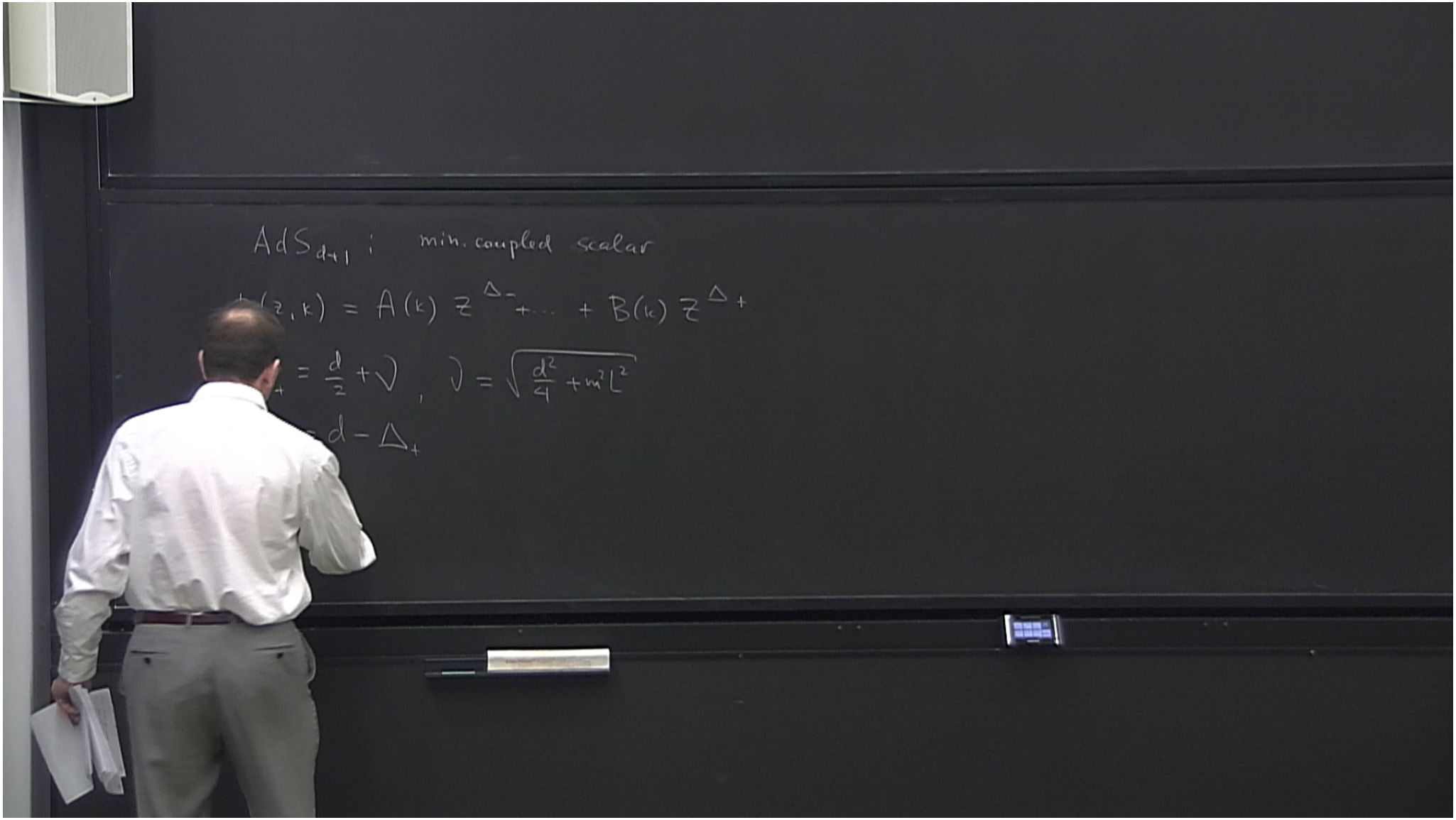
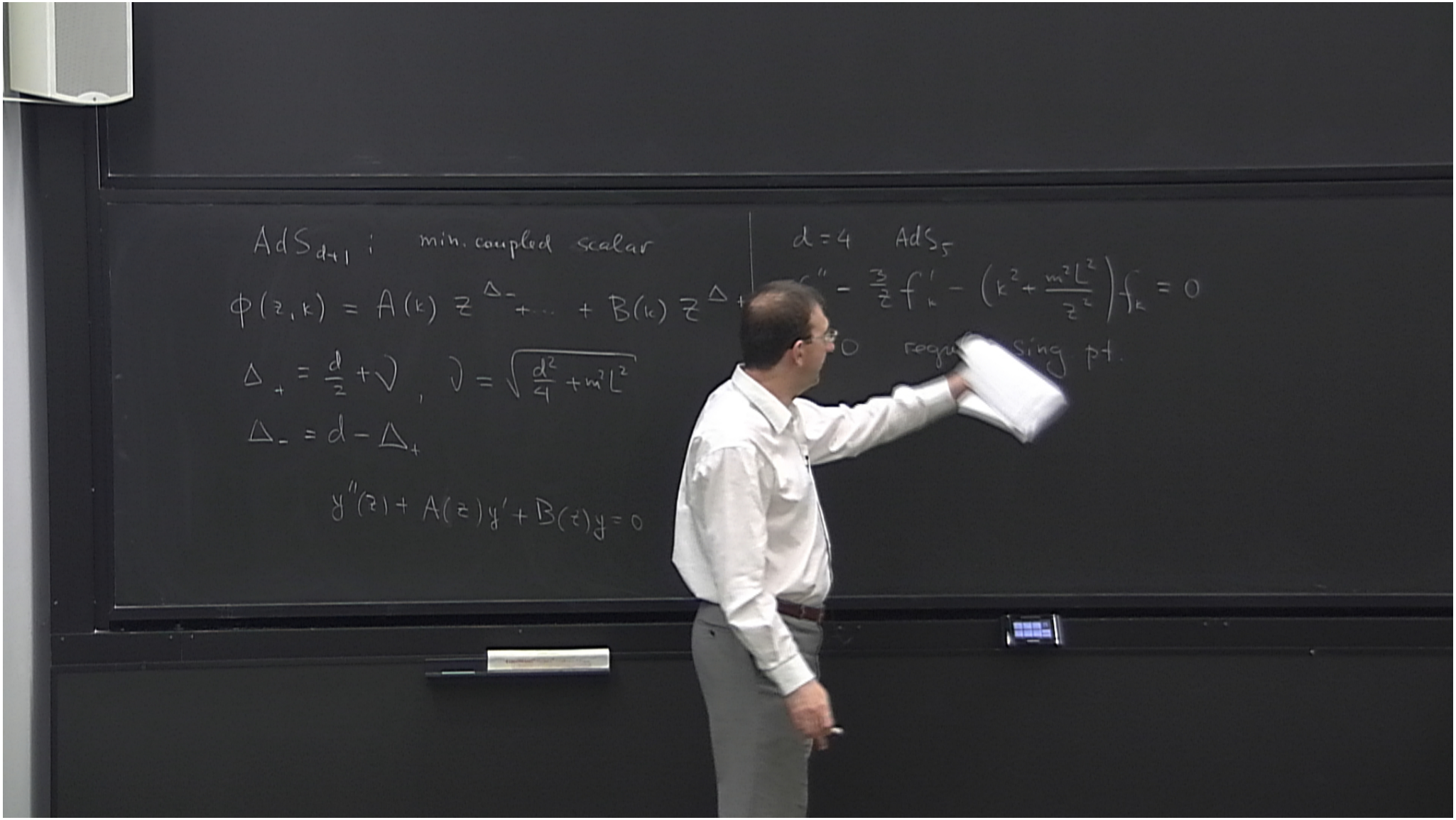
Title: 13/14 PSI - Explorations in String Theory - Lecture 13

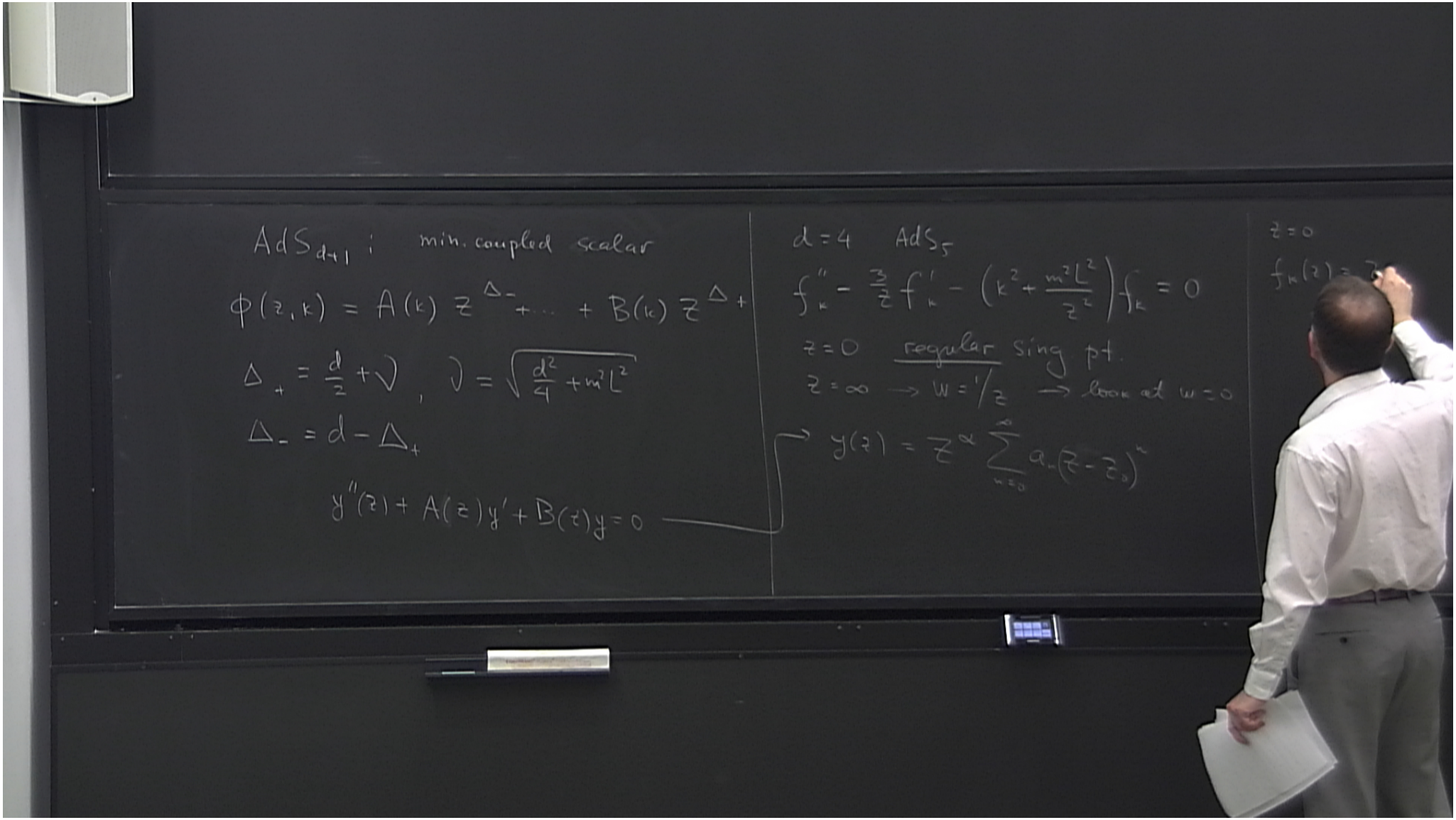
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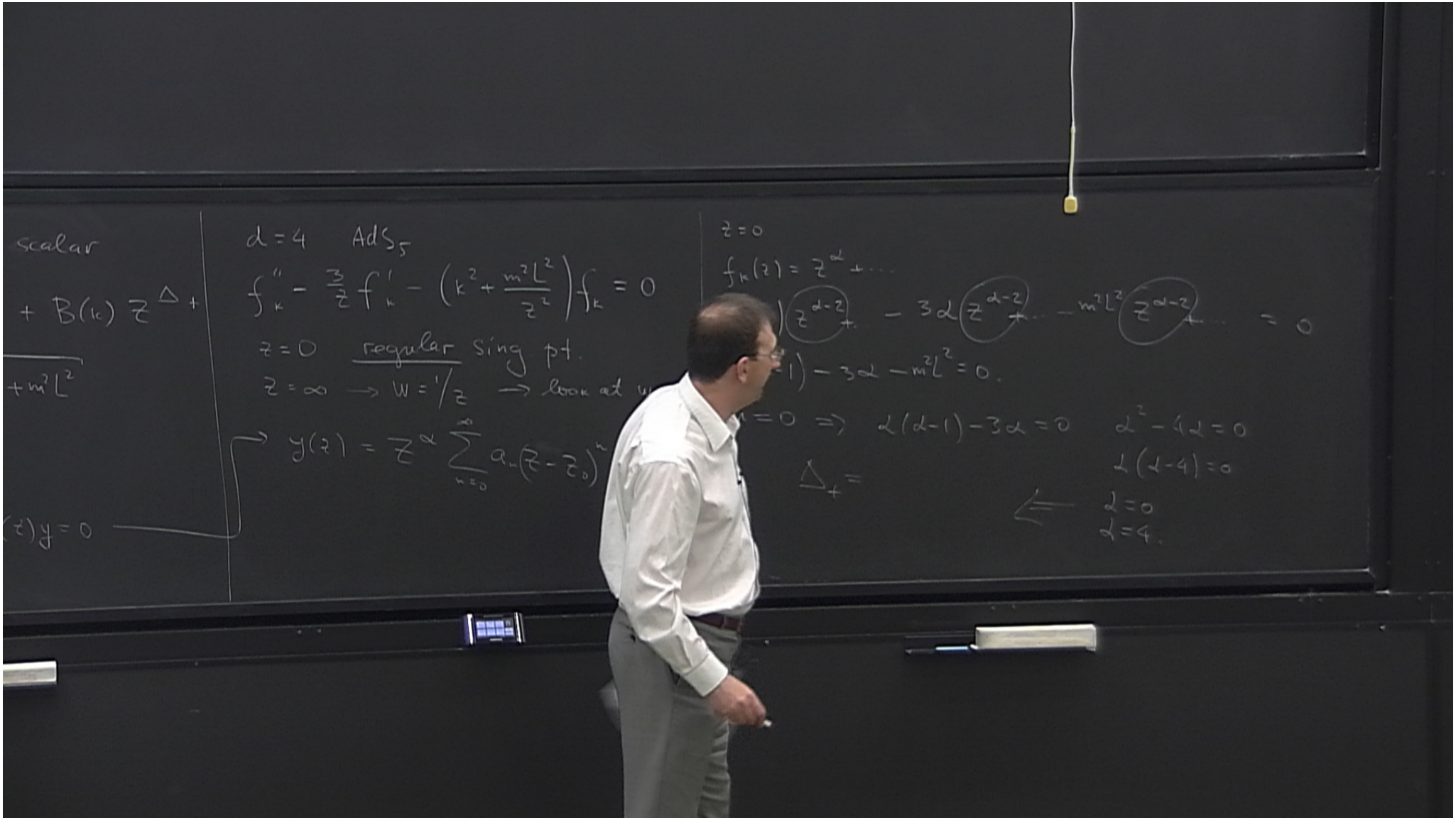
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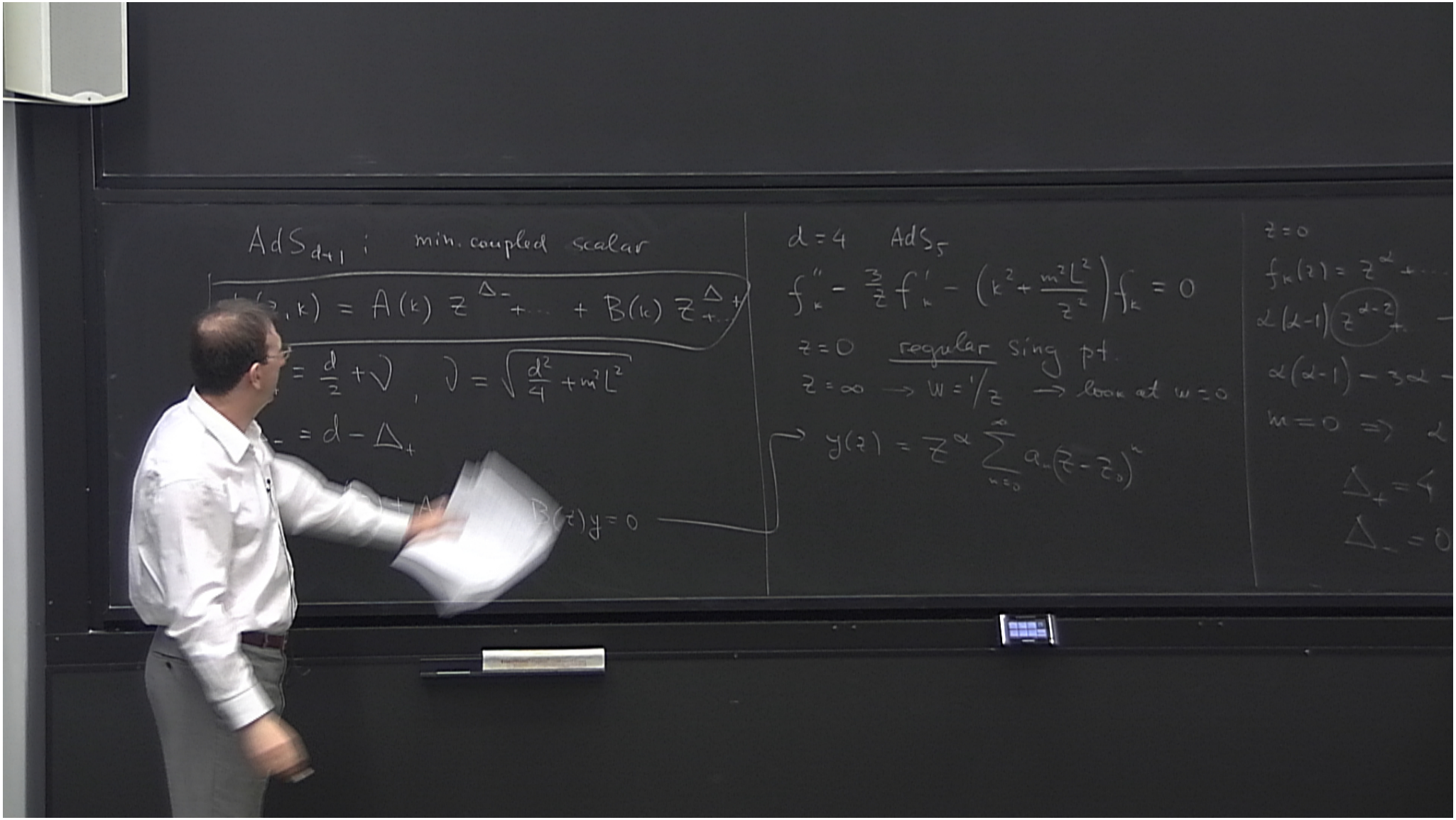
Abstract:

