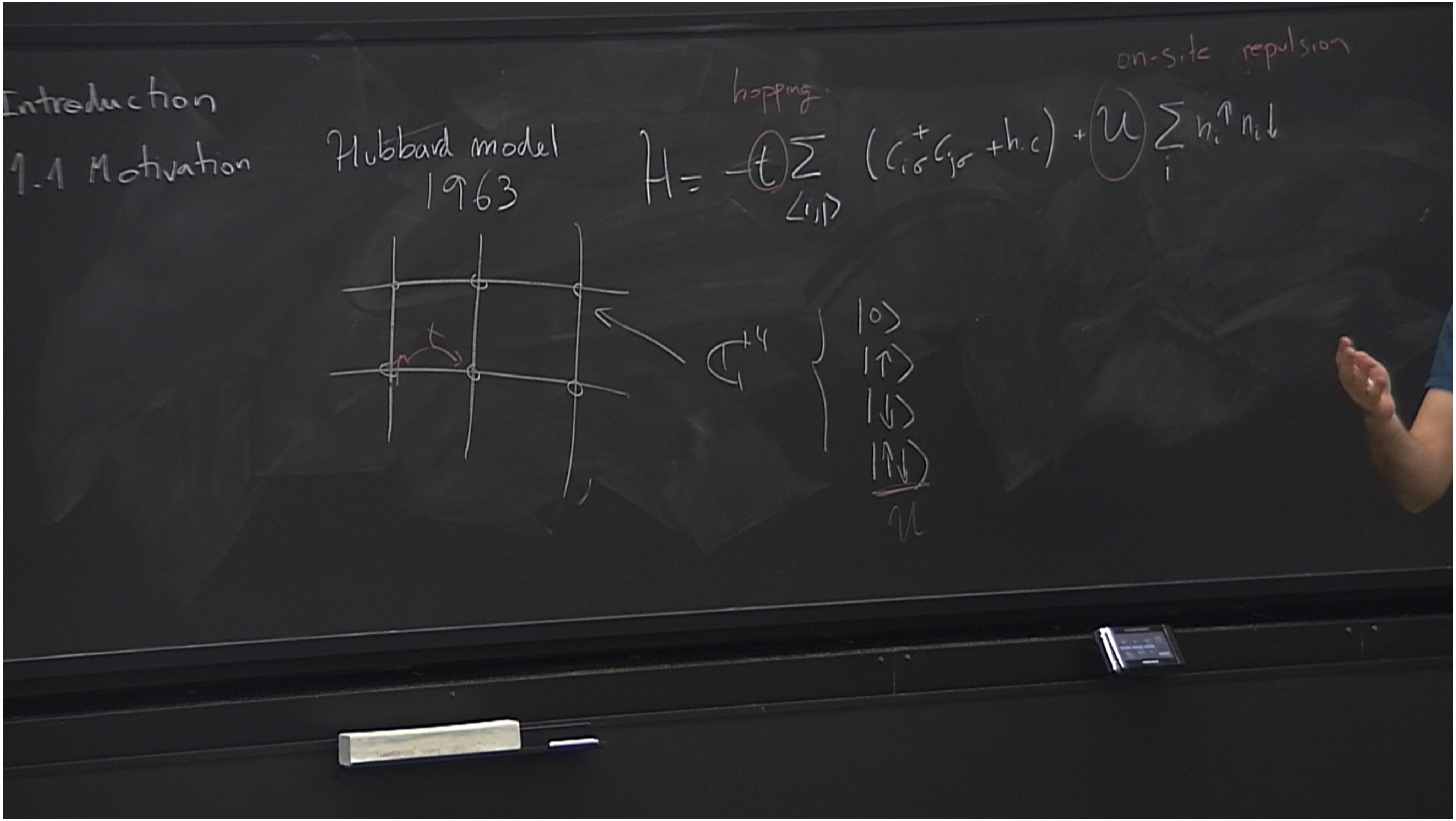
7- Matlab IV

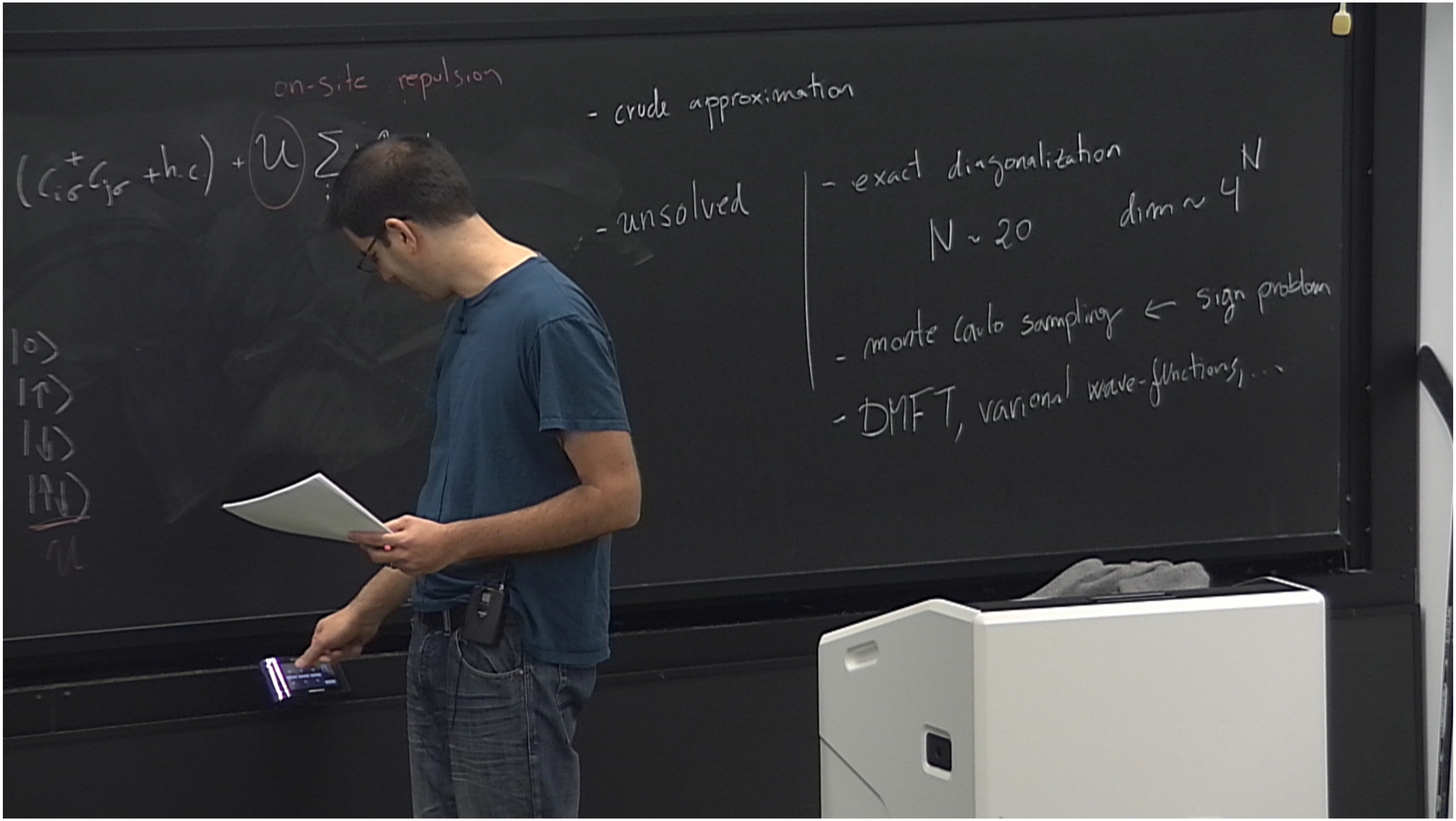
8- Matlab V

9- Boundary


NSOR NETWORKS
GUIFRE VIDAL
Part II: Entanglement in many-body ground states
challenge
6- Basics of entanglement
7 - Matlab IV: entanglement in quantum spin chains
xact diagonalization
8- Matlab V: entanglement in a system of free fermions
ower method
9- Boundary law


1- Introduction
1.1 Motivation
Hubbard model
$H = -t \sum_{\langle i,j \rangle} (c^{+}_{i\sigma} c_{j\sigma} + h.c.) + U \sum_{i} n$


Introduction
1.1 Motivation
Hubbard model
1963
hopping
on-site repulsion
$H = -t \sum_{\langle i,j \rangle} (c_{i\sigma}^{+} c_{j\sigma} + h.c) + U \sum_i n_{i\uparrow} n_{i\downarrow}$
t
|0⟩
|↑⟩
|↓⟩
|↑↓⟩
U


