Title: Seeing is Believing: Direct Observation of a General Quantum State - Jeff Lundeen

Date: Mar 17, 2014 02:00 PM

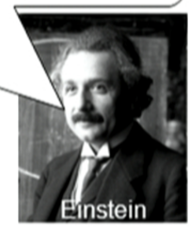
URL: http://pirsa.org/14030110

Abstract: Central to quantum theory, the wavefunction is a complex distribution associated with a quantum system. Despite its fundamental role, it is typically introduced as an abstract element of the theory with no explicit definition. Rather, physicists come to a working understanding of it through its use to calculate measurement outcome probabilities through the Born Rule. Tomographic methods can reconstruct the wavefunction from measured probabilities. In contrast, I present a method to directly measure the wavefunction so that its real and imaginary components appear straight on our measurement apparatus. I will also present new work extending this concept to mixed quantum states. This extension directly measures a little-known proposal by Dirac for a classical analog to a quantum operator. Furthermore, it reveals that our direct measurement is a rigorous example of a quasi-probability phase-space (i.e. x,p) distribution that is closely related to the Q, P, and Wigner functions. Our direct measurement method gives the quantum state a plain and general meaning in terms of a specific set of simple operations in the lab.


Recruiting undergrads, graduate students, and post-docs
www.photonicquantum.info for more info
Bob Boyd
Jacob Krich
Anne Broadbent
Ksenia Dolgaleva
Jeff Lundeen
At least one more
?

What is the wavefunction?

The wave function does not describe a single system; it relates rather to many systems, to an ‘ensemble of systems.’



$$|\psi\rangle$$

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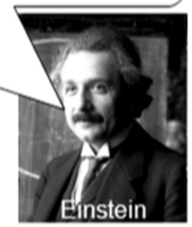
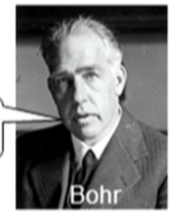


The wave function represents an observer's knowledge of the system.

The state function is purely symbolic.



Shut up and calculate!

Shut up and measure!

No-Cloning Theorem: one cannot copy a particle's wavefunction

Corollary: It is impossible to determine an arbitrary wavefunction of a single particle.

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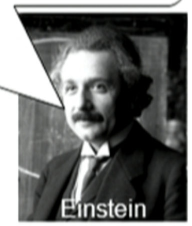

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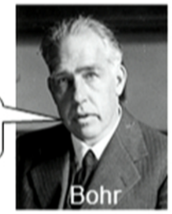

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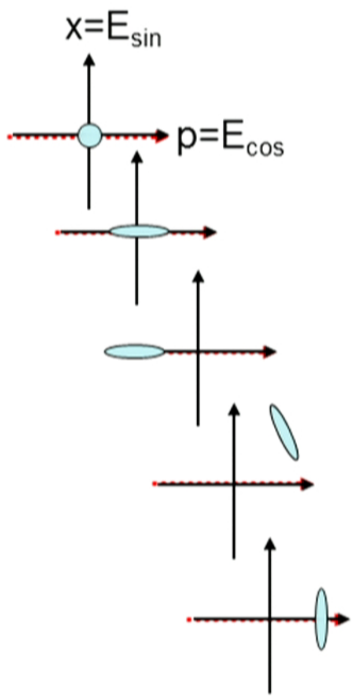
Quantum State Tomography

Wavefunction of an Electric Field



Homodyne
Detection

Gerdenbach, Science (2000)

- Reconstruction is effective and well developed but indirect.

Quantum State Tomography

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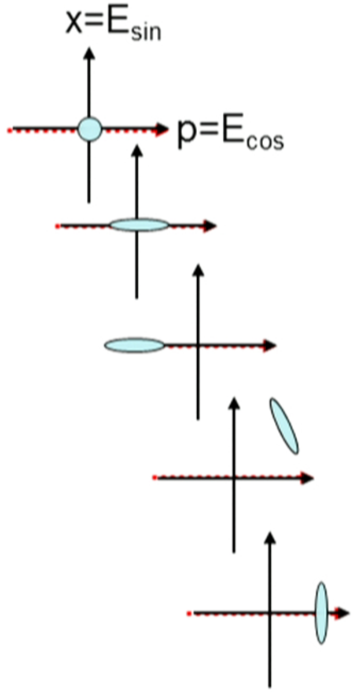
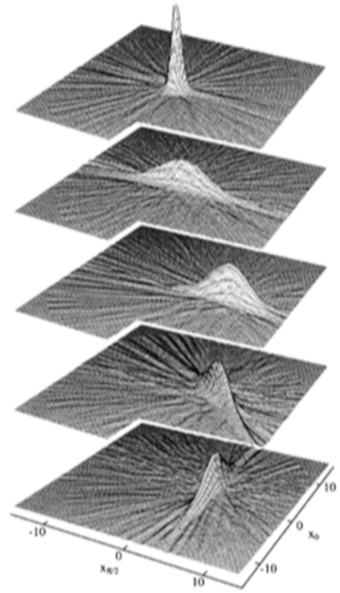


Homodyne Detection

Find the Wavefunction (or Wigner Function) most compatible with those measurements.



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Quantum State Tomography

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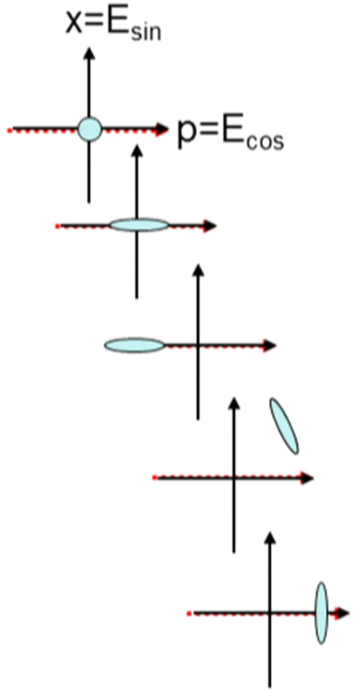

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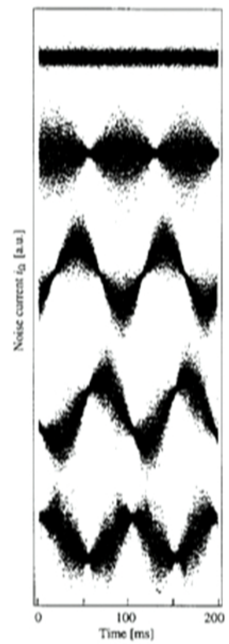

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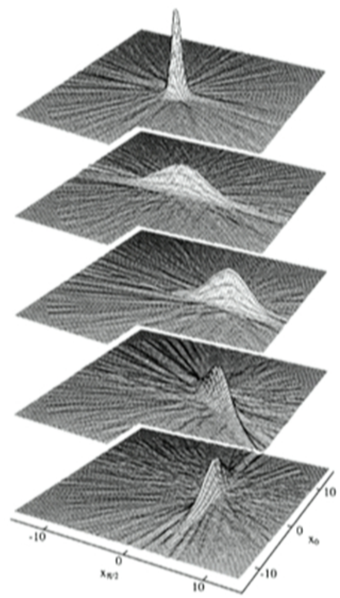
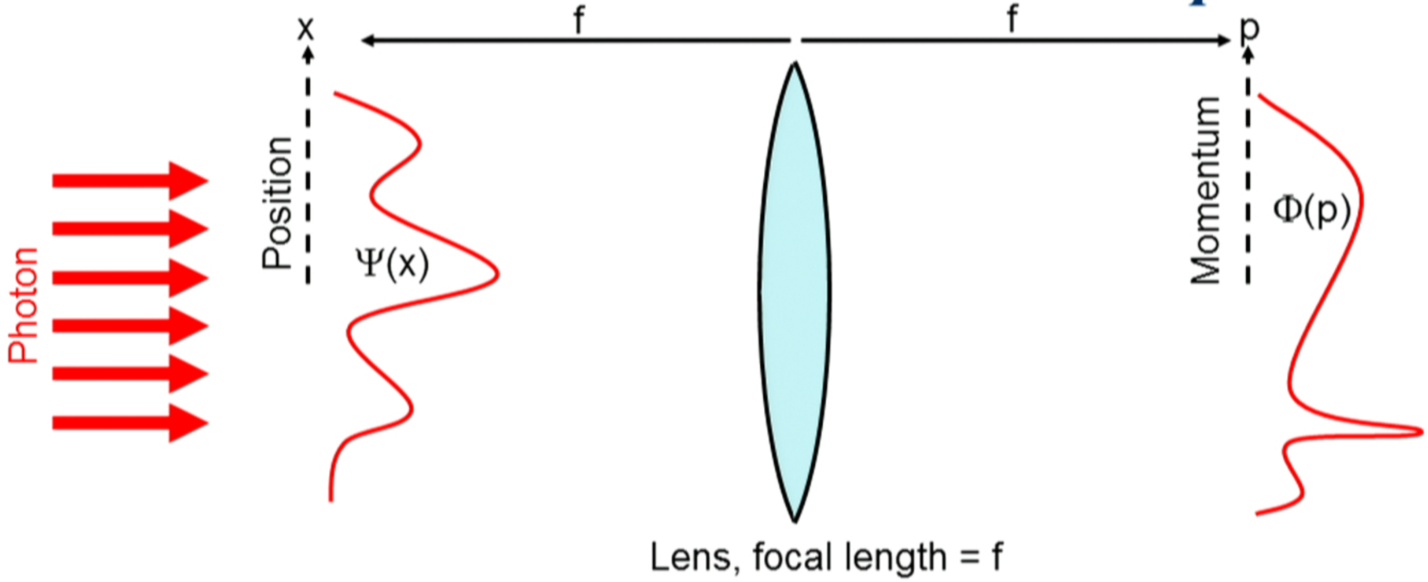


• Reconstruction is effective and well developed but indirect.

Simultaneous Measurement of x and p



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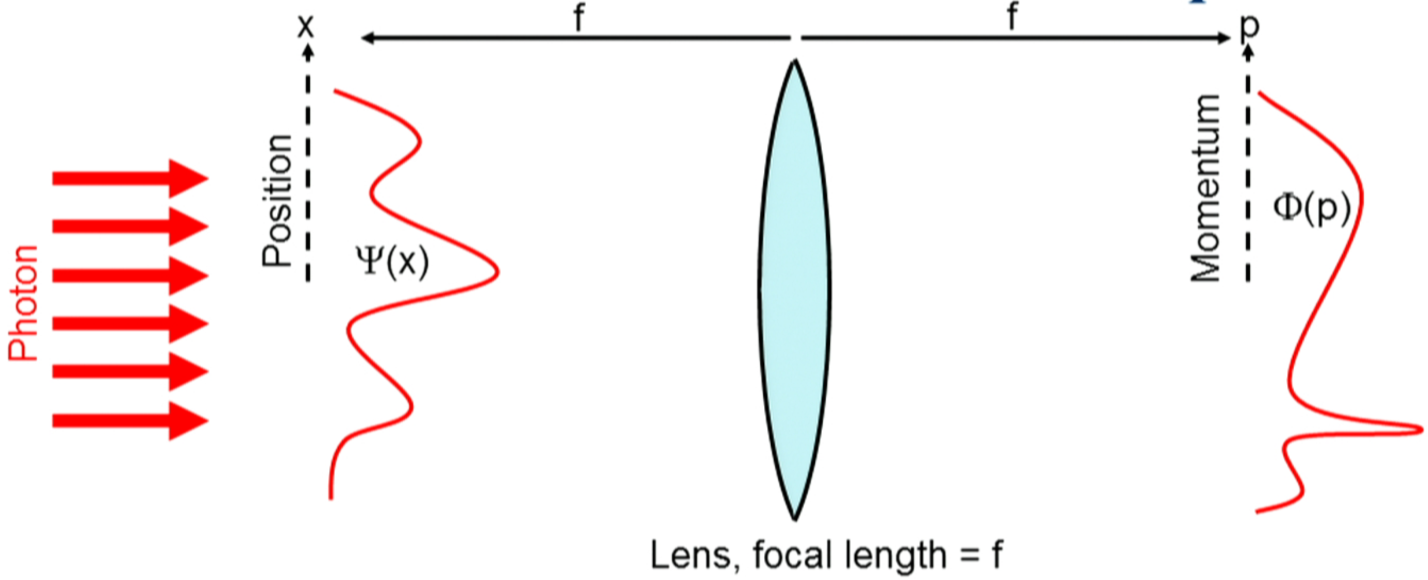
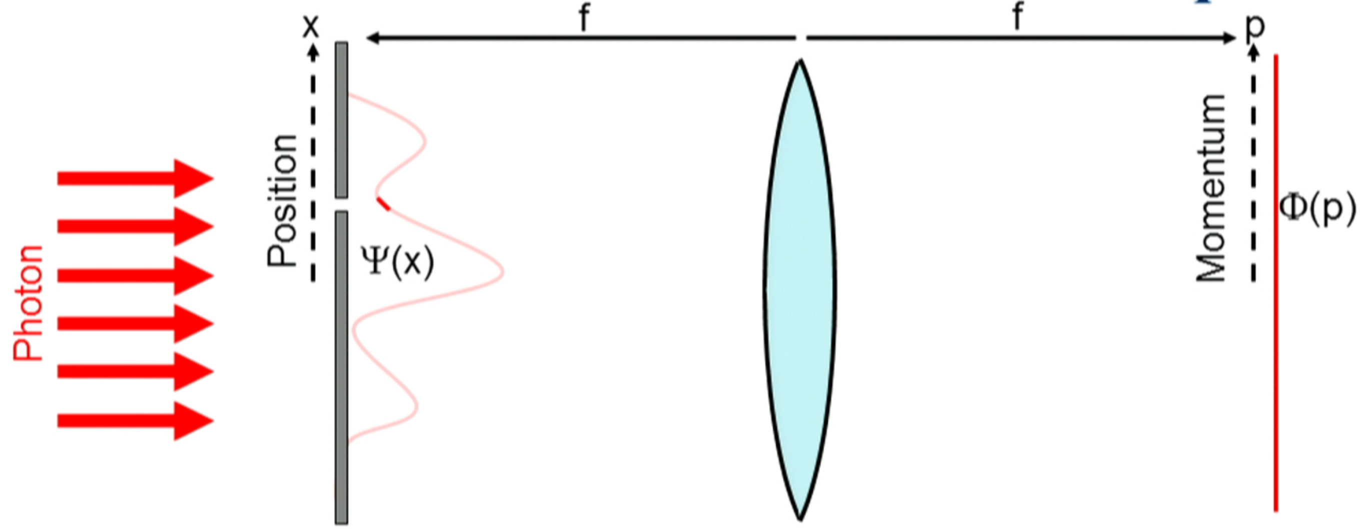