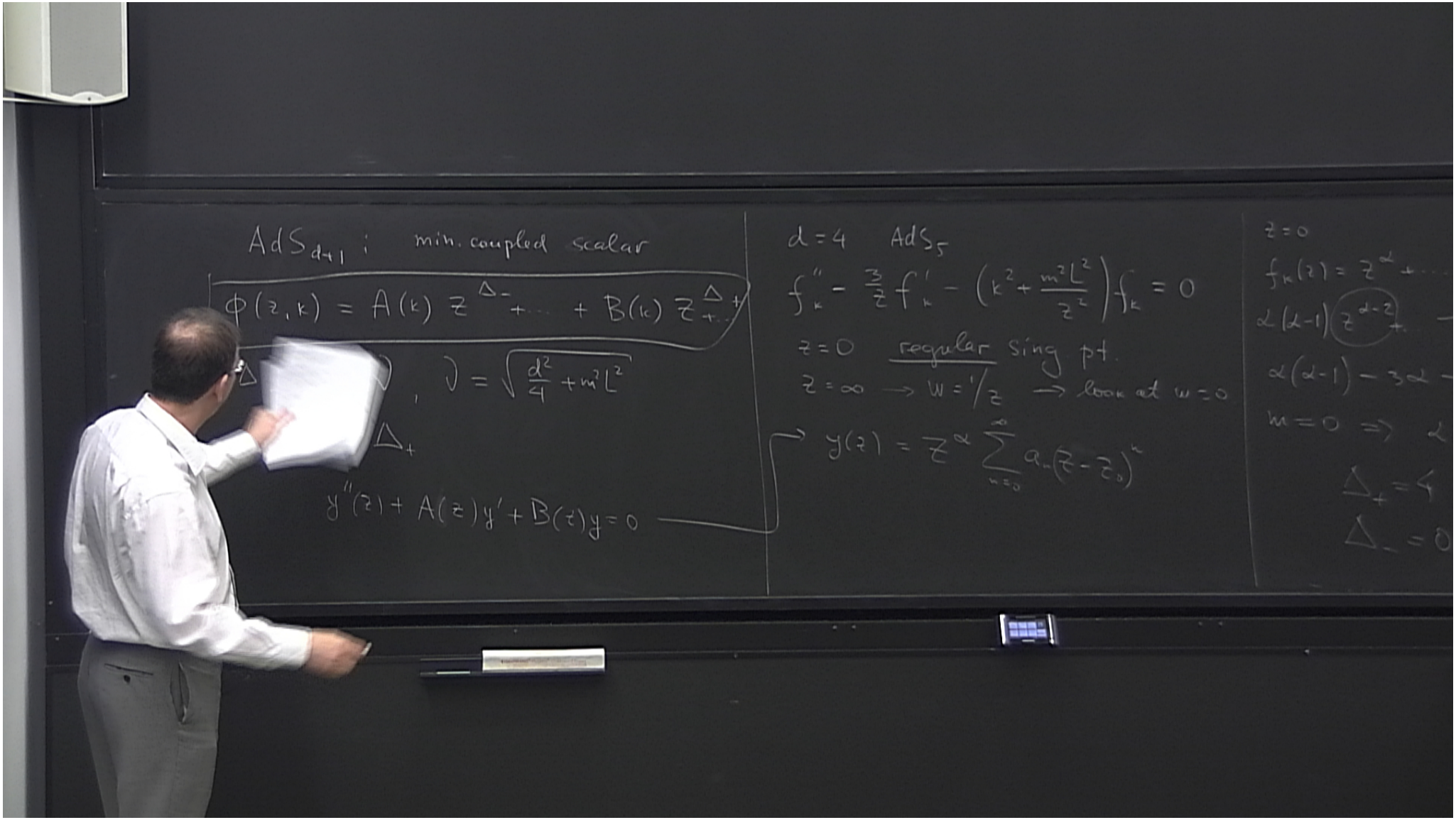

AdS_{d+1} ; min. coupled scalar
$\Delta_+ = \frac{d}{2} + \nu$, $\nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$d - \Delta_+$

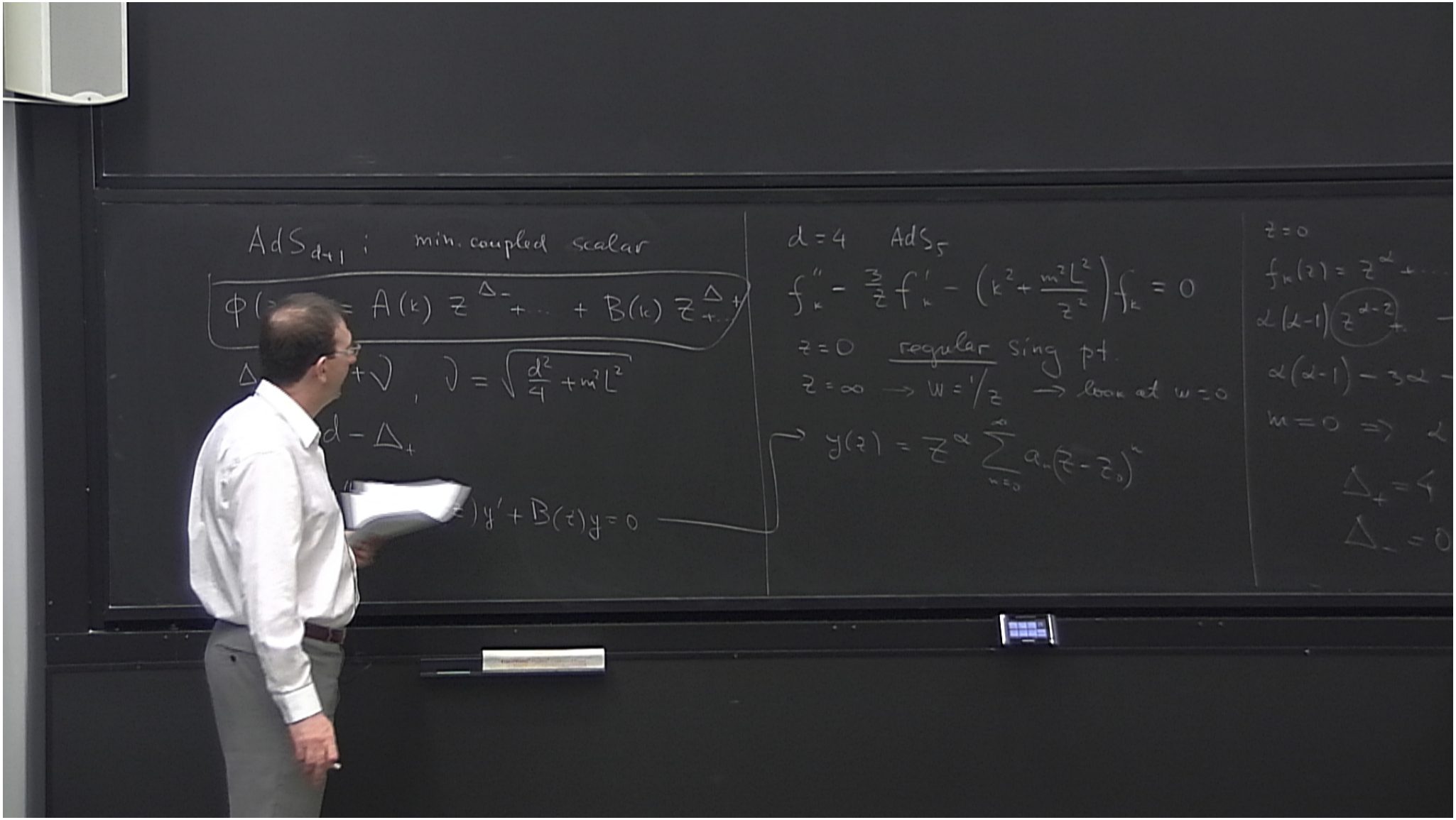

AdS_{d+1} ; min. coupled scalar
$\phi(z,k) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+}$
$\Delta_+ = \frac{d}{2} + \nu \,, \quad \nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$\Delta_- = d - \Delta_+$
$y''(z) + A(z)y' + B(z)y = 0$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
regu sing pt.

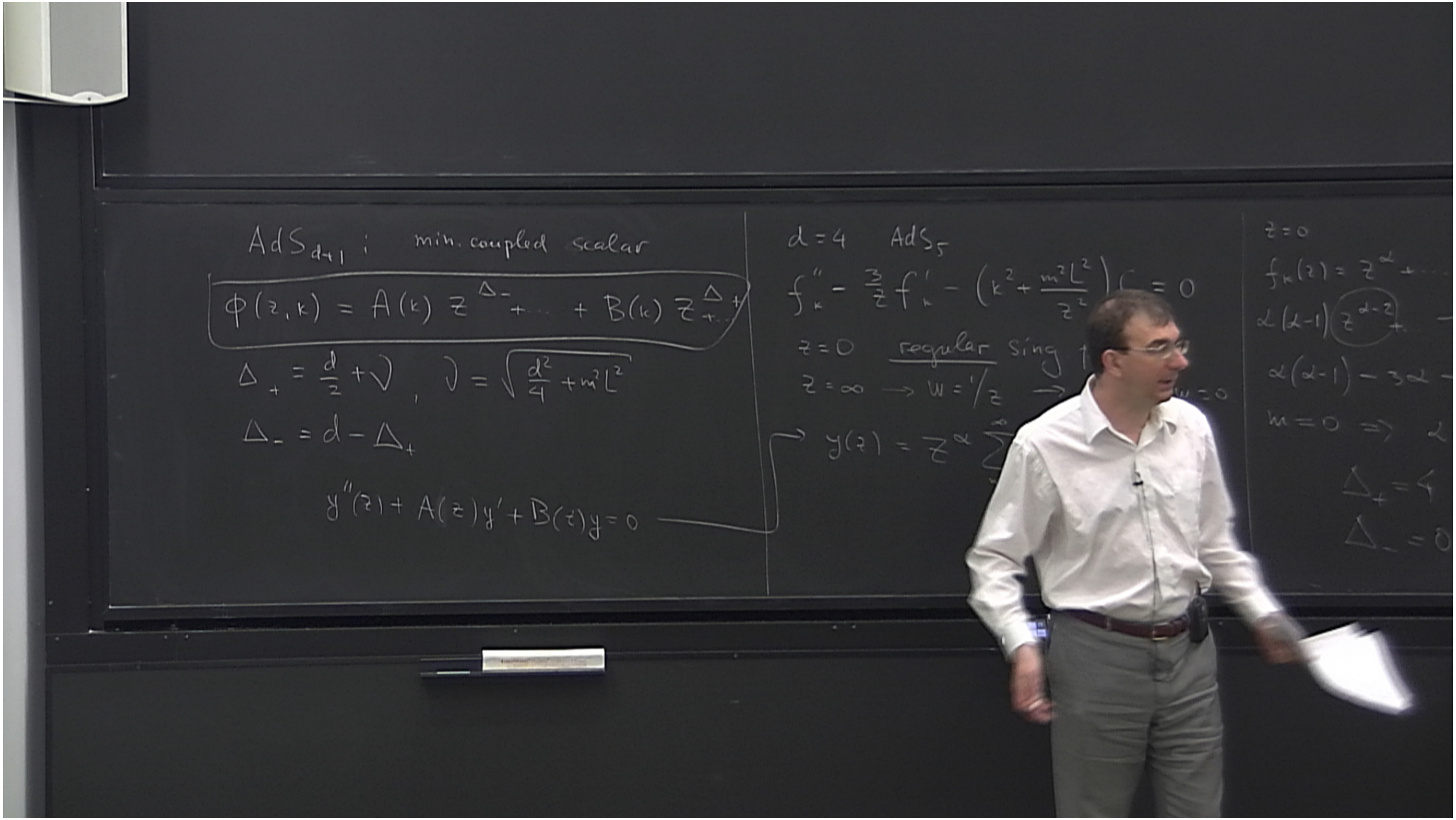

AdS_{d+1} ; min. coupled scalar
$\phi(z,k) = A(k)\, z^{\Delta_-} + \dots + B(k)\, z^{\Delta_+}$
$\Delta_+ = \frac{d}{2} + \nu \,, \quad \nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$\Delta_- = d - \Delta_+$
$y''(z) + A(z)y' + B(z)y = 0$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
$z = 0$ regular sing pt.
$z = \infty \rightarrow w = 1/z \rightarrow$ look at $w = 0$
$y(z) = z^\alpha \sum_{n=0}^{\infty} a_n (z - z_0)^n$
$z = 0$
$f_k(z) =$

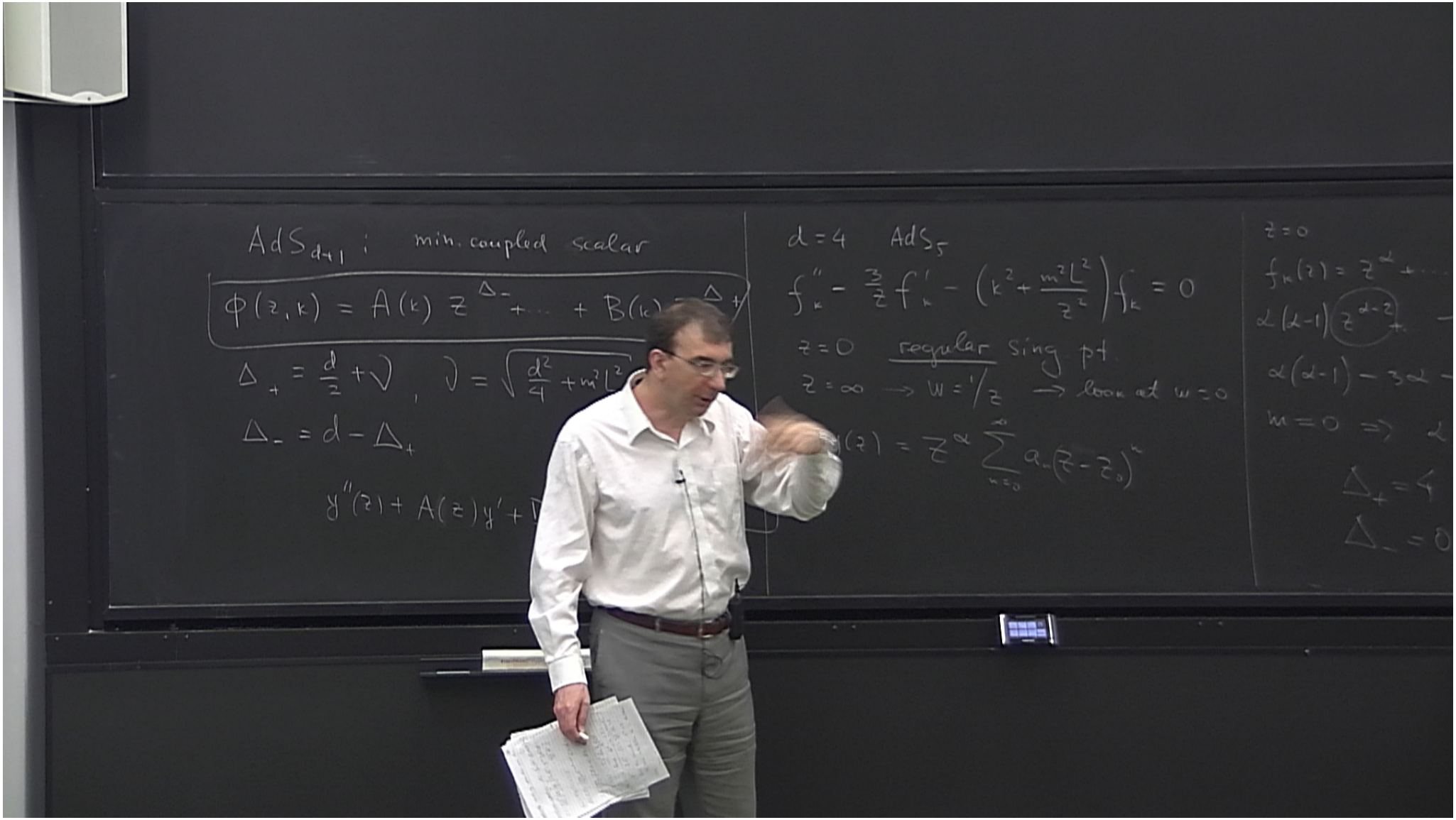

scalar
+ B(k) z^Δ+
+m²L²
(z)y = 0
d = 4 AdS5
f_k'' − (3/z) f_k' − (k² + m²L²/z²) f_k = 0
z = 0 regular sing pt.
z = ∞ → w = 1/z → look at
y(z) = z^α Σ_{n=0}^{∞} a_n (z − z_0)^n
z = 0
f_k(z) = z^α + ...
z^{α-2} − 3α z^{α-2} − m²L² z^{α-2} = 0
α(α−1) − 3α − m²L² = 0.
α = 0 ⇒ α(α−1) − 3α = 0
α² − 4α = 0
α(α−4) = 0
α = 0
α = 4.
Δ+ =

