Title: TBA

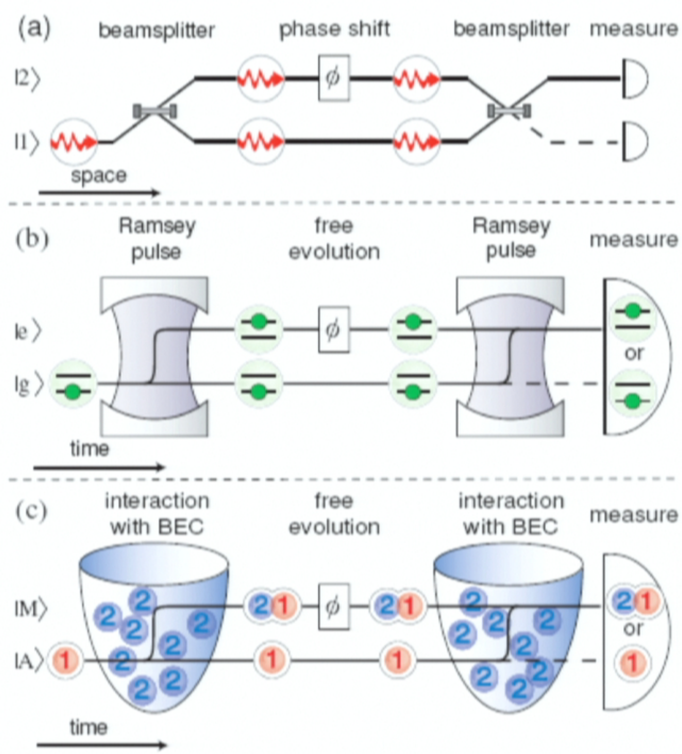
Date: Mar 14, 2014 10:00 AM

URL: http://pirsa.org/14030114

Abstract:

Are superselection rules fundamental?

Robert Spekkens



March 14, 2011

CMP group meeting, PI

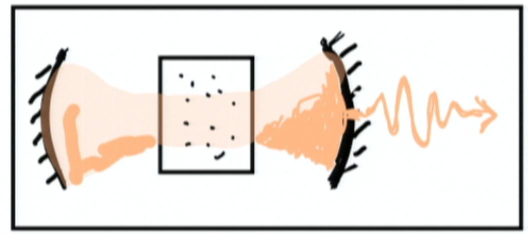
The optical coherence controversy

Optical coherence: a convenient myth?

K. Molmer, Phys. Rev. A. 55, 3195 (1997)

Standard assumption for field: $\rho = |\alpha\rangle\langle\alpha|$

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{e^{-|\alpha|^2/2}\alpha^n}{\sqrt{n!}}|n\rangle$$



coherence is **fact**

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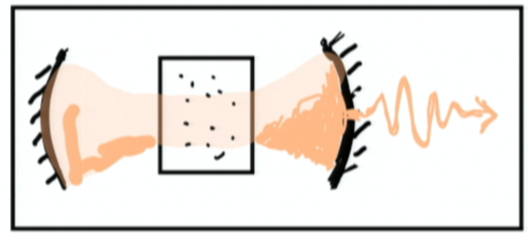

coherence is **fact**

But if we quantize the atoms in the gain medium, and

- assume incoherent mixture of energy eigenstates
- apply energy conservation

$\rightarrow$ For a given n, atoms and field evolve to an entangled state

$$|e\rangle|n\rangle \longrightarrow a(t)|e\rangle|n\rangle + b(t)|g\rangle|n+1\rangle$$

$$\rho = |a(t)|^2 \, |n\rangle\langle n| + |b(t)|^2 \, |n+1\rangle\langle n+1|$$

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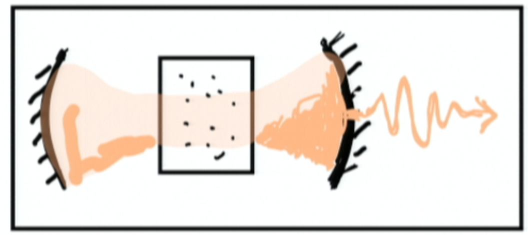
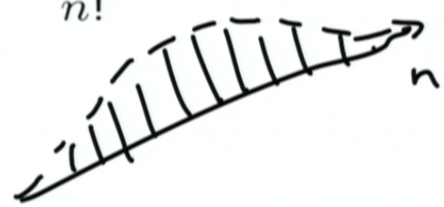


coherence is **fact**

But if we quantize the atoms in the gain medium, and

- assume incoherent mixture of energy eigenstates (thermal state)
- apply energy conservation

$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n| \qquad \text{where} \qquad p_n = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}$$



Thus, coherence is **fiction**!

Optical coherence: a convenient myth?

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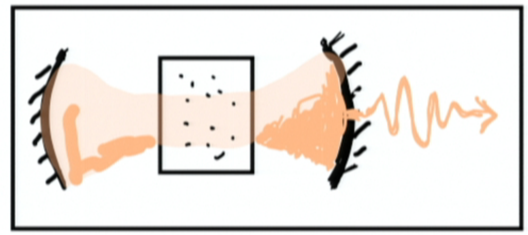

coherence is **fact**

Density operator is **not** block-diagonal in photon number eigenspaces

But if we quantize the atoms in the gain medium, and

- assume incoherent mixture of energy eigenstates (thermal state)
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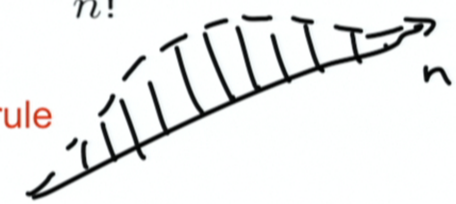
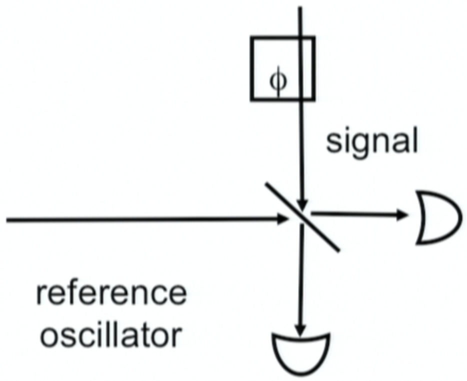
Thus, coherence is **fiction**!

Density operator is block-diagonal = Superselection rule for photon number



Homodyne measurement



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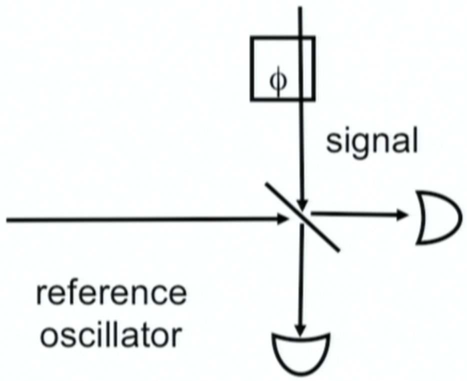
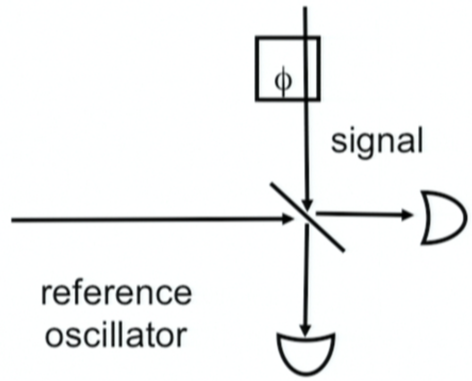


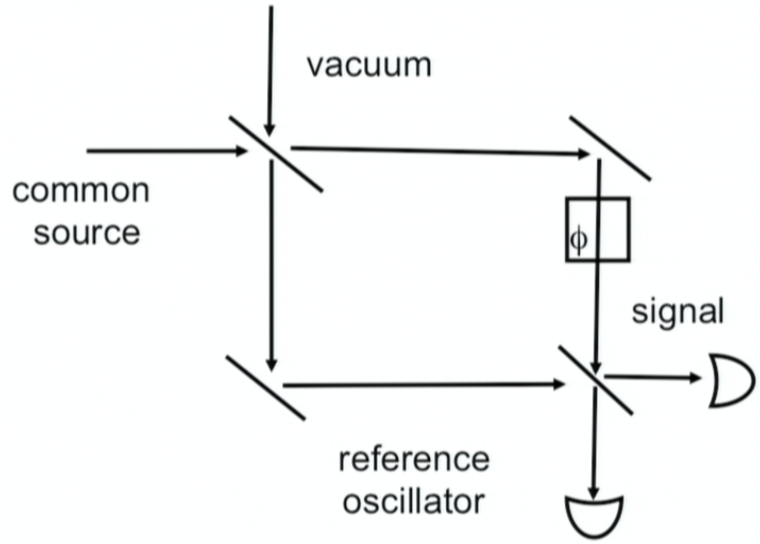
$$\rho_S = |\alpha\rangle_S\langle\alpha|$$

$$|\alpha\rangle_S = \sum_{n=0}^{\infty} \sqrt{p_n} e^{in\phi} |n\rangle_S$$

$$\Delta I = Tr_S[\rho_S(\beta^* a_S + \beta a_S^\dagger)]$$

Homodyne measurement





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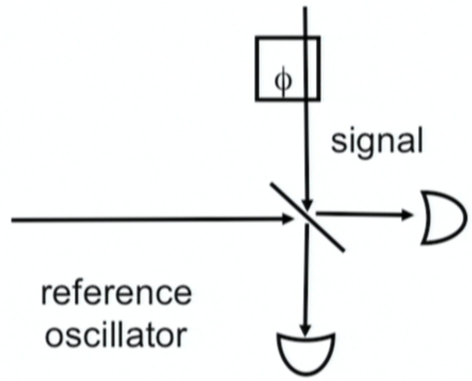

