Title: Exact holographic mapping and emergent space-time geometry

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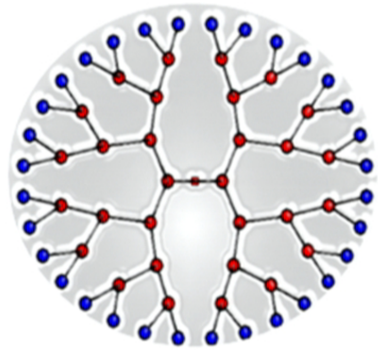
Abstract: Holographic duality is a duality between quantum many-body systems (boundary) and gravity systems with one additional spatial dimension (bulk). In this talk, I will describe a new approach to holographic duality for lattice systems, called the exact holographic mapping. The key idea of this approach can be summarized by two points: 1) The bulk theory is nothing but the boundary theory viewed in a different basis. 2) Space-time geometry is determined by the structure of correlations and quantum entanglement in a quantum state. For free fermion boundary theories, I will show how different bulk geometries including AdS space, black holes and worm-holes emerge. I will also discuss the generalization of this approach in more generic interacting systems.



Exact holographic mapping and emergent space-time geometry

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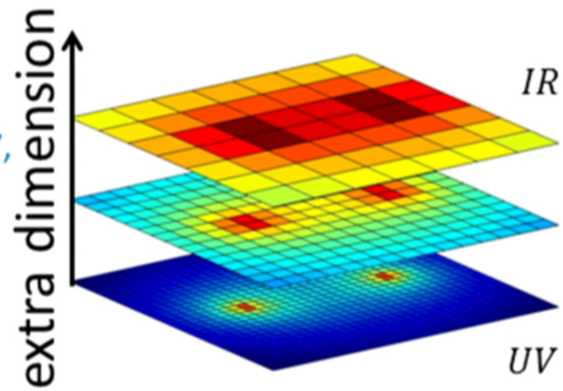
Outline

- Motivation of this work: A poor man's understanding to holography. (Some thoughts about space-time geometry in quantum systems)
- The definition of exact holographic mapping (EHM)
- EHM for 1+1d Dirac fermion
- Geometry dual to different states
- Probing the black hole physics
- Discussion on the application of EHM to interacting systems

Ref: XLQ, **arXiv:1309.6282 (2013)**

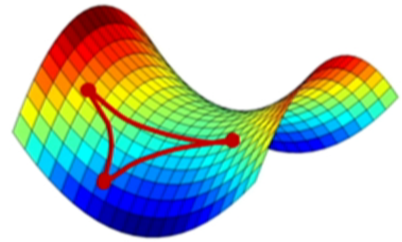
Holographic duality

- Holographic duality was obtained in string theory context. (Maldacena '97, Witten '98, Gubser, Klebanov & Polyakov '98)



- The extra dimension can be interpreted as energy scale. Bulk equation of motion ←→ renormalization group flow. (E. Akhmedov, '98, Heemskerk & Polchinski '10)
- Holographic duality provides an alternative approach to strongly correlated electron systems. For a review, see S. Sachdev, Annual Review of Condensed Matter Physics **3**, 9 (2012)
- Constructive approach starting from the boundary theory. (S.-S. Lee, Nucl. Phys. B 832 (2010) [0912.5223] and more recent papers)
- My attempt: a constructive approach based on quantum states

A poor man's understanding to holographic duality

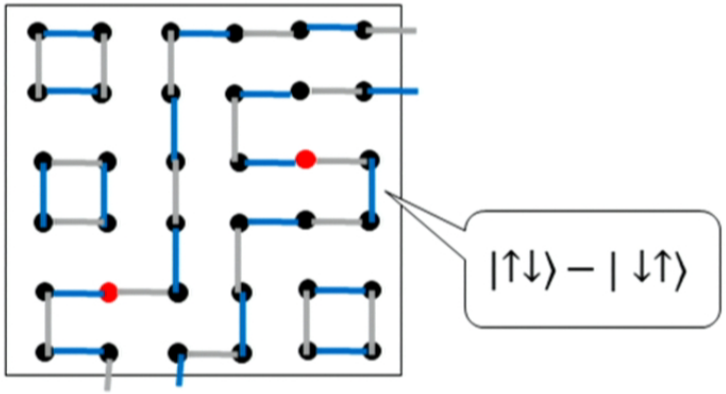


- In the classical world, the space-time geometry tells us a space-time manifold consisting of points, and the distance between points given by geodesics.
- How to know the geometry of the space-time that a quantum system lives in?
- **Points:** A basis choice in the Hilbert space, in which the physics is local.
- **Distance:** Distance between points shall be measured by physical correlation functions.

How to define distance between points?

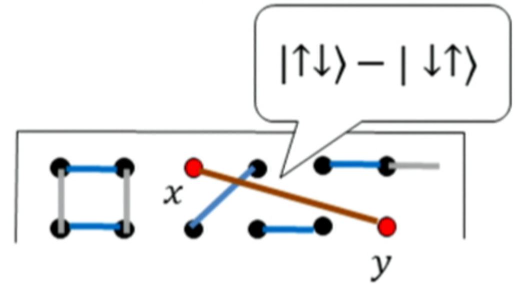
- Different basis choices are distinguished by locality.
- Once we find a suitable local basis, we defined "points".
- In a *many-body state*, the distance should be related to correlation functions.

Example: A short-range resonance valence bond (RVB) state with spin singlet pairs



- First consider the ground state of a gapped system.
- Two-point correlation function $C_{xy} \equiv \langle O_x O_y \rangle - \langle O_x \rangle \langle O_y \rangle \simeq C_0 \left(d_{xy}\right)^{\nu} e^{-d_{xy}/\xi}$ depends exponentially on the distance d_{xy}

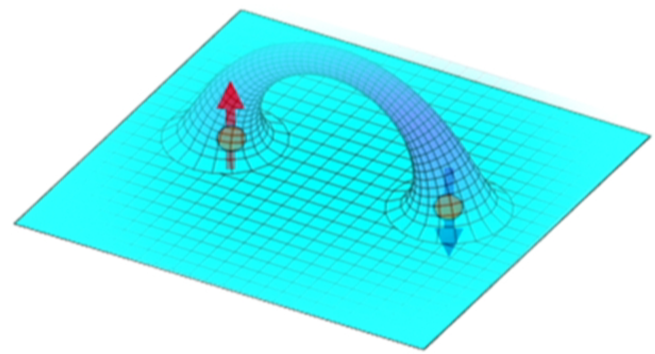
ER=EPR



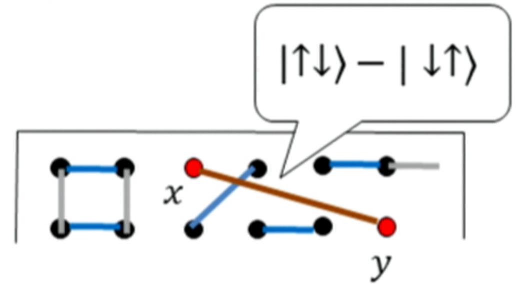
- Distance $d_{xy} = -\xi \log \frac{I_{xy}}{I_0}$

more mutual information	$\Longleftrightarrow$	shorter distance
Maximal mutual information $S_x = S_y = \log D\,, S_{xy} = 0$ $\Rightarrow I_{xy} = I_0 = 2\log D$	$\Longleftrightarrow$	$d_{xy} = 0$ between a EPR pair $|\Psi\rangle = \sum_{i=1}^{D} |i\rangle_x |i\rangle_y$

- Maximal entanglement=Worm hole (Einstein-Rosen bridge)
- Accessing x is equivalent to accessing y ➔ $d_{xy} = 0$
- A realization of the "ER=EPR" principle (J. Maldacena, L. Susskind '13)



ER=EPR



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$\Leftrightarrow$

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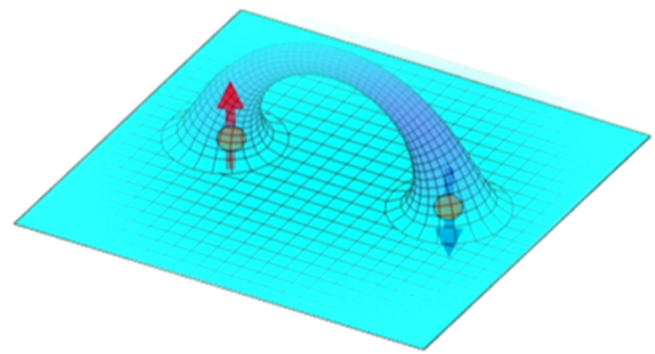
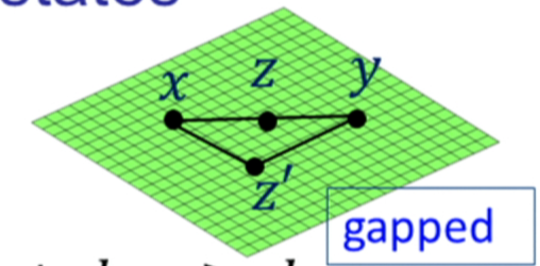
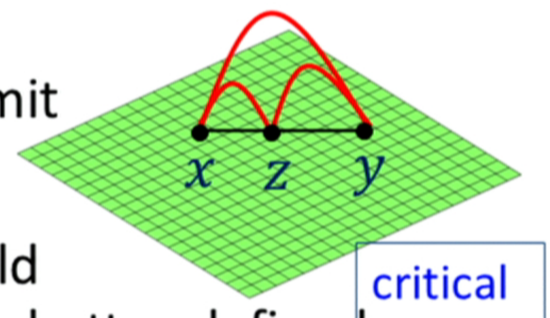


From gapped states to critical states

- **Gapped states:** the distance defined in this way agrees with the geodesic distance in long distance limit.