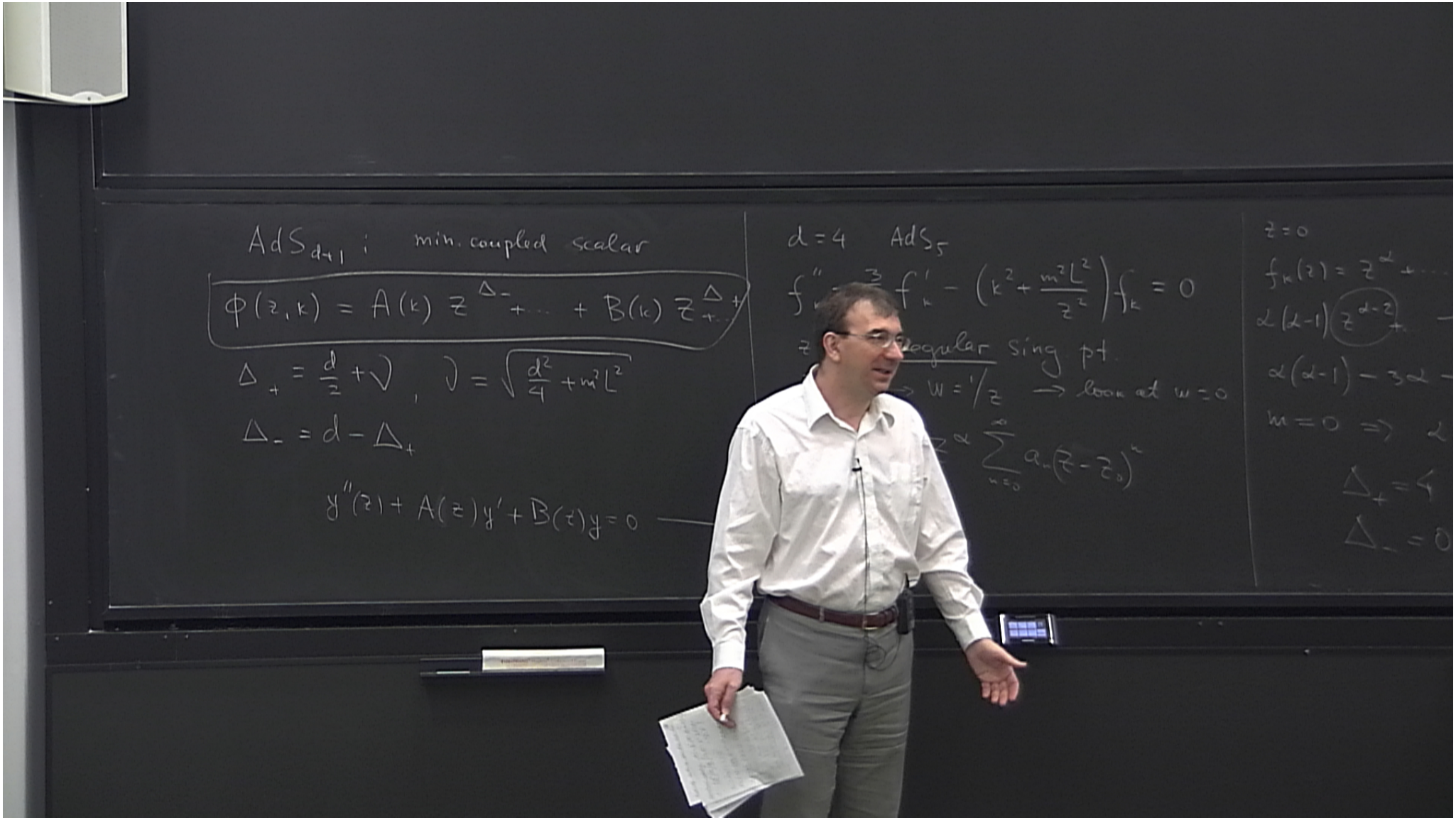

AdS_{d+1} ; min. coupled scalar
$= A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+}$
$= \frac{d}{2} + \nu \,, \quad \nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$= d - \Delta_+$
$B(z) y = 0$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
$z = 0$ regular sing pt.
$z = \infty \to w = 1/z$ → look at $w = 0$
$y(z) = z^\alpha \sum_{n=0}^{\infty} a_n (z - z_0)^n$
$z = 0$
$f_k(z) = z^\alpha + \ldots$
$\alpha(\alpha-1) z^{\alpha-2} + \ldots$
$\alpha(\alpha-1) - 3\alpha$
$w = 0 \Rightarrow \alpha$
$\Delta_+ = 4$
$\Delta_- = 0$

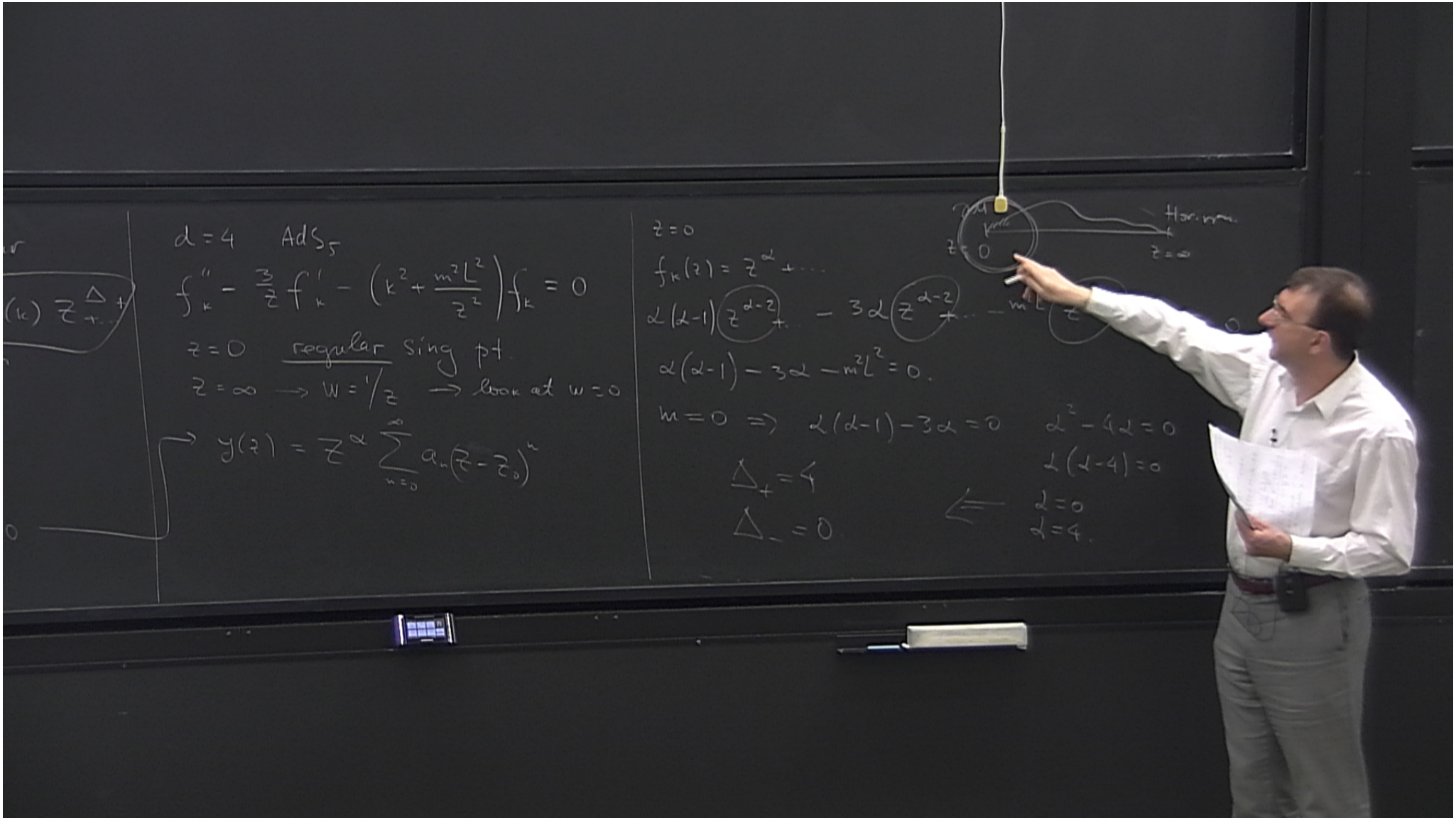

AdS_{d+1} ; min. coupled scalar
φ(z,k) = A(k) z^{Δ-} + ... + B(k) z^{Δ+} + ...
ν = √(d²/4 + m²L²)
Δ+
y''(z) + A(z) y' + B(z) y = 0
d = 4 AdS5
f_k'' - (3/z) f_k' - (k² + m²L²/z²) f_k = 0
z = 0 regular sing pt.
z = ∞ → w = 1/z → look at w = 0
y(z) = z^α Σ_{n=0}^{∞} a_n (z - z_0)^n
z = 0
f_k(z) = z^α + ...
α(α-1) z^{α-2} + ...
α(α-1) - 3α + ...
m = 0 ⇒ α
Δ+ = 4
Δ- = 0

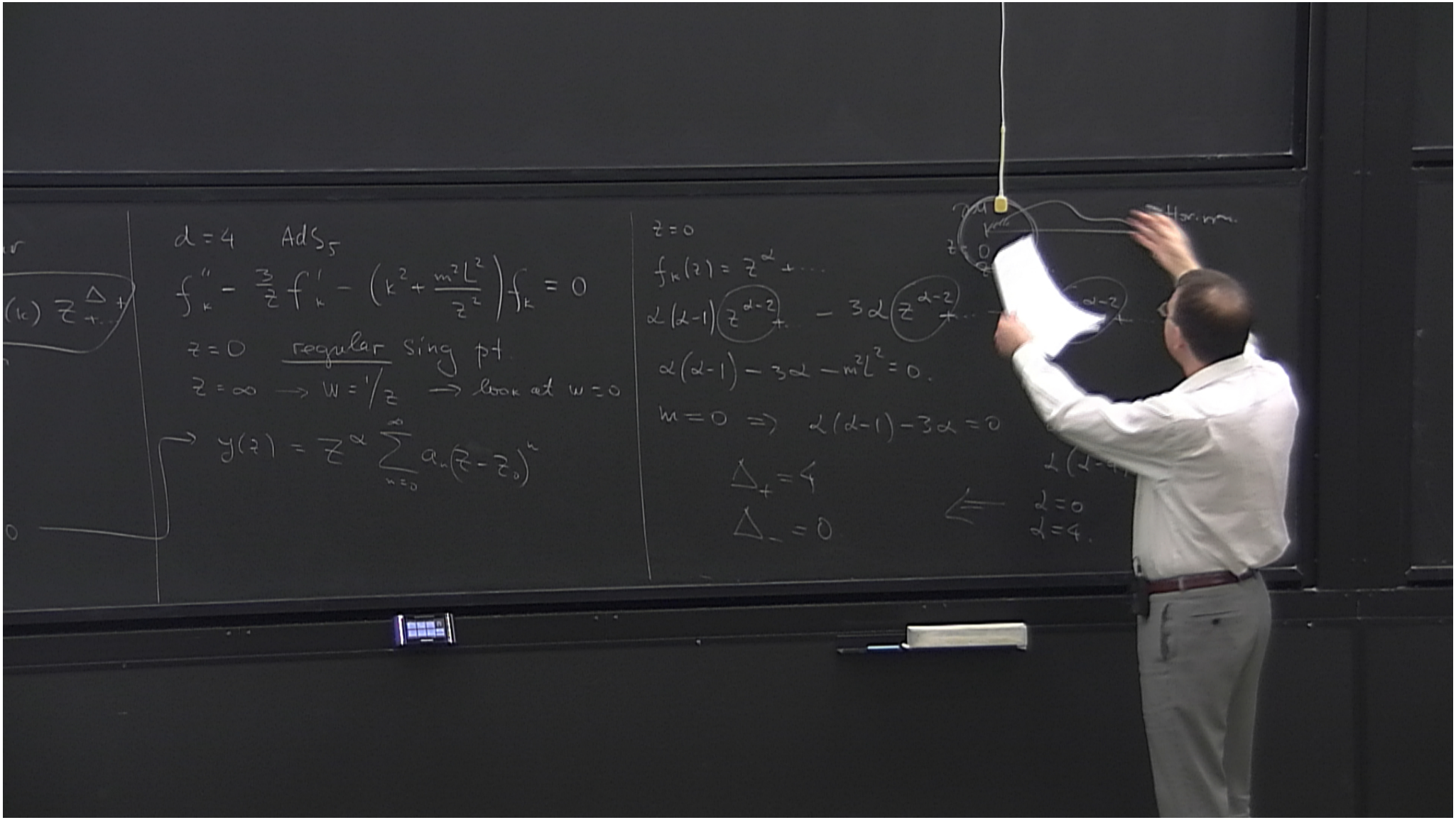

AdS_{d+1} ; min. coupled scalar
$\phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$\Delta_\pm = \frac{d}{2} \pm \nu$, $\nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$d - \Delta_+$
$z y' + B(z) y = 0$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
$z = 0$ regular Sing pt.
$z = \infty \rightarrow w = 1/z$ → look at $w = 0$
$y(z) = z^\alpha \sum_{n=0}^{\infty} a_n (z - z_0)^n$
$z = 0$
$f_k(z) = z^\alpha + \ldots$
$\alpha(\alpha-1) z^{\alpha-2} + \ldots$
$\alpha(\alpha-1) - 3\alpha$
$w = 0 \Rightarrow \alpha$
$\Delta_+ = 4$
$\Delta_- = 0$

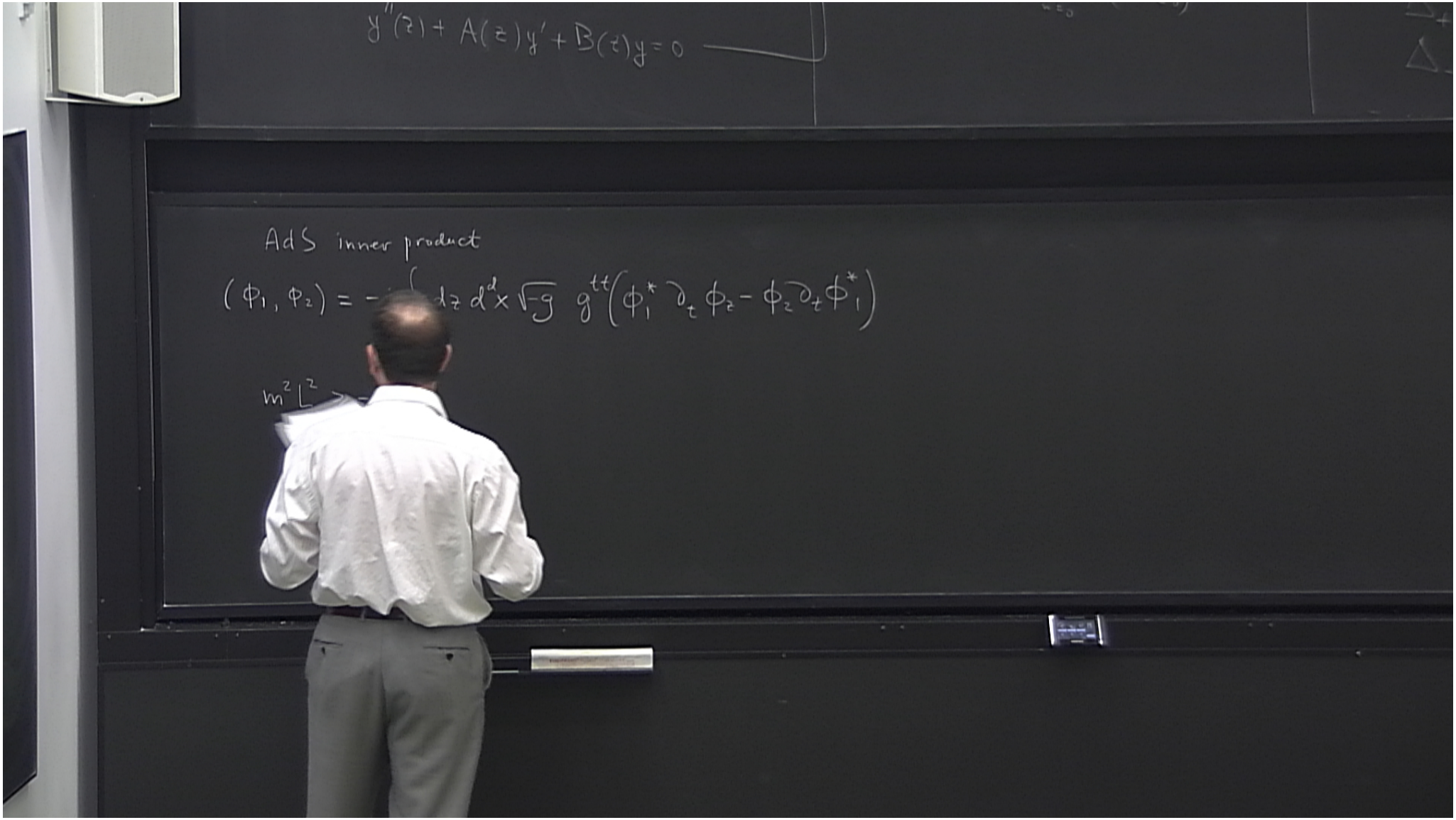

AdS_{d+1} ; min. coupled scalar
$\phi(z,k) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$\Delta_+ = \frac{d}{2} + \nu$, $\nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$\Delta_- = d - \Delta_+$
$y''(z) + A(z)y' + B(z)y = 0$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
$z = 0$ regular sing
$z = \infty \rightarrow w = 1/z$
$y(z) = z^{\alpha}$
$z = 0$
$f_k(z) = z^{\alpha} + \ldots$
$\alpha(\alpha-1) z^{\alpha-2} + \ldots$
$\alpha(\alpha-1) - 3\alpha$
$m = 0 \Rightarrow$
$\Delta_+ = 4$
$\Delta_- = 0$