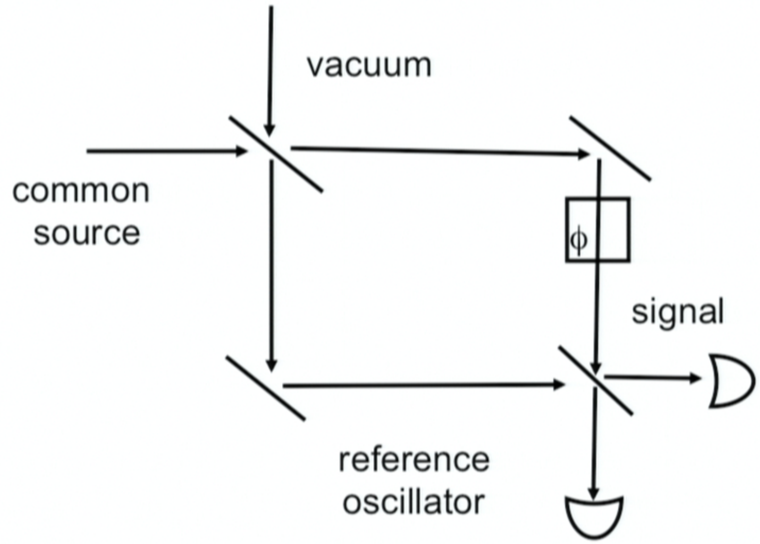

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$$\Delta I = Tr_S[\rho_S(\beta^* a_S + \beta a_S^\dagger)]$$

$$\sigma_{RS} = \sum_n q_n |\psi_{n,\phi}\rangle_{RS}\langle\psi_{n,\phi}|$$

$$|\psi_{n,\phi}\rangle \equiv \sum_m c_m^{(n)} e^{im\phi} |n-m\rangle|m\rangle$$

$$\sigma_S = \sum_{n=0}^{\infty} p_n |n\rangle_S\langle n|$$

$$\Delta I = Tr_{RS}[\sigma_{RS}(b_R^\dagger a_S + b_R a_S^\dagger)]$$

The ensuing controversy

Quantum optics

- K. Molmer, Phys. Rev. A. 55, 3195 (1997)
- R. W. Spekkens and J. E. Sipe, in *Coherence and Quantum Optics* VIII, eds. N. Bigelow et al. (Kluwer Academic, New York, 2003) p. 465.
- T. Rudolph and B. C. Sanders, Phys. Rev. Lett. 87, 077903 (2001)
- H. M. Wiseman, J. Mod. Opt. 50, 1797 (2003); arXiv:quant-ph/0104004
- S. J. van Enk and C. A. Fuchs, Phys. Rev. Lett. 88, 027902 (2002)
- S. J. van Enk and C. A. Fuchs, Quantum Information and Computation 2, 151 (2002)
- T. Rudolph and B. C. Sanders, quant-ph/0112020 (2001)
- K. Nemoto and S. L. Braunstein, quant-ph/0207135 (2002)
- B. C. Sanders, S. D. Bartlett, T. Rudolph, P. L. Knight, Phys. Rev. A 68, 042329 (2003)
- J. Smolin, quant-ph/0407009
- …

Other areas in which the coherence controversy has arisen

Nonlocality of a single photon

D. M. Greenberger, M. A. Horne and A. Zeilinger, Phys. Rev. Lett. 75 (1995) 2064.
L. Hardy, Phys. Rev. Lett. 75 (1995) 2065.

Bose-Einstein condensation

J. Javanainen and S. M. Yoo, Phys. Rev. Lett. 76 (1996) 161.
W. Hoston and L. You, Phys. Rev. A 53 (1996) 4254.
S. M. Yoo, J. Ruostekoski and J. Javanainen, J. Mod. Opt. 44 (1997) 1763.
Y. Castin and J. Dalibard, Phys. Rev. A 55 (1997) 4330.

Superconductivity

P. W. Anderson, in The Lesson of Quantum Theory, eds. J. D. Boer, E. Dal, O. Ulfbeck (Elsevier, Amsterdam, 1986), pp. 2333.
R. Haag, Il Nuovo Cimento XXV (1962) 2695.
D. Kershaw and C. H. Woo, Phys. Rev. Lett. 33 (1974) 918.

Whether there are superselection rules for charge, baryon number, etc.

G. C. Wick, A. S. Wightman and E. P. Wigner, Phys. Rev. 88 (1952) 101.
Y. Aharonov and L. Susskind, Phys. Rev. 155 (1967) 1428.

For a synopsis of the issues (as a dialogue) and our proposed solution, see:

Bartlett, Rudolph, and Spekkens, Int. J. Quantum Information 4, 17 (2006)

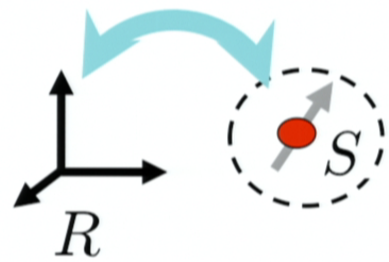
The proposed resolution

The debate presumes:
quantum states only contain information about
the intrinsic properties of the system

But in fact:
quantum states also contain information about
the extrinsic properties of the system
Specifically: the relation to other systems external to it.

Whether or not coherences are applicable depends on the external system to which one is comparing

What does it mean to say that the spin is up along the z axis?



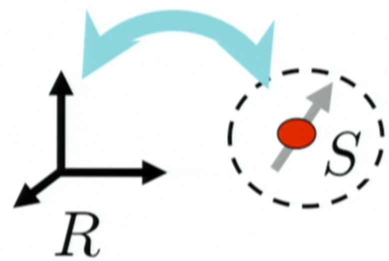
It means spin up relative to another physical system, such as gyroscopes in the lab, that define the z axis (i.e. act as a Cartesian reference frame)

What does it mean to say that a mode has a particular phase?

It means that it has that phase relative to another physical system, such as another oscillator in the lab (i.e. one that acts as a phase reference)

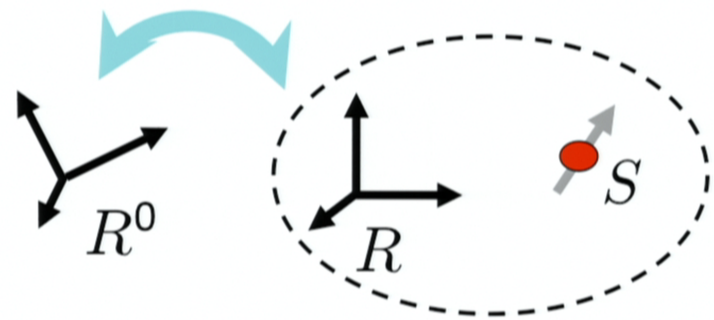
Implicated RF treated externally

$$\rho_S \in \mathcal{L}(\mathcal{H}_S)$$



Implicated RF treated internally

$$\sigma_{RS} \in \mathcal{L}(\mathcal{H}_R) \otimes \mathcal{L}(\mathcal{H}_S)$$



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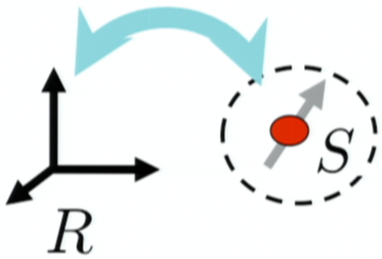

So, the two states need not be the same

Implicated RF treated internally

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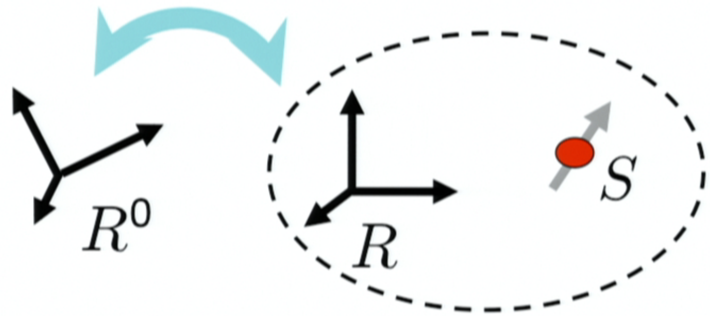

