Title: Before the Bang

Date: Mar 18, 2014 01:00 PM

URL: http://pirsa.org/14030113

Abstract: The simplest black hole solution in asymptotic AdS spacetime, the eternal 3-dimensional BTZ black hole, is studied from the viewpoint of AdS/CFT duality. We identify a class of non-local correlators on the CFT side that allow us to generalize the notion of quantum gravity "S-matrix" to scattering inside the horizon. Since the interior of the horizon is a cosmological spacetime with a big bang/crunch-like singularity, our construction can be interpreted as identifying generally coordinate invariant observables of quantum gravity in a simple cosmology. We show in a certain precise sense, that these holographic observables lie "before the big bang" in a region of spacetime containing closed timelike curves. A central tool in our analysis is the realization of BTZ as a quotient of 3-dimensional AdS.

BEFORE THE BANG

with Anton de la Fuente
based on hep-th 1307.7738

Raman Sundrum
University of Maryland

QUANTUM GRAVITY

If I only knew what the question was!

Usual QFT questions $\sim \langle \mathcal{O}(x)\, \mathcal{O}(x')\, \mathcal{O}(x'') \rangle$

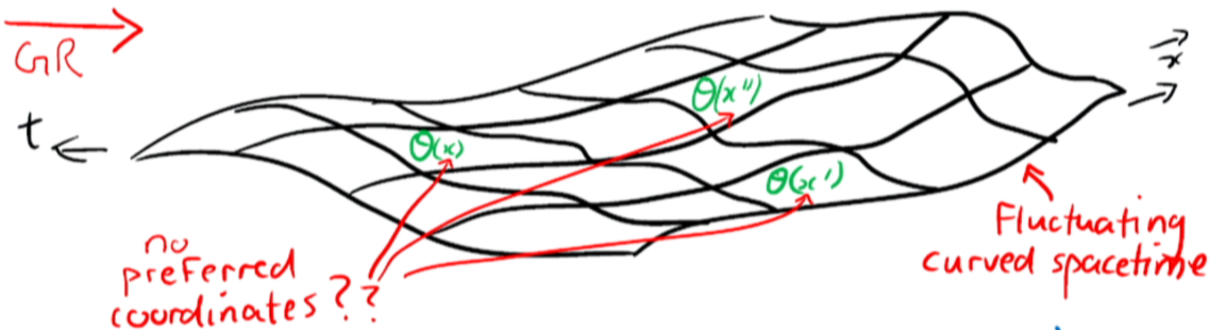
Eg. local gauge-invariant $G^a_{\mu\nu}(x)\, G^a_{\rho\sigma}(x)$

QUANTUM GRAVITY

If I only knew what the question was!

Usual QFT questions $\sim \langle \mathcal{O}(x)\, \mathcal{O}(x')\, \mathcal{O}(x'') \rangle$

$\xrightarrow{GR}$



$\Rightarrow$ no fundamental local observables are generally coordinate invariant

GAUGE-FIXED LOCAL OPERATORS

Eg. A_μ^a: $\quad \partial_\mu A^{a\mu} = 0$ gauge

requires solving

$$\partial_\mu D^\mu \omega^a = \partial_\mu A^{a\mu} \text{ non-locally:}$$

$$\partial_\mu A_a'^\mu = 0,$$

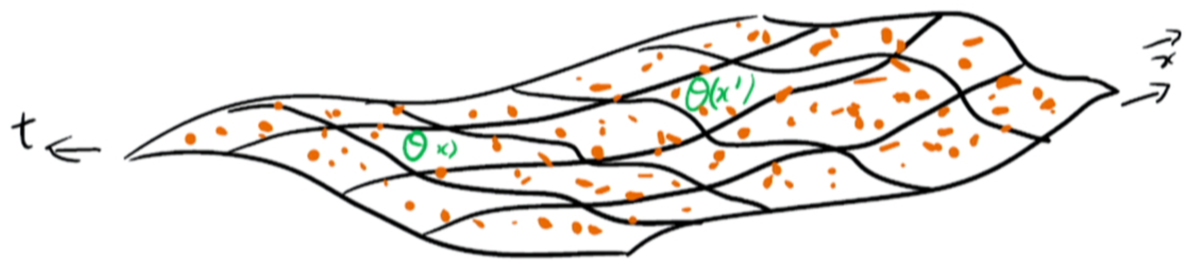
$$A^{a\prime\mu} \equiv A^{a\mu} - D^\mu \omega^a$$

But only perturbatively useful

HIGGSED

COORDINATE INVARIANCE

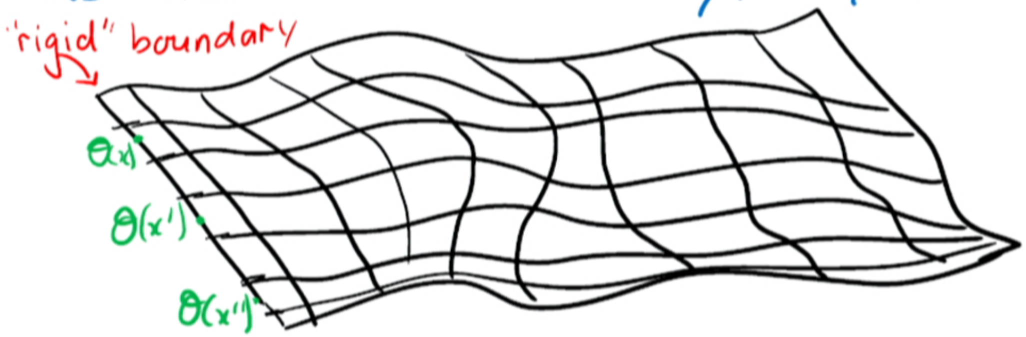
& "UNITARY" GAUGE



$\Rightarrow \quad \langle\, \Theta(\text{Sun})\, \Theta(\text{Alpha Centauri}) \,\rangle$

VIRTUE OF SPACETIME BOUNDARY/ASYMPTOTIC GEOMETRY

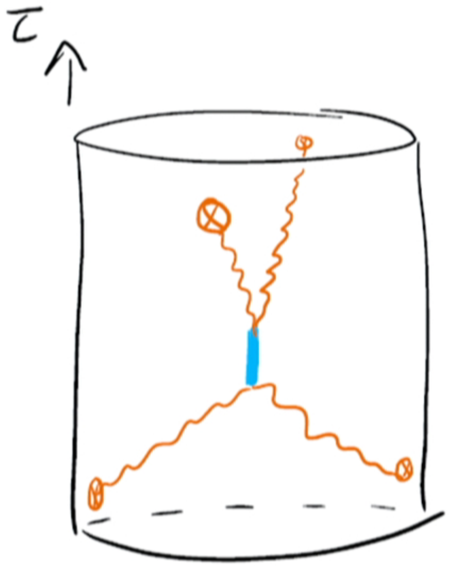
IF spacetime becomes non-dynamical as one → boundary/asymptotic infinity



⇒ meaningful local boundary observables Eg. AdS

or asymptotic "S-matrix" Eg. Minkowski

QUANTUM GRAVITY IN AdS



AdS_{2+1}

(conformal to time x hemisphere)

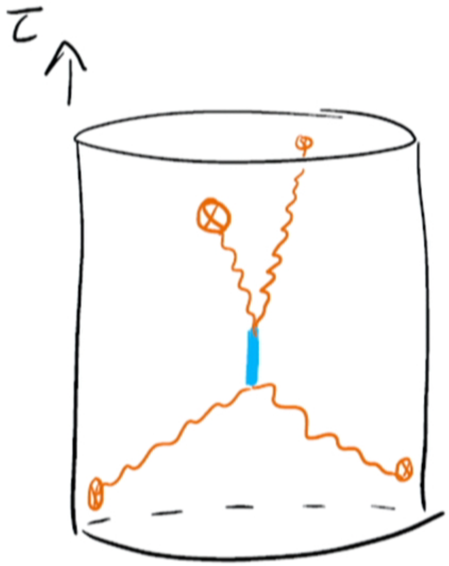
Witten Diagrams
(Feynman diagrams for gravitons & particles ending on ∂AdS)

≡ AdS generalization of S-matrix,
general coordinate invariant.