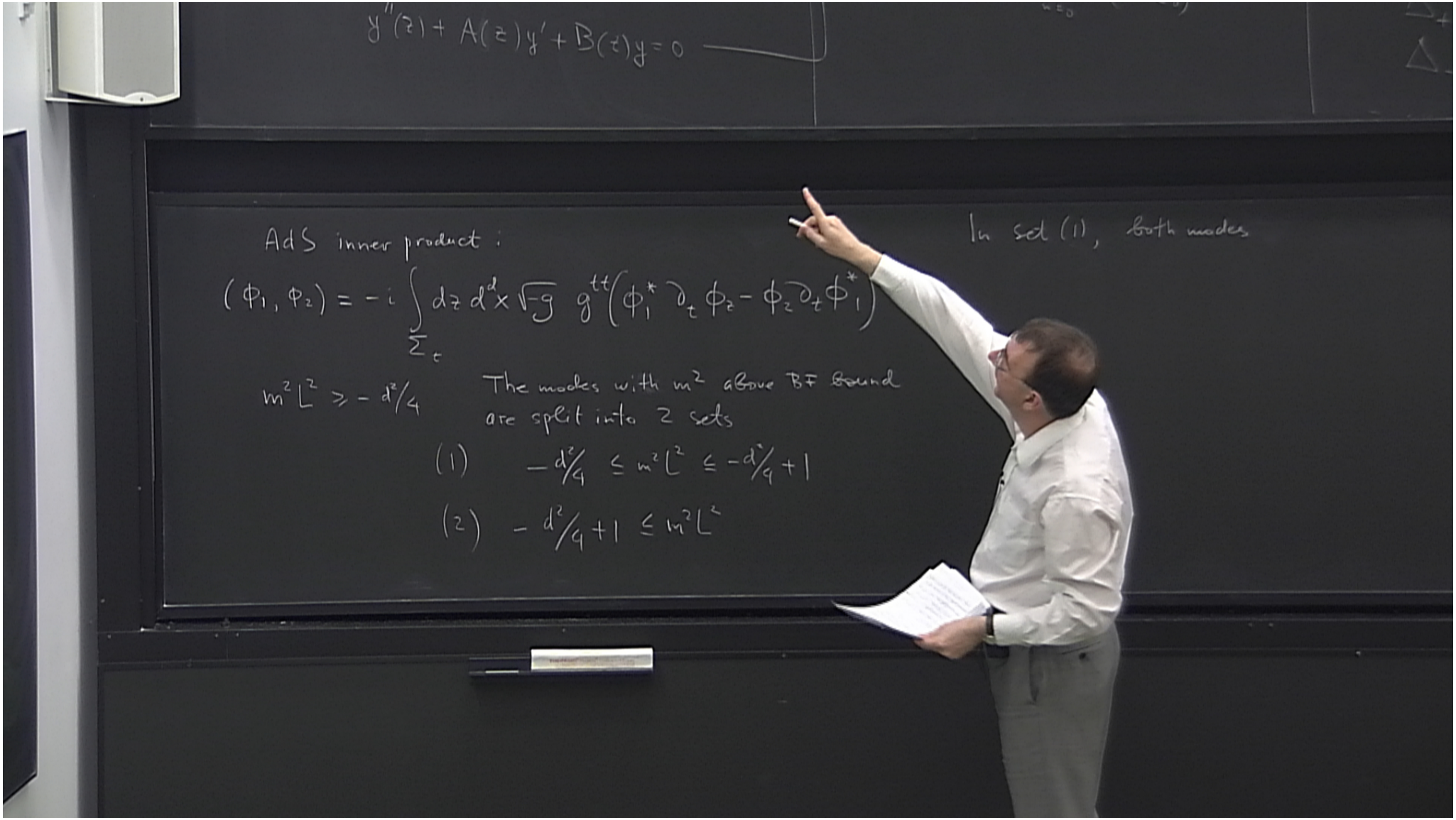

AdS_{d+1} ; min. coupled scalar
$\phi(z,k) = A(k)\, z^{\Delta_-} + \dots + B(k)\, z^{\Delta_+}$
$\Delta_+ = \frac{d}{2} + \nu$, $\nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$\Delta_- = d - \Delta_+$
$y''(z) + A(z) y' +$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
$z = 0$ regular Sing pt
$z = \infty \to w = 1/z \to$ look at $w = 0$
$z = 0$
$f_k(z) = z^{\alpha} +$
$\alpha(\alpha-1) z^{\alpha-2} +$
$\alpha(\alpha-1) - 3\alpha -$
$w = 0 \Rightarrow \alpha$
$\Delta_+ = 4$
$\Delta_- = 0$

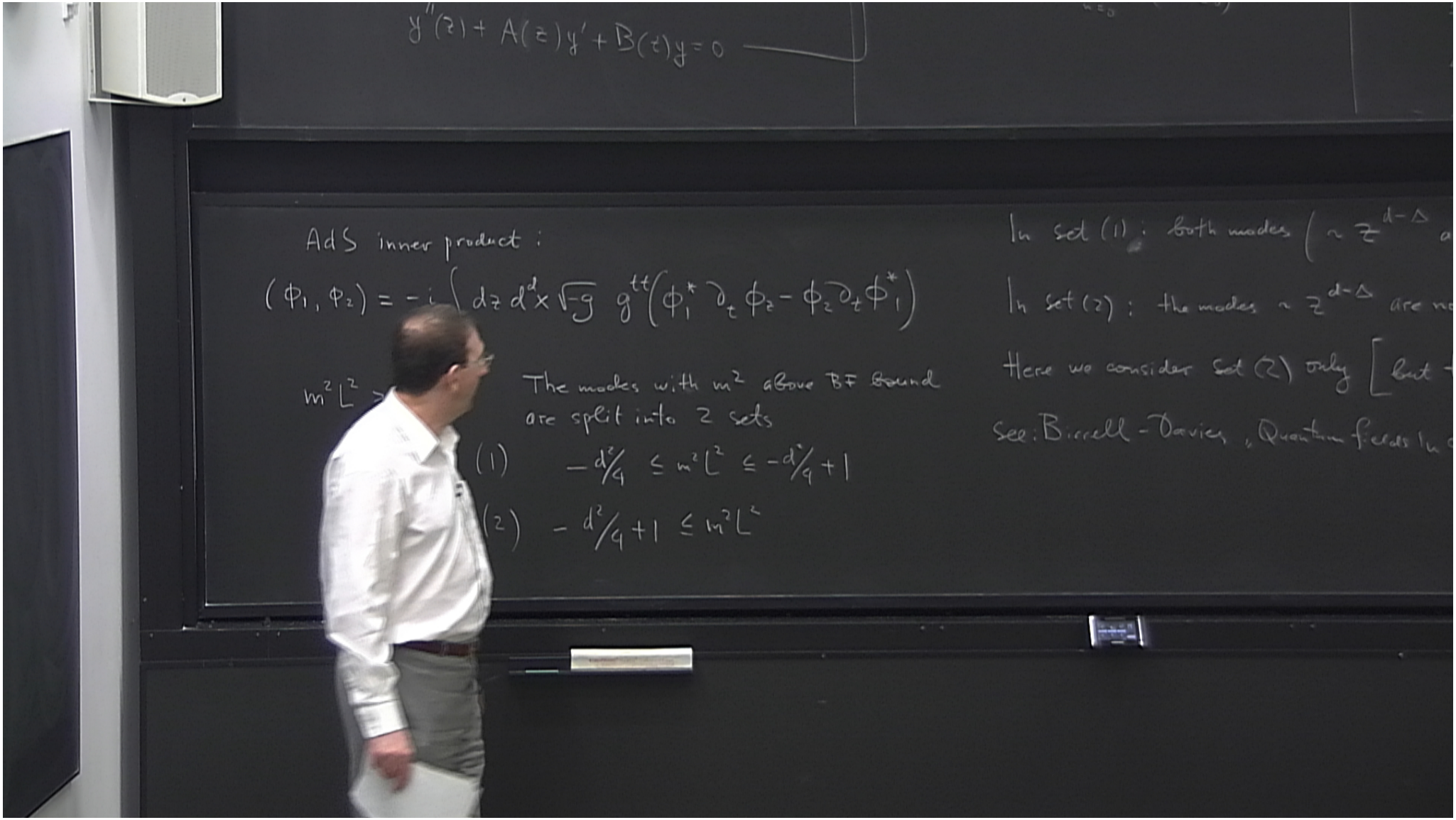

AdS_{d+1} ; min. coupled scalar
$\phi(z,k) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$\Delta_+ = \frac{d}{2} + \nu \,, \quad \nu = \sqrt{\frac{d^2}{4} + m^2 L^2}$
$\Delta_- = d - \Delta_+$
$y''(z) + A(z)y' + B(z)y = 0$
$d = 4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2 L^2}{z^2}\right) f_k = 0$
regular sing pt
$\rightarrow w = 1/z$ $\rightarrow$ look at $w = 0$
$z^{\alpha} \sum_{n=0}^{\infty} a_n (z - z_0)^n$
$z = 0$
$f_k(z) = z^{\alpha} + \ldots$
$\alpha(\alpha-1) z^{\alpha-2} + \ldots$
$\alpha(\alpha-1) - 3\alpha$
$w = 0 \Rightarrow \alpha$
$\Delta_+ = 4$
$\Delta_- = 0$

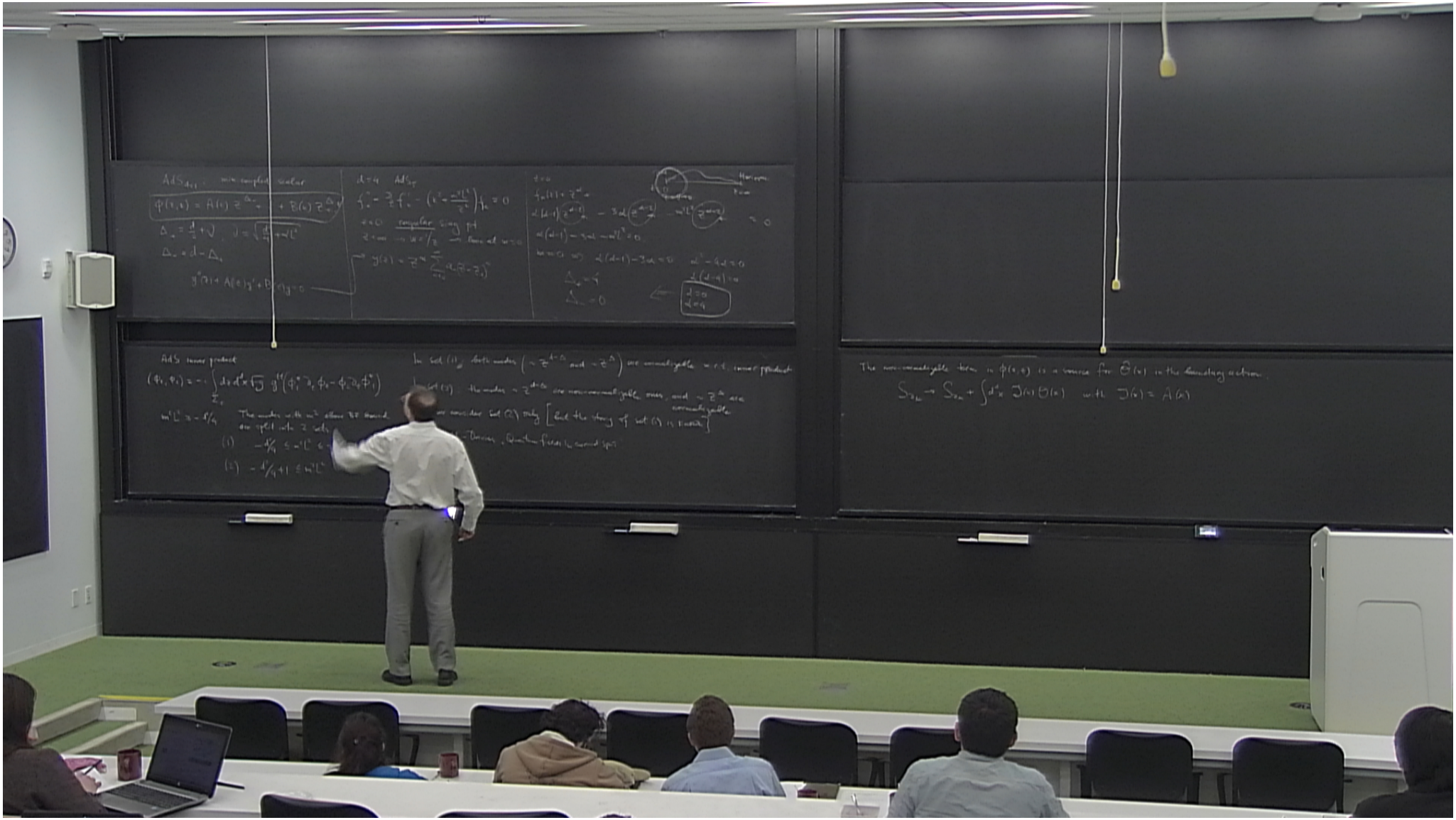

$d=4$ AdS_5
$f_k'' - \frac{3}{z} f_k' - \left(k^2 + \frac{m^2L^2}{z^2}\right) f_k = 0$
$z=0$ regular Sing pt
$z=\infty \rightarrow w = 1/z \rightarrow$ look at $w=0$
$y(z) = z^\alpha \sum_{n=0}^{\infty} a_n (z-z_0)^n$
$z=0$
$f_k(z) = z^\alpha + \dots$
$\alpha(\alpha-1) z^{\alpha-2} + \dots - 3\alpha z^{\alpha-2} + \dots - m^2L^2 z^\alpha$
$\alpha(\alpha-1) - 3\alpha - m^2L^2 = 0.$
$m=0 \Rightarrow \alpha(\alpha-1) - 3\alpha = 0$
$\alpha^2 - 4\alpha = 0$
$\alpha(\alpha-4) = 0$
$\alpha = 0$
$\alpha = 4.$
$\Delta_+ = 4$
$\Delta_- = 0$
Horizon
$z=0$
$z=\infty$

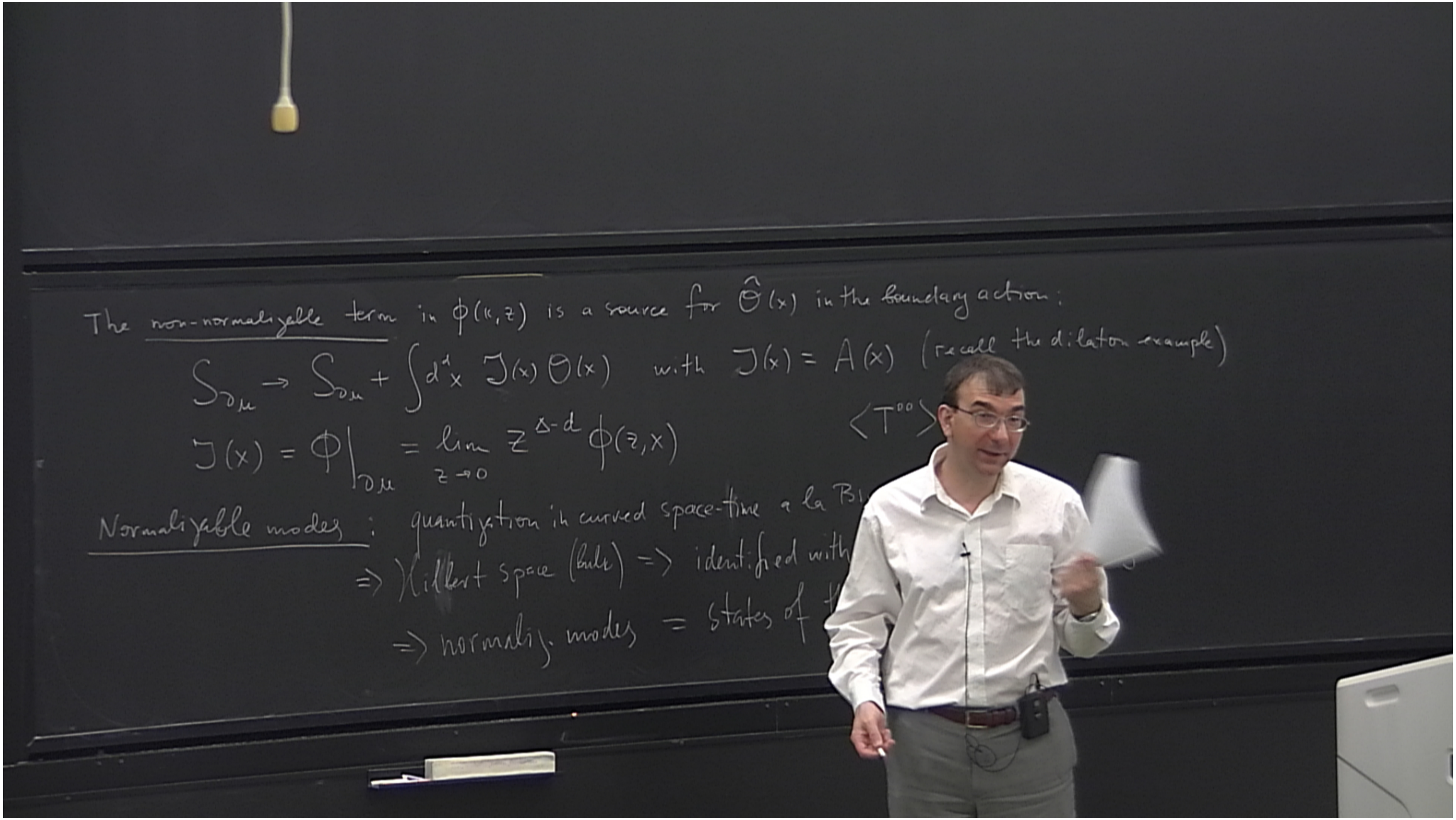

d = 4 AdS5
f_k'' − (3/z) f_k' − (k² + m²L²/z²) f_k = 0
z = 0 regular Sing pt.
z = ∞ → w = 1/z → look at w = 0
y(z) = z^α Σ_{n=0}^{∞} a_n (z − z_0)^n
z = 0
f_k(z) = z^α + ...
α(α−1) z^{α−2} + ... − 3α z^{α−2} + ...
α(α−1) − 3α − m²L² = 0.
m = 0 ⇒ α(α−1) − 3α = 0
Δ+ = 4
Δ− = 0
α = 0
α = 4.