- Can easily measure Prob(x)=$|\Psi(x)|^2$ and then Prob(p)= $|\Phi(p)|^2$
- We don't see the phase, i.e. the θ in $\Psi=|\Psi|e^{i\theta}$

Simultaneous Measurement of x and p



Lens, focal length = f

- Can easily measure Prob(x)=$|\Psi(x)|^2$ and then Prob(p)= $|\Phi(p)|^2$
- We don't see the phase, i.e. the θ in $\Psi=|\Psi|e^{i\theta}$

- Measure x and we cause $\Delta p \rightarrow \infty$
  - "Heisenberg Uncertainty **Relation**"
  - Can not know x and p perfectly at the same time

Why not gently measure x and then strongly measure p?

A Brief History of Weak Measurement

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

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www.sciencemag.org SCIENCE VOL 319 8 FEBRUARY 2008 787

Observation of the Spin Hall Effect of Light via Weak Measurements

Onur Hosten* and Paul Kwiat

Selected for a Viewpoint in *Physics*

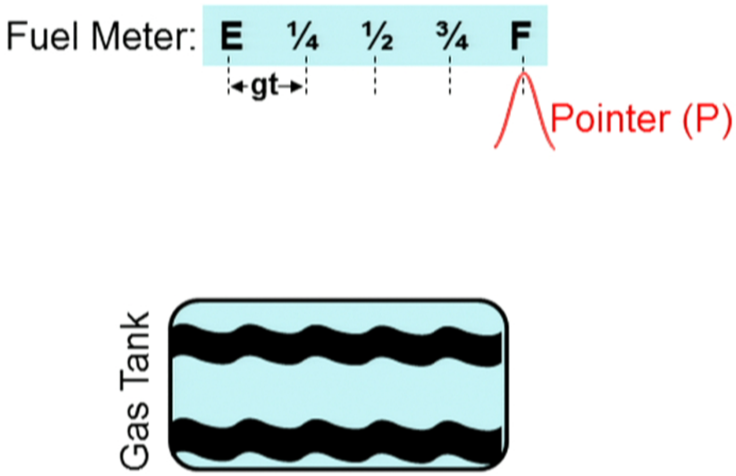
PRL **102,** 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell

Quantum Measurement

Strong Measurement



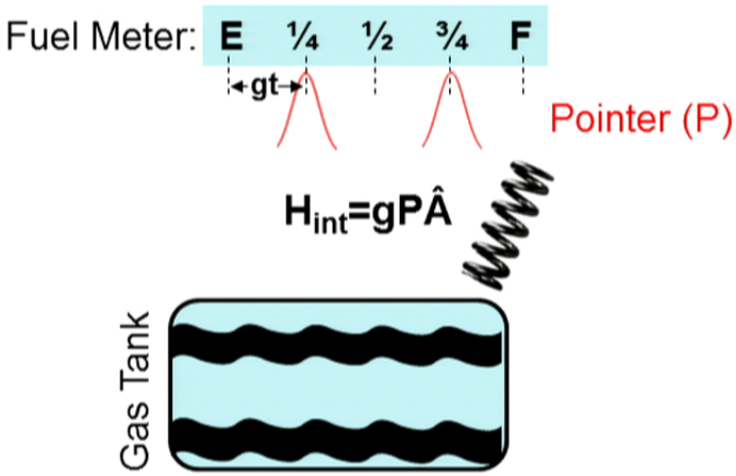
System=$\sum c_i|a_i\rangle$

Model both the measured system and the measurement apparatus as quantum systems.

e.g. The pointer needle on a fuel gauge has a wavefunction and so does the gas tank.

Quantum Measurement

Strong Measurement



System+Pointer=$\sum c_i|a_i\rangle|P_i\rangle$

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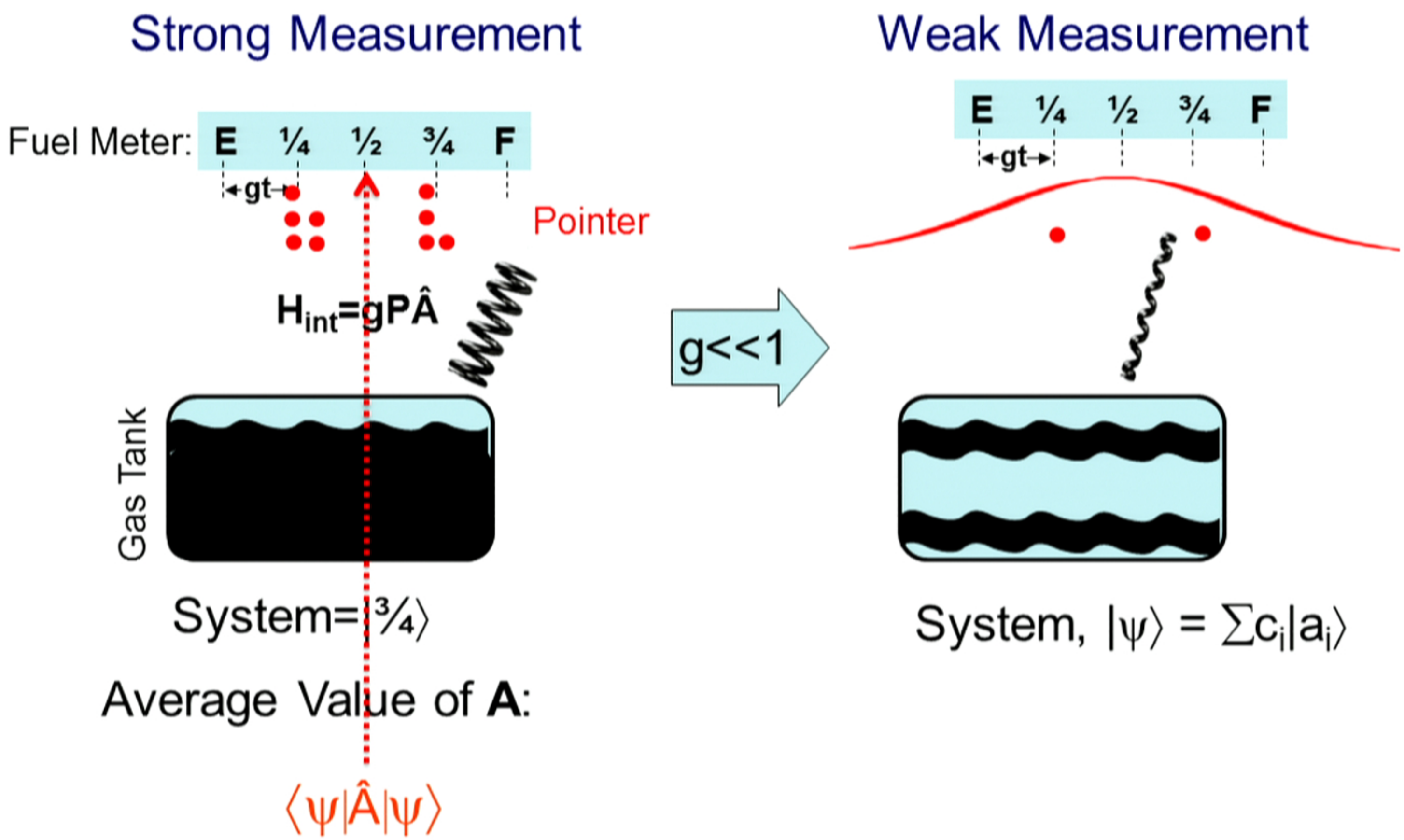
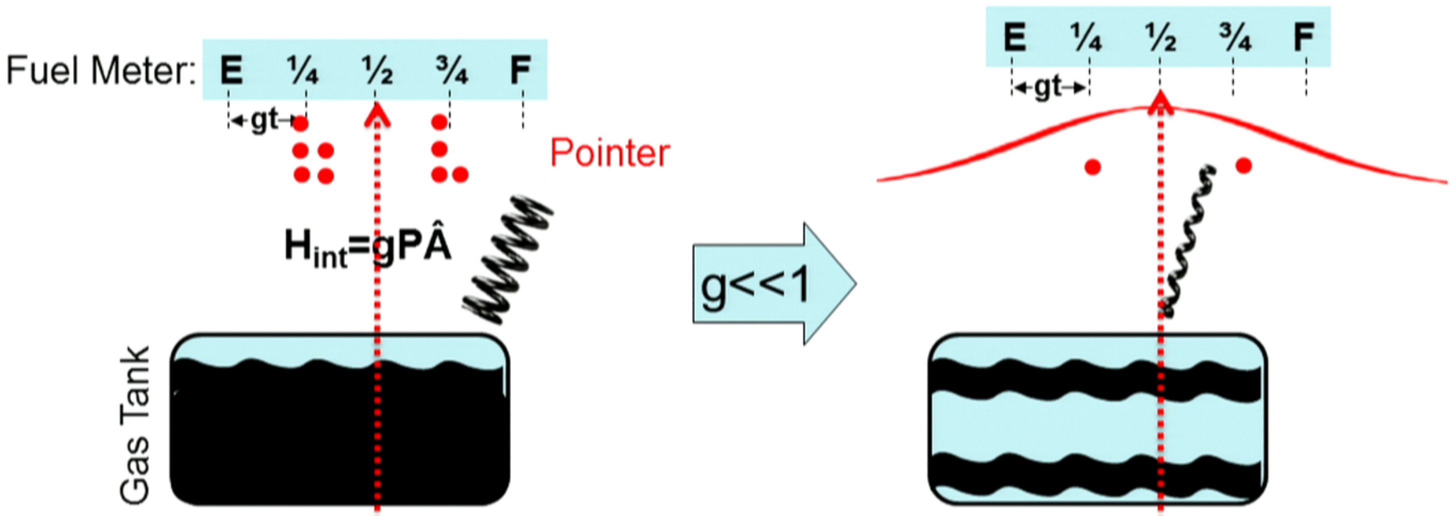


Quantum Measurement

Strong Measurement

Weak Measurement



System=$|¾\rangle$

Average Value of **A**:

$\langle\psi|\hat{A}|\psi\rangle$

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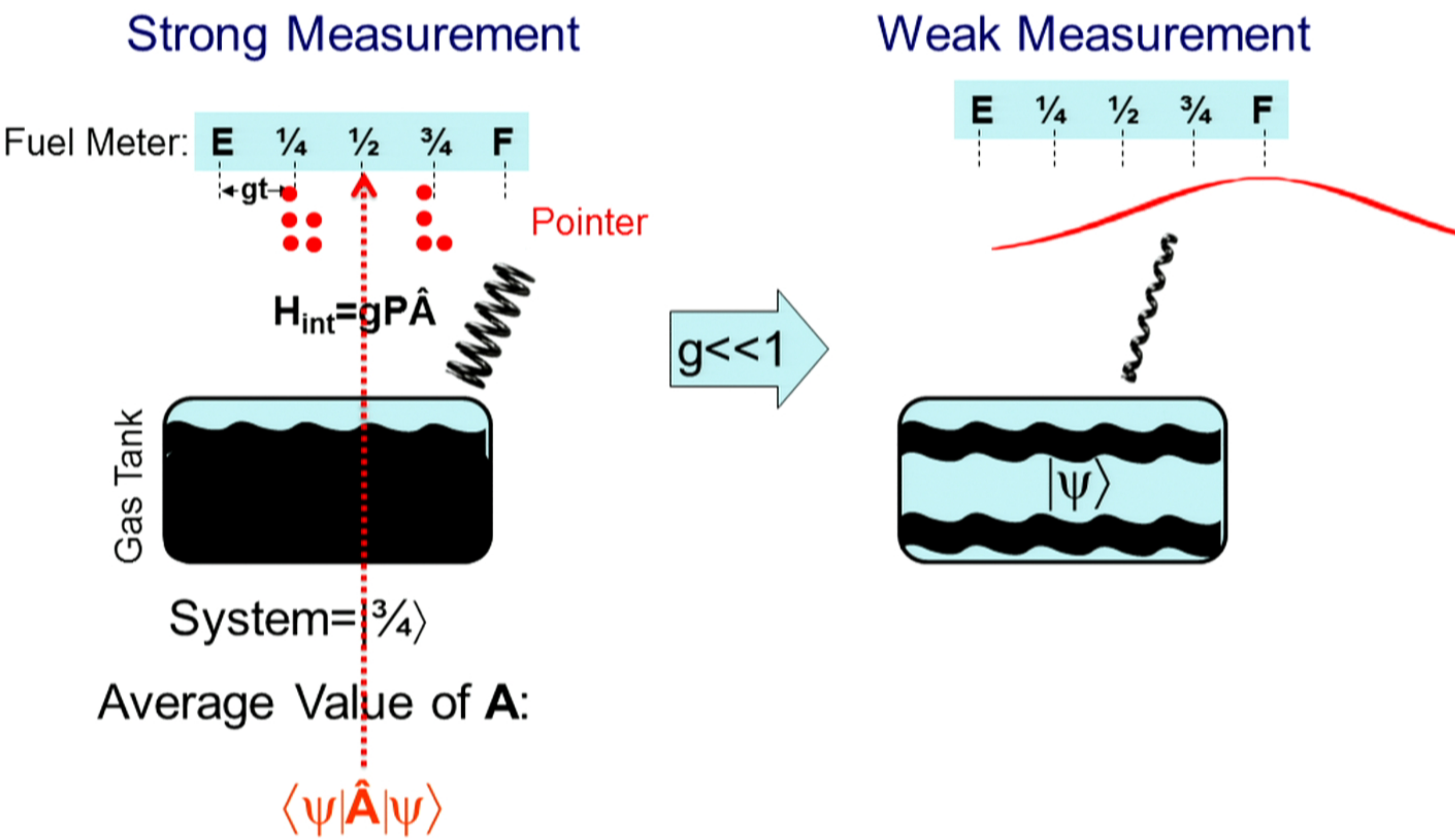
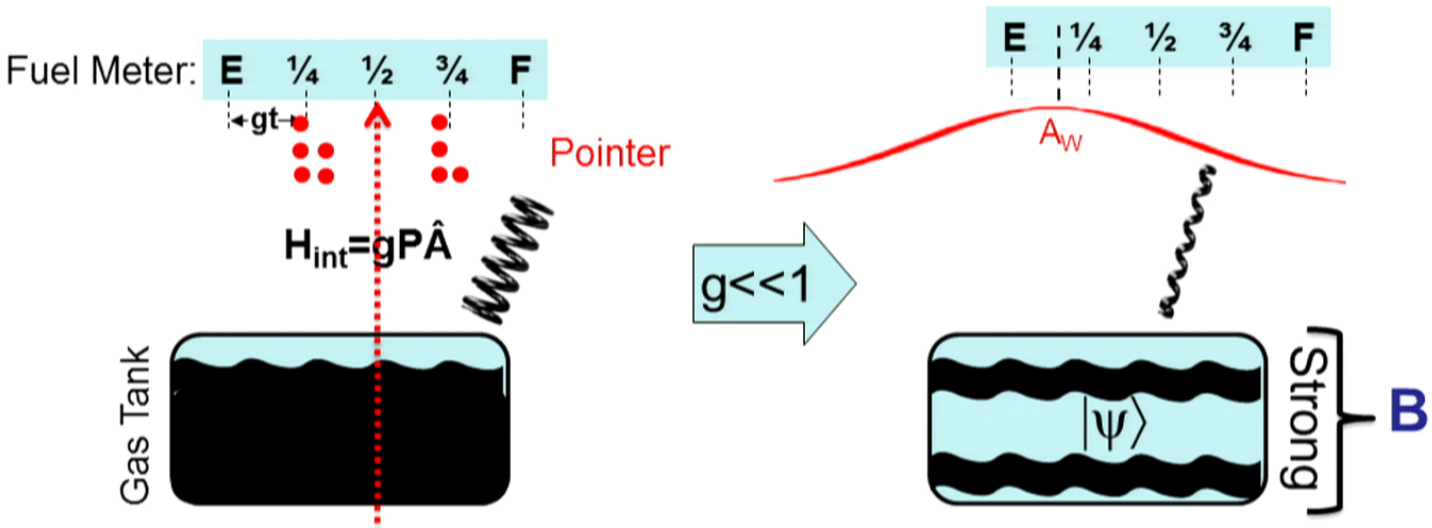
Quantum Measurement



Quantum Measurement

Strong Measurement | Weak Measurement



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In the cases where result of **B** is b

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$$A_W=\frac{\langle b|A|\psi\rangle}{\langle b|\psi\rangle}$$

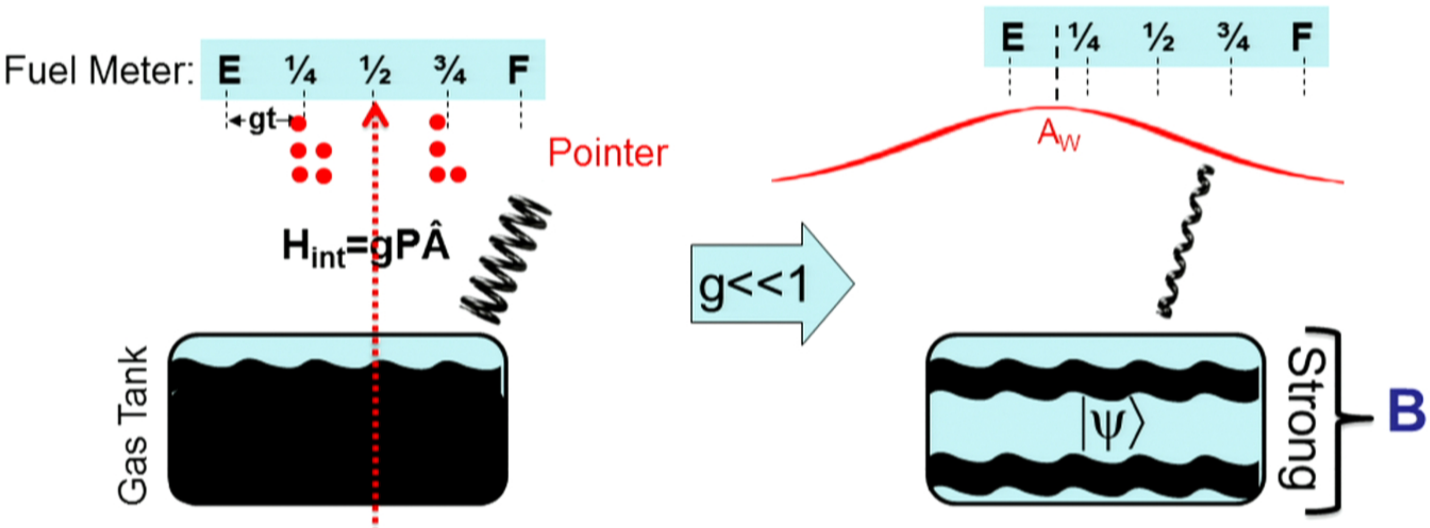
Real part of A_W is the position shift of the pointer

Imaginary part of A_w is the momentum shift of the pointer

Quantum Measurement

Strong Measurement

Weak Measurement



System=$|\frac{3}{4}\rangle$

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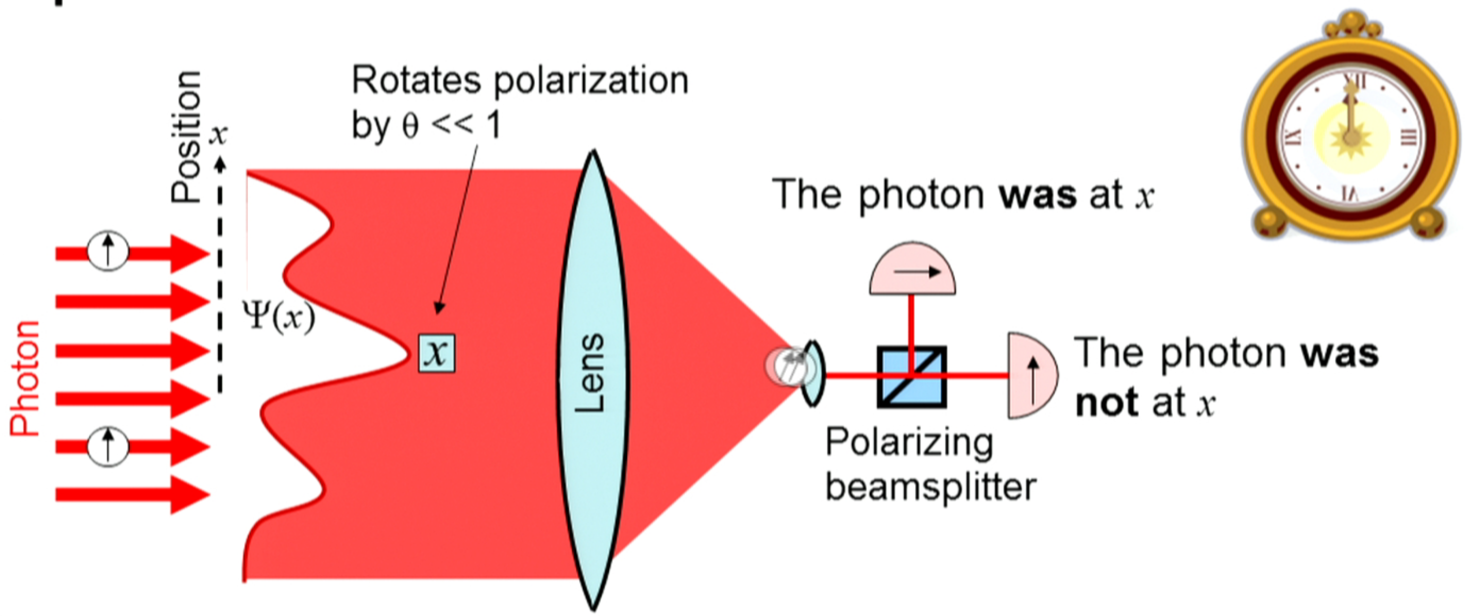
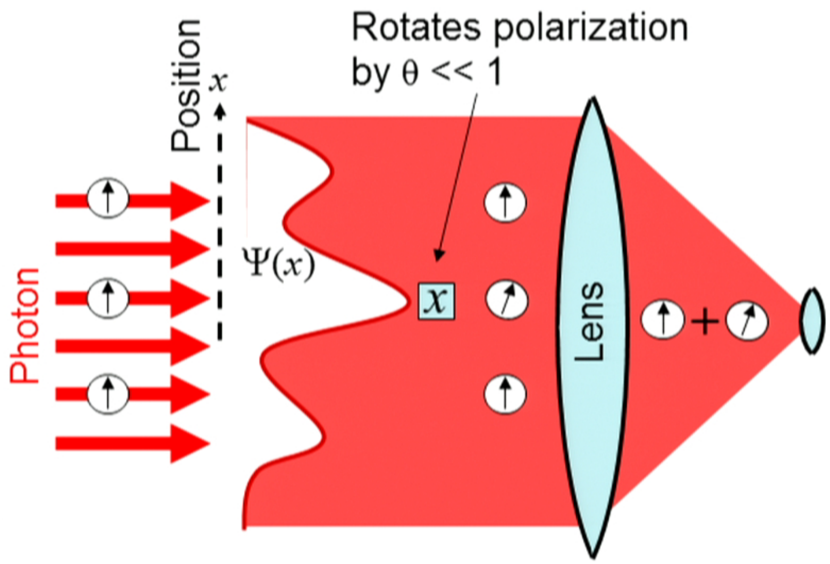
Weak Measurement Example

- **For a weak measurement we reduce the rotation of the polarization**



Weak Measurement Example

• The average result of the weak measurement is the final rotation of our pointer: the linear polarization.