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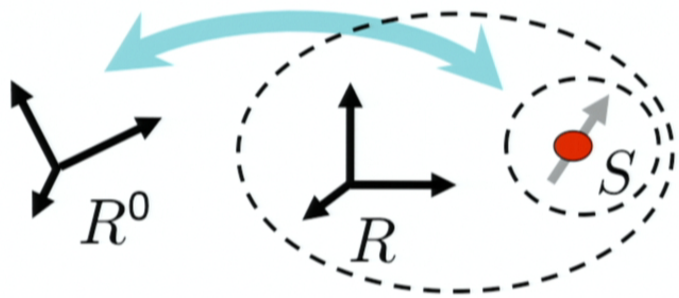


SSR for photon number = symmetric under phase shifts

or, equivalently

Quantum coherence = asymmetric under phase shifts

$$U(\phi) = e^{-i\phi\hat{N}} \quad \phi \in \mathsf{U}(1)$$

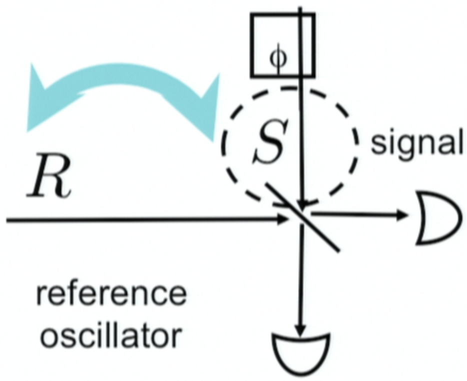
$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n| \leftrightarrow U(\phi)\rho U^{\dagger}(\phi) = \rho$$

More generally, for any ρ

$$\mathcal{G}(\rho) \equiv \int \mathrm{d}\phi \, U(\phi)\rho U^{\dagger}(\phi) = \sum_{n=0}^{\infty} \Pi_n \rho \Pi_n$$

$$\leftrightarrow U(\phi)\mathcal{G}(\rho)U^{\dagger}(\phi) = \mathcal{G}(\rho)$$

Implicated RF treated externally



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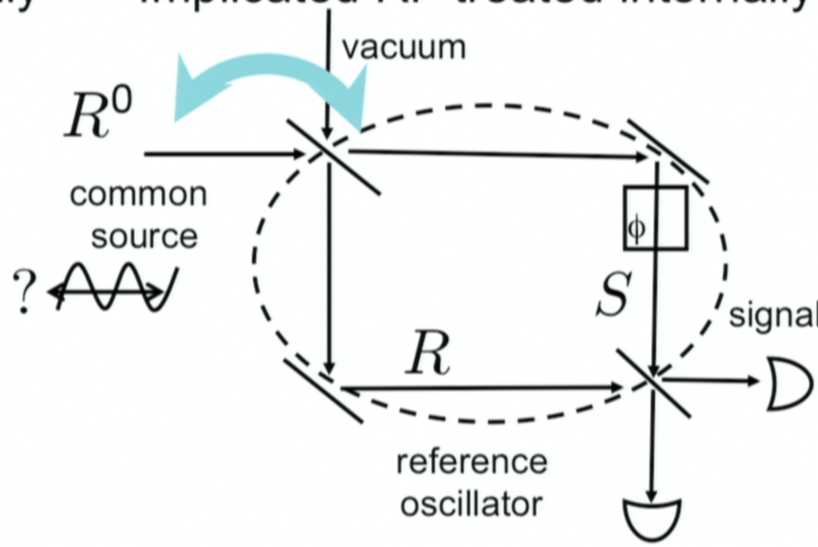

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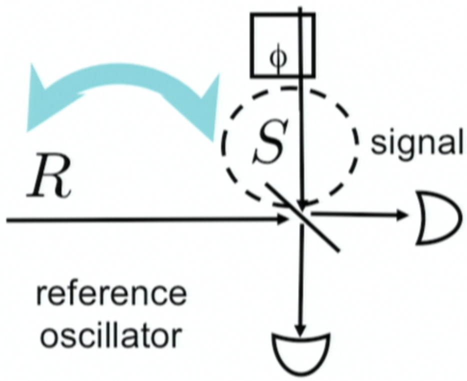

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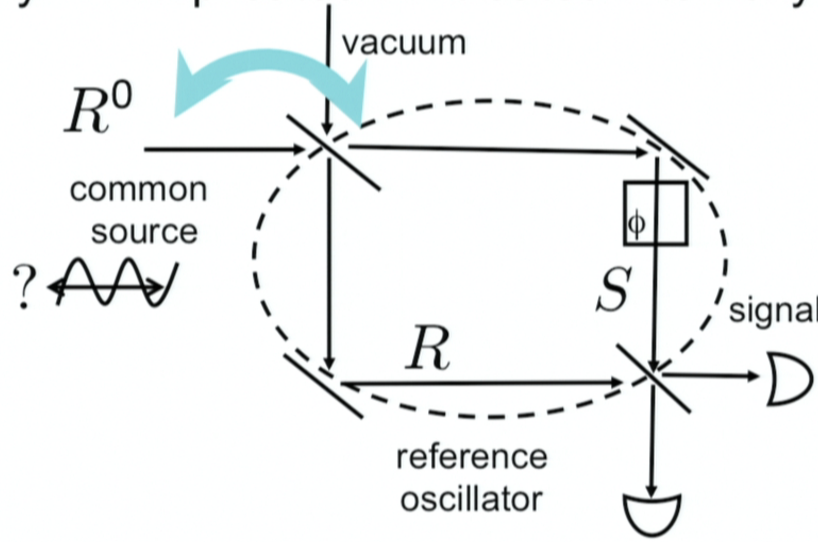
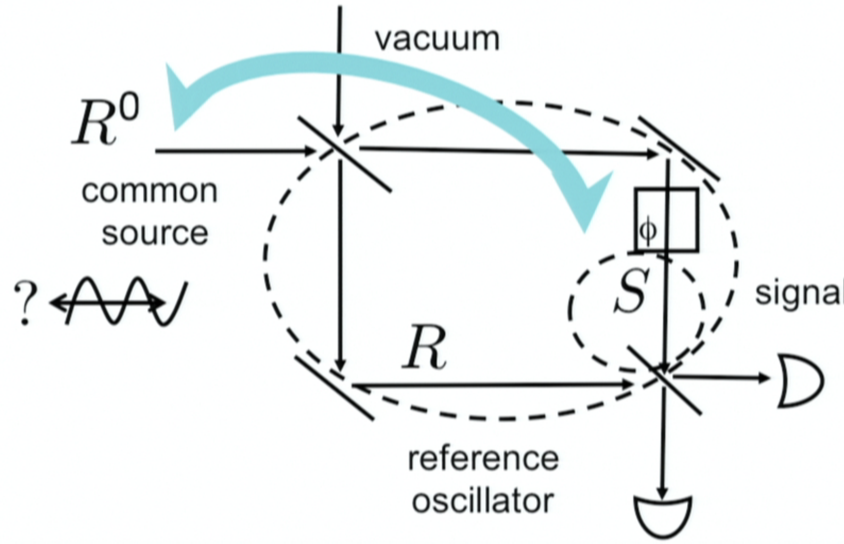


