Title: Universal topological quantum computation from a superconductor/Abelian quantum Hall heterostructure

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Abstract: Non-Abelian anyons promise to reveal spectacular features of quantum mechanics that could ultimately provide the foundation for a decoherence-free quantum computer. The Moore-Read quantum Hall state and a (relatively simple) two-dimensional p+ip superconductor both support Ising non-Abelian anyons, also referred to as Majorana zero modes. Here we construct a novel two-dimensional superconductor in which charge-2e Cooper pairs are built from fractionalized quasiparticles, and like the Z3 Read-Rezayi state, harbors Fibonacci anyons that--unlike Ising anyons--allow for universal topological quantum computation solely through braiding.

Universal topological quantum computation from a superconductor/ Abelian quantum Hall heterostructure

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Caltech

Perimeter Institute Mar 19, 2014

Phys. Rev. X 4, 011036 [arXiv:1307.4403]

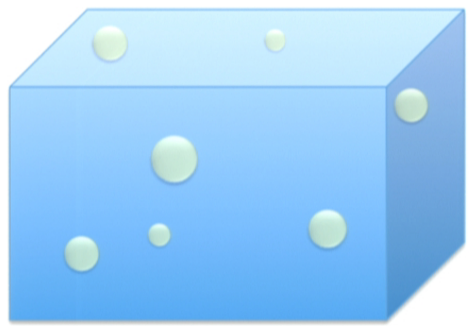
Outline

- Non-Abelian anyons: Why & Where?
- Exotic 2D superconductivity from conventional superconductor/quantum Hall heterostructures
  - Integer case ("Ising anyons")
  - Fractional case ("Fibonacci anyons")
- Outlook

Exchange statistics

Describes how wavefunctions transform when indistinguishable particles exchange positions

$$\psi(\mathbf{r_1}, \ldots, \mathbf{r_N})$$



Possible particle types

I. Bosons/Fermions

(all fundamental particles)

$$\psi \rightarrow \pm\psi$$

II. "Anyons"

(*emergent* particles)

Abelian

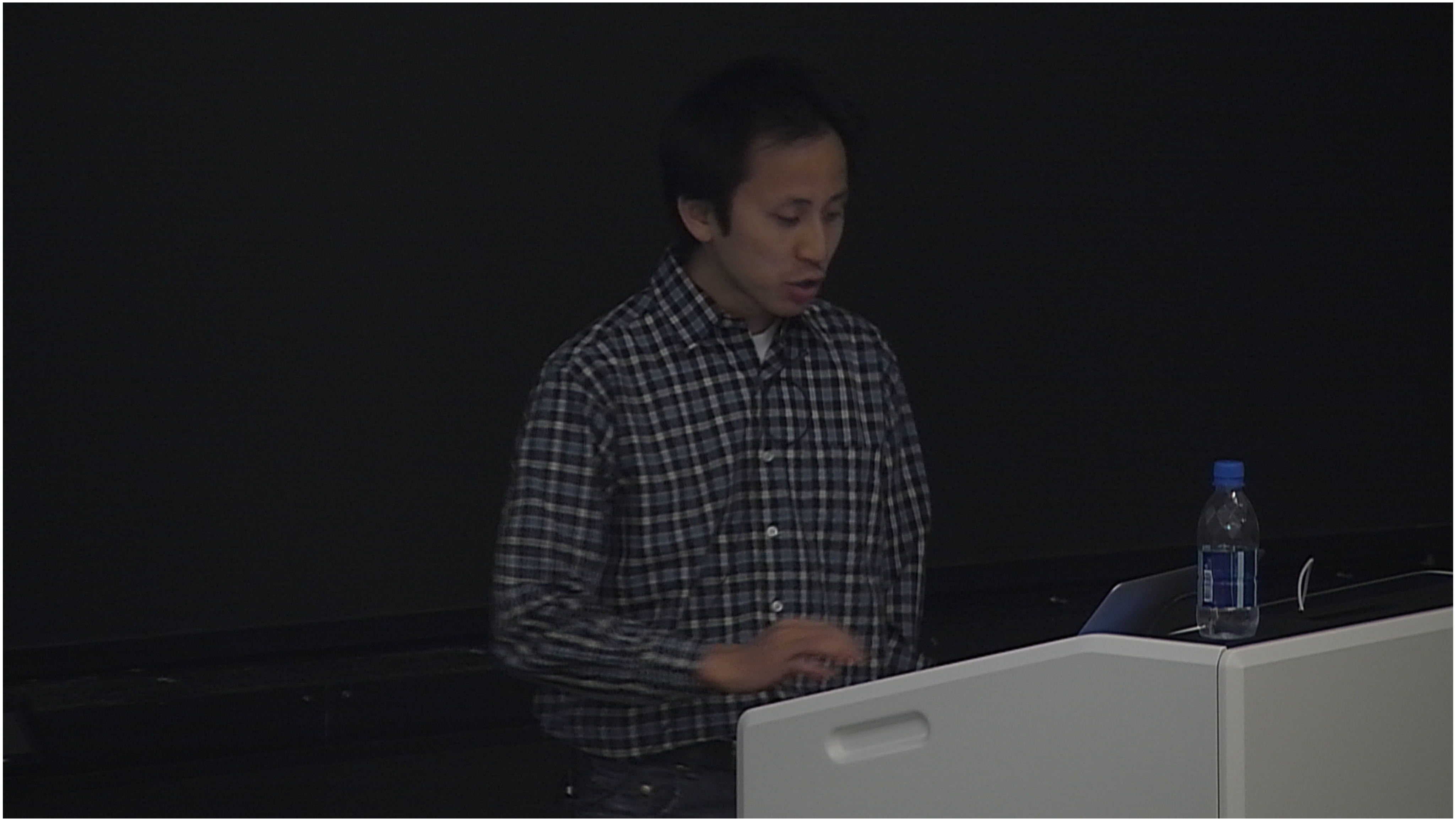
Wilczek (1982)

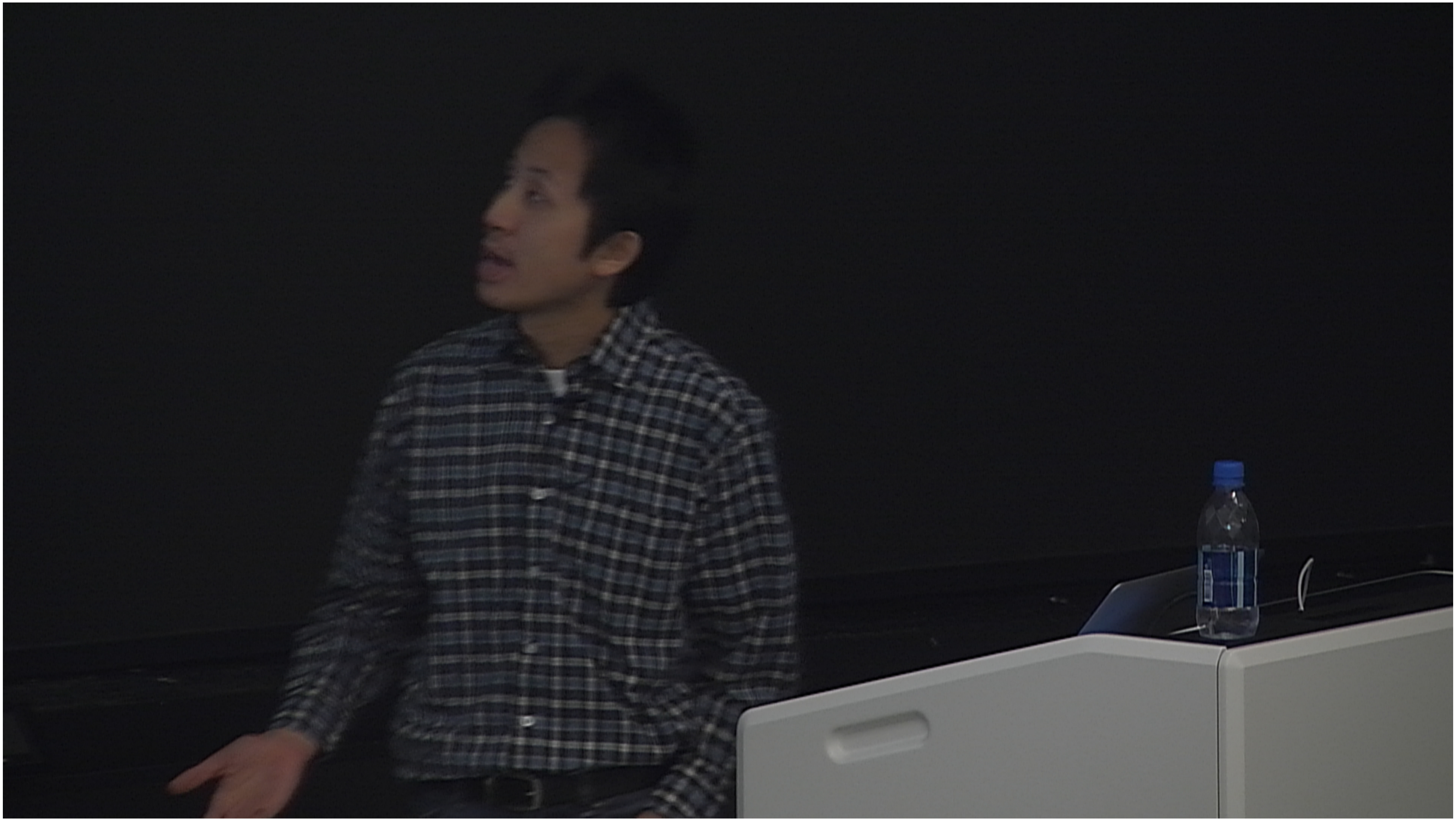
$$\psi \longrightarrow e^{i\theta}\psi$$