on-site repulsion
- crude approximation
- unsolved
- exact diagonalization
N ~ 20
dim ~ 4^N
- monte Carlo sampling ← sign problem
- DMFT, varional wave-functions, ...
|0⟩
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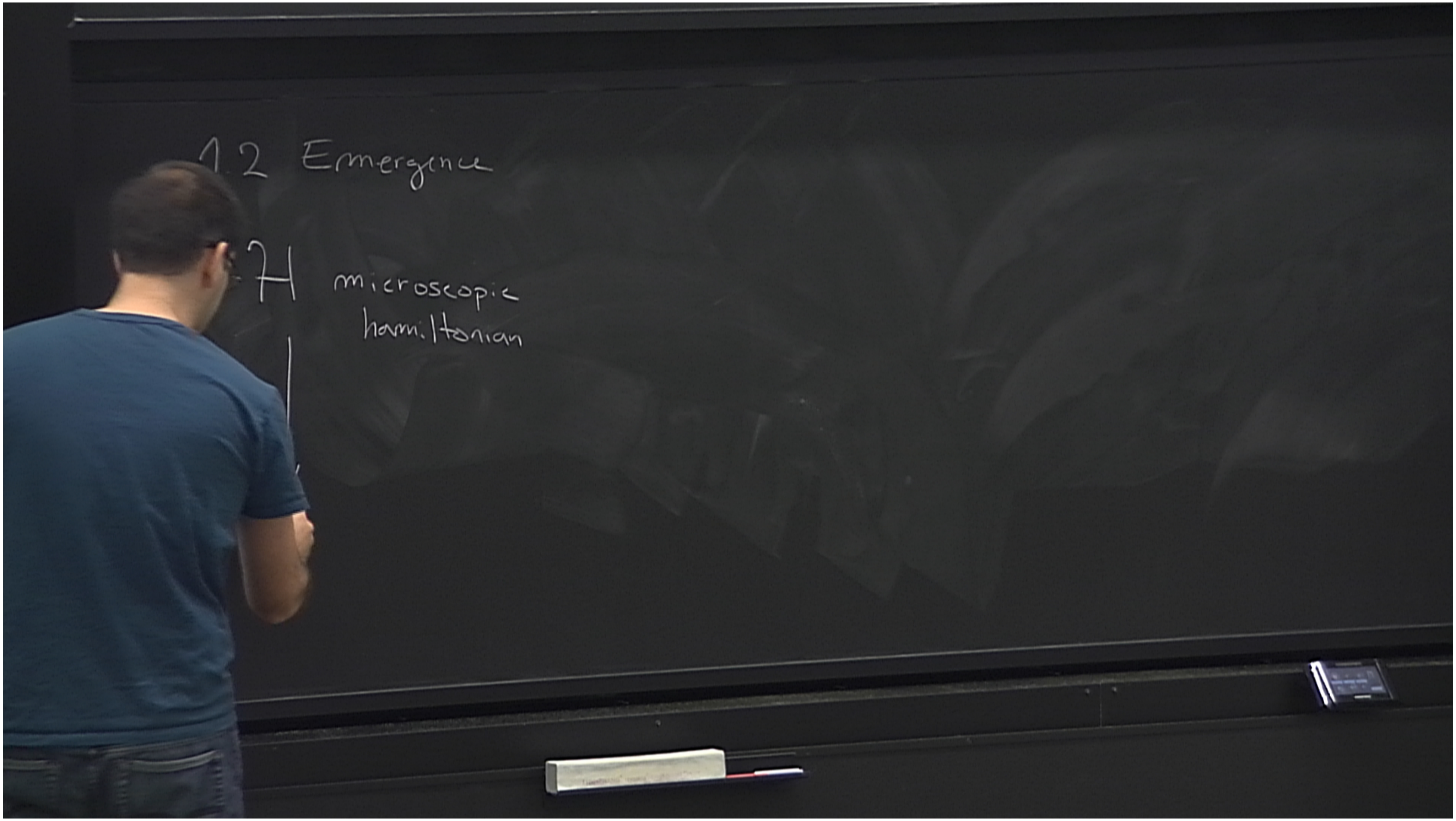
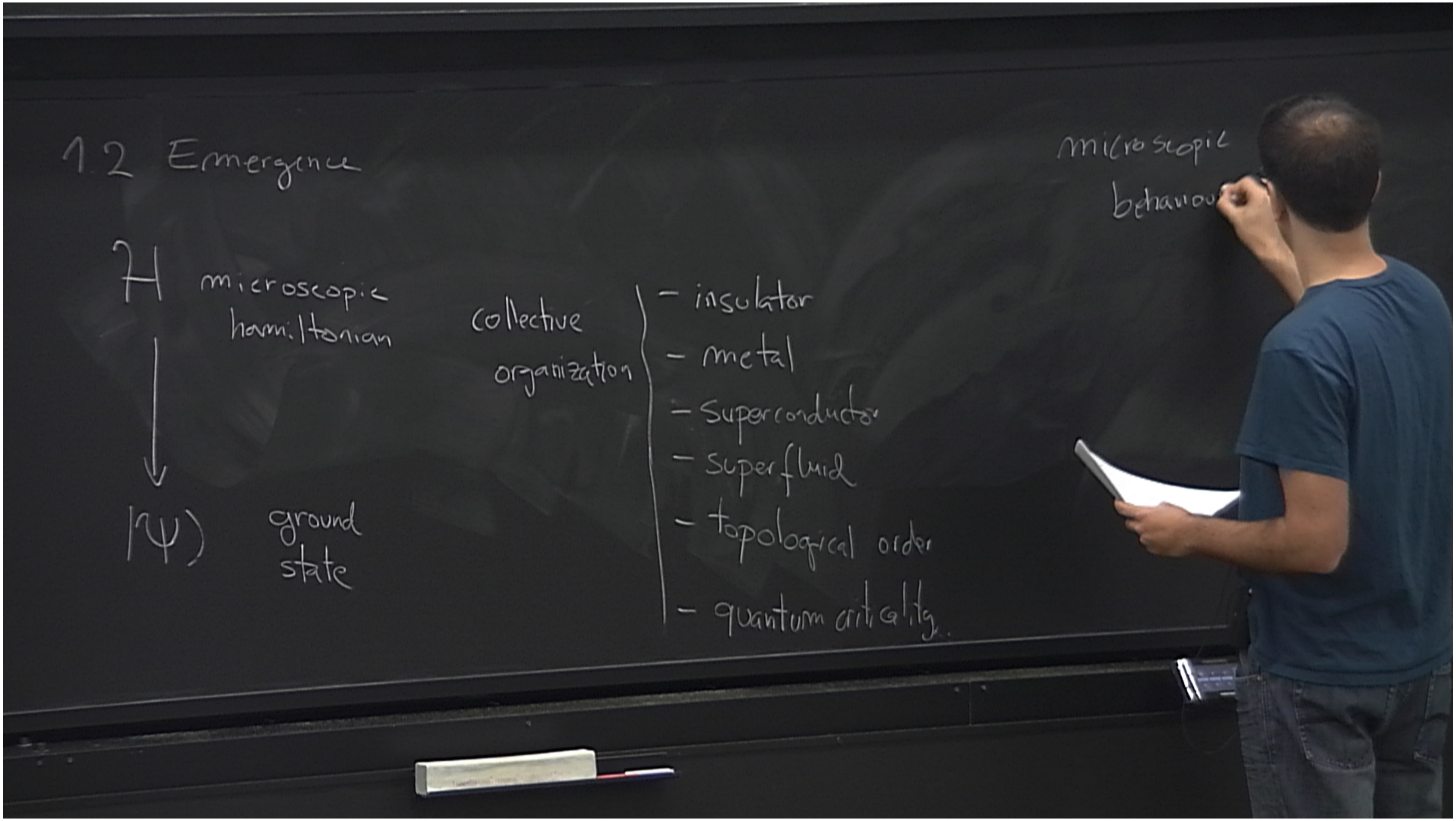
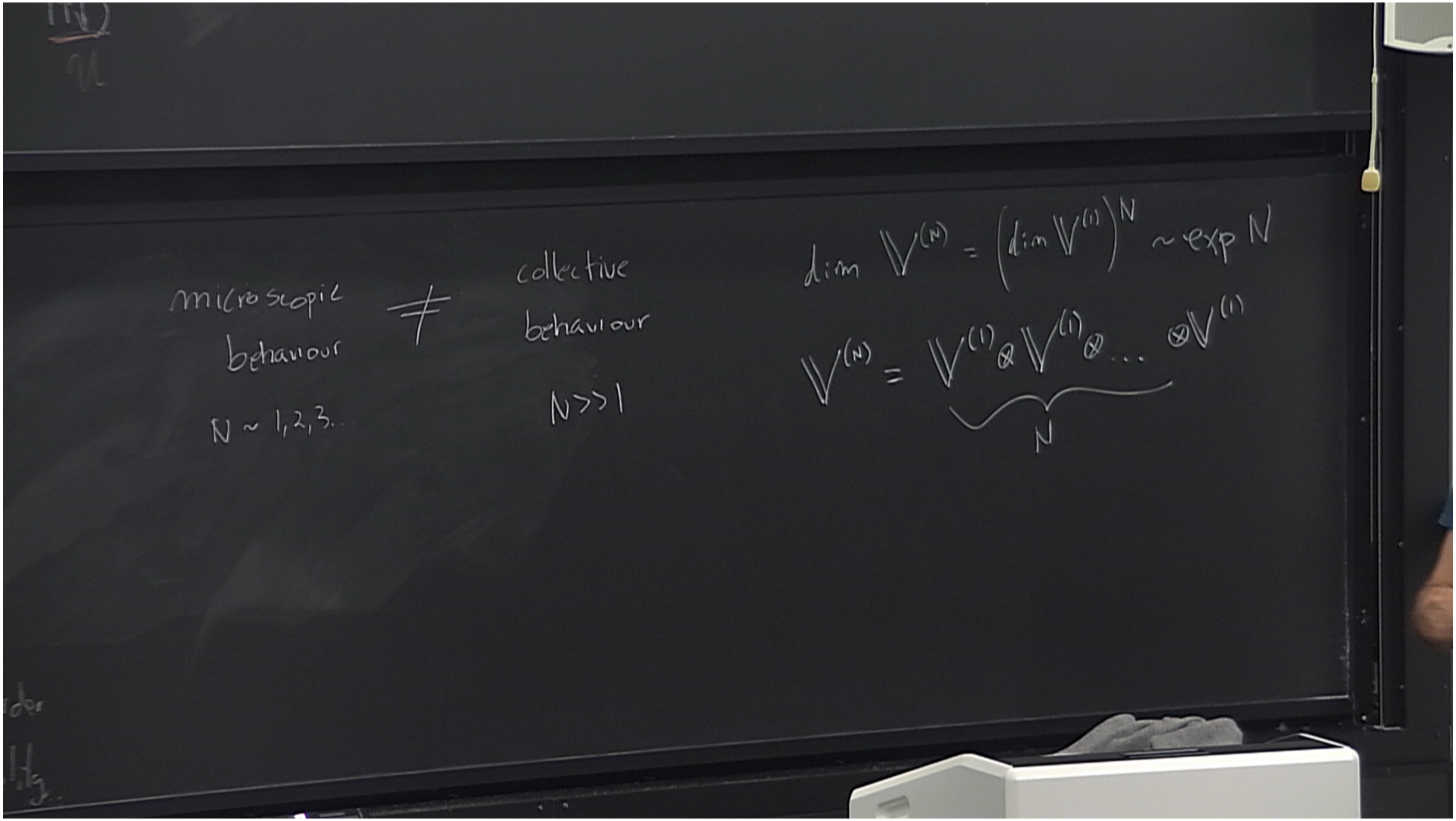
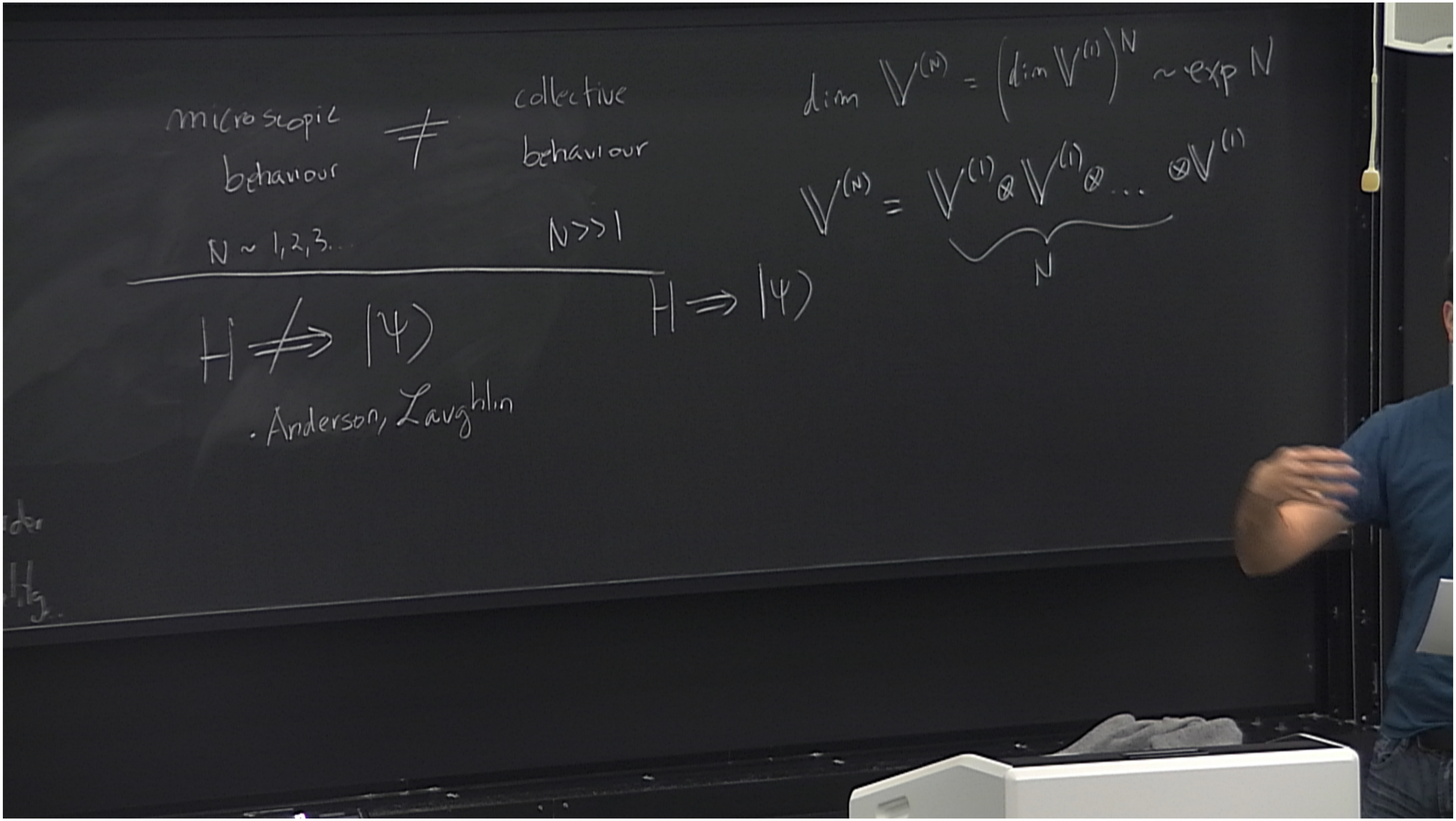