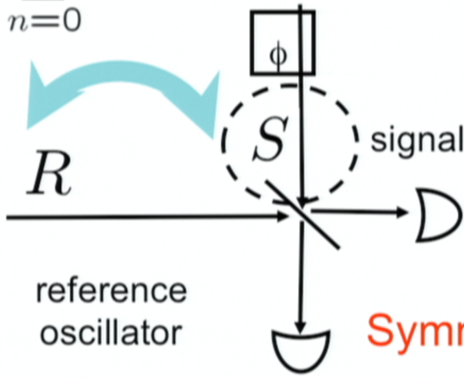


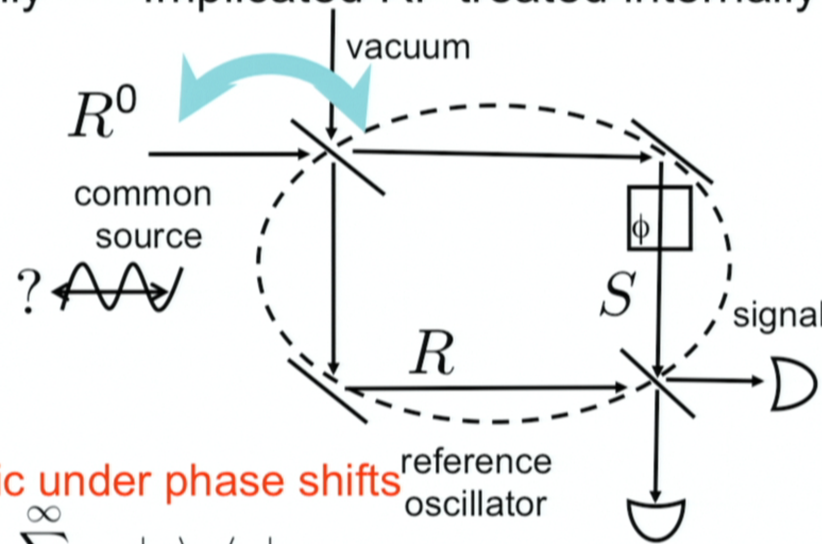
Implicated RF treated externally

Implicated RF treated internally

Asymmetric under phase shifts

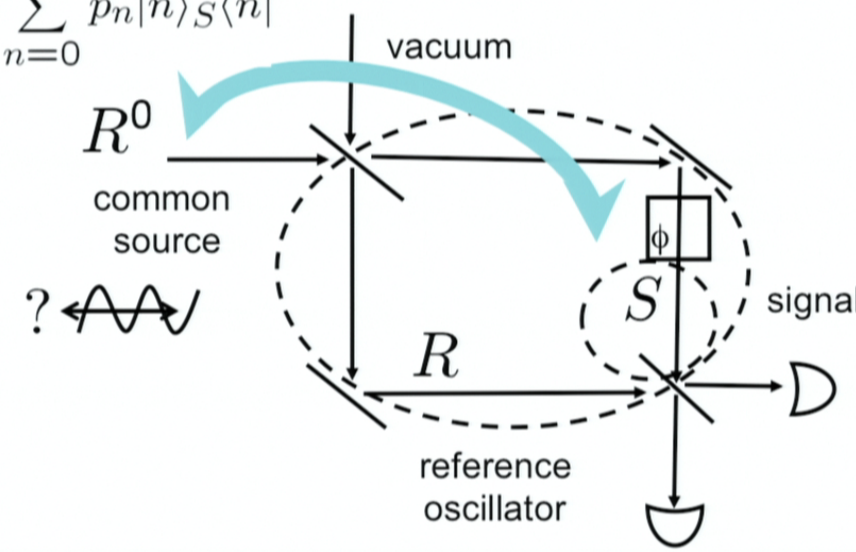
$$|\alpha\rangle_S = \sum_{n=0}^{\infty} \sqrt{p_n} e^{in\phi} |n\rangle_S$$





Symmetric under phase shifts

$$\sigma_S = \sum_{n=0}^{\infty} p_n |n\rangle_S \langle n|$$



Coherence as fact

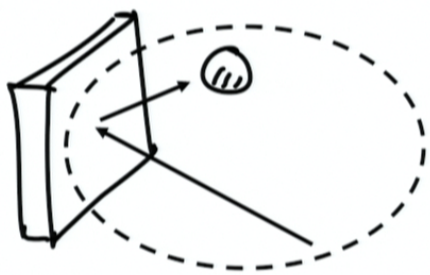
Always treat the implicated RFs externally

Coherence as fiction

Always treat the implicated RFs internally

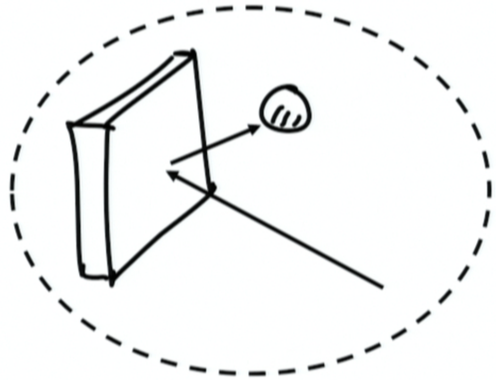
External versus internal reference frames in classical physics

External RF



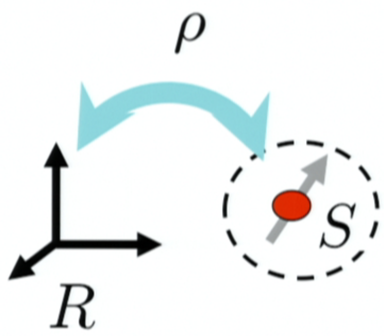
- Failure of conservation of linear momentum
- Failure of symmetry under translations

Internal RF



- Conservation of linear momentum
- Symmetry under translations

External RF paradigm



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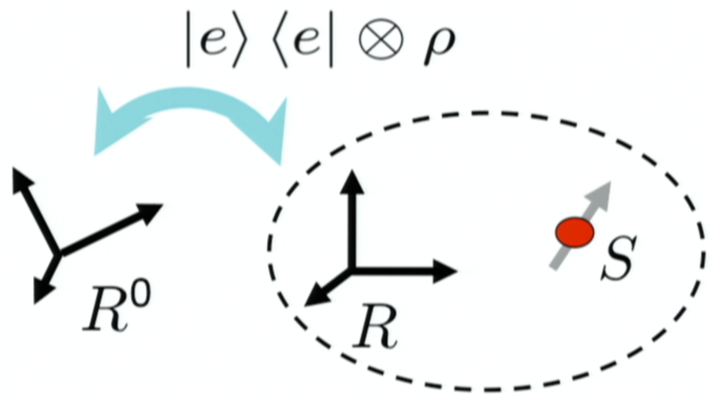

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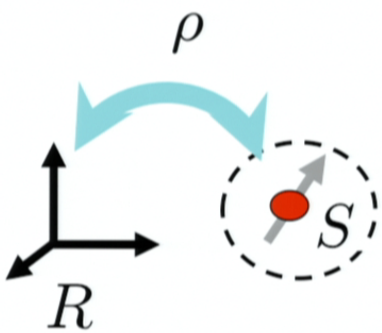
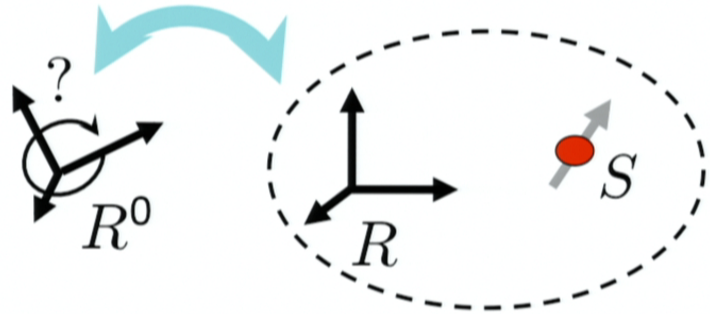


Internal RF paradigm

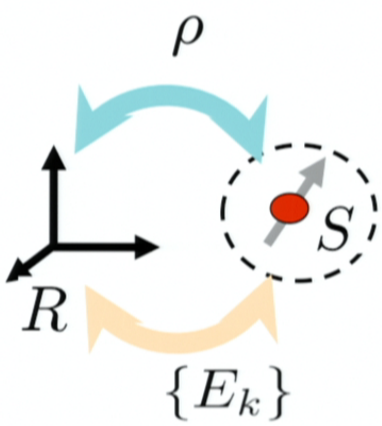
$$d^{-1}\mathcal{G}(|e\rangle\langle e| \otimes \rho)$$



$$\mathcal{G}(\cdot) \equiv \int d\Omega (U_R(\Omega) \otimes U(\Omega))(\cdot)(U_R^\dagger(\Omega) \otimes U^\dagger(\Omega))$$

State of RS is rotationally-invariant

External RF paradigm

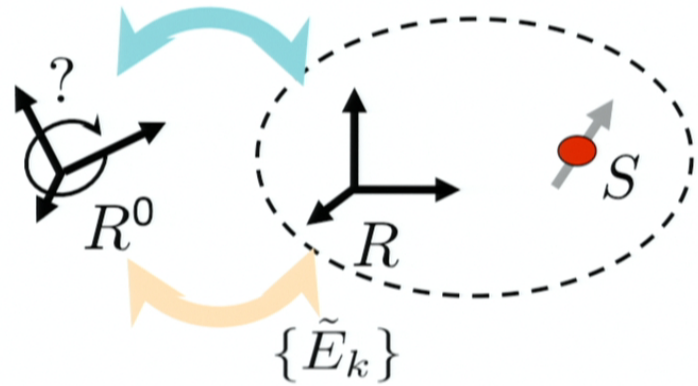


defined on $\mathcal{H}_S$

$\mathrm{Tr}_S[\rho E_k]$

Internal RF paradigm

$\tilde{\rho} = \frac{1}{d}\mathcal{G}(|e\rangle\langle e| \otimes \rho)$

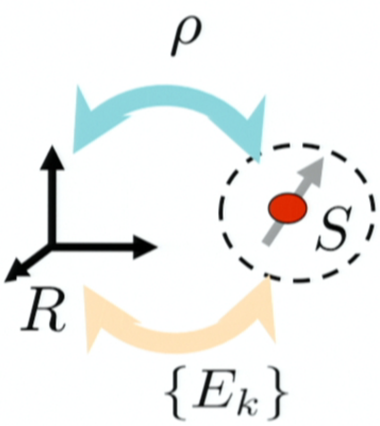


defined on $\mathcal{H}_R \otimes \mathcal{H}_S$

and rotationally-invariant

$\mathrm{Tr}_{RS}[\tilde{\rho}\tilde{E}_k]$

External RF paradigm



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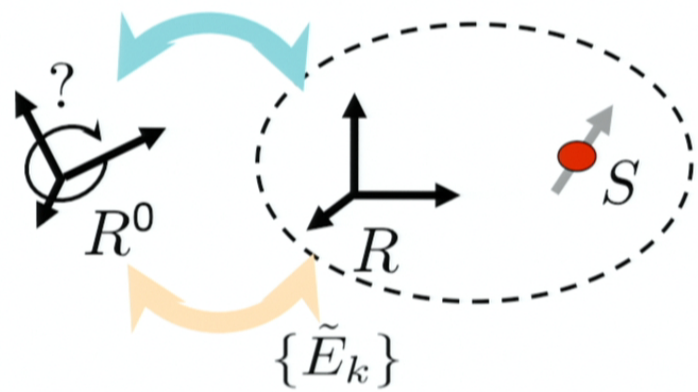


