Title: Naturalness and the Weak Gravity Conjecture

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Abstract: The weak gravity conjecture (WGC) asserts a powerful consistency condition on gauge theories coupled to quantum gravity: an Abelian, long-range force requires a state of charge q and mass m such that q > m/mPl. Failure of this condition implies the existence of stable black hole remnants and is in tension with no-hair theorems. In this paper, we argue that the WGC creates a non- perturbative obstruction to naturalness, which is the notion that dimensionless coefficients should take on O(1) values in the absence of enhanced symmetry. As an illustration, we show that for scalar quantum electrodynamics, a natural spectrum can actually be forbidden by the WGC, which bounds a radiatively unstable quantity, m, by a radiatively stable quantity, q. More generally, the WGC can be at odds with naturalness in any theory containing charged fundamental scalars. We extend the conditions of the WGC to more complicated theories with multiple gauge symmetries and particles. Finally, we discuss implications for the hierarchy problem and construct a simple model in which the natural value of the electroweak scaleâ€”at the cutoffâ€”is forbidden by the WGC.

Naturalness and the Weak Gravity Conjecture

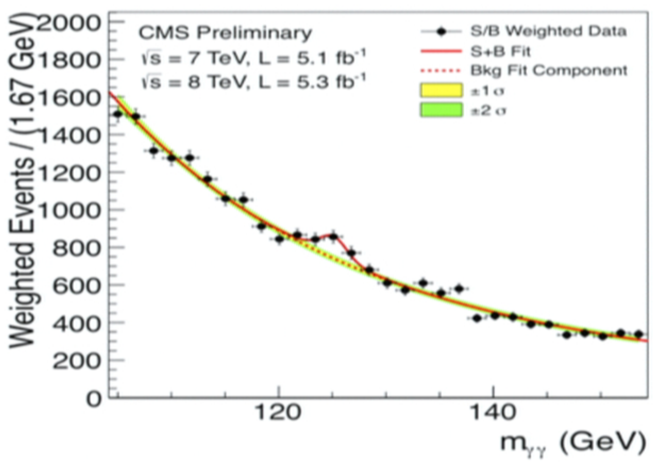
Clifford Cheung

California Institute of Technology 1891

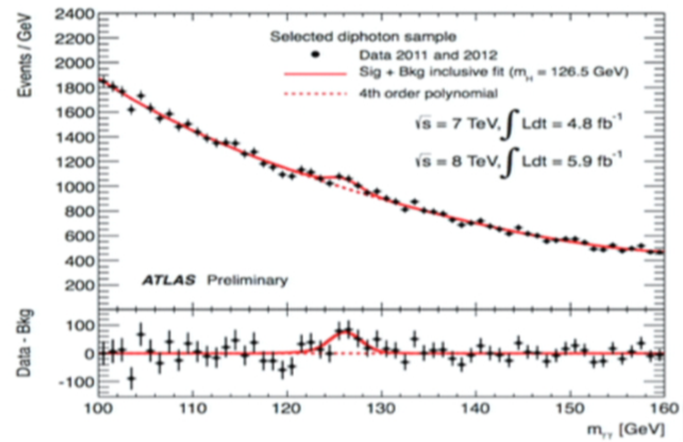
C. Cheung, G. N. Remmen (1402.2287, 14xx.xxxx)

LHC has discovered a new scalar.

CMS



ATLAS



Naturalness has been the foremost guide for new physics for decades.

But age-old ideas are now being revisited:

- regulator abra cadabra
- modified naturalness principles

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- regulator abra cadabra ⟵ (denial)
- modified naturalness principles
- ultraviolet conformal symmetry
- meso-tuning

(bargaining) — modified naturalness principles, ultraviolet conformal symmetry

weak gravity conjecture (WGC)

(Arkani-Hamed, Motl, Nicolis, Vafa)

A long-range U(1) coupled consistently to gravity *requires* a state with

$$q > m/m_{\mathrm{Pl}}$$

which is a non-perturbative, highly non-trivial criterion for healthy theories. In short:

"Gravity is the weakest force."

evidence #1

The WGC is satisfied by a litany of healthy field and string theories.