1.2 Emergence
H microscopic
hamiltonian


1.2 Emergence
H microscopic hamiltonian
|Ψ) ground state
collective organization
- insulator
- metal
- superconductor
- superfluid
- topological order
- quantum criticality
microscopic


microscopic behaviour
$\neq$
collective behaviour
$N \sim 1, 2, 3 \ldots$
$N \gg 1$
$\dim \mathbb{V}^{(N)} = \left(\dim \mathbb{V}^{(1)}\right)^N \sim \exp N$
$\mathbb{V}^{(N)} = \underbrace{\mathbb{V}^{(1)} \otimes \mathbb{V}^{(1)} \otimes \ldots \otimes \mathbb{V}^{(1)}}_{N}$


microscopic behaviour ≠ collective behaviour
N ~ 1,2,3...
N>>1
$H \not\Rightarrow |\psi\rangle$
$H \Rightarrow |\psi\rangle$
. Anderson, Laughlin
$\dim \mathbb{V}^{(N)} = (\dim \mathbb{V}^{(1)})^N \sim \exp N$
$\mathbb{V}^{(N)} = \underbrace{\mathbb{V}^{(1)} \otimes \mathbb{V}^{(1)} \otimes \ldots \otimes \mathbb{V}^{(1)}}_{N}$

microscopic behaviour $\neq$ collective behaviour

$N \sim 1, 2, 3 \ldots$ $\qquad$ $N \gg 1$

$H \not\Rightarrow |\psi\rangle$ $\qquad$ $H \Rightarrow |\psi\rangle$

· Anderson, Laughlin

$$\dim \mathbb{V}^{(N)} = \left(\dim \mathbb{V}^{(1)}\right)^N \sim \exp N$$

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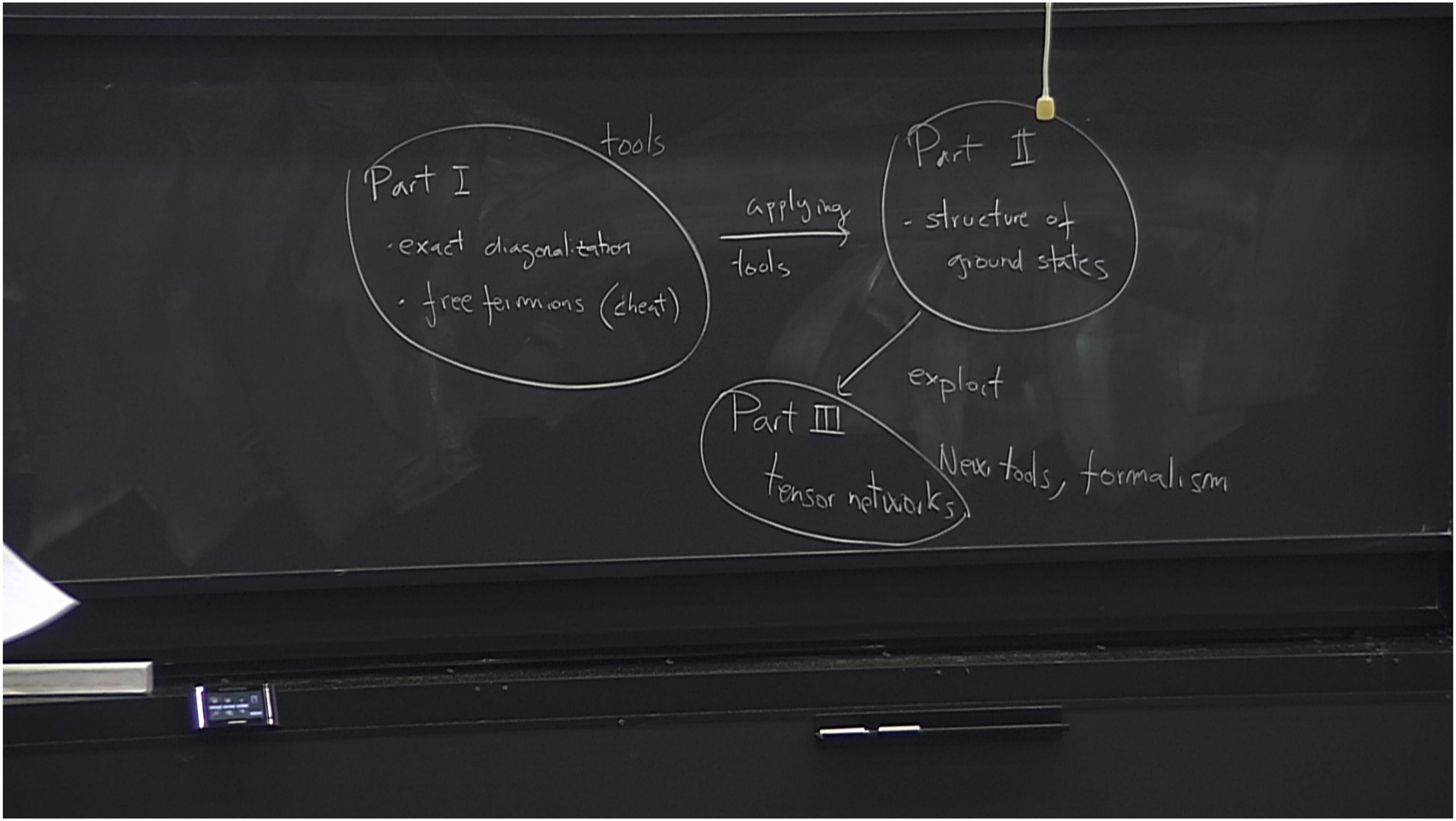
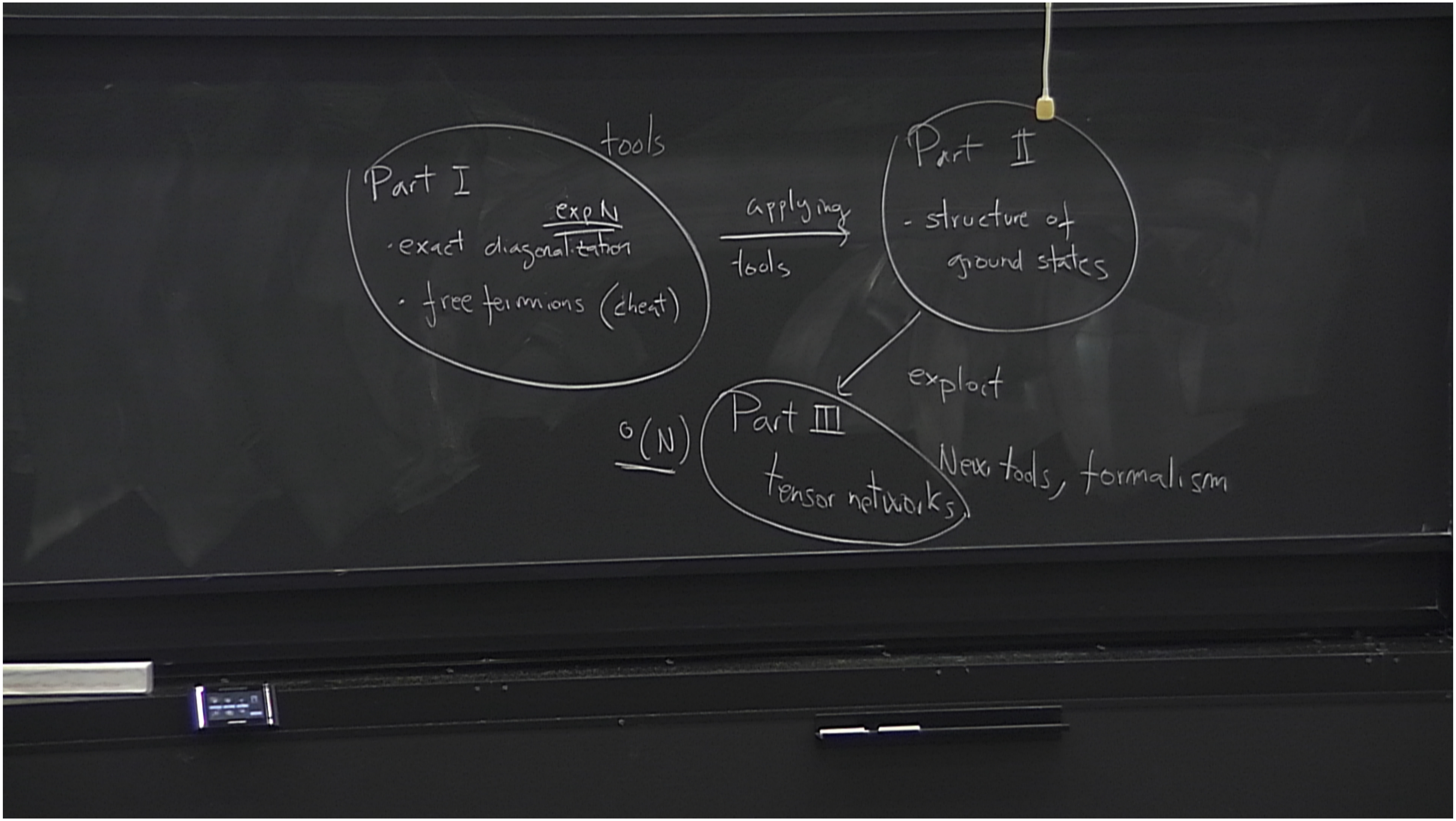

Part I
· exact diagonalization
· free fermions (cheat)
tools
applying
tools
Part II
· structure of ground states
exploit
Part III
tensor networks
New tools, formalism