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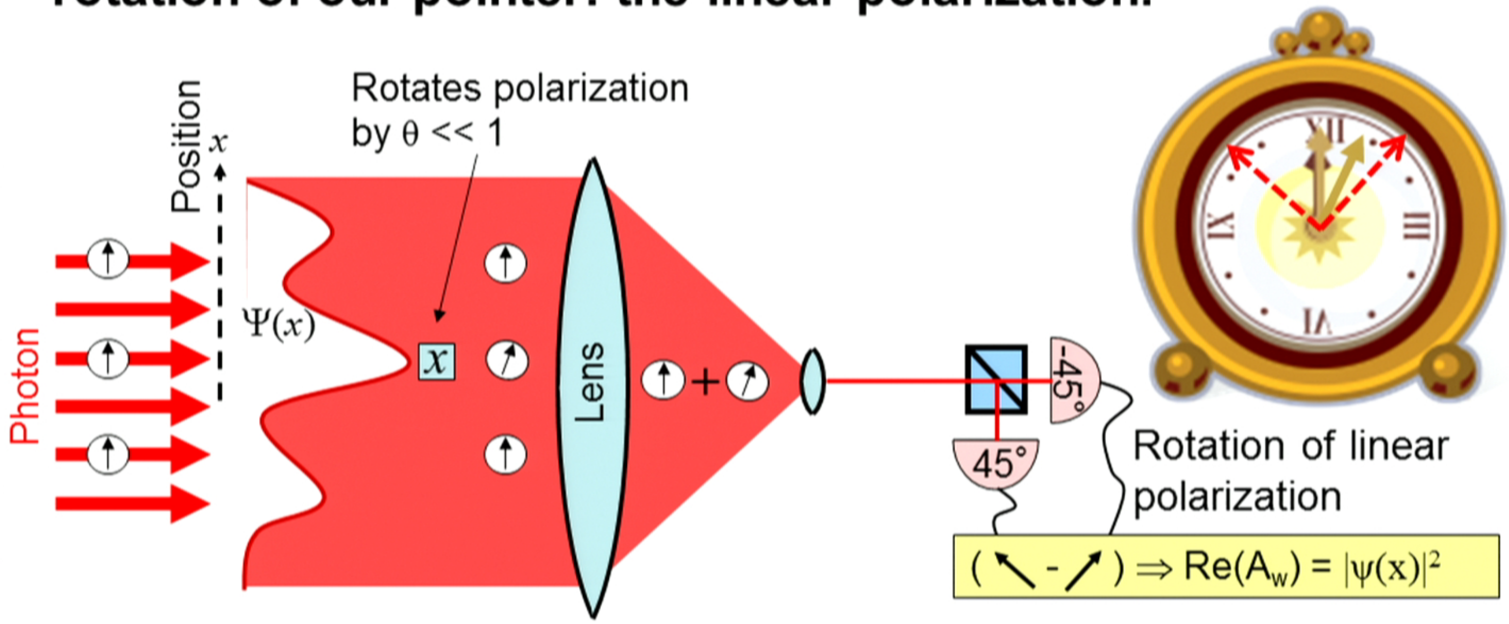

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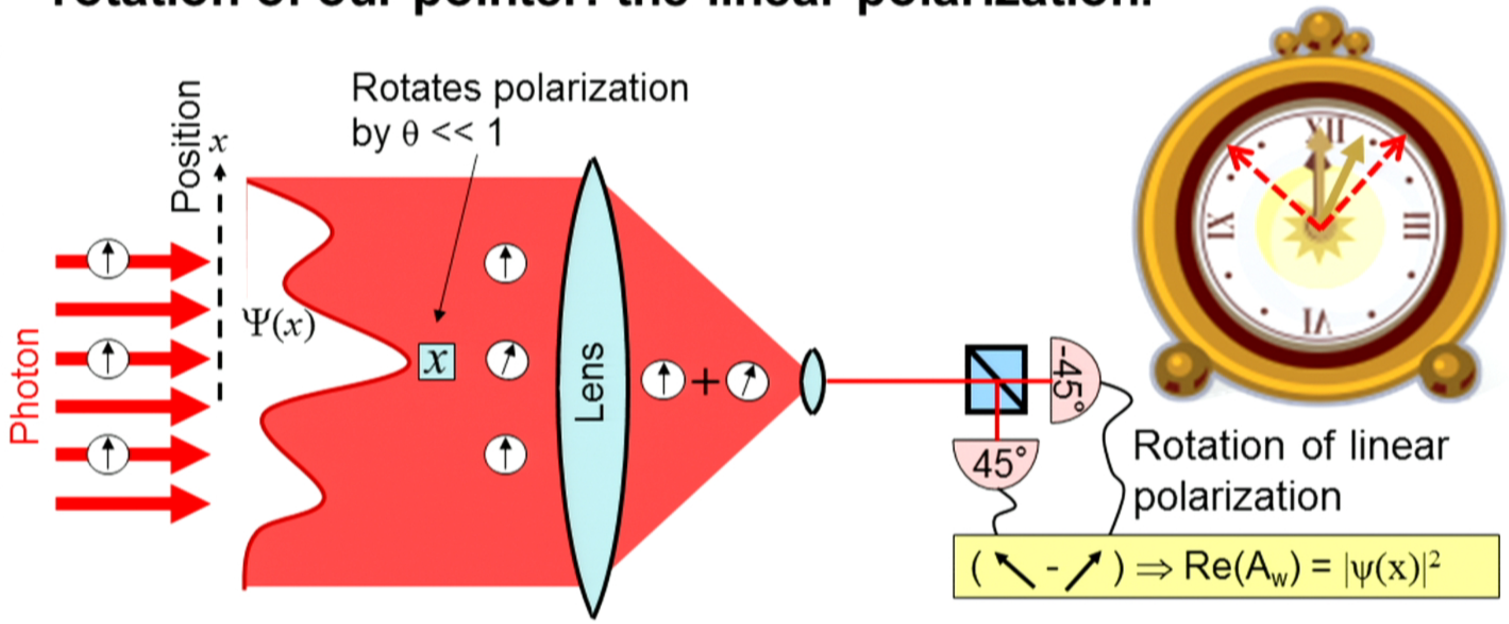
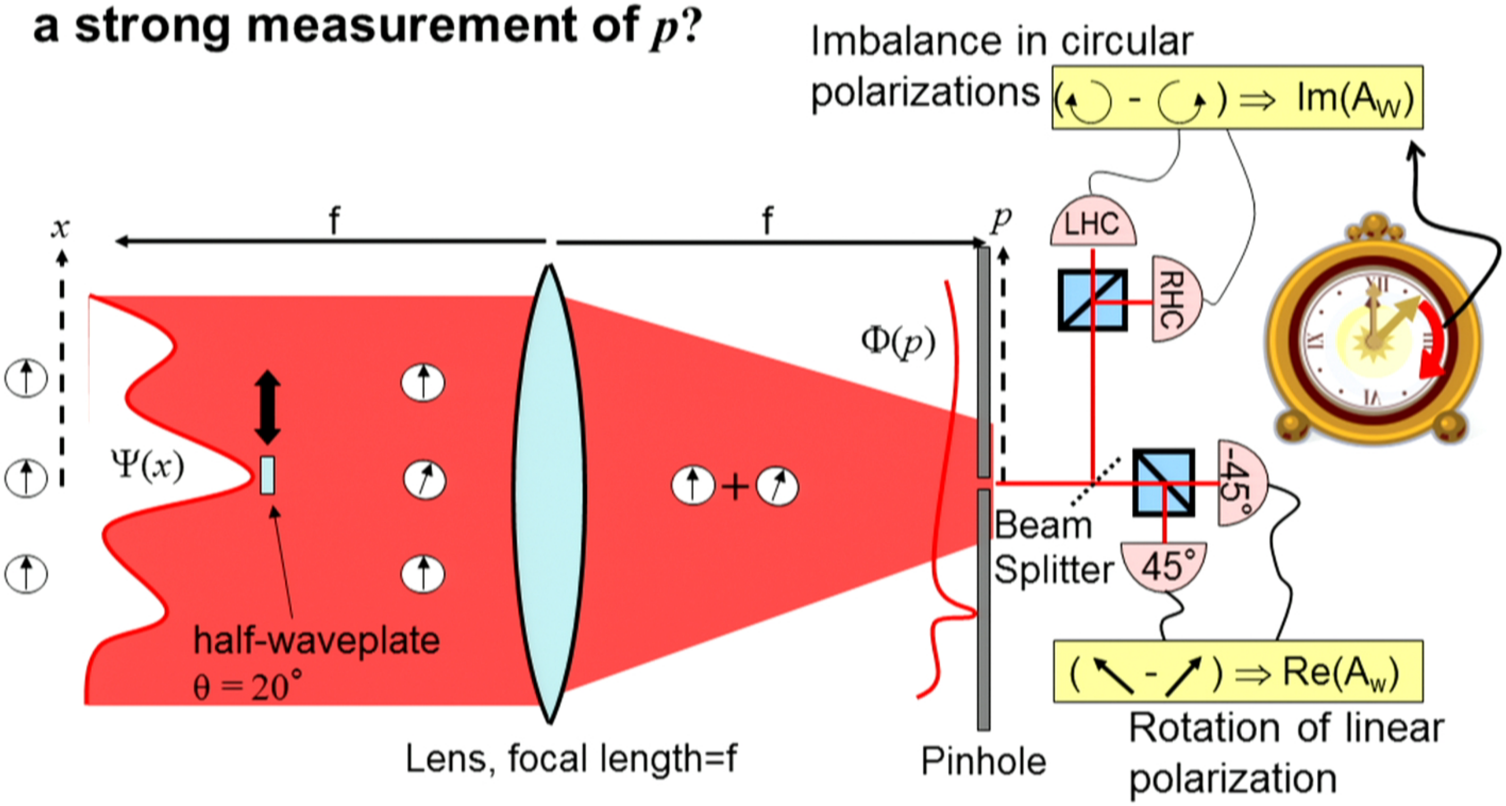


Weak then Strong Measurement

- What if we do a weak measurement of x, and then make a strong measurement of p?