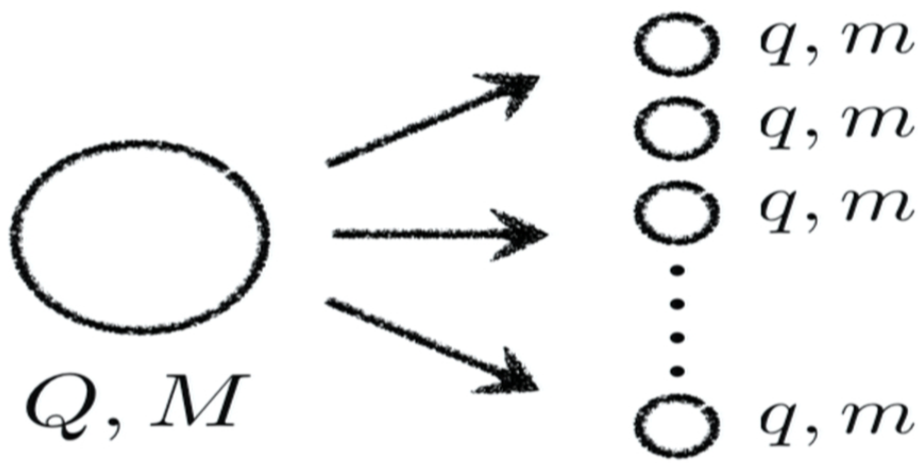
For example, for SU(2) → U(1) gauge theory,

$$g > m_W/m_{\mathrm{Pl}} \xrightarrow{(m_W = gv)} m_{\mathrm{Pl}} > v$$

and similarly for the monopoles.

evidence #3

The authors of the WGC justified it with a Gedanken experiment with black holes:



number of particles in final state $= \quad Q/q$ conservation of charge

total rest mass in final state $= \quad mQ/q < M$ conservation of energy

For an extremal black hole, $Q = M/m_{\rm Pl}$, so

$$q > m/m_{\rm Pl}$$

When the WGC criterion fails, extremal black holes are exactly stable.

In such a theory there will be a huge number of stable black hole remnants.

This yields serious pathologies:

- thermodynamic catastrophes
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The story is the same with many charged species. We define convenient notation:

$$z_i = q_i m_{\rm Pl}/m_i$$ (particle species i)

$$Z = Q\, m_{\rm Pl}/M = 1$$ (extremal black hole)

So the WGC states there there must exist a particle species i for which:

$$z_i > 1$$

evidence #4 (new)

Failure of the WGC should yield pathologies which are visible at low energies.

Integrate out all but the photon and graviton:

$$\mathcal{L}_F = a_1 (F_{\mu\nu}F^{\mu\nu})^2 + a_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

$$\mathcal{L}_{FR} = b_1 F_{\mu\nu}F^{\mu\nu}R + b_2 F_{\mu\rho}F_{\nu}{}^{\rho}R^{\mu\nu} + b_3 F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma}$$

$$\mathcal{L}_R = c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Crucially, the coefficients of the effective action encode important information.

$$aF^4 \qquad a \sim z^4$$

$$bF^2 R \qquad b \sim z^2$$

$$cR^2 \qquad c \sim 1$$

How are a, b, c constrained at low energies?

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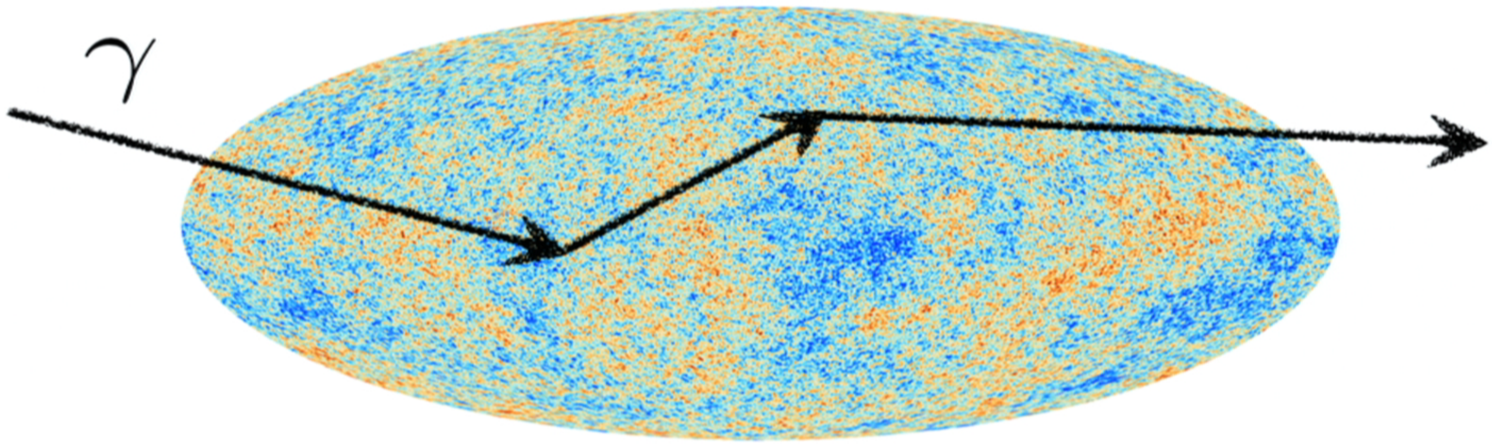
Photon propagation is modified in a radiation dominated FRW universe.