Part I
tools
exp N
· exact diagonalization
· free fermions (cheat)
applying
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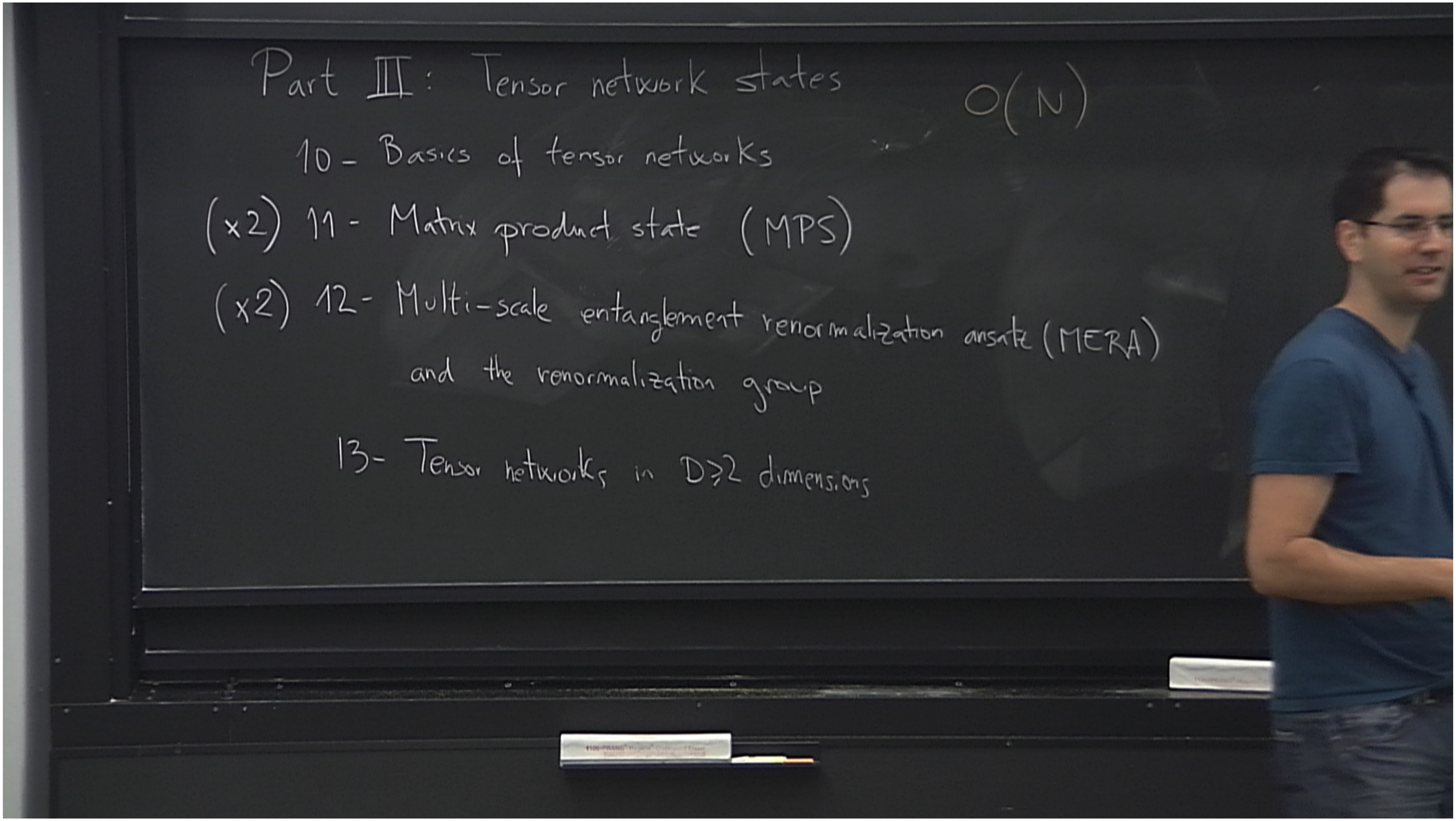
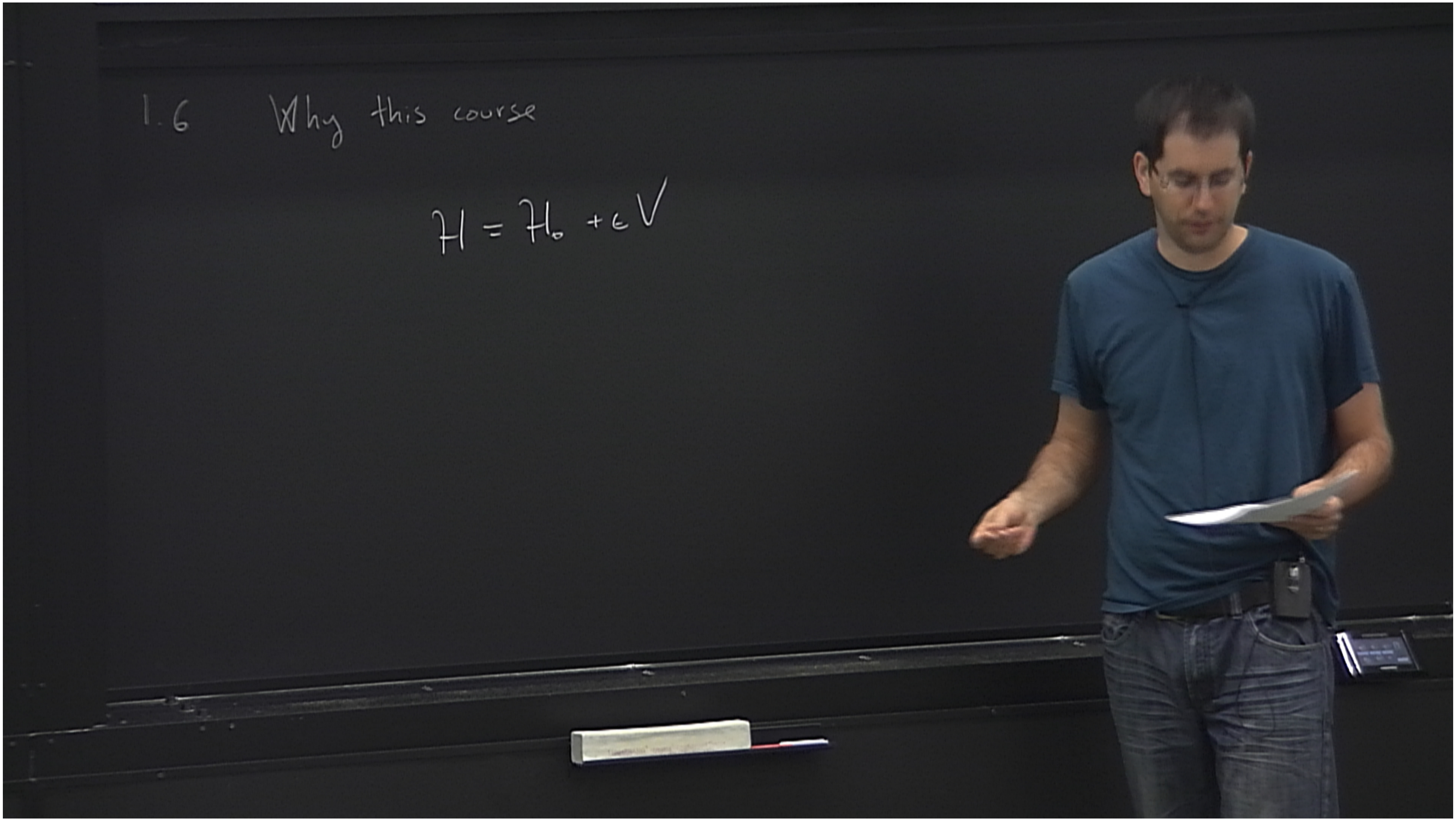
Part II
· structure of ground states
exploit
O(N)
Part III
tensor networks
New tools, formalism


Part III: Tensor network states
O(N)
10 - Basics of tensor networks
(x2) 11 - Matrix product state (MPS)
(x2) 12 - Multi-scale entanglement renormalization ansatz (MERA)
and the renormalization group
13 - Tensor networks in D≥2 dimensions


1.6 Why this course
$\mathcal{H} = \mathcal{H}_0 + \epsilon V$

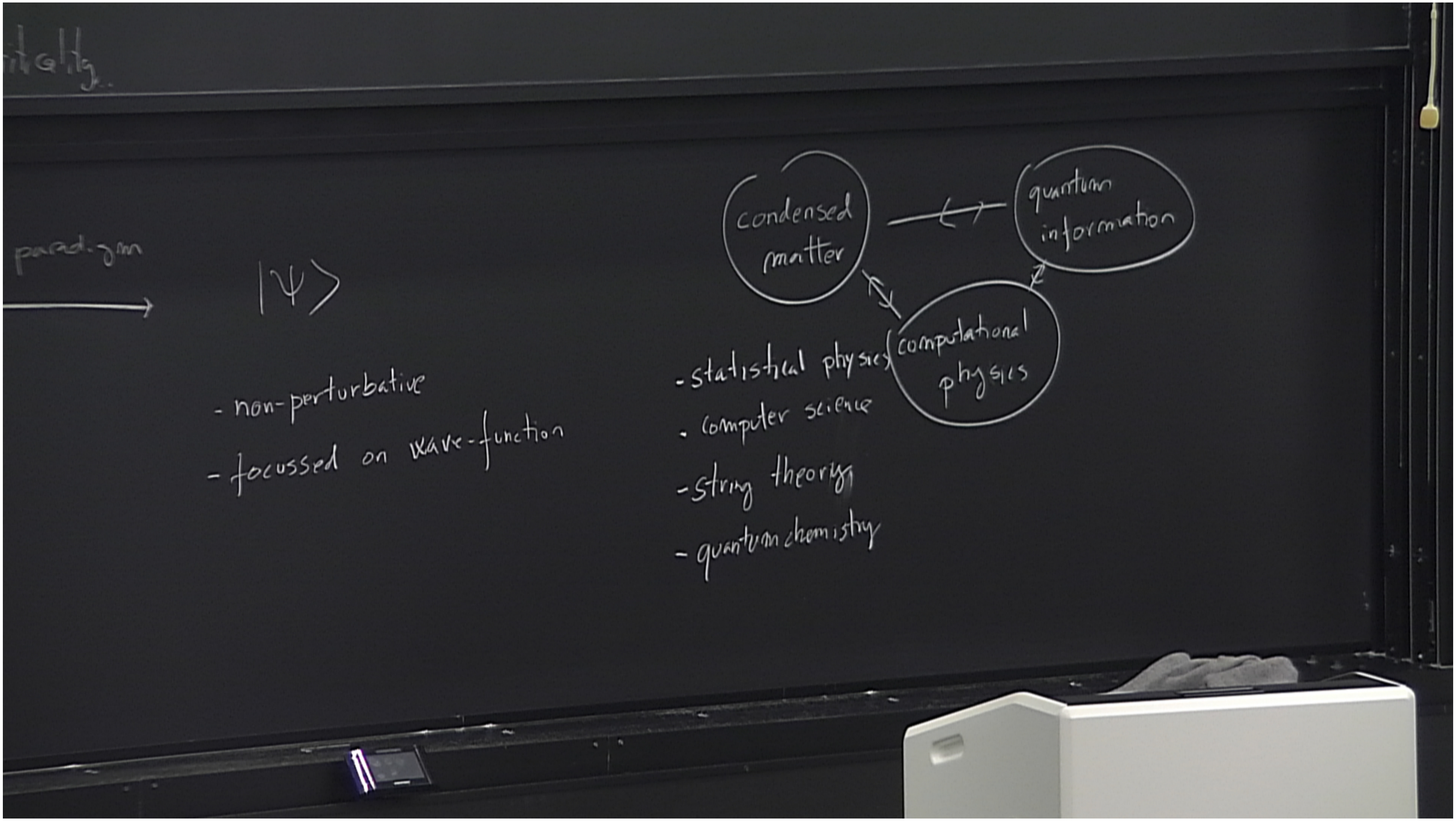
1.6 Why this course

$$\mathcal{H} = \mathcal{H}_0 + \epsilon V$$

change of paradigm $\longrightarrow$ $|\Psi\rangle$

QFT

- perturbative
- focussed on operators

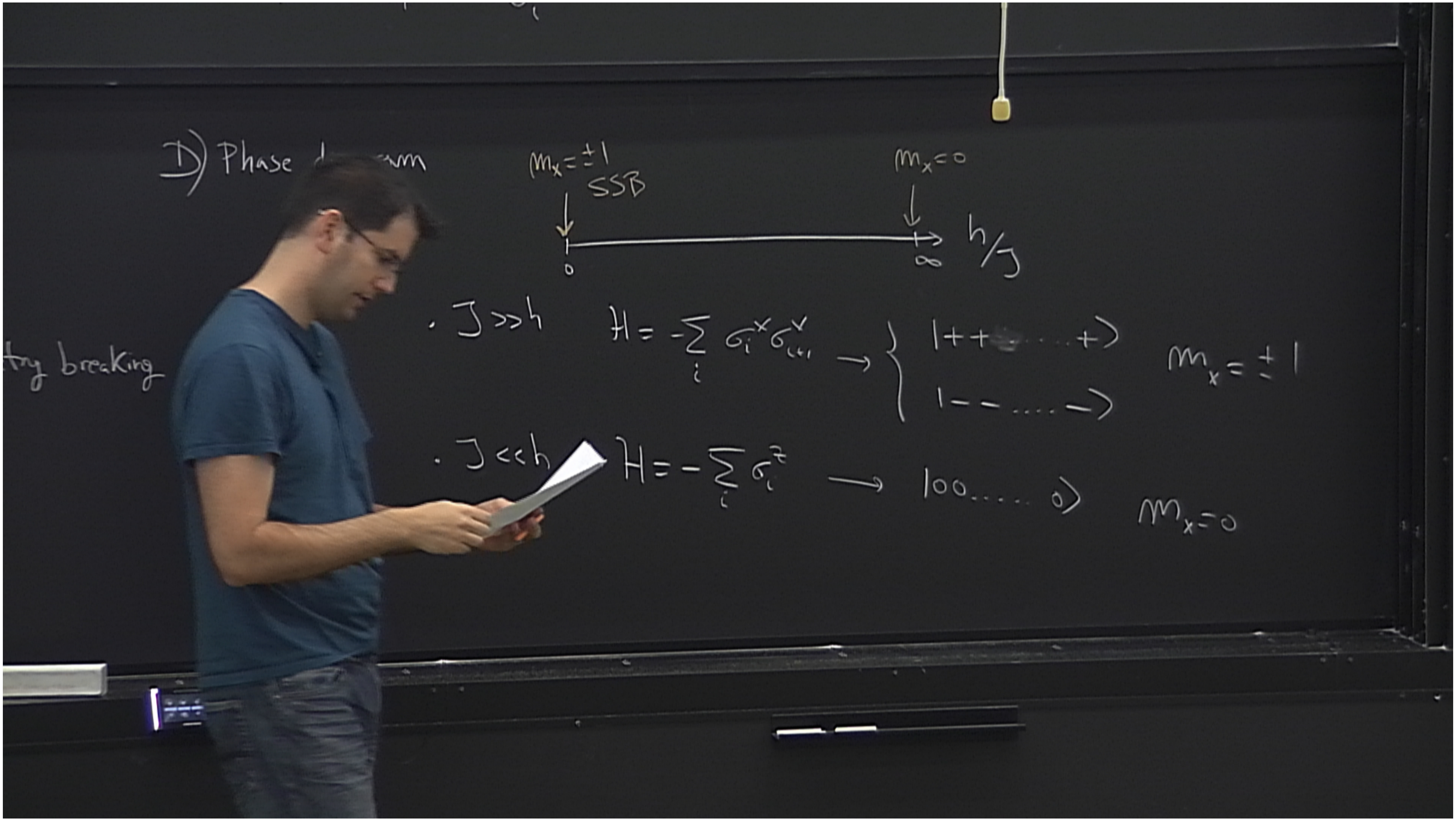

paradigm
|ψ>
- non-perturbative
- focussed on wave-function
condensed matter
quantum information
computational physics
- statistical physics
- computer science
- string theory
- quantum chemistry