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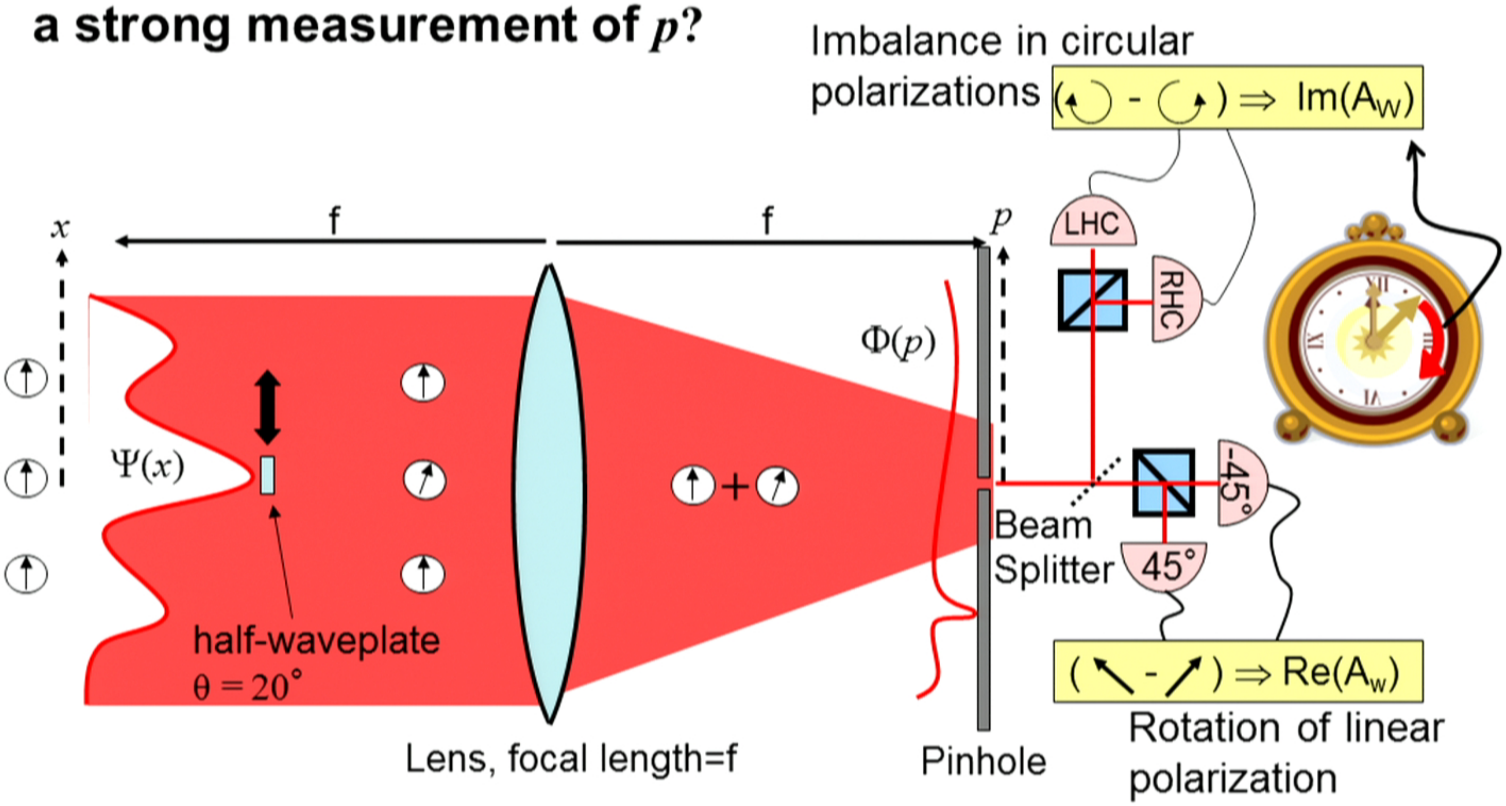


- **Real and Imaginary parts of the weak measurement average appear in the linear and circular polarization rotations.**

The idea

• What if we do a weak measurement of $\boldsymbol{X}$, and then make a strong measurement of $\boldsymbol{P}$?

i.e. $\mathbf{A} = |x\rangle\langle x| = \pi$, Initial state= $|\psi\rangle$, Strong measurement result $P=p$

Average shift of the pointer: $A_W = \frac{\langle b|A|\psi\rangle}{\langle b|\psi\rangle}$

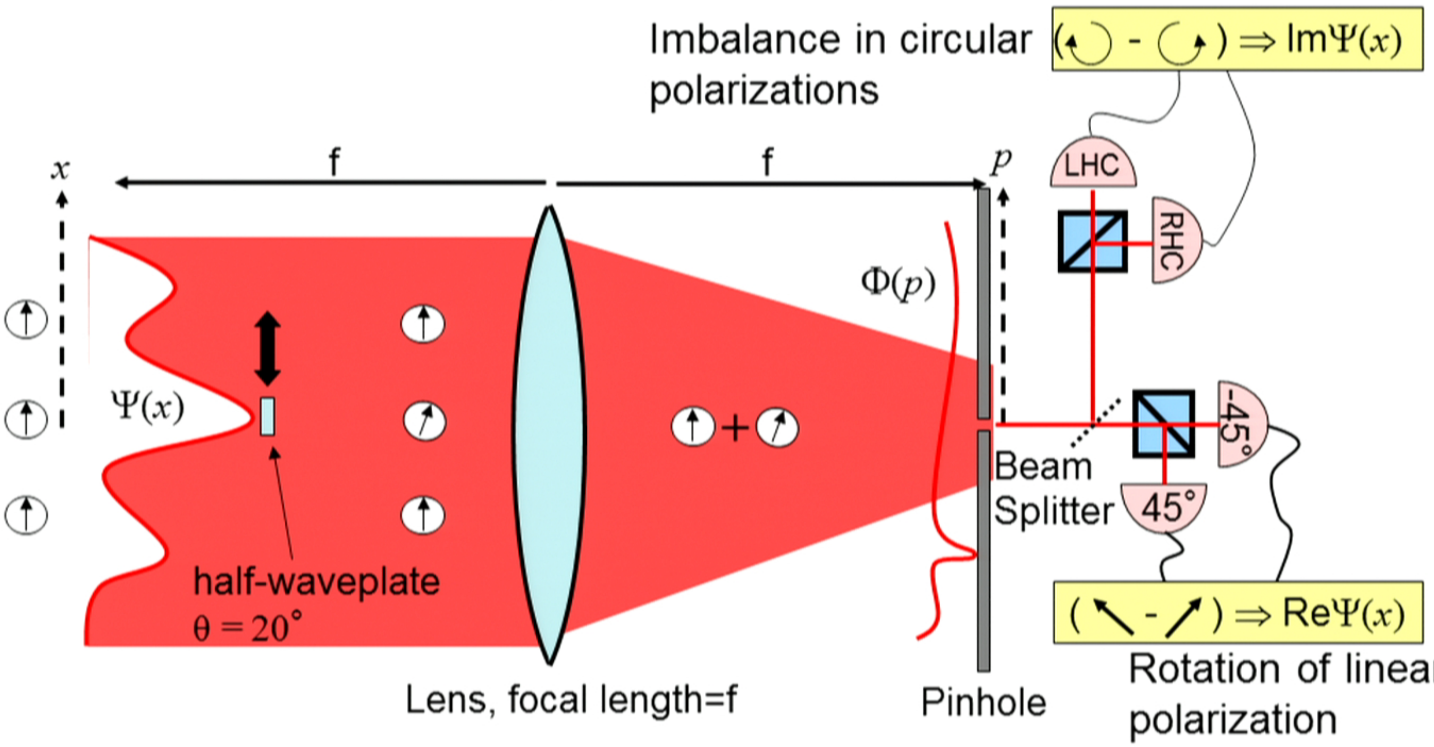
$$\pi_w = \frac{\langle p|x\rangle\langle x|\psi\rangle}{\langle p|\psi\rangle}$$

And if $p=0$, $\pi_w = \frac{1/\sqrt{2\pi}\cdot\langle x|\psi\rangle}{\sqrt{\mathrm{Prob}(p=0)}} = \boxed{k\cdot\psi(x)}$

• **The average shift of the pointer (i.e. rotation of the polarization) is proportional to the wavefunction**

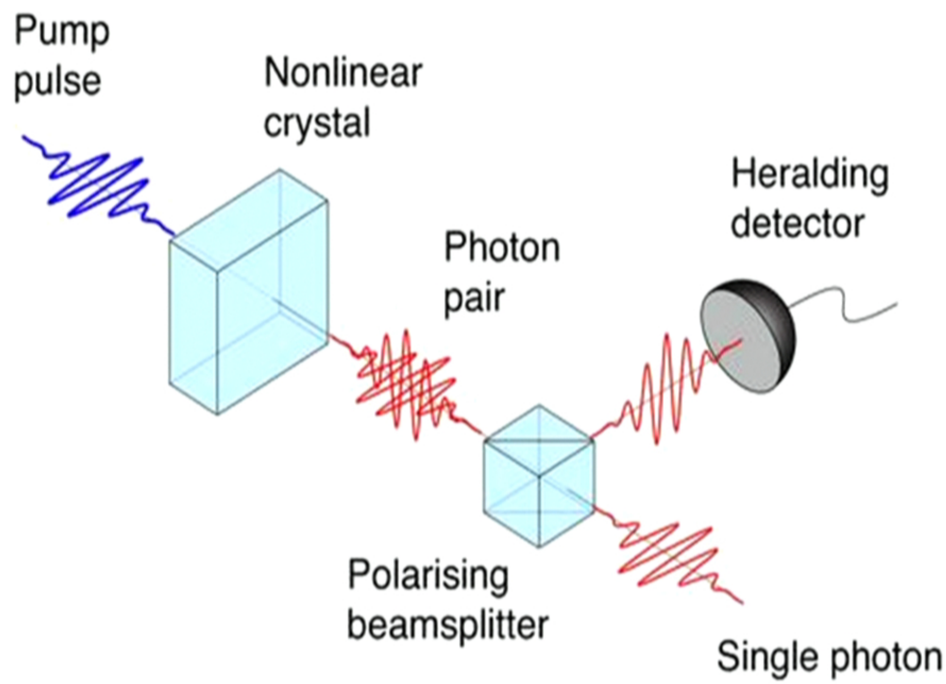
Direct Measurement of the Wavefunction

•Weakly measure $|x\rangle\langle x|$ then strongly measure p, and keep only the photons found with p=0 .



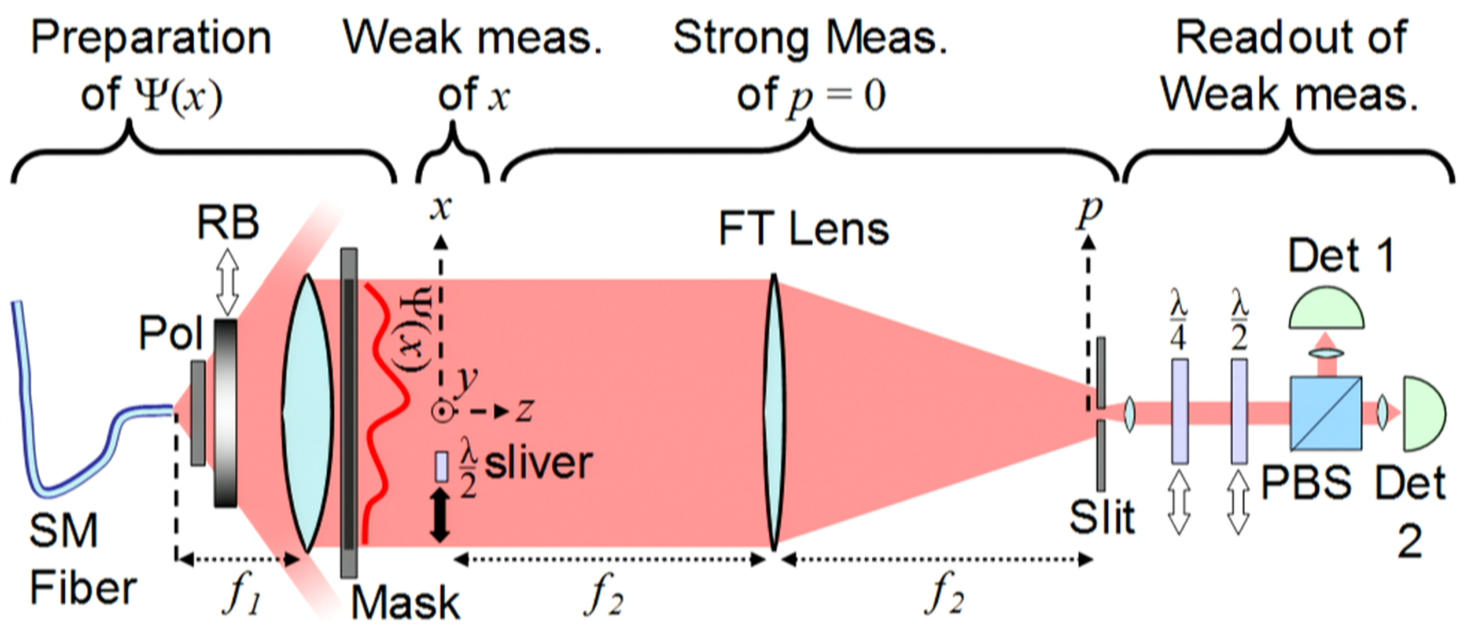
Our Source of Single Photons

- A pump photon is spontaneously converted into two lower frequency photons in a nonlinear optical material