- Exchanges commute with one another
- Most fractional quantum Hall states

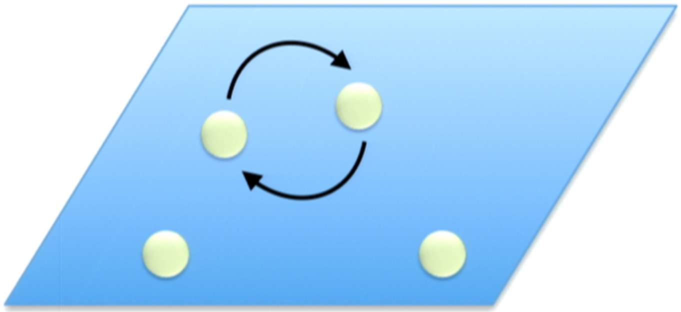
Non-Abelian

- Much more exotic and elusive!





Non-Abelian anyons

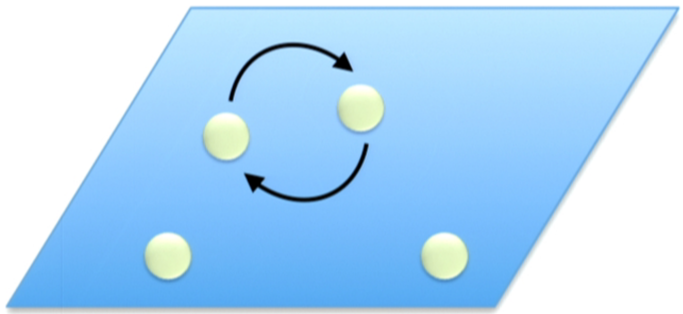


$$\psi_a \longrightarrow U_{ab}\psi_b$$

Interesting for 2 reasons:

- Fundamental physics
- Decoherence-free quantum computation

Non-Abelian anyons



$$\psi_a \longrightarrow U_{ab}\psi_b$$

Qubits — Quantum gates

Need sufficiently dense braid matrices for computational universality!

Interesting for 2 reasons:

- Fundamental physics
- Decoherence-free quantum computation

Kitaev; Freedman; etc.
Nayak, Simon, Stern, Freedman, & Das Sarma, RMP **80**, 1083 (2008)

Storing quantum information with non-Abelian anyons

Ising anyons (e.g., p+ip superconductor, Moore-Read)

3 anyon species: $1, \psi, \sigma$

1 Vacuum (all bosonic excitations)

ψ Fermion

σ Vortex or charge e/4 quasiparticle

Storing quantum information with non-Abelian anyons

Ising anyons (e.g., p+ip superconductor, Moore-Read)

3 anyon species: $1, \psi, \sigma$

Fusion rules: $1 \times \psi = \psi \qquad 1 \times \sigma = \sigma$

$$\psi \times \psi = 1 \qquad \psi \times \sigma = \sigma$$

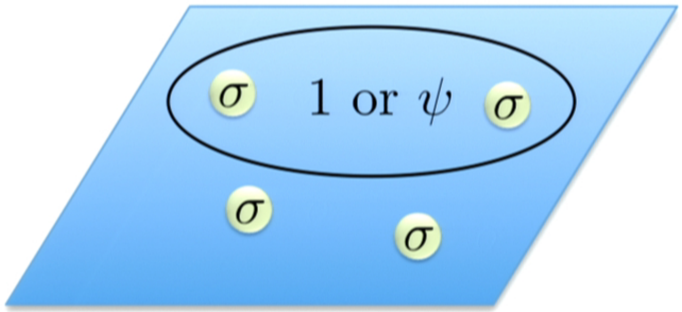
$$\sigma \times \sigma = 1 + \psi$$

Wavefunction degeneracy!

Storing quantum information with non-Abelian anyons

Ising anyons (e.g., p+ip superconductor, Moore-Read)

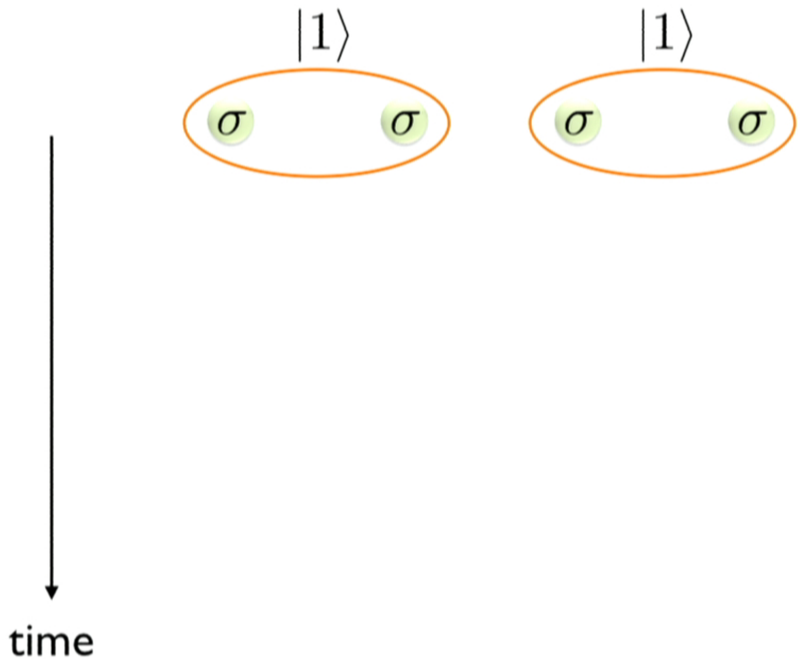
$$\sigma \times \sigma = 1 + \psi$$



The only way to measure the resulting fusion channel is look at both anyons at the same time.

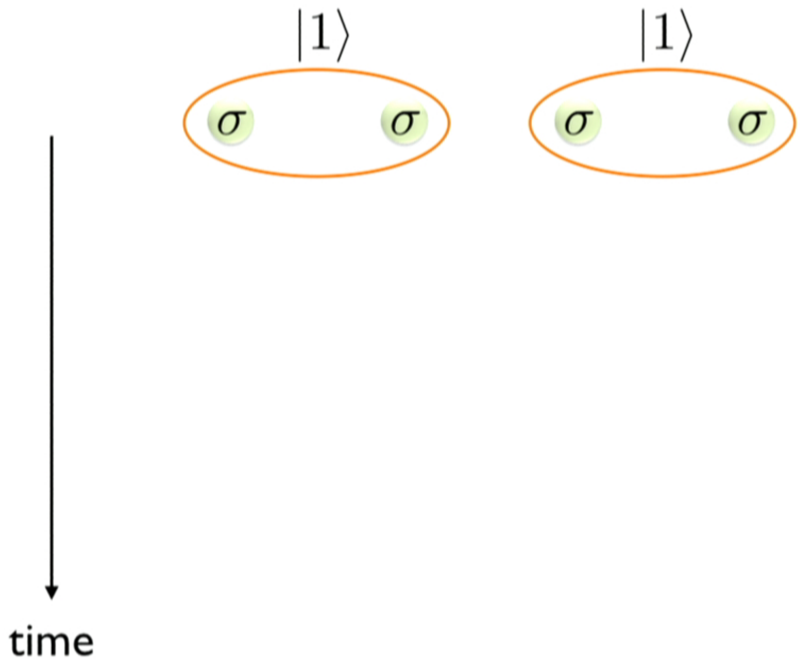
- Energy degeneracy
- Spatial separation provide protection from decoherence