Incoming/Outgoing "beams" can sharply probe inside bulk.

~ Polchinski '99;
Susskind '99;
⋮
Fitzpatrick, Kaplan '11

QUANTUM GRAVITY IN AdS



AdS_{2+1}

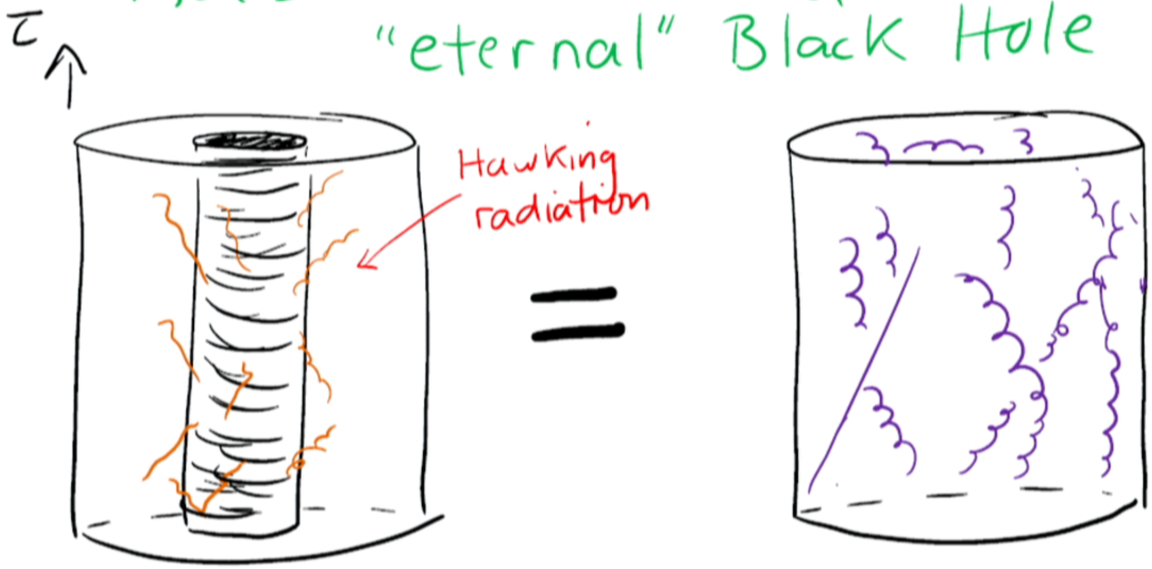
(conformal to time x hemisphere)

Witten Diagrams
(Feynman diagrams for gravitons & particles ending on ∂AdS)
≡ AdS generalization of S-matrix,
general coordinate invariant.

Incoming/Outgoing "beams" can sharply probe inside bulk.

~ Polchinski '99;
Susskind '99;
⋮
Fitzpatrick, Kaplan '11

AdS Schwarzchild
"eternal" Black Hole

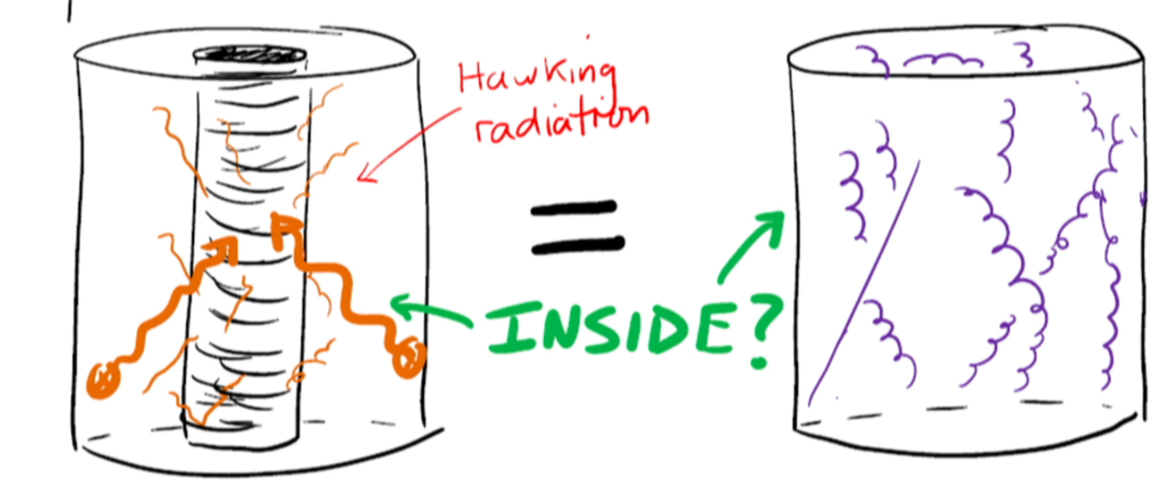


Hot CFT $T \neq 0$

$$ds^2_{BTZ} = \frac{r^2 - r_s^2}{R_{AdS}^2} d\tau^2 - \frac{R_{AdS}^2}{r^2 - r_s^2} dr^2 - r^2 d\varphi^2 \quad \varphi \in (-\pi, \pi]$$

$$\underset{R_{AdS} \equiv 1}{\equiv} (r^2 - 1) d\tau^2 - \frac{dr^2}{r^2 - 1} - r^2 d\sigma^2 \quad \sigma \in \left(-\frac{\ln\lambda}{2}, \frac{\ln\lambda}{2}\right], \quad \lambda \equiv e^{r_s / R_{AdS}}$$

AdS Schwarzchild
"eternal" Black Hole



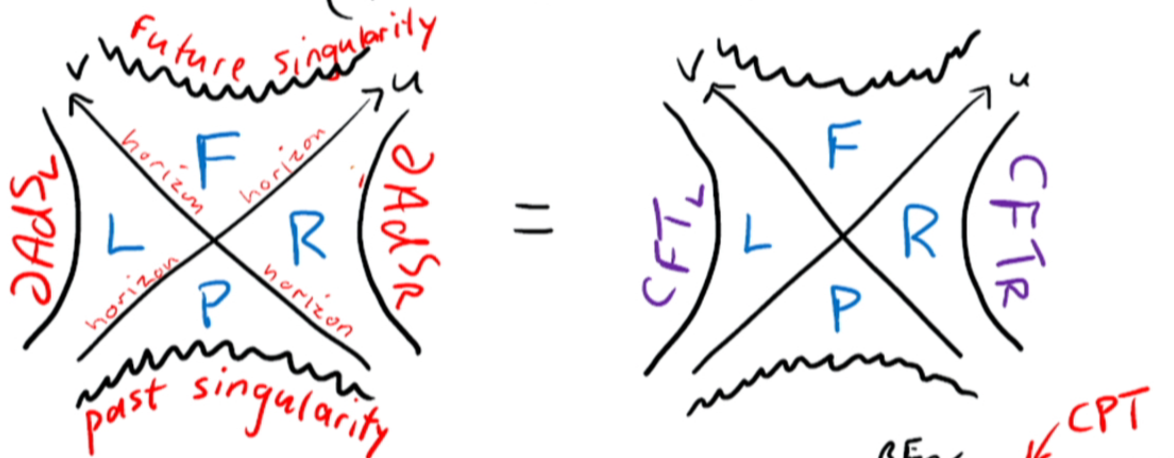
Hot CFT $T \neq 0$

$$ds^2_{BTZ} = \frac{r^2 - r_s^2}{R_{AdS}^2}\, d\tau^2 - \frac{R_{AdS}^2}{r^2 - r_s^2}\, dr^2 - r^2 d\varphi^2_{\nwarrow (-\pi,\, \pi]}$$