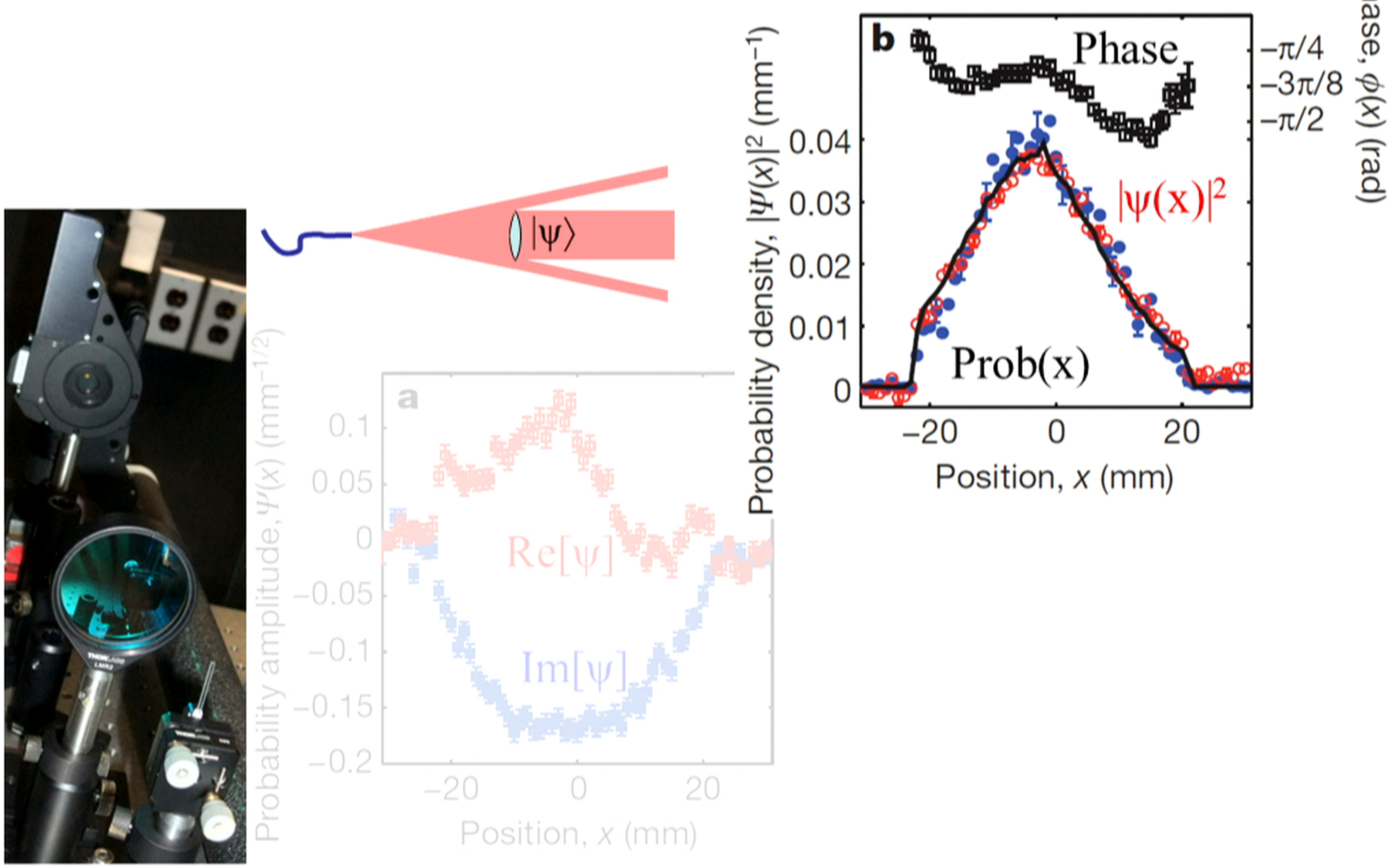
- Photons are produced rarely but always in pairs
  → Detection of one photon 'Heralds' the presence of its twin

Experimental Setup



- A mask (horizontal aperture) makes the experiment one dimensional along x.

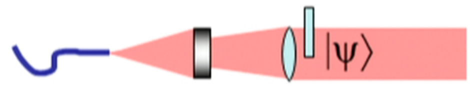
Direct Measurement of the Wavefunction

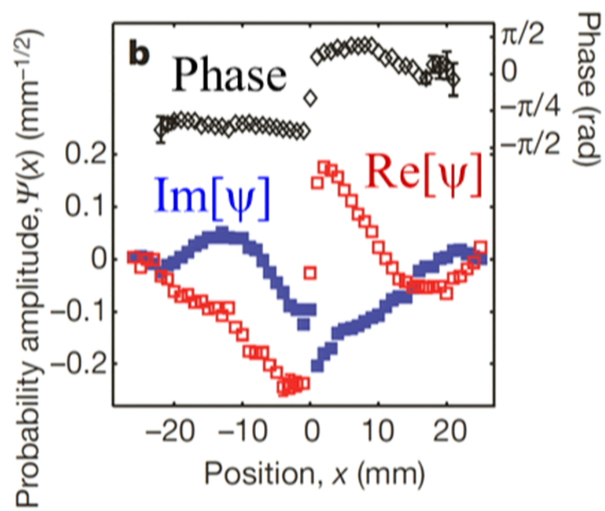


Testing another wavefunction shape

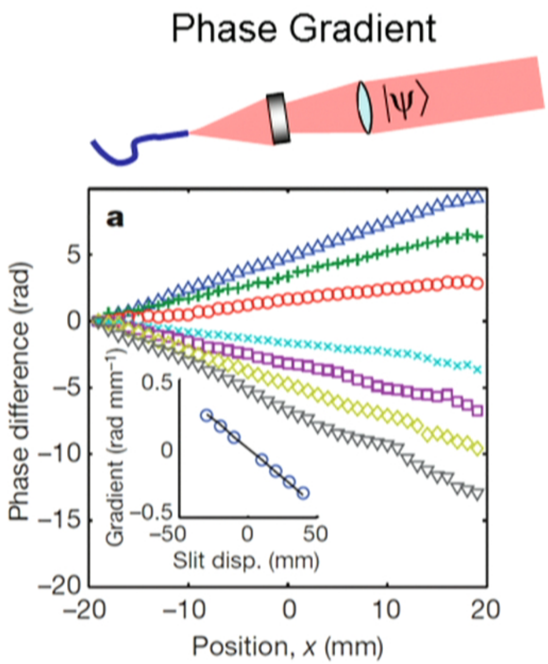
• Created new transverse wavefunction with a reverse bullseye filter

• Phase Discontinuity: Placed a glass square across half the wavefunction





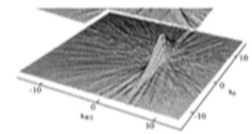
Testing other wavefunctions phase profiles



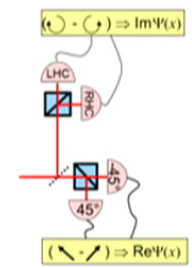
Why it is Direct

1. It is local - measures $\psi(x)$ at x

2. No complicated mathematical reconstruction



3. The value of $\psi(x)$ appears right on our measurement apparatus



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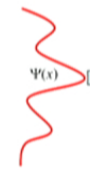

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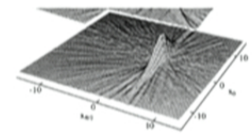

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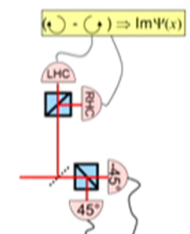
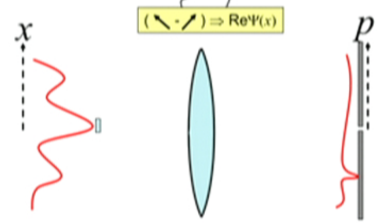
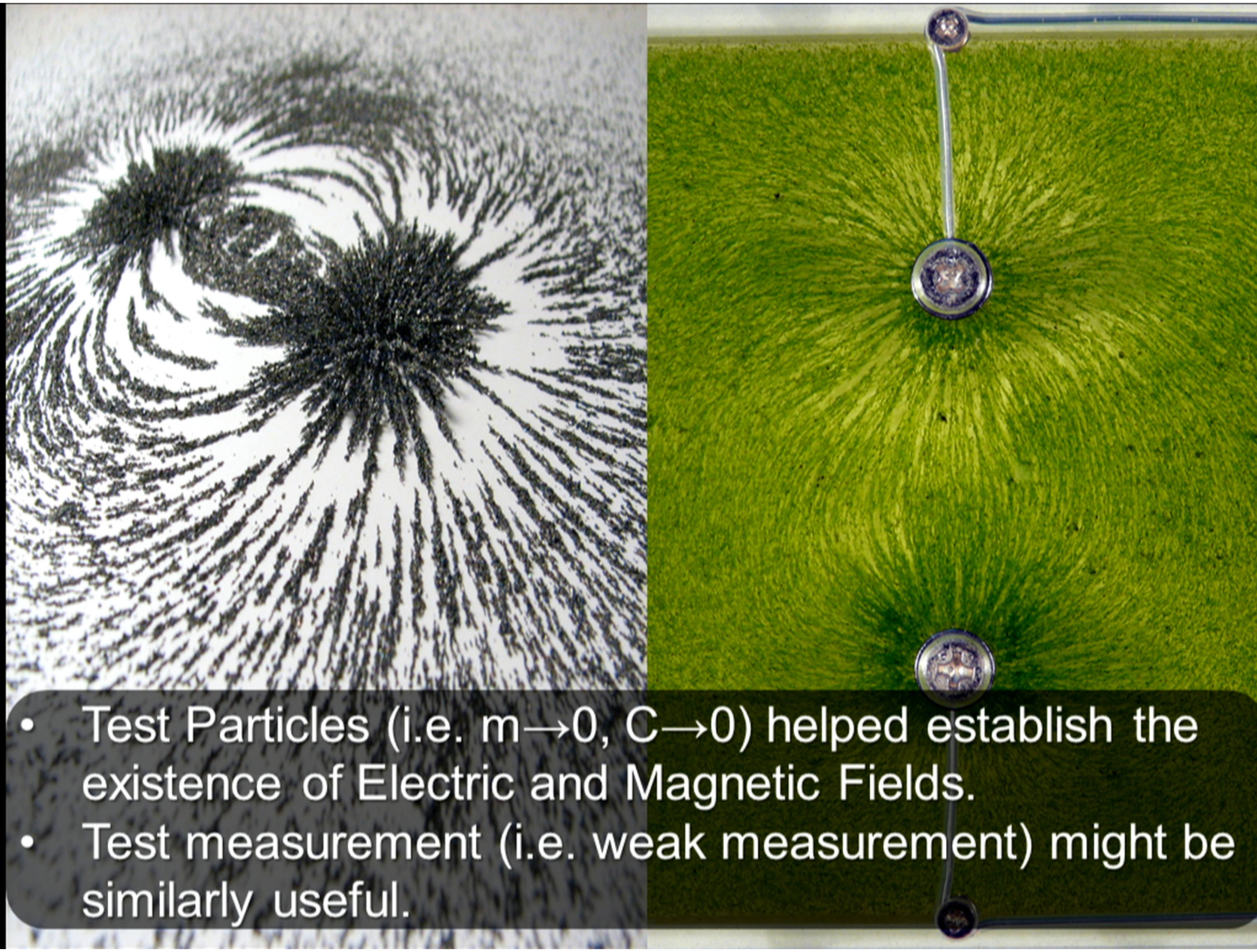


4. The procedure is simple and
- measure x and then p





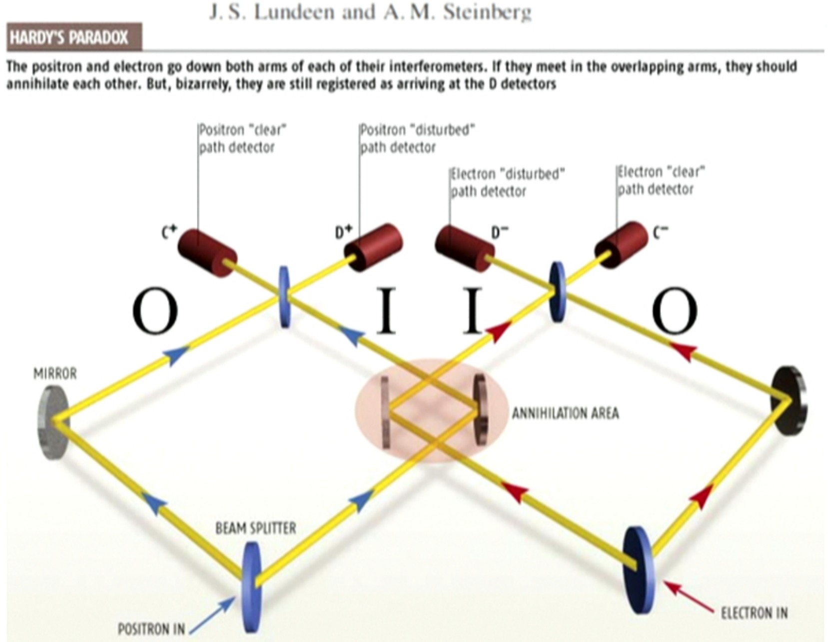
- Test Particles (i.e. m→0, C→0) helped establish the existence of Electric and Magnetic Fields.
- Test measurement (i.e. weak measurement) might be similarly useful.