defined on $\mathcal{H}_R \otimes \mathcal{H}_S$

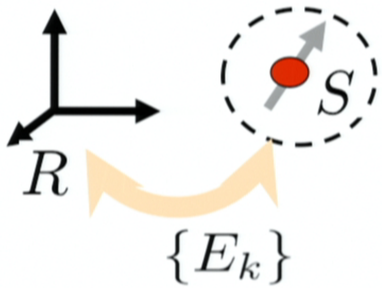
and rotationally-invariant

$\mathrm{Tr}_{RS}[\tilde{\rho}\tilde{E}_k]$

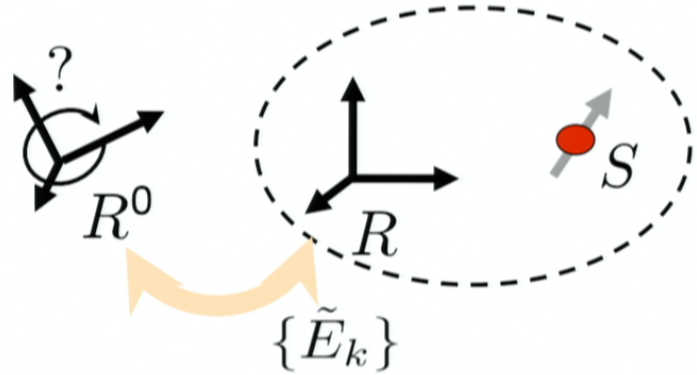
Can we find a measurement scheme such that

$$\mathrm{Tr}_{RS}[\tilde{\rho}\tilde{E}_k] = \mathrm{Tr}_S[\rho E_k] \quad ?$$

External RF paradigm



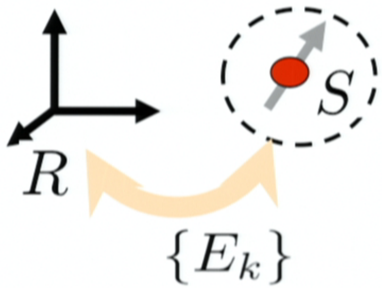
Internal RF paradigm



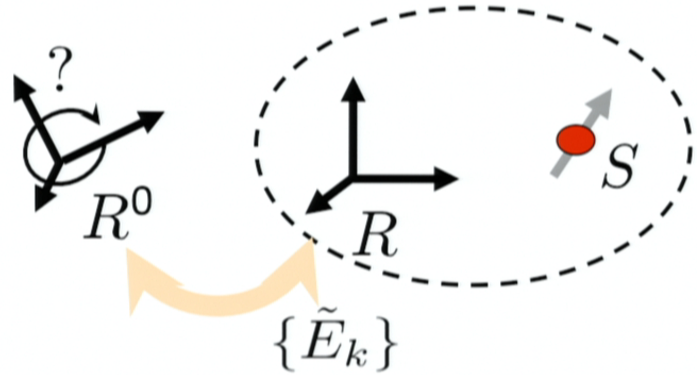
Measure a covariant POVM on R

$$\{U_R(\Omega)|e\rangle\langle e|U_R^\dagger(\Omega)\}_\Omega$$

External RF paradigm



Internal RF paradigm



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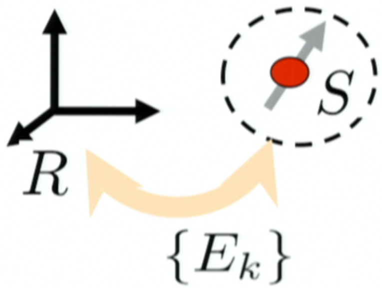
Upon obtaining Ω, measure on S

$$\{U(\Omega)E_k U^\dagger(\Omega)\}_k$$

External RF paradigm



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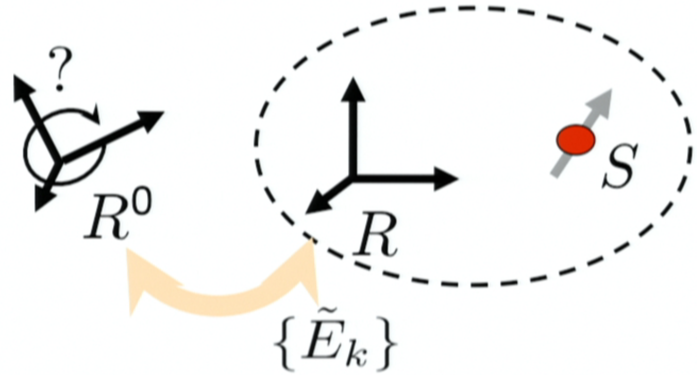

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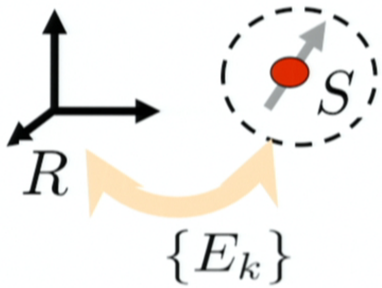
But R^0 has an unknown orientation

$$\tilde{E}_k = \int d\Omega U_R(\Omega)|e\rangle\langle e|U_R^\dagger(\Omega) \otimes U(\Omega)E_kU^\dagger(\Omega)$$

External RF paradigm



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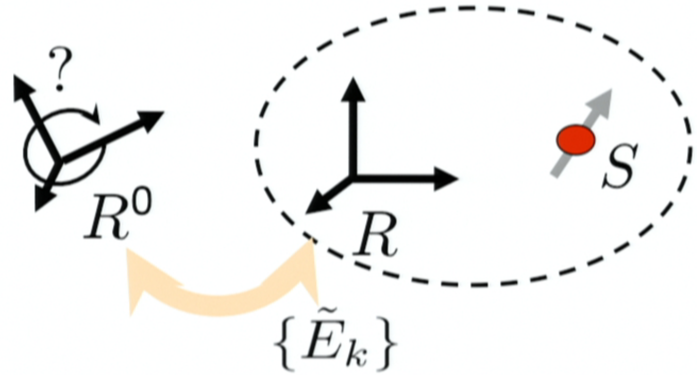

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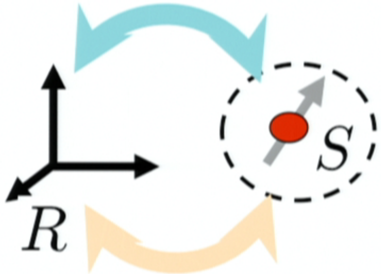
But R^0 has an unknown orientation

$$\tilde{E}_k = \int d\Omega U_R(\Omega)|e\rangle\langle e|U_R^\dagger(\Omega) \otimes U(\Omega)E_k U^\dagger(\Omega)$$
$$= \mathcal{G}(|e\rangle\langle e| \otimes E_k)$$

External RF paradigm

ρ

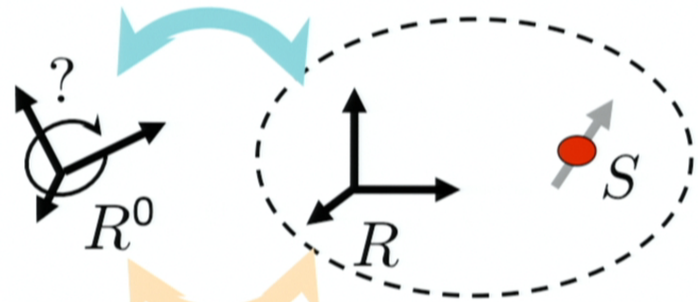


$\{E_k\}$

$\mathrm{Tr}_S[\rho E_k]$

Internal RF paradigm

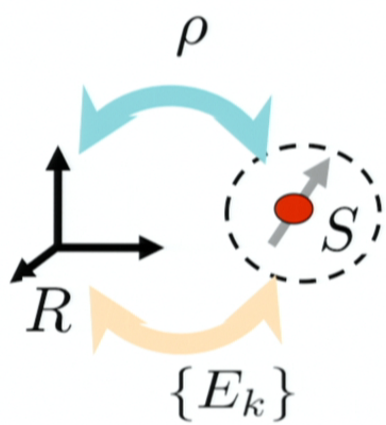
$\tilde{\rho} = \frac{1}{d}\mathcal{G}(|e\rangle\langle e| \otimes \rho)$