Setting $\langle F^2 \rangle, \langle R \rangle \neq 0$, we find that photons are superluminal unless $z \gtrsim \mathrm{const}$!

But we have ignored a crucial effect, which is that charges and masses are loop corrected!

$$q(\mu) > m(\mu)/m_{\rm Pl}$$

renormalized quantities

We should evaluate quantities at pole mass.

Note: WGC can bound a radiatively *unstable* quantity (mass) by a *stable* one (charge).

scalar QED

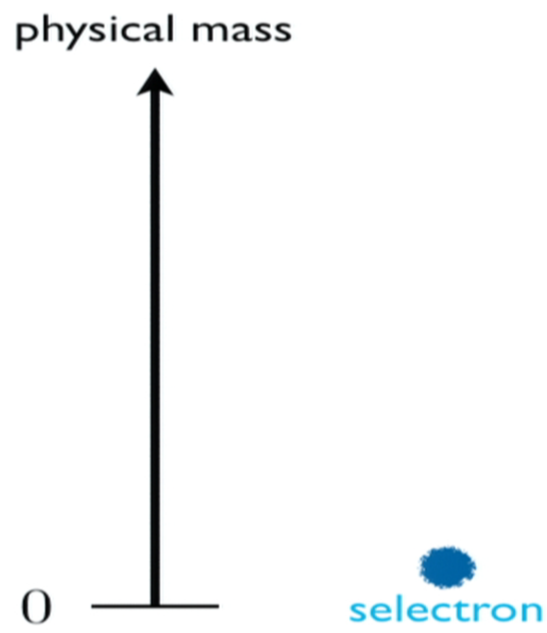
Take the very simplest case of a U(1) charged particle with a hierarchy problem:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu \phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

where the "selectron" has charge q:

$$D_\mu = \partial_\mu + iqA_\mu$$

There is fundamental tension between naturalness and the WGC.



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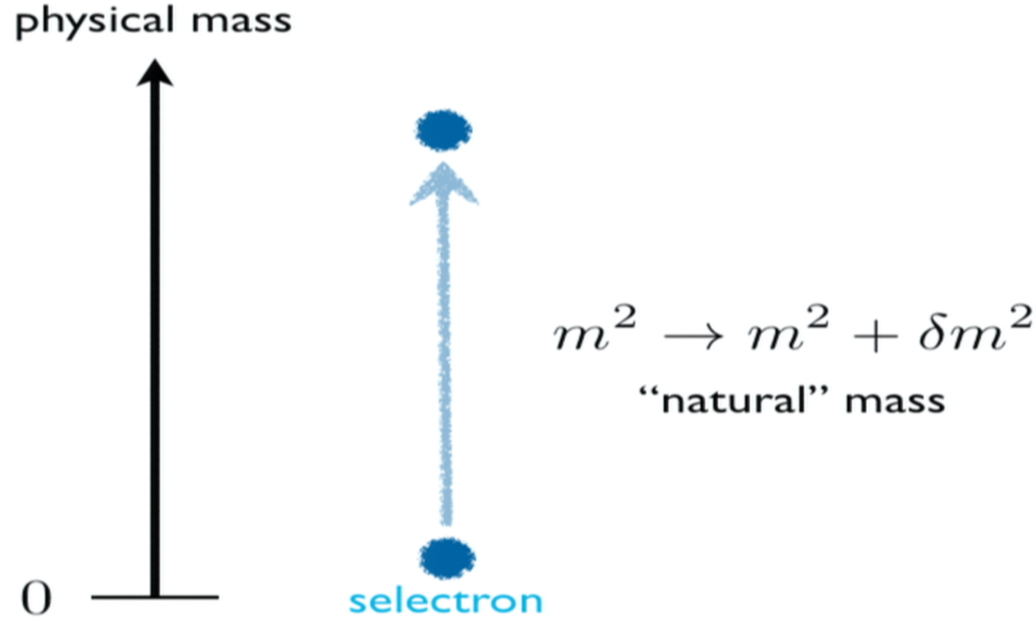