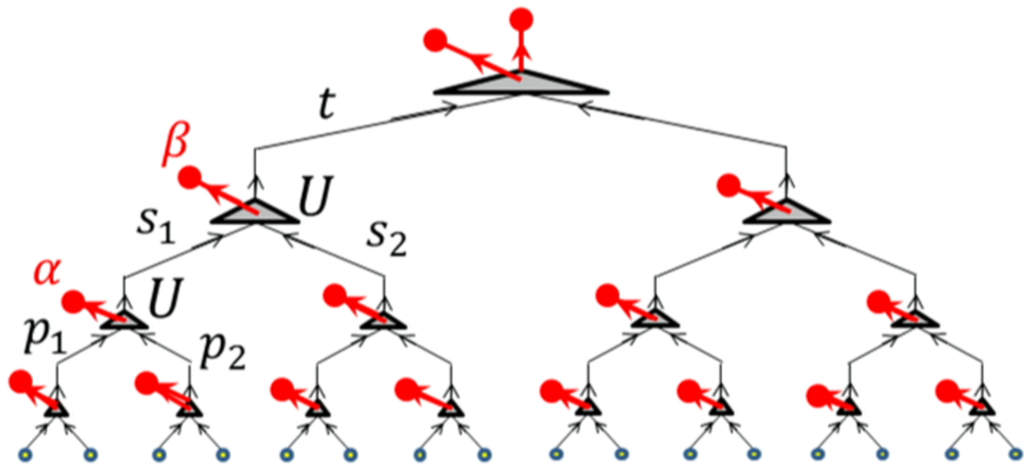
- ➔Triangle inequality is satisfied $d_{xz'} + d_{z'y} \geq d_{xy}$
- **Critical states:** Power law correlation $I_{xy} \propto |x-y|^{-2\Delta}$
- Geometric interpretation of an "intrinsic observer": $d_{xy} \propto \log|x-y|$
- $d_{xz} + d_{yz} > d_{xy}$ in long distance limit
- d_{xy} is not a geodesic distance
- This suggests that a new basis should be defined, in which the geometry is better defined.

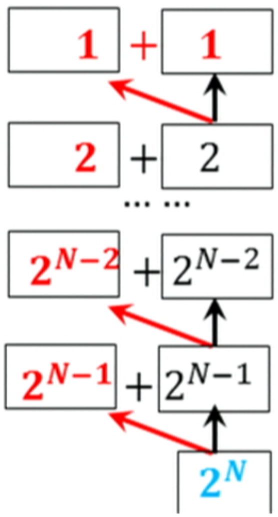


- The new geometry is hyperbolic in $d+1$ dimension.
- The transformation to the new basis defines the duality.

Definition of the exact holographic mapping (EHM)

- 1. Starting from 2^N sites, U maps each two neighboring sites to a "low energy" site and a "high energy" site
  2. Repeat step 1 the 2^{N-1} low energy sites.
- Unitary mapping $M = \prod_{\text{network}} U$ maps boundary (2^N sites) to bulk ($1 + 1 + 2 + \cdots + 2^{N-1} = 2^N$ sites)
- A modification of Multiscale Entanglement Renormalization Ansatz (MERA) (Vidal '07)





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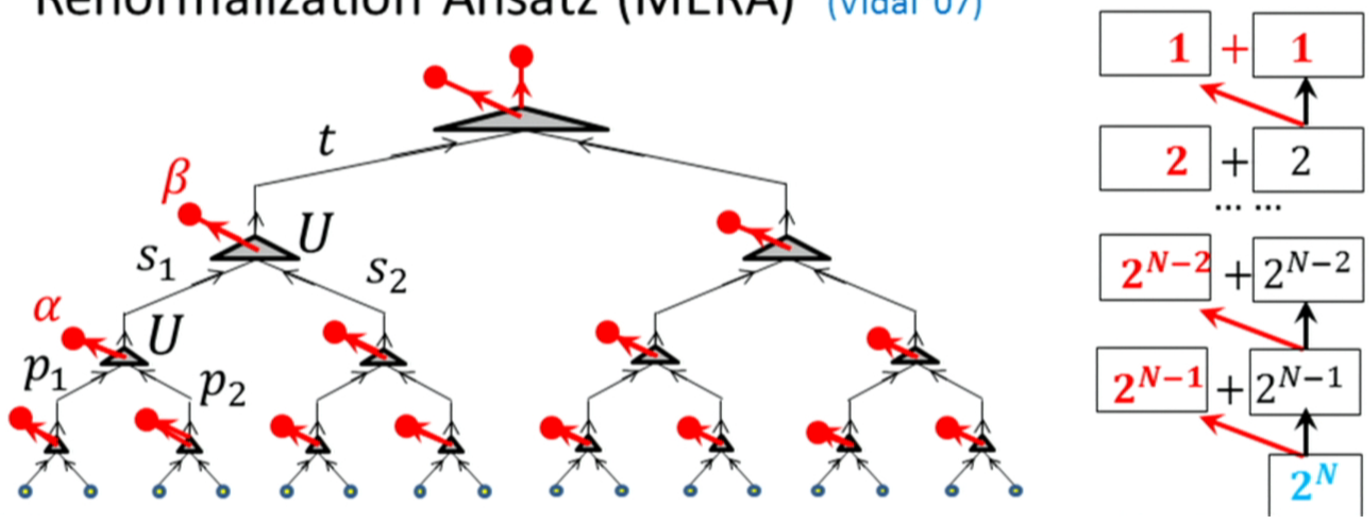