$$\underset{R_{AdS} \equiv 1}{\equiv} (r^2 - 1)\, d\tau^2 - \frac{dr^2}{r^2 - 1} - r^2 d\sigma^2_{\nearrow \left(-\frac{\ln\lambda}{2},\, \frac{\ln\lambda}{2}\right]}, \quad \lambda \equiv e^{r_s / R_{AdS}}$$

Extended Black Hole Spacetime

$$ds^2_{\text{Kruskal}} = \frac{4\,du\,dv}{(1+uv)^2} - \left(\frac{1-uv}{1+uv}\right)^2 d\sigma^2, \quad |uv|<1$$



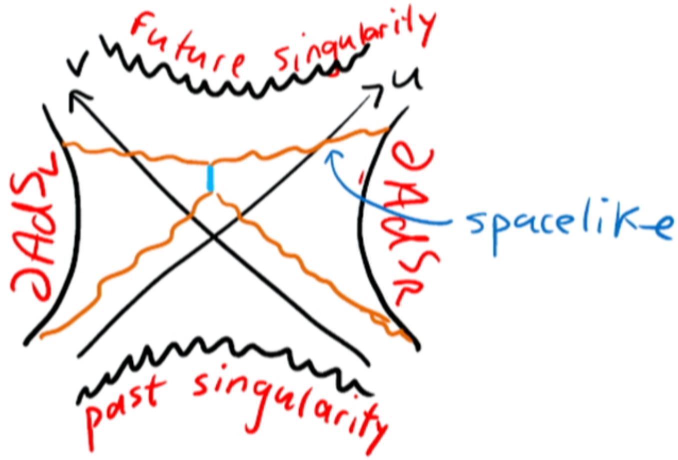
CPT

$$|\text{Hartle-Hawking}\rangle = |\psi\rangle \equiv \sum_n e^{-\beta E_n/2}\, |\bar{n}\rangle \otimes |n\rangle$$

$$\in CFT_L \otimes CFT_R$$

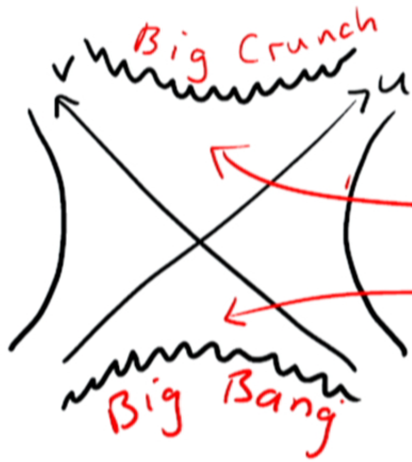
Israel '76; ... Maldacena '01

STILL CANNOT SCATTER
IN & OUT OF HORIZON "ON-SHELL"
FROM $\partial AdS_{L,R}$



$\Rightarrow$ DIFFUSE, DIM "VIEW"
INSIDE HORIZON

Inside horizons:
(Anisotropic) Cosmological Spacetimes



fuelled by vacuum energy < 0.

"contracting" cosmology

"expanding" cosmology

GOALS

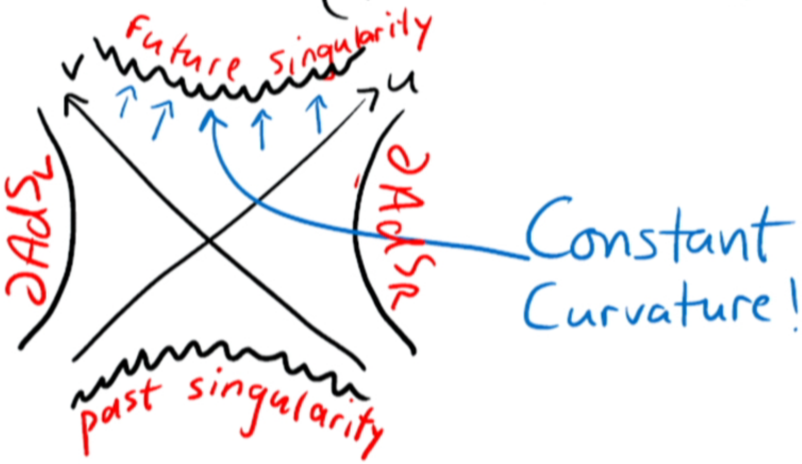
- New class of simple CFT observables to more sharply probe inside horizon of BTZ black hole, generalize quantum gravity S-matrix
- Translation of local correlations inside horizon to non-local correlations outside, where translating procedure is non-perturbative (in $1/N_{CFT}$, GR EFT, ...)

OUTLINE

- BTZ & its singularity
- BTZ/CFT & Rindler AdS/CFT
- "S-matrix" ≡ boundary correlators behind singularity
- Finiteness of such correlators, $i\varepsilon$
- $x \leftrightarrow t$ inside horizon & in CFT
- boundary correlators behind singularity ≡ non-local CFT correlators
- local correlators inside horizon as non-local correlators outside

2+1 BTZ "SINGULARITY" NOT SO SINGULAR...

$$ds^2_{\text{Kruskal}} = \frac{4\,du\,dv}{(1+uv)^2} - \left(\frac{1-uv}{1+uv}\right)^2 d\sigma^2, \quad |uv|<1$$



SPACETIME CAN BE FURTHER EXTENDED BEYOND SINGULARITIES...

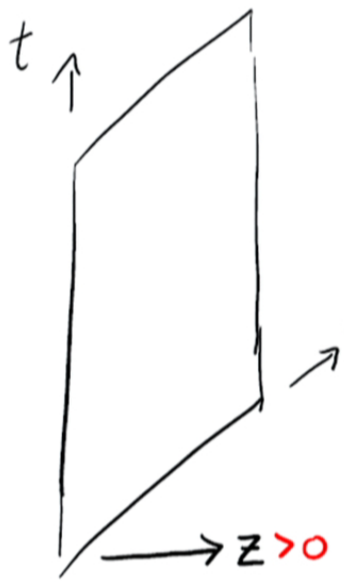
MAXIMAL EXTENSION

$$BTZ = AdS_{2+1}^{\text{global}} \Big/ \text{discrete isometry}$$

Here, we restrict to an INTERMEDIATE EXTENSION > Kruskal:

$$BTZ = AdS_{2+1}^{\text{Poincare}} \Big/ \text{discrete isometry}$$

$AdS_{2+1}^{\text{Poincare}}$



$\subset AdS_{2+1}^{\text{global}}$

$$ds^2 = \frac{dt^2 - dx^2 - dz^2}{z^2} R_{AdS}^2$$

$R_{AdS} = 1$

AdS boundary $z = 0$ is infinitely far away, but light can go there & back in finite time.