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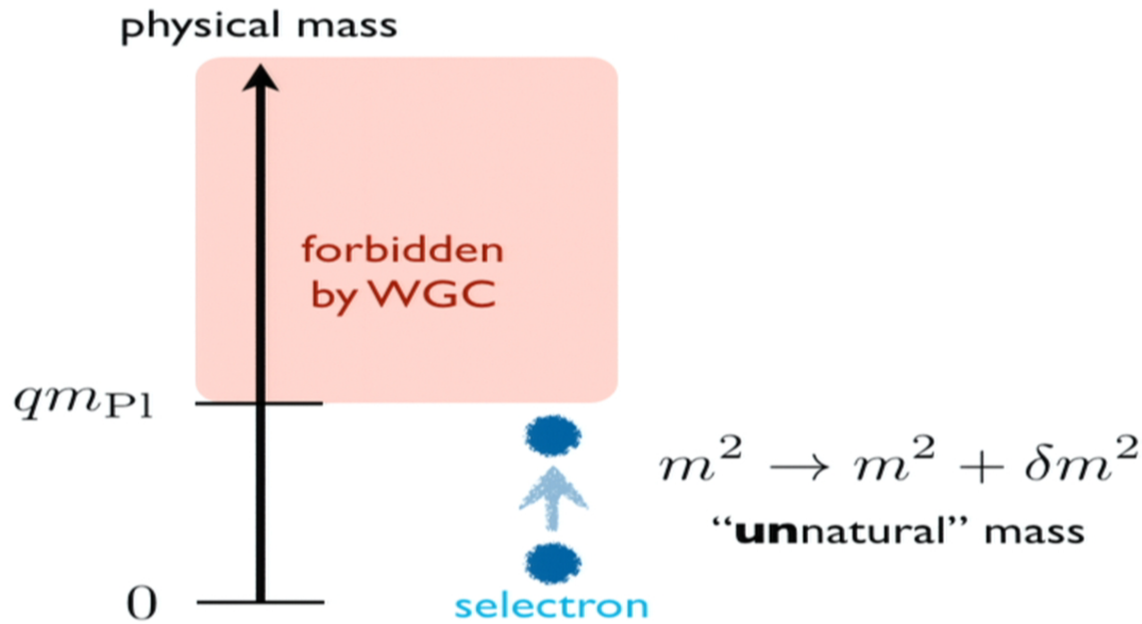

Let's quantify the tension.

$$m^2 \to m^2 + \delta m^2$$

$$\delta m^2 = \frac{\Lambda^2}{16\pi^2}(aq^2 + b\lambda)$$

incalculable coefficients

Naturalness principle: absent symmetries, the physical mass squared is $\sim \delta m^2$, so a, b are $\mathcal{O}(1)$ coefficients.

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So, the loop cutoff is bounded from above.

$$\Lambda < \frac{4\pi m_{\rm Pl}}{\sqrt{a}} \qquad q^2 \gg \lambda$$

$$\Lambda < 4\pi m_{\rm Pl}\sqrt{\frac{q^2}{b\lambda}} \qquad q^2 \ll \lambda$$

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reasonable: cutoff below Planck

