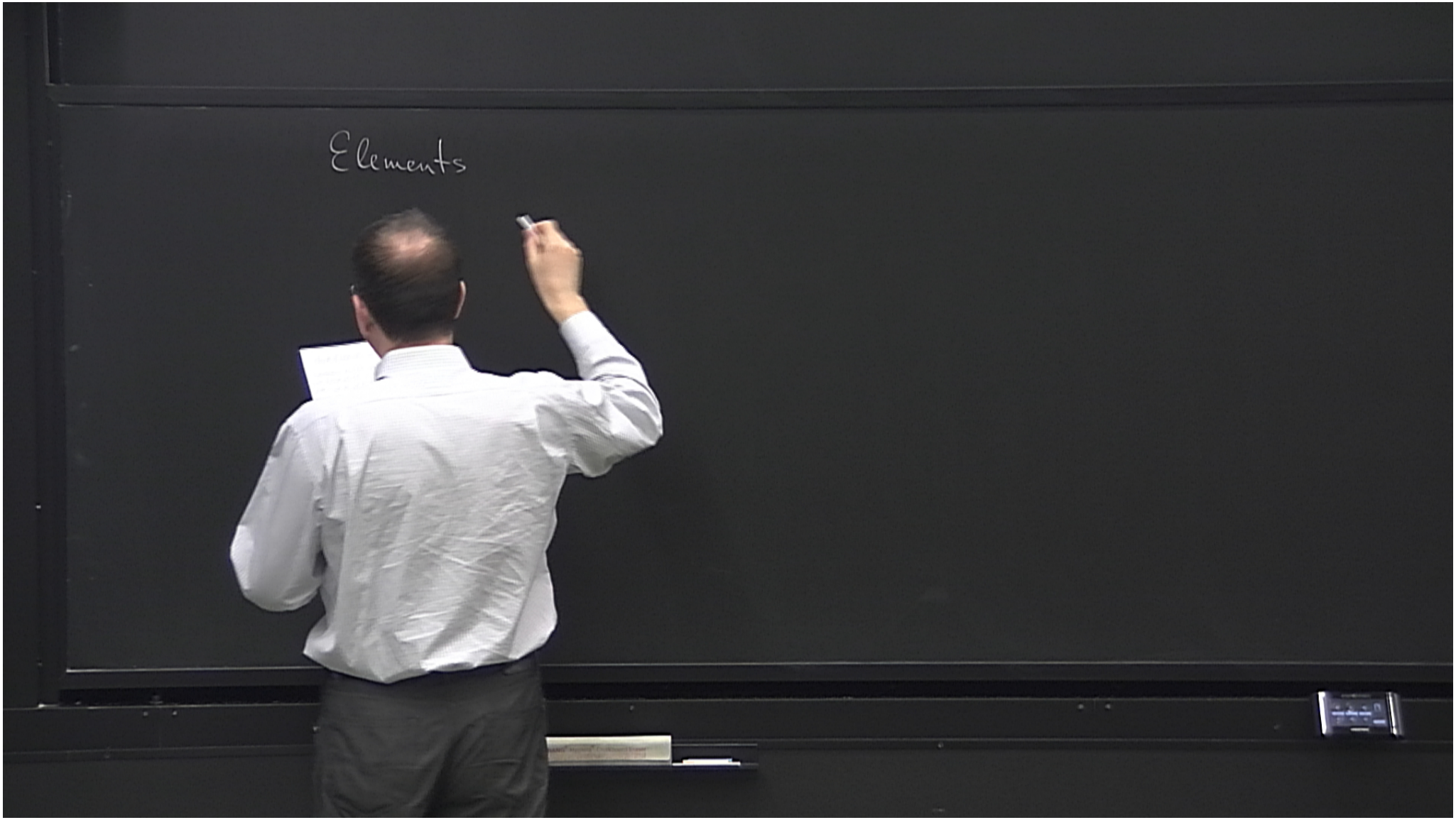
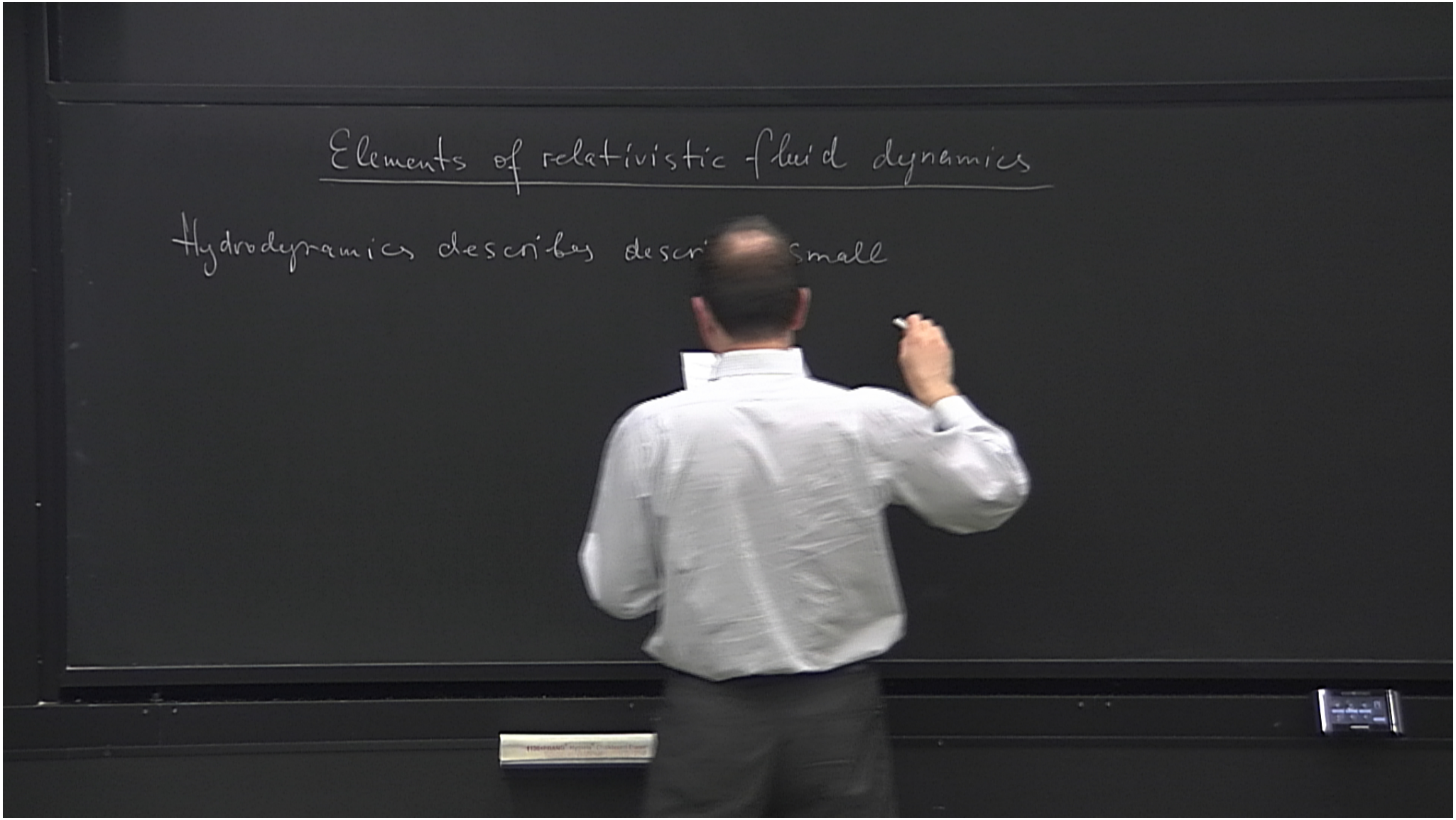
Title: 13/14 PSI - Explorations in String Theory - Lecture 14

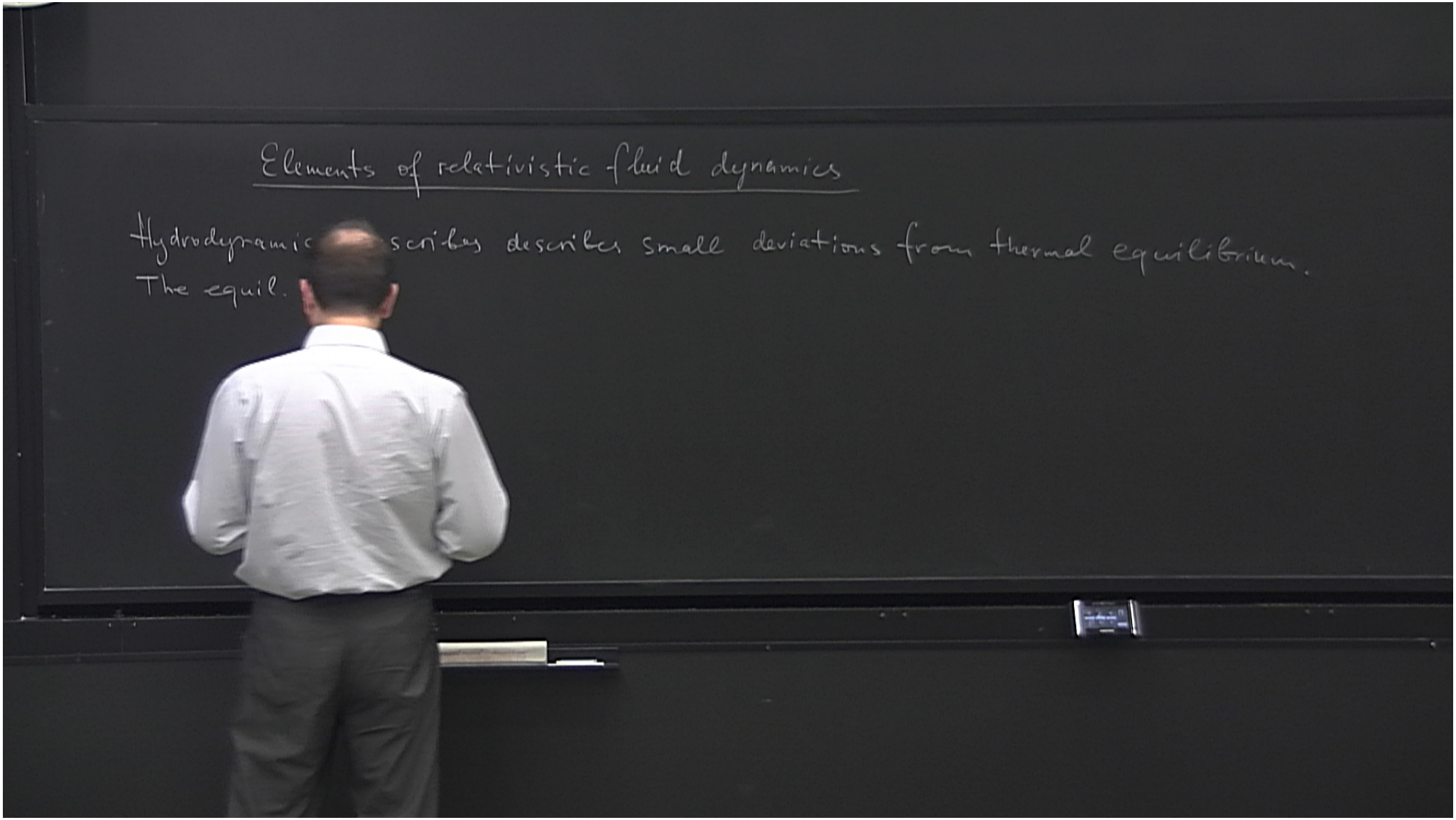
Date: Apr 03, 2014 11:30 AM

URL: http://pirsa.org/14040020

Abstract:


Elements


Elements of relativistic fluid dynamics
Hydrodynamics describes


Elements of relativistic fluid dynamics
Hydrodynamic
describes small deviations from thermal equilibrium.
The equil.