Direct Measurement of an Entangled Quantum State

PRL 102, 020404 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

Experimental Joint Weak Measurement on a Photon Pair as a Probe of Hardy's Paradox

J. S. Lundeen and A. M. Steinberg

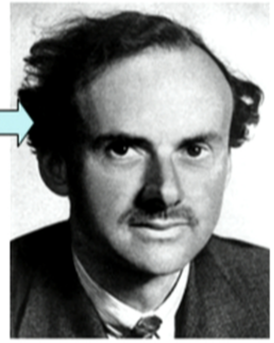


Theoretical Quantum State:

$$|\psi\rangle = \mathbf{1}\ |IO\rangle + \mathbf{1}\ |OI\rangle\ \mathbf{-1}\ |OO\rangle + \mathbf{0}\ |II\rangle$$

Dirac's Distribution

José Moyal re-invented the Wigner function



Paul Dirac thought it was a poor idea.

REVIEWS OF MODERN PHYSICS VOLUME 17, NUMBERS 2 AND 3 APRIL–JULY, 1945

On the Analogy Between Classical and Quantum Mechanics

P. A. M. DIRAC
St. John's College, Cambridge, England

$$D_\rho(x,p) = \langle p||x\rangle\langle x|\,\boldsymbol{\rho}\,|p\rangle$$

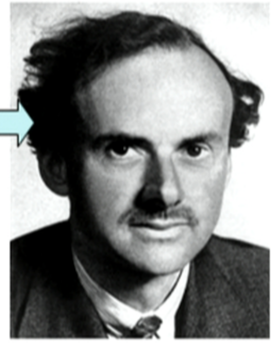
(But first discussed by McCoy in 1932)

- In physics, the Dirac Distribution was forgotten as a theoretical novelty (There was no way to measure it!)

The distribution is complex!

Dirac's Distribution

José Moyal re-invented the Wigner function



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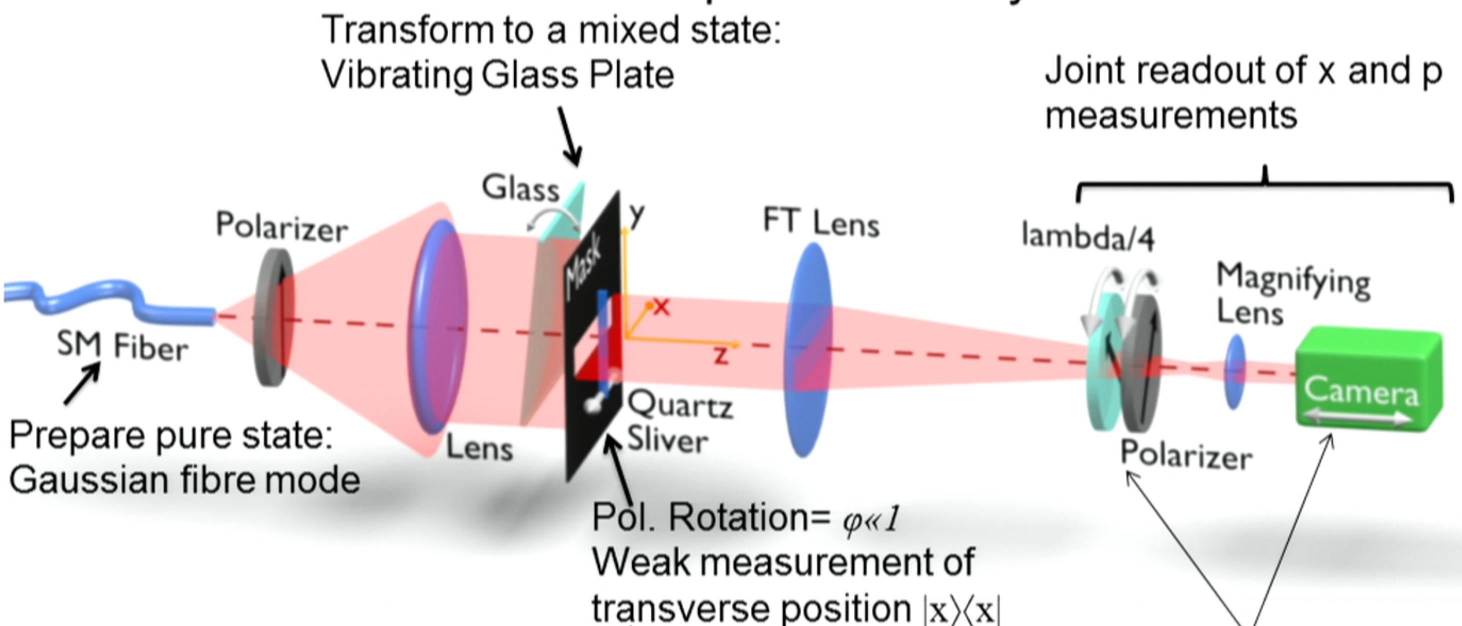
Measurement of the Dirac Distribution

- We measured the transverse state of a photon
- Make a weak-strong joint measurement of X and P
- For each x measure all p with an array.



$$D_\rho(x,p)\sin\varphi = \frac{N_{p,45^\circ} - N_{p,-45^\circ}}{N_T} - i\,\frac{N_{p,\circlearrowleft} - N_{p,\circlearrowright}}{N_T}$$

- Not a weak value (not post-selected) but still complex

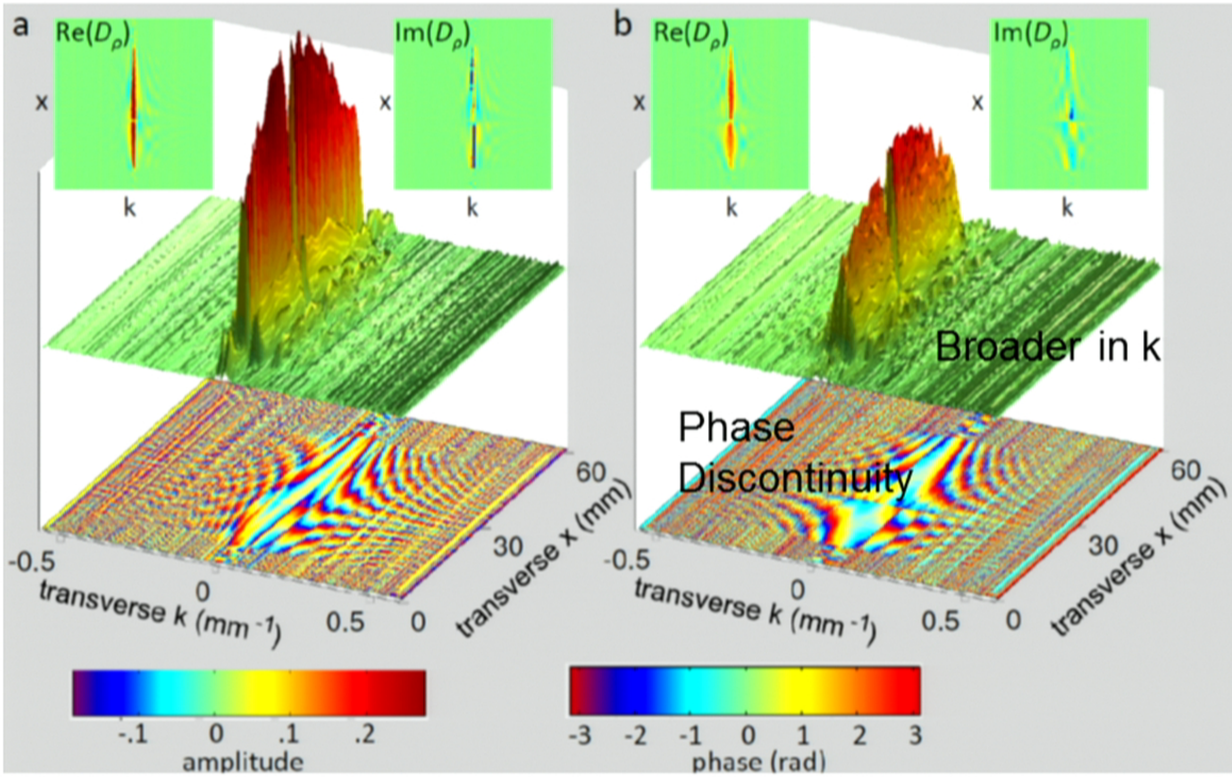
Experimental Dirac Distributions, D_ρ

Pure State

$$D_\rho = \Psi(x)\Phi^*(p)\exp(ipx/\hbar)$$

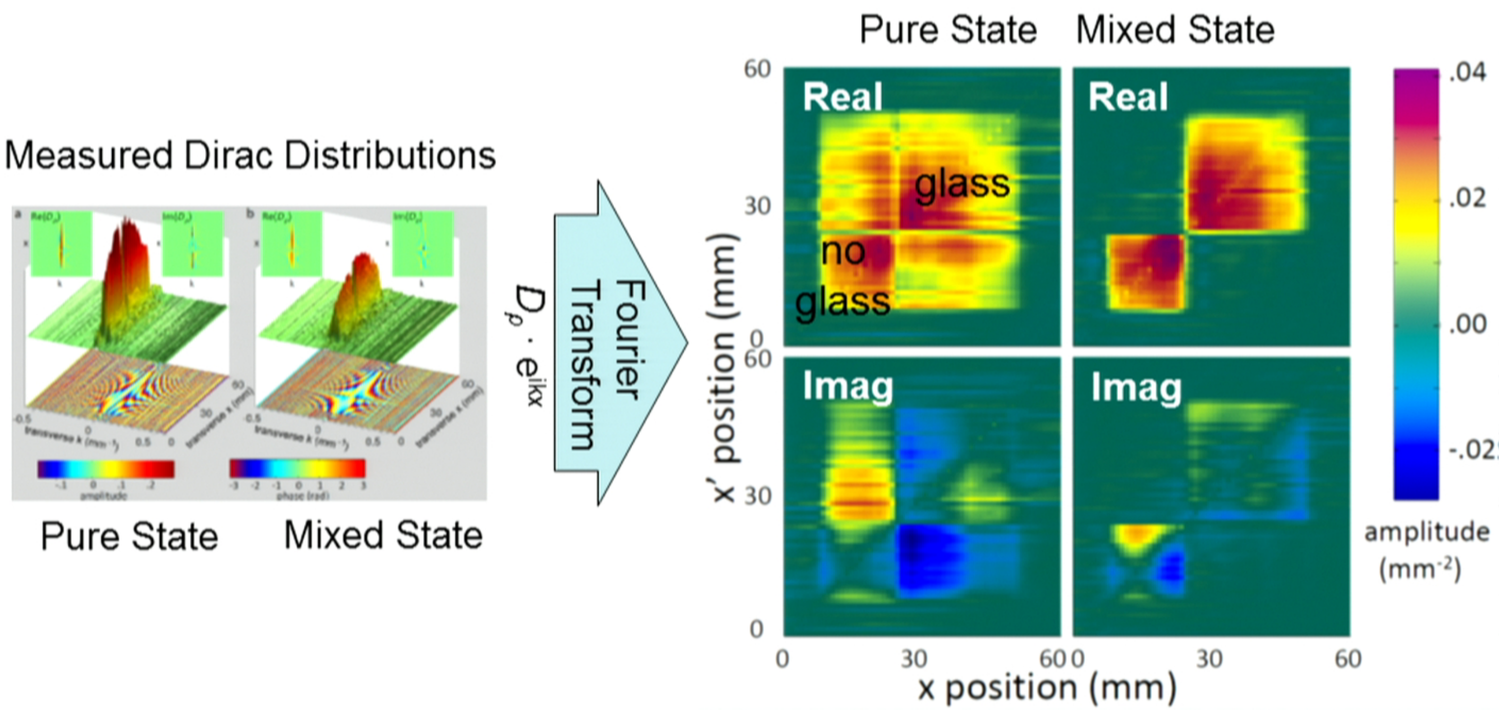
Mixed State

$$D_\rho = [\sum \Psi_j(x)\Phi^*_j(p)]\cdot\exp(ipx/\hbar)$$



- The Dirac distribution can represent both pure and mixed states

Relationship to the Density Matrix



- The density matrices are approx. Hermitian (not guaranteed)
- The off-diagonals between glass and no glass are zero
  - The state exhibits no coherence between the two regions