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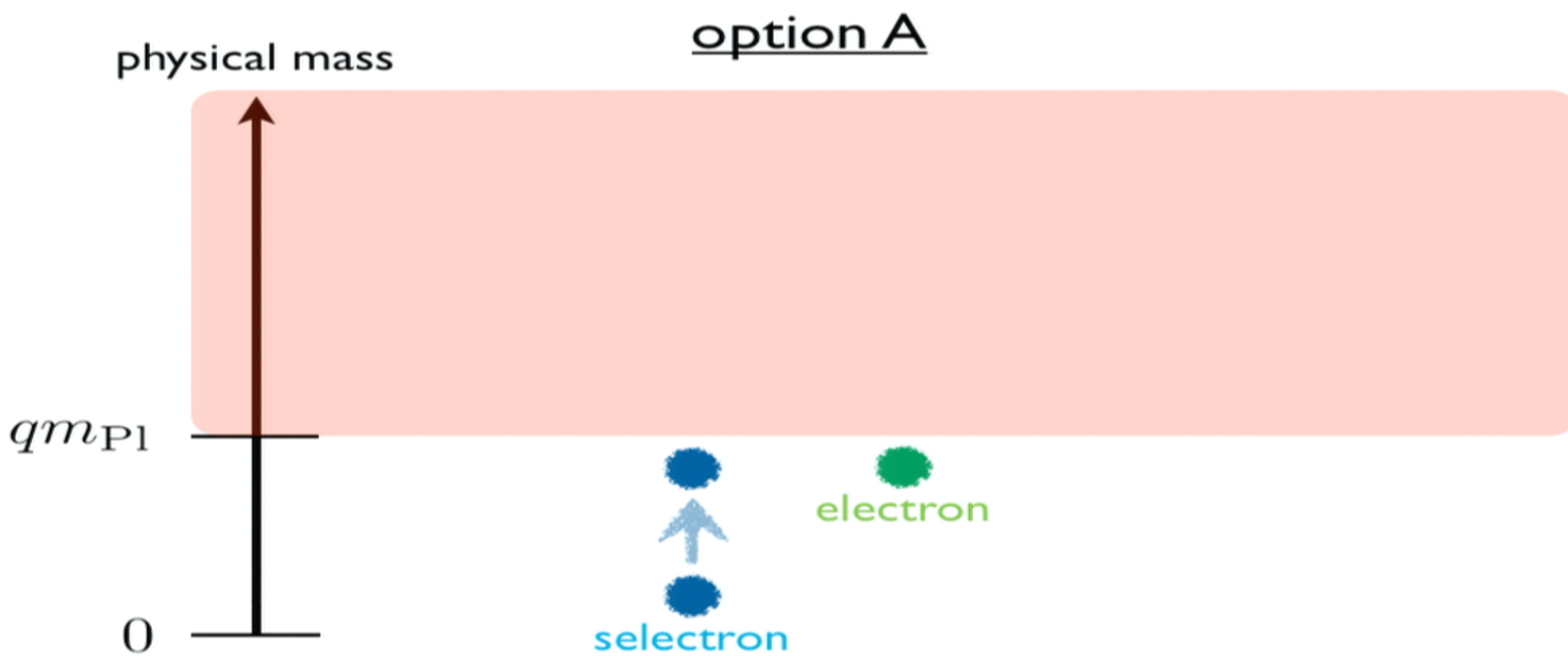
(technically natural)

weird: cutoff parametrically below Planck!

Naturalness and WGC *can* be reconciled if we revisit and *modify* our premises.

There are three obvious strategies i), ii), iii).

i) Add new degrees of freedom below cutoff.



ii) Hierarchical couplings are forbidden.

$$q^2 \not\ll \lambda \longrightarrow q^2 \sim \lambda$$ (e.g. SUSY D-terms)

Of course, something more than SUSY must be required to justify this possibility.

Still, WGC implies that "little hierarchical" couplings will impose "little hierarchies".

iii) Theory is driven to a Higgs phase.

$$\delta m^2 < 0$$

The WGC is ambiguous in the Higgs phase because $[q, m] \neq 0$. Whose mass, charge?

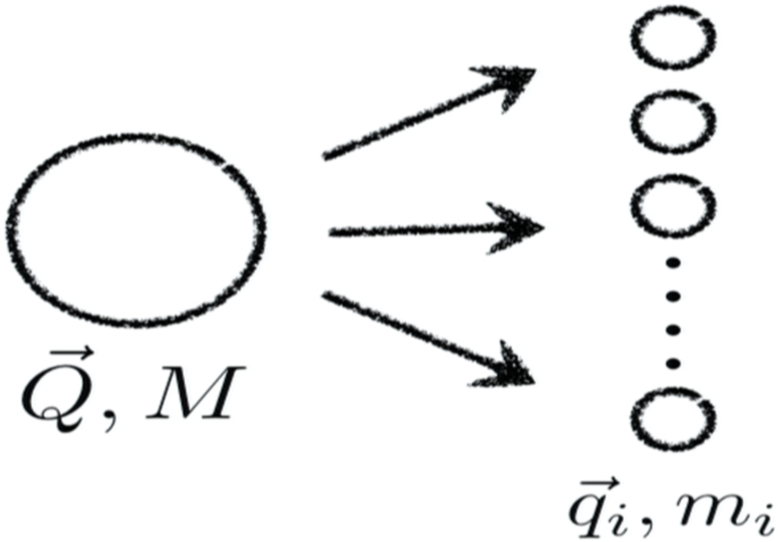
More importantly, black holes do not have Higgsed U(1) hair. No justification for WGC!

Ideally, we'd like to connect WGC to our universe, which exhibits *multiple* forces and *multi-charged* states.

What is the generalization of the WGC?

We can try to guess the generalized WGC.
The condition should be SO(N) invariant.

Let's just derive the generalized WGC.



charge conservation

$$\vec{Q} = \sum_i n_i \vec{q}_i$$

energy conservation

$$M > \sum_i n_i m_i$$

Black hole decays to n_i particles of species i.