Elements of relativistic fluid dynamics

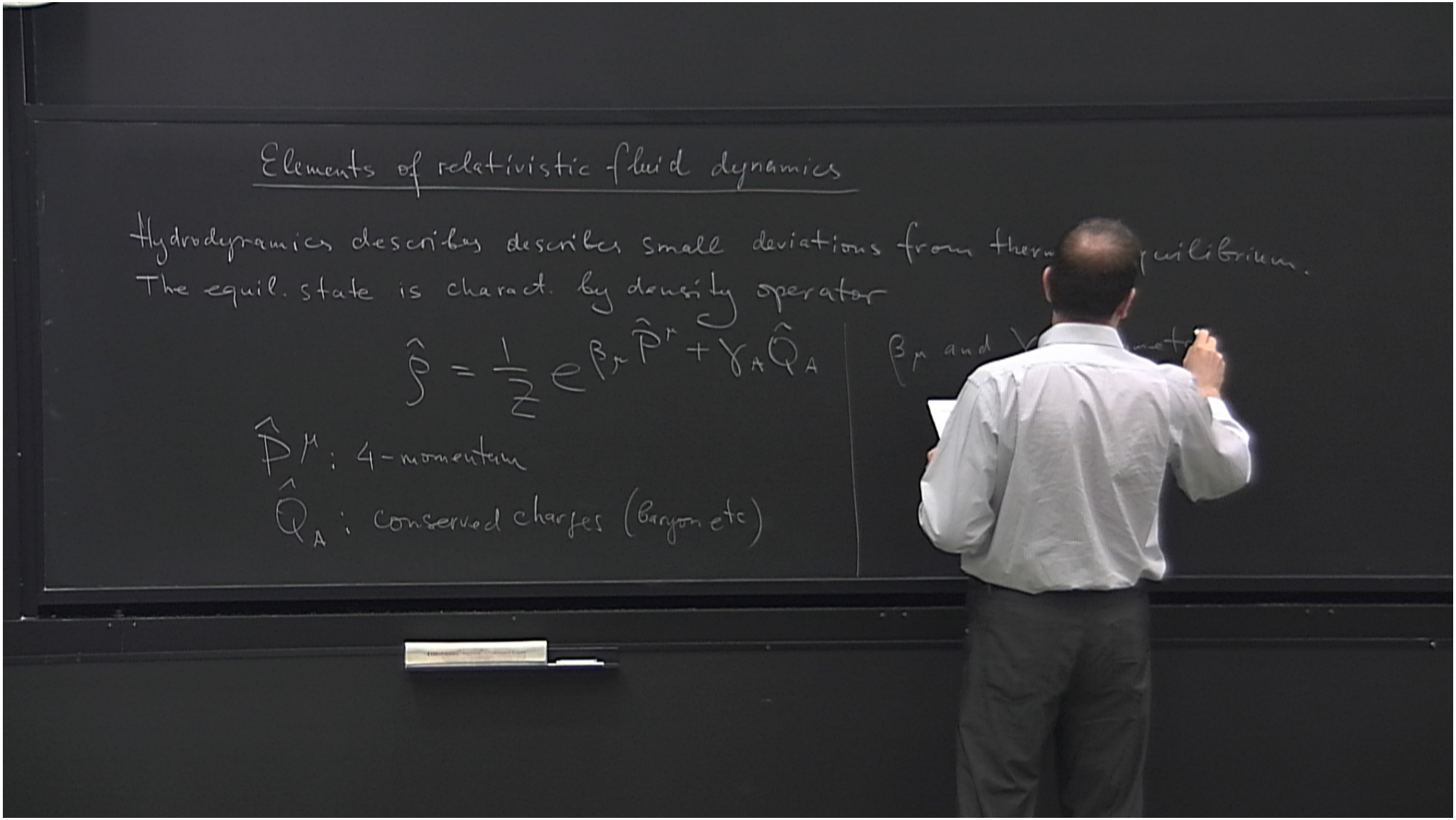
Hydrodynamics describes describes small deviations from thermal equilibrium.
The equil. state is charact. by density operator

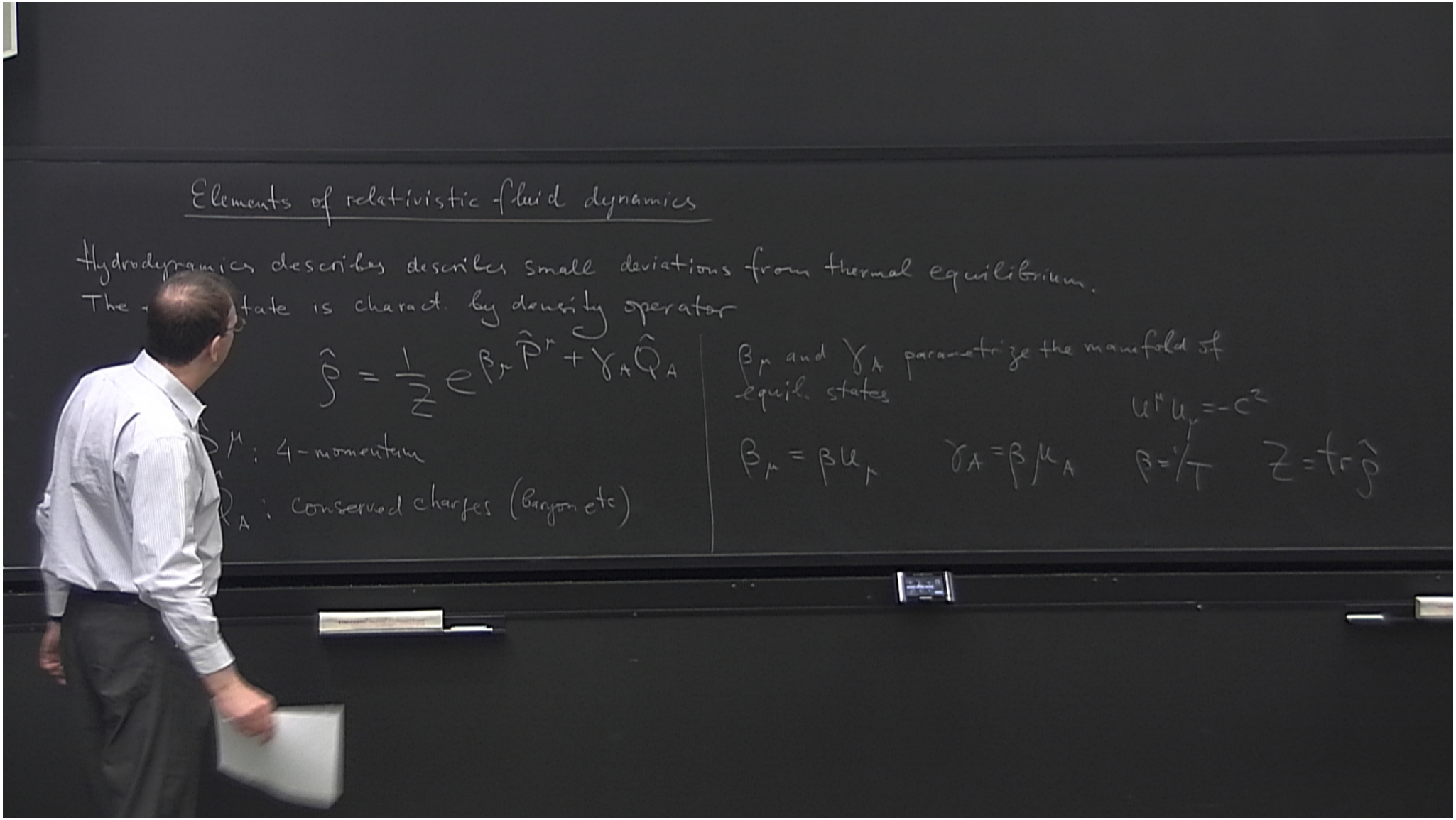
$$\hat{\rho} = \frac{1}{Z} e^{\beta_\mu \hat{P}^\mu + \gamma_A \hat{Q}_A}$$

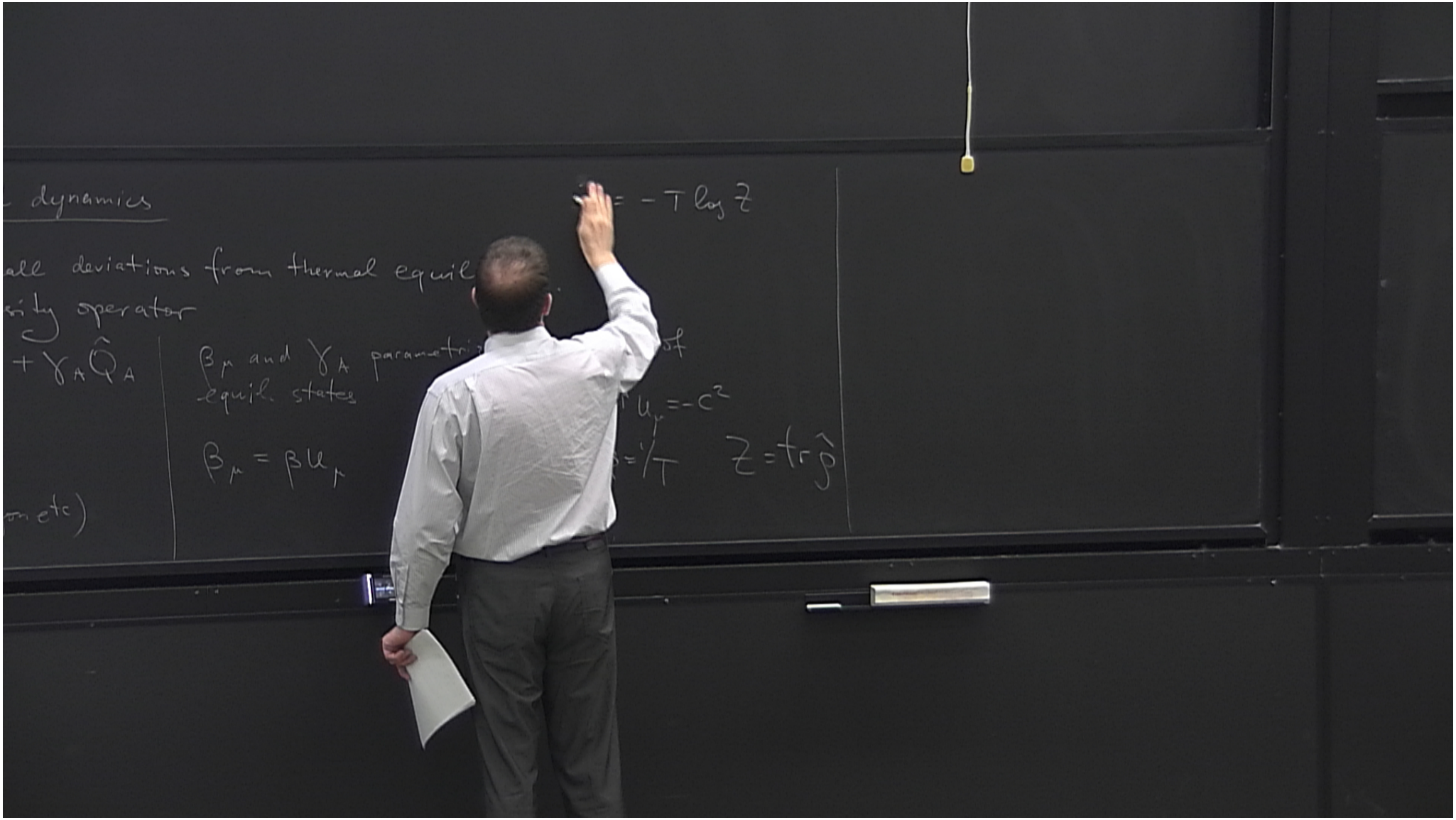
β_μ and γ [illegible]