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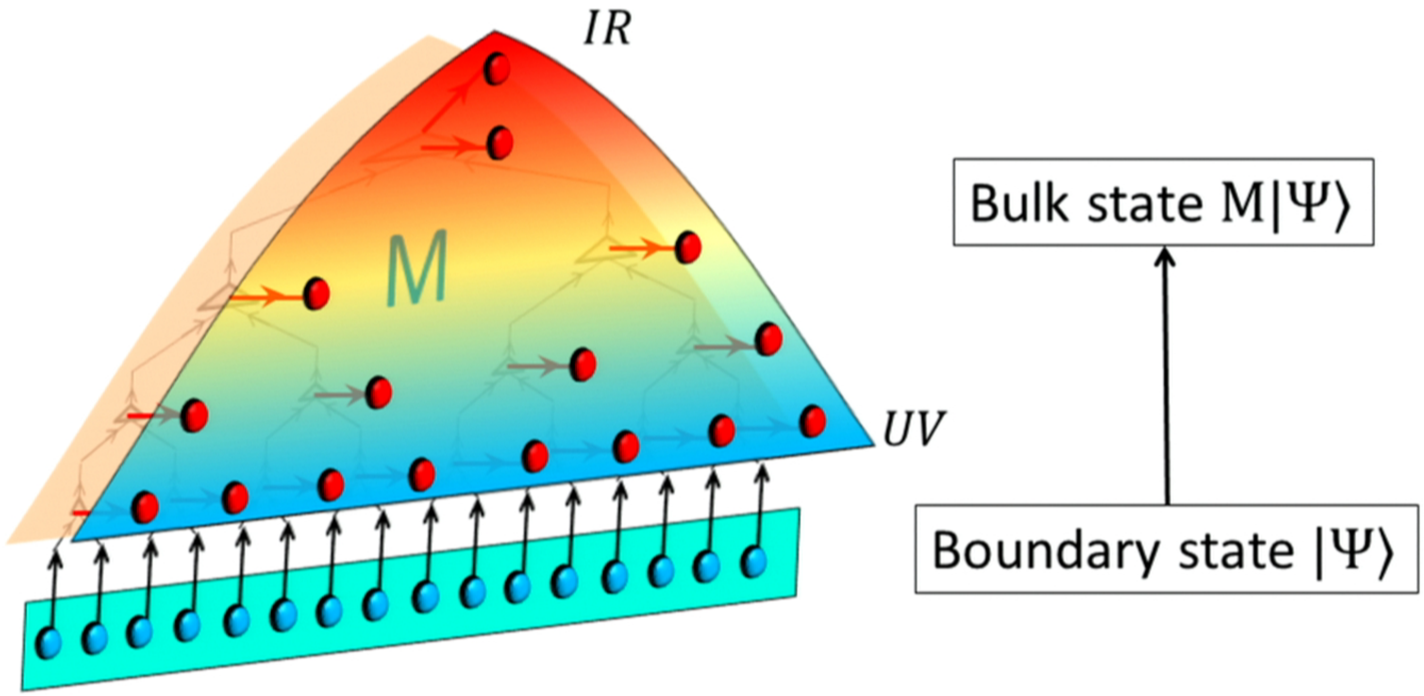
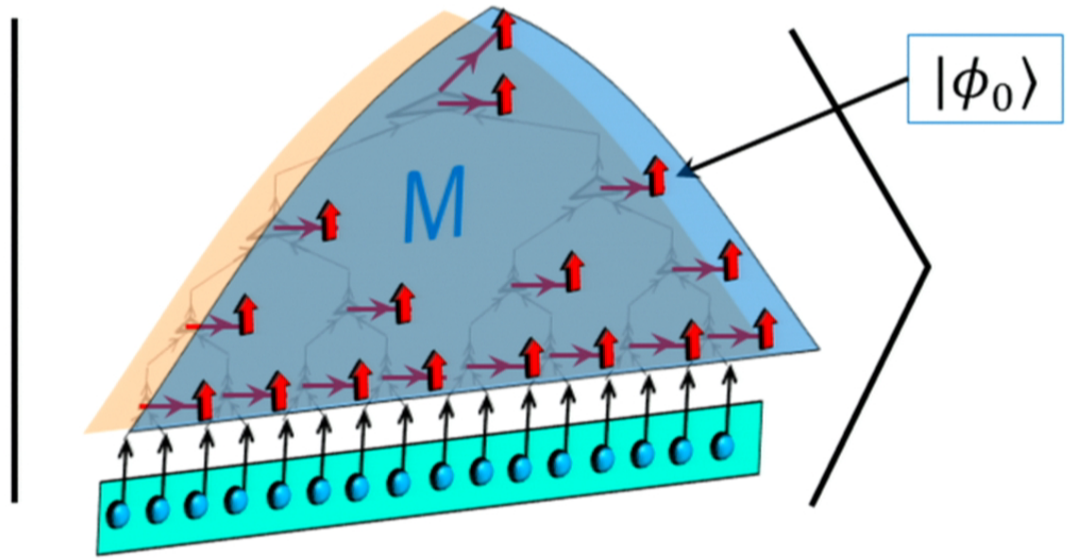


- An exact form of real space RG.
- Degrees of freedom at different energy scales are all kept, and they can entangle with each other.

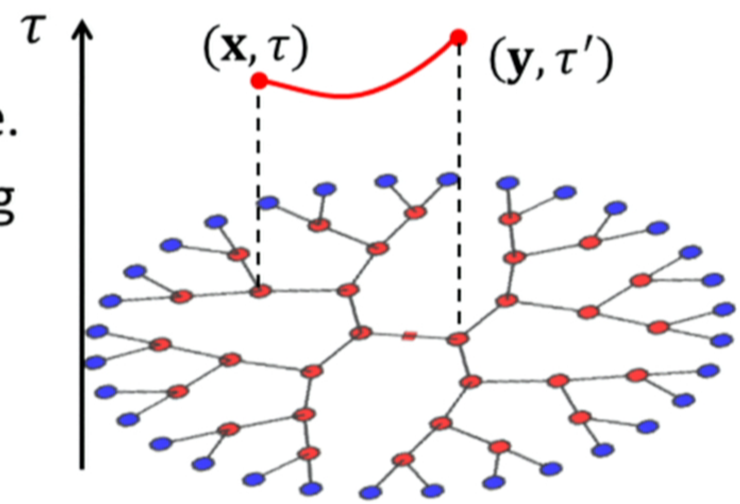
Relation of EHM and MERA

- A MERA state corresponds to acting the reverse EHM of a direct product bulk state $|MERA\rangle = M^{-1} \prod^{\otimes} |\phi_0\rangle$
- MERA has been proposed to be related to AdS/CFT (Swingle '10, Evenbly&Vidal '11, Haegeman et al '11, Nozaki et al '12)
- The goal of EHM is to *allow bulk states to entangle* and use that to probe the geometry.

$|MERA\rangle =$ 

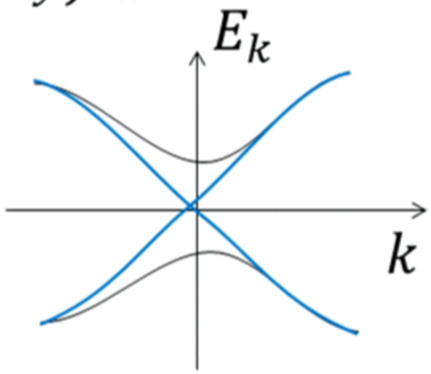
Exact holographic mapping: bulk geometry

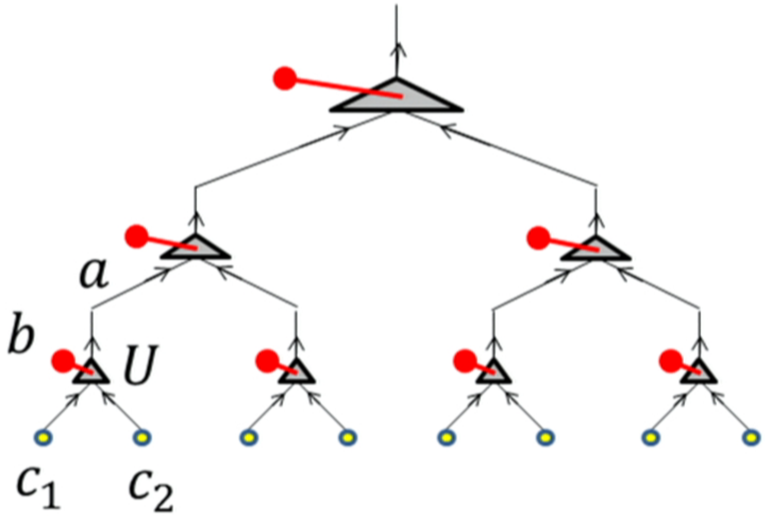
- Bulk space-time geometry determined by bulk correlation functions $d_{(x,\tau),(y,\tau\prime)} = -\xi \log \frac{\langle O_{(x,\tau)} O_{(y,\tau')} \rangle}{C_0}$.
  (τ is the imaginary time)
- Mutual information can be used for equal time distance.
- Different choices of mapping U correspond to different choices of "classical background geometry".
- The quantum geometry is generically different from the classical structure of the network.



Exact holographic mapping: free fermion

- 1+1d lattice Dirac fermion
  $H = \sum_k c_k^+\left(\sin k\, \sigma_x + (m + 1 - \cos k)\sigma_y\right)c_k,$
- Unitary mapping
- $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \to \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} c_1 + c_2 \\ c_1 - c_2 \end{pmatrix}$





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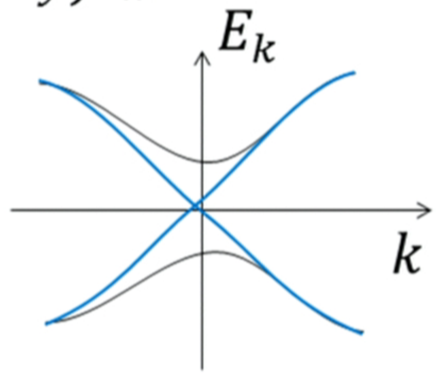
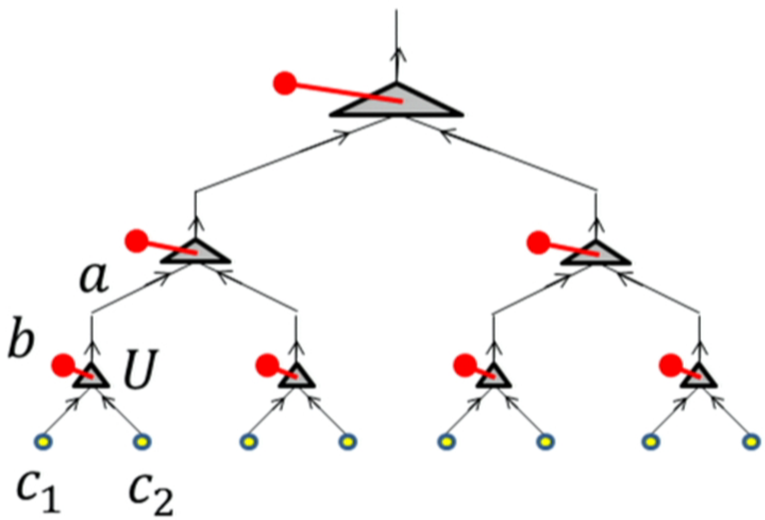