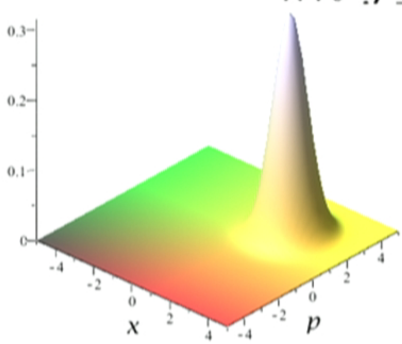
Quasi-Probability Distributions

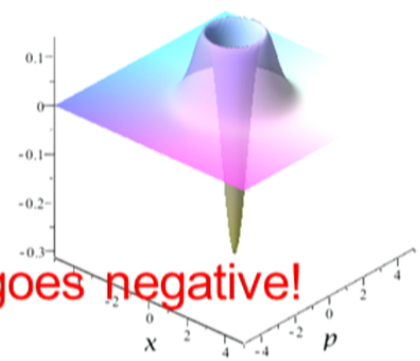
- In classical physics we have the *Liouville* Distribution, Prob(x,p), a phase space (i.e. position-momentum) distribution for an ensemble of particles.
- Any quantum analog will not satisfy some of the standard laws of probability (e.g. Prob>0)

  → ***Quasi-Probability*** Distribution

- 1932, Eugene Wigner: **Wigner Function**

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \langle x+y|\hat{\rho}|x-y\rangle e^{-2ipy/\hbar}\,dy$$



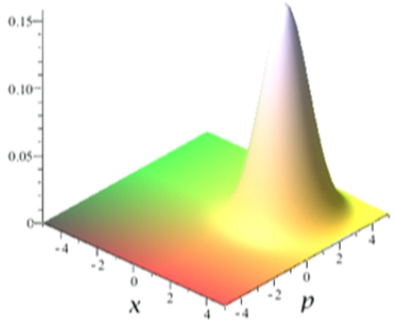


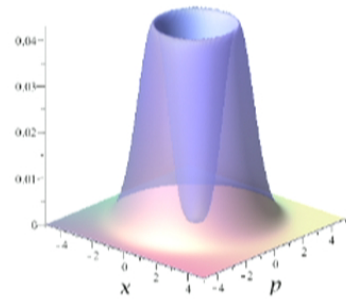
It goes negative!

Other Quasi-Probability Distributions

- 1940, Kodi Husimi: **Q function**

$$Q(\alpha = x + ip) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle$$

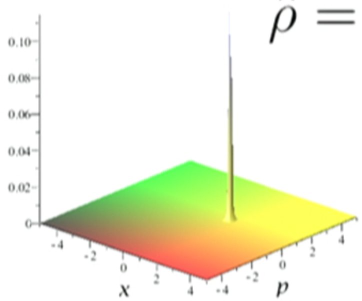


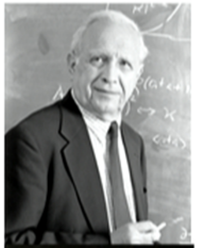


Marginals are not correct, e.g. $\int Q(x,p)\mathrm{d}p \neq \mathrm{Prob}(x)$

- 1963: R. Glauber, G. Sudarshan: **P function**

$$\hat{\rho} = \int P(\alpha)|\alpha\rangle\langle\alpha|\, d^2\alpha,$$





$P(x,p)$ is highly singular for most non-classical states

An issue of how to quantize phase-space

- The Q-function, Wigner function, and P-function reflect different operator orderings

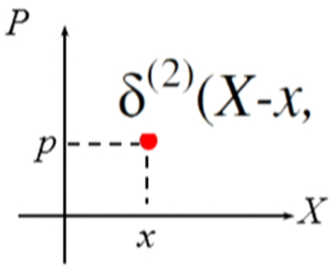
- Using $\mathbf{X} = (\mathbf{a} + \mathbf{a}^\dagger)/\sqrt{2}$, $\mathbf{P} = i(\mathbf{a} - \mathbf{a}^\dagger))/\sqrt{2}$
  $\rightarrow \alpha = x + ip$

1. Expand the density matrix in a particular ordering O
2. Put $\mathbf{a} \rightarrow \alpha$ and $\mathbf{a}^\dagger \rightarrow \alpha^*$
3. The result is the O ordered quasi-prob. Distribution, $\mathrm{Pq}_O(\mathrm{x,p})$

Direct Measurements of Quasi Probability distributions

- Classical measurement of a phase-space point is a Dirac delta
- How does on translate this to a quantum measurement?

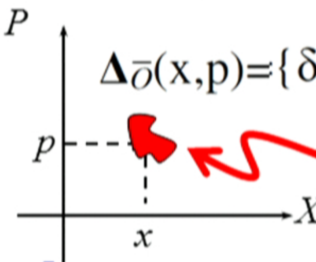
Classical



$\delta^{(2)}(X\text{-}x,\ P\text{-}p)$

Operator anti-ordering $\bar{O}$

Quantum



$\boldsymbol{\Delta}_{\bar{O}}(\mathrm{x,p})=\{\delta^{(2)}(\mathbf{X}\text{-}x,\ \mathbf{P}\text{-}p)\}_{\bar{O}}$

What is this observable?

$\mathrm{Pq}_O\,(x,p) = \mathrm{Tr}[\boldsymbol{\Delta}_{\bar{O}}\,(x,p)\,\boldsymbol{\rho}]$

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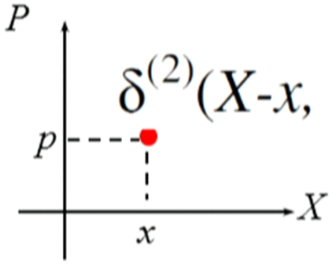

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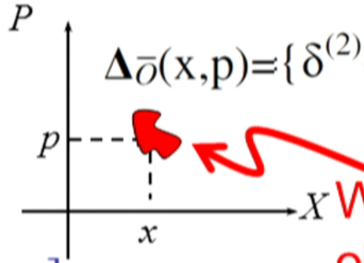

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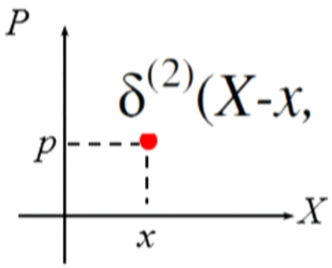

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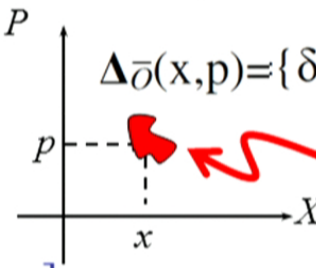

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Quasi-Prob, Pq_O	Ordering O	Dirac Delta, $\Delta_{\bar{O}}(x,p)$	Experiments & Theory
Q	Normal, N	$\boldsymbol{\Delta}_{AN}(x,p) = \lvert\alpha\rangle\langle\alpha\rvert$	Shapiro, Yuen
Wigner	Symmetric, W	$\boldsymbol{\Delta}_W(x,p)= \boldsymbol{\Pi}(x,p)$ parity about (x,p)	Banaszek, Haroche, Silberhorn, Smith
P	Anti-N,AN	$\boldsymbol{\Delta}_N(x,p)\neq$observable	

G. S. Agarwal and E. Wolf, *Phys. Rev. D*,**2** (1970) pp. 2161–2186.

X-P ordered Quasi-Prob Distributions

- Two more orderings:

Standard S: **X** to the left of **P**

Anti-Standard AS: **P** to the left of **X**

For the Standard ordering, following our quantization procedure the corresponding Quasi-Probability distribution is:

$$\mathrm{Pq}_S\,(x,p) = \mathrm{Tr}[\boldsymbol{\Delta}_{AS}(x,p)\,\boldsymbol{\rho}]$$

$$\begin{aligned}\boldsymbol{\Delta}_{AS}(x,p) &= \{\delta^{(2)}(\mathbf{X}\text{-}x,\ \mathbf{P}\text{-}p)\}_S \\ &= \delta(\mathbf{P}\text{-}p)\delta(\mathbf{X}\text{-}x,) \\ &= |\mathrm{p}\rangle\langle\mathrm{p}||\mathrm{x}\rangle\langle\mathrm{x}|\end{aligned}$$

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$$\boxed{\mathrm{Pq}_S\,(x,p) = \mathrm{Tr}[|\mathrm{p}\rangle\langle\mathrm{p}||\mathrm{x}\rangle\langle\mathrm{x}|\ \rho] = \langle\mathrm{p}||\mathrm{x}\rangle\langle\mathrm{x}|\ \rho\ |\mathrm{p}\rangle = \mathrm{D}_\rho(\mathrm{x,p})}$$

1. The standard ordered distribution is the Dirac distribution!
2. Expectation values = overlap integral, $\langle\mathbf{B}\rangle = \int \mathrm{Pq}_{AS}\cdot \mathrm{Pq}_S\,dxdp$
3. Marginals are equal to Prob(x) and Prob(p)

Bayes' Law and Weak Measurement

A. M. Steinberg, Phys. Rev. A, 52, 32 (1995):
Weakly measured probabilities (e.g. Dirac Dist.) satisfy Bayes' Law.

H. F. Hofmann, New Journal of Physics, 14, 043031 (2012):
Use Baye's law to propagate the Dirac Distribution (like in classical physics!)

1. Generalize Dirac Distribution (no longer anti-standard ordered):

$$\mathrm{Pq}_D(x, q, k, p) = \langle \delta(\mathbf{P} - p)\delta(\mathbf{K} - k)\delta(\mathbf{Q} - q)\delta(\mathbf{X} - x)\rangle$$

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2. Use Baye's Law to propagate the Dirac Dist:

$$\begin{aligned}\mathrm{Pq}_{AS}(x,k) &= \sum_{x,p} \mathrm{Pq}_{\mathrm{D}}(x,q,k,p) \\ &= \sum_{x,p} \mathrm{Pq}_{\mathrm{D}}(q,k|x,p)\cdot \mathrm{Pq}_{AS}(x,p)\end{aligned}$$

3. Use Eq 1 and the formula for the Dirac Dist to find the propagator:

$$\mathrm{Pq}_{\mathrm{D}}(q,k|x,p) = \frac{\mathrm{Pq}_{\mathrm{D}}(x,q,k,p)}{\mathrm{Pq}_{AS}(x,p)} = \frac{\langle p|k\rangle\langle k|q\rangle\langle q|x\rangle}{\langle p|x\rangle}$$

- The propagator is a weak conditional probability, made up of state overlaps

Bayesian Propagation of the Dirac Distribution

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Classical Bayes' Law: $P(x,k') = \sum_p P(x,k'|p)P(x,p)$

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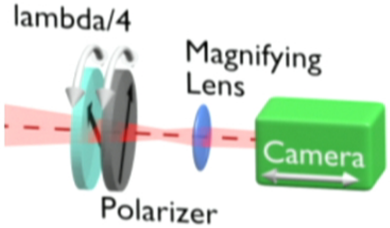

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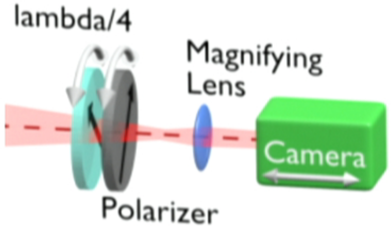

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