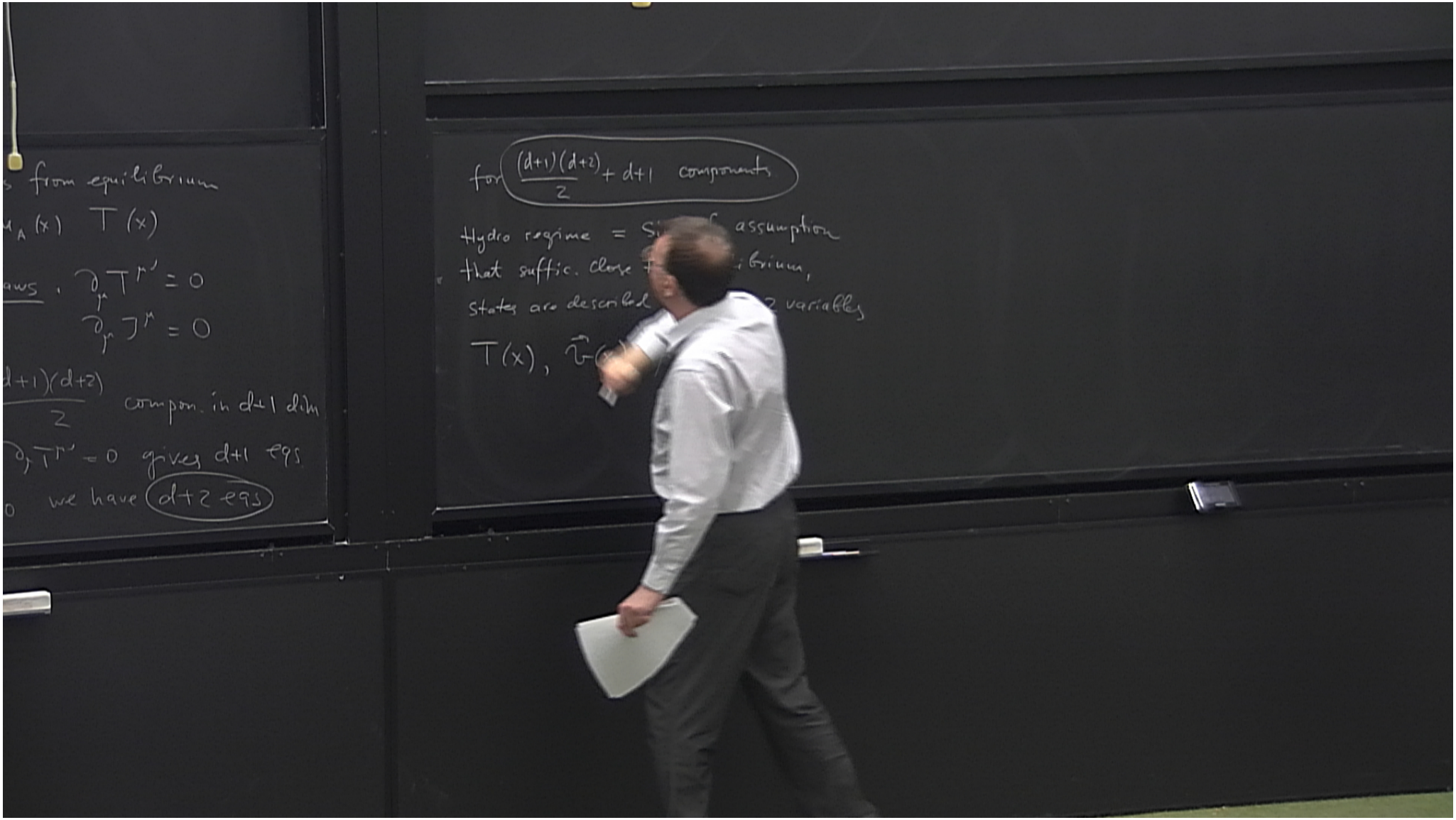
$\hat{P}^\mu$: 4-momentum

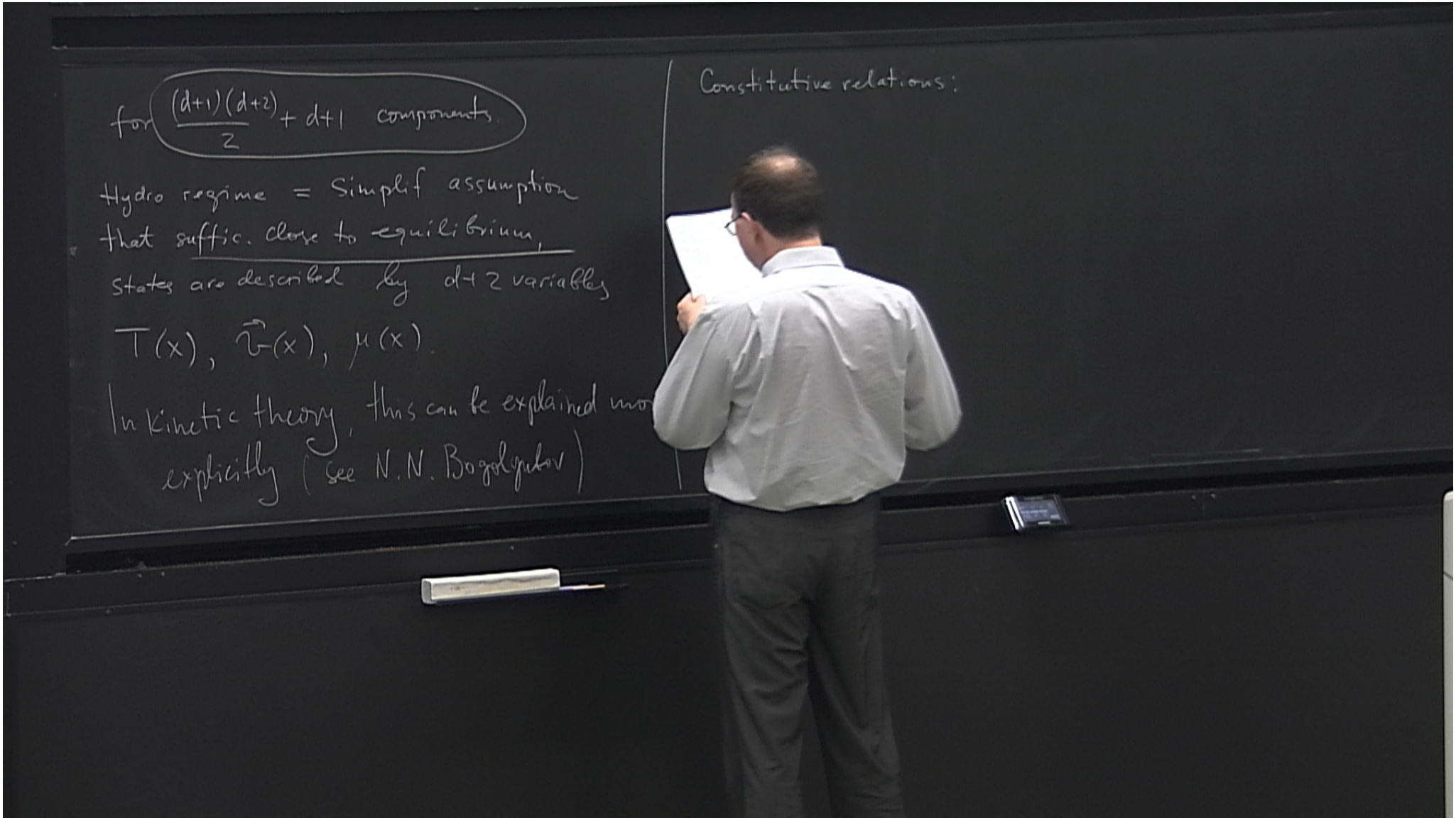
$\hat{Q}_A$: conserved charges (baryon etc)




Elements of relativistic fluid dynamics
Hydrodynamics describes small deviations from thermal equilibrium.
$\hat{\rho} = \frac{1}{Z} e^{\beta_\mu \hat{P}^\mu + \gamma_A \hat{Q}_A}$
β_μ and γ_A parametrize the manifold of equil. states
$u^\mu u_\mu = -c^2$
$\beta_\mu = \beta u_\mu$
$\gamma_A = \beta \mu_A$
$\beta = 1/T$
$Z = \mathrm{tr}\,\hat{\rho}$
$\hat{P}^\mu$: 4-momentum
$\hat{Q}_A$: Conserved charges (Baryon etc)


dynamics
= - T log Z
all deviations from thermal equil
sity operator
+ γ_A Q̂_A
β_μ and γ_A parametri
equil. states
β_μ = β u_μ
u_ν = -c²
= 1/T
Z = tr ρ̂
on etc)


from equilibrium
$T(x)$
laws. $\partial_\mu T^{\mu\nu} = 0$
$\partial_\mu J^\mu = 0$
$\frac{(d+1)(d+2)}{2}$ compon. in $d+1$ dim
$\partial_i T^{ri} = 0$ gives $d+1$ eqs
we have $d+2$ eqs
for $\frac{(d+1)(d+2)}{2} + d+1$ components
Hydro regime = Si... assumption
that suffic. close t... ibrium,
states are described ... variables
$T(x)$,


for $\frac{(d+1)(d+2)}{2} + d+1$ components.
Hydro regime = Simplif assumption
that suffic. close to equilibrium,
states are described by d+2 variables
$T(x)$, $\vec{v}(x)$, $\mu(x)$
In kinetic theory, this can be explained mo
explicitly (see N.N. Bogolyubov)
Constitutive relations:

for $\frac{(d+1)(d+2)}{2} + d+1$ components.

Hydro regime = Simplif assumption that suffic. close to equilibrium, states are described by $d+2$ variables