Braiding anyons



$$|\Psi\rangle = |11\rangle$$

Braiding anyons



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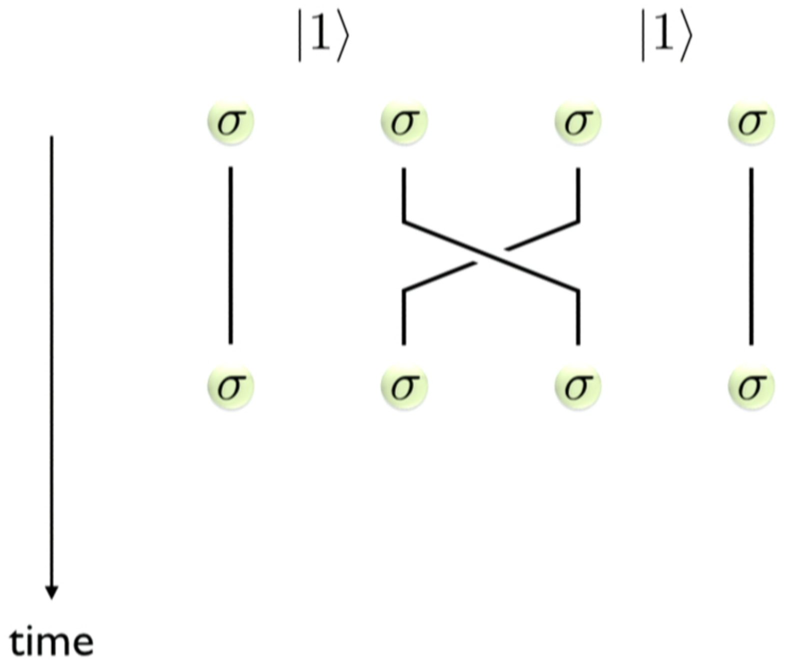
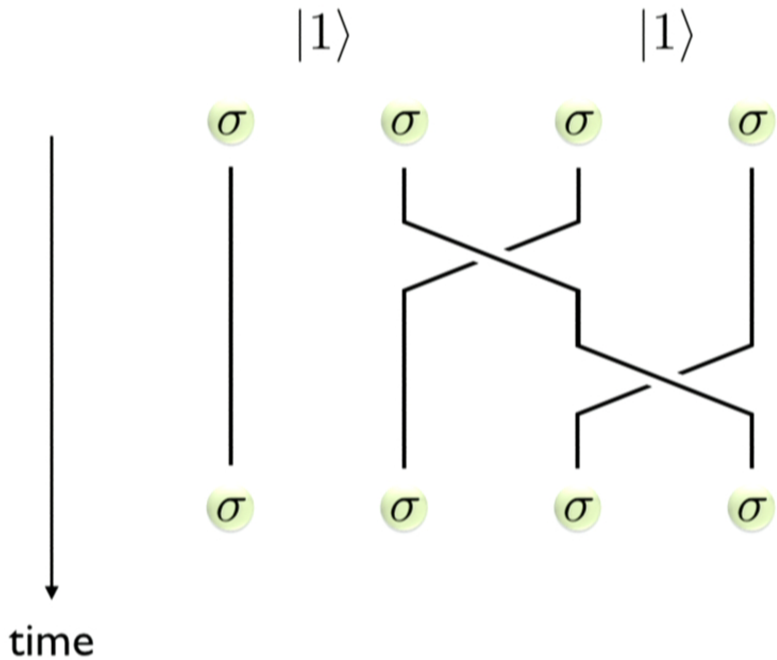
Braiding anyons



$$|\Psi\rangle = |11\rangle$$

$$|\Psi\rangle \sim \frac{|11\rangle - i|\psi\psi\rangle}{\sqrt{2}}$$

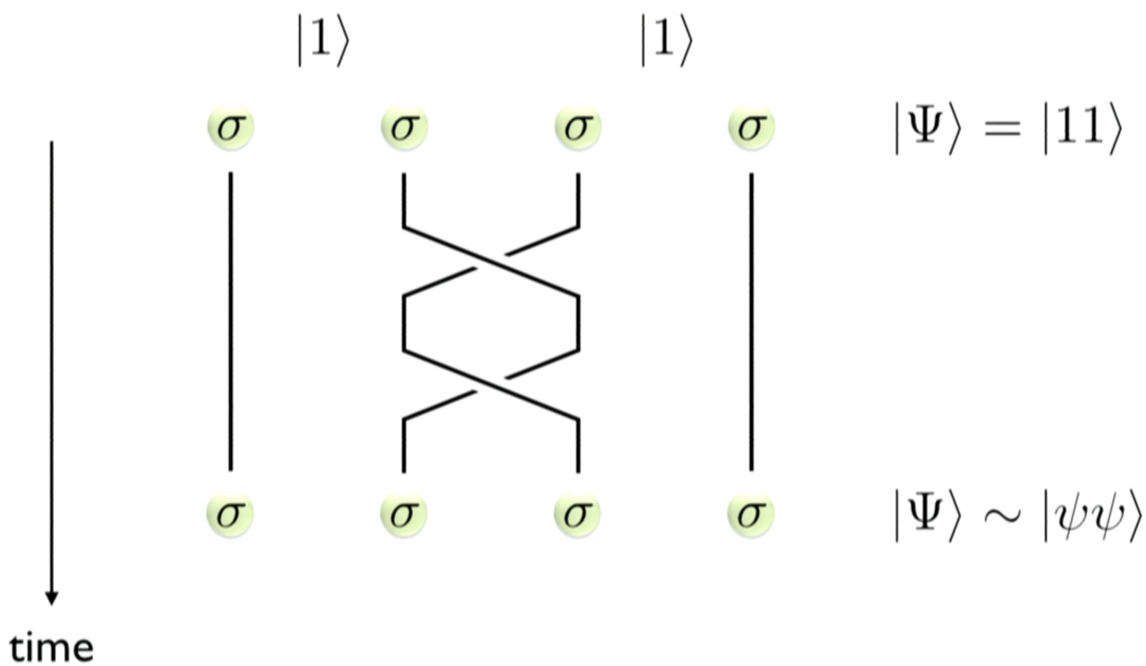
Braiding anyons



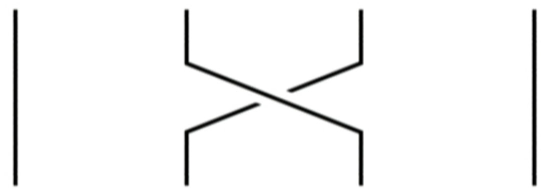
$$|\Psi\rangle = |11\rangle$$

$$|\Psi\rangle \sim \frac{|11\rangle + |\psi\psi\rangle}{\sqrt{2}}$$

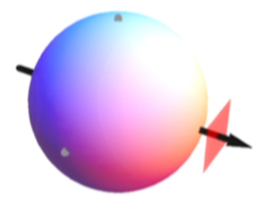
Braiding anyons

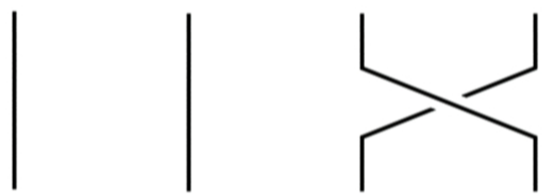


Quantum gates

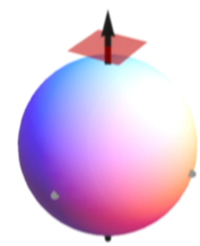


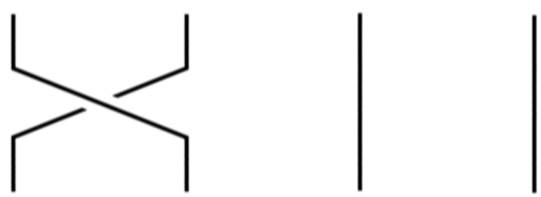
$$\propto \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$



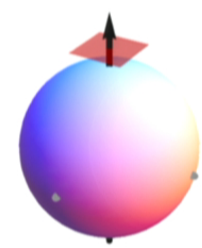


$$\propto \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

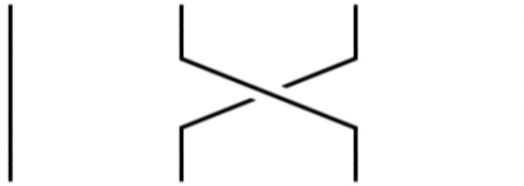




$$\propto \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$



Quantum gates



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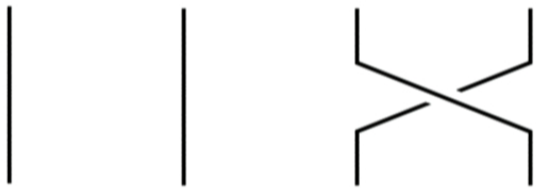

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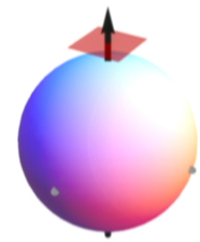