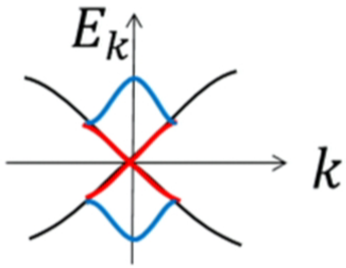
Exact holographic mapping: free fermion

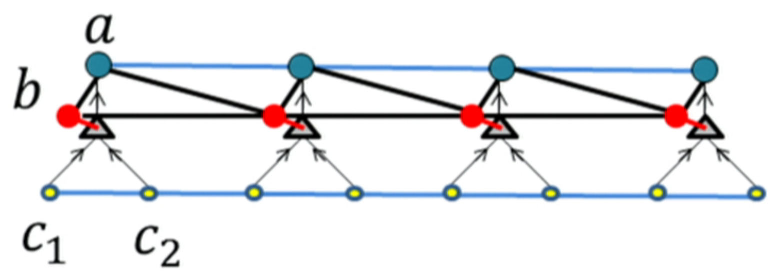
- Bulk Hamiltonian after first layer of the mapping:

$$H = \sum_k \left(\underline{a_k^+ h_a^{(1)}(k) a_k} + b_k^+ h_b(k) b_k + [a_k^+ T(k) b_k + h.c.] \right)$$

$$h_a^{(1)}(k) = \frac{1}{2}[\sigma_x \sin k + (2m + 1 - \cos k)\sigma_y]$$

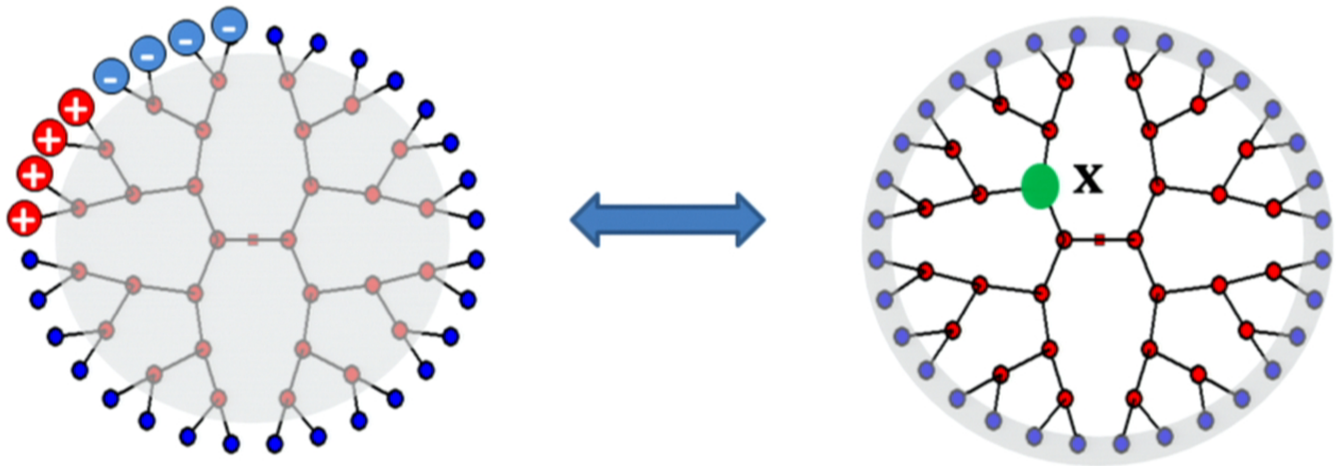
- Next layer $h_a^{(2)}(k) = \frac{1}{4}[\sigma_x \sin k + (4m + 1 - \cos k)\sigma_y]$
- For $m = 0$, the low energy Hamiltonian is "on the fix point". ➔ Bulk Hamiltonian is scaling invariant.
- Iteration leads to $h_a^{(n)}(k) = 2^{-n} h_c(k)$.



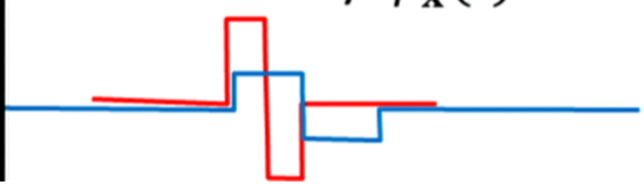


Exact holographic mapping: free fermion

- Basis transformation between boundary and bulk



"Haar wavelet" wavefunctions at the boundary $\phi_{\mathbf{x}}(i)$

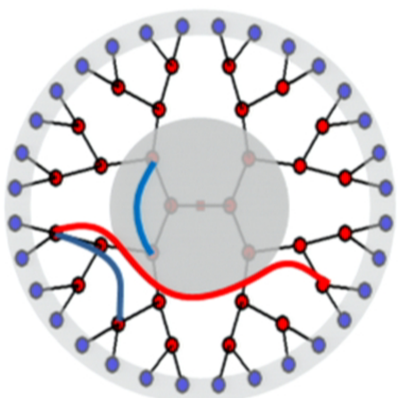


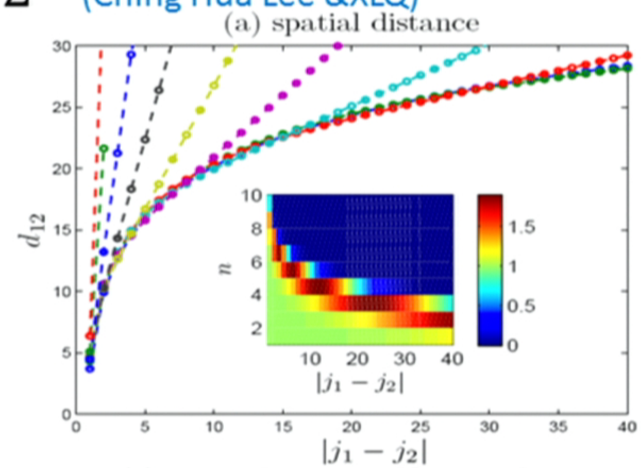
Local basis in the bulk

$$b_{\mathbf{x}} = \sum_i \phi^*_{\mathbf{x}}(i) c_i$$

Finite temperature and black hole

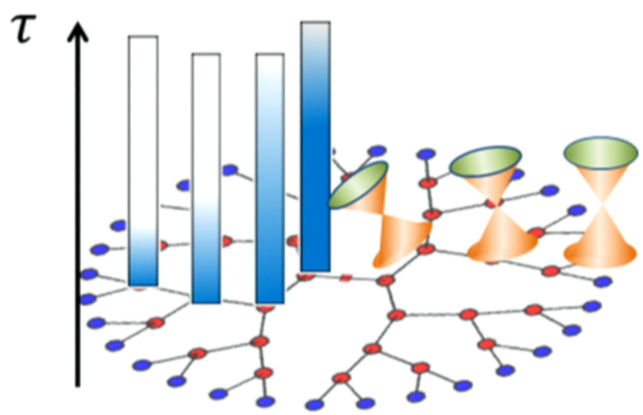
- $T > 0, m = 0$. Geometry is modified for non-critical systems, even if the same mapping is chosen.
- **Spatial direction:** $d_{(x,n),(y,n)}$ Cross-over from AdS ($\propto \log|x-y|$) to Euclidean ($\propto |x-y|$).
- IR limit $n \to N$, *Stretched horizon* region.
- angle-direction distance $d_{0n,xn} \simeq (1-2\pi T)^{2^{n+1}x} \Rightarrow$ Perimeter $2\pi\rho \simeq 4\pi T \cdot 2^N$ (Ching Hua Lee &XLQ)