$\{\tilde{E}_k = \mathcal{G}(|e\rangle\langle e| \otimes E_k)\}_k$

$\mathrm{Tr}_{RS}[\tilde{\rho}\tilde{E}_k]$

External RF paradigm



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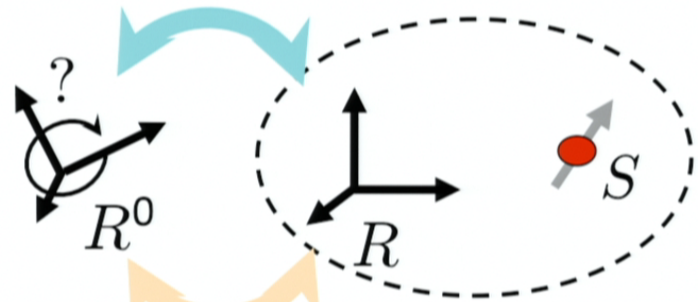


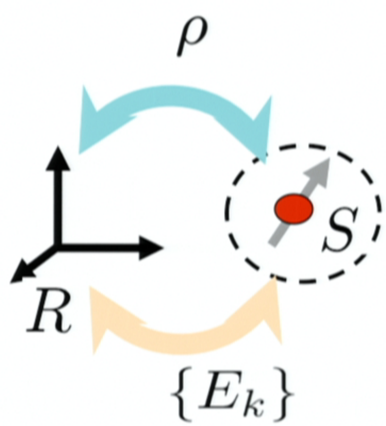
$$\{\tilde{E}_k = \mathcal{G}(|e\rangle\langle e| \otimes E_k)\}_k$$

$$\mathrm{Tr}_{RS}[\tilde{\rho}\tilde{E}_k] = \mathrm{Tr}_S[\mathcal{D}(\rho)E_k]$$

where

$$\mathcal{D}(\cdot) = \frac{1}{d}\int d\Omega |\langle e|U_R(\Omega)|e\rangle|^2 U(\Omega)(\cdot)U^\dagger(\Omega)$$

External RF paradigm



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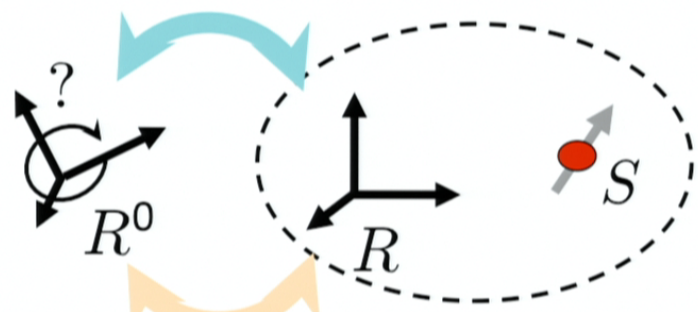

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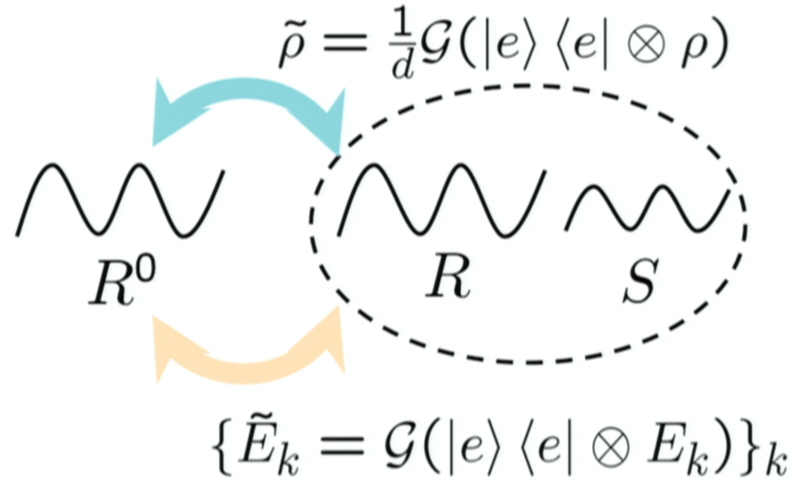
where

$$\mathcal{D}(\cdot) = \frac{1}{d}\int d\Omega |\langle e|U_R(\Omega)|e\rangle|^2 U(\Omega)(\cdot)U^\dagger(\Omega)$$

RF of unbounded size: $\mathcal{D} = \mathrm{id}$

RF of bounded size: $\mathcal{D} \neq \mathrm{id}$

A phase reference



$$\mathrm{Tr}_{RS}[\tilde{\rho}\tilde{E}_k] = \mathrm{Tr}_S[\mathcal{D}(\rho)E_k]$$

where

$$\mathcal{D}(\cdot) = \frac{1}{d}\int d\phi |\langle e|U_R(\phi)|e\rangle|^2 U(\phi)(\cdot)U^\dagger(\phi)$$

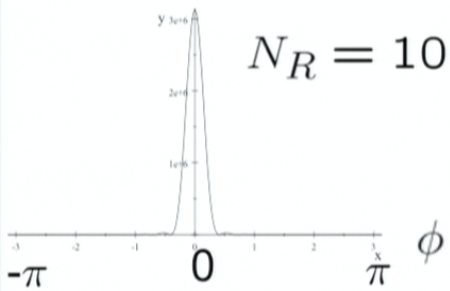
A phase reference

Assume:

$$|e\rangle = \frac{1}{\sqrt{N_R+1}}\sum_{n=0}^{N_R}|n\rangle$$

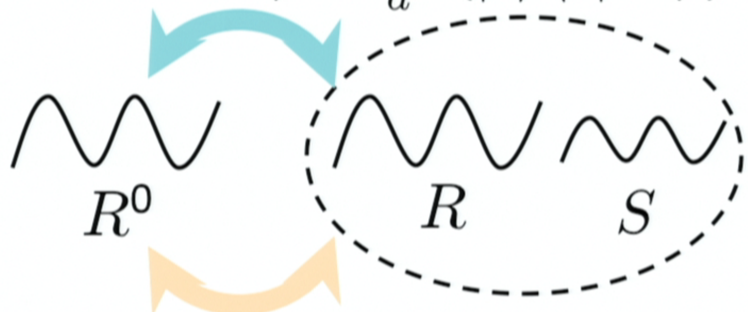
$$|\langle e|U_R(\phi)|e\rangle|^2$$

$$N_R = 10$$



$$U_R(\phi) = e^{i\phi N}$$

$$\tilde{\rho} = \frac{1}{d}\mathcal{G}(|e\rangle\langle e| \otimes \rho)$$



$$\{\tilde{E}_k = \mathcal{G}(|e\rangle\langle e| \otimes E_k)\}_k$$

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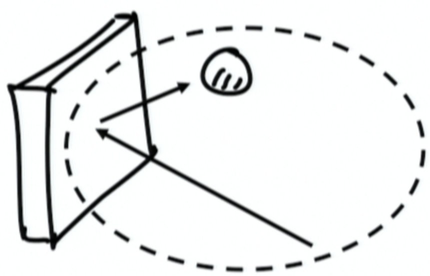
where

$$\mathcal{D}(\cdot) = \frac{1}{d}\int d\phi|\langle e|U_R(\phi)|e\rangle|^2 U(\phi)(\cdot)U^\dagger(\phi)$$

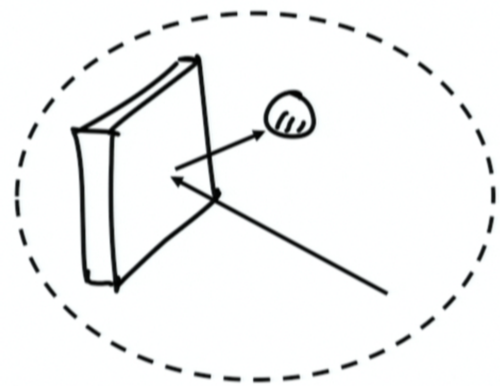
$$\mathcal{D}[\rho] = (\frac{N_R}{N_R+1}\mathrm{id} + \frac{1}{N_R+1}\mathcal{G})[\rho]$$

From internal RF to external RF paradigm: identifying the relational degrees of freedom