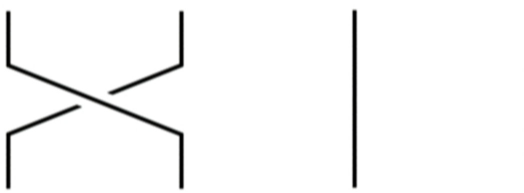

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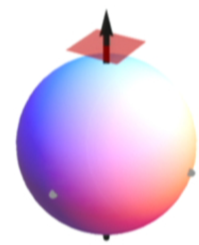
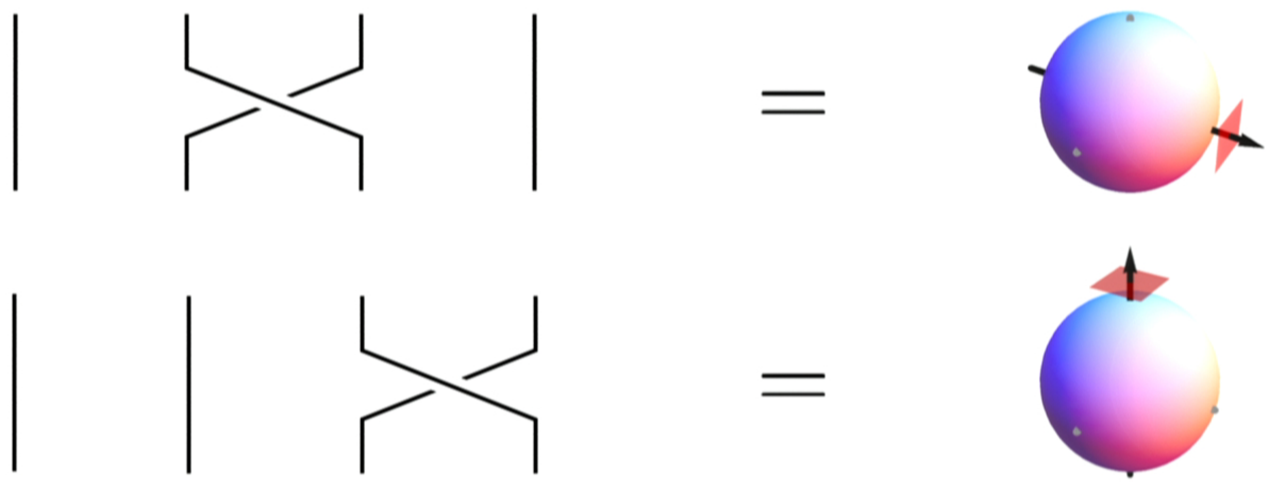
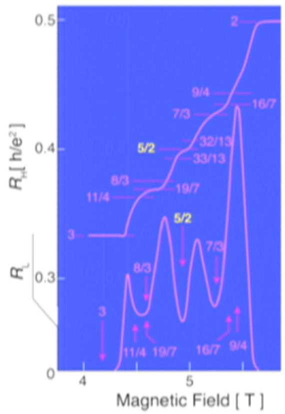


Quantum gates



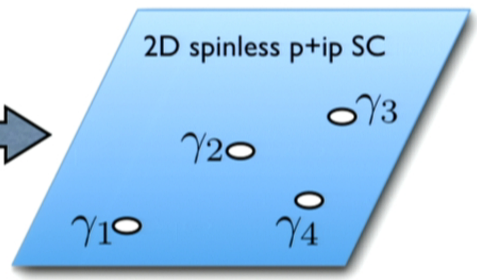
Ising anyons/Majoranas are not
sufficient to construct any
quantum gate!

Plausible non-Abelian anyons platforms

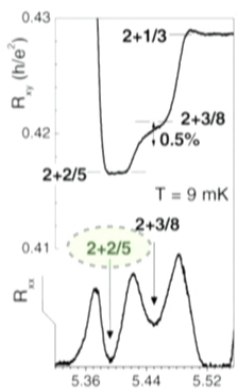


Willet, Eisenstein, et al. (1987)
Moore & Read (1991)

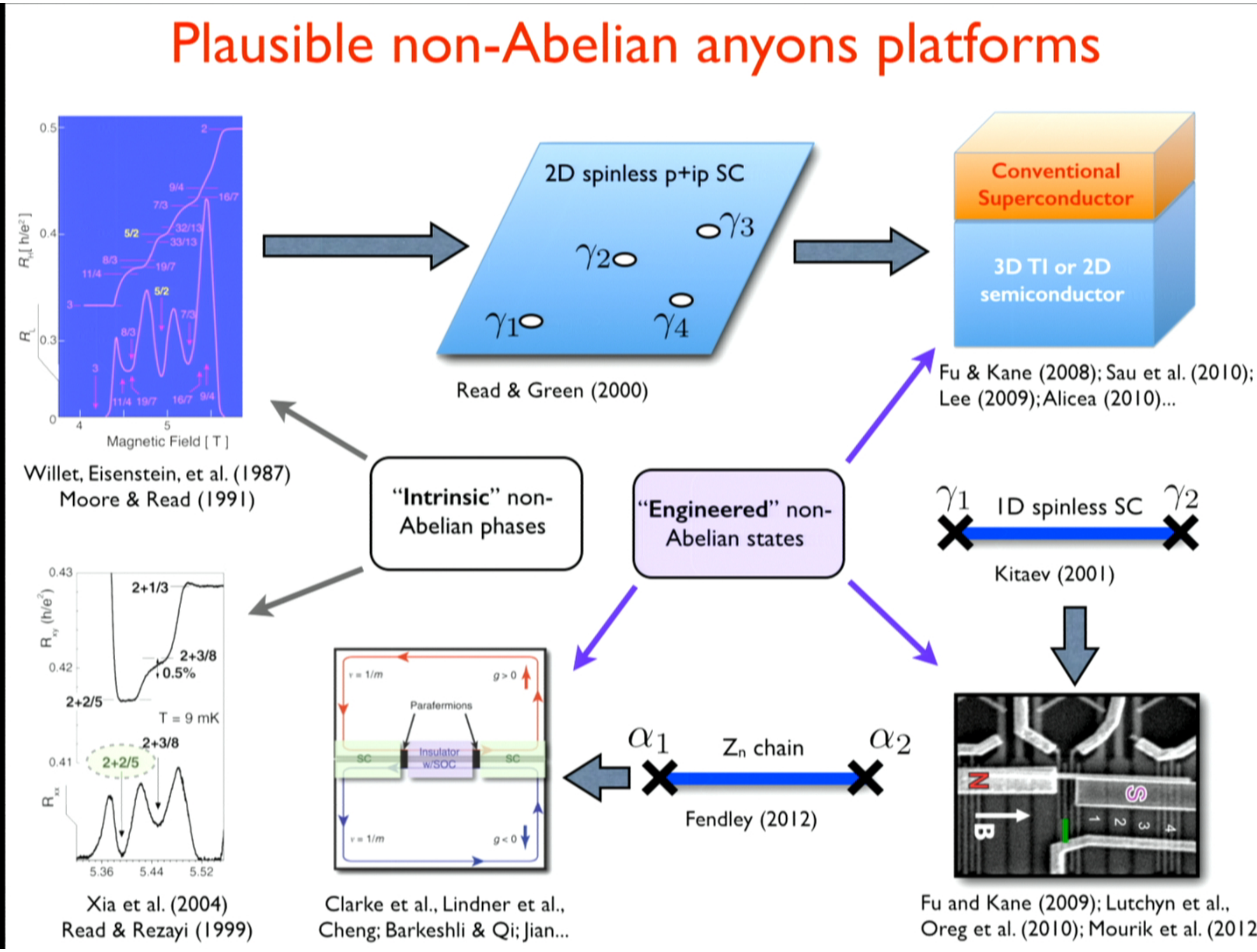
Key advance



Read & Green (2000)



Xia et al. (2004)
Read & Rezayi (1999)

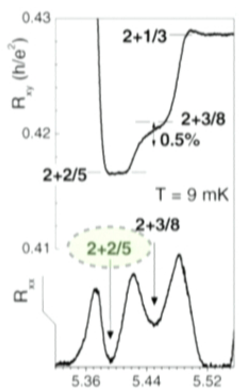

Plausible non-Abelian anyons platforms
Magnetic Field [T]
Willet, Eisenstein, et al. (1987)
Moore & Read (1991)
2D spinless p+ip SC
γ_1 γ_2 γ_3 γ_4
Read & Green (2000)
Conventional Superconductor
3D TI or 2D semiconductor
Fu & Kane (2008); Sau et al. (2010); Lee (2009); Alicea (2010)...
"Intrinsic" non-Abelian phases
"Engineered" non-Abelian states
γ_1 1D spinless SC γ_2
Kitaev (2001)
2+1/3
2+3/8
0.5%
2+2/5
T = 9 mK
2+3/8
2+2/5
Xia et al. (2004)
Read & Rezayi (1999)
Parafermions
SC
Insulator w/SOC
SC
Clarke et al., Lindner et al., Cheng; Barkeshli & Qi; Jian...
α_1 Z_n chain α_2
Fendley (2012)
N
S
B
Fu and Kane (2009); Lutchyn et al., Oreg et al. (2010); Mourik et al. (2012)