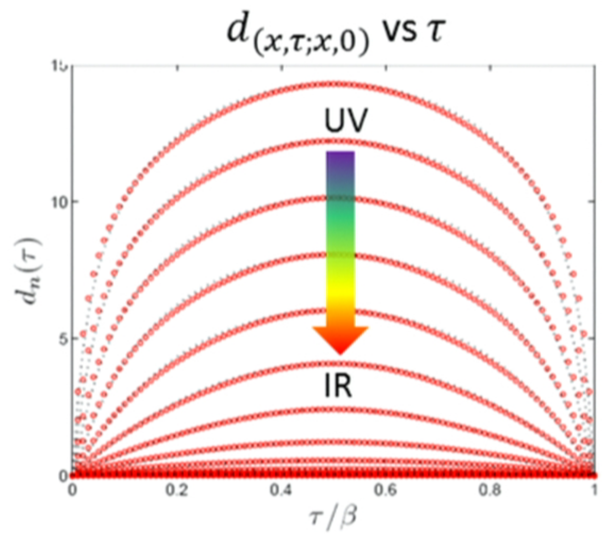




Finite temperature and black hole

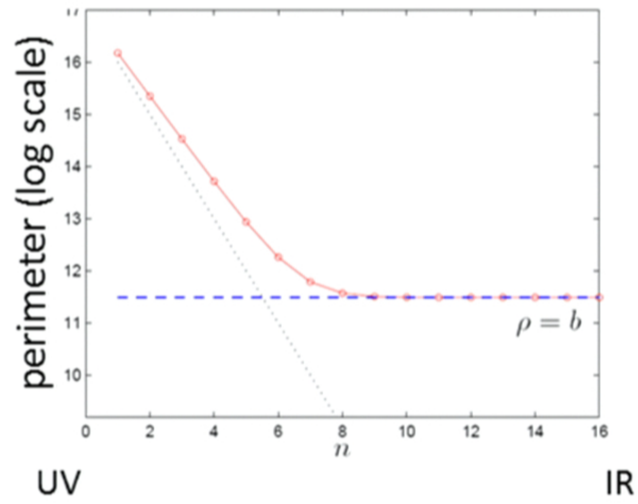
- **Time direction**: In IR region, fermion bandwidth$\ll T$
- The time dependence of correlation function exponentially slows down. $d_{(x,\tau),(x,0)} \to 0$ in IR
- General reason: reduced density matrix of the stretched horizon region $\rho_{IR} \propto \mathrm{I}$ ➔ Trivial time evolution.





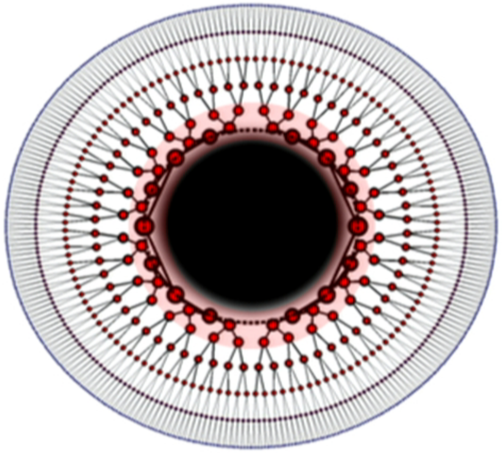
Finite temperature and black hole

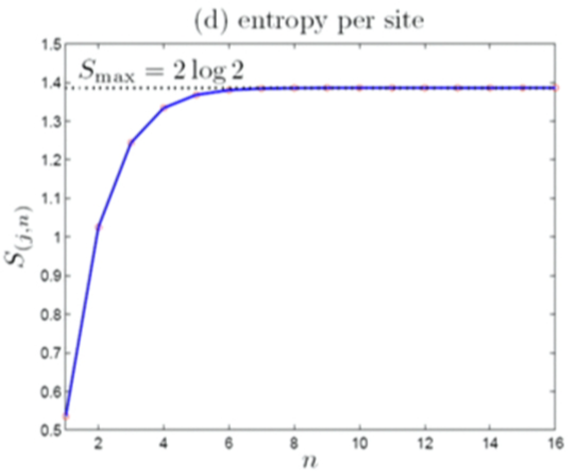
- Fitting $d_{(x,\tau),(x,0)}$ with the AdS formula
$$d_{(x,\tau),(x,0)} = R\,\mathrm{acosh}\left[\frac{\rho^2}{b^2} - \left(\frac{\rho^2}{b^2} - 1\right)\cos\left(\frac{2\pi}{\beta}\tau\right)\right],$$
black hole radius can be determined. R value is different from the spatial direction.
- The infinite red shift in IR sees generic for thermal state, due to maximal entanglement with thermal bath.



Black-hole entropy

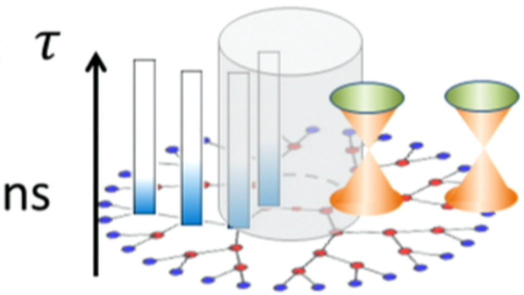
- The IR region is the “stretched horizon” region of black-hole.
- Each bulk-site carries a finite entanglement entropy with the rest of the system and the thermal bath.
- Each site in the stretched horizon region still carries maximal entropy $S = \log D = \log 4$.
- The entropy in this region shall be considered as the black-hole entropy

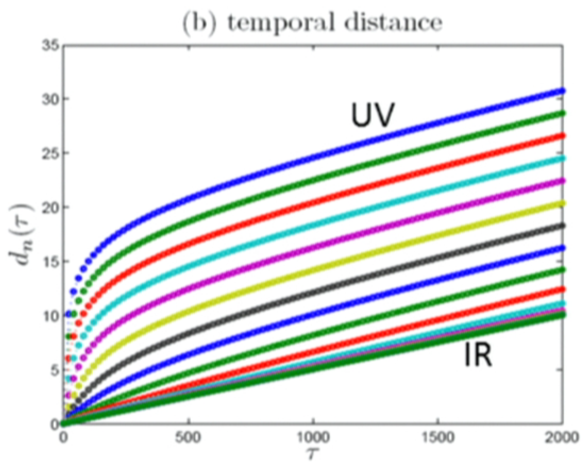


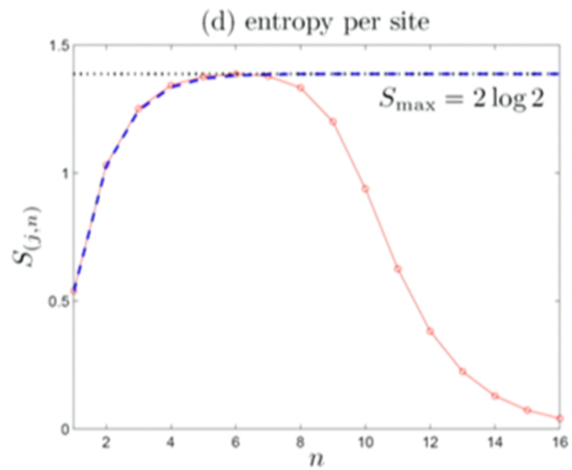


Space-time geometry for a massive state

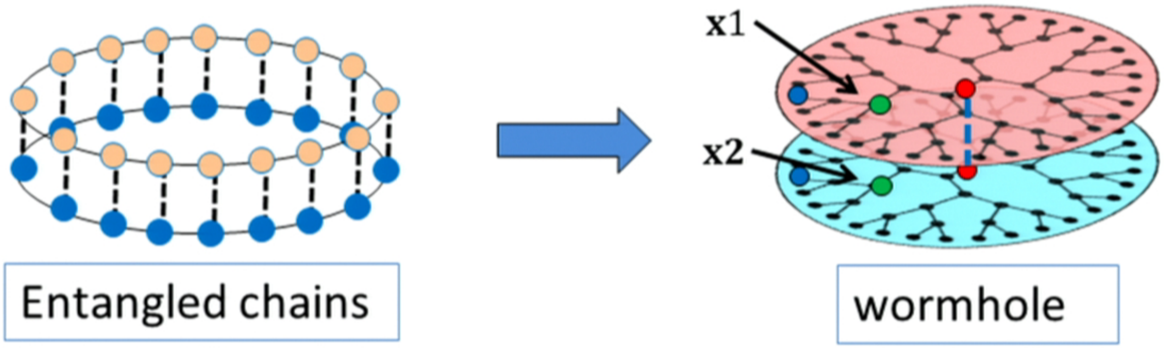
- $T = 0, m \neq 0$. Spatial distance behaves similarly to black-hole
- Time direction correlation length remains finite in IR. $\langle T c_{x,\tau} c^{+}_{x,0} \rangle \propto e^{-m|\tau|}$
- The space terminates, but the time direction remains finite. IR boundary perimeter $2\pi\rho \simeq 2m \cdot 2^N$ (Lee&Qi)
- Entropy vanishes in IR region. Consistent with no horizon





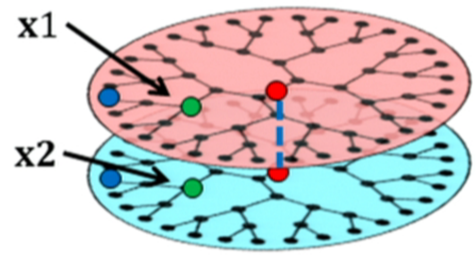


Thermal field double and wormhole geometry



- More explicit understanding to the thermal state
- The thermal field double state of entangled chains $|\Psi\rangle = \sum_n e^{-\beta E_n/2}\, |\bar{n}\rangle_L |n\rangle_R,$
- Holographic mapping defined separately for each chain
- Each chain is mapped to a black hole.
- Free fermion Hamiltonian $H = H_R - H_L + H_{\text{tunneling}}$ can be designed, with $|\Psi\rangle$ a ground state.