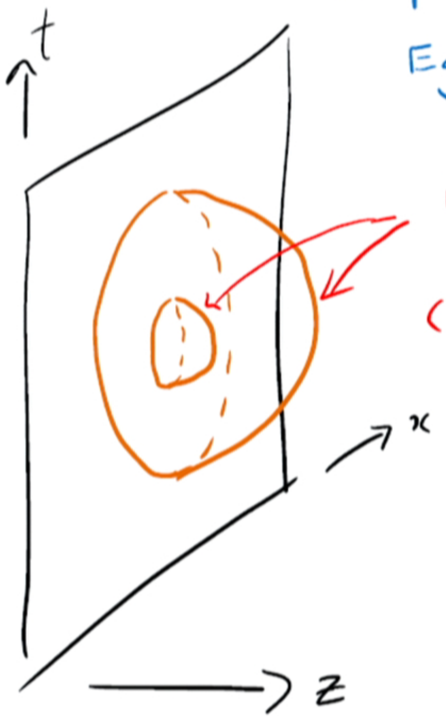
BTZ BLACK HOLE

Banados, Teitelboim, Zanelli '92; Banados, Henneaux, Teitelboim, Zanelli '93

$$= AdS_{2+1}(\times \mathcal{M})/\Gamma$$

Eg. $\mathcal{M} = S^3 \times T^4$ IIB String Theory

Maldacena, Strominger '98



identify hemispherical surfaces (same physical size) $\Rightarrow$ Constant curvature geometry without t-like symmetry

$$(t, x, z) \longleftrightarrow (\lambda t, \lambda x, \lambda z)$$

identify

$\Gamma \equiv$ discrete dilatation

COORDINATES

$$t + x \equiv \frac{2e^{\sigma} v}{1 - uv} \equiv \begin{cases} +\sqrt{1 - 1/r^2}\; e^{\tau+\sigma}, & r > 1 \\ +\sqrt{1/r^2 - 1}\; e^{\tau+\sigma}, & r < 1 \end{cases}$$

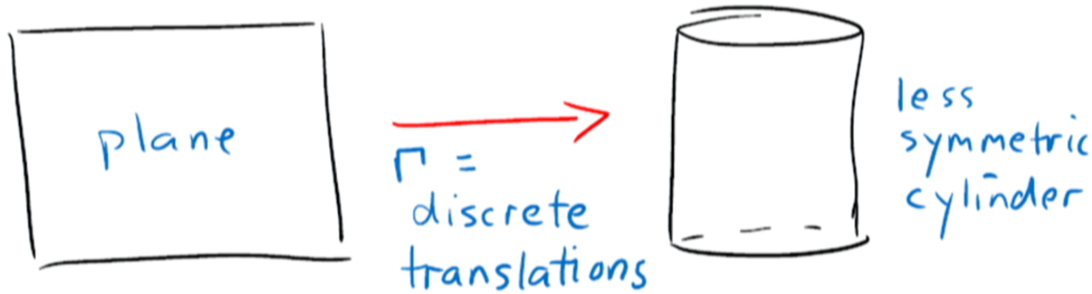
$$t - x \equiv \frac{2e^{\sigma} u}{1 - uv} \equiv \begin{cases} -\sqrt{1 - 1/r^2}\; e^{\sigma-\tau}, & r > 1 \\ +\sqrt{1/r^2 - 1}\; e^{\sigma-\tau}, & r < 1 \end{cases}$$

$$z \equiv \frac{1 + uv}{1 - uv} e^{\sigma} \equiv e^{\sigma}/r$$

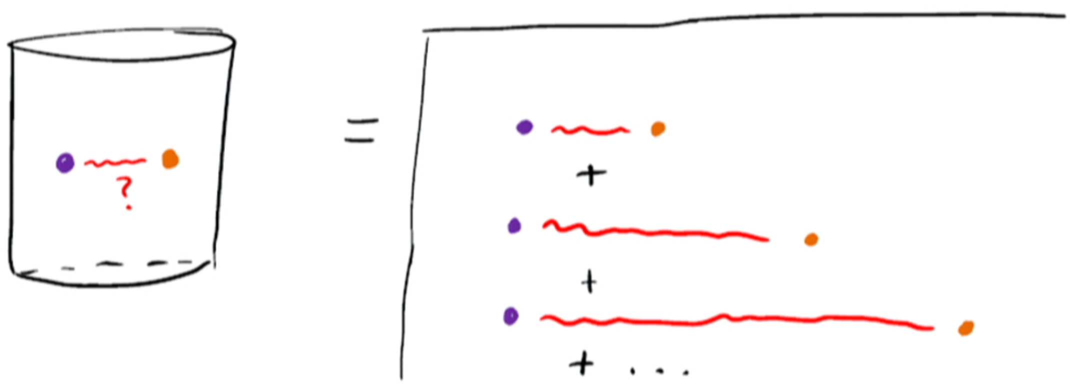
Poincare Kruskal Schwarzchild

QUOTIENTING SPACETIME

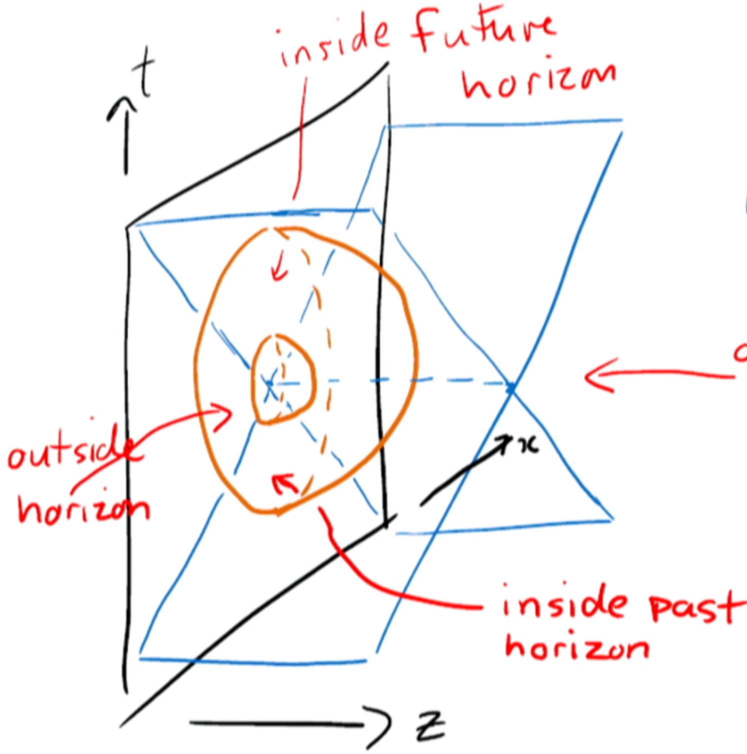
Eg.