Going to charge to mass ratio variables:

$$\vec{Q} = \sum_i n_i \vec{q}_i \qquad M > \sum_i n_i m_i$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\vec{Z} = \sum_i \sigma_i \vec{z}_i \qquad 1 > \sum_i \sigma_i$$

convex hull
spanned by vectors

where $\sigma_i = n_i m_i / M$ is the fractional mass.

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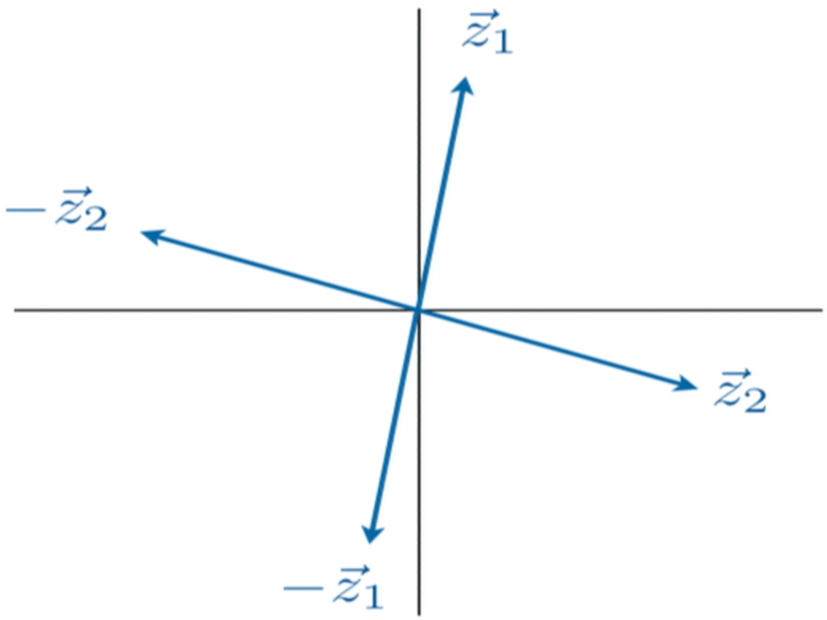
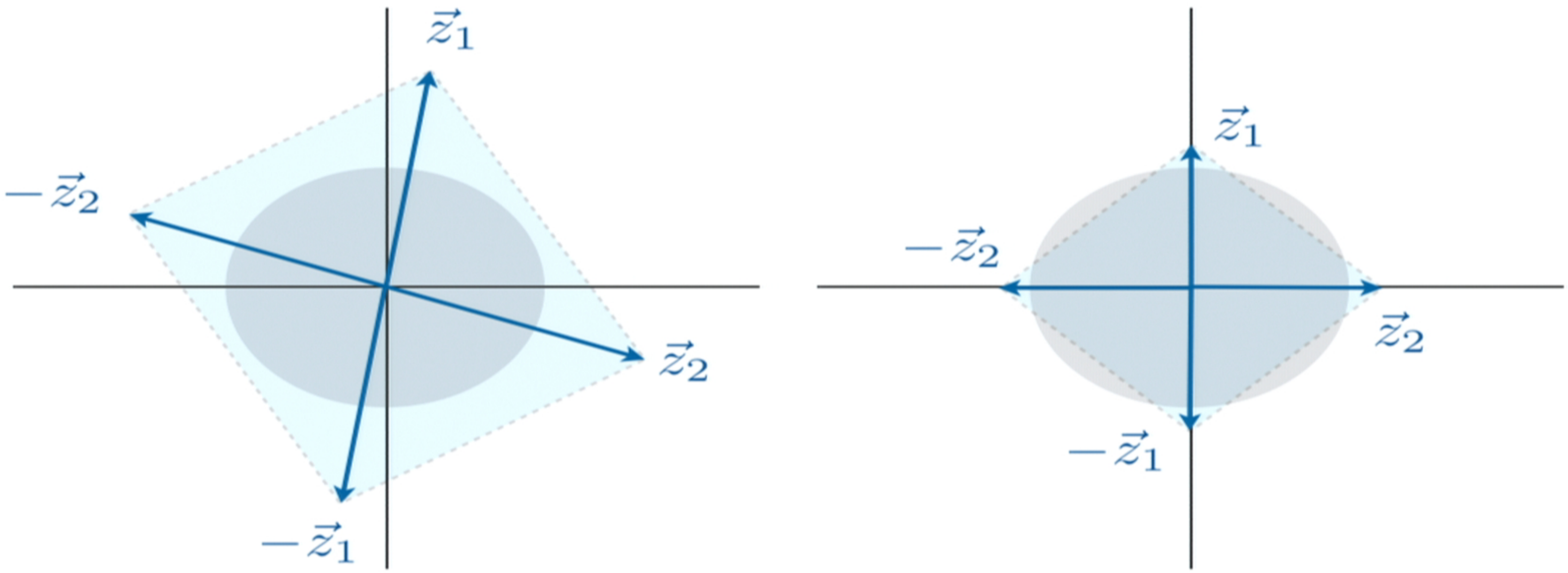