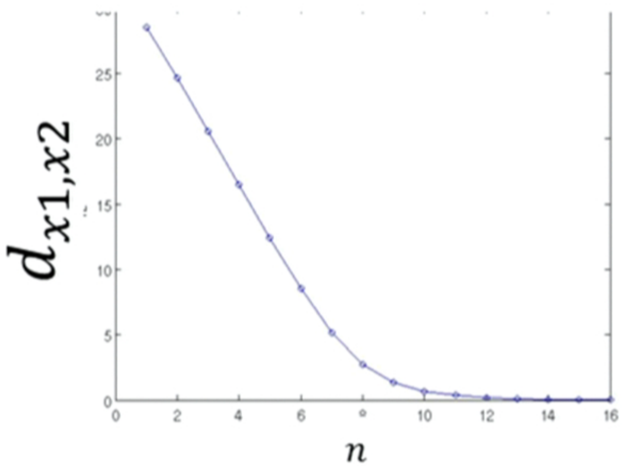
Wormhole geometry

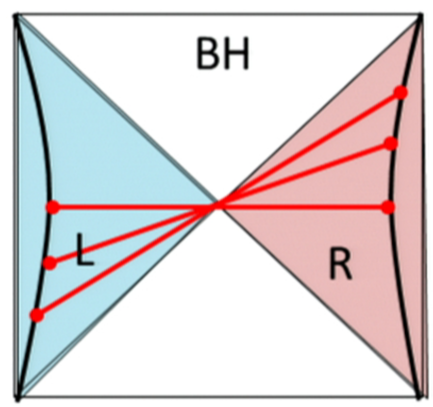


- "Vertical" distance $d_{\mathbf{x1x2}}$ vanishes exponentially in IR.
- The two stretched horizons are maximally entangled.

$$d_{\mathbf{x1,x2}} = -\xi \log \frac{I_{\mathbf{x1x2}}}{I_0},$$

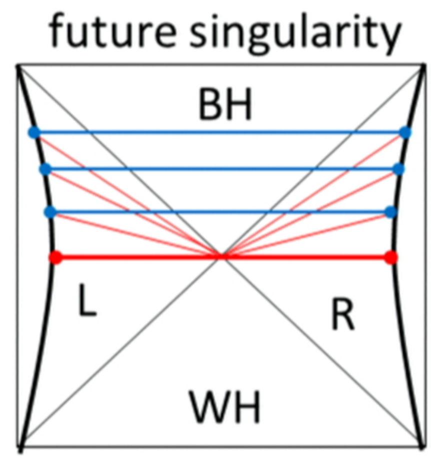
- ➔Einstein-Rosen bridge connecting the two AdS regions
- The dual geometry contain outside regions of the eternal black-hole (Maldacena'03)





Quantum quench on the wormhole geometry

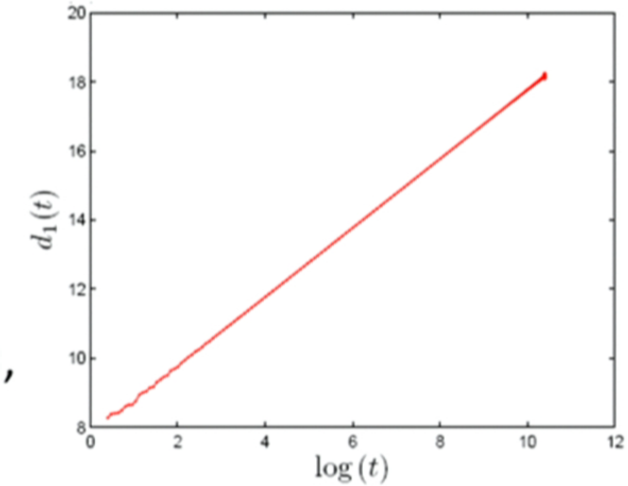
- The blackhole interior can be indirectly probed by the distance between outside points (Hartman & Maldacena JHEP '13)

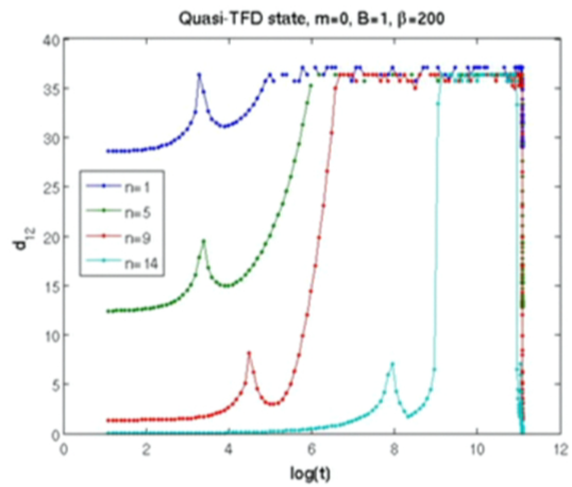


- Time evolution by $H_R + H_L$ preserves the thermal field double state.
- Time evolution by $H_R - H_L$ gives
  $|\Psi(\mathrm{t})\rangle = \sum_n e^{-\left(2it+\frac{\beta}{2}\right)E_n} |\bar{n}\rangle_L |n\rangle_R,$
  and changes the entanglement structure between the two sides.
- Hartman&Maldacena obtains that the distance between two points $d(t) \simeq at + C$ (for AdS_3, the same as the minimal surface area) increases linearly

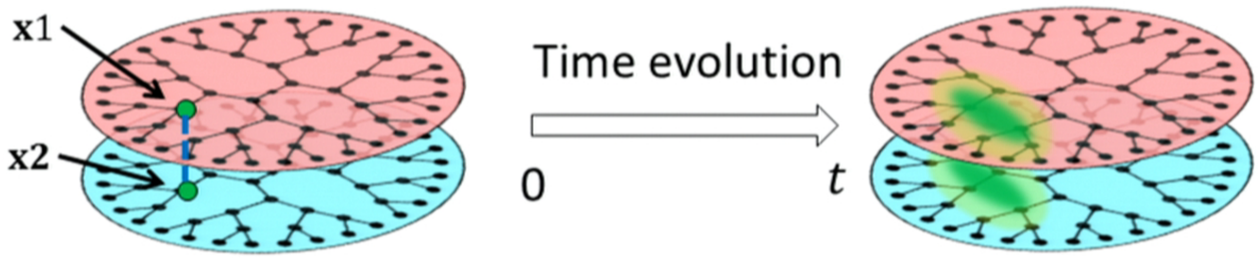
Quantum quench on the wormhole geometry

- The quantum quench for free fermion case
- $H = H_R - H_L + \lambda(t) H_{\mathrm{t}}$
- $H_{R(L)} = \sum_{k,a} \gamma_{ka}^{R(L)+} \gamma_{ka}^{R(L)} E_{ka}$,
  $H_t = \sum_{k,a} \frac{E_{ka}}{\sinh\left(\frac{\beta E_{ka}}{2}\right)} \left(\gamma_{ka}^{R+} \gamma_{ka}^{L} + h.c.\ \right)$,
- Switch off λ at $t = 0$
- Worm-hole shrinks,
  $I_{x1x2} \propto \frac{1}{t},\ d_{x1x2} \propto \log t.$
- Different from classical space-time
  $d_{x1x2} \propto t, (I_{x1x2} \propto e^{-at}).$