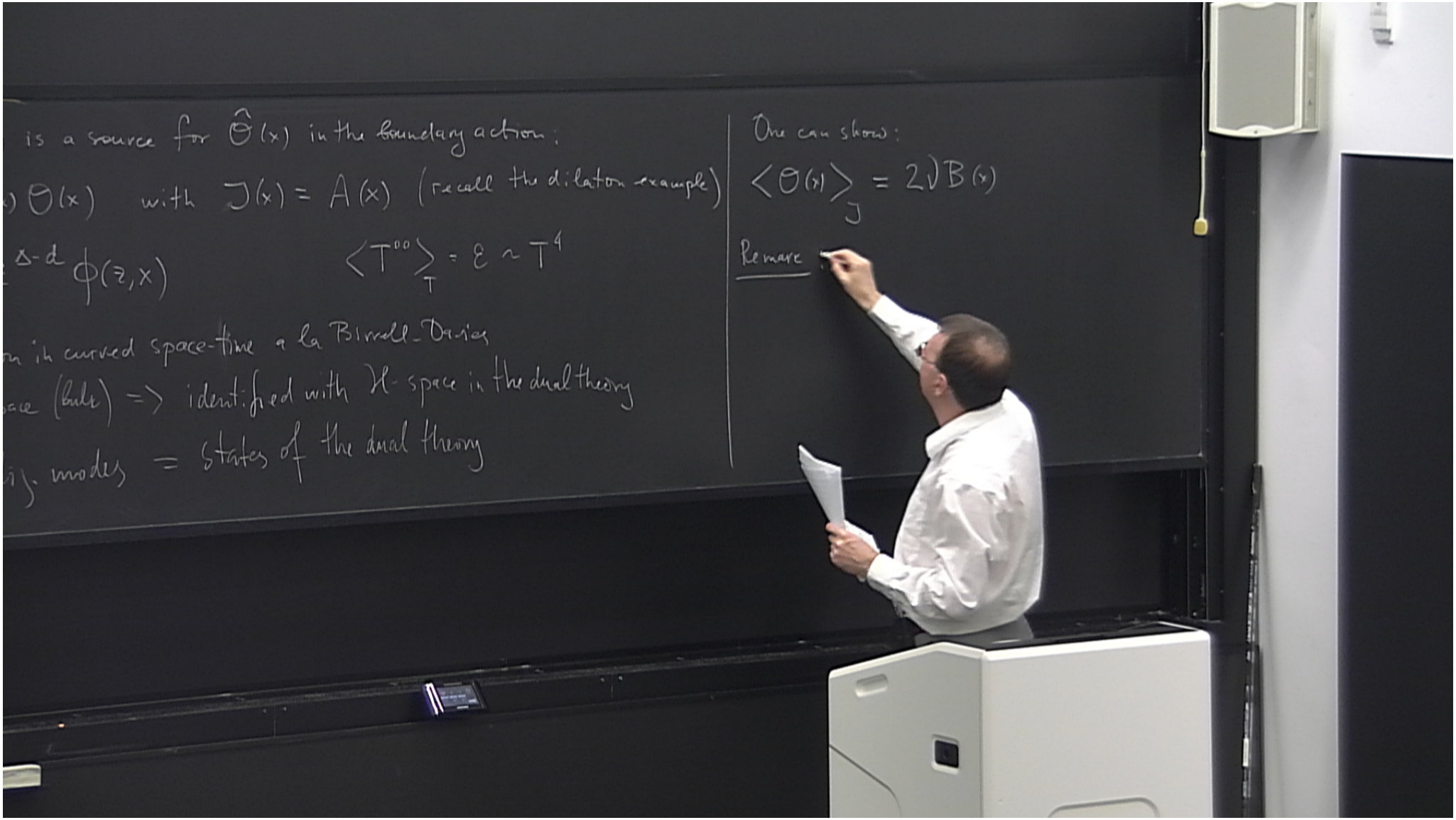

$y''(z) + A(z)y' + B(z)y = 0$
AdS inner product
$(\phi_1, \phi_2) = -\int dz\, d^d x \sqrt{-g}\; g^{tt}\left(\phi_1^* \partial_t \phi_2 - \phi_2 \partial_t \phi_1^*\right)$
$m^2 L^2 >$

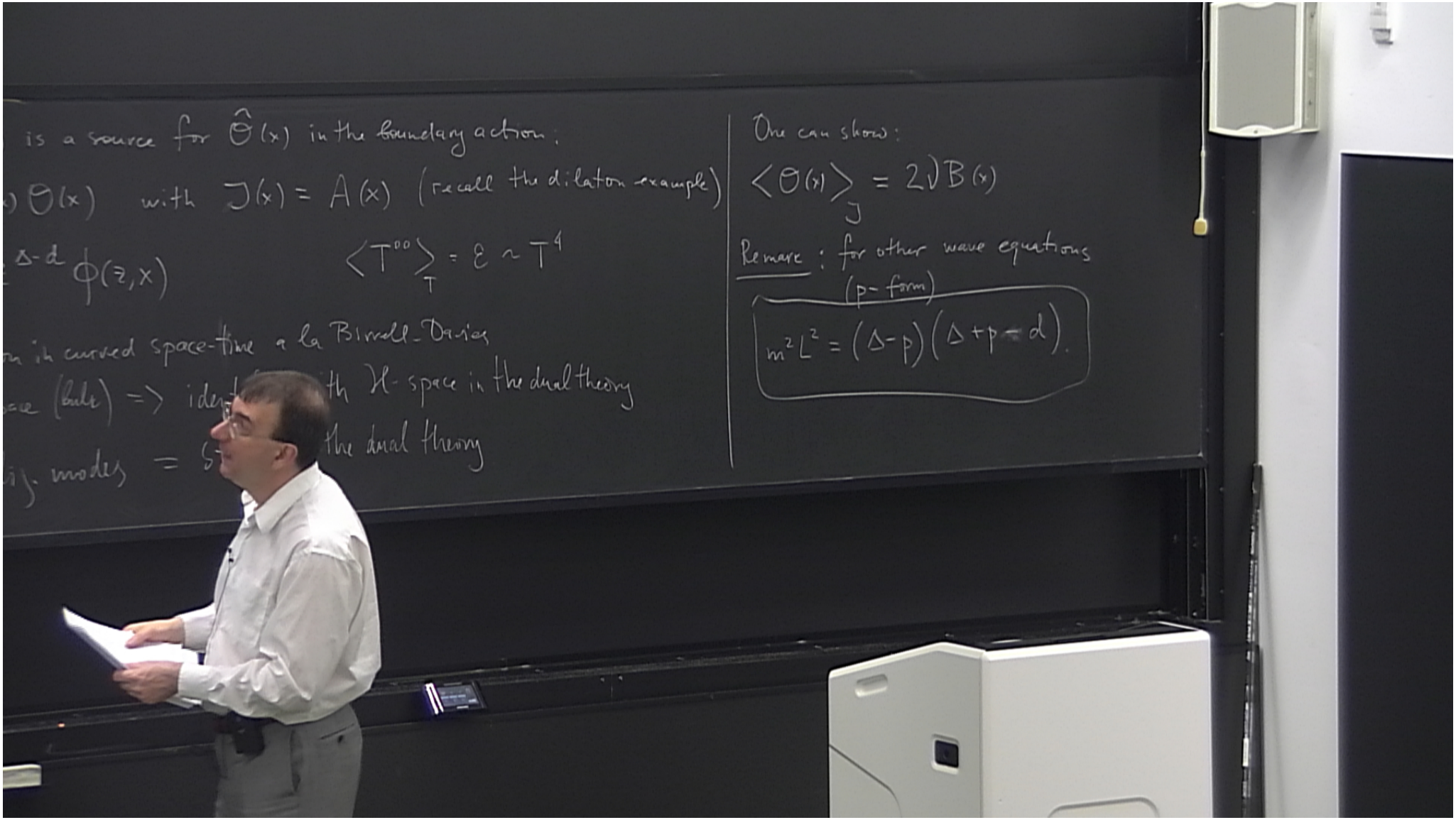

$y''(z) + A(z)y' + B(z)y = 0$
AdS inner product:
$(\phi_1, \phi_2) = -i \int_{\Sigma_t} dz\, d^d x \sqrt{-g}\; g^{tt} \left(\phi_1^* \partial_t \phi_2 - \phi_2 \partial_t \phi_1^*\right)$
$m^2 L^2 \geq -d^2/4$
The modes with m^2 above BF bound are split into 2 sets
(1) $-d^2/4 \leq m^2 L^2 \leq -d^2/4 + 1$
(2) $-d^2/4 + 1 \leq m^2 L^2$
In set (1), both modes

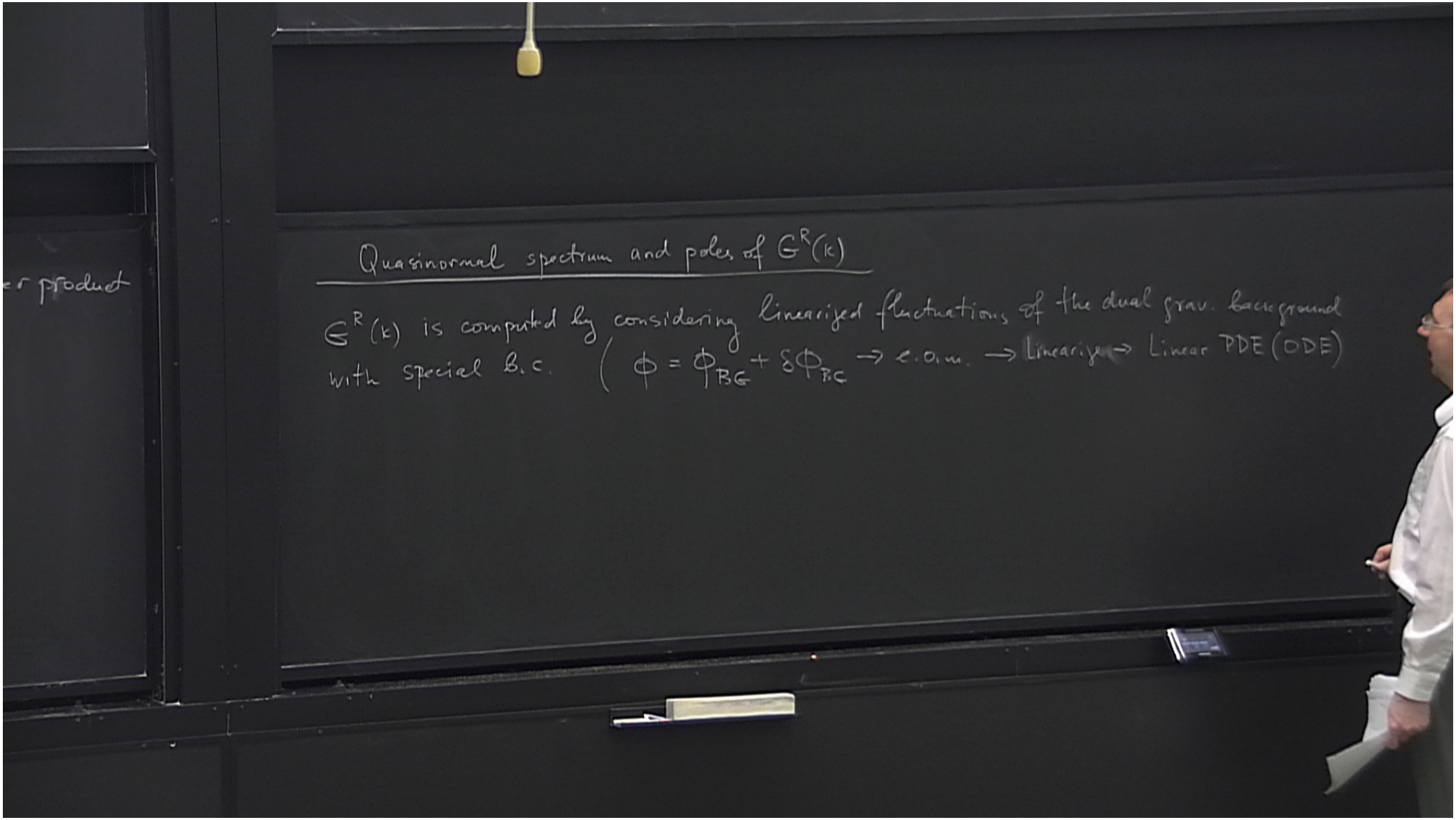

$y''(z) + A(z)y' + B(z)y = 0$
AdS inner product :
$(\phi_1, \phi_2) = -i \int dz\, d^d x \sqrt{-g}\; g^{tt} \left(\phi_1^* \partial_t \phi_2 - \phi_2 \partial_t \phi_1^* \right)$
$m^2 L^2 >$
The modes with m^2 above BF bound are split into 2 sets
(1) $-d^2/4 \leq m^2 L^2 \leq -d^2/4 + 1$
(2) $-d^2/4 + 1 \leq m^2 L^2$
In set (1) : both modes ($\sim z^{d-\Delta}$
In set (2) : the modes $\sim z^{d-\Delta}$ are
Here we consider set (2) only [
See: Birrell - Davies, Quantum fields

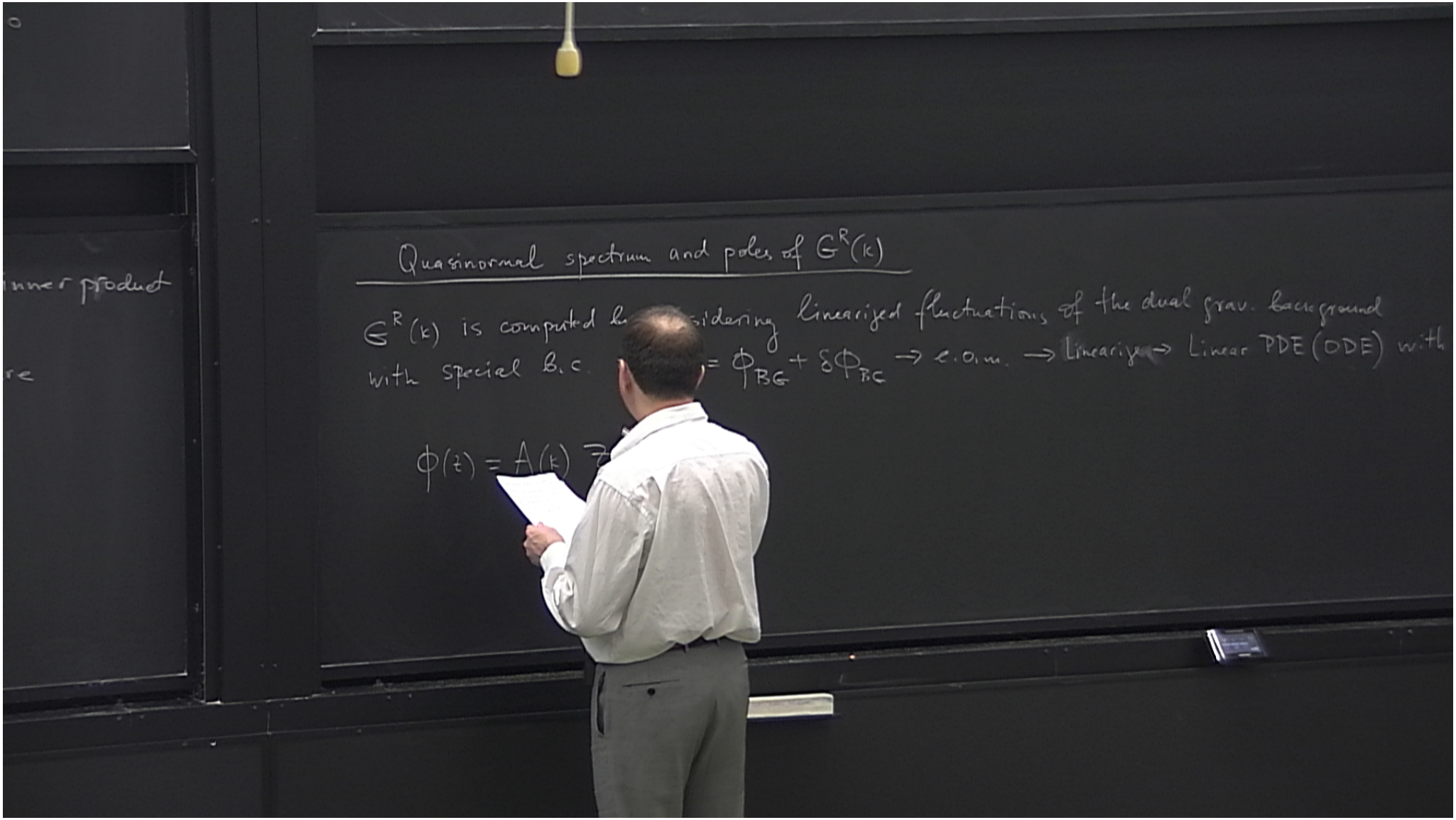


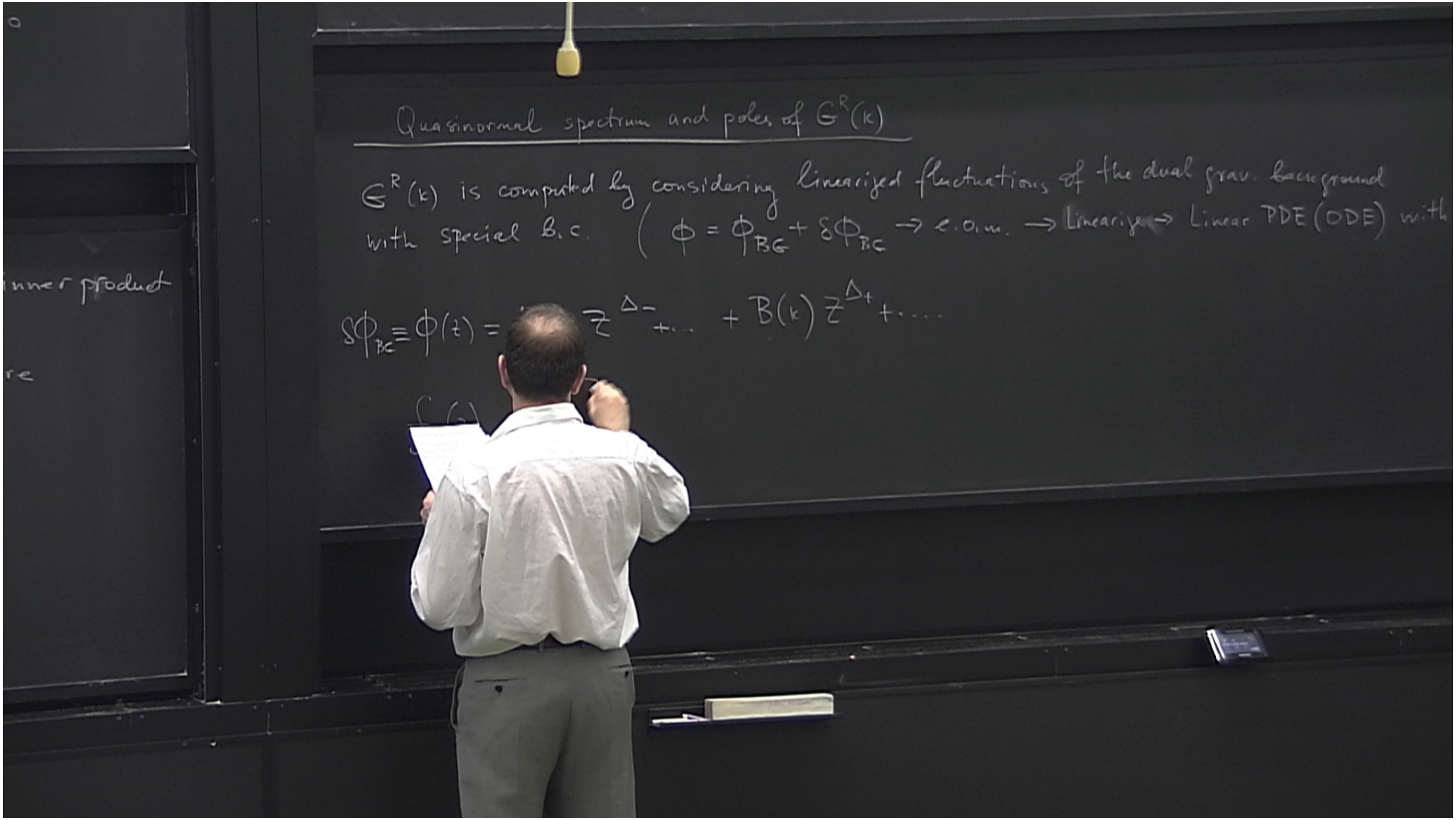

The non-normalizable term in $\phi(x,z)$ is a source for $\hat{\mathcal{O}}(x)$ in the boundary action:
$S_{\partial M} \to S_{\partial M} + \int d^d x\, J(x)\,\mathcal{O}(x)$ with $J(x) = A(x)$ (recall the dilaton example)
$J(x) = \phi|_{\partial M} = \lim_{z\to 0} z^{\Delta - d}\,\phi(z,x)$
$\langle T^{00} \rangle$
Normalizable modes: quantization in curved space-time a la Bi
$\Rightarrow$ Hilbert space (bulk) $\Rightarrow$ identified with
$\Rightarrow$ normaliz. modes = states of t