C) order parameter

$$m_x \equiv \frac{1}{N}\sum_i \langle \sigma_i^x \rangle = \begin{cases} 0 & \text{symmetric phase} \\ \neq 0 & \text{spontaneous symmetry breaking} \end{cases}$$

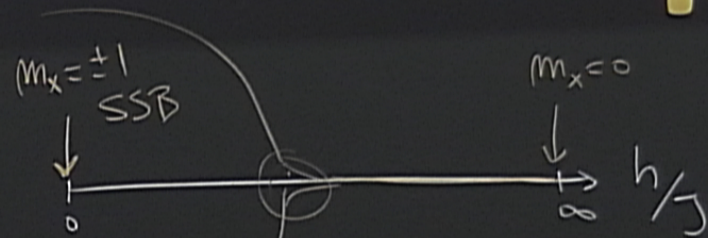
ground state

D) Phase diagram


D) Phase
$m_x = \pm 1$
SSB
$m_x = 0$
0
∞
h/J
breaking
$\cdot\ J \gg h$
$H = -\sum_i \sigma_i^x \sigma_{i+1}^x \rightarrow |++\ldots+\rangle,\ |--\ldots-\rangle$
$m_x = \pm 1$
$\cdot\ J \ll h$
$H = -\sum_i \sigma_i^z \rightarrow |00\ldots0\rangle$
$m_x = 0$