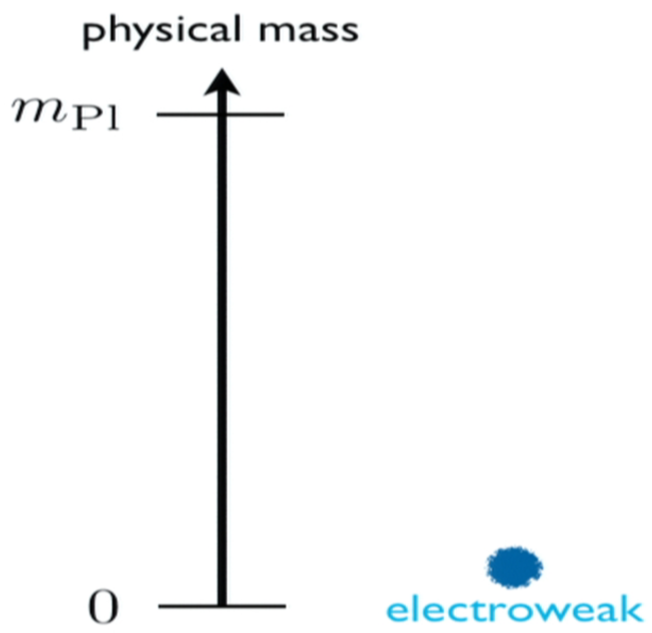
For example, take $U(1)^2$ + 2 charged states.

consistent with WGC



implications for physics beyond the standard model

What about the "big" hierarchy problem?