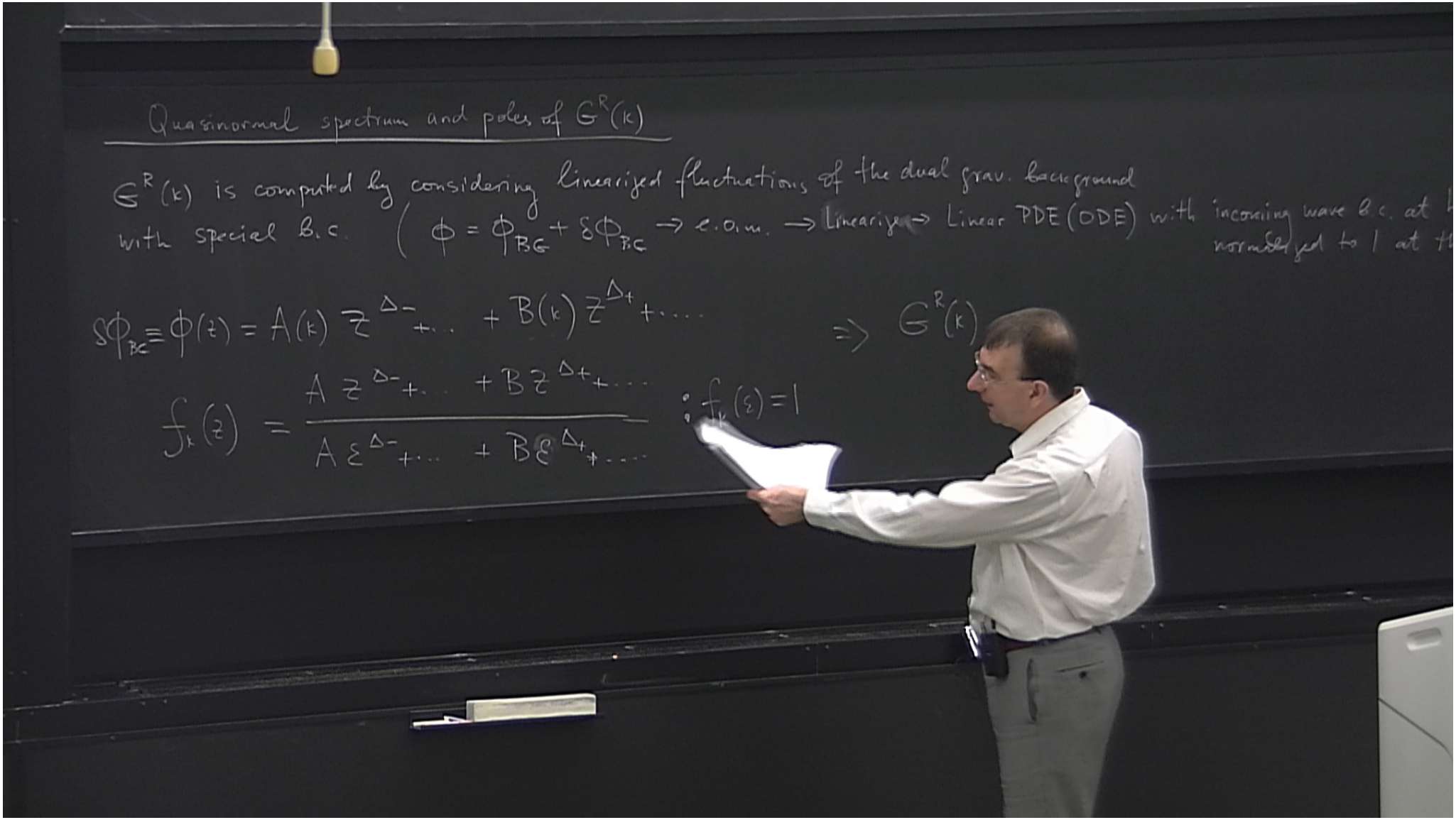

is a source for $\hat{\Theta}(x)$ in the boundary action:
$\Theta(x)$ with $J(x) = A(x)$ (recall the dilaton example)
Δ-d $\phi(z,x)$
$\langle T^{00} \rangle_T = \mathcal{E} \sim T^4$
in curved space-time a la Birrell-Davies
(bulk) => identified with $\mathcal{H}$-space in the dual theory
modes = states of the dual theory
One can show:
$\langle \Theta(x) \rangle_J = 2\sqrt{B}(x)$
Remark


is a source for $\hat{\mathcal{O}}(x)$ in the boundary action:
$\mathcal{O}(x)$ with $J(x) = A(x)$ (recall the dilaton example)
$\Delta - d$ $\phi(z,x)$
$\langle T^{00} \rangle_T = \mathcal{E} \sim T^4$
in curved space-time a la Birrell-Davies
(bulk) => ide th H-space in the dual theory
the dual theory
modes = S
One can show:
$\langle \mathcal{O}(x) \rangle_J = 2\sqrt{B}(x)$
Remark: for other wave equations
(p-form)
$m^2 L^2 = (\Delta - p)(\Delta + p - d)$.


Quasinormal spectrum and poles of $G^R(k)$
$G^R(k)$ is computed by considering linearized fluctuations of the dual grav. background
with special B.C. ($\phi = \phi_{BG} + \delta\phi_{BC}$ → e.o.m. → Linearize → Linear PDE (ODE)


Quasinormal spectrum and poles of $G^R(k)$
$G^R(k)$ is computed by considering linearized fluctuations of the dual grav. background
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$\phi(z) = A(k)$


Quasinormal spectrum and poles of $G^R(k)$
$G^R(k)$ is computed by considering linearized fluctuations of the dual grav. background
with special B.c. ($\phi = \phi_{BG} + \delta\phi_{BC}$ → e.o.m. → Linearity → Linear PDE (ODE) with
$\delta\phi_{BC} \equiv \phi(z) = \ldots z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
inner product


Quasinormal spectrum and poles of $G^R(k)$
$G^R(k)$ is computed by considering linearized fluctuations of the dual grav. background
with special B.c. ($\phi = \phi_{BG} + \delta\phi_{BC}$ → e.o.m. → Linearize → Linear PDE (ODE) with incoming wave B.c. at
normalized to 1 at
$\delta\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$ ⇒ $G^R(k)$
$f_k(z) = \dfrac{A\, z^{\Delta_-} + \ldots + B\, z^{\Delta_+} + \ldots}{A\, \varepsilon^{\Delta_-} + \ldots + B\, \varepsilon^{\Delta_+} + \ldots}$: $f_k(\varepsilon) = 1$

Quasinormal spectrum and poles of $G^R(k)$

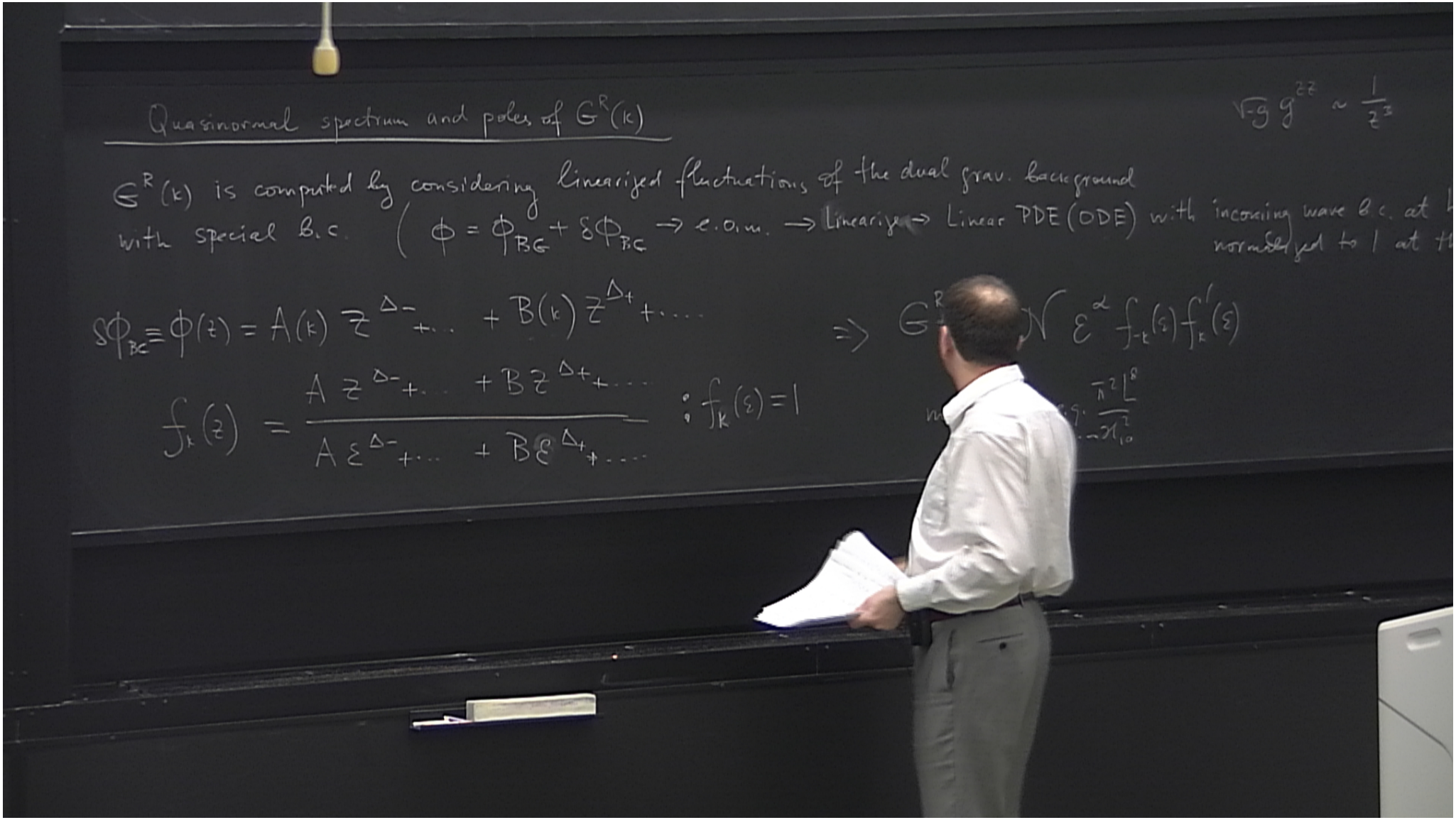
$\sqrt{-g}\, g^{zz} \sim \frac{1}{z^3}$

$G^R(k)$ is computed by considering linearized fluctuations of the dual grav. background with special B.c. ($\phi = \phi_{BG} + \delta\phi_{BC} \rightarrow$ e.o.m. $\rightarrow$ Linearize $\rightarrow$ Linear PDE (ODE) with incoming wave B.C. normalized to 1 at

$$\delta\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \dots + B(k)\, z^{\Delta_+} + \dots \quad \Rightarrow \quad G^R(k) \sim \mathcal{N}\, \varepsilon^{\alpha}$$

$$f_k(z) = \frac{A\, z^{\Delta_-} + \dots + B\, z^{\Delta_+} + \dots}{A\, \varepsilon^{\Delta_-} + \dots + B\, \varepsilon^{\Delta_+} + \dots} \quad ; f_k(\varepsilon) = 1$$

normalization e.g. $\frac{\bar{h}^2 L^8}{\kappa_{10}^2}$


Quasinormal spectrum and poles of $G^R(k)$
$\sqrt{-g}\, g^{zz} \sim \frac{1}{z^3}$
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$\delta\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$\Rightarrow G^R \sim \mathcal{N}\, \varepsilon^{\alpha} f_{-k}(\varepsilon) f_k'(\varepsilon)$
$f_k(z) = \frac{A z^{\Delta_-} + \ldots + B z^{\Delta_+} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots}$; $f_k(\varepsilon) = 1$

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$$\Rightarrow G^R(k) \sim \mathcal{N} \left(\varepsilon^{\alpha} f_{-k}(\varepsilon) \right) f_k'(\varepsilon)$$

normalization e.g. $\frac{\pi^2 L^8}{\ldots \pi_{10}^2}$

Quasinormal spectrum and poles of $G^R(k)$

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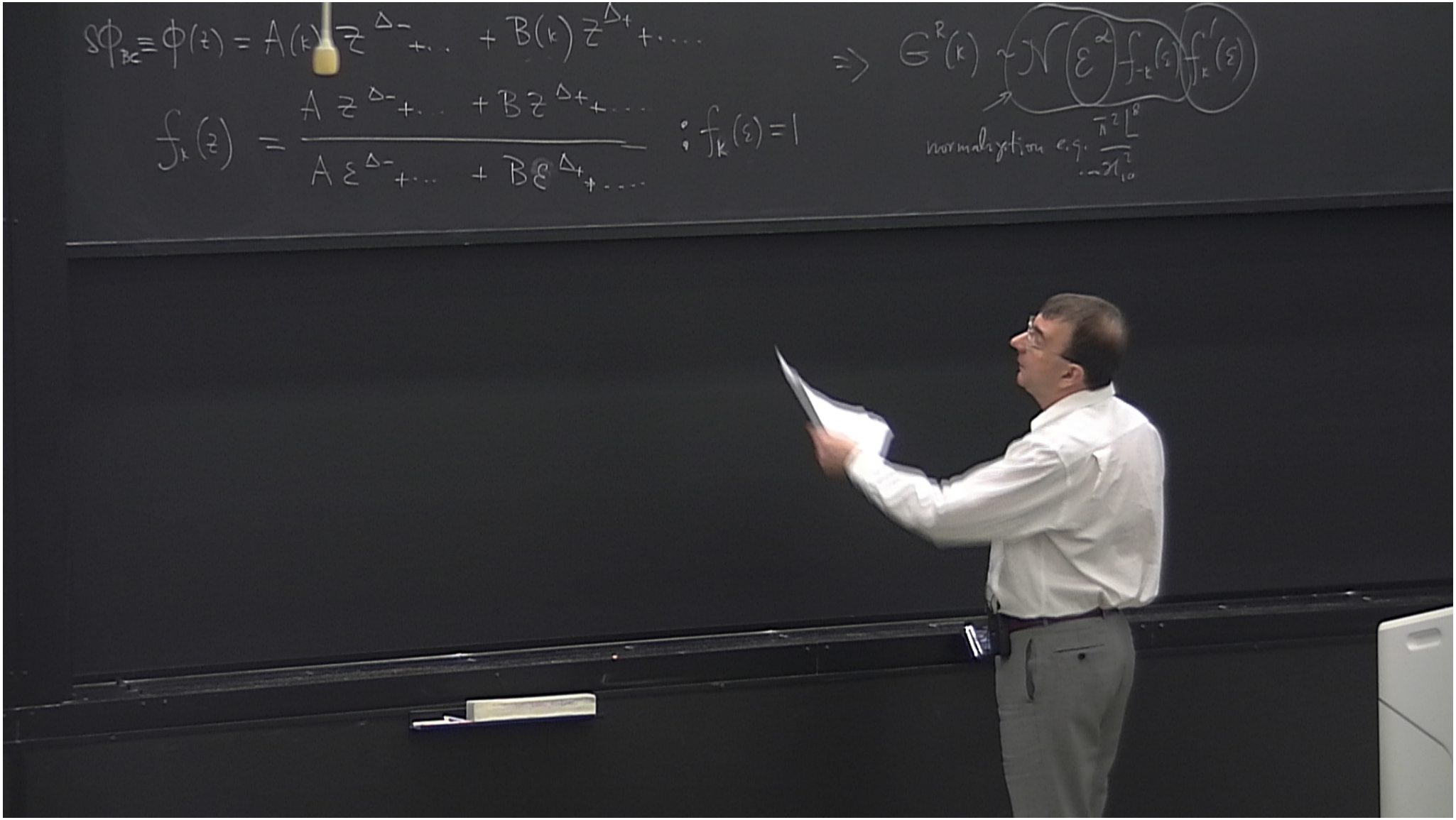
normalized to 1 at th