Phases with universal braid statistics

Fibonacci anyons

$$\varepsilon \times \varepsilon = 1 + \varepsilon$$

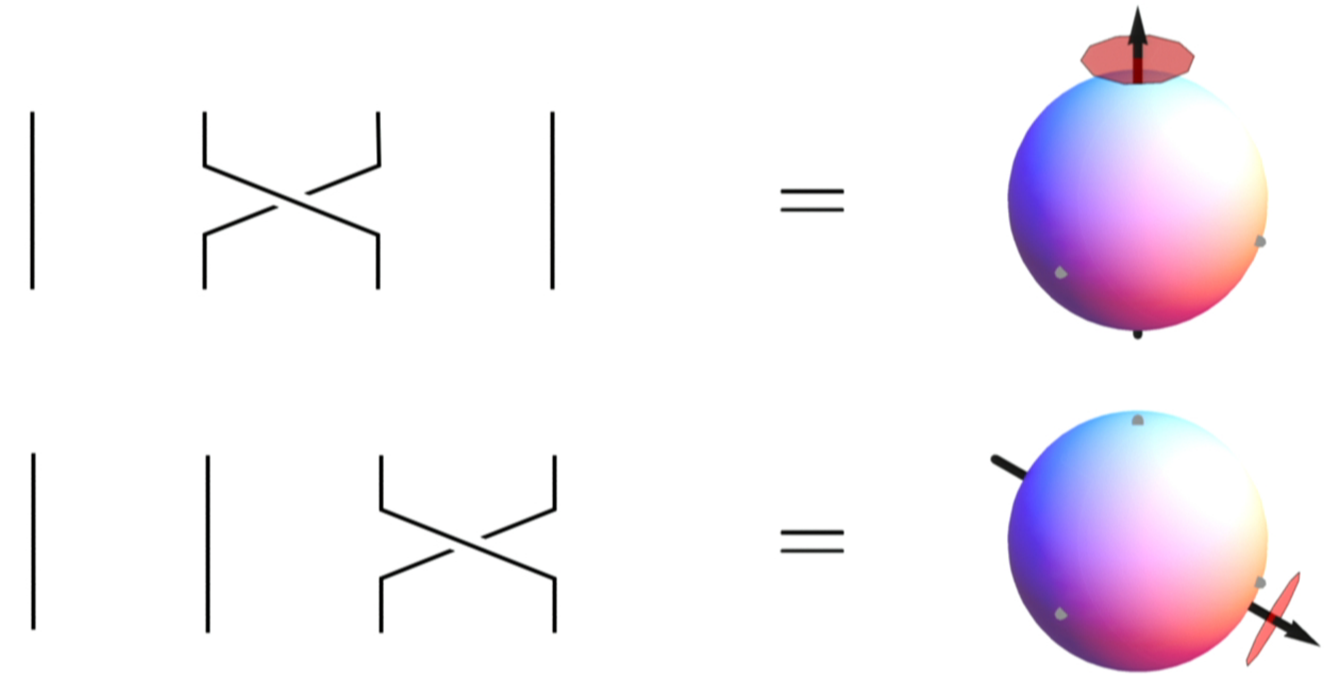
# of fusion channels follow the Fibonacci sequence



Xia et al. (2004)
Read & Rezayi (1999)

Read-Rezayi state only plausible candidate for anyons with **universal braid statistics**...

Quantum gates with Fibonacci



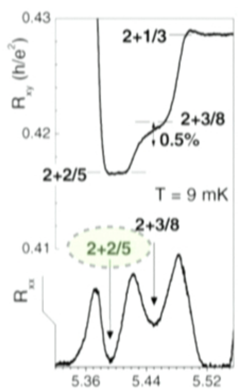
Fibonacci anyons can be used for universal quantum computing

Freedman, Larsen, Wang (2002)

Resources required for topological quantum factoring

M. Baraban,[1] N. E. Bonesteel,[2] and S. H. Simon[3]
[1]*Department of Physics, Yale University, 217 Prospect Street, New Haven, Connecticut 06511, USA*
[2]*Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32310, USA*
[3]*Rudolf Peierls Centre for Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom*
(Received 24 March 2010; published 16 June 2010)

We consider a hypothetical topological quantum computer composed of either Ising or Fibonacci anyons. For each case, we calculate the time and number of qubits (space) necessary to execute the most computationally expensive step of Shor's algorithm, modular exponentiation. For Ising anyons, we apply Bravyi's distillation method [S. Bravyi, Phys. Rev. A **73**, 042313 (2006)] which combines topological and nontopological operations to allow for universal quantum computation. With reasonable restrictions on the physical parameters we find that factoring a 128-bit number requires approximately 10^3 Fibonacci anyons versus at least 3×10^9 Ising anyons. Other distillation algorithms could reduce the resources for Ising anyons substantially.



Xia et al. (2004)
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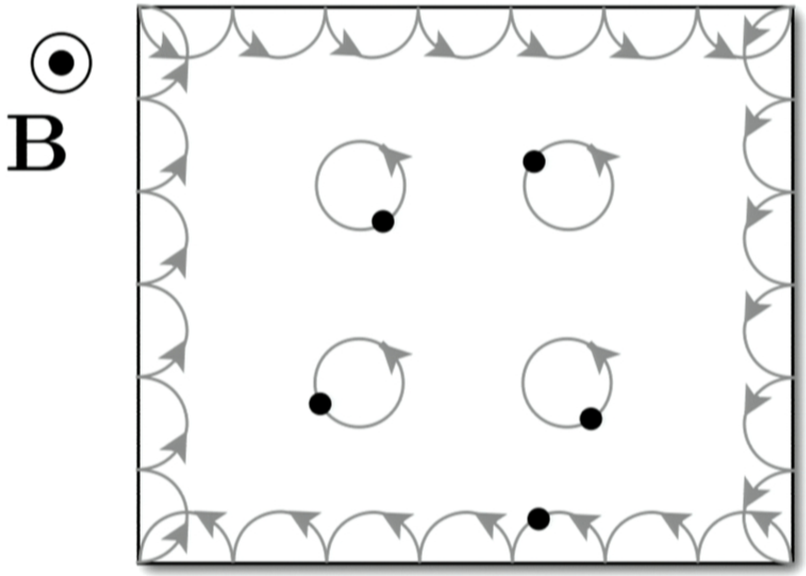
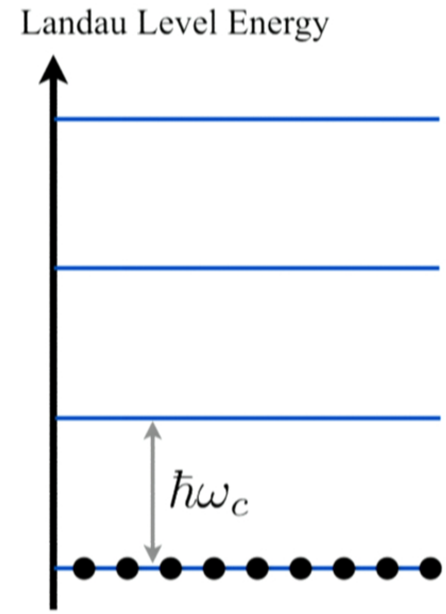
Read-Rezayi state only plausible candidate for anyons with **universal braid statistics**...