External RF

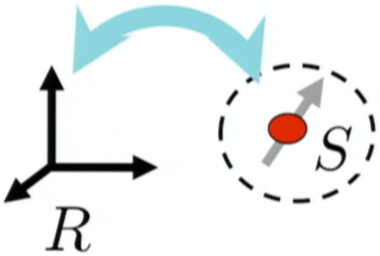


Internal RF



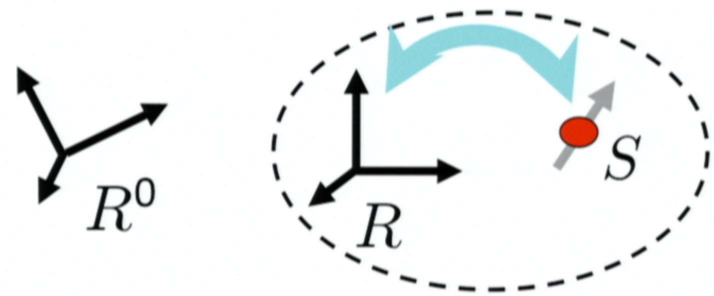
Implicated RF treated externally

$$\rho_S \in \mathcal{L}(\mathcal{H}_S)$$

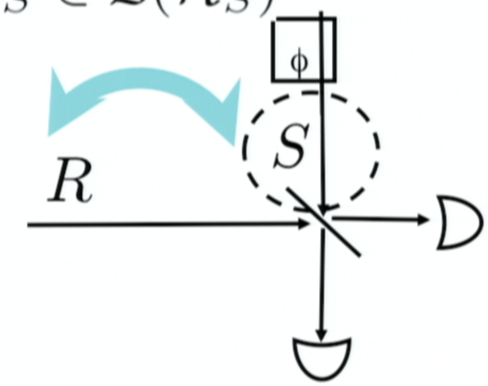


Implicated RF treated internally

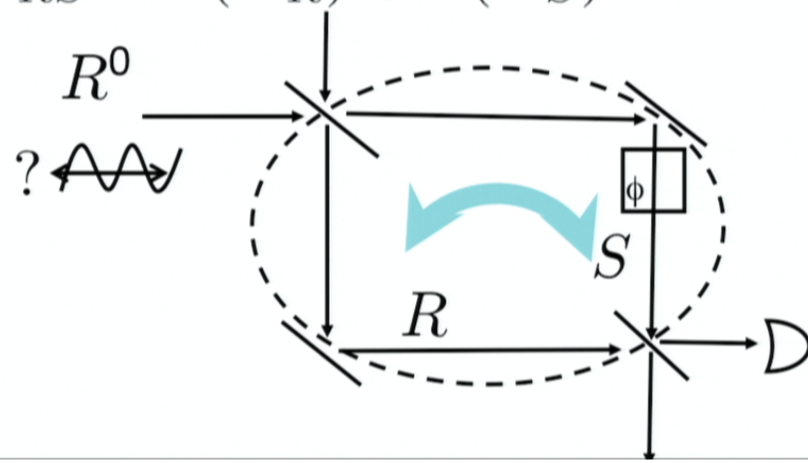
$$\sigma_{RS} \in \mathcal{L}(\mathcal{H}_R) \otimes \mathcal{L}(\mathcal{H}_S)$$



$$\rho_S \in \mathcal{L}(\mathcal{H}_S)$$



$$\sigma_{RS} \in \mathcal{L}(\mathcal{H}_R) \otimes \mathcal{L}(\mathcal{H}_S)$$



Where does the relation between R and S live in the Hilbert space?

The relational degrees of freedom between R and S are
invariant under collective action of group
This property defines virtual subsystems

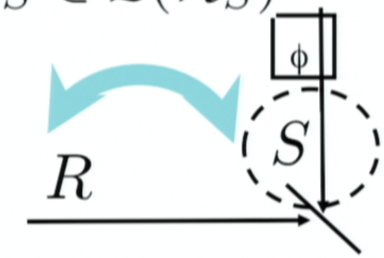
The gauge spaces
Carry the irreps of G

The multiplicity spaces
Carry trivial rep'ns of G

$$\mathcal{H}_{RS} = \sum_q \mathcal{M}_q \otimes \mathcal{N}_q$$

See: Bartlett, Rudolph, Spekkens, Turner, New J. Phys. 11 063013 (2009)

$\rho_S \in \mathcal{L}(\mathcal{H}_S)$



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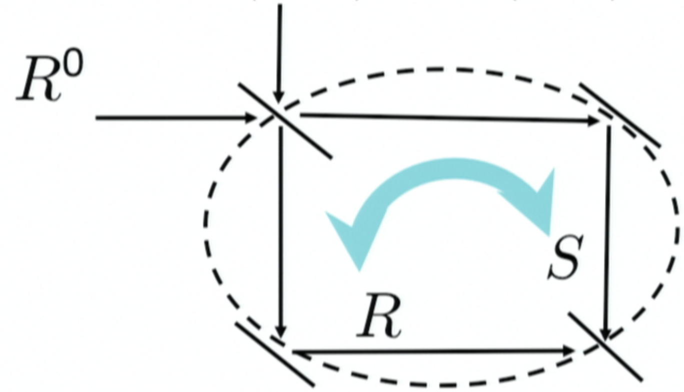


U(1) has only 1d irreps

$\mathcal{M}_n \equiv \mathbb{C}$

The multiplicity spaces are the eigenspaces of total number

$$\mathcal{H}_{RS} = \sum_n \mathcal{M}_n \otimes \mathcal{N}_n$$
$$= \bigoplus_n \mathcal{N}_n$$

External RF　　Internal RF

$$|0\rangle_S \leftrightarrow |n\rangle_R |0\rangle_S$$
$$|1\rangle_S \leftrightarrow |n-1\rangle_R |1\rangle_S$$

$$a|0\rangle_S + b|1\rangle_S \;\leftrightarrow\; a\,|n\rangle_R|0\rangle_S + b\,|n-1\rangle_R|1\rangle_S$$

In practice how do we prepare appropriate reference frames?

Superselection rule	Reference frame
angular momentum	orientation frame
linear momentum	spatial frame
energy	clock
Cooper pair number	superconductor
atom number	Bose-Einstein condensate

Experiments that demonstrate that one has effectively lifted a superselection rule

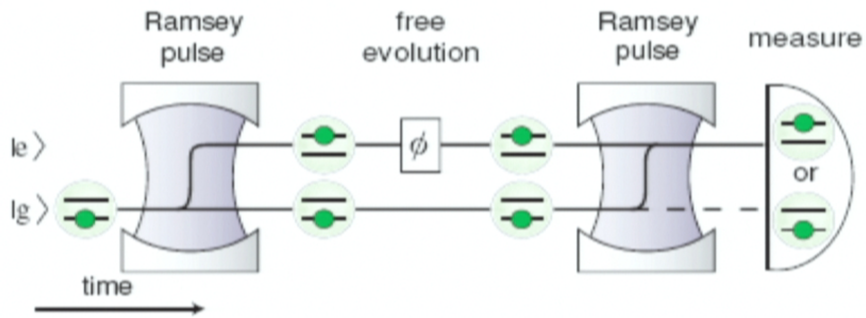
How, in practice, do we distinguish

$$|a\rangle + |b\rangle$$

from

$$|a\rangle\langle a| + |b\rangle\langle b| \quad ?$$

Answer: Interference experiments



$$H_{\text{Ram}} = \frac{\hbar\Omega}{2}\left(|g\rangle\langle e| + |e\rangle\langle g|\right)$$

$$\text{for } \Delta t = \frac{\pi}{2\Omega}$$

$$H_{\text{free}} = \hbar\omega|e\rangle\langle e|$$

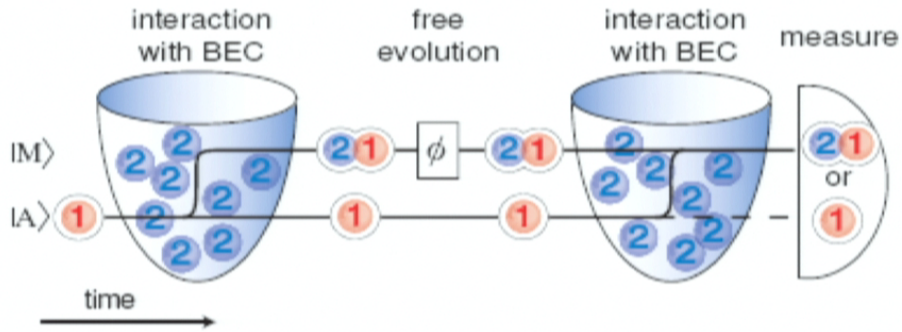
$$\text{for } \Delta t = \phi/\omega$$

$$|g\rangle$$

$$\rightarrow |g\rangle + |e\rangle$$

$$\rightarrow |g\rangle + e^{i\phi}|e\rangle$$

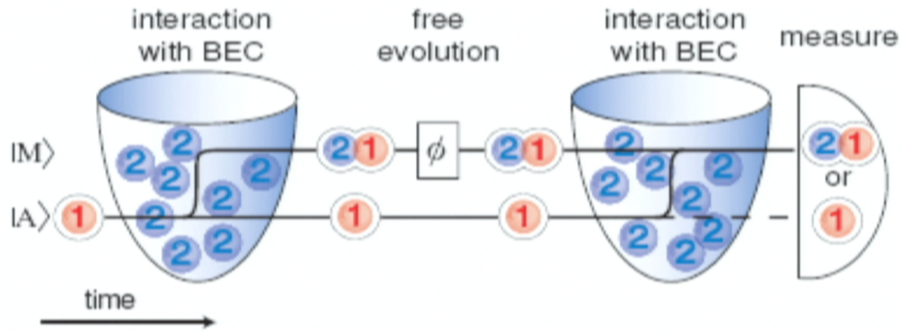
$$\rightarrow \cos(\phi/2)|g\rangle + \sin(\phi/2)|e\rangle$$