$\Rightarrow$ PHYSICS BY METHOD OF IMAGES



BTZ BLACK HOLE

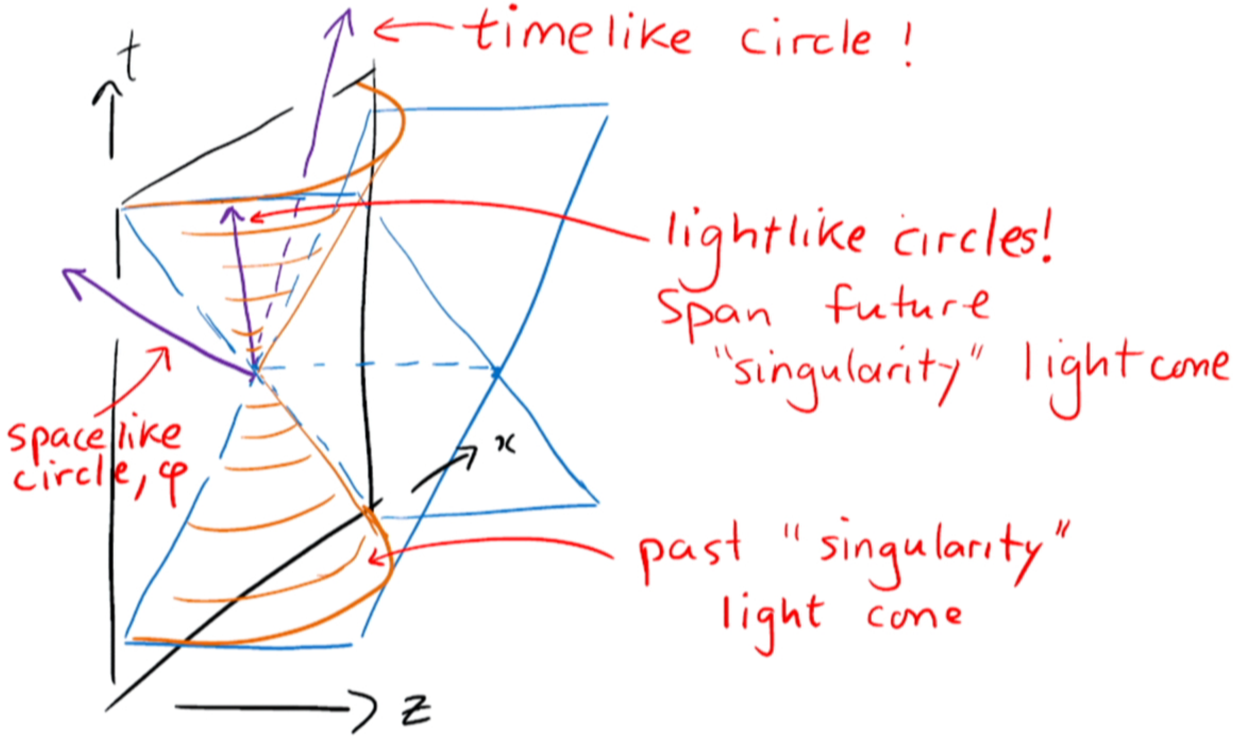


Before quotienting we start with mere Rindler horizons in (warped) Minkowski space.

After quotienting these get "locked in" globally to "real" horizons.

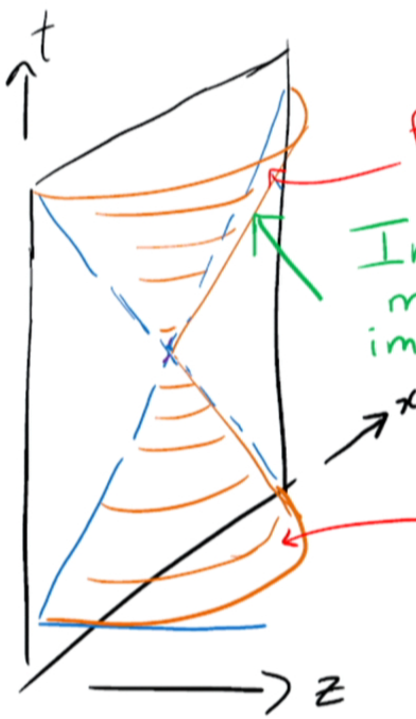
BACK TO THE FUTURE

& BACK AGAIN, & ...



"Singularity" + particle = Curvature Singularity!

GR argument: Horowitz, Polchinski '02



Future "singularity" lightcone

Infalling particle (+ infinitely many, arbitrarily energetic images) back reacts → real singularity (Curvature → ∞)

past "singularity" light cone

General light circle pathologies: Steif '94; Lifschytz, Ortiz '94; Kay, Radzikowski, Wald '97

String scattering, backreaction: " . Review: Berkooz, Reichmann '07

WHY? BTZ (Scalar)
GREEN'S FUNCTION

$$G_{AdS_{2+1}}(\xi), \quad \xi \equiv \frac{2zz'}{z^2+z'^2+(x-x')^2-(t-t')^2(1-i\varepsilon)}$$

simple function of proper distance

$$G_{BTZ} = \sum_{n=-\infty}^{\infty} G_{AdS}\left(\left\{\frac{2zz'\lambda^n}{\lambda^{2n}z^2+z'^2+(\lambda^n x-x')^2-(\lambda^n t-t')^2(1-i\varepsilon)}\right\}\right)$$

images

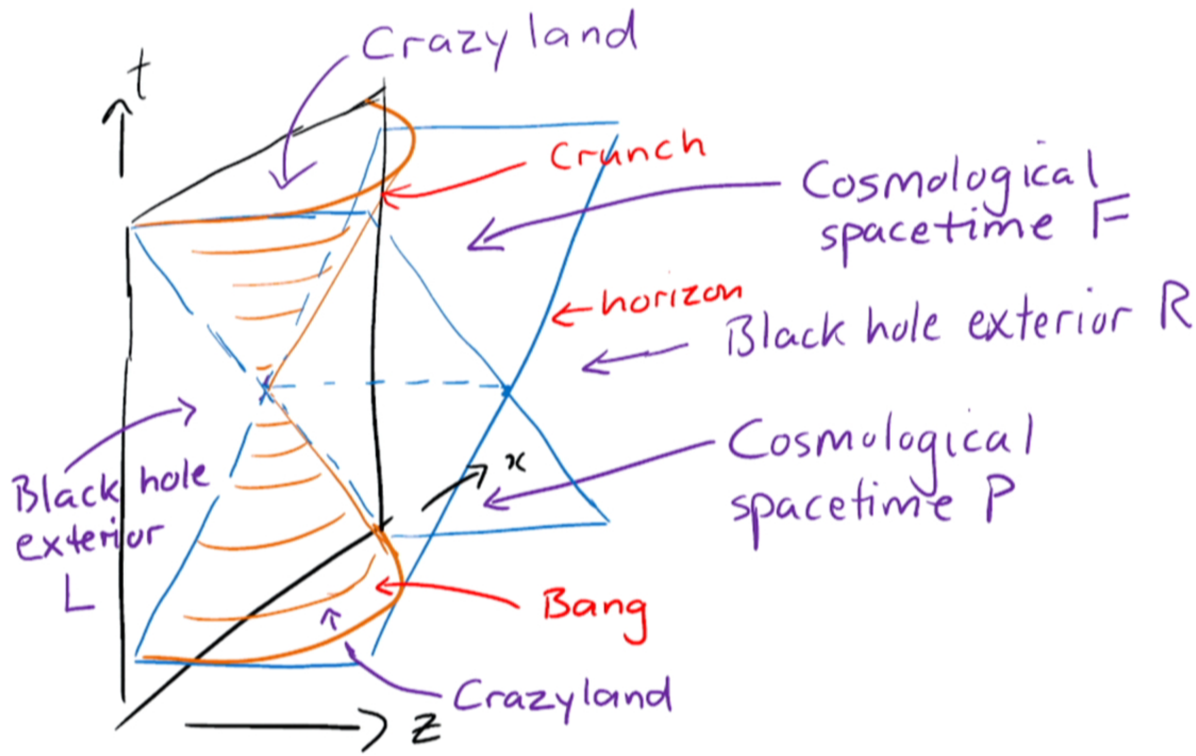
$\lambda \equiv e^{r_s} > 1$

$\sim \begin{cases} \lambda^{-|n|} & \text{as } |n| \to \infty, \text{ generically} \\ n\text{-independent as } n \to \infty, & \text{if } z^2+x^2-t^2=0 \end{cases}$