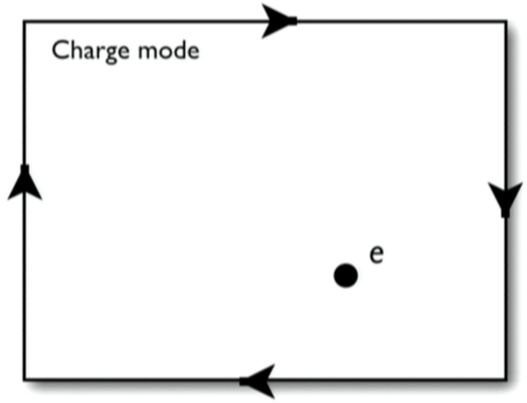
Quantum Hall



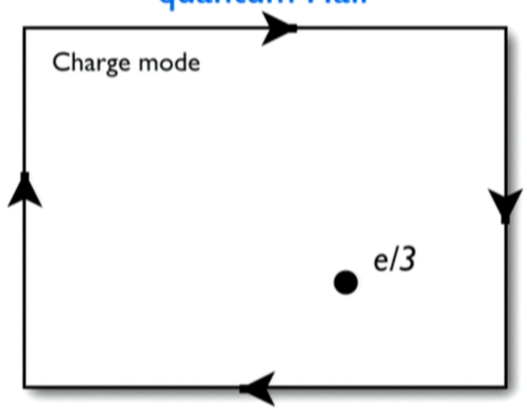


More on quantum Hall

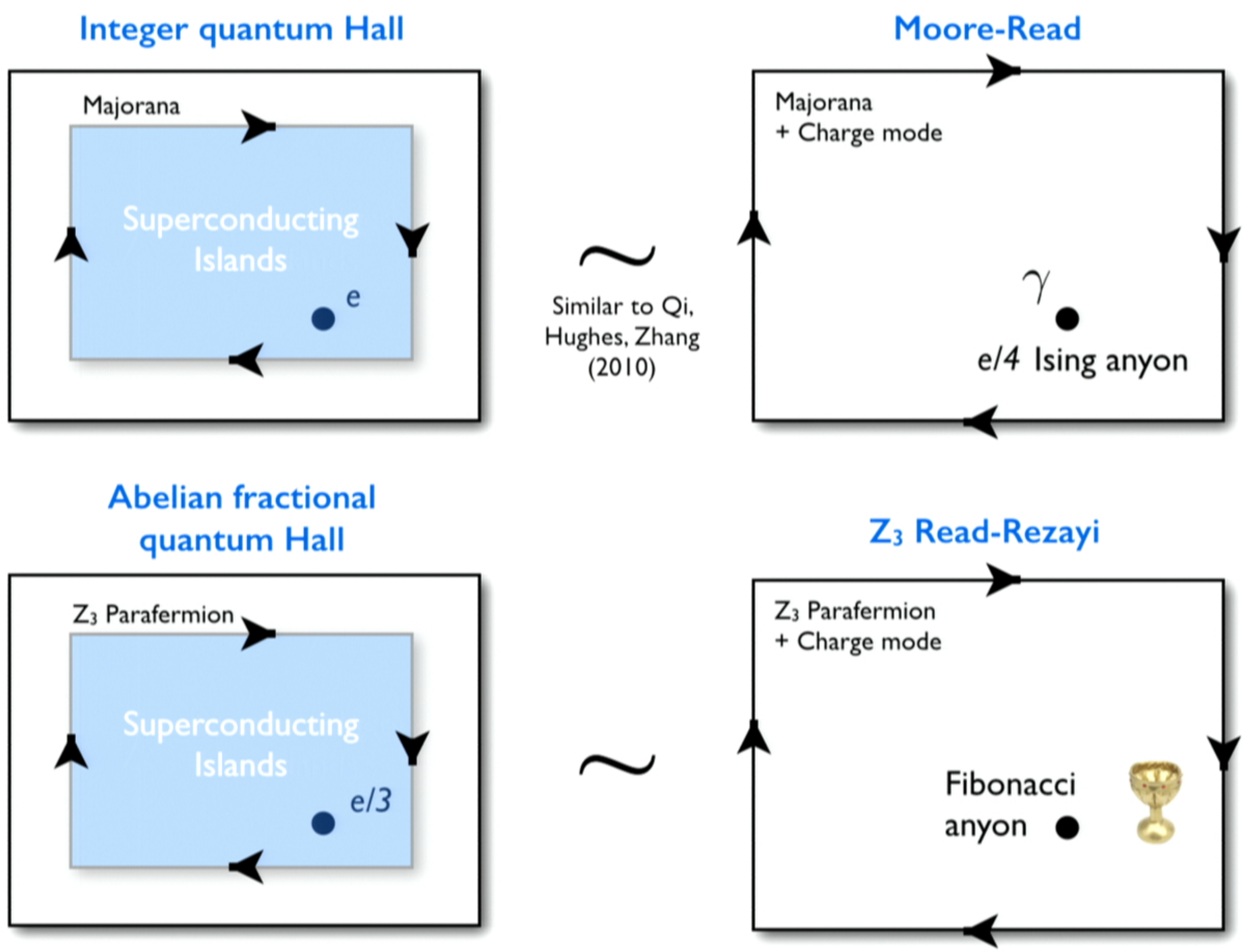
Integer quantum Hall



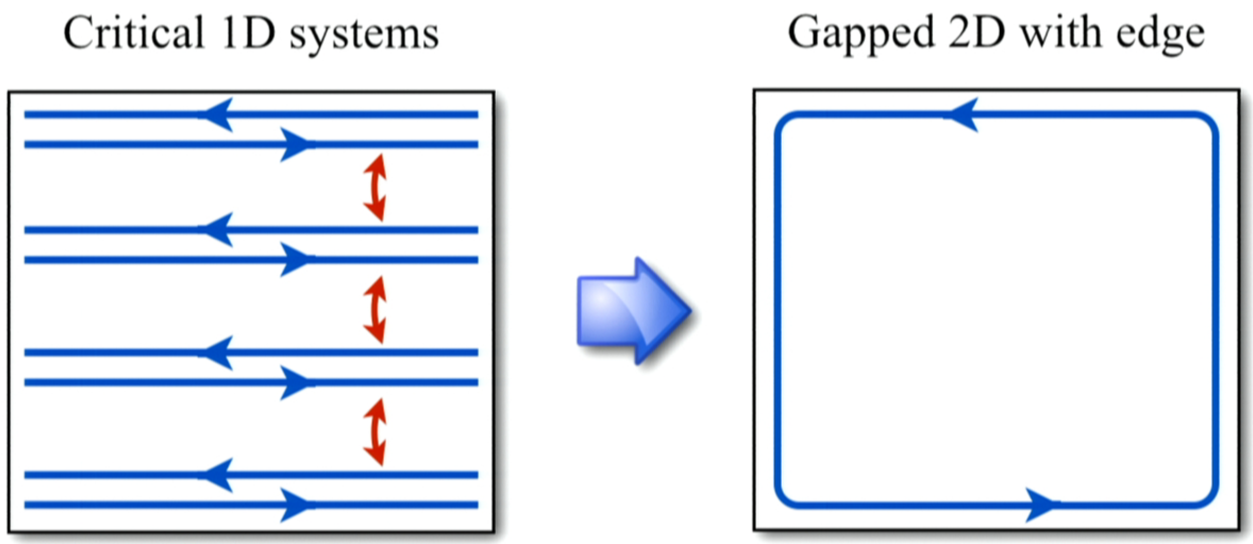
Abelian fractional quantum Hall



2D non-Abelian SC's from simple QH states



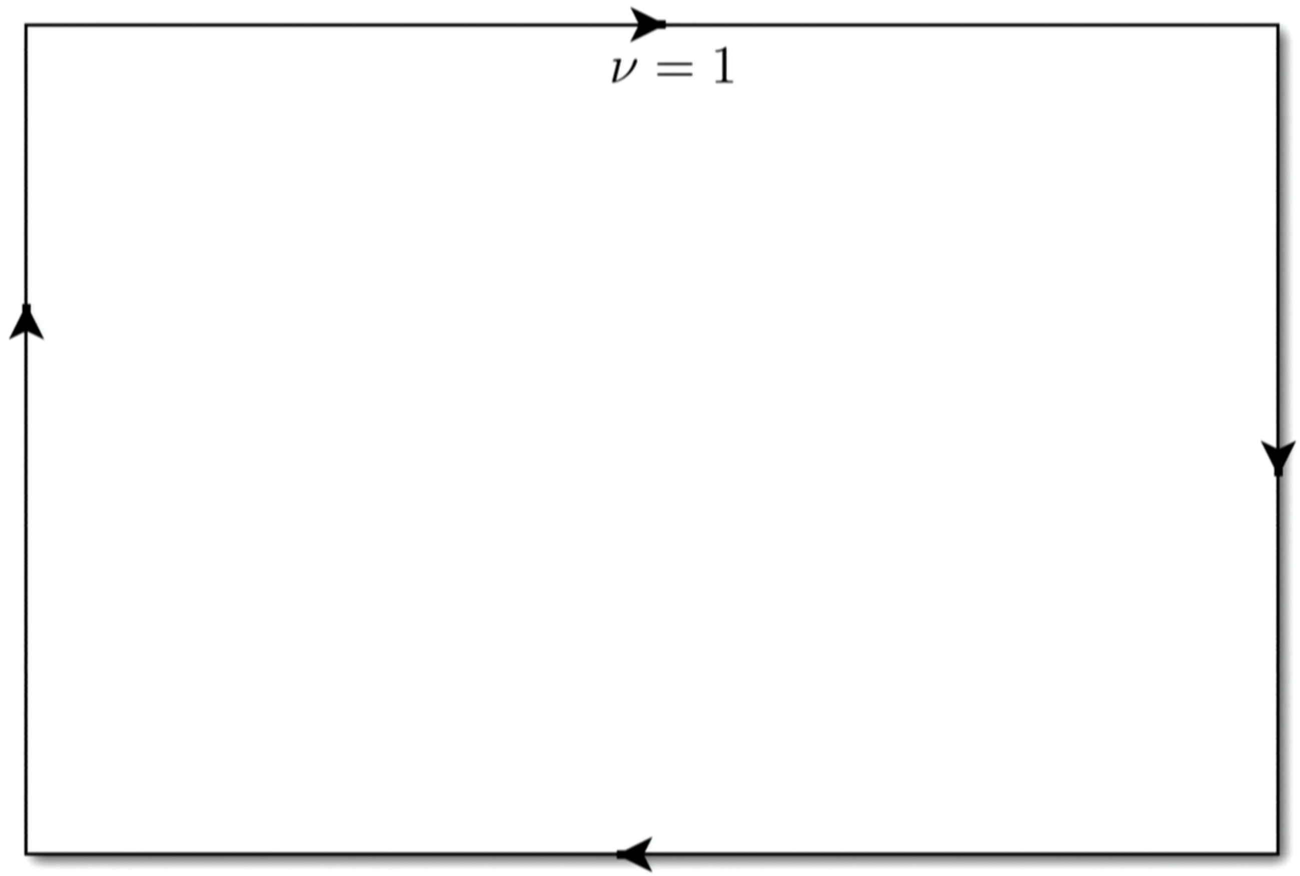
Constructing edge states



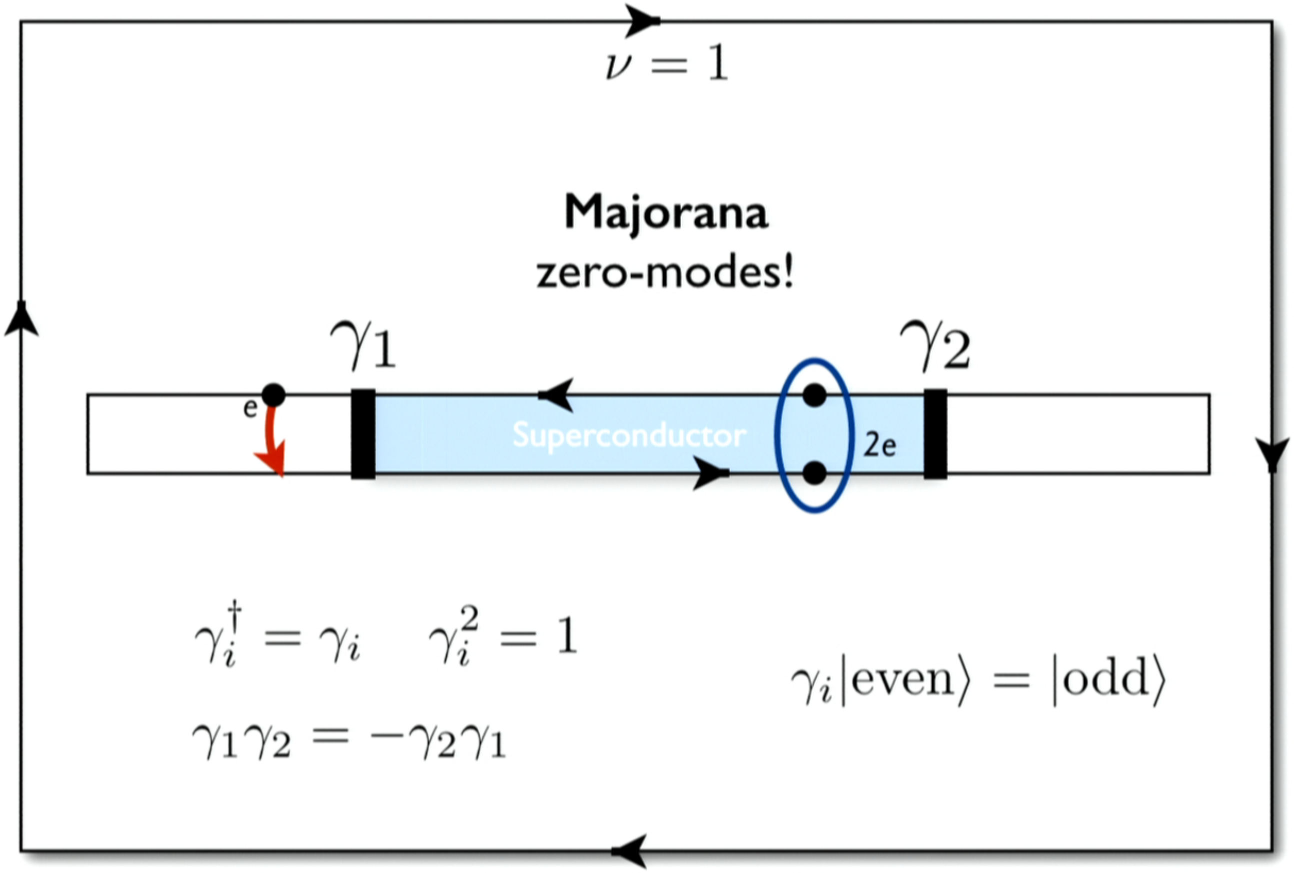