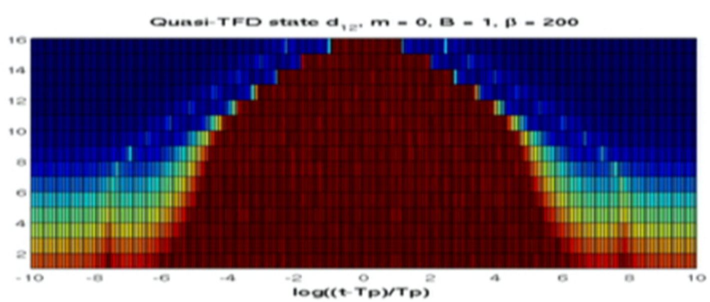


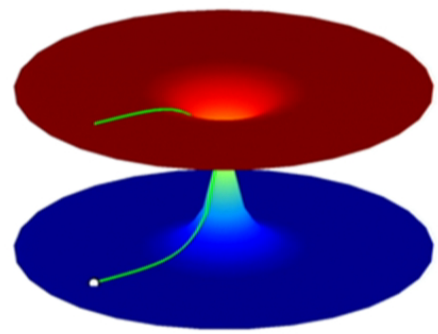


Wormhole geometry and quantum quench



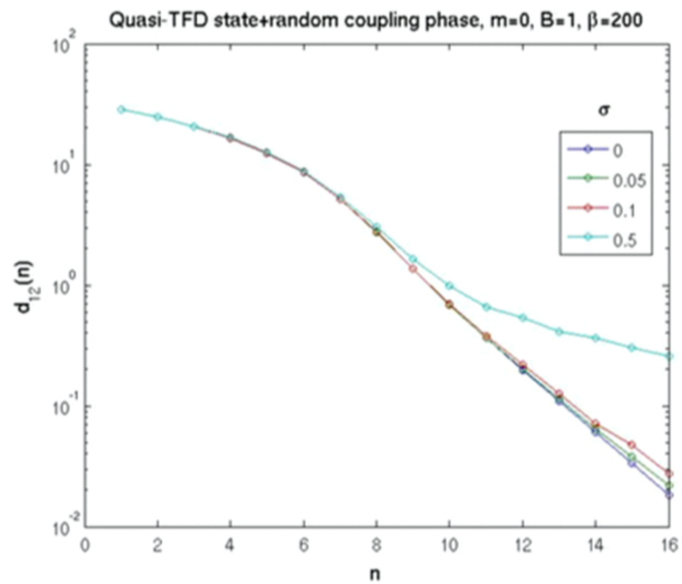
- Free system: mutual information is smeared to a region with area $V \propto t$
- Interaction system: mutual information is distributed in the many body states in that region. # of states $\propto e^{at}$
- The wormhole is restored in time $\simeq L = 2^N$



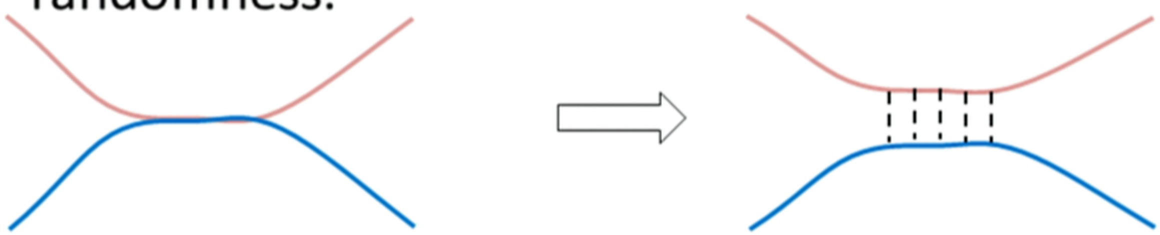


Random perturbation to the thermal field double state

- The eternal black hole is a very special black hole geometry.
- A random perturbation can destroy the worm hole between the two spaces. (e.g. Shenker&Stanford '13)



- Effect of a random phase perturbation $\gamma_{k\sigma}^{L} \to \gamma_{k\sigma}^{L} e^{i\theta_k}$
- Distance between two spaces increase with the randomness.



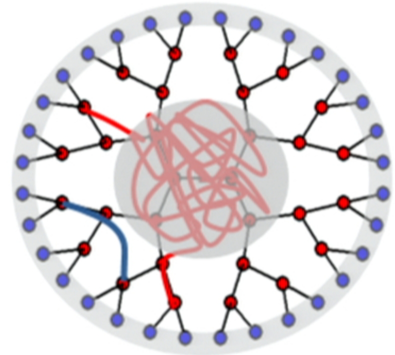
Bulmash&Qi

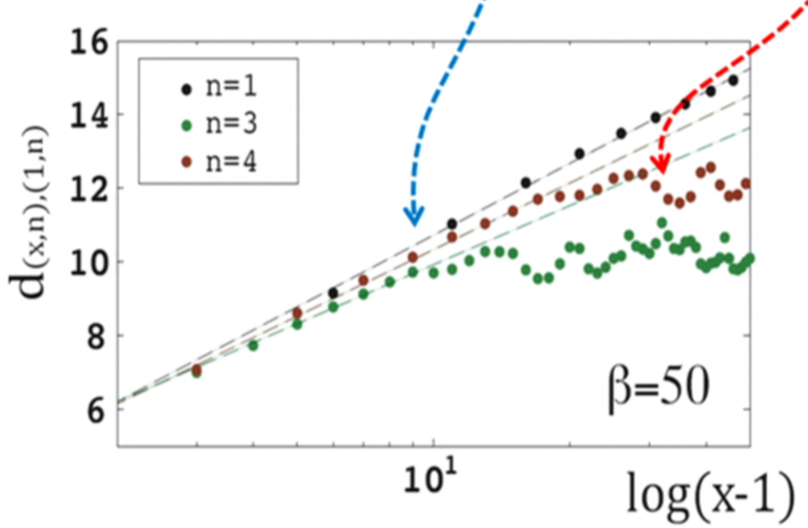
Black hole microstates

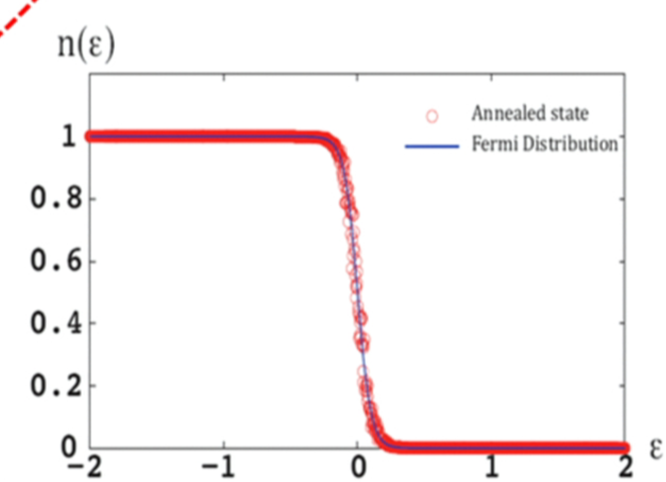
- For a generic interacting system, according to the eigenstate thermalization hypothesis (ETH) (Deutsch '91, Srednicki '94) A generic finite energy state looks like a thermal ensemble in simple correlation functions.
- Free fermion is integrable, so thermalization does not happen. However, we can still consider the geometry dual to a random (Slater determinant) state.
- A completely random Slater determinant state $|U = e^{iT}\rangle = e^{ic^{+}Tc} \prod_i c_{i\downarrow}^{+} |0\rangle$. Equivalent to an infinite temperature ensemble. Bulk correlation $\langle b_{x\alpha}^{+} b_{y\beta}\rangle \simeq \overline{\langle b_{x\alpha}^{+} b_{y\beta}\rangle} = \frac{1}{2}\delta_{xy}\delta_{\alpha\beta}$.
- An ``annealed" random state $|U, \beta\rangle = e^{-\frac{\beta H}{2}}|U\rangle$

Black hole microstates

- Geometry corresponding to the annealed random state $|U, \beta\rangle$
- AdS-like region in UV
- Random correlation in the stretched horizon region. Breakdown of locality. (Jian&Qi)