$\longrightarrow \infty$ if (t,x,z) on "singularity"

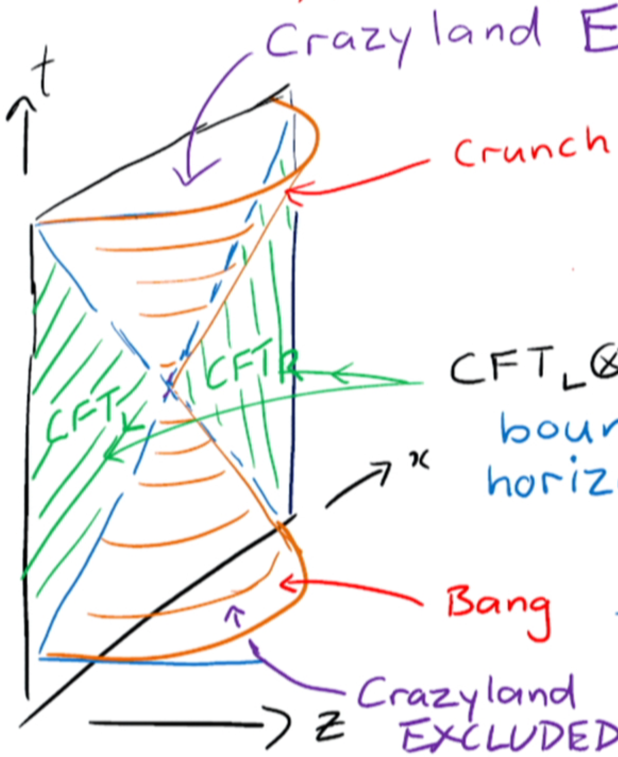
Kraus, Ooguri, Shenker '03

MAP OF UNIVERSE



BTZ / CFT DUALITY

Israel '76; .. ; Maldacena, Strominger '98; ... ; Maldacena '01; ..



$CFT_L \otimes CFT_R$ on $z=0$

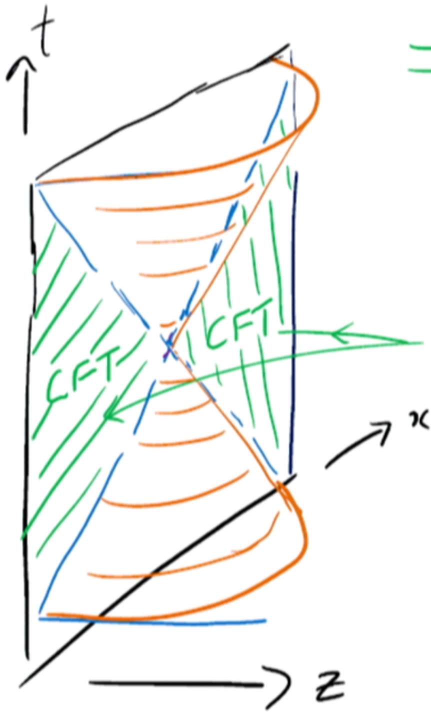
boundary wedges outside horizon, quotiented,

$\equiv CFT_{\mathbb{R}\times S^1} \times CFT_{\mathbb{R}\times S^1}$

Thermofield entangled state dual to bulk outside singularities.

⇐ BTZ "BLACK STRING"

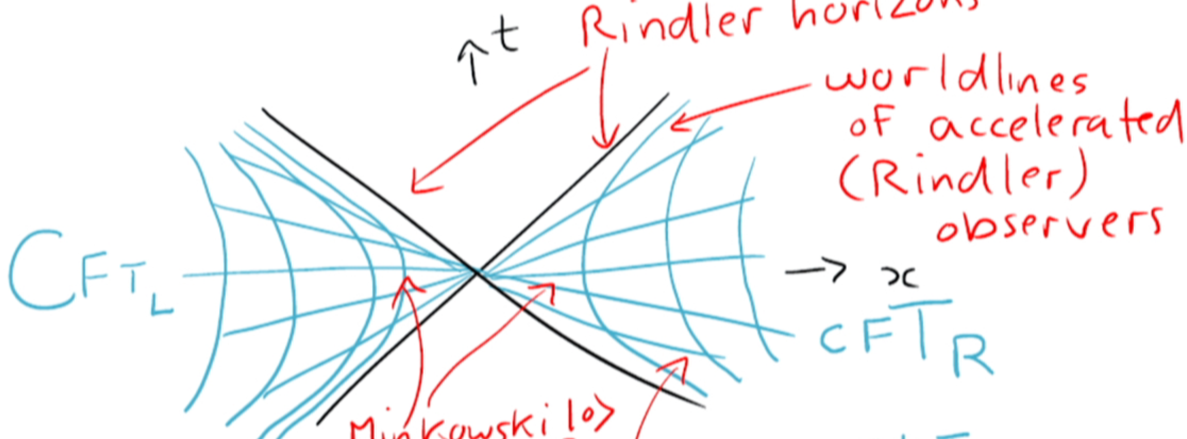
$\lambda = \infty$ (no quotienting)



= RINDLER VIEW OF $\text{AdS}^{\text{Poincare}}$

CFT quanta from different wedges are entangled in thermofield state dual to AdS quantum gravity outside (non-singular) cones.

$\lambda = \infty$: RINDLER VIEW OF $\mathrm{CFT}_{\mathbb{R}_t \times \mathbb{R}_x}$



superposition of left & right Rindler states

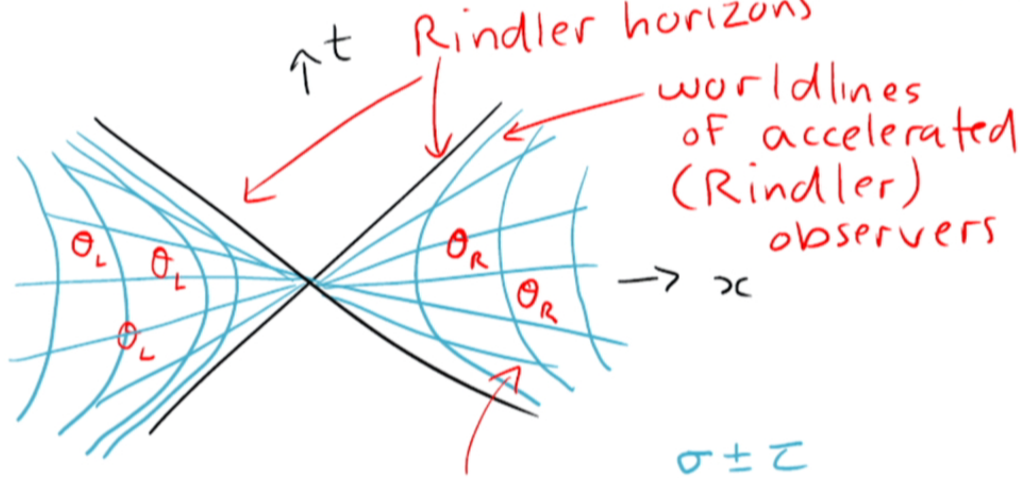
$$x \pm t \equiv e^{\sigma \pm \tau}$$

a conformal transformation

Accelerating Geiger counter goes off in vacuum:

$$|0\rangle_{\text{Minkowski}} = \sum_n e^{-\beta E_n/2} |\bar{n}\rangle_L \otimes |n\rangle_R$$