Bulk-Edge correspondence:

Bulk anyons are determined by edge physics

Ising anyons from the integer QHE



Ising anyons from the integer QHE



$$\gamma_i^\dagger = \gamma_i \quad \gamma_i^2 = 1$$

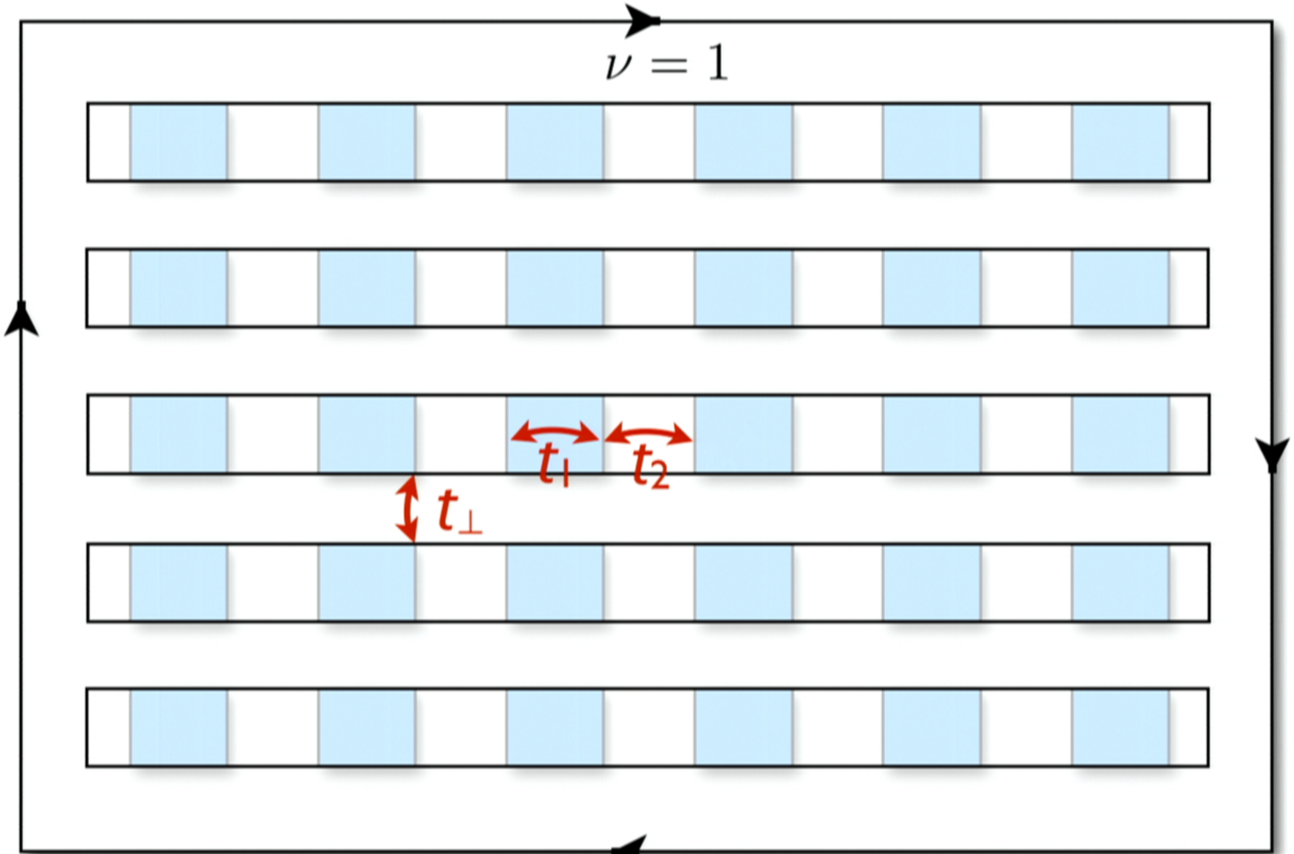
$$\gamma_1\gamma_2 = -\gamma_2\gamma_1$$

$$\gamma_i|\text{even}\rangle = |\text{odd}\rangle$$

[Same physics as 2D topological insulator edges; Fu & Kane (2009)]