How to prepare a coherent superposition of an atom and a molecule

or

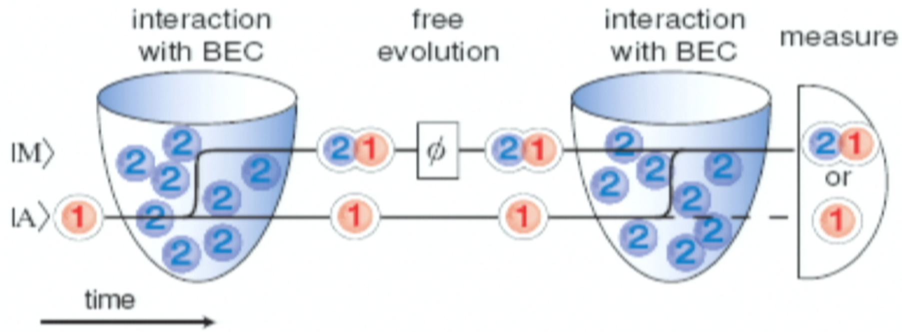
How to violate the baryon number superselection rule



$$H_{\text{Fesh}} = \frac{\hbar\kappa}{2}\left(|M\rangle\langle A| \otimes a_2 + |A\rangle\langle M| \otimes a_2^\dagger\right)$$

for $\Delta t = \frac{\pi}{2\kappa \bar{N}_2}$

$$|A\rangle \equiv a_1^\dagger |\text{vac}\rangle$$
$$|M\rangle \equiv a_M^\dagger |\text{vac}\rangle$$



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$$\text{for } \Delta t = \frac{\pi}{2\kappa \bar{N}_2}$$

$$|A\rangle\langle A| \otimes \sum_n p_n |n\rangle_2\langle n|$$

$$H_{\text{free}} = \hbar\omega |M\rangle\langle M|$$

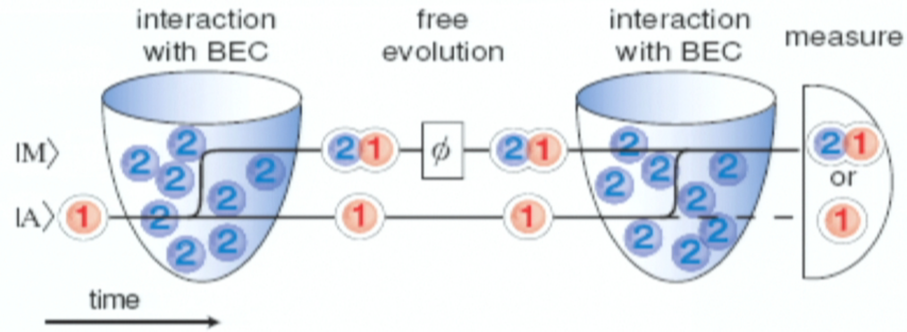
$$|A\rangle|n\rangle_2$$

$$\text{for } \Delta t = \phi/\omega$$

$$\rightarrow |A\rangle|n\rangle_2 + |M\rangle|n-1\rangle_2$$

$$\rightarrow |A\rangle|n\rangle_2 + e^{i\phi}|M\rangle|n-1\rangle_2$$

$$\rightarrow \cos(\phi/2)|A\rangle|n\rangle_2 + \sin(\phi/2)|M\rangle|n-1\rangle_2$$



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$$|A\rangle$$

$$\rightarrow |A\rangle + |M\rangle$$

$$\rightarrow |A\rangle + e^{i\phi}|M\rangle$$

$$\rightarrow \cos(\phi/2)|A\rangle + \sin(\phi/2)|M\rangle$$

Dowling, Bartlett, Rudolph, Spekkens, Phys. Rev. A 74, 052113 (2006)

In practice how do we prepare appropriate reference frames?

Superselection rule	Reference frame
angular momentum	orientation frame
linear momentum	spatial frame
energy	clock
Cooper pair number	superconductor
atom number	Bose-Einstein condensate

Charge and baryon number are not distinguished from angular momentum, linear momentum and energy.

Whether a SSR applies is a matter of convention

To lift a SSR for a conserved quantity, it is necessary to have an appropriate reference frame for the conjugate quantity

In practice how do we prepare appropriate reference frames?

Superselection rule	Reference frame
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univalence	degenerate Fermi gas?

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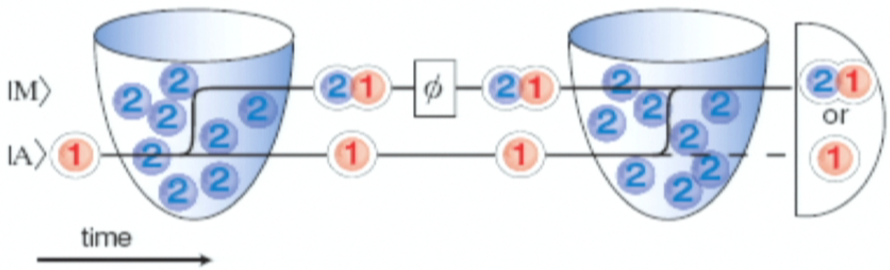
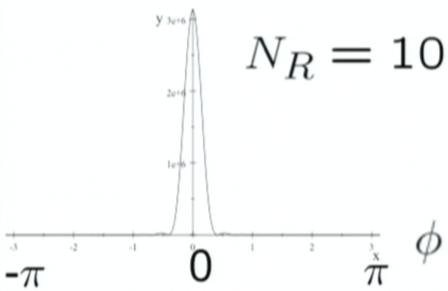


A phase reference

Assume:

$$|e\rangle = \frac{1}{\sqrt{N_R+1}}\sum_{n=0}^{N_R}|n\rangle$$

$$|\langle e|U_R(\phi)|e\rangle|^2$$



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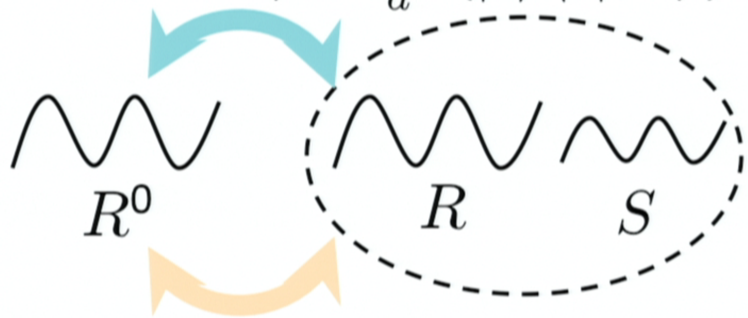

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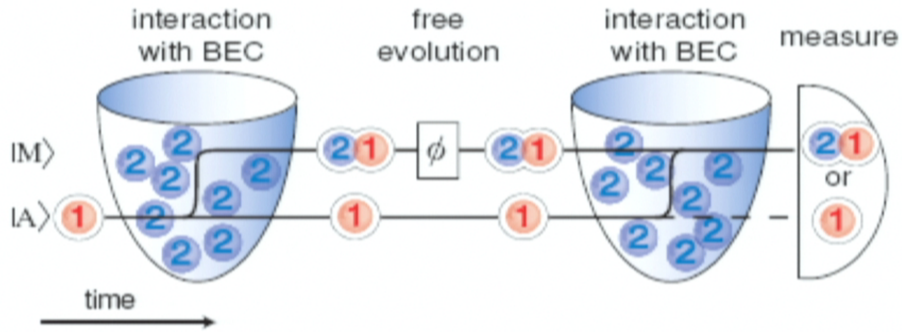

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$$\text{for } \Delta t = \frac{\pi}{2\kappa\bar{N}_2}$$

$$H_{\text{free}} = \hbar\omega_M a_M^\dagger a_M + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_M^\dagger a_2$$

$$\text{for } \Delta t = \phi/(\omega_M - (\omega_1 + \omega_2))$$

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