$\lambda = \infty$: RINDLER VIEW OF $CFT_{\mathbb{R}_t \times \mathbb{R}_x}$



$$x \pm t \equiv e^{\sigma \pm \tau}$$

$$\langle 0 | T_t\, \mathcal{O}_{L_1} \cdots \mathcal{O}_{L_\ell}\, \mathcal{O}_{R_1} \cdots \mathcal{O}_{R_r} | 0 \rangle$$

conformal transformation

$$= \sum_{n,m} e^{-\beta E_n/2} e^{-\beta E_m/2} \langle \bar{n} | T_\tau \mathcal{O}'_{L_1} \cdots \mathcal{O}'_{L_\ell} | \bar{m} \rangle \langle n | T_\tau \mathcal{O}'_{R_1} \cdots \mathcal{O}'_{R_r} | m \rangle$$

$\lambda < \infty$ BTZ BLACK HOLE

BOUNDARY CORRELATORS in L, R exterior regions follow by method of images

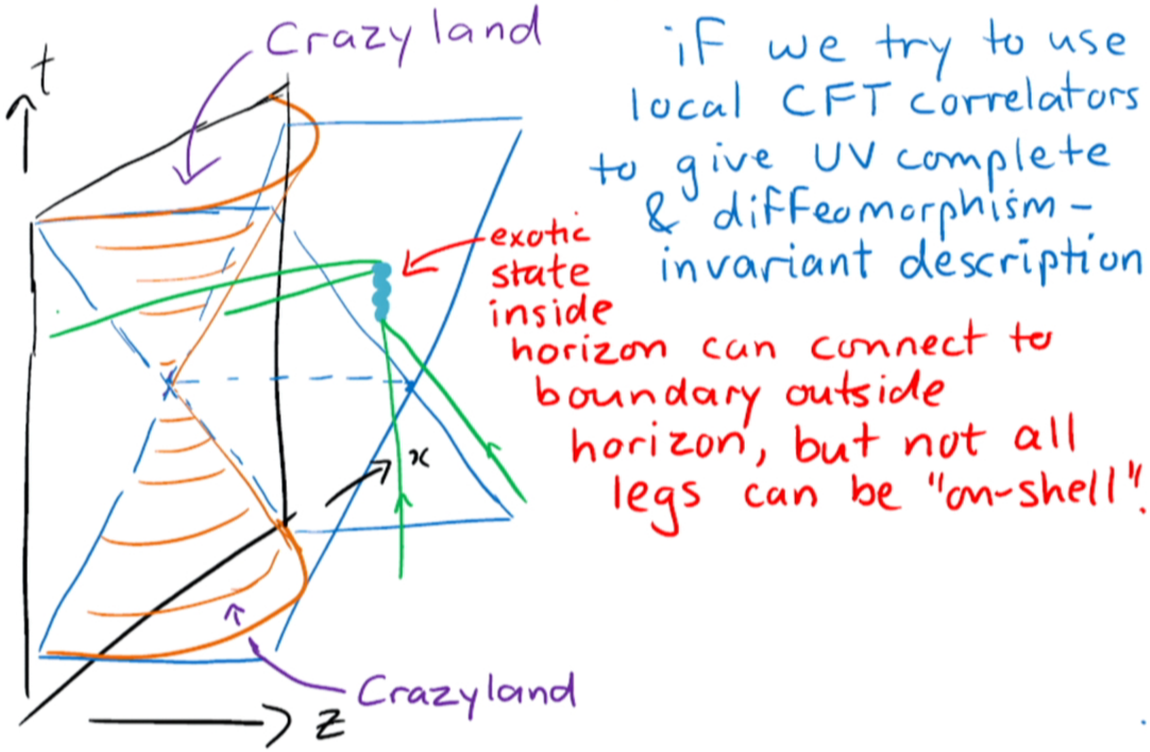
$$= \sum_{n,m} e^{-\beta E_n/2} e^{-\beta E_m/2} \langle \bar{n} | T_\tau \mathcal{O}_{L_1} \cdots \mathcal{O}_{L_\ell} | \bar{m} \rangle \times \langle n | T_\tau \mathcal{O}_{R_1} \cdots \mathcal{O}_{R_r} | m \rangle$$

where $|n\rangle$ are energy eigenstates of $CFT_{\mathbb{R}_\tau \times S^1_\sigma}$ $\leftarrow$ boundary of Schwarzchild spacetime (τ, σ, r).

since $\sigma \to \sigma + \ln \lambda$ under dilatations

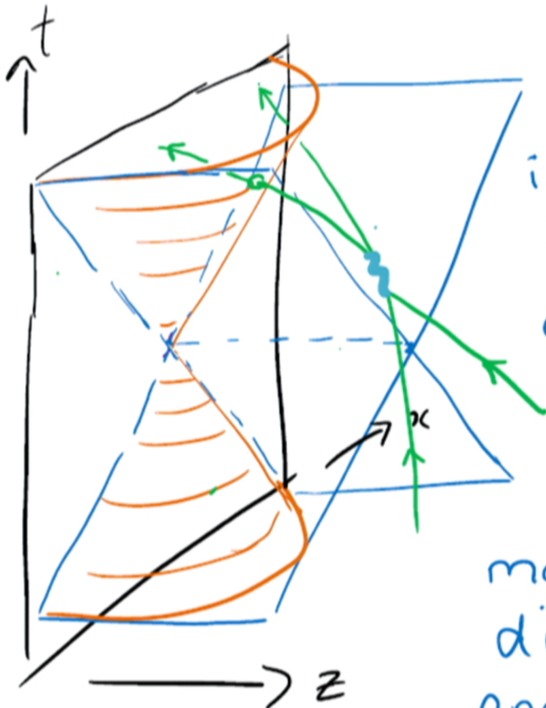
$$\Rightarrow |BTZ \text{ ``vacuum''}\rangle = \sum_n e^{-\beta E_n/2} |\bar{n}\rangle \otimes |n\rangle$$

ONLY "OFF-SHELL" PROBES INSIDE HORIZON



if we try to use local CFT correlators to give UV complete & diffeomorphism-invariant description

COMPARISON WITH $\lambda = \infty$

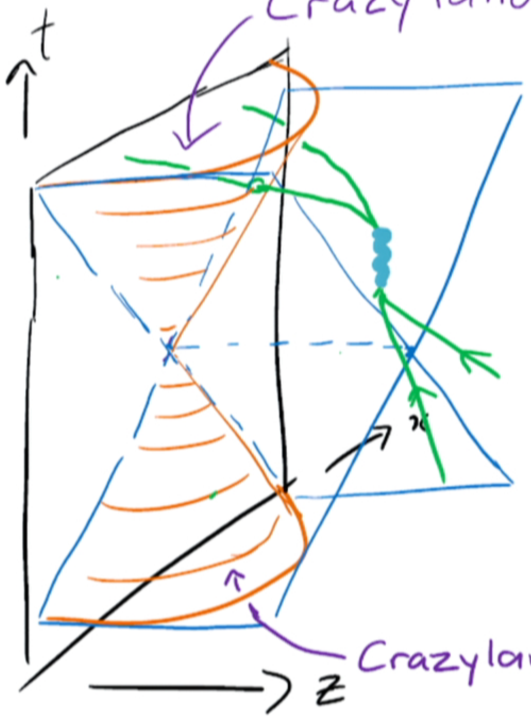


With $CFT_{\mathbb{R}_t \times \mathbb{R}_x}$ on all of $z=0$, Witten diagrams can probe inside (Rindler) horizon "on-shell", by ending on boundary inside cones

This suggests we make sense of analogous diagrams for $\lambda < \infty$ ending on Crazyland Boundary suggests

A CENTRAL ROLE FOR CRAZYLAND $\lambda < \infty$

Crazy land after the Crunch



Crazyland before the Bang

Analogous Witten diagrams despite ending on ∂Crazyland?