Ising anyons from the integer QHE

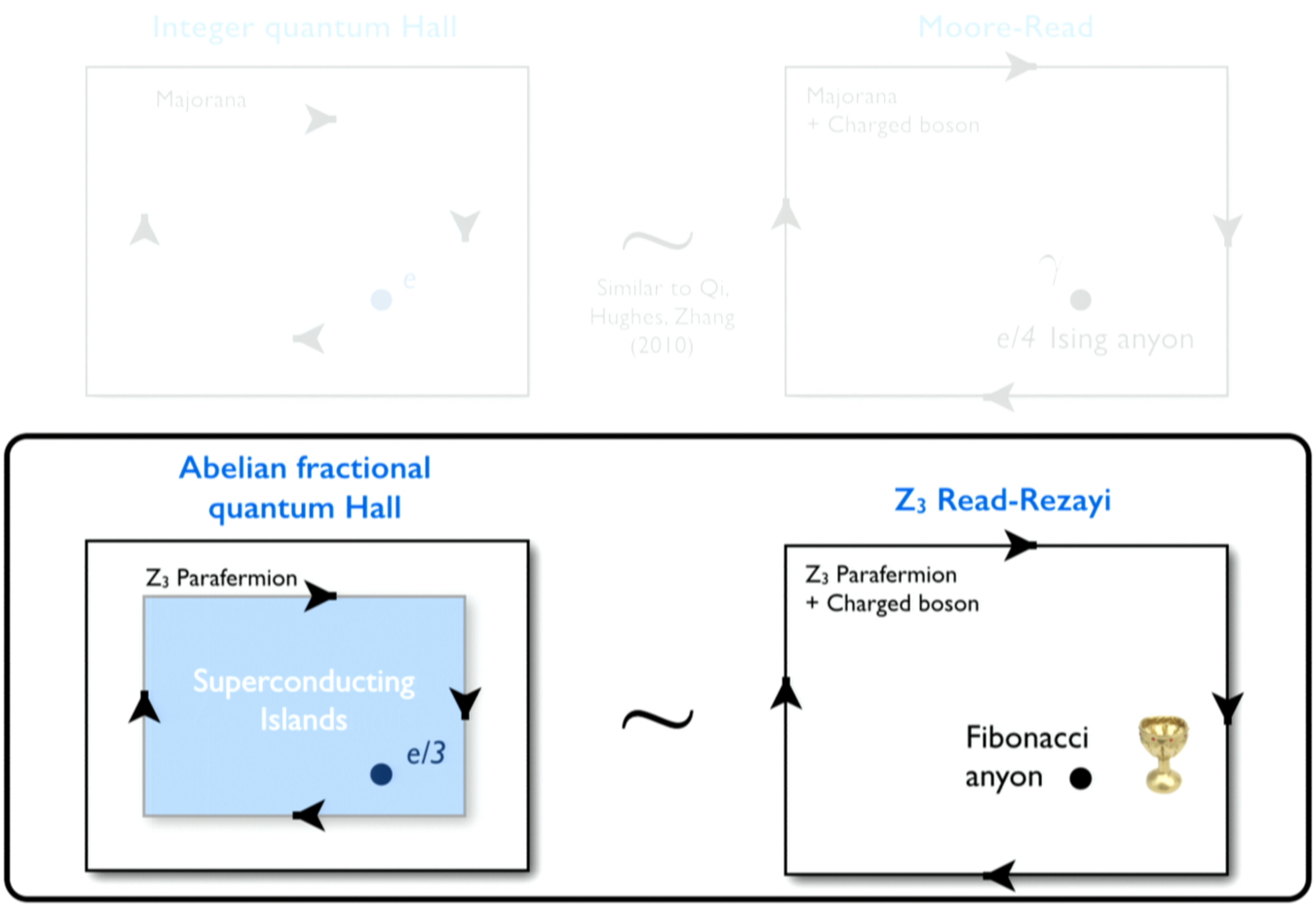
Macroscopic ground-state degeneracy...



...resolve in **weakly coupled chain limit** $t_{1,2} \gg t_\perp$

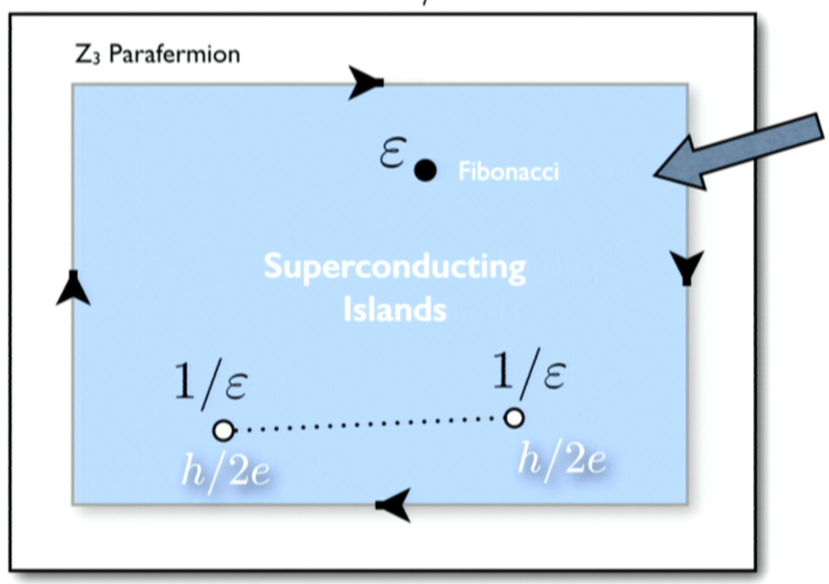

Ising anyons from the integer QHE
Physics of a single chain
$\nu = 1$
0
Majorana end states
Trivial
t_1/t_2
γ_1
γ_2
t_2
$H = i\sum_j [t_1\gamma_{2j-1}\gamma_{2j} + t_2\gamma_{2j}\gamma_{2j+1}]$
Kitaev (2001)

2D non-Abelian SC's from simple QH states



Summary & Outlook

$\nu = 2/3$

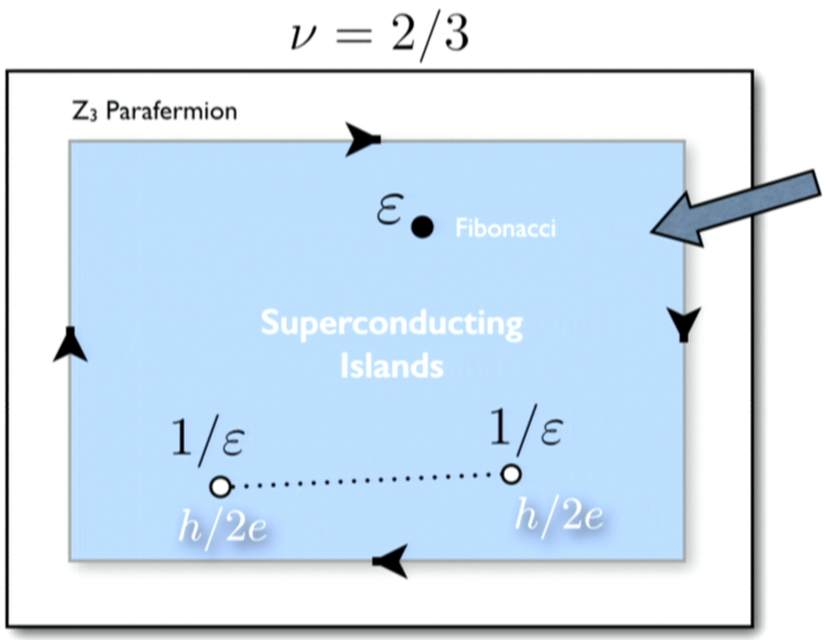


Superconducting phase with universal braid statistics

Our construction required much fine-tuning, but may be relaxed because the phase is gapped.

- Can we trap the non-Abelian anyon in a vortex?
- Can we find a more exotic anyon beyond the Fibonacci?

Summary & Outlook



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