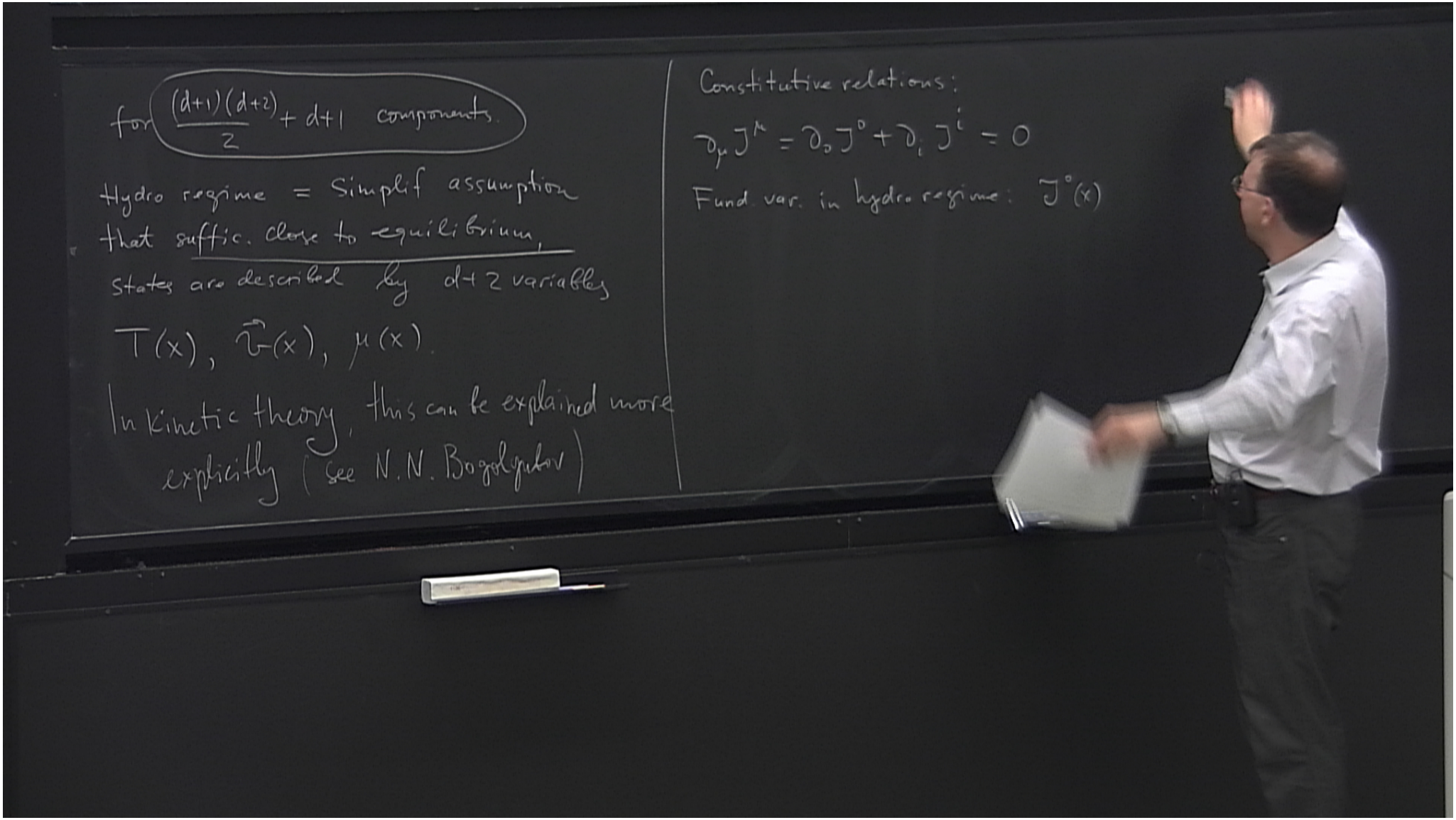
$T(x)$, $\vec{v}(x)$, $\mu(x)$

In kinetic theory, this can be explained more explicitly (see N.N. Bogolyubov)

Constitutive relations:

$\partial_\mu J^\mu = \partial_0 J^0 + \partial_i J^i = 0$

Fund. var. in hydro regime: $J^0(x)$



for $\frac{(d+1)(d+2)}{2} + d+1$ components.

Hydro regime = Simplif assumption that suffic. close to equilibrium, states are described by $d+2$ variables

$T(x)$, $\vec{v}(x)$, $\mu(x)$

In kinetic theory, this can be explained more explicitly (see N.N. Bogolyubov)

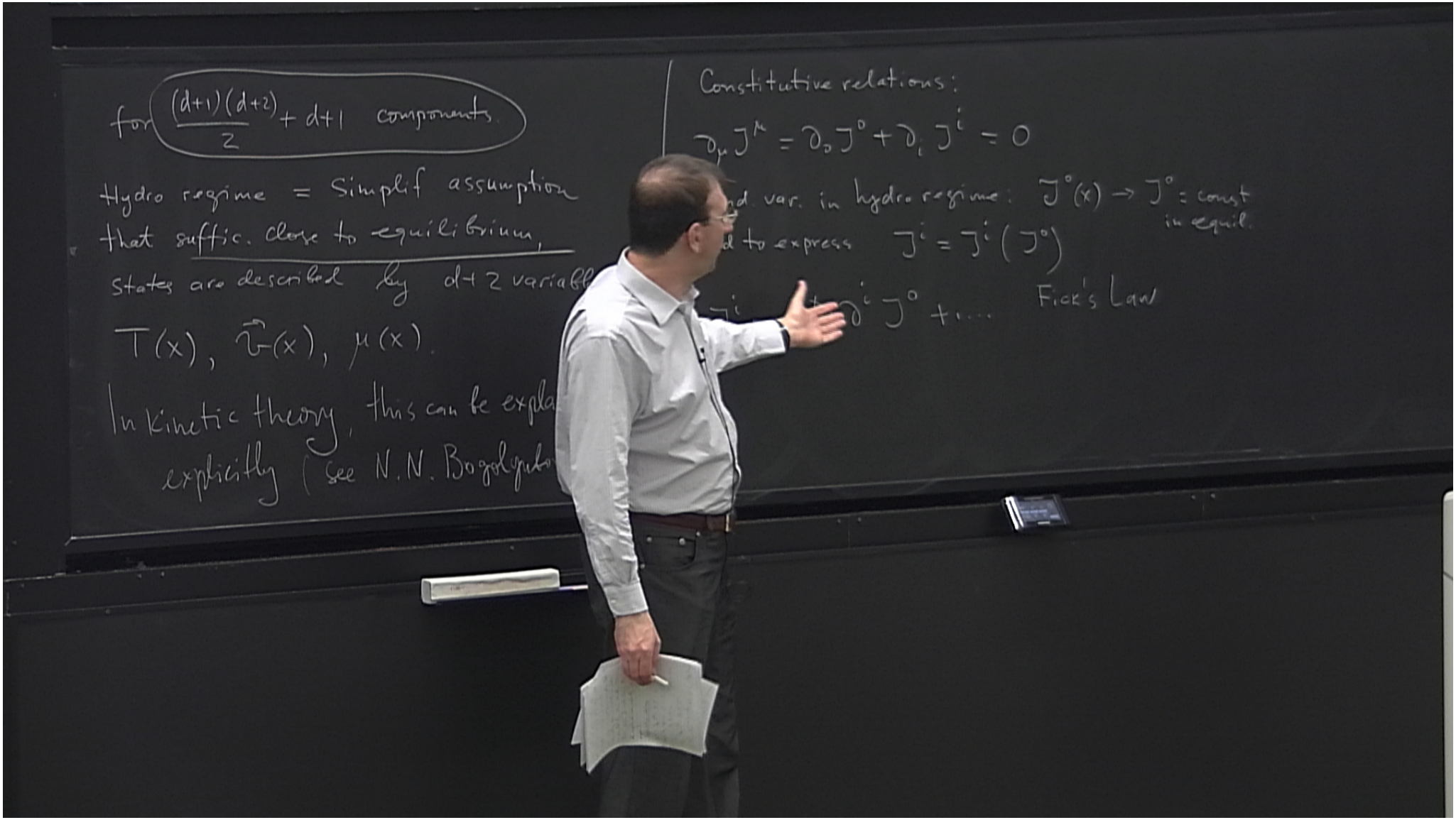
Constitutive relations:

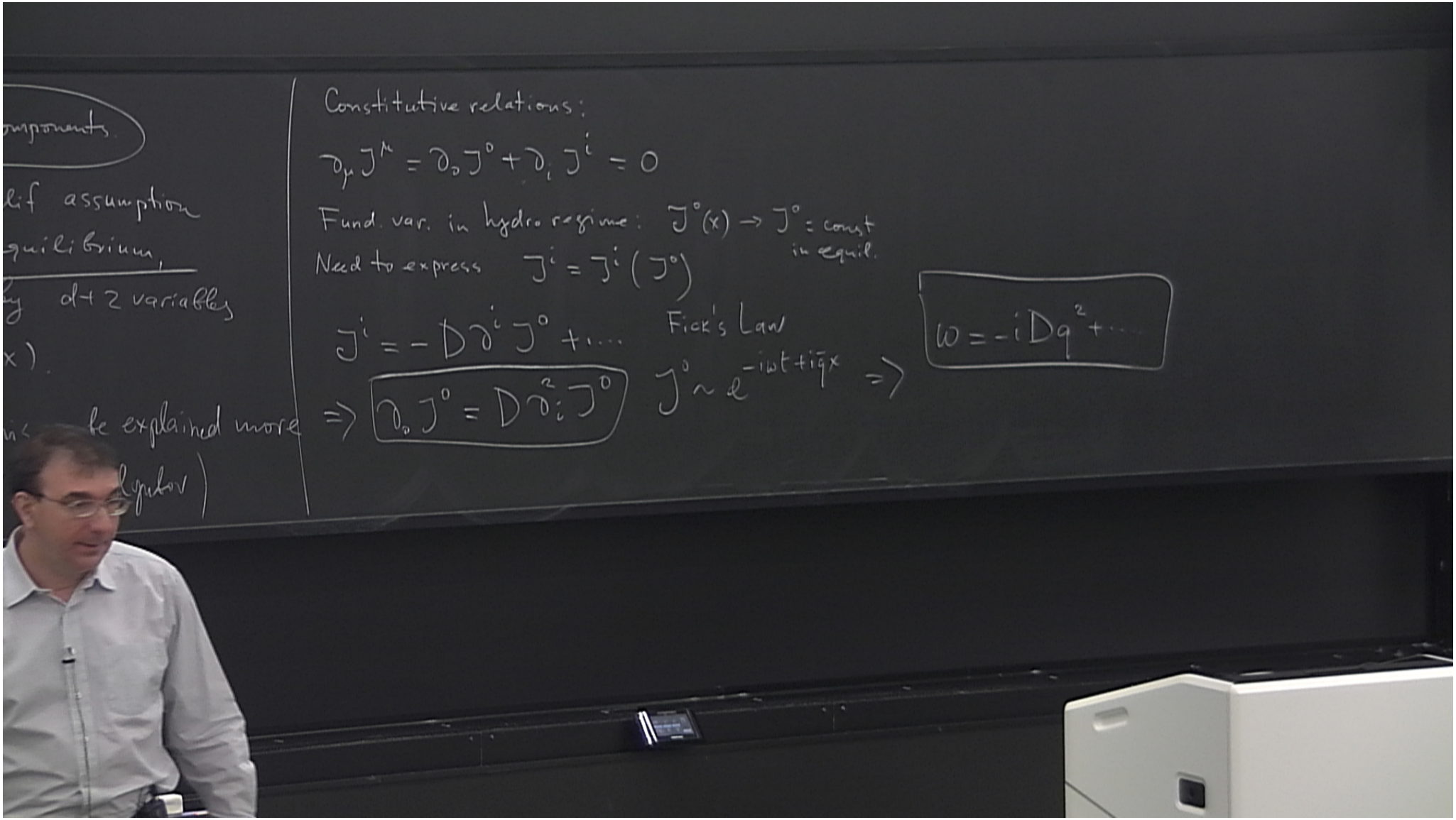
$\partial_\mu J^\mu = \partial_0 J^0 + \partial_i J^i = 0$