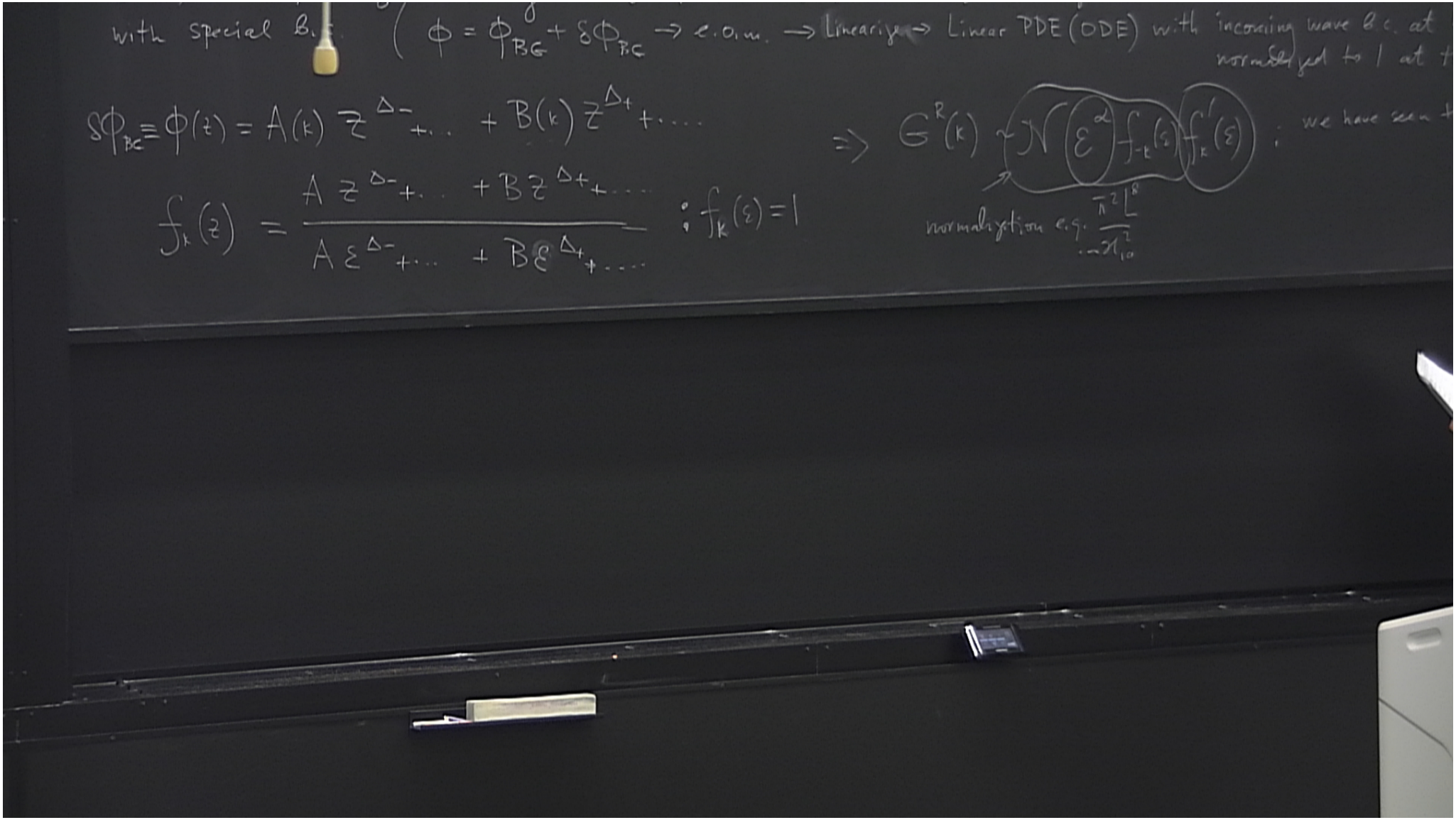
$$\delta\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$$

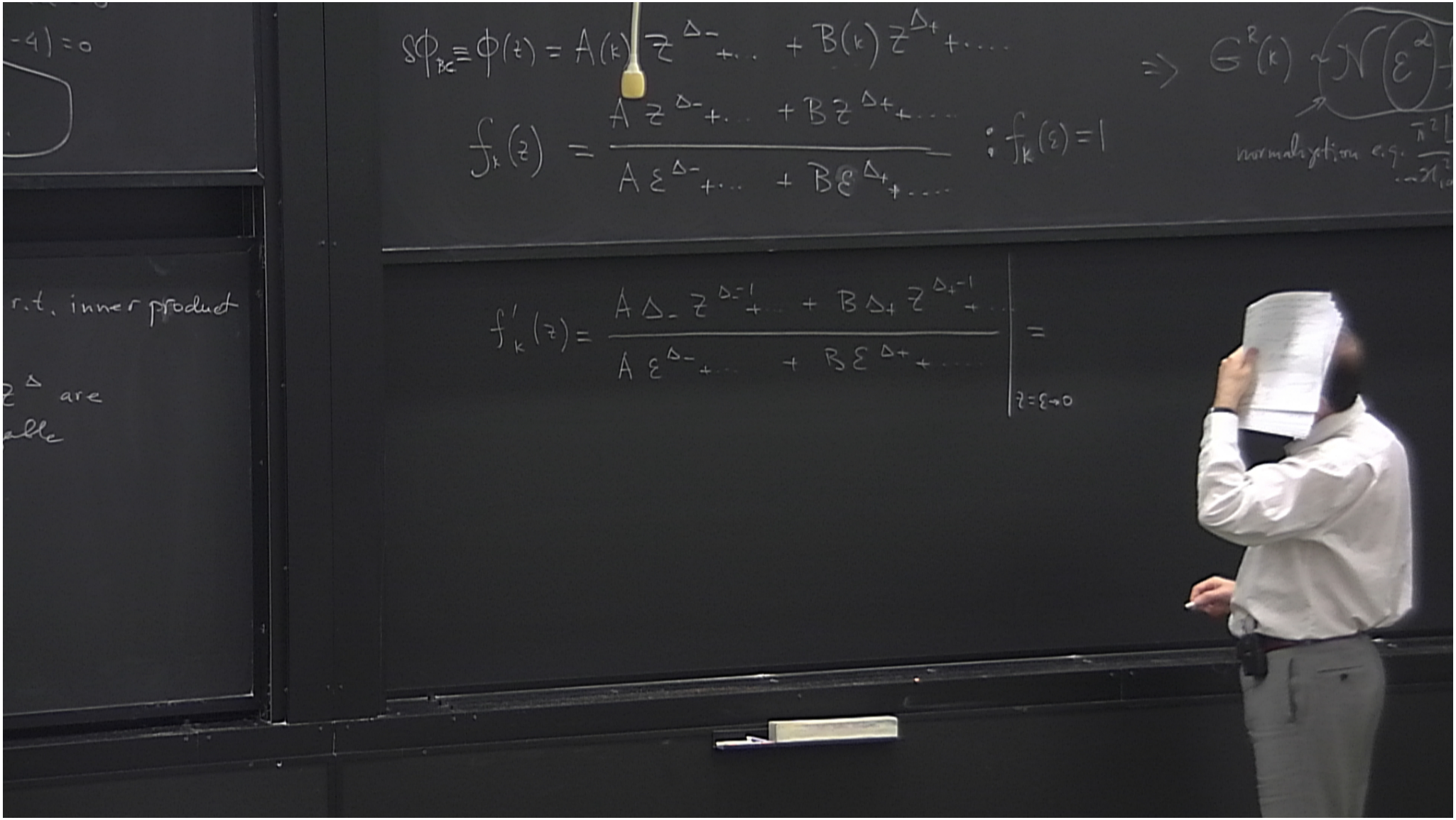
$$\Rightarrow G^R(k) \sim \mathcal{N} \left(\varepsilon^{\alpha} f_{-k}(\varepsilon) \right)$$

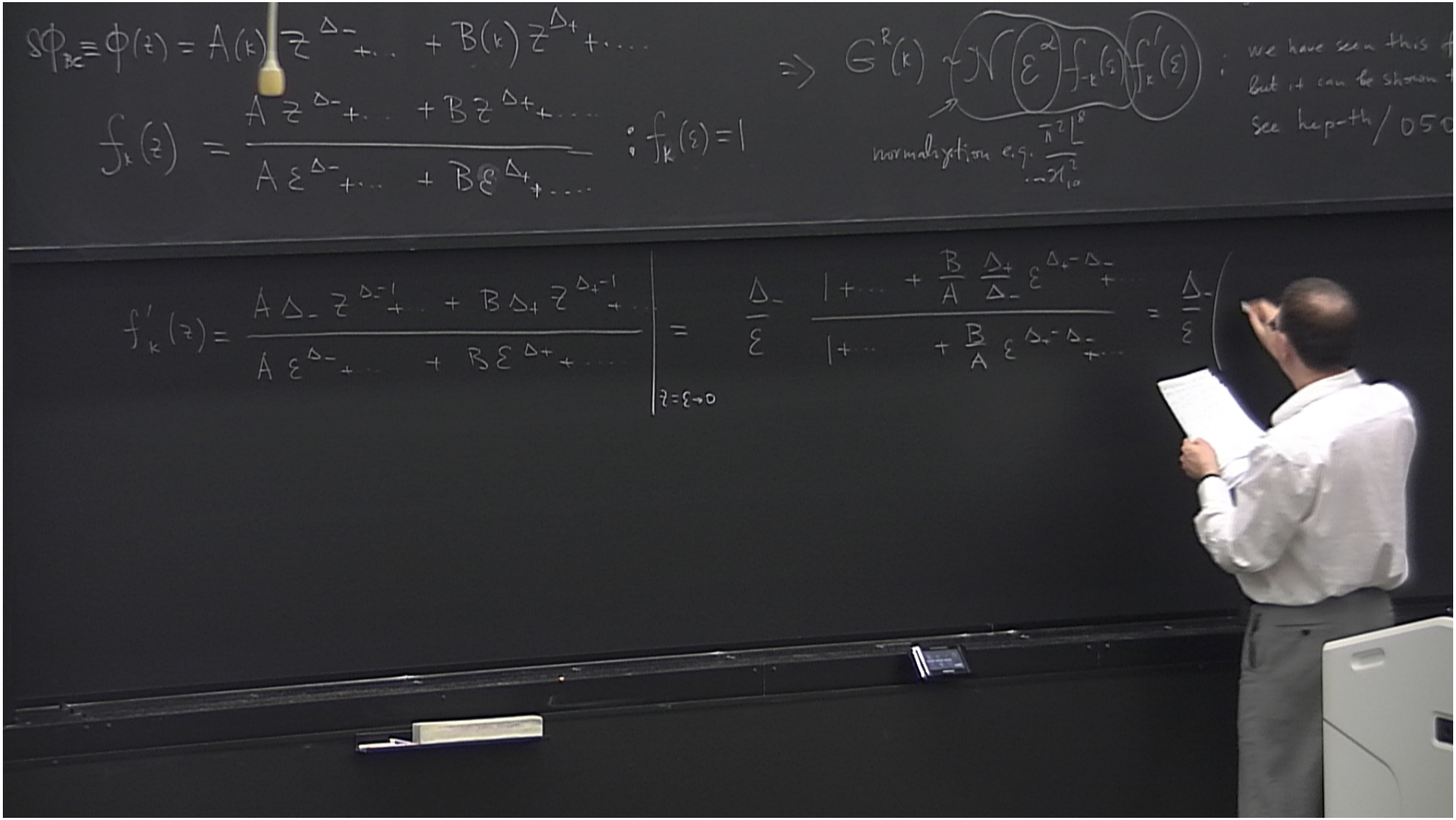
$$f_k(z) = \frac{A\, z^{\Delta_-} + \ldots + B\, z^{\Delta_+} + \ldots}{A\, \varepsilon^{\Delta_-} + \ldots + B\, \varepsilon^{\Delta_+} + \ldots} \quad ; f_k(\varepsilon) = 1$$

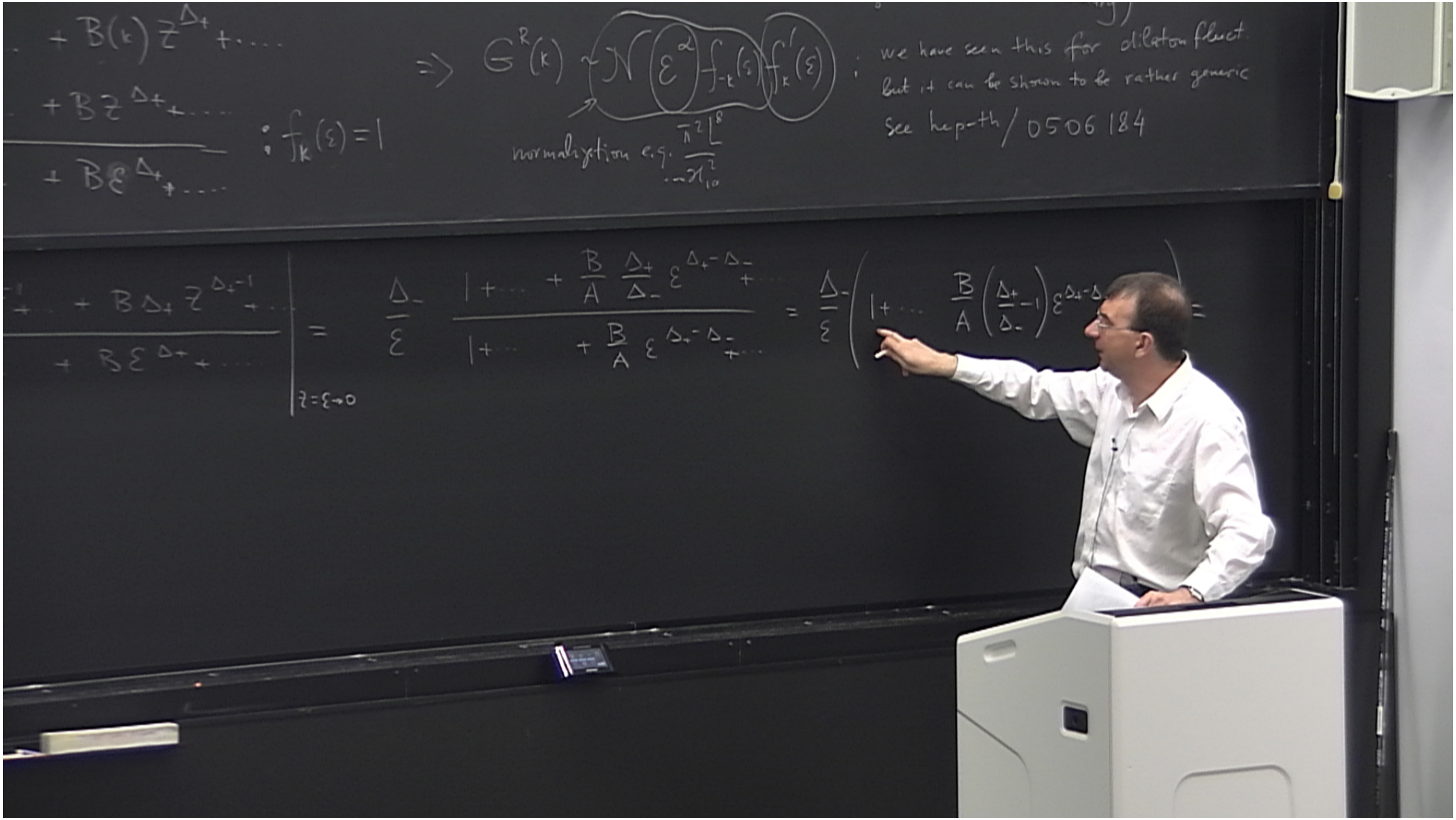
normalization e.g. $\frac{\bar{h}^2 L^8}{\ldots \pi_{10}^2}$

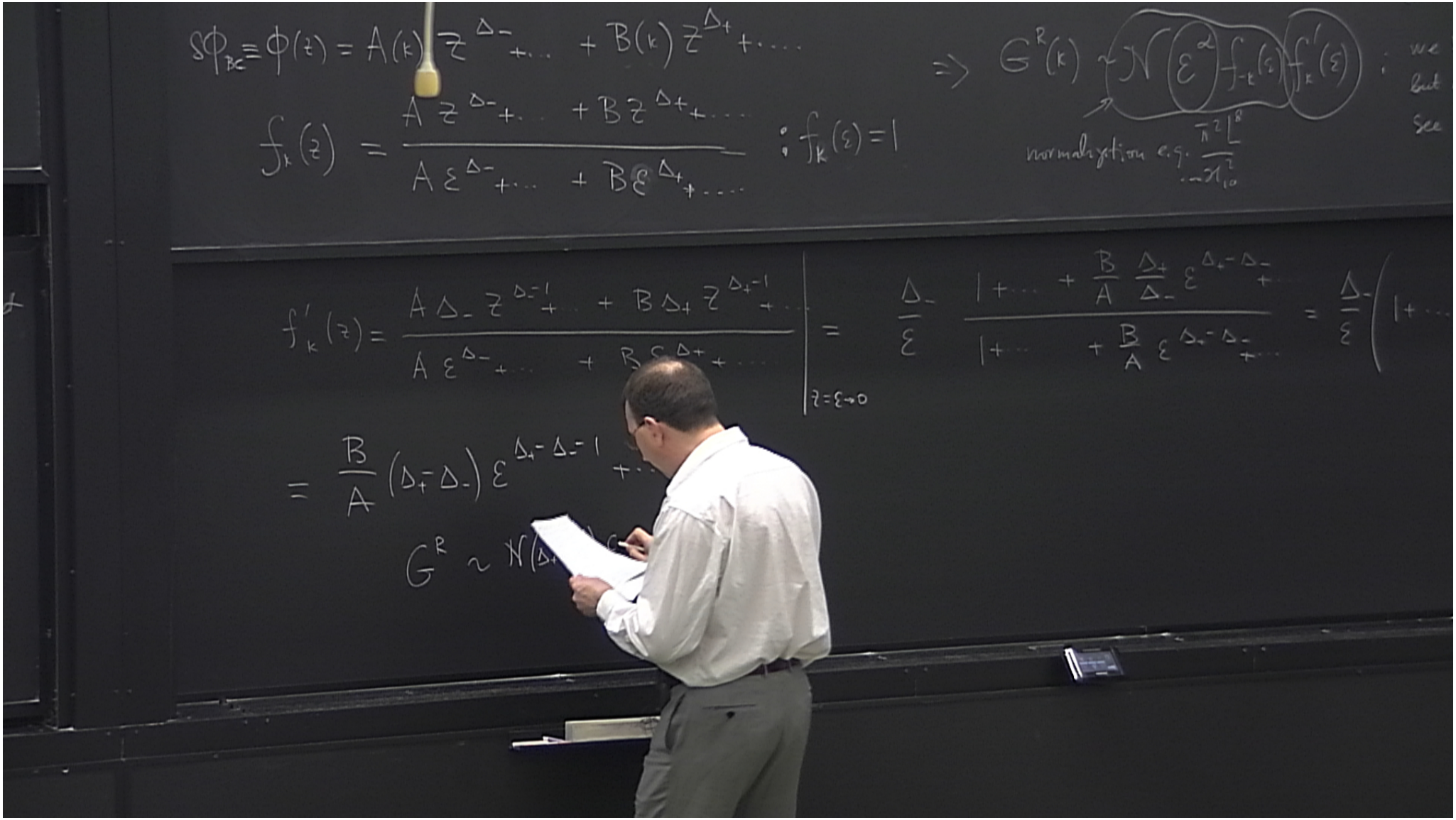

$S\phi_{BC} \equiv \phi(z) = A(k) z^{\Delta_-} + \ldots + B(k) z^{\Delta_+} + \ldots$
$\Rightarrow G^R(k) \sim \mathcal{N} \left(\varepsilon^{\alpha} f_{-k}(\varepsilon) f_k'(\varepsilon) \right)$
$f_k(z) = \frac{A z^{\Delta_-} + \ldots + B z^{\Delta_+} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots}$; $f_k(\varepsilon) = 1$
normalization e.g. $\frac{\bar{n}^2 L^8}{\ldots x_{12}^2}$