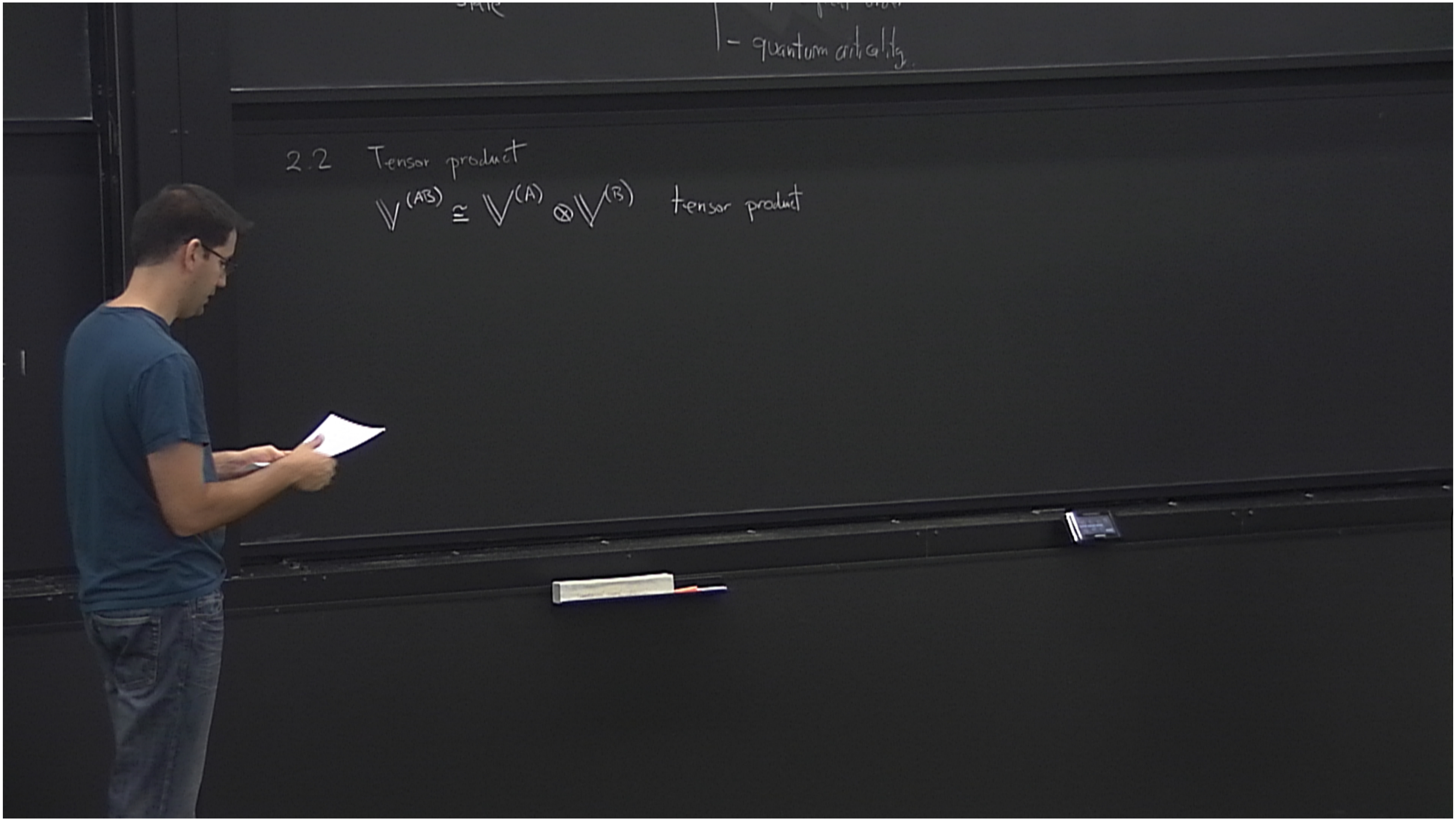
…try breaking

D) Phase diagram

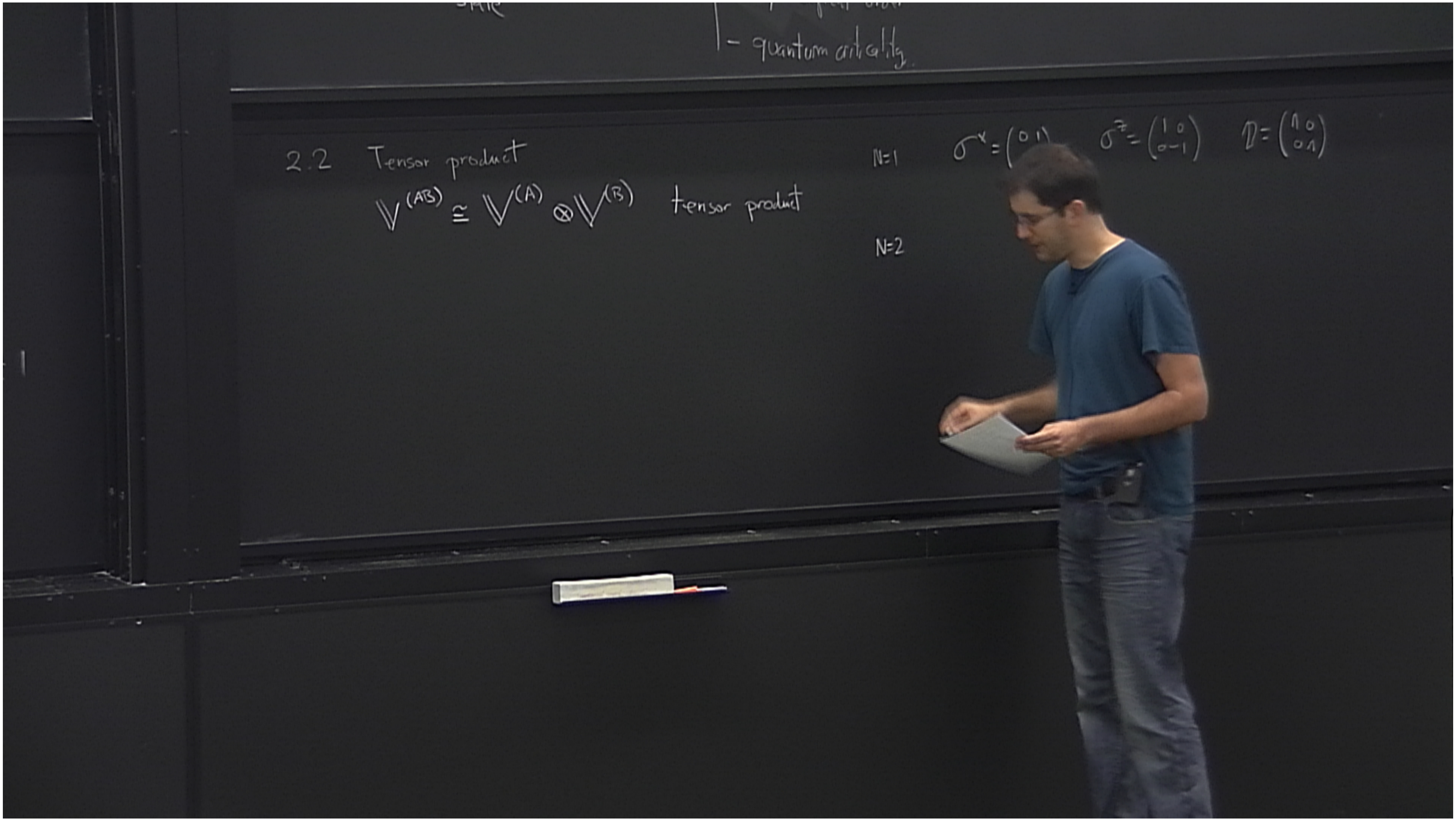


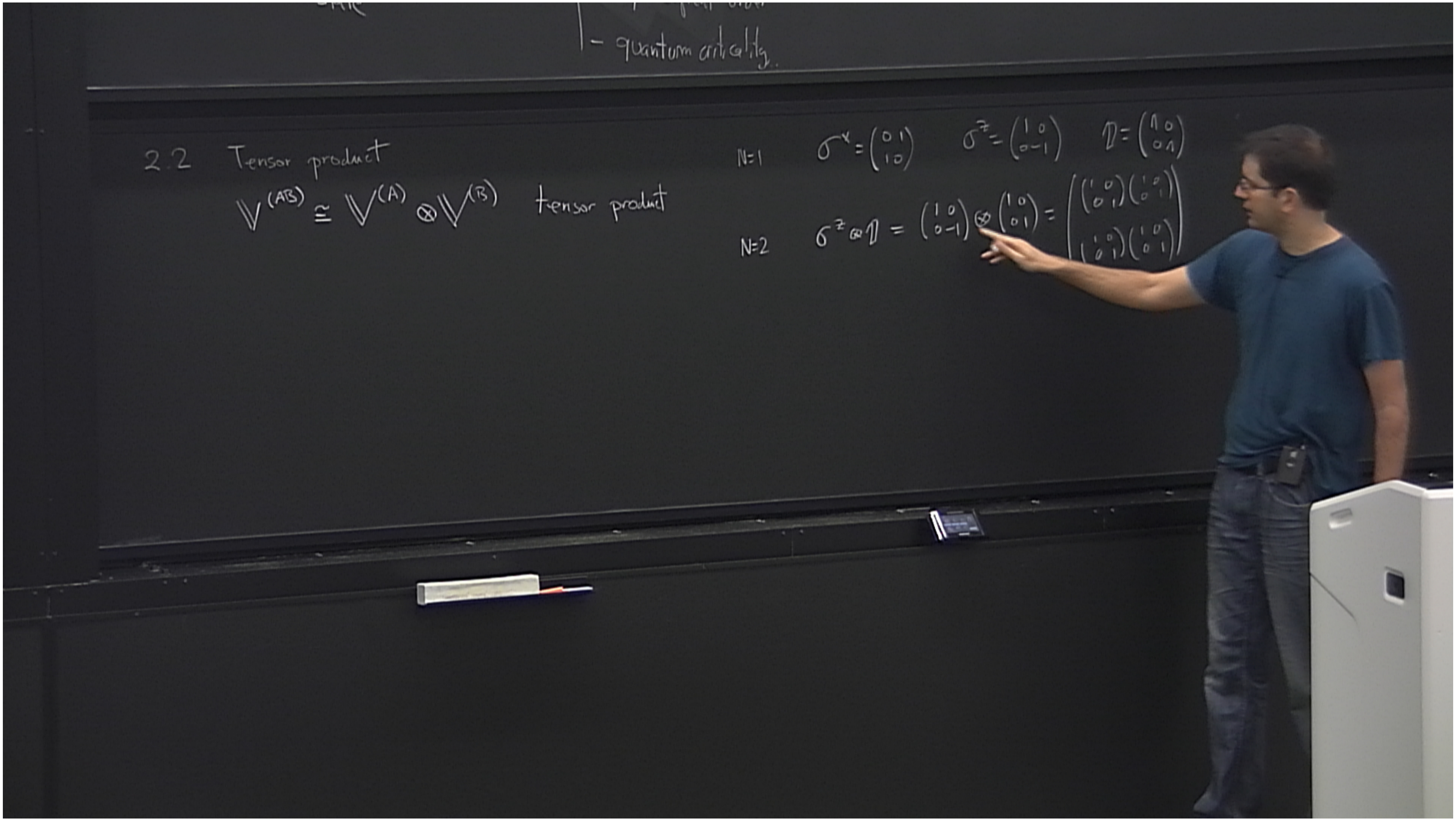
• $J \gg h$ $\quad H = -\sum_i \sigma_i^x \sigma_{i+1}^x \rightarrow \begin{cases} |++\ldots+\rangle \\ |--\ldots-\rangle \end{cases}$ $\quad m_x = \pm 1$

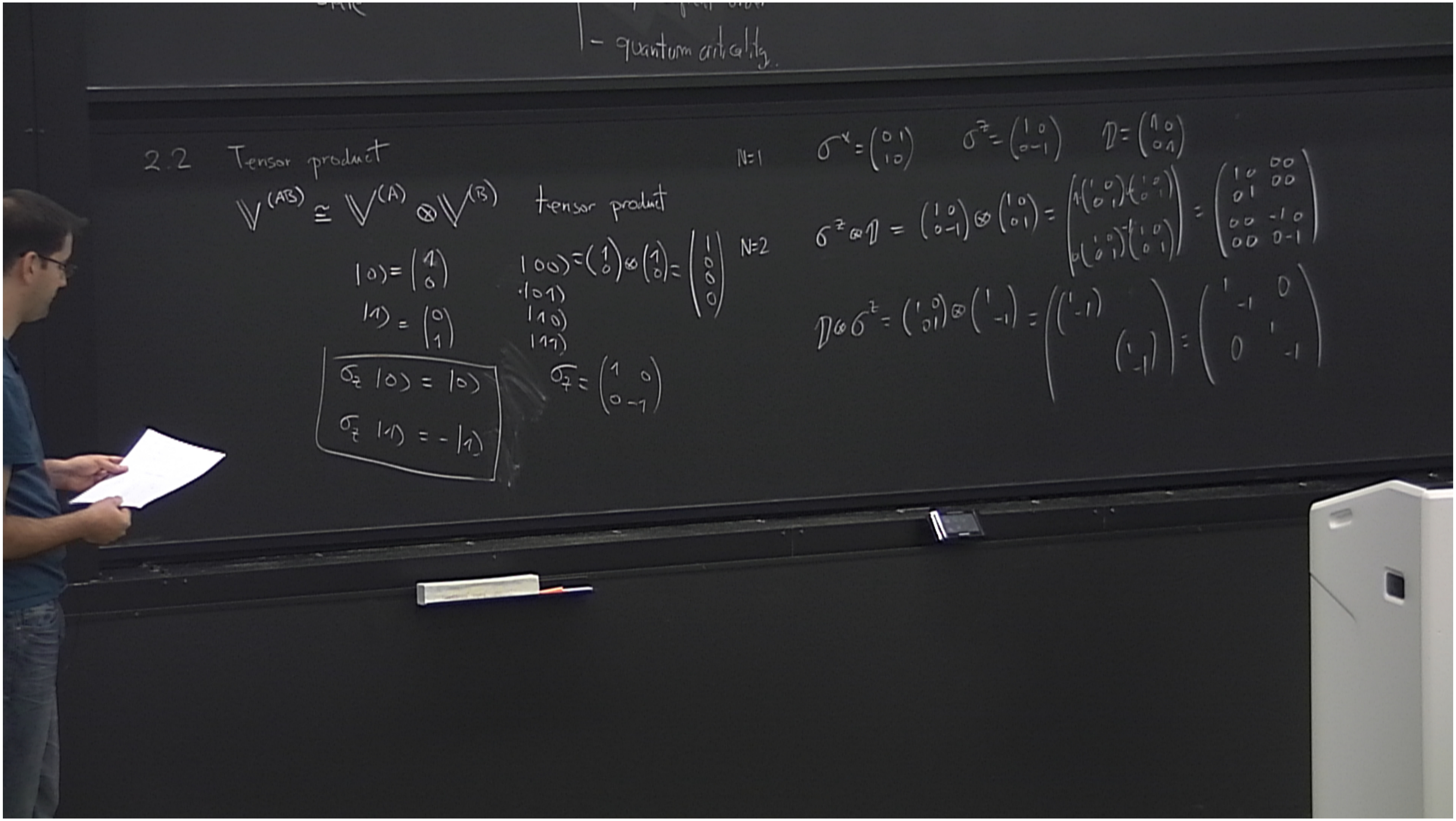
• $J \ll h$ $\quad H = -\sum_i \sigma_i^z \rightarrow |00\ldots0\rangle$ $\quad m_x = 0$