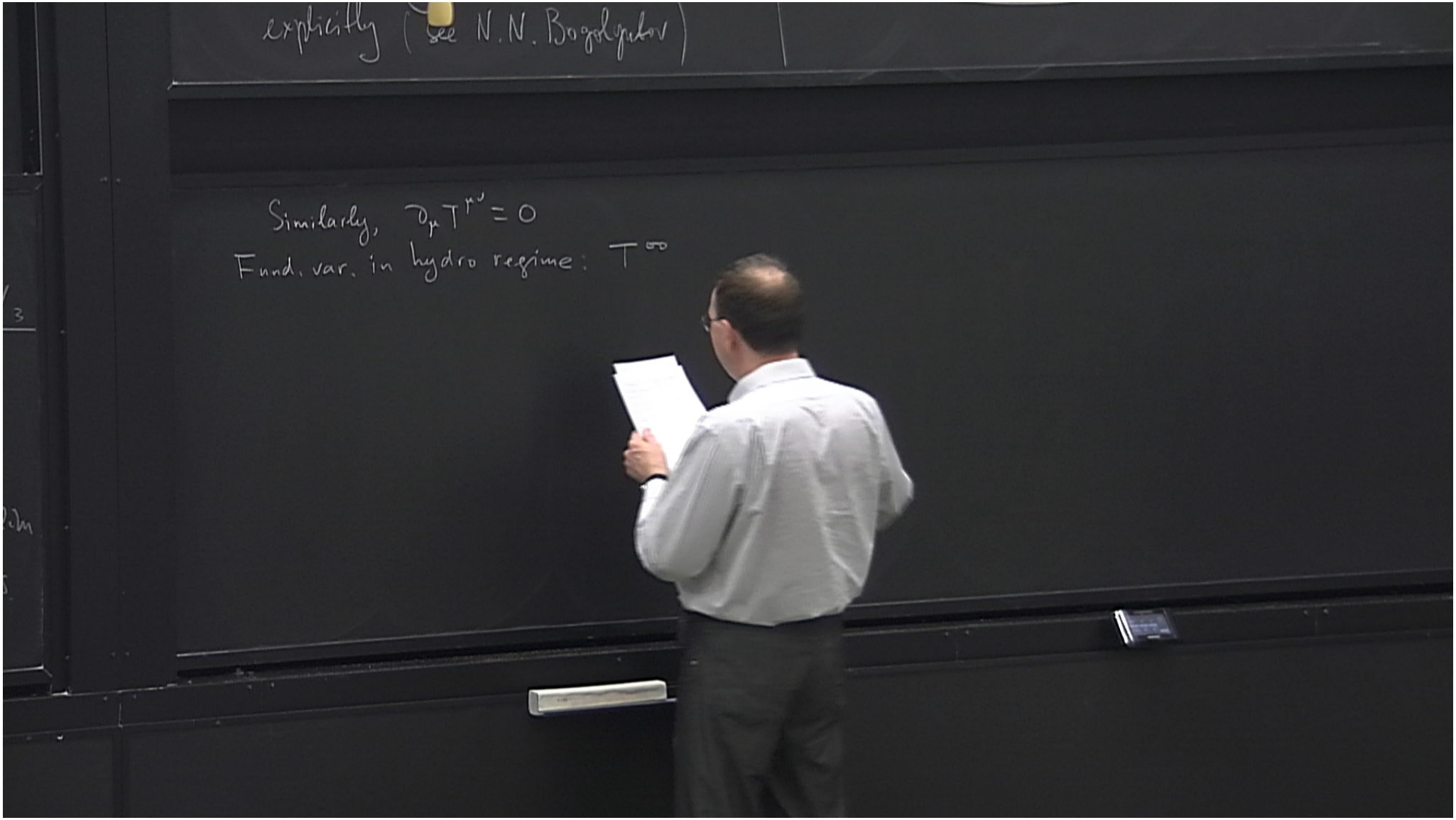
Fund. var. in hydro regime: $J^0(x) \to J^0$ in

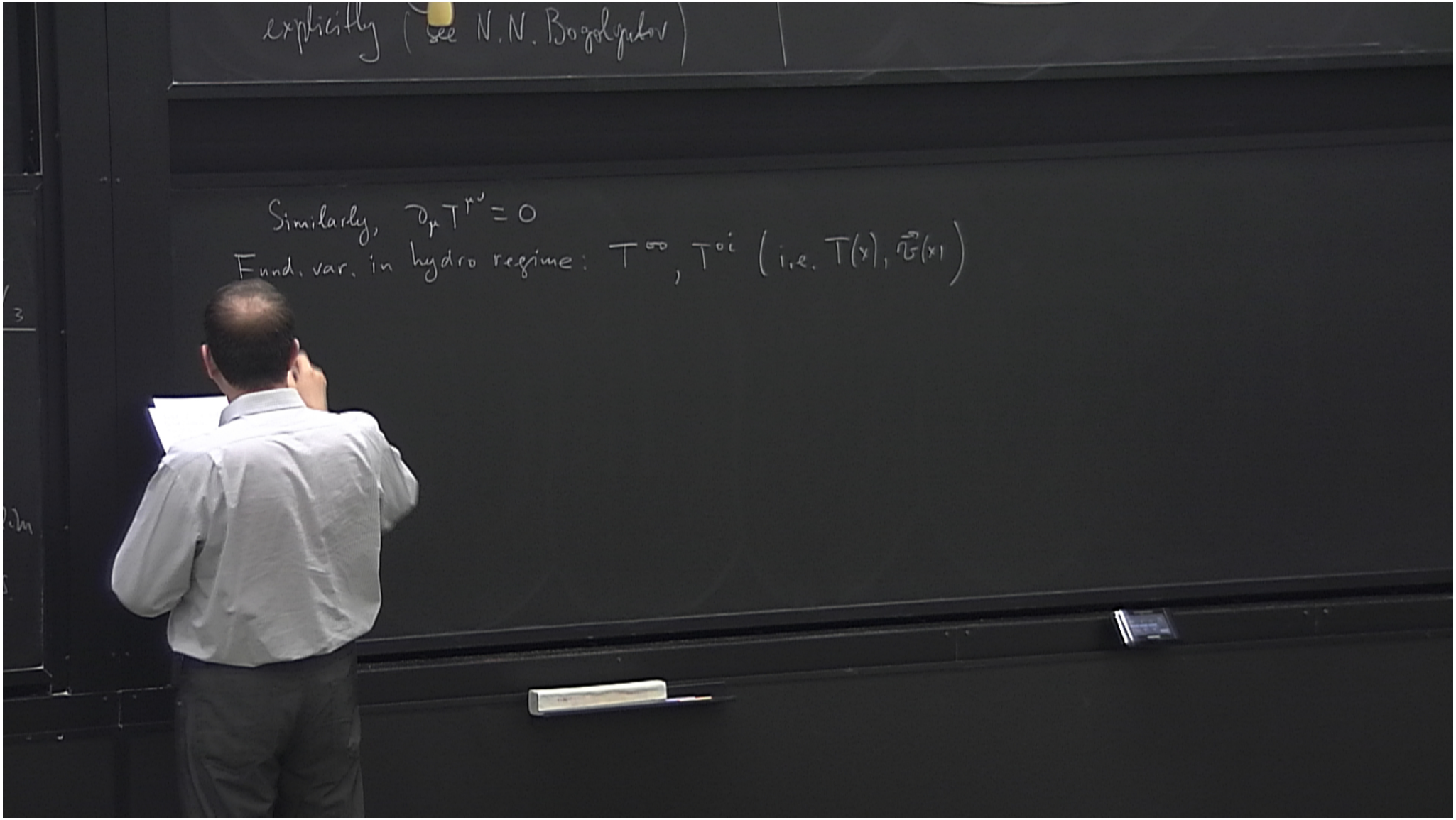
Need to express $J^i = J^i(J^0)$

$J^i = -D\partial^i J^0 + \dots$ Fick's La


for $\frac{(d+1)(d+2)}{2} + d+1$ components.
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Constitutive relations:
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nd. var. in hydro regime: $J^0(x) \to J^0$ = const in equil.
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$\partial^i J^0 + \ldots$ Fick's Law


Constitutive relations:
$\partial_\mu J^\mu = \partial_0 J^0 + \partial_i J^i = 0$
Fund. var. in hydro regime: $J^0(x) \to J^0$ = const in equil.
Need to express $J^i = J^i(J^0)$
$J^i = -D\partial^i J^0 + \dots$ Fick's Law
$\Rightarrow \boxed{\partial_0 J^0 = D\partial_i^2 J^0}$
$J^0 \sim e^{-i\omega t + i\vec{q}x} \Rightarrow$
$\boxed{\omega = -iDq^2 + \dots}$


explicitly (see N.N. Bogolyubov)
Similarly, $\partial_\mu T^{\mu\nu} = 0$
Fund. var. in hydro regime: T^{∞}


explicitly (see N.N. Bogolyubov)
Similarly, $\partial_\mu T^{\mu\nu} = 0$
Fund. var. in hydro regime: T^{00}, T^{0i} (i.e. $T(x)$, $\vec{v}(x)$)

explicitly (see N.N. Bogolyubov)

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Fund. var. in hydro regime: T^{00}, T^{0i} (i.e. $T(x)$, $\vec{v}(x)$)

$T^{\mu\nu} = \mathrm{diag}(\varepsilon, P, P, P)$, where $P = P(\varepsilon)$ (eos)



explicitly (see N.N. Bogolyubov)

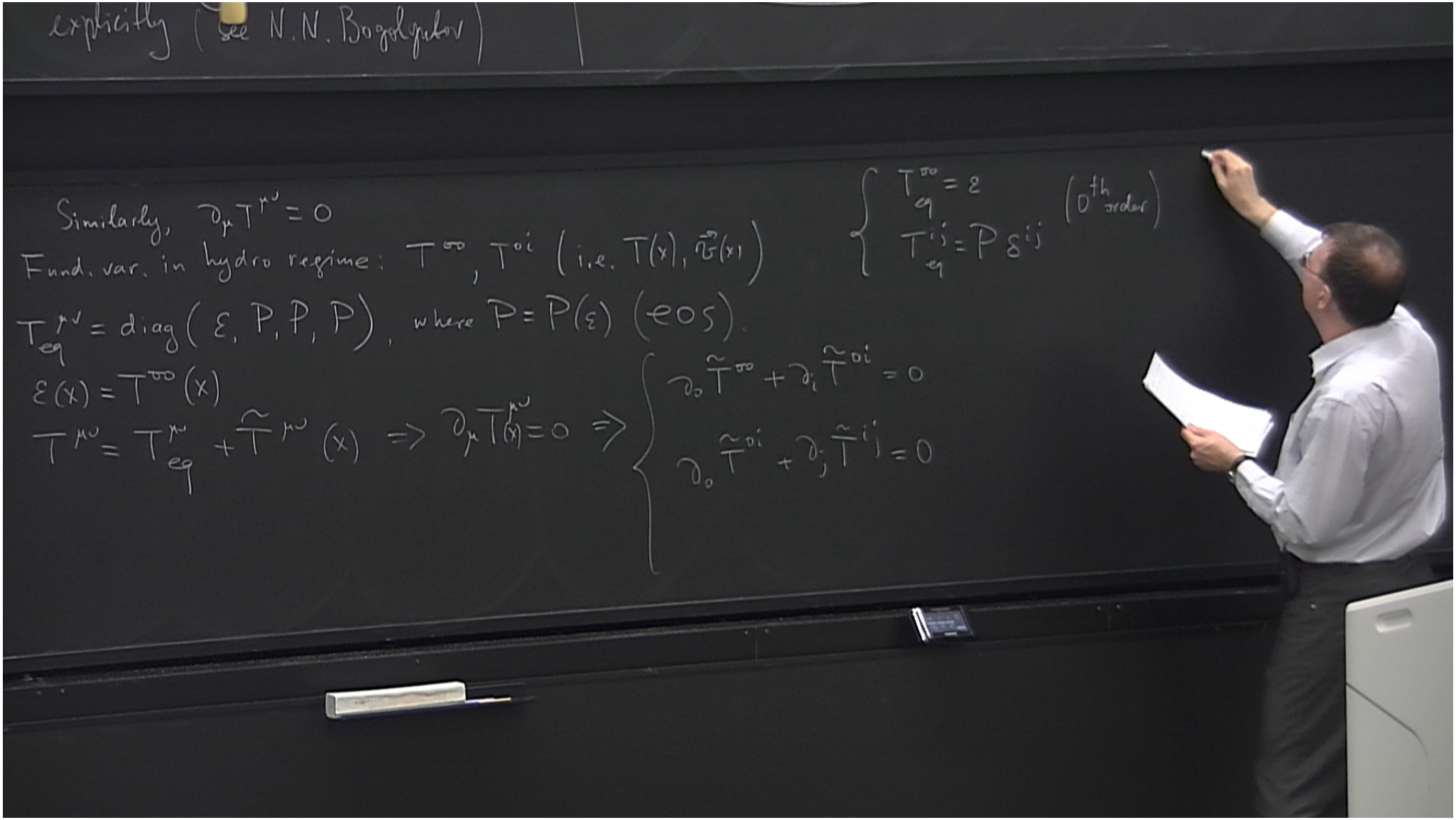
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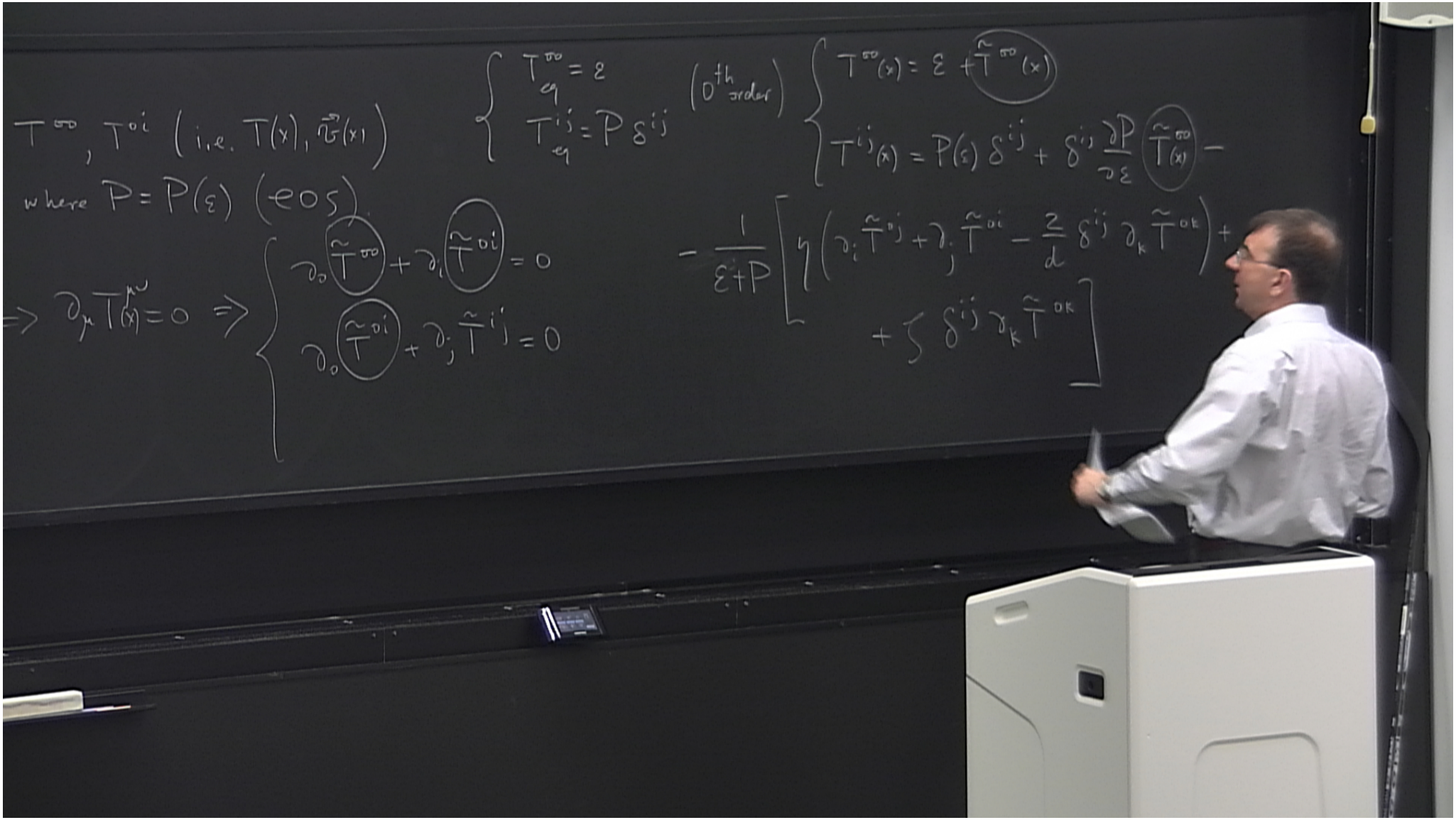
Fund. var. in hydro regime: T^{00}, T^{0i} (i.e. $T(x)$, $\vec{v}(x)$)