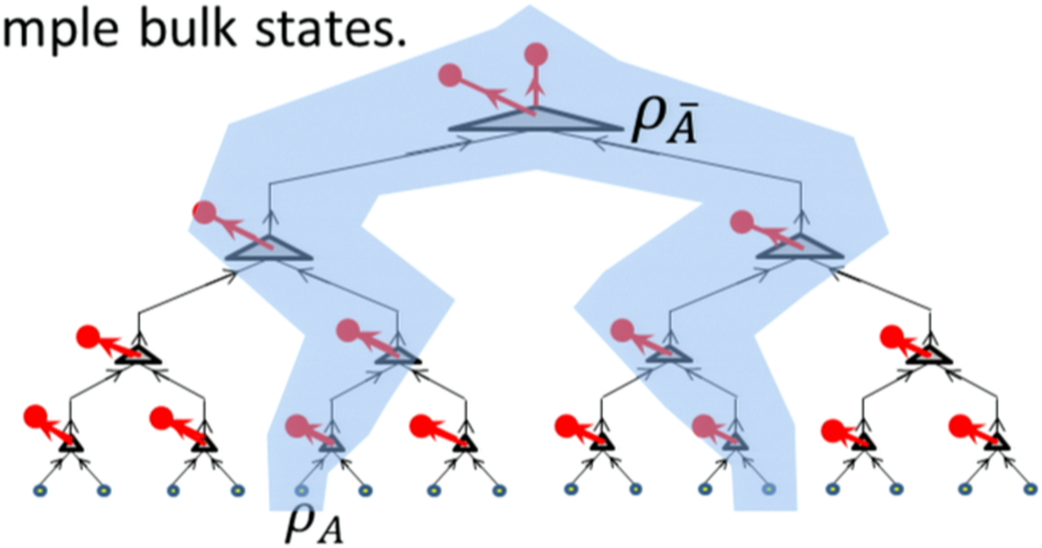






Causal cone structure

- The reduced density matrix of a boundary region A is uniquely determined by the bulk reduced density matrix in a causal cone region $\bar{A}$. (Size of $\bar{A}$) $\simeq$ (Size of A)$\times$ log(length of the system).
- This property is inherited from MERA (Vidal '08). It's possible to efficiently compute boundary correlation functions for simple bulk states.



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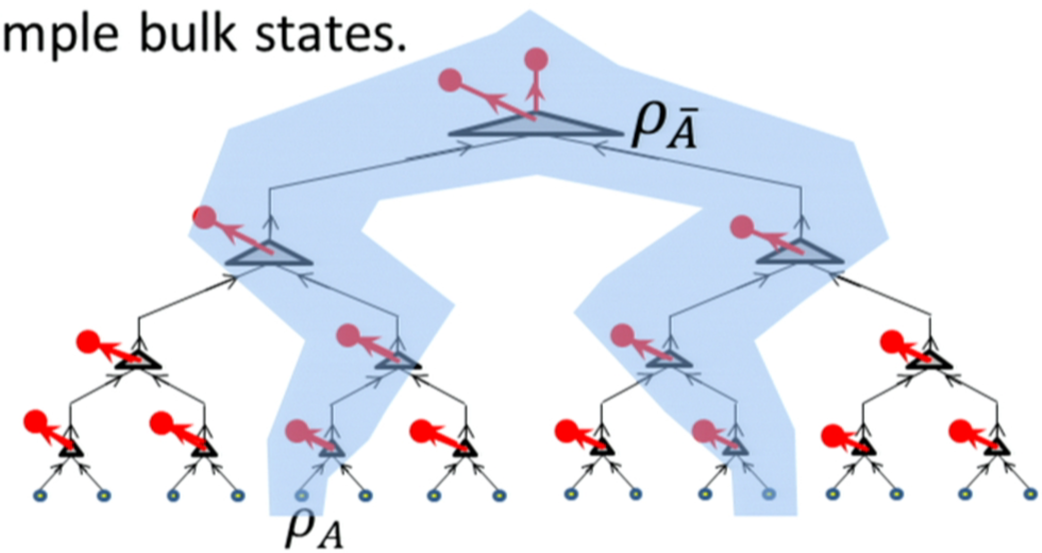
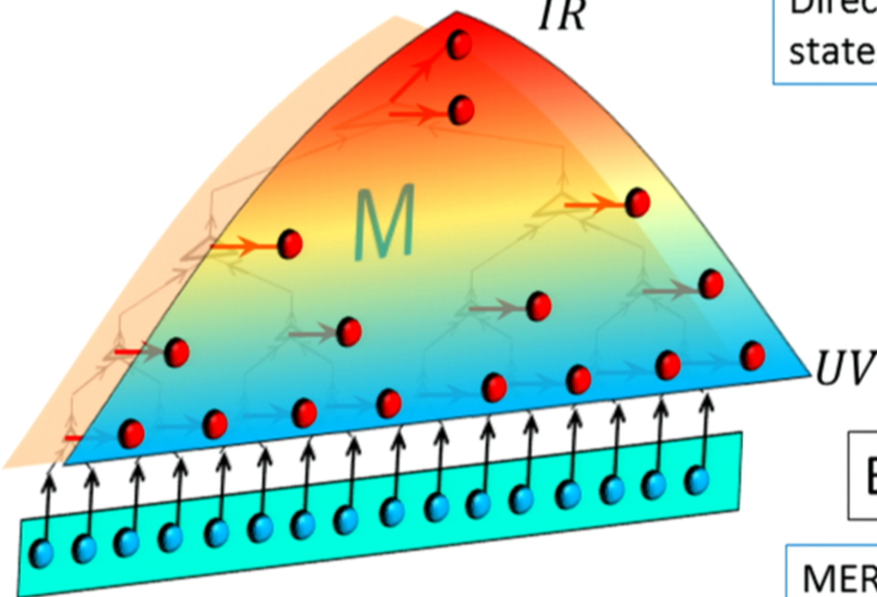
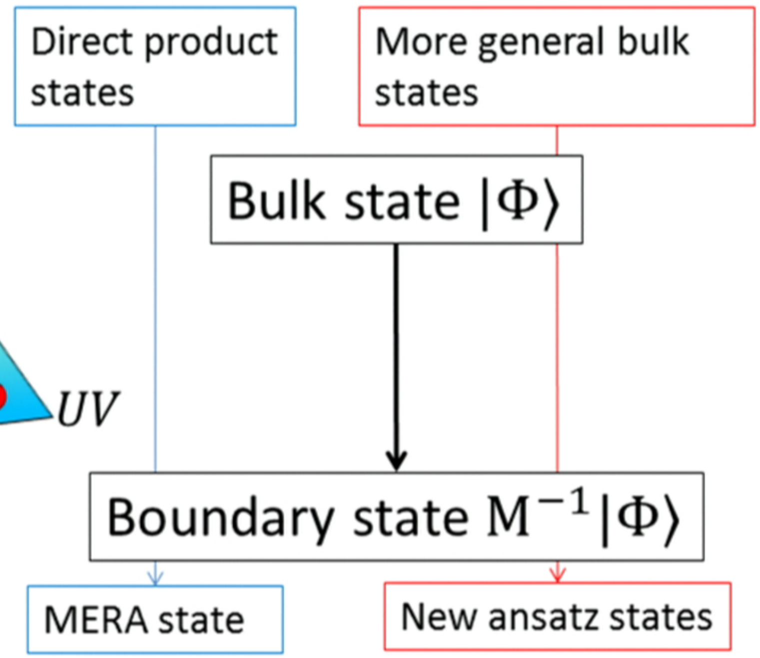


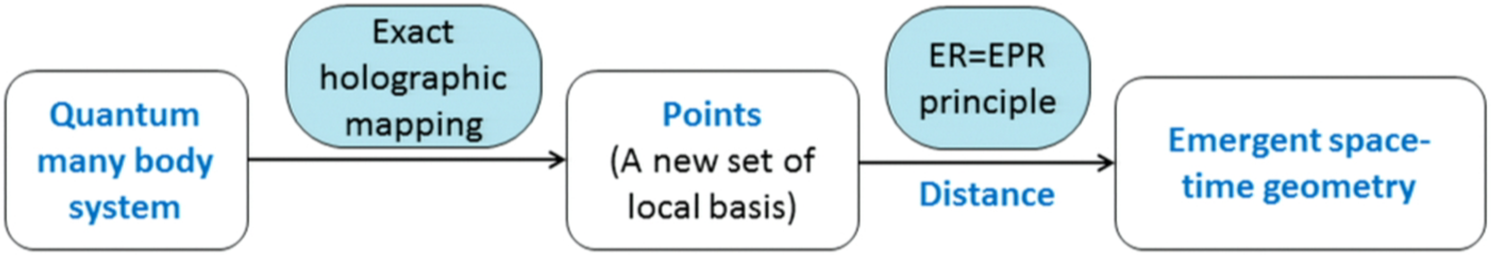
Towards interacting systems

- We can use the (reverse) exact holographic mapping to generate new variational states for the boundary system.





Summary and open questions



- Relation between quantum entanglement and space-time geometry.
- A purely quantum model for blackholes.
- A possible new approach to strongly correlated systems.
- Open questions:
  ◊ Explicit examples of interacting systems.
  ◊ A guiding principle for finding one of the "optimal" mappings
  ◊ Role of large N limit and continuum limit
  ◊ Relation to known AdS/CFT correspondence.
  ◊ Can we access the interior of a black-hole?