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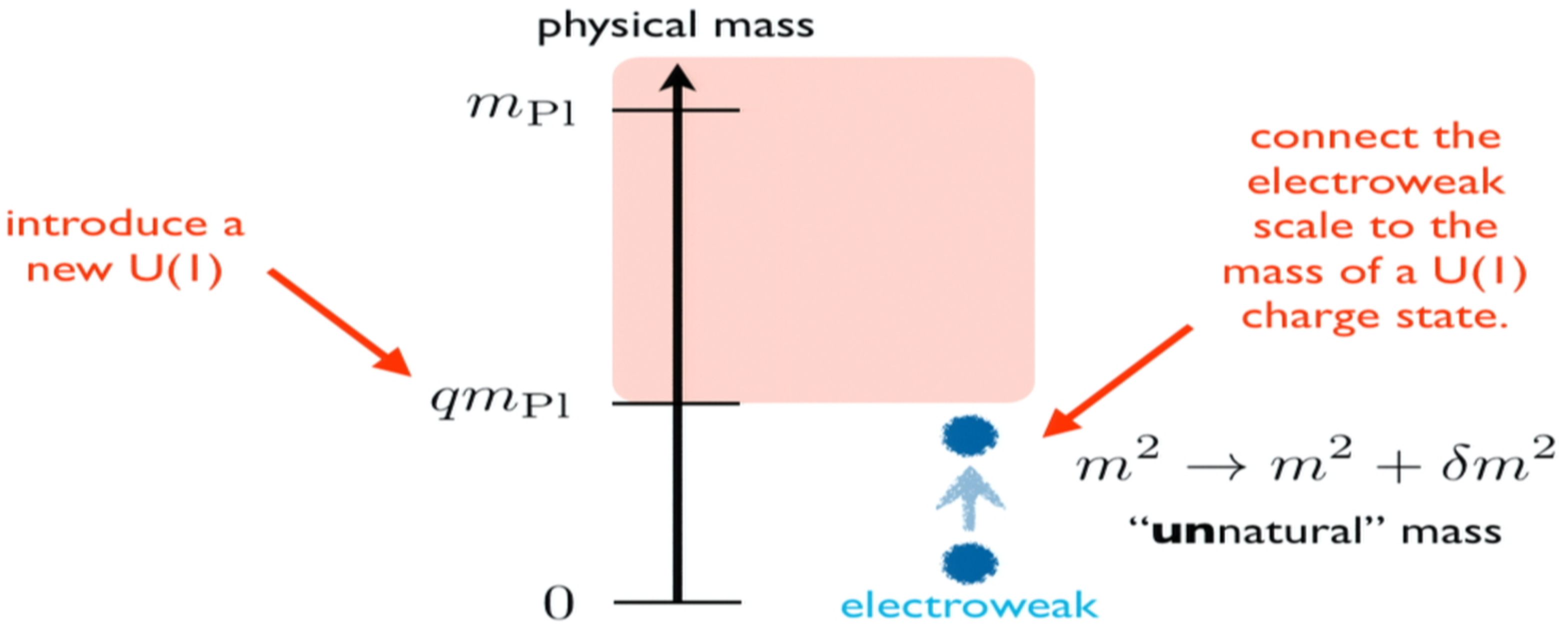
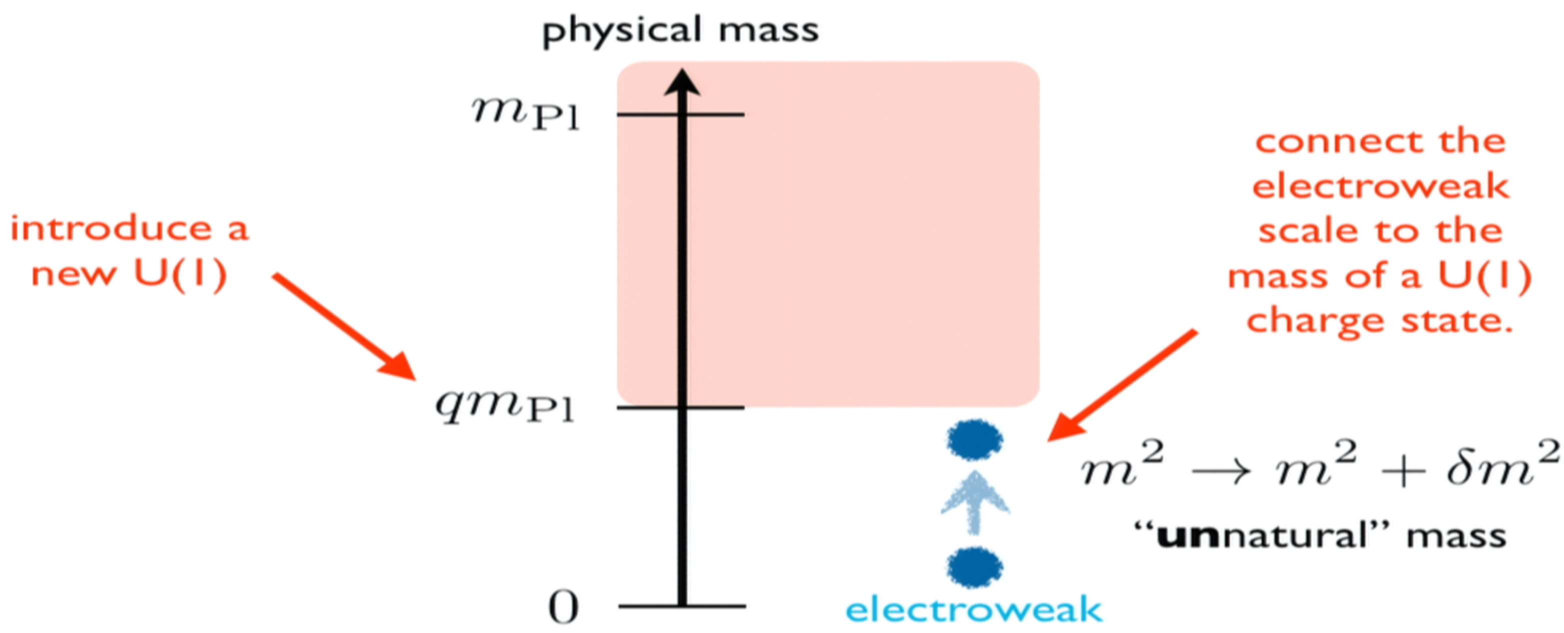


The electroweak scale is unnatural, but only because a natural value is forbidden!



model #1

Weakly gauge $U(1)_{\mathrm{B-L}}$ with Dirac neutrinos.

$$-\mathcal{L} = m_\nu \bar{\nu}_L \nu_R + \mathrm{h.c.} \qquad m_\nu \sim y_\nu v$$

Assuming that $m_\nu \sim 0.1\ \mathrm{eV}$, we fix

$$q \sim 10^{-29} \quad (\sim m_\nu / m_{\mathrm{Pl}})$$

so that the WGC is marginally satisfied.

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The model is a proof of concept but it has has a prediction: a massless gauge boson.

There are very stringent limits of fifth forces and violation of equivalence principle:

$$q \lesssim 10^{-24}$$ (torsion balance)

The model may yet be probed in the future.

thanks!