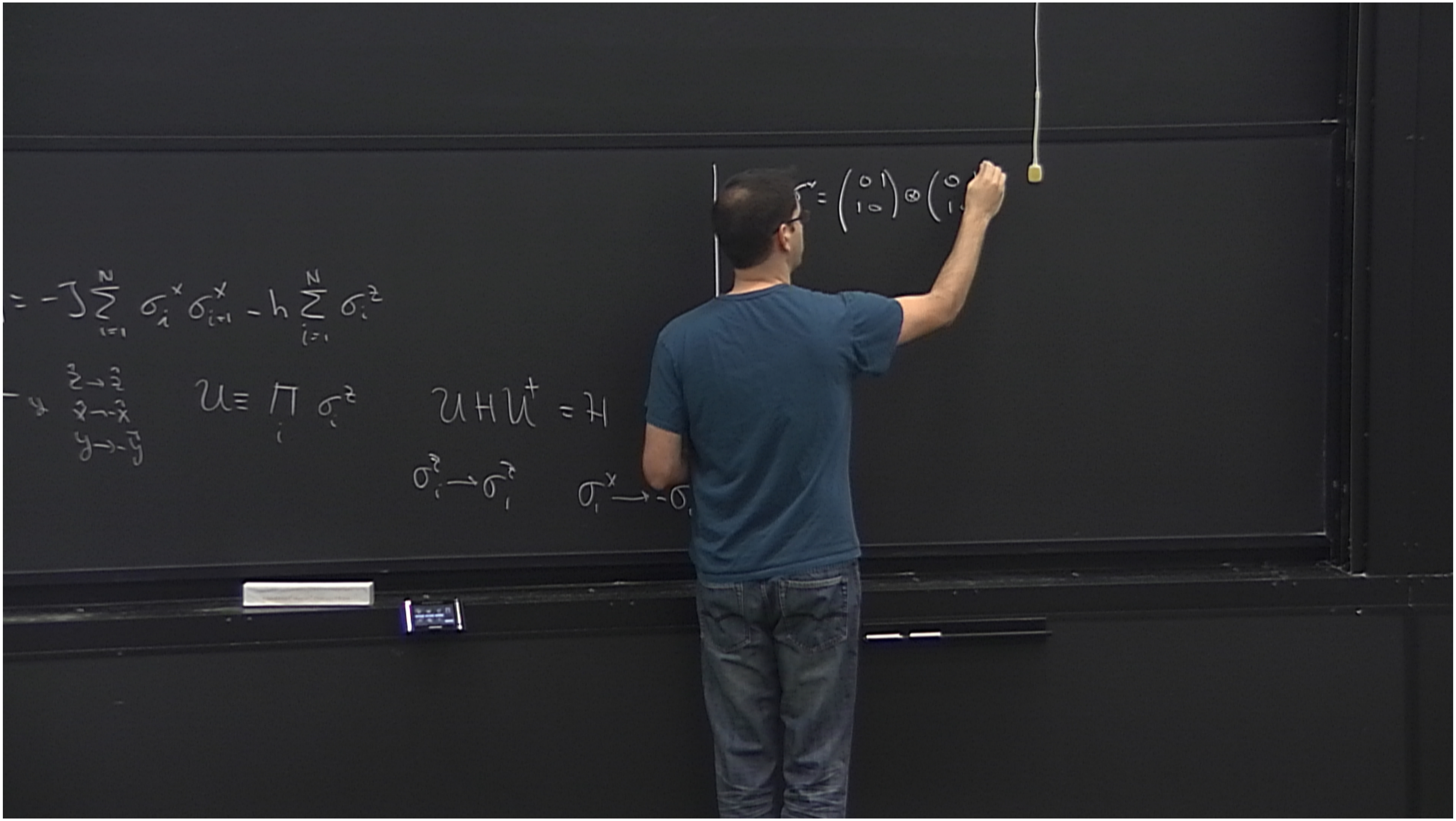


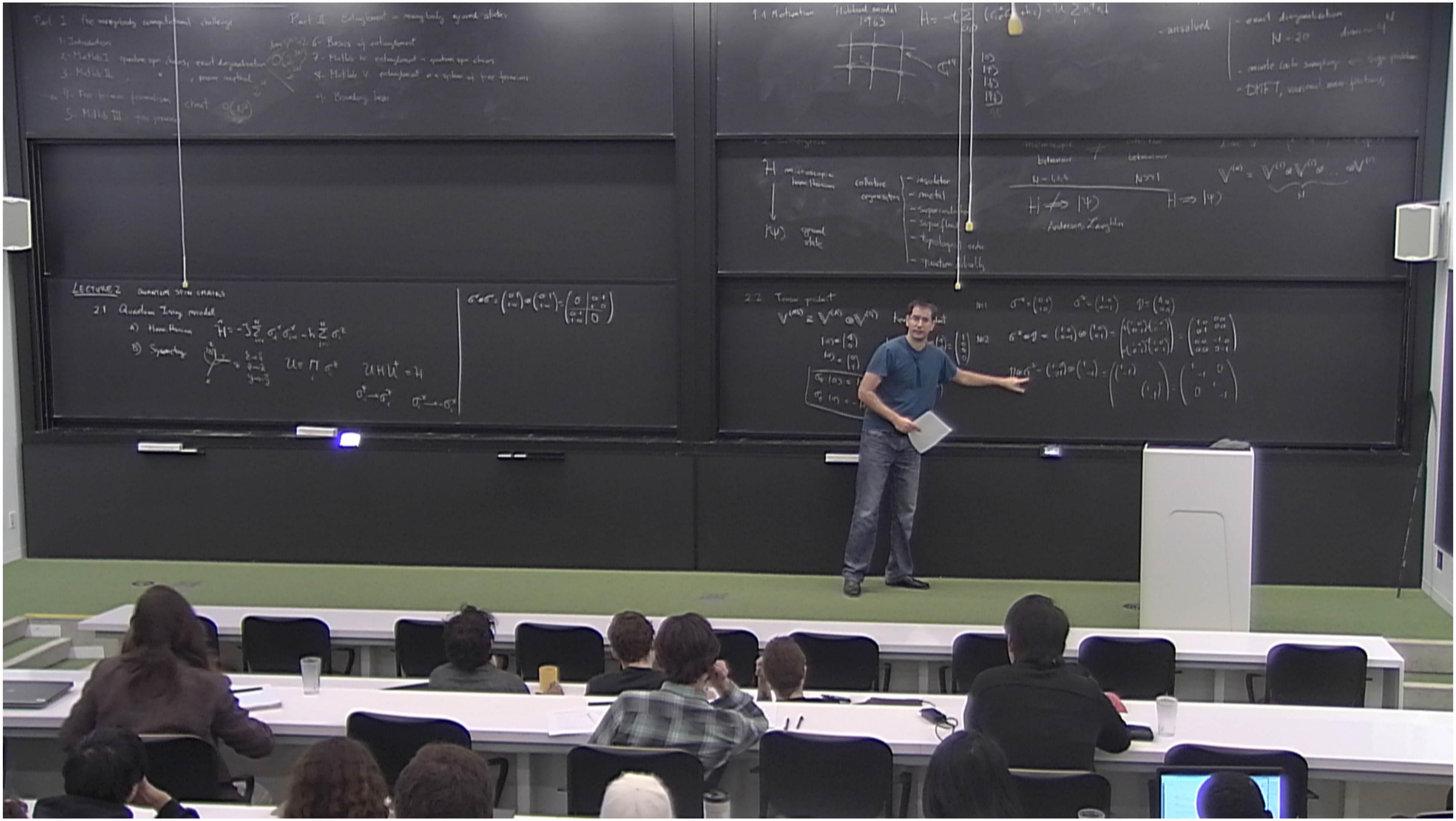
- quantum criticality
2.2 Tensor product
$\mathbb{V}^{(AB)} \cong \mathbb{V}^{(A)} \otimes \mathbb{V}^{(B)}$ tensor product

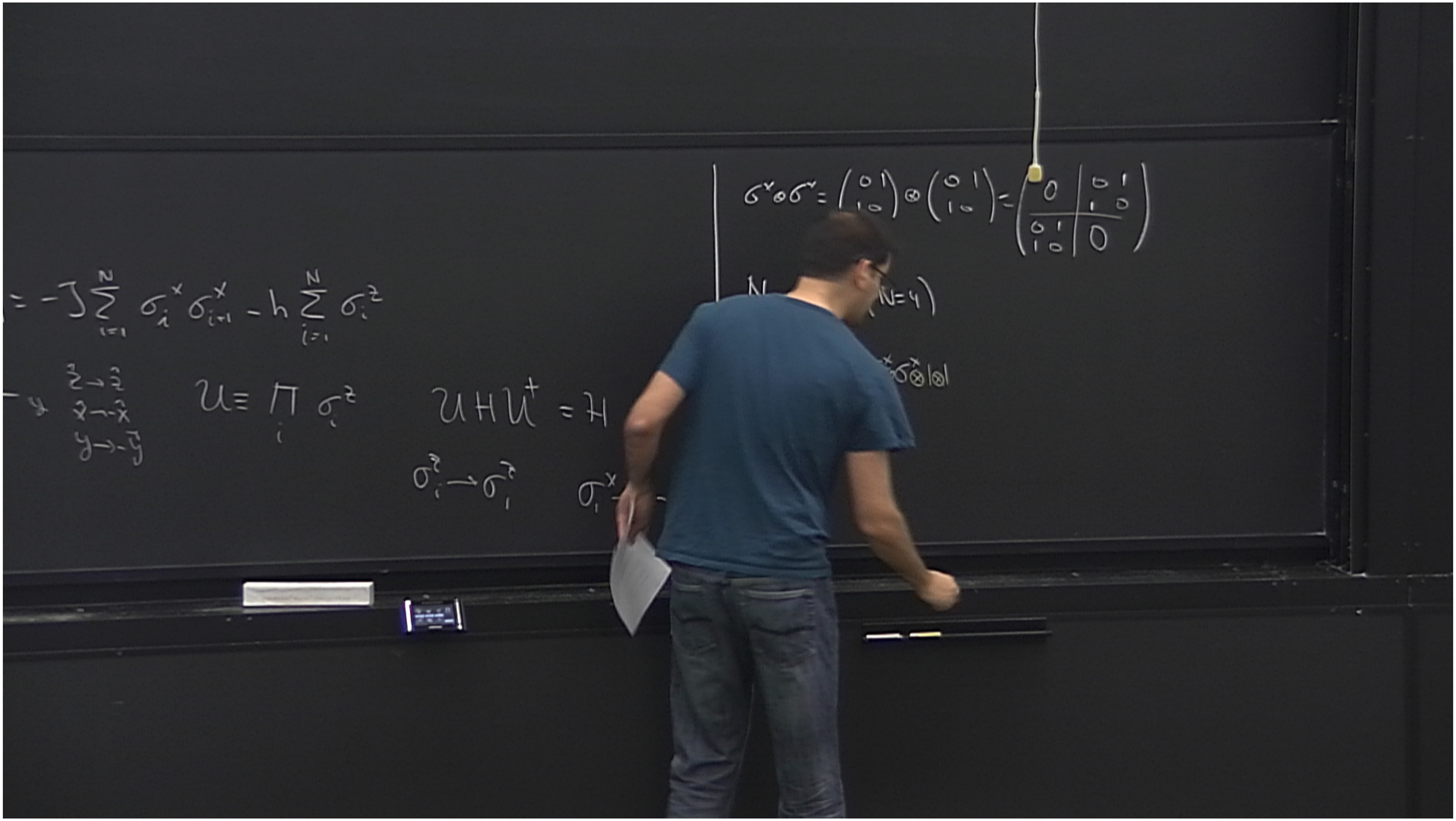

- quantum criticality
2.2 Tensor product
$\mathbb{V}^{(AB)} \cong \mathbb{V}^{(A)} \otimes \mathbb{V}^{(B)}$ tensor product
N=1
$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
N=2

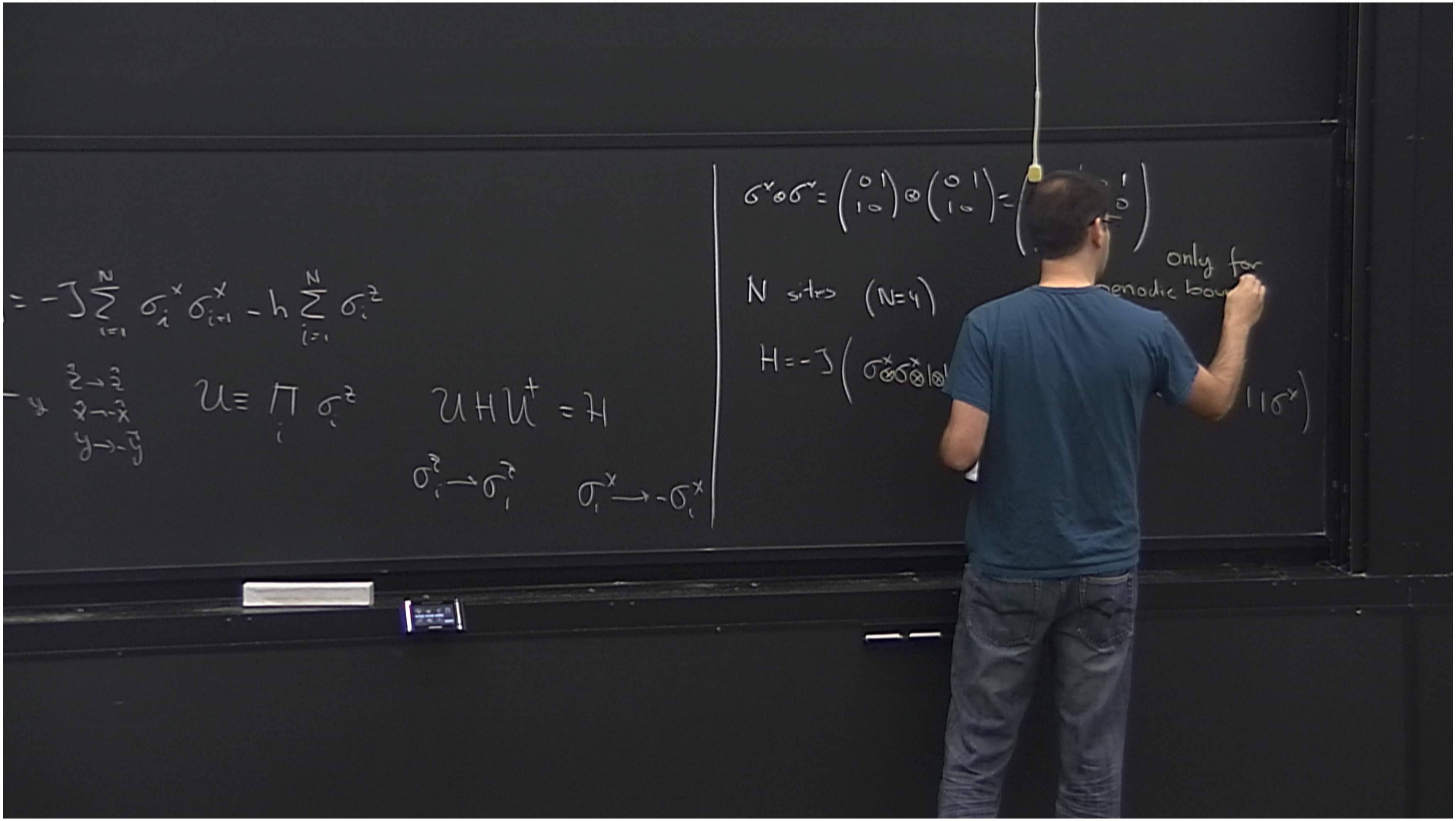

- quantum criticality
2.2 Tensor product
V^(AB) ≅ V^(A) ⊗ V^(B) tensor product
N=1
σ^x = (0 1 / 1 0)
σ^z = (1 0 / 0 −1)
1 = (1 0 / 0 1)
N=2
σ^z ⊗ 1 = (1 0 / 0 −1) ⊗ (1 0 / 0 1) =


- quantum criticality
2.2 Tensor product
$\mathbb{V}^{(AB)} \cong \mathbb{V}^{(A)} \otimes \mathbb{V}^{(B)}$ tensor product
$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$
$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$
$|00\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$
$|01\rangle$
$|10\rangle$
$|11\rangle$
$\boxed{\sigma_z |0\rangle = |0\rangle \quad \sigma_z |1\rangle = -|1\rangle}$
$\sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$
N=1
$\sigma^x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ $\sigma^z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ $\mathbb{1} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$
N=2
$\sigma^z \otimes \mathbb{1} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \otimes \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}1\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & 0\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\\0\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & -1\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & -1\end{pmatrix}$
$\mathbb{1} \otimes \sigma^z = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \otimes \begin{pmatrix}1 & \\ & -1\end{pmatrix} = \begin{pmatrix}\begin{pmatrix}1 & \\ & -1\end{pmatrix} & \\ & \begin{pmatrix}1 & \\ & -1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1\end{pmatrix}$