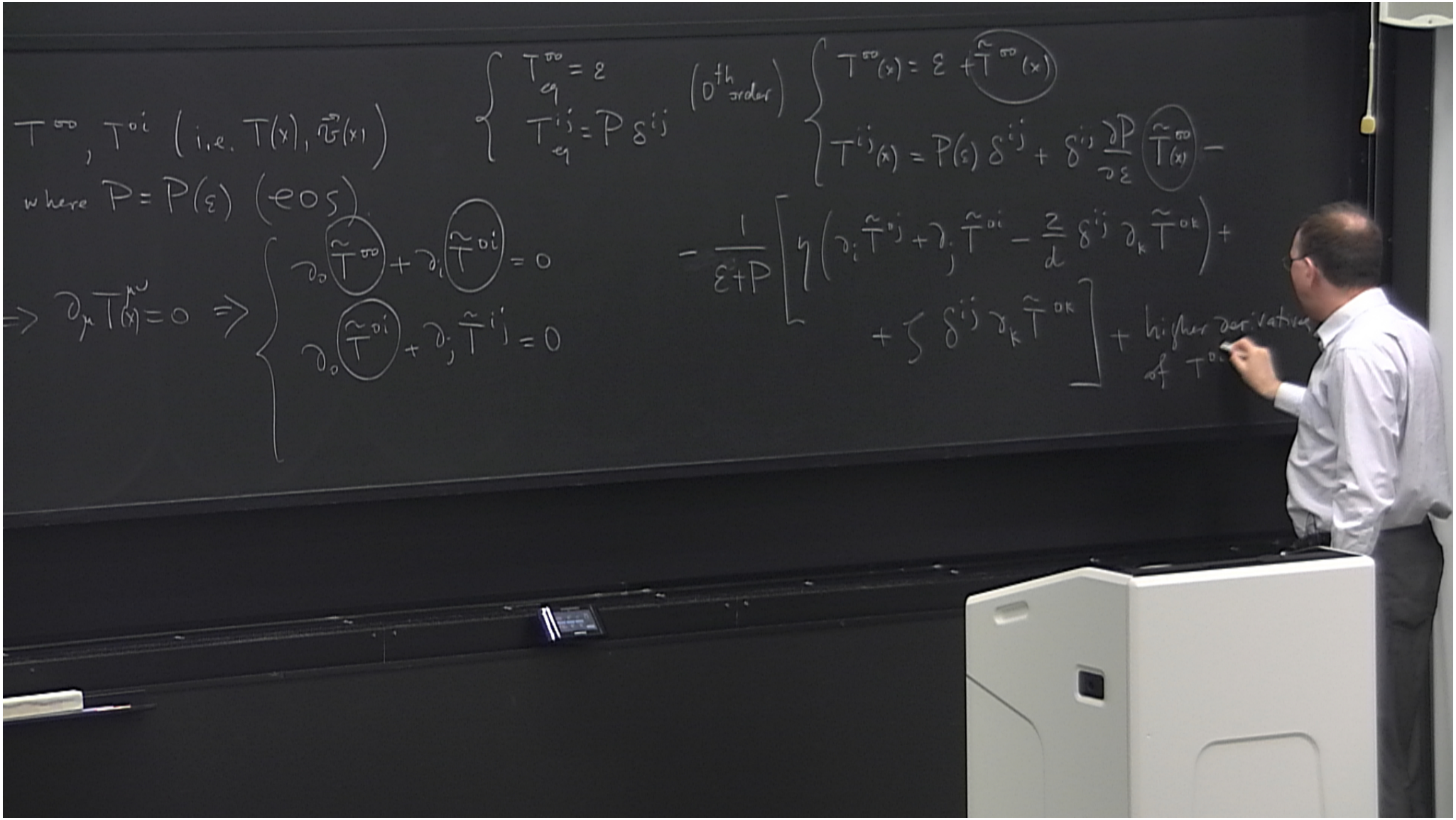
$T^{\mu\nu}_{eq} = \mathrm{diag}(\mathcal{E}, P, P, P)$, where $P = P(\mathcal{E})$ (eos)