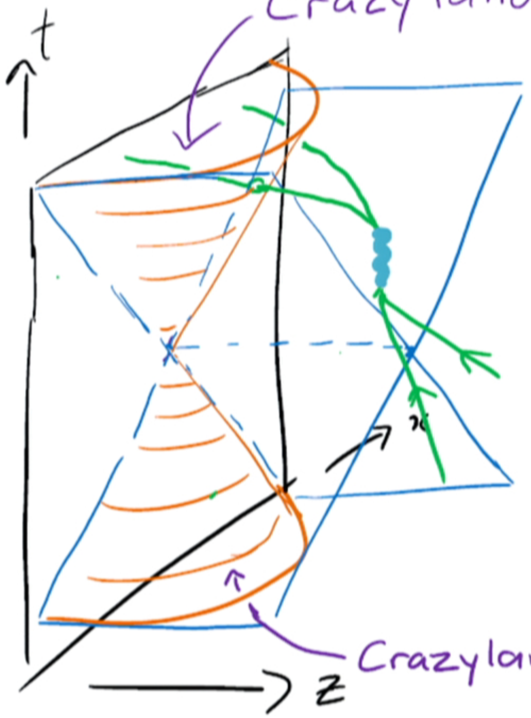
a) $< \infty$ despite traversing singularity?

b) Physically sensible?

c) Part of originally proposed $CFT_L \otimes CFT_R$ duality?

suggests

A CENTRAL ROLE FOR CRAZYLAND $\lambda < \infty$



Analogous Witten diagrams despite ending on ∂Crazyland?

a) $< \infty$ despite traversing singularity?

b) Physically sensible?

c) Part of originally proposed $CFT_L \otimes CFT_R$ duality?

YES, YES, YES!

MORE DIRECTLY . . .

$$G_{BTZ} = \sum_{n\in\mathbb{Z}} G_{AdS}\left(\frac{2zz'\lambda^n}{(\lambda^n x - x')^2 - (\lambda^n t - t')^2 + \lambda^{2n} z^2 + z'^2 + \underbrace{i\varepsilon}(\lambda^n t - t')^2}\right)$$

$\varepsilon \ll 1$, but still important for large n.

$$\underset{t \sim \sqrt{x^2+z^2}}{\sim} \ln\left(x^2 + z^2 - t^2 + i\varepsilon t^2\right)$$

$$\underset{\text{diagrams}}{\Longrightarrow} \int_{<\sqrt{x^2+z^2}}^{>\sqrt{x^2+z^2}} dt \; \frac{\ln^k(x^2+z^2-t^2+i\varepsilon t^2)}{(x^2+z^2-t^2+i\varepsilon t^2)^\ell}$$

ℓ from derivative interactions

$< \infty$ due to $i\varepsilon$

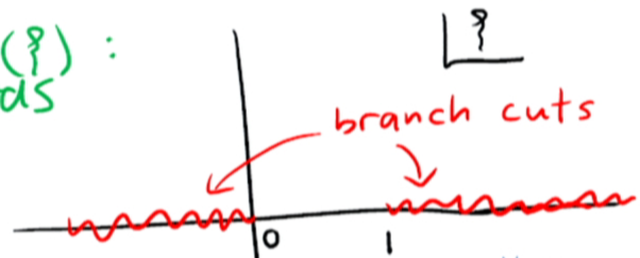
c/w Kraus, Ooguri, Shenker '03

Oddly, does not work with AdS^{global} $i\varepsilon$!?

MORE EFFICIENTLY...

$z>0$ only appears in (integrated) interaction vertices of Witten diagrams.

$G_{AdS}(\rho)$:



Proper "Distance" measure

$$\rho = \frac{2 z' z}{z^2+z'^2+\Delta x^2-\Delta t^2(1-i\varepsilon)}$$

Rotate integration contour

$$z \to z e^{i\beta}, \quad 0<\beta<\pi/2$$

$\Longrightarrow$

$$\xi \longrightarrow \frac{2zz'e^{2i\beta}}{(z^2+z'^2)e^{2i\beta}+\Delta x^2-\Delta t^2}$$

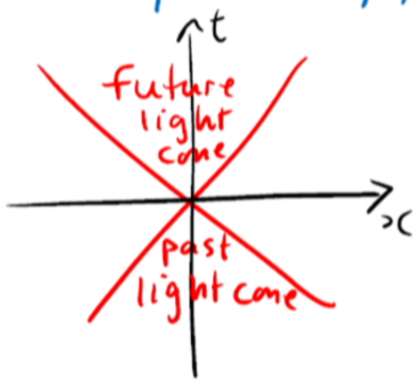
$$G_{BTZ}(\xi) \longrightarrow \sum_{n\in\mathbb{Z}} G_{AdS}\left(\frac{2zz'e^{2i\beta}\lambda^n}{(z^2\lambda^{2n}+z'^2)e^{2i\beta}+(\lambda^n x-x')^2-(\lambda^n t-t')^2}\right)$$

Sum converges, even at singularity $z^2+x^2=t^2$

Conclusion: Correlators ending near singularity diverge.
But there are cancelations for straddling correlators.

$\lambda=\infty$: SPACE & TIME TRADE PLACES IN $CFT_{\mathbb{R}_t \times \mathbb{R}_x}$

In 1+1 dimensions, causal structure has more symmetry, $x \rightleftarrows t$



Conformally invariant physics exhibits this as an "improper" conformal symmetry

$$ds^2 = dt^2 - dx^2 \xrightarrow[x \leftrightarrow t]{} (-1)[dt^2 - dx^2]$$

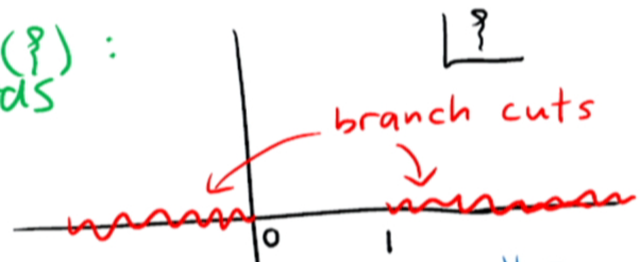
Classical example: Non-linear σ-model

$$\partial_t[k_{ij}(\vec{\chi})\partial_t \chi_j] = \partial_x[k_{ij}(\vec{\chi})\partial_x \chi_j]$$

MORE EFFICIENTLY...

$z>0$ only appears in (integrated) interaction vertices of Witten diagrams.

$G_{AdS}(\xi)$:



Proper "Distance" measure

$$\xi = \frac{2 z' z}{z^2+z'^2+\Delta x^2-\Delta t^2(1-i\varepsilon)}$$

Rotate integration contour

$$z \to z e^{i\beta}, \quad 0<\beta<\pi/2$$