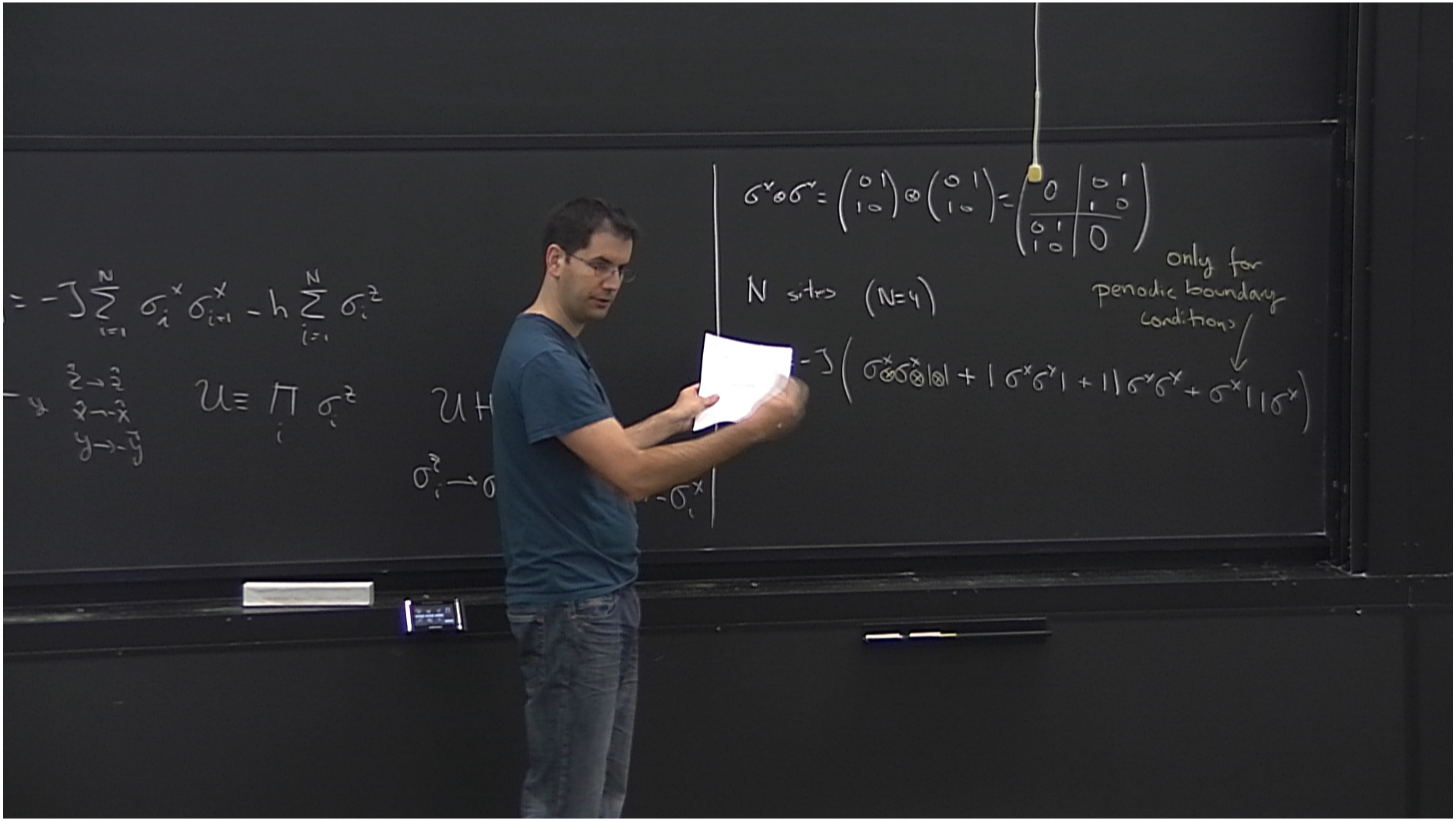

$= -J \sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x - h \sum_{i=1}^{N} \sigma_i^z$
$U \equiv \prod_i \sigma_i^z$
$U H U^+ = H$
$\sigma_i^z \to \sigma_i^z$


LECTURE 2 QUANTUM SPIN CHAINS
2.1 Quantum Ising model
1.1 Motivation
Hubbard model
1963
2.2 Tensor product
- insulator
- metal
Anderson, Laughlin

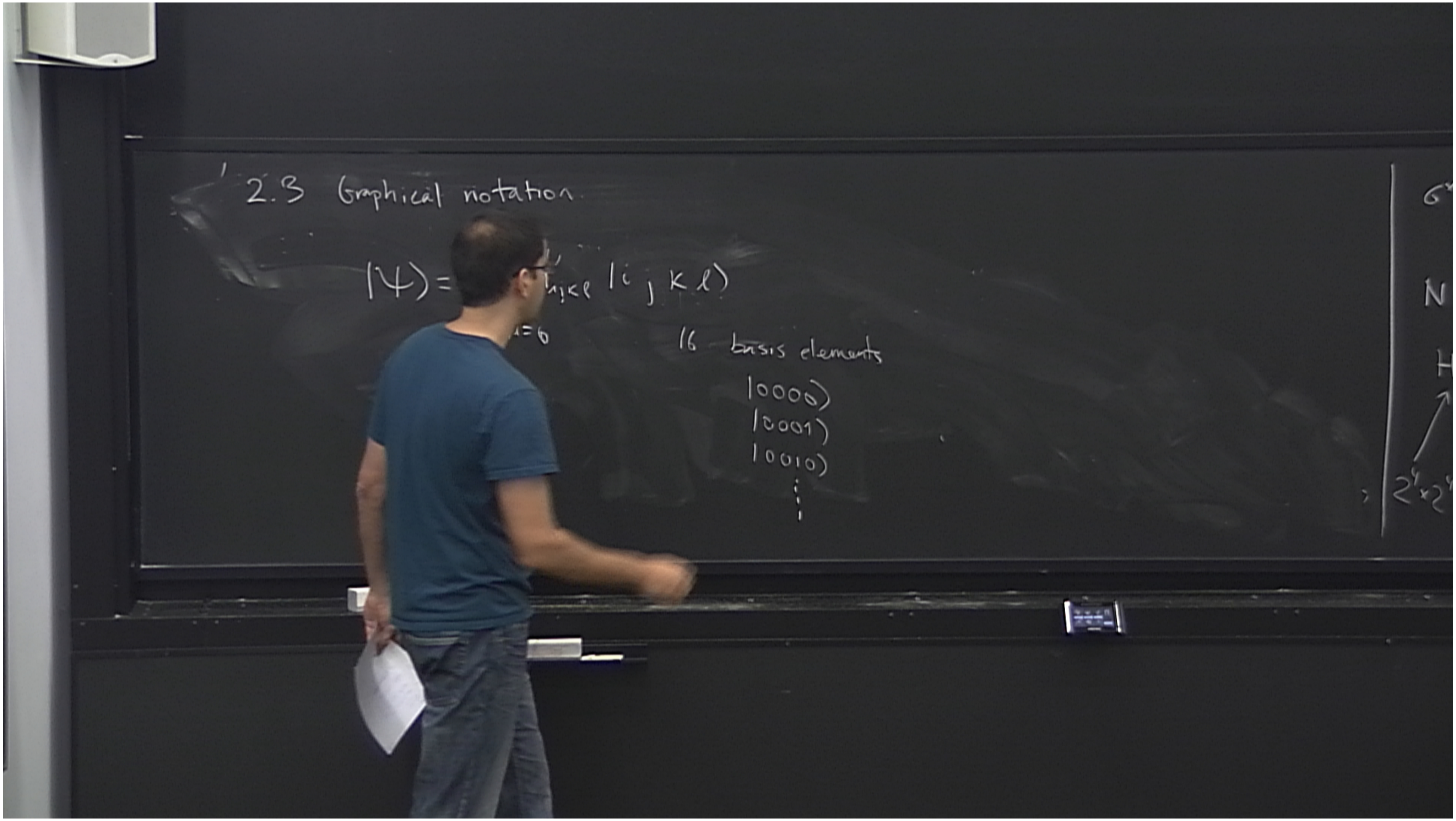

$\sigma^x \otimes \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \\ \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} & 0 \end{pmatrix}$
$= -J \sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x - h \sum_{i=1}^{N} \sigma_i^z$
$\hat{z} \to \hat{z}$
$\hat{x} \to -\hat{x}$
$y \to -\hat{y}$
$\mathcal{U} \equiv \prod_i \sigma_i^z$
$\mathcal{U} H \mathcal{U}^+ = H$
$\sigma_i^z \to \sigma_i^z$

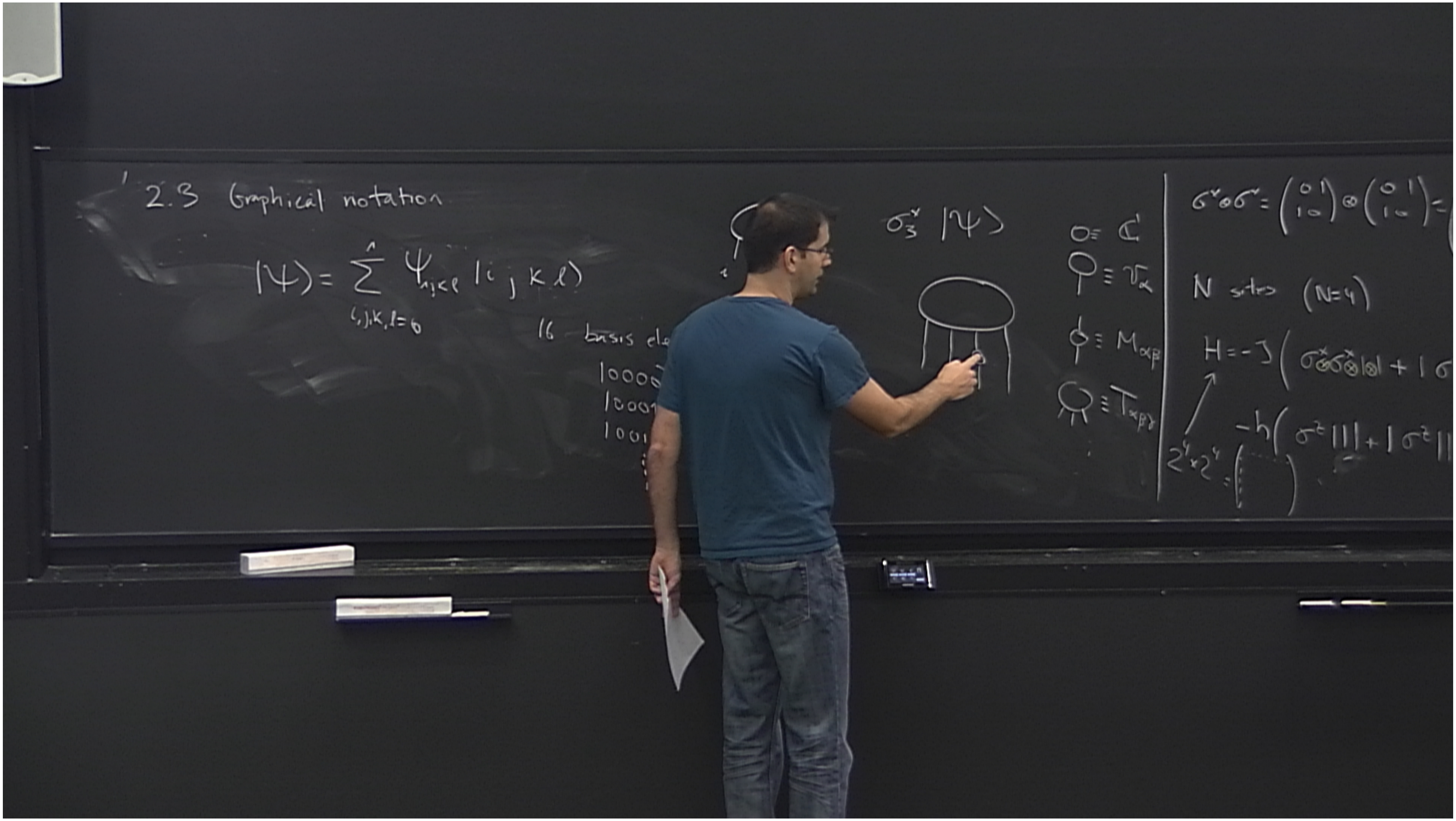


N sites (N=4)
only for
UHU+ = H
U ≡ Π σ



only for
periodic boundary
conditions
N sites (N=4)


2.3 Graphical notation
16 basis elements
|0000⟩
|0001⟩
|0010⟩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