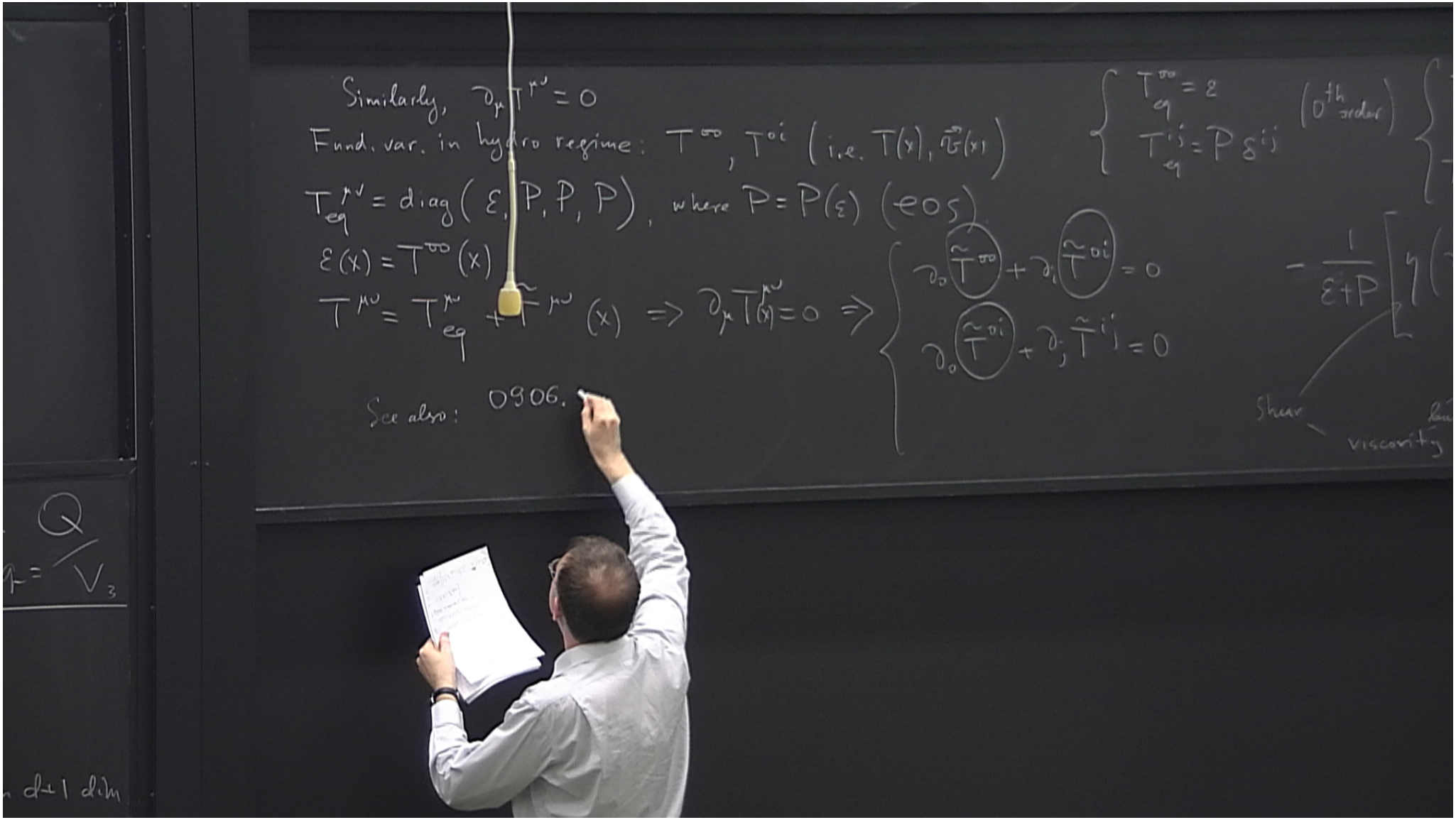
$\mathcal{E}(x) = T^{00}(x)$

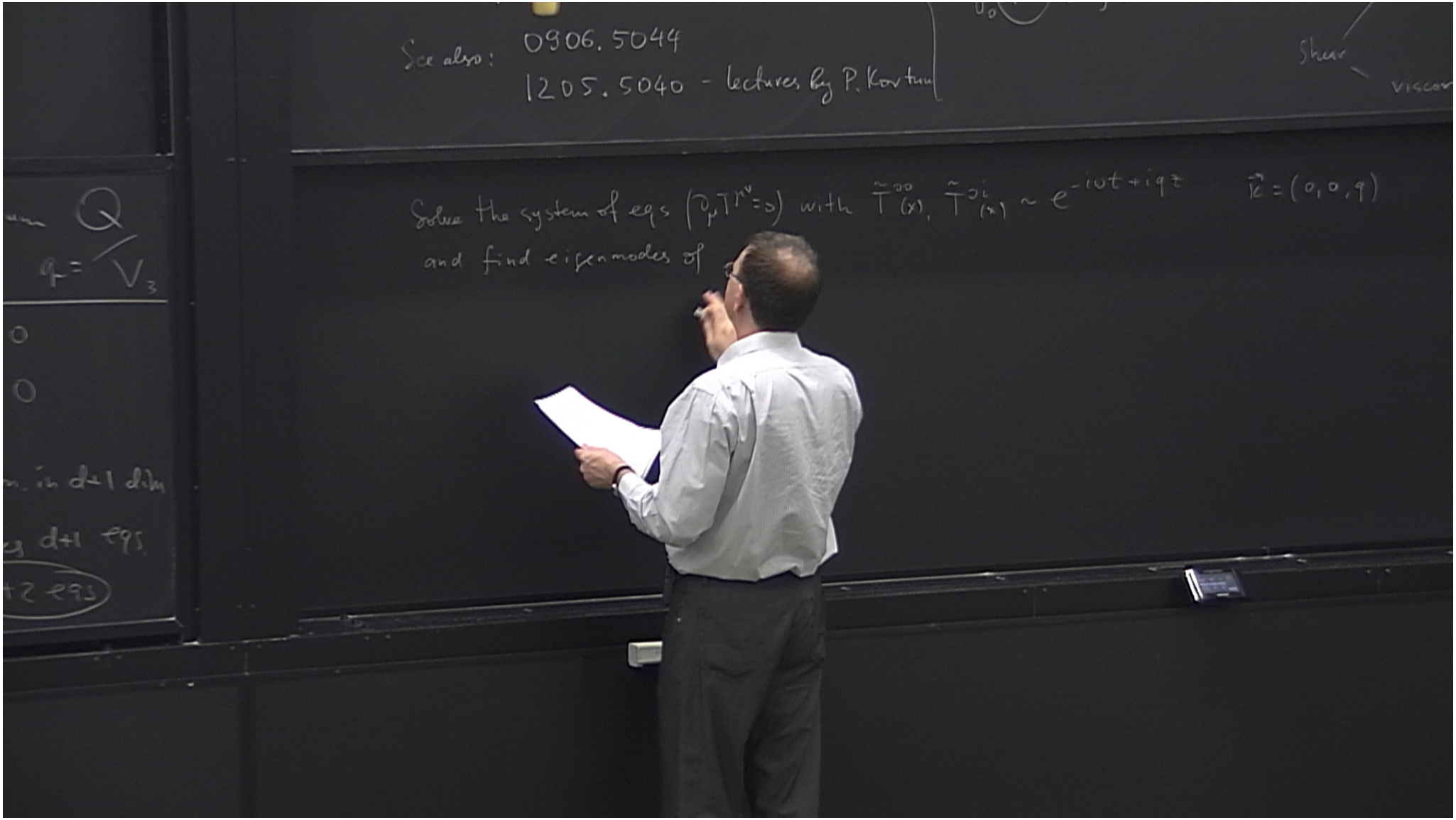
$T^{\mu\nu} = T^{\mu\nu}_{eq} + \tilde{T}^{\mu\nu}(x) \Rightarrow \partial_\mu T^{\mu\nu}(x) = 0 \Rightarrow$ { $\partial_0 \tilde{T}^{00}$

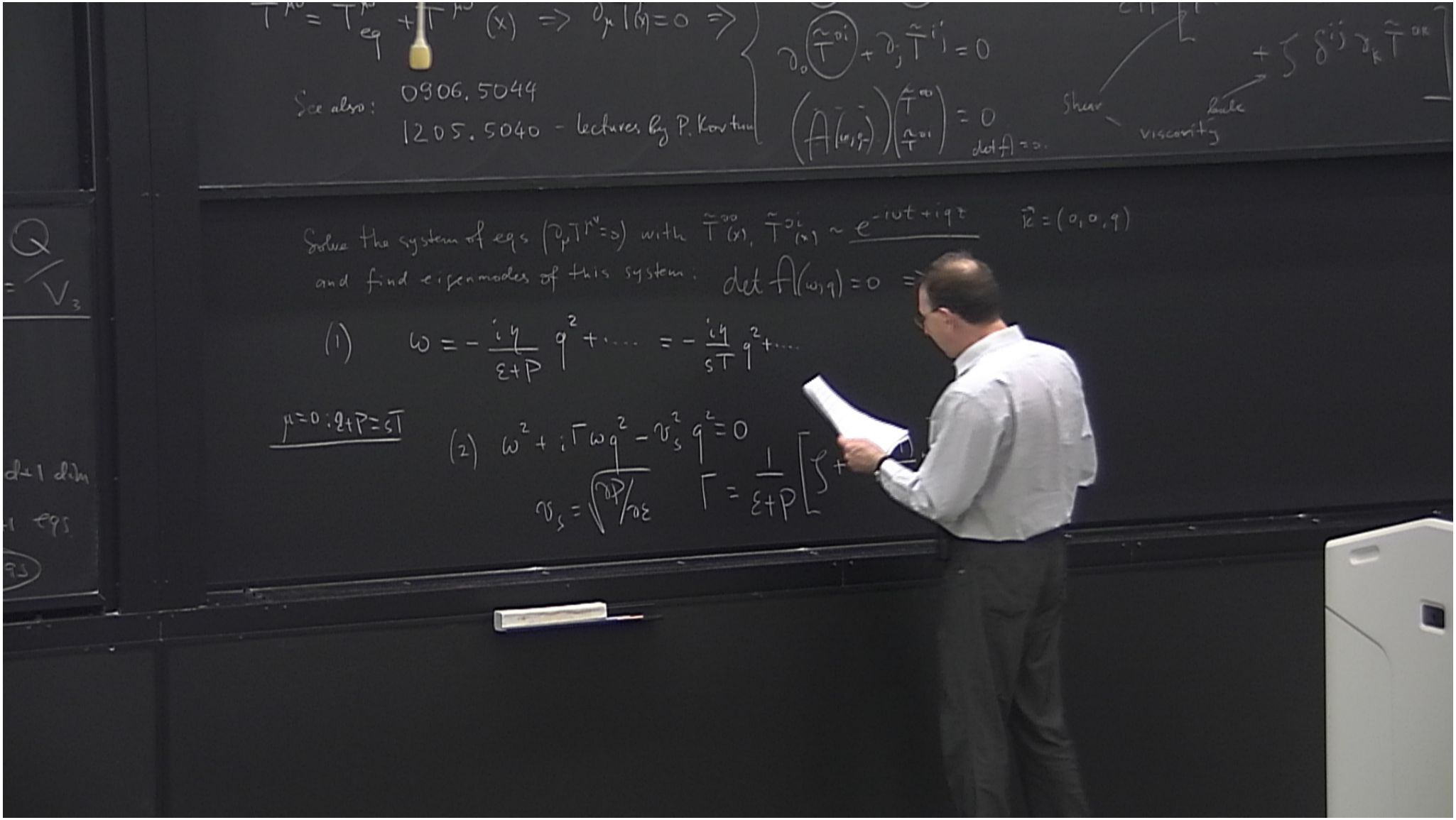

explicitly (see N.N. Bogolyubov)
Similarly, $\partial_\mu T^{\mu\nu} = 0$
Fund. var. in hydro regime: T^{00}, T^{0i} (i.e. $T(x), \vec{v}(x)$)
$\begin{cases} T^{00}_{eq} = \varepsilon \\ T^{ij}_{eq} = P\,\delta^{ij} \end{cases}$ (0^{th} order)
$T^{\mu\nu}_{eq} = \mathrm{diag}(\varepsilon, P, P, P)$, where $P = P(\varepsilon)$ (eos)
$\varepsilon(x) = T^{00}(x)$
$T^{\mu\nu} = T^{\mu\nu}_{eq} + \tilde{T}^{\mu\nu}(x) \Rightarrow \partial_\mu T^{\mu\nu} = 0 \Rightarrow$
$\begin{cases} \partial_0 \tilde{T}^{00} + \partial_i \tilde{T}^{0i} = 0 \\ \partial_0 \tilde{T}^{0i} + \partial_j \tilde{T}^{ij} = 0 \end{cases}$

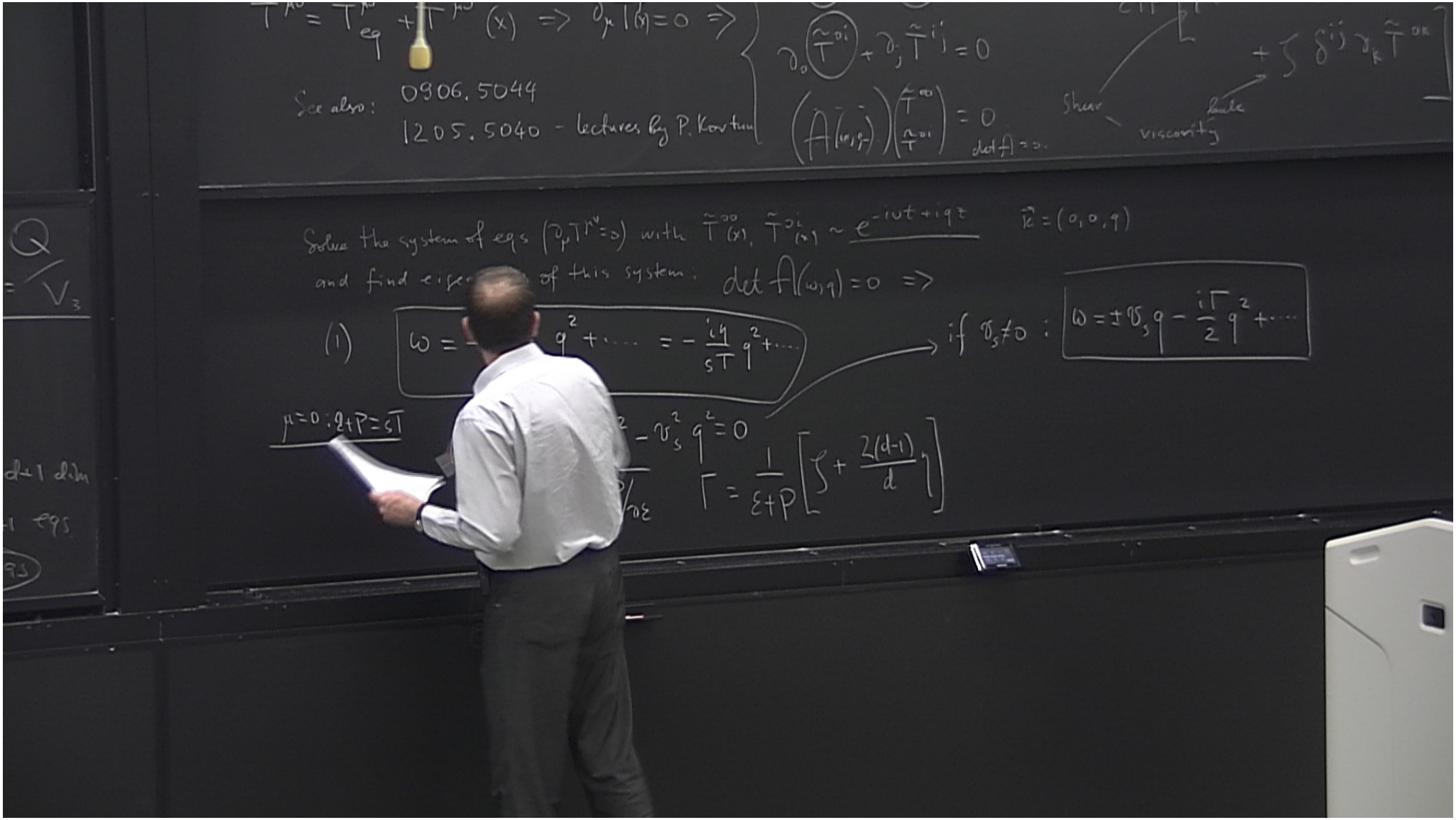

T^{00}, T^{0i} (i.e. $T(x), \vec{\pi}(x)$)
where $P = P(\varepsilon)$ (eos)
$\Rightarrow \partial_\mu T^{\mu\nu}(x) = 0 \Rightarrow$
$\partial_0 \tilde{T}^{00} + \partial_i \tilde{T}^{0i} = 0$
$\partial_0 \tilde{T}^{0i} + \partial_j \tilde{T}^{ij} = 0$
$T^{00}_{eq} = \varepsilon$
$T^{ij}_{eq} = P \delta^{ij}$ (0^{th} order)
$T^{00}(x) = \varepsilon + \tilde{T}^{00}(x)$
$T^{ij}(x) = P(\varepsilon)\delta^{ij} + \delta^{ij}\frac{\partial P}{\partial \varepsilon}\tilde{T}^{00}(x) -$
$- \frac{1}{\varepsilon + P}\left[\eta\left(\partial_i \tilde{T}^{0j} + \partial_j \tilde{T}^{0i} - \frac{2}{d}\delta^{ij}\partial_k \tilde{T}^{0k}\right) + \right.$
$\left. + \zeta\, \delta^{ij}\partial_k \tilde{T}^{0k}\right]$

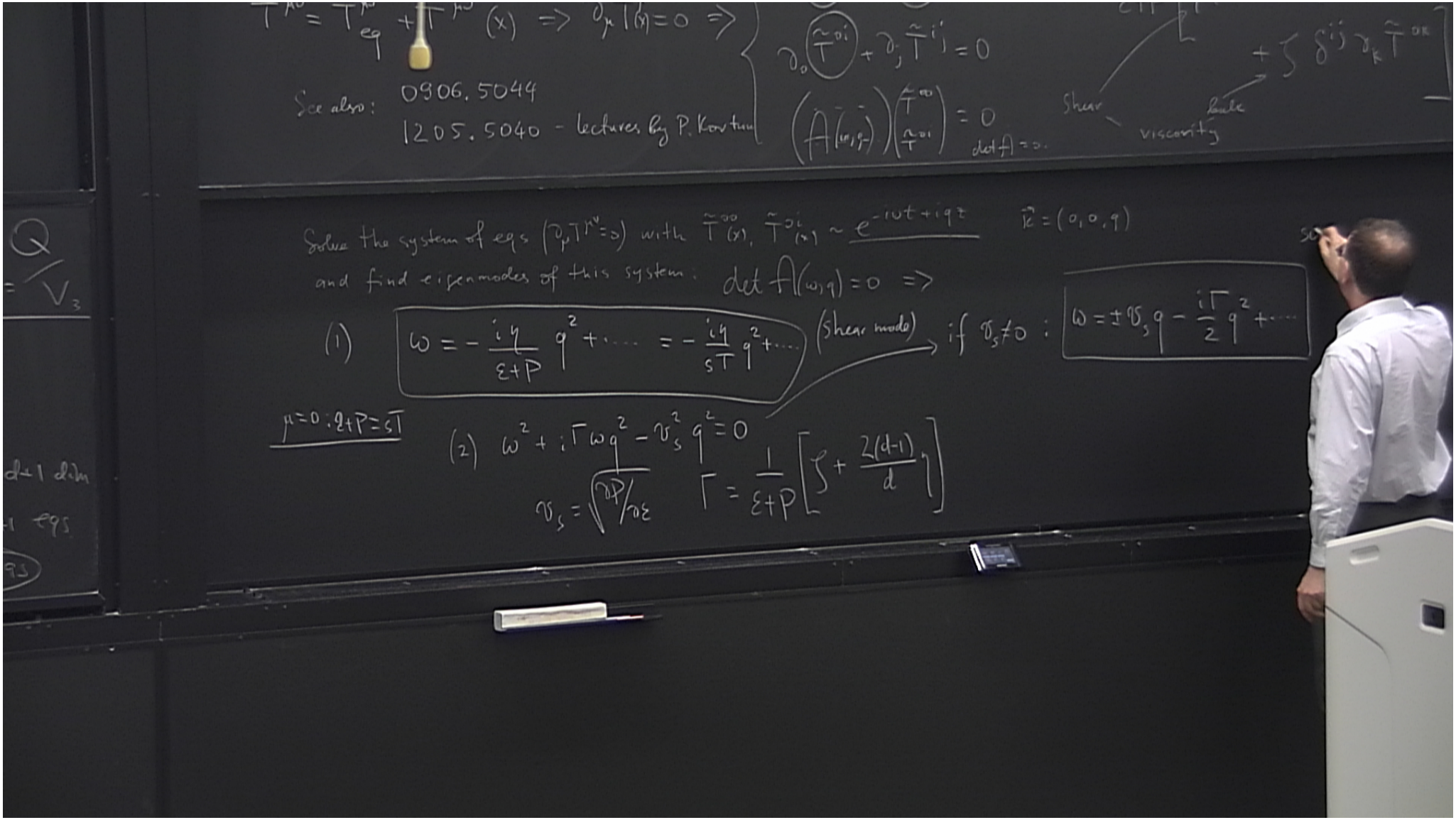

T^{00}, T^{0i} (i.e. $T(x), \vec{v}(x)$)
where $P = P(\varepsilon)$ (eos)
$\Rightarrow \partial_\mu T^{\mu\nu}(x) = 0 \Rightarrow$
$\partial_0 \tilde{T}^{00} + \partial_i \tilde{T}^{0i} = 0$
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$- \frac{1}{\varepsilon + P}\left[\eta\left(\partial_i \tilde{T}^{0j} + \partial_j \tilde{T}^{0i} - \frac{2}{d}\delta^{ij}\partial_k \tilde{T}^{0k}\right) + \zeta\,\delta^{ij}\partial_k \tilde{T}^{0k}\right]$ + higher derivative
of T^{0}


Similarly, $\partial_\mu T^{\mu\nu} = 0$
Fund. var. in hydro regime: T^{00}, T^{0i} (i.e. $T(x)$, $\vec{v}(x)$)
$T^{\mu\nu}_{eq} = diag(\mathcal{E}, P, P, P)$, where $P = P(\mathcal{E})$ (eos)
$\mathcal{E}(x) = T^{00}(x)$
$T^{\mu\nu} = T^{\mu\nu}_{eq} + \tilde{T}^{\mu\nu}(x) \Rightarrow \partial_\mu T^{\mu\nu}(x) = 0 \Rightarrow$
$\partial_0 \tilde{T}^{00} + \partial_i \tilde{T}^{0i} = 0$
$\partial_0 \tilde{T}^{0i} + \partial_j \tilde{T}^{ij} = 0$
$T^{00}_{eq} = \mathcal{E}$ (0th order)
$T^{ij}_{eq} = P \delta^{ij}$
$- \frac{1}{\mathcal{E}+P} [\eta($
See also: 0906.
Shear
viscosity
$\eta = \frac{Q}{V_3}$
d+1 dim


See also: 0906.5044
1205.5040 - lectures By P. Kovtun
Shear
Visco
Solve the system of eqs $(\partial_\mu T^{\mu\nu}=0)$ with $\tilde{T}^{00}(x)$, $\tilde{T}^{0i}(x) \sim e^{-i\omega t+iqz}$ $\vec{k}=(0,0,q)$
and find eigenmodes of
$q=$
Q
V_3
0
0
in d+1 dim
d+1 eqs
+2 eqs


See also:
0906.5044
1205.5040 - lectures by P. Kovtun
Shear
Bulk
viscosity
Solve the system of eqs $(\partial_\mu T^{\mu\nu}=0)$ with $\tilde{T}^{00}(x)$, $\tilde{T}^{0i}(x) \sim e^{-i\omega t+iqz}$ $\vec{k}=(0,0,q)$
and find eigenmodes of this system: $\det f(\omega,q)=0$ =>
(1) $\omega = -\frac{i\eta}{\varepsilon+P}q^2+\dots = -\frac{i\eta}{sT}q^2+\dots$
$\mu=0$: $\varepsilon+P=sT$
(2) $\omega^2 + i\Gamma\omega q^2 - v_s^2 q^2 = 0$
$v_s=\sqrt{\partial P/\partial\varepsilon}$ $\Gamma=\frac{1}{\varepsilon+P}\left[\zeta+\right.$
d+1 dim
eqs


See also:
0906.5044
1205.5040 - lectures by P. Kovtun
Shear
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Solve the system of eqs
and find
of this system:
det fl(ω,q) = 0 =>
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See also: 0906.5044
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and find eigenmodes of this system: $\det f(\omega,q)=0 \Rightarrow$
(1) $\omega = -\frac{i\eta}{\varepsilon+P}q^2+\dots = -\frac{i\eta}{sT}q^2+\dots$ (shear mode) if $v_s\neq 0$: $\omega = \pm v_s q - \frac{i\Gamma}{2}q^2+\dots$
$\mu=0: \varepsilon+P=sT$
(2) $\omega^2 + i\Gamma\omega q^2 - v_s^2 q^2 = 0$
$v_s = \sqrt{\partial P/\partial\varepsilon}$ $\Gamma = \frac{1}{\varepsilon+P}\left[\zeta + \frac{2(d-1)}{d}\eta\right]$

$\hat{P}^\mu$: 4-momentum

$\hat{Q}_A$: conserved charges (Baryon etc)

$\beta_A = \beta u_A \quad \gamma_A = \beta/\mu_A \quad \beta = 1/T \quad Z = \mathrm{tr}\,\hat{\rho}$

whereas $\partial_\mu T^{\mu\nu} = 0$ gives d+1 eqs with $\partial_\mu J^\mu = 0$ we have d+2 eqs

Now consider the hydro limit of AdS/CFT : expect correlators such as $\langle J^\mu(k) J^\nu(-k)\rangle_T$ and $\langle T^{\mu\nu}(k) T^{\rho\sigma}(-k)\rangle_T$ to have poles at $\omega = -iDq^2 + \ldots$ $\omega = -i\frac{\eta}{sT}q^2 + \ldots$ $\omega = \pm c_s q - i\Gamma q^2 + \ldots$ in the limit where hydro approx. is valid.

At $T \neq 0$: $K^\mu = (\omega, \vec{q})$

$\langle J^\mu($

$\hat{P}^\mu$: 4-momentum

$\hat{Q}_A$: Conserved charges (Baryon etc)

$\beta_A = \beta u_\mu \quad \gamma_A = \beta/\mu_A \quad \beta = 1/T \quad Z = \mathrm{tr}\,\hat{\rho}$

Whereas $\partial_\mu T^{\mu\nu} = 0$ gives d+1 eqs with $\partial_\mu J^\mu = 0$ we have d+2 eqs

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at $\omega = -iDq^2 + \dots \quad \omega = -i\frac{\eta}{sT}q^2 + \dots \quad \omega = \pm c_s q - i\Gamma q^2 + \dots$ in the limit when hydro approx. is valid.

At $T \neq 0$: $K^\mu = (\omega, \vec{q})$

$$\langle J^\mu(\omega,\vec{q})\, J^\nu(-\omega,-\vec{q})\rangle_T = G^R(\omega,q)$$

$L_{\text{size of system}} \gg l \gg l$

$T_{\text{life time of system}} \gg t$

CFT at $T \neq 0 \Rightarrow T$ is the only scale