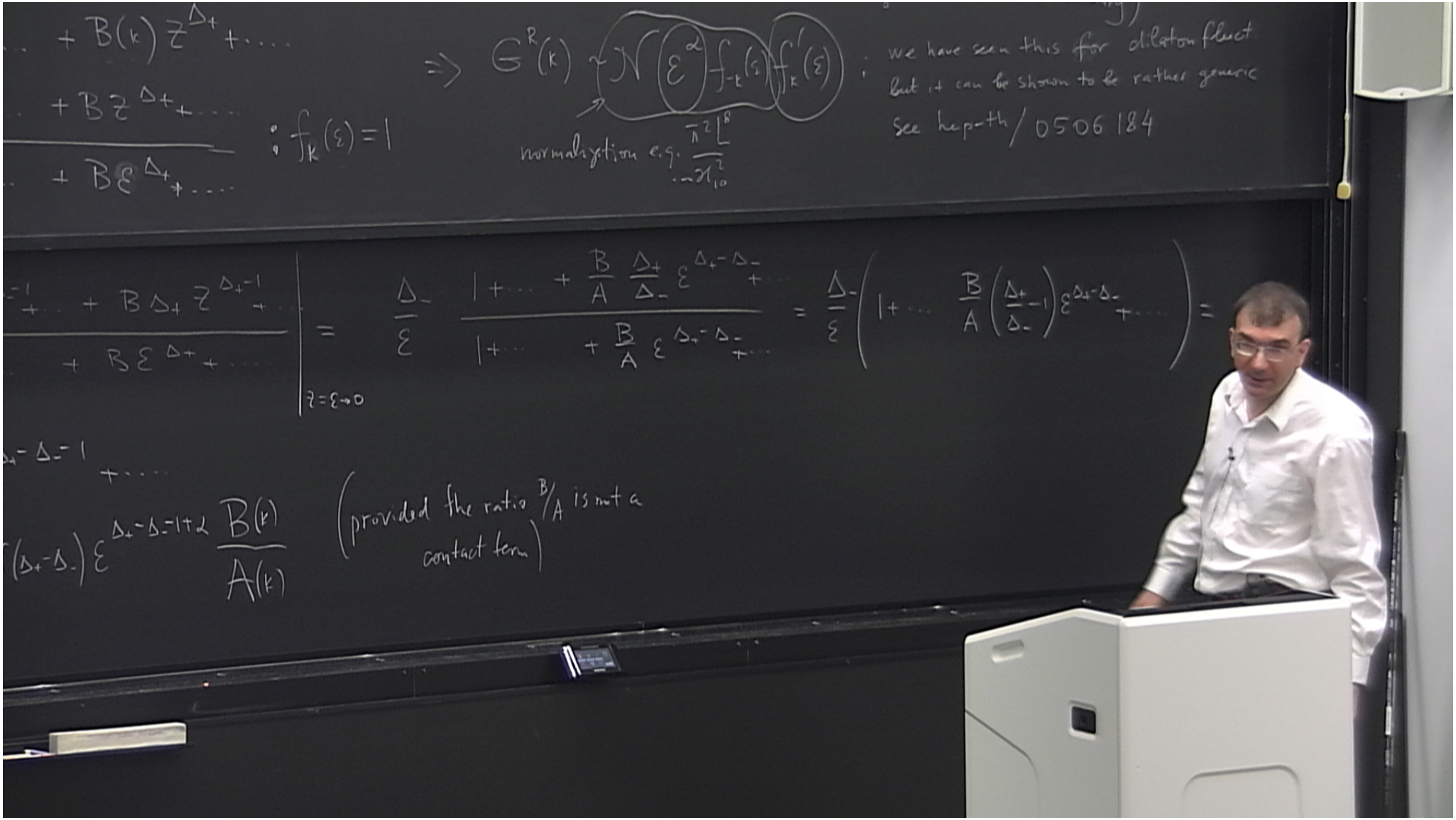

with special B.C.
$\phi = \phi_{BG} + \delta\phi_{B.C.}$ → e.o.m. → Linearize → Linear PDE (ODE) with incoming wave B.C. at
normalized to 1 at
$\delta\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$\Rightarrow G^R(k) \sim \mathcal{N}\left(\varepsilon^{\alpha}\, f_{-k}(\varepsilon)\, f_k(\varepsilon)\right)$; we have seen
$f_k(z) = \dfrac{A\, z^{\Delta_-} + \ldots + B\, z^{\Delta_+} + \ldots}{A\, \varepsilon^{\Delta_-} + \ldots + B\, \varepsilon^{\Delta_+} + \ldots}$; $f_k(\varepsilon) = 1$
normalization e.g. $\frac{\hbar^2 L^8}{\ldots \pi_{10}^2}$

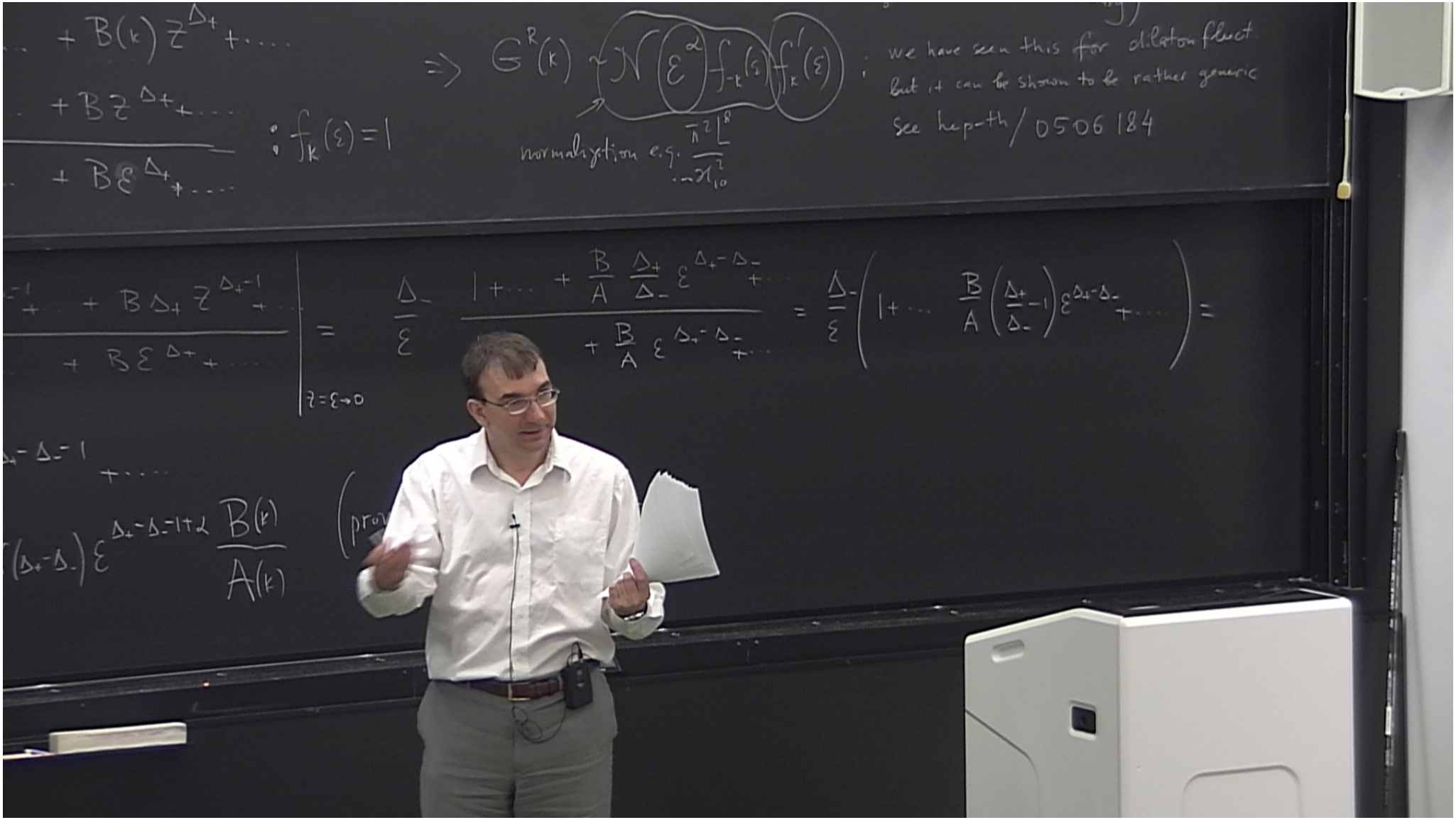

$\delta\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$\Rightarrow G^R(k) \sim \mathcal{N}\left(\varepsilon^{2}\right)$
$f_k(z) = \dfrac{A z^{\Delta_-} + \ldots + B z^{\Delta_+} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots} \; ; f_k(\varepsilon) = 1$
normalization e.g.
$f_k'(z) = \dfrac{A\Delta_- z^{\Delta_- - 1} + \ldots + B\Delta_+ z^{\Delta_+ - 1} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots} \Bigg| = \; z = \varepsilon \to 0$
r.t. inner product
z^{Δ} are


$s\phi_{BC} \equiv \phi(z) = A(k) z^{\Delta_-} + \ldots + B(k) z^{\Delta_+} + \ldots$
$f_k(z) = \frac{A z^{\Delta_-} + \ldots + B z^{\Delta_+} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots}$; $f_k(\varepsilon) = 1$
$\Rightarrow G^R(k) \sim \mathcal{N} \left(\varepsilon^2 f_{-k}(\varepsilon) \right) f_k'(\varepsilon)$
normalization e.g. $\frac{\bar{\Lambda}^2 L^8}{x_{10}^2}$
we have seen this
but it can be shown
see hep-th/05
$f_k'(z) = \frac{A\Delta_- z^{\Delta_- - 1} + \ldots + B\Delta_+ z^{\Delta_+ - 1} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots}$
$z = \varepsilon \to 0$
$= \frac{\Delta_-}{\varepsilon} \frac{1 + \ldots + \frac{B}{A} \frac{\Delta_+}{\Delta_-} \varepsilon^{\Delta_+ - \Delta_-} + \ldots}{1 + \ldots + \frac{B}{A} \varepsilon^{\Delta_+ - \Delta_-} + \ldots} = \frac{\Delta_-}{\varepsilon}$


$+B(k)z^{\Delta_+}+\cdots$
$+Bz^{\Delta_+}+\cdots$
$+B\varepsilon^{\Delta_+}+\cdots$
$\Rightarrow G^R(k)\sim\mathcal{N}\left(\varepsilon^{\alpha}f_{-k}(\varepsilon)f_k(\varepsilon)\right)$
$:f_k(\varepsilon)=1$
normalization e.g. $\frac{\bar{\Lambda}^2L^8}{\cdots\pi_{10}^2}$
we have seen this for dilaton fluct.
But it can be shown to be rather generic
See hep-th/0506184
$\frac{\cdots+B\Delta_+z^{\Delta_+-1}+\cdots}{\cdots+B\varepsilon^{\Delta_+}+\cdots}\Big|_{z=\varepsilon\to 0}=\frac{\Delta_-}{\varepsilon}\frac{1+\cdots+\frac{B}{A}\frac{\Delta_+}{\Delta_-}\varepsilon^{\Delta_+-\Delta_-}+\cdots}{1+\cdots+\frac{B}{A}\varepsilon^{\Delta_+-\Delta_-}+\cdots}=\frac{\Delta_-}{\varepsilon}\left(1+\cdots\frac{B}{A}\left(\frac{\Delta_+}{\Delta_-}-1\right)\varepsilon^{\Delta_+-\Delta_-}\right)=$


$S\phi_{BC} \equiv \phi(z) = A(k)\, z^{\Delta_-} + \ldots + B(k)\, z^{\Delta_+} + \ldots$
$f_k(z) = \dfrac{A z^{\Delta_-} + \ldots + B z^{\Delta_+} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots}$; $f_k(\varepsilon) = 1$
$\Rightarrow G^R(k) \sim \mathcal{N}\left(\varepsilon^2 f_{-k}(\varepsilon) f_k'(\varepsilon)\right)$; we but see
normalization e.g. $\frac{\pi^2 L^8}{\ldots \pi_{10}^2}$
$f_k'(z) = \dfrac{A\Delta_- z^{\Delta_- - 1} + \ldots + B\Delta_+ z^{\Delta_+ - 1} + \ldots}{A \varepsilon^{\Delta_-} + \ldots + B \varepsilon^{\Delta_+} + \ldots}$ $\Big|_{z=\varepsilon \to 0} = \dfrac{\Delta_-}{\varepsilon} \dfrac{1 + \ldots + \frac{B}{A}\frac{\Delta_+}{\Delta_-} \varepsilon^{\Delta_+ - \Delta_-} + \ldots}{1 + \ldots + \frac{B}{A} \varepsilon^{\Delta_+ - \Delta_-} + \ldots} = \dfrac{\Delta_-}{\varepsilon}\left(1 + \ldots\right.$
$= \dfrac{B}{A}(\Delta_+ - \Delta_-)\, \varepsilon^{\Delta_+ - \Delta_- - 1} + \ldots$
$G^R \sim \mathcal{N}(\Delta$


$+B(k)z^{\Delta_+}+\dots$
$+Bz^{\Delta_+}+\dots$
$+B\varepsilon^{\Delta_+}+\dots$
$\Rightarrow G^R(k) \sim \mathcal{N}\left(\varepsilon^{\alpha} f_{-k}(\varepsilon) f_k'(\varepsilon)\right)$
$: f_k(\varepsilon)=1$
normalization e.g. $\frac{\bar{\Lambda}^2 L^\beta}{\dots x_{10}^2}$
we have seen this for dilaton fluct
but it can be shown to be rather generic
See hep-th/0506184
$\frac{\Delta_-}{\varepsilon} \frac{1+\dots+\frac{B}{A}\frac{\Delta_+}{\Delta_-}\varepsilon^{\Delta_+-\Delta_-}+\dots}{1+\dots+\frac{B}{A}\varepsilon^{\Delta_+-\Delta_-}+\dots} = \frac{\Delta_-}{\varepsilon}\left(1+\dots\frac{B}{A}\left(\frac{\Delta_+}{\Delta_-}-1\right)\varepsilon^{\Delta_+-\Delta_-}+\dots\right)=$
$z=\varepsilon\to 0$
$\Delta_+-\Delta_-=1$
$(\Delta_+-\Delta_-)\varepsilon^{\Delta_+-\Delta_-=1+\alpha}\frac{B(k)}{A(k)}$
(provided the ratio B/A is not a contact term)


$+B(k)z^{\Delta_+}+\dots$
$+Bz^{\Delta_+}+\dots$
$+B\varepsilon^{\Delta_+}+\dots$
$\Rightarrow G^R(k)\sim \mathcal{N}\left(\varepsilon^{\alpha} f_{-k}(\varepsilon) f_k'(\varepsilon)\right)$
$: f_k(\varepsilon)=1$
normalization e.g. $\frac{\bar{\Lambda}^2 L^\delta}{x_{10}^2}$
we have seen this for dilaton fluct
but it can be shown to be rather generic
See hep-th/0506184
$z=\varepsilon\to 0$
$= \frac{\Delta_-}{\varepsilon}\frac{1+\dots+\frac{B}{A}\frac{\Delta_+}{\Delta_-}\varepsilon^{\Delta_+-\Delta_-}+\dots}{+\frac{B}{A}\varepsilon^{\Delta_+-\Delta_-}+\dots} = \frac{\Delta_-}{\varepsilon}\left(1+\dots\frac{B}{A}\left(\frac{\Delta_+}{\Delta_-}-1\right)\varepsilon^{\Delta_+-\Delta_-}+\dots\right)=$
$\Delta_+-\Delta_-=1$
$(\Delta_+-\Delta_-)\varepsilon^{\Delta_+-\Delta_-=1+\alpha}\frac{B(k)}{A(k)}$

$m^2L^2 \geq -d^2/4$

The modes with m^2 above BF bound are split into 2 sets

(1) $\quad -d^2/4 \leq m^2L^2 \leq -d^2/4 + 1$

(2) $\quad -d^2/4 + 1 \leq m^2L^2$

See: Birrell-Davies, Quan

But zeros of $A(k)$ have other meaning in GR: these are Dirichlet & c. on $\phi(z)$ a

$$\phi = A z^{\Delta_-} \ldots + B z^{\Delta_+} \ldots$$

Quasin

...es with m^2 above BF bound
...it into 2 sets

$-d^2/4 \leq m^2L^2 \leq -d^2/4 + 1$

$-d^2/4 + 1 \leq m^2L^2$

Here we consider set (2) only [but the story of set (1) is known]

See: Birrell-Davies, "Quantum fields in curved space"

...ther meaning in GR: these are Dirichlet b.c. on $\phi(z)$ at ∂M

$A z^{\Delta_-} + \ldots + B z^{\Delta_+} \ldots$

solutions of linearized EOM satisfying the inc. wave b.c. at the horizon and Dirichlet cond. at ∂M (or sometimes other b.c. — e.g. outgoing waves in asympt. flat case)

determined by $A(k) = 0$ or $A(\omega, \vec{k}) = 0 \Rightarrow \boxed{\omega = \omega(k)}$

... usually, discrete spectrum

... ω ... $= 1, \ldots \infty$

e.g. ...

above BF bound

$-d^2/4 + 1$

Here we consider set (2) only [but the story of set (1) is known]

see: Birrell-Davies, "Quantum fields in curved space"

in GR: these are Dirichlet b.c. on $\phi(z)$ at ∂M

$+ B z^{\Delta_+} \ldots$

linearized BG e.o.m. satisfying the inc. wave b.c. at the horizon

+ cond. at ∂M (or sometimes other b.c. — e.g. outgoing

symptot. flat case).

by $A(k) = 0$ or $A(\omega, \vec{k}) = 0 \Rightarrow \boxed{\omega = \omega(k)}$

- usually, discrete spectrum
- ω_n $n = 1, \ldots \infty$

e.g. in $d=4$ Sch[illegible]ld:

