Title: Line Defects and Tropicalization

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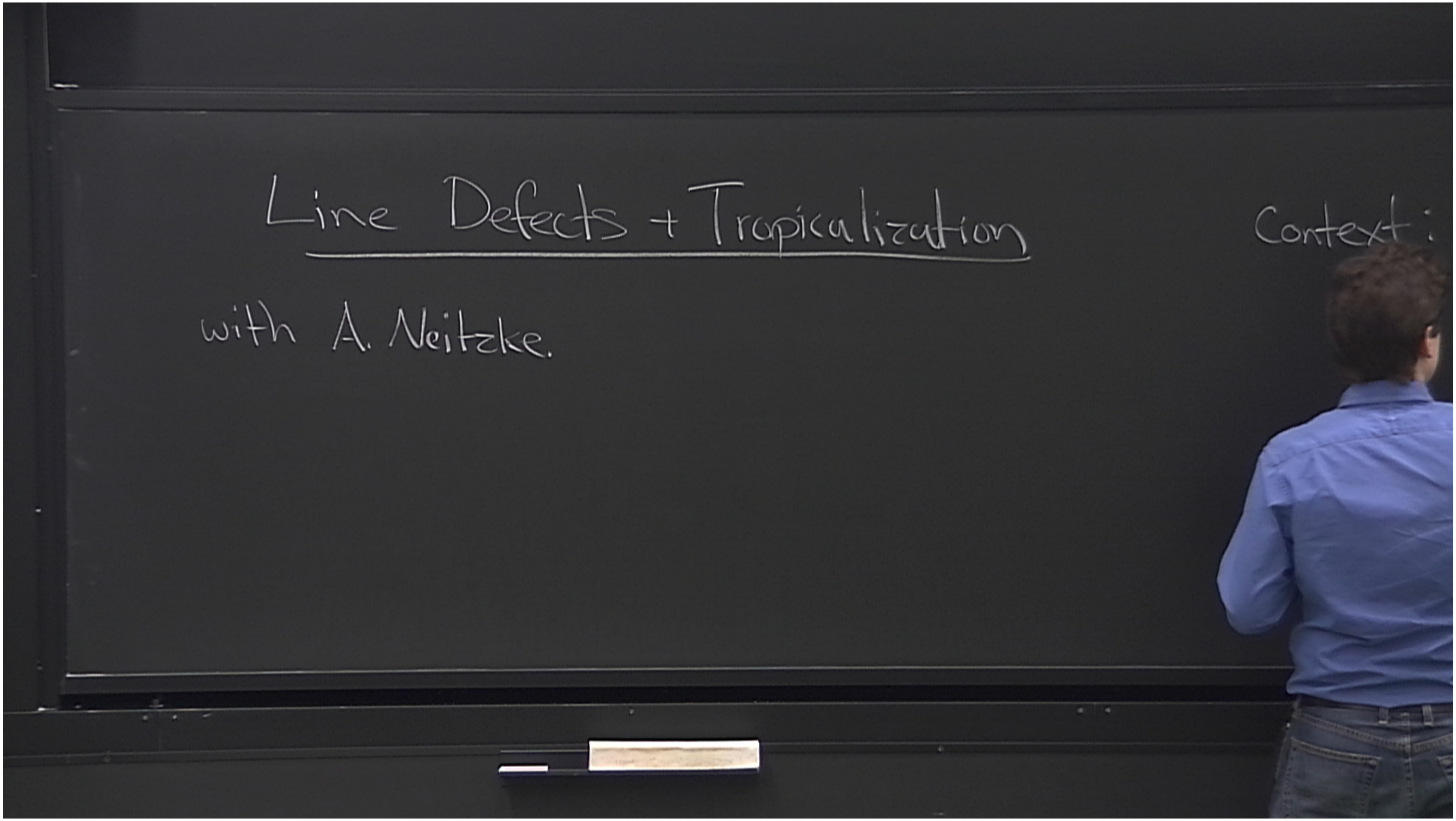
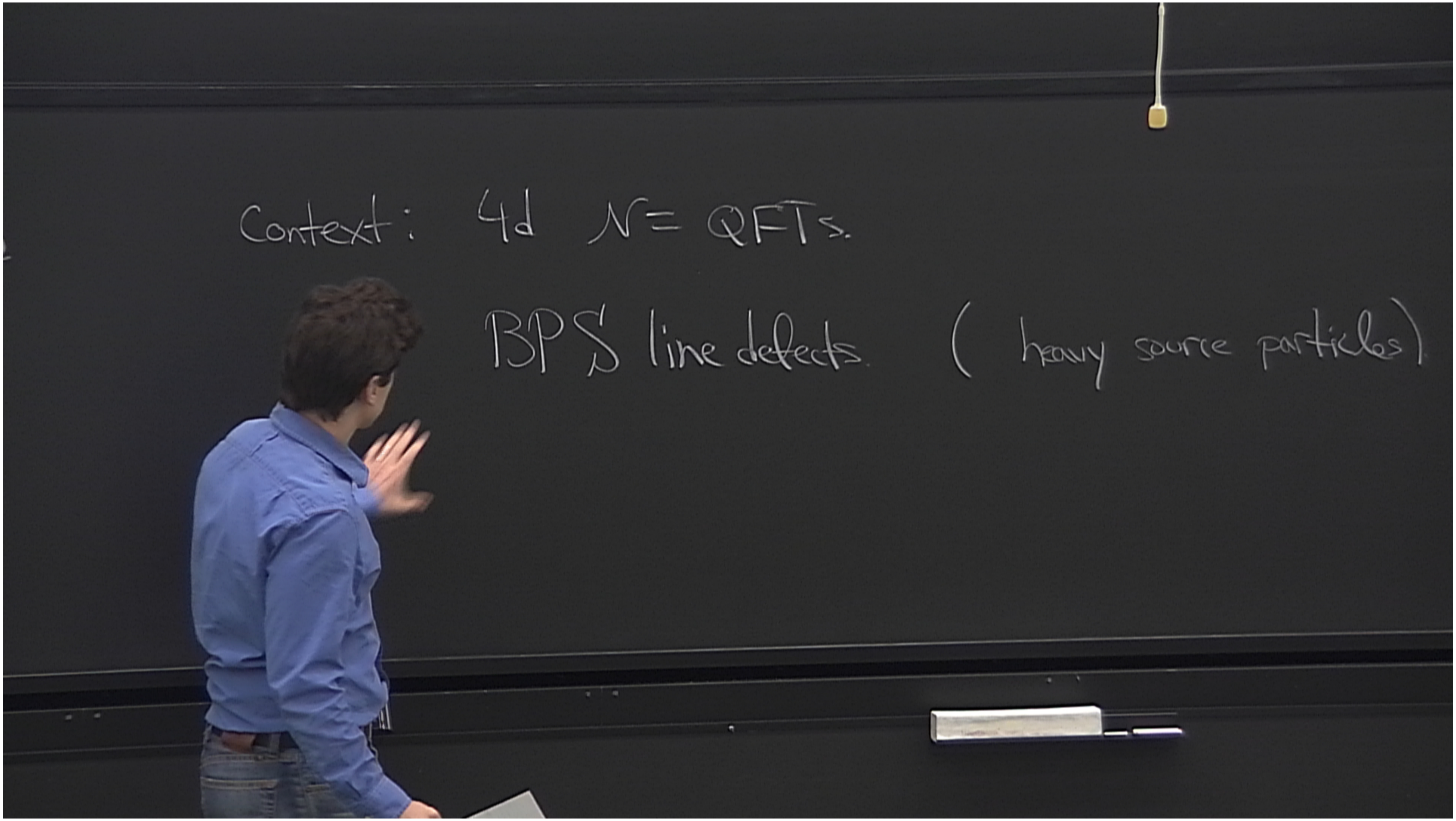
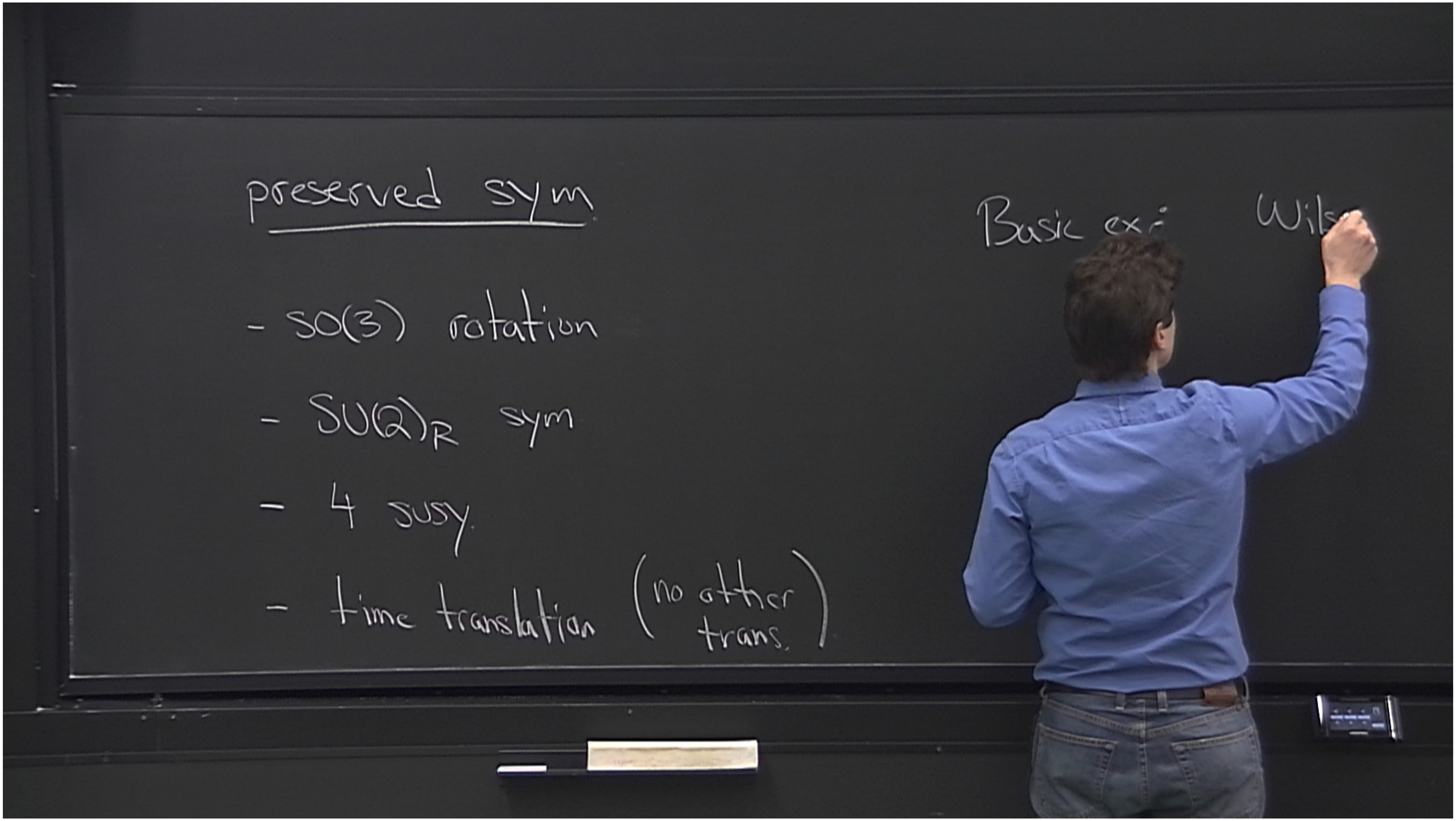
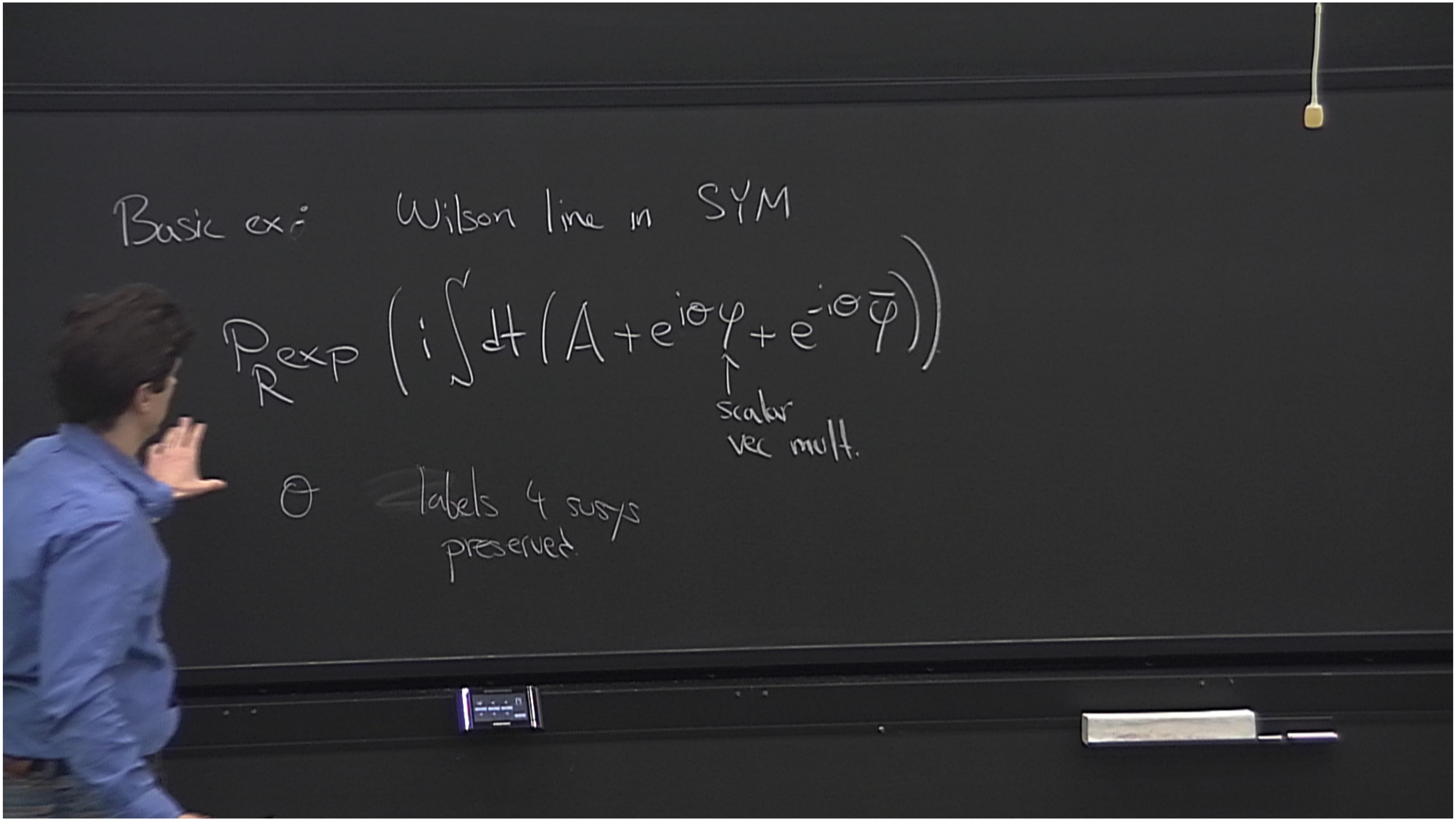
URL: http://pirsa.org/14030106

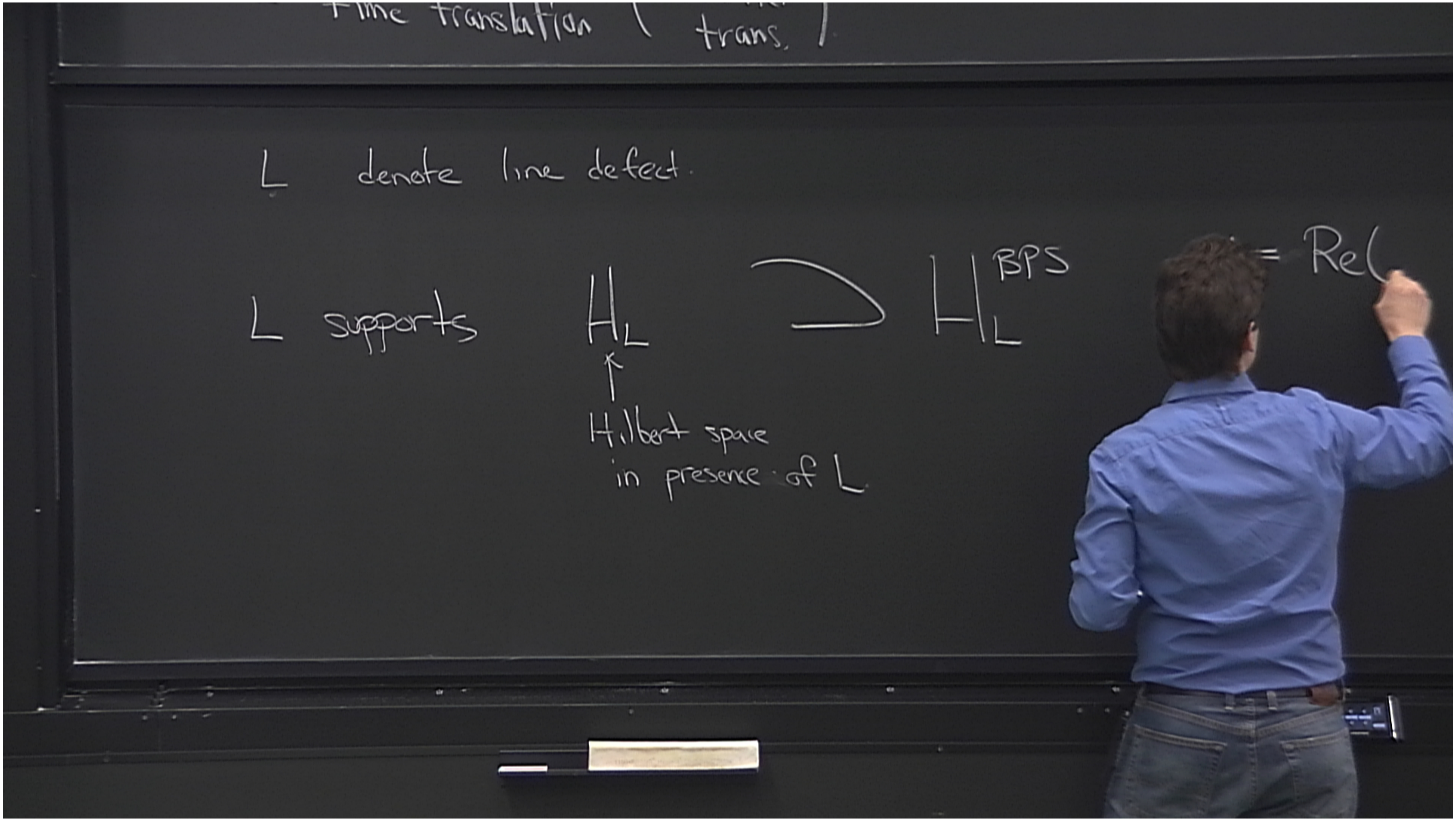
Abstract: I will review BPS line defects in 4d N=2 field theories and describe a technique for calculating their spectra using quiver quantum mechanics.


Line Defects + Tropicalization
Context:
with A. Neitzke.


Context: 4d N= QFTs.
BPS line defects. (heavy source particles)


preserved sym
- SO(3) rotation
- SU(2)$_R$ sym
- 4 susy
- time translation (no other trans.)
Basic ex:
Wils

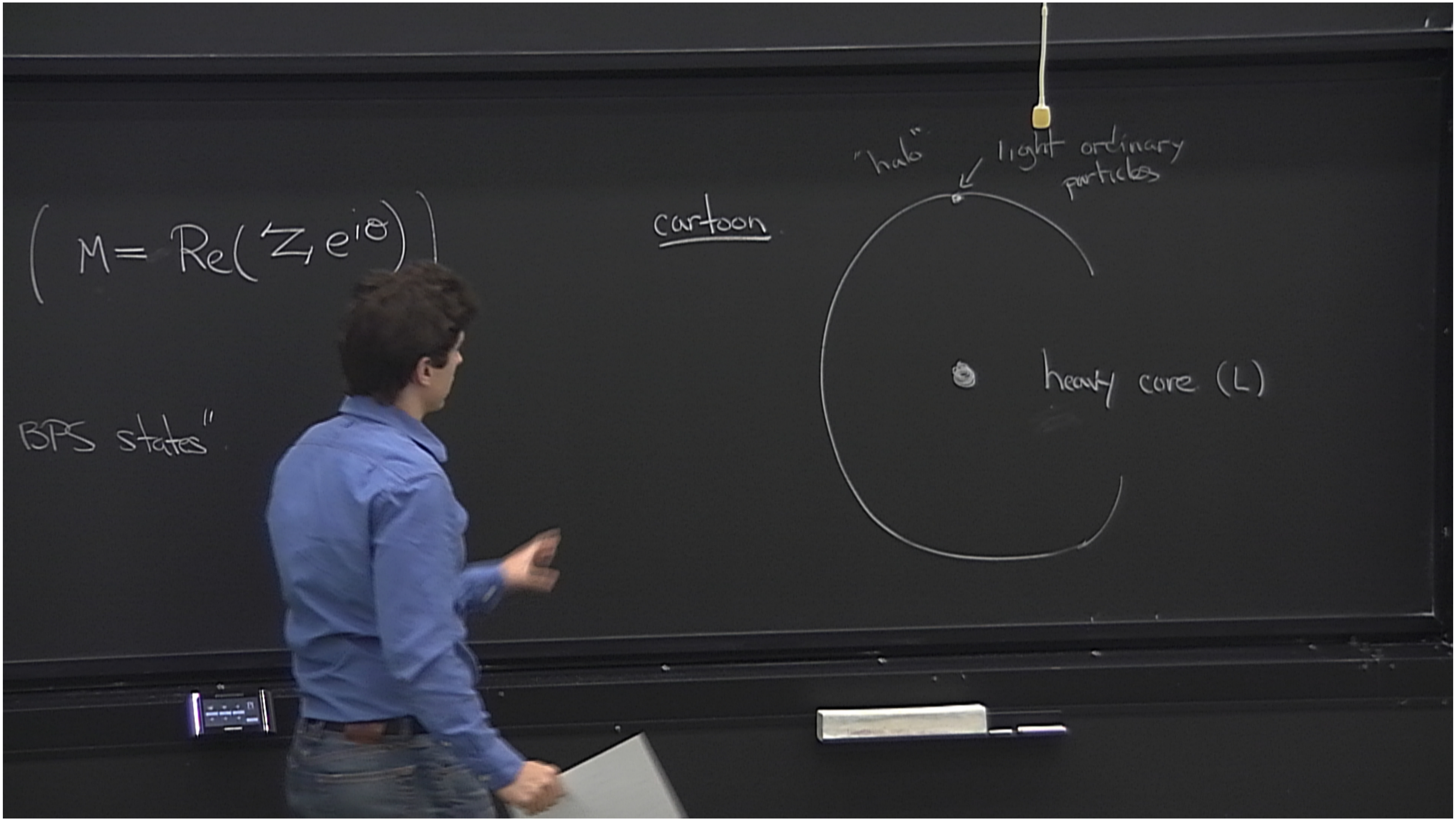

Basic ex:
Wilson line in SYM
$P_R \exp\left(i\int dt\left(A+e^{i\theta}\varphi+e^{-i\theta}\bar{\varphi}\right)\right)$
scalar
vec mult.
θ labels 4 susys
preserved.

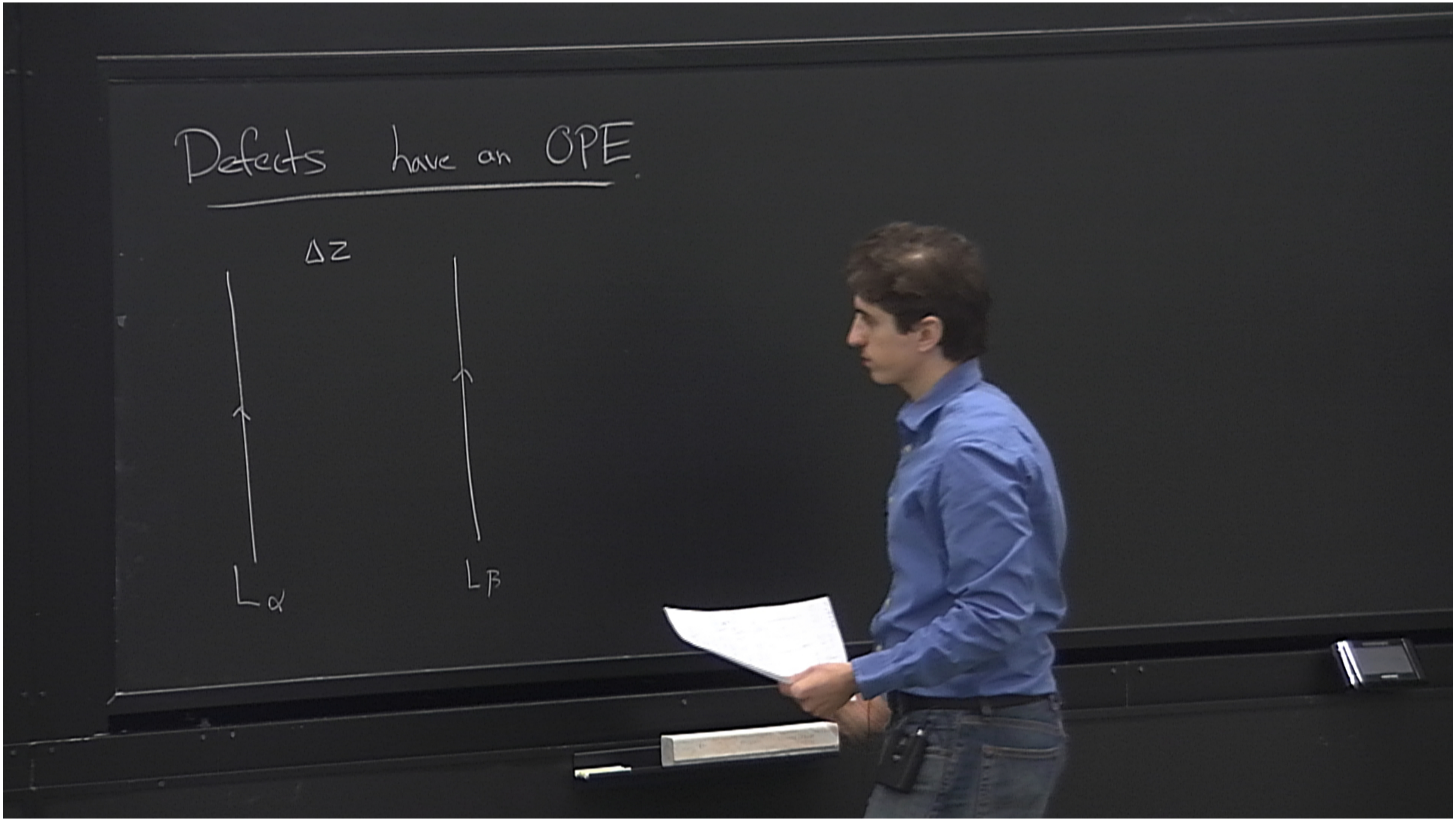

time translation
trans.
L denote line defect.
L supports $\mathcal{H}_L \supset \mathcal{H}_L^{BPS}$
= Re(
Hilbert space
in presence of L

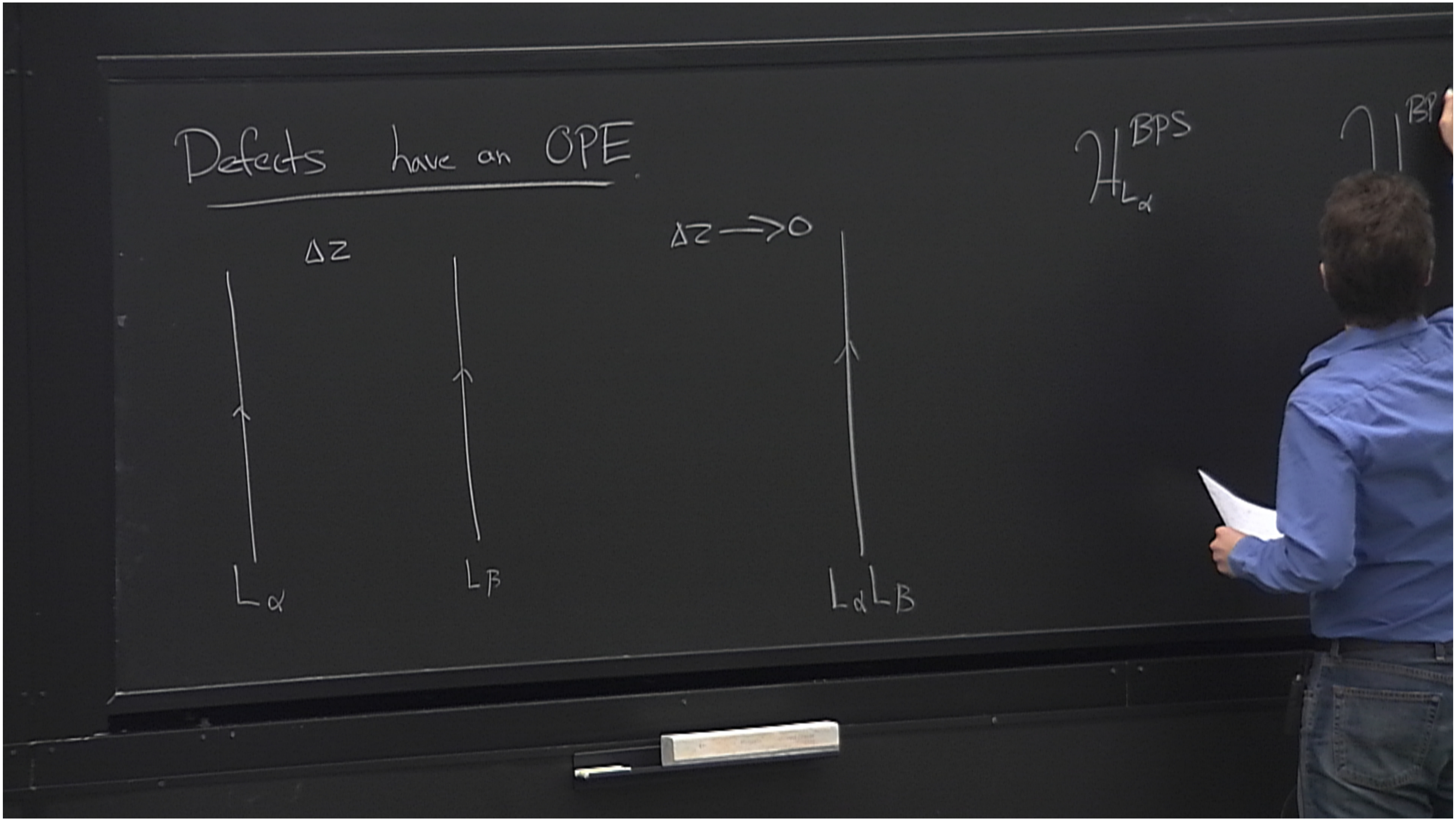
$$\mathcal{H}_L^{BPS} \quad \left(M = \mathrm{Re}\left(Z e^{i\theta} \right) \right)$$

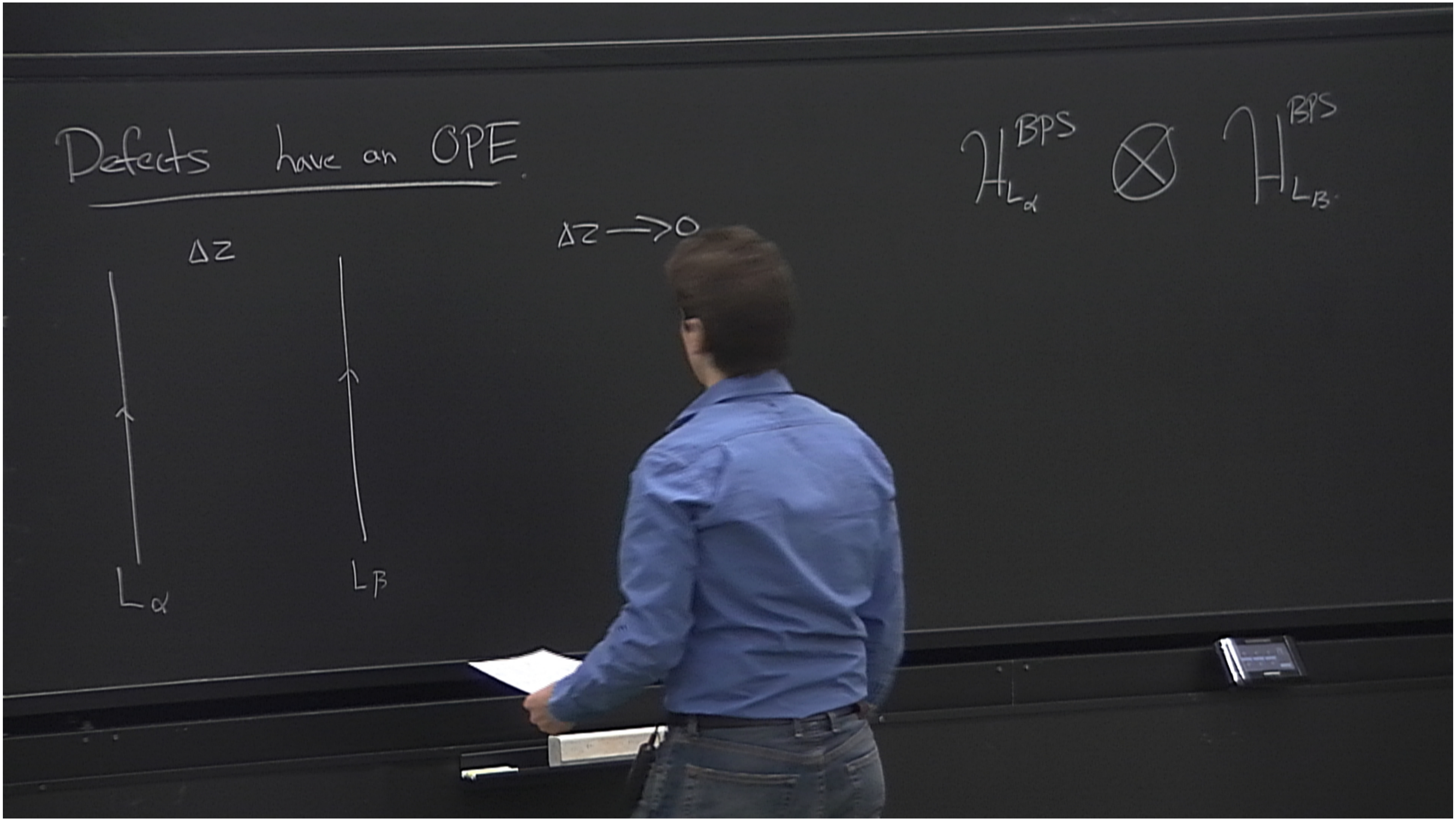
"framed BPS states"

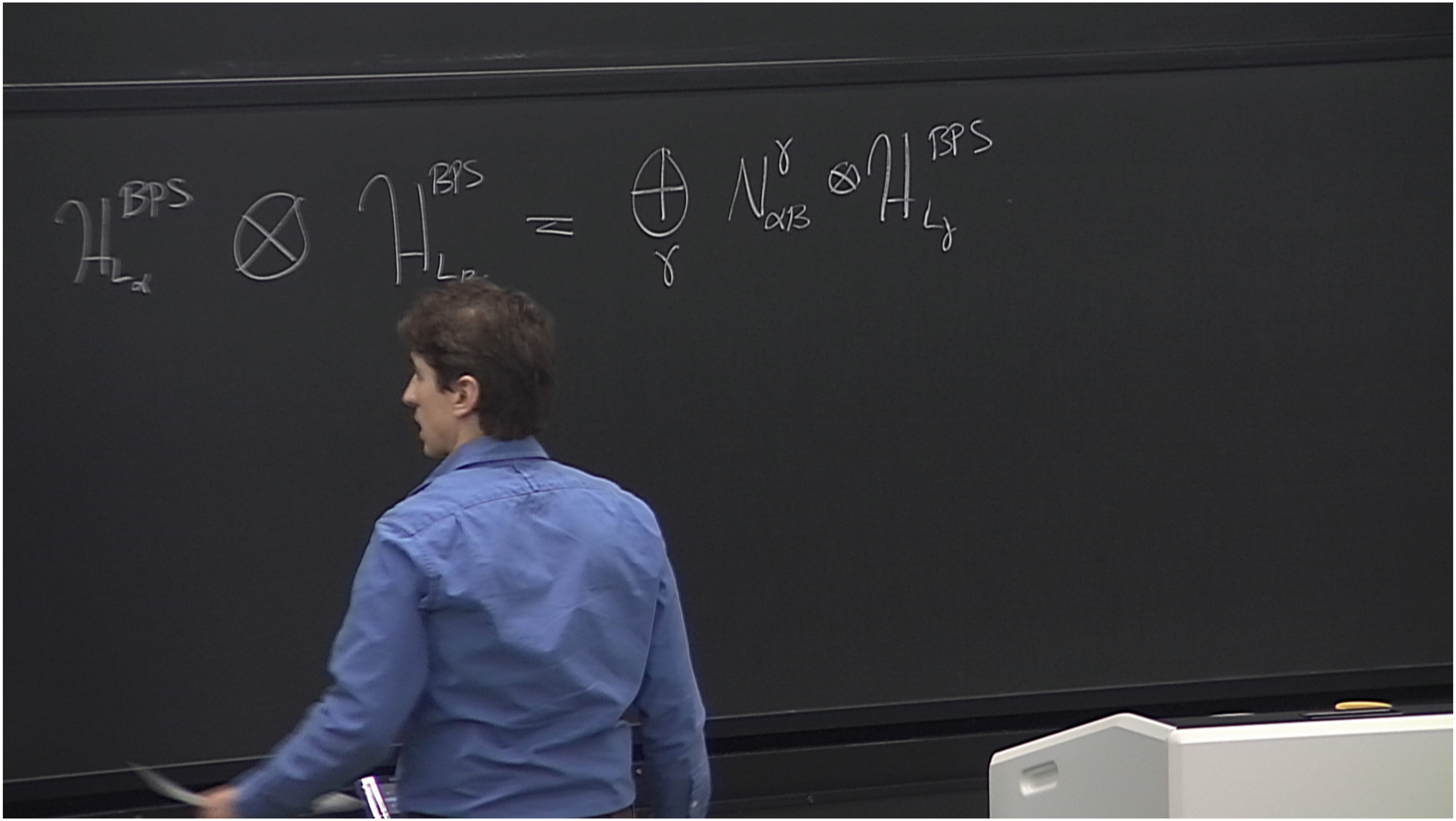
f L

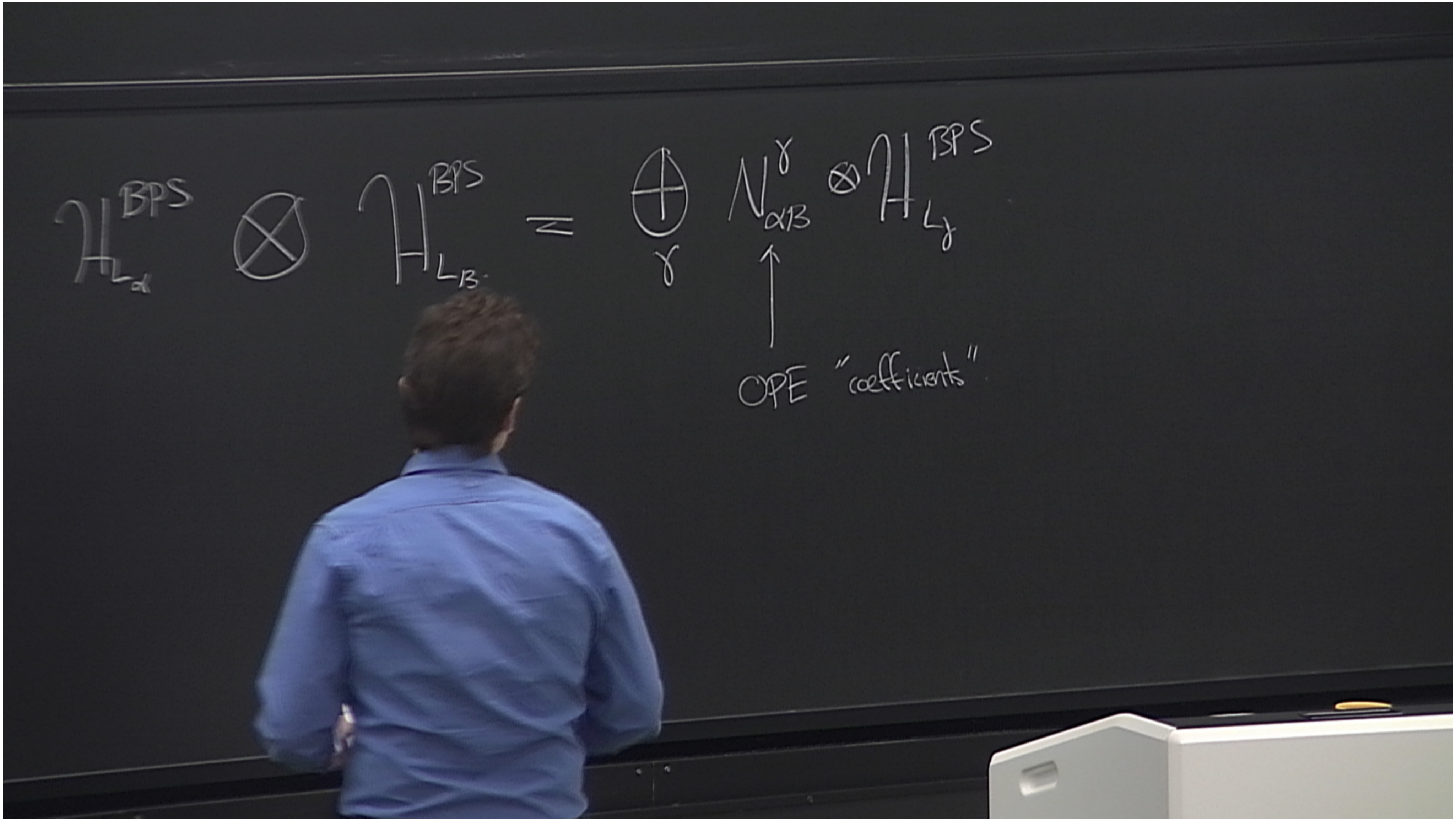

(M = Re(Z, e^{iθ}))
"BPS states"
cartoon
"halo"
light ordinary particles
heavy core (L)

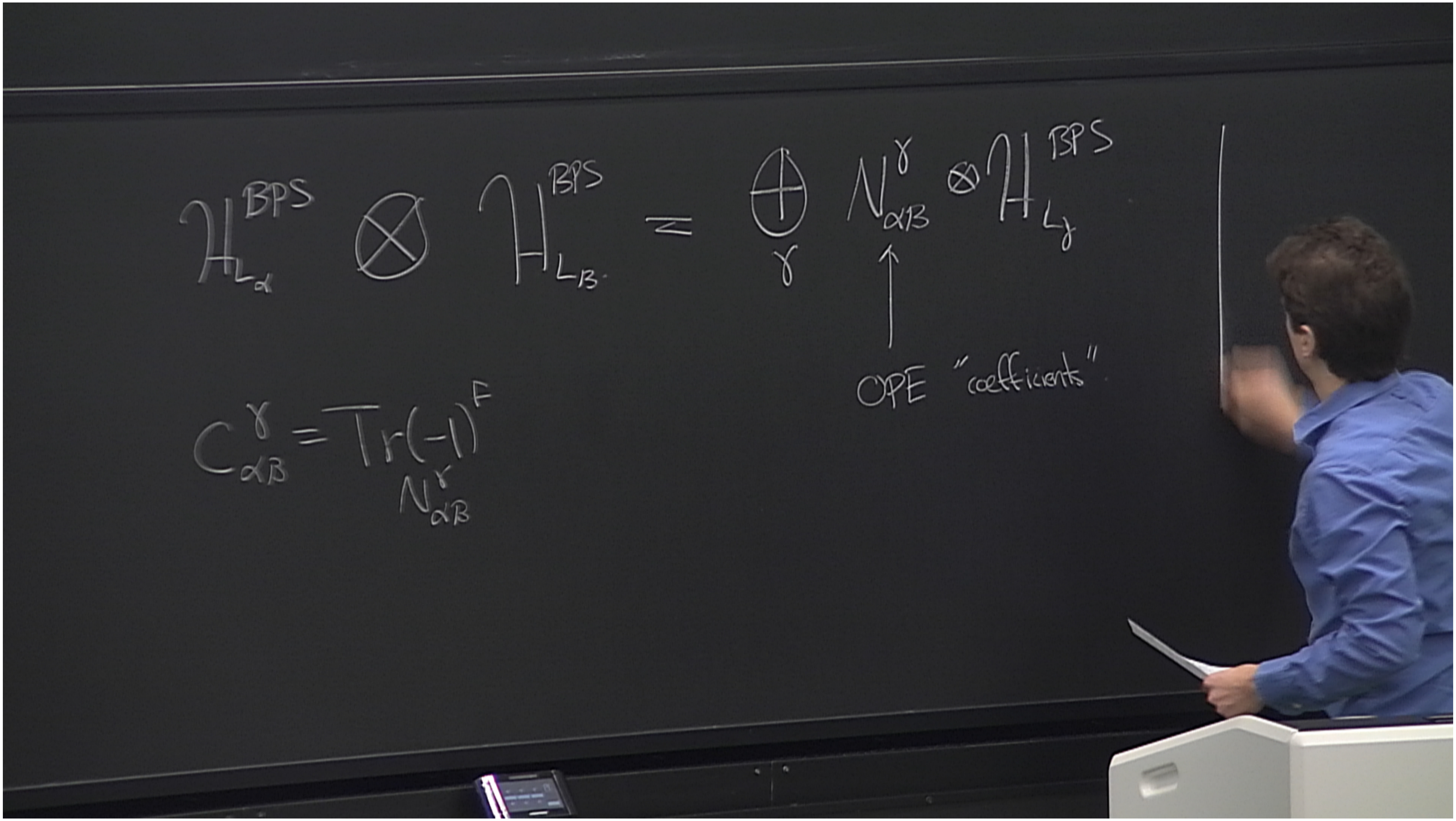

Defects have an OPE
ΔZ
L_α
L_β

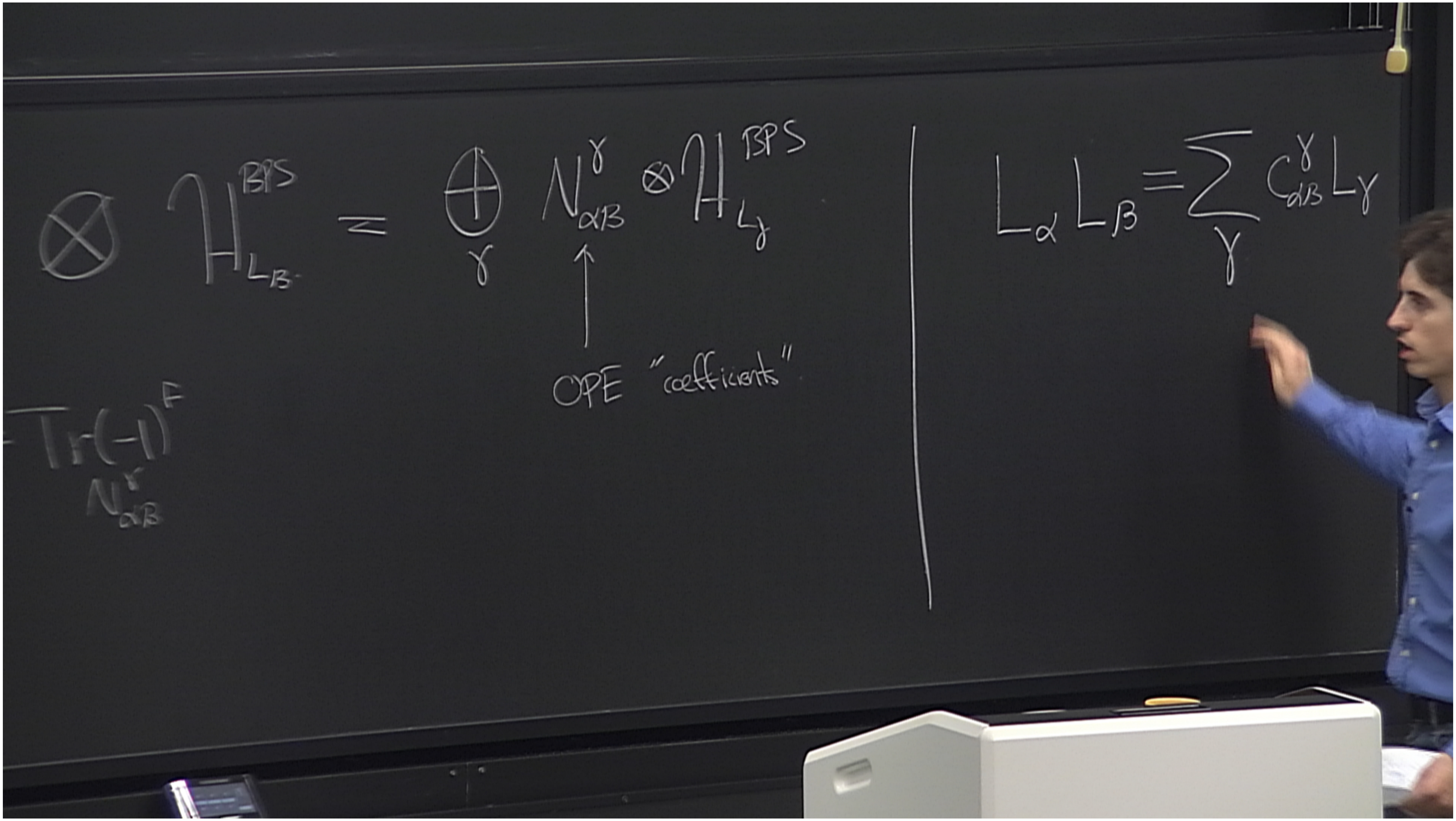

Defects have an OPE
ΔZ
ΔZ → 0
Lα
Lβ
LαLβ
H^BPS_Lα

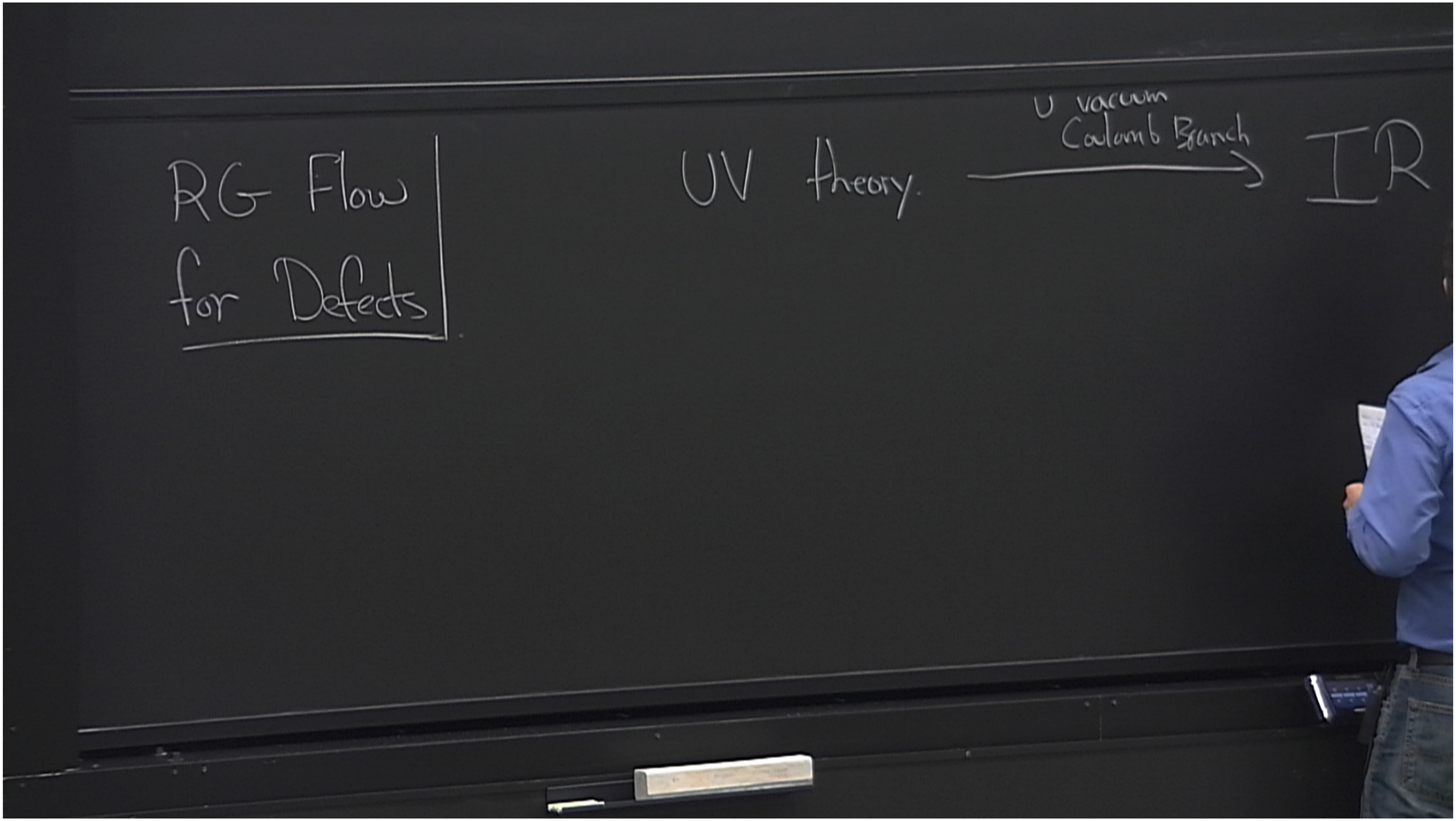

Defects have an OPE
$\mathcal{H}^{BPS}_{L_\alpha} \otimes \mathcal{H}^{BPS}_{L_\beta}$
$\Delta z \longrightarrow 0$
Δz
L_α
L_β

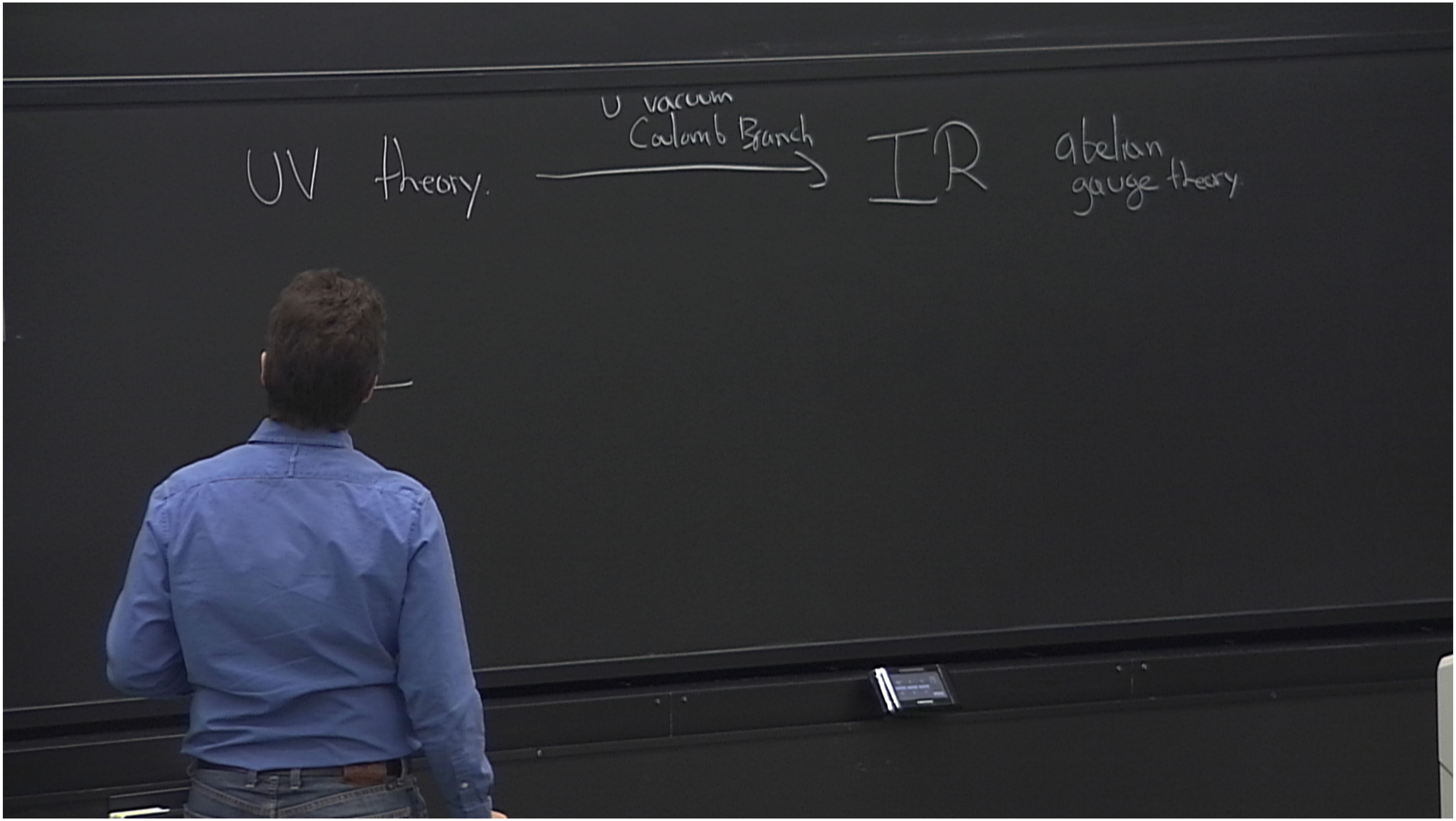

$\mathcal{H}^{BPS}_{L_\alpha} \otimes \mathcal{H}^{BPS}_{L_\beta} = \bigoplus_\gamma N^{\gamma}_{\alpha\beta} \otimes \mathcal{H}^{BPS}_{L_\gamma}$

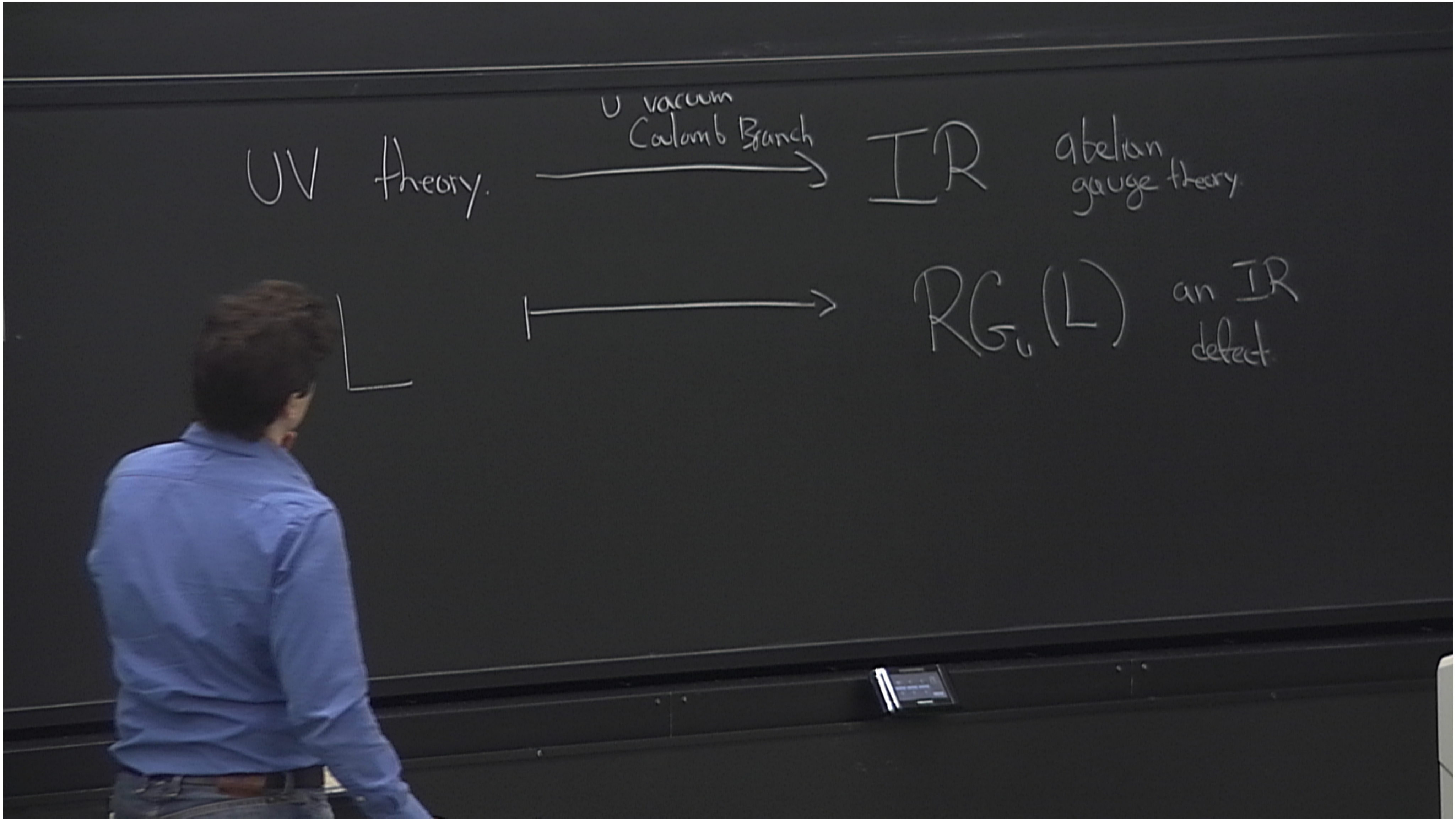

$\mathcal{H}^{BPS}_{L_\alpha} \otimes \mathcal{H}^{BPS}_{L_\beta} = \bigoplus_{\gamma} N^{\gamma}_{\alpha\beta} \otimes \mathcal{H}^{BPS}_{L_\gamma}$
OPE "coefficients"

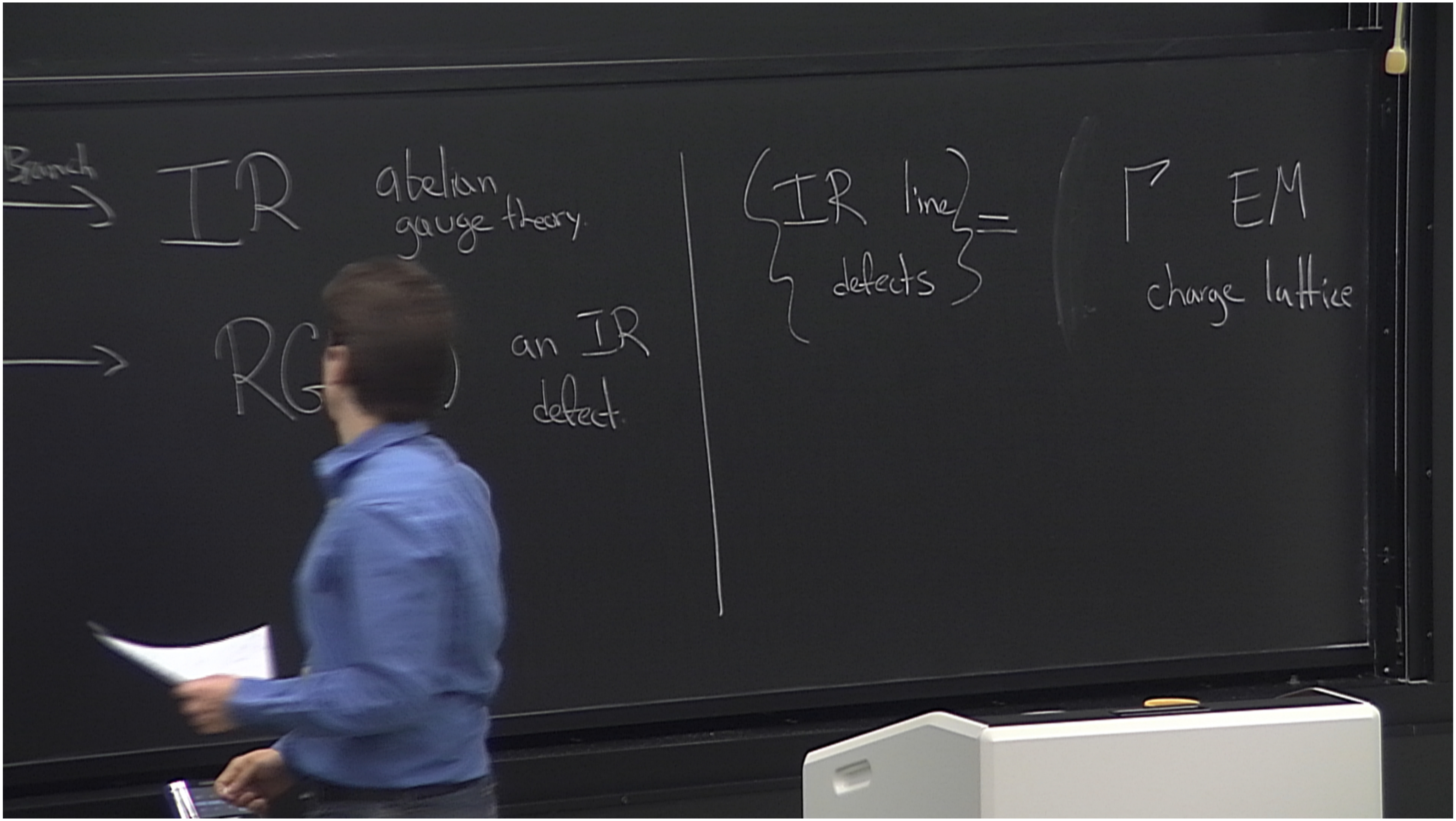

$\mathcal{H}^{BPS}_{L_\alpha} \otimes \mathcal{H}^{BPS}_{L_\beta} = \bigoplus_\gamma N^\gamma_{\alpha\beta} \otimes \mathcal{H}^{BPS}_{L_\gamma}$
OPE "coefficients"
$C^\gamma_{\alpha\beta} = \mathrm{Tr}_{N^\gamma_{\alpha\beta}} (-1)^F$

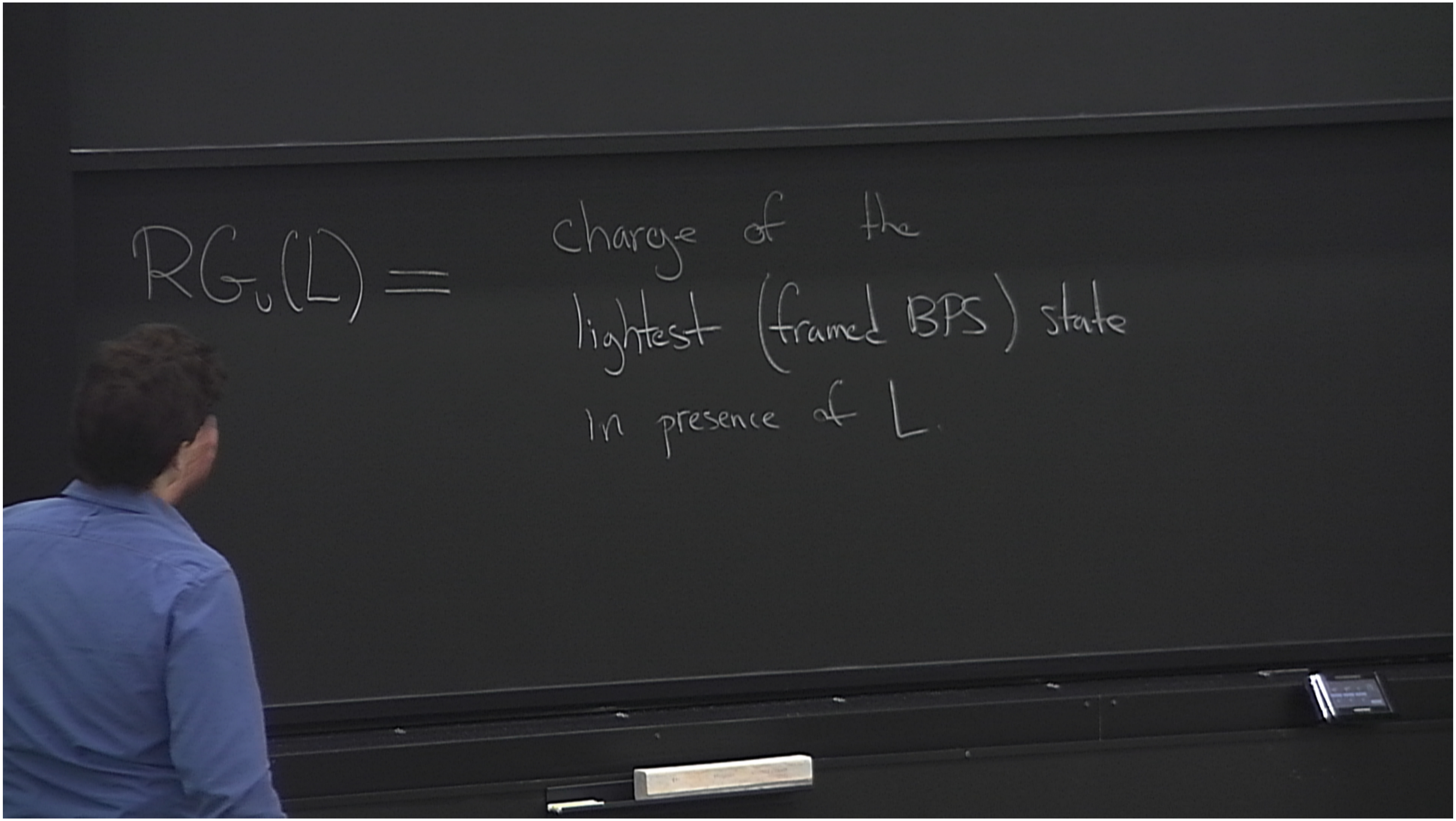

$\bigotimes \mathcal{H}^{BPS}_{L_\beta} = \bigoplus_\gamma N^\gamma_{\alpha\beta} \otimes \mathcal{H}^{BPS}_{L_\gamma}$
OPE "coefficients"
$Tr(-1)^F$
$N^\gamma_{\alpha\beta}$
$L_\alpha L_\beta = \sum_\gamma C^\gamma_{\alpha\beta} L_\gamma$

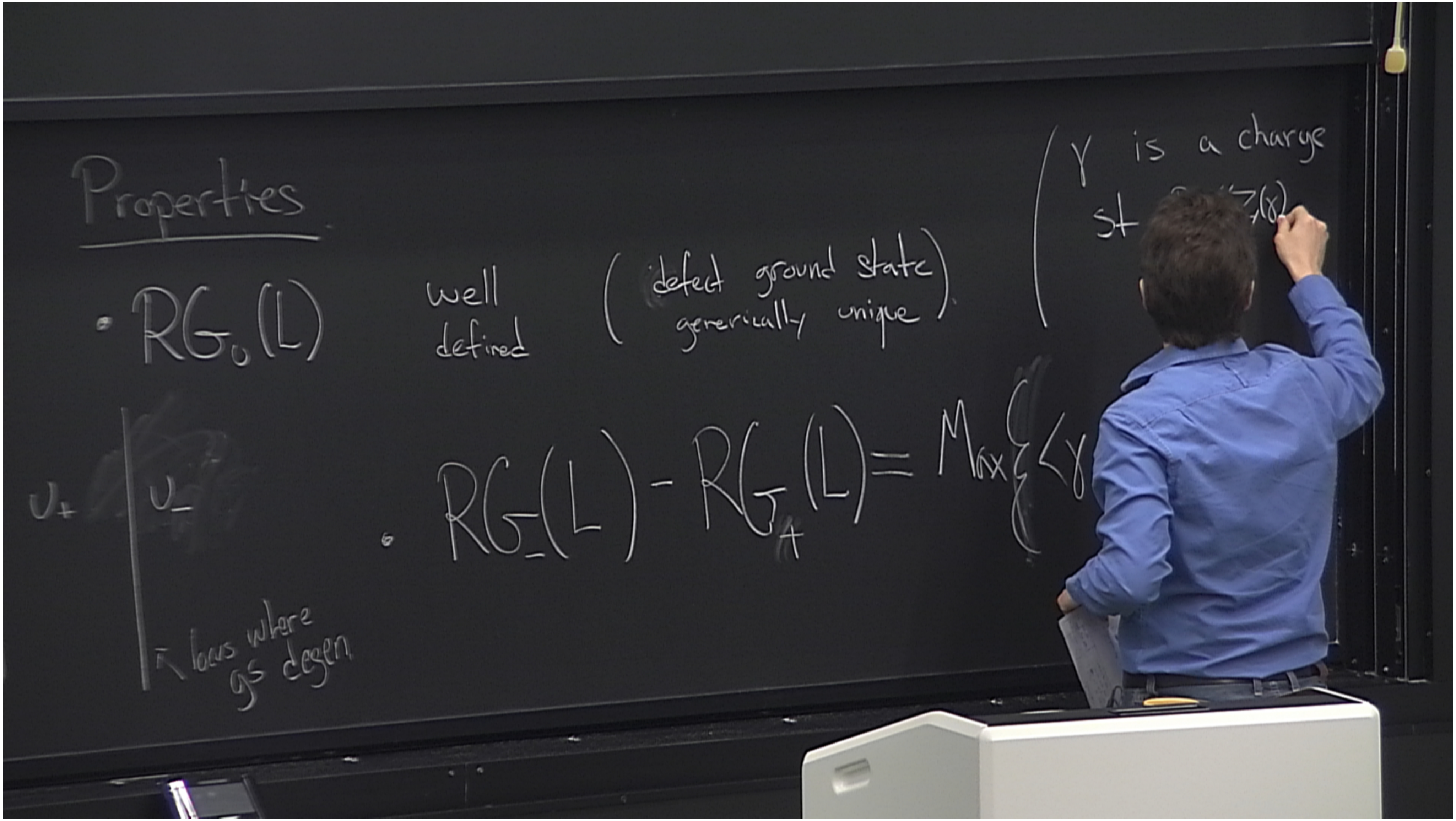

RG Flow
for Defects
UV theory. $\xrightarrow[\text{Coulomb Branch}]{\text{U vacuum}}$ IR

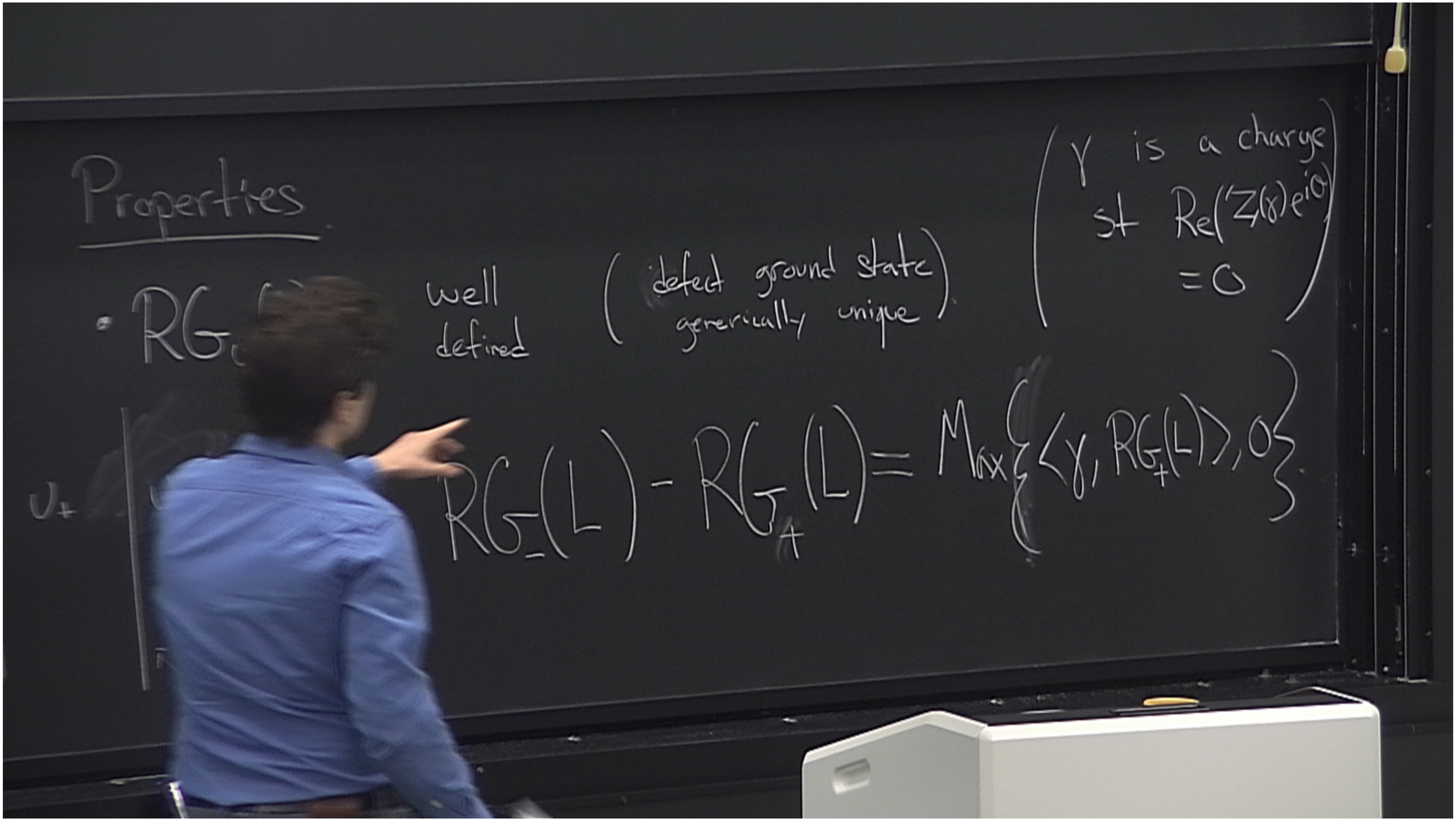

U vacuum
Coulomb Branch
UV theory.
IR
abelian
gauge theory.


u vacuum
Coulomb Branch
UV theory.
IR
abelian gauge theory.
L
RG_u(L)
an IR defect


Branch
IR
abelian gauge theory
RG
an IR defect
IR line defects =
Γ
EM charge lattice


$RG_{\vartheta}(L) =$
charge of the
lightest (framed BPS) state
in presence of L.


Properties
• RG0(L)
well defined
(defect ground state generically unique)
γ is a charge
st
• RG−(L) − RG+(L) = Max{⟨γ
u+
u−
locus where gs degen


Properties
• RG
well
defined
(defect ground state
generically unique)
(γ is a charge
st Re(Z(γ)e^{iθ})
= 0)
RG_−(L) − RG_+(L) = Max{⟨γ, RG_+(L)⟩, 0}.

Properties

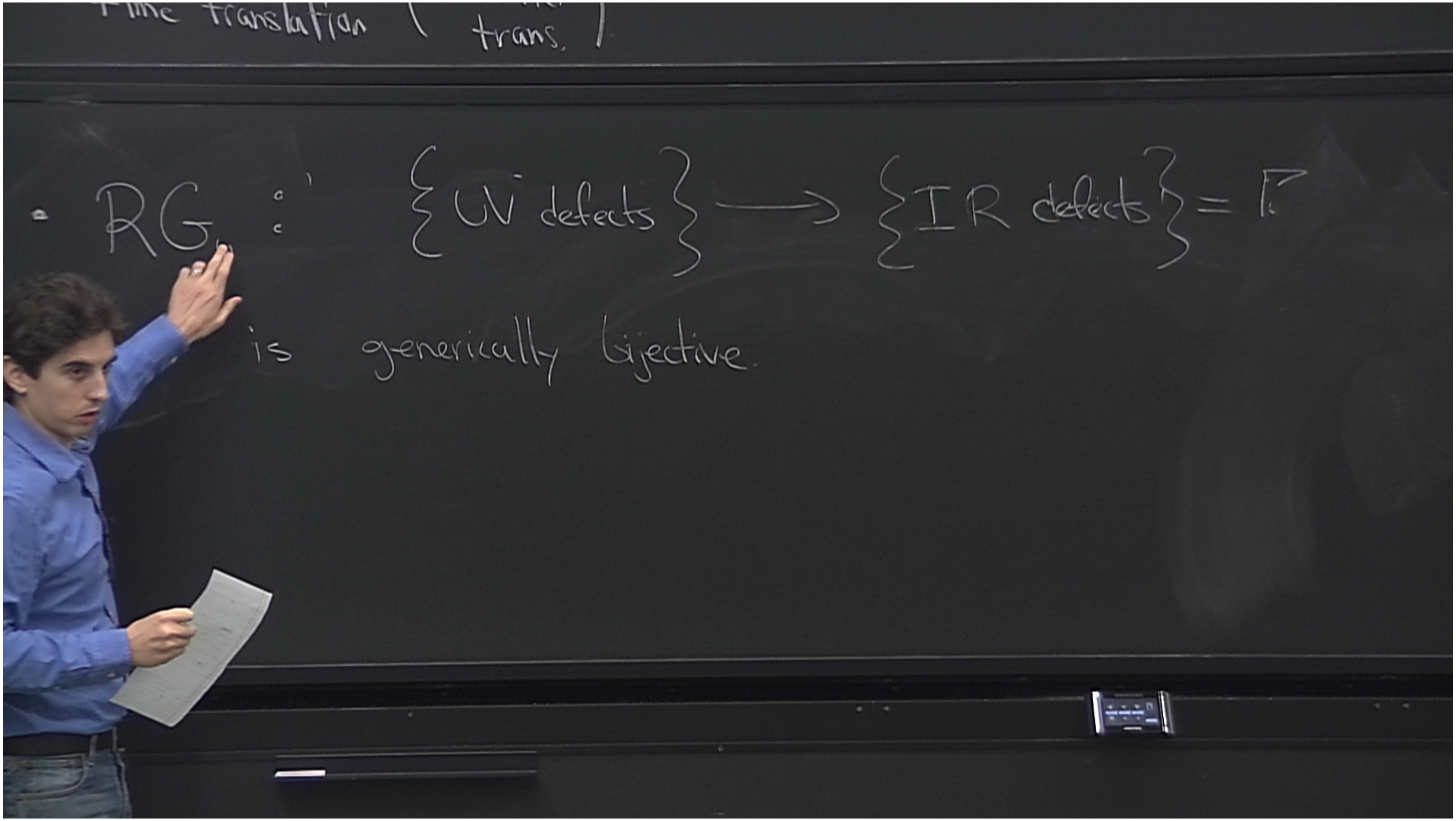
- $RG_0(L)$ well defined (defect ground state generically unique).

(γ is a charge st $\mathrm{Re}(Z(\gamma)e^{i\theta}) = 0$)

U_+ | U_-

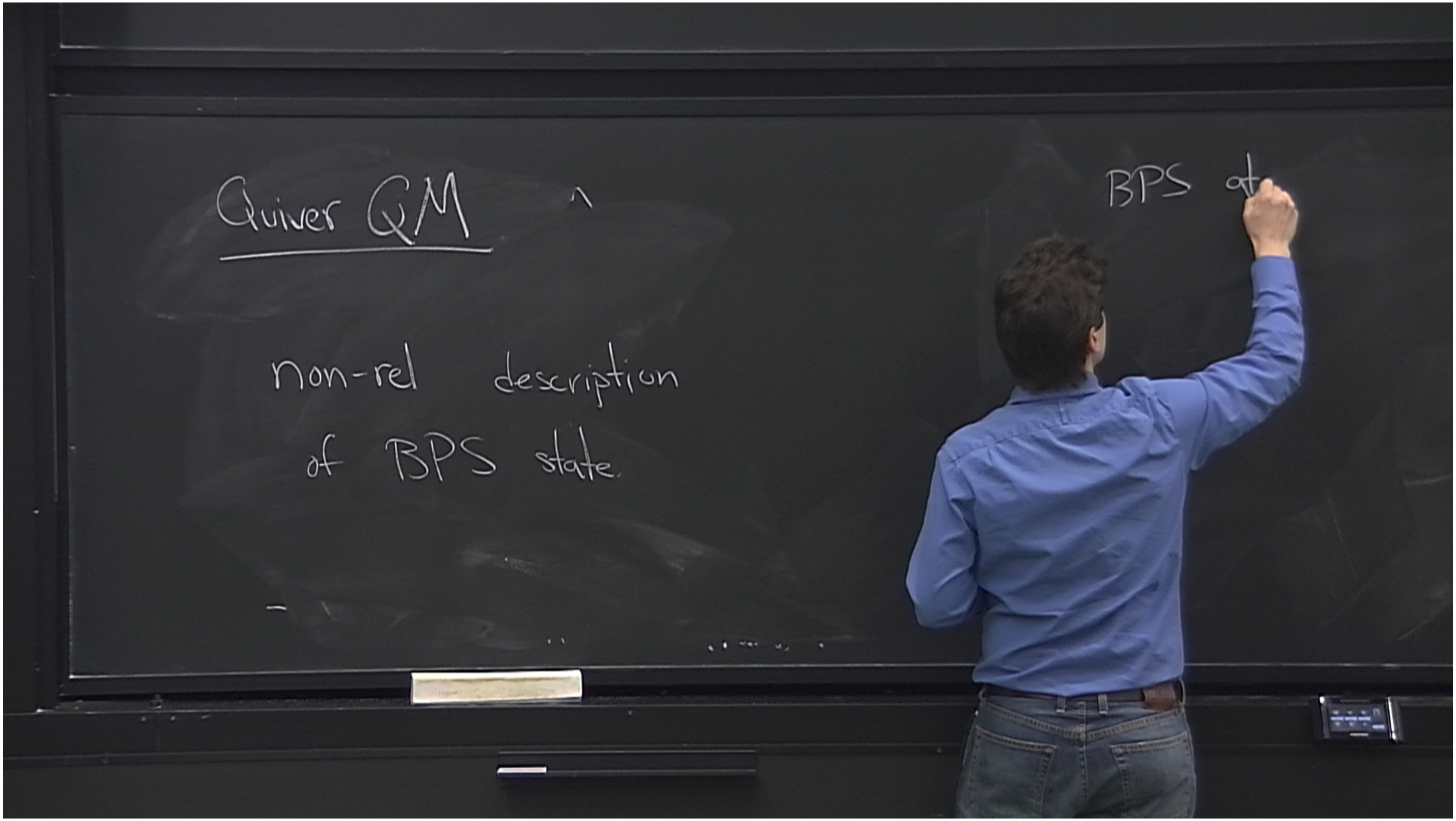
locus where gs degen

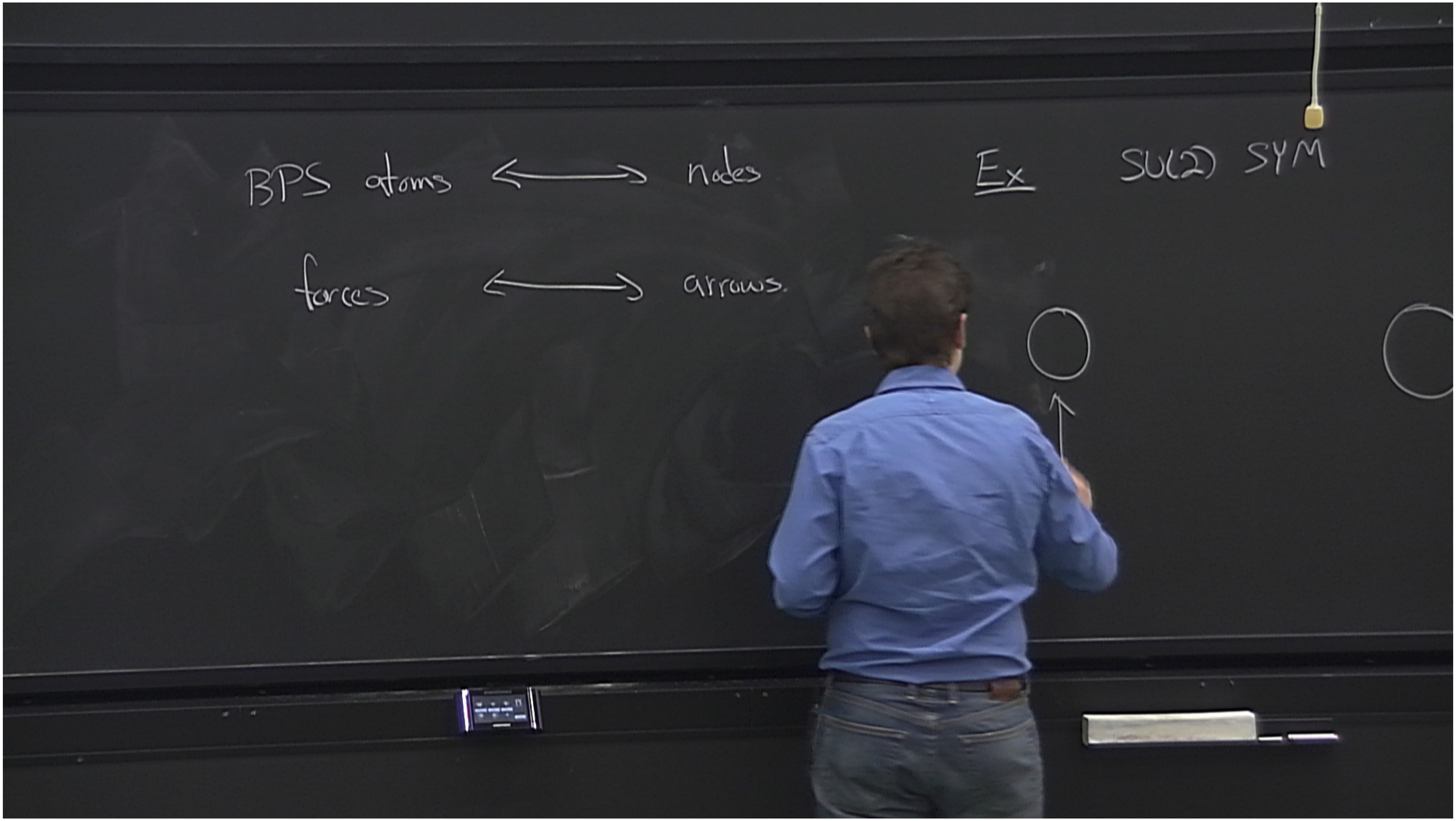
- $RG_-(L) - RG_+(L) = \mathrm{Max}\{\langle \gamma, RG_+(L)\rangle, 0\}\gamma$

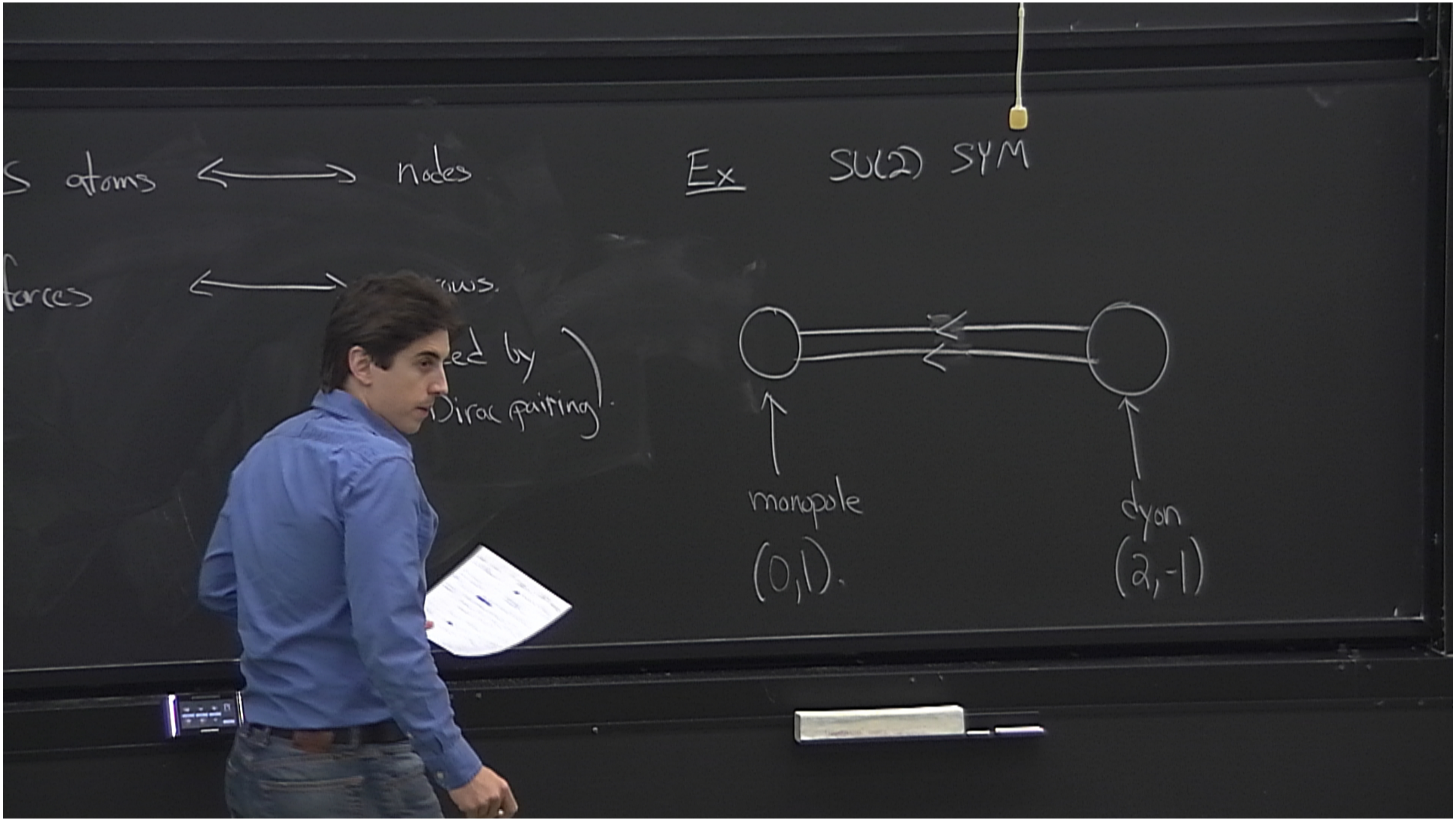

trans.
RG :
{UV defects} ⟶ {IR defects} =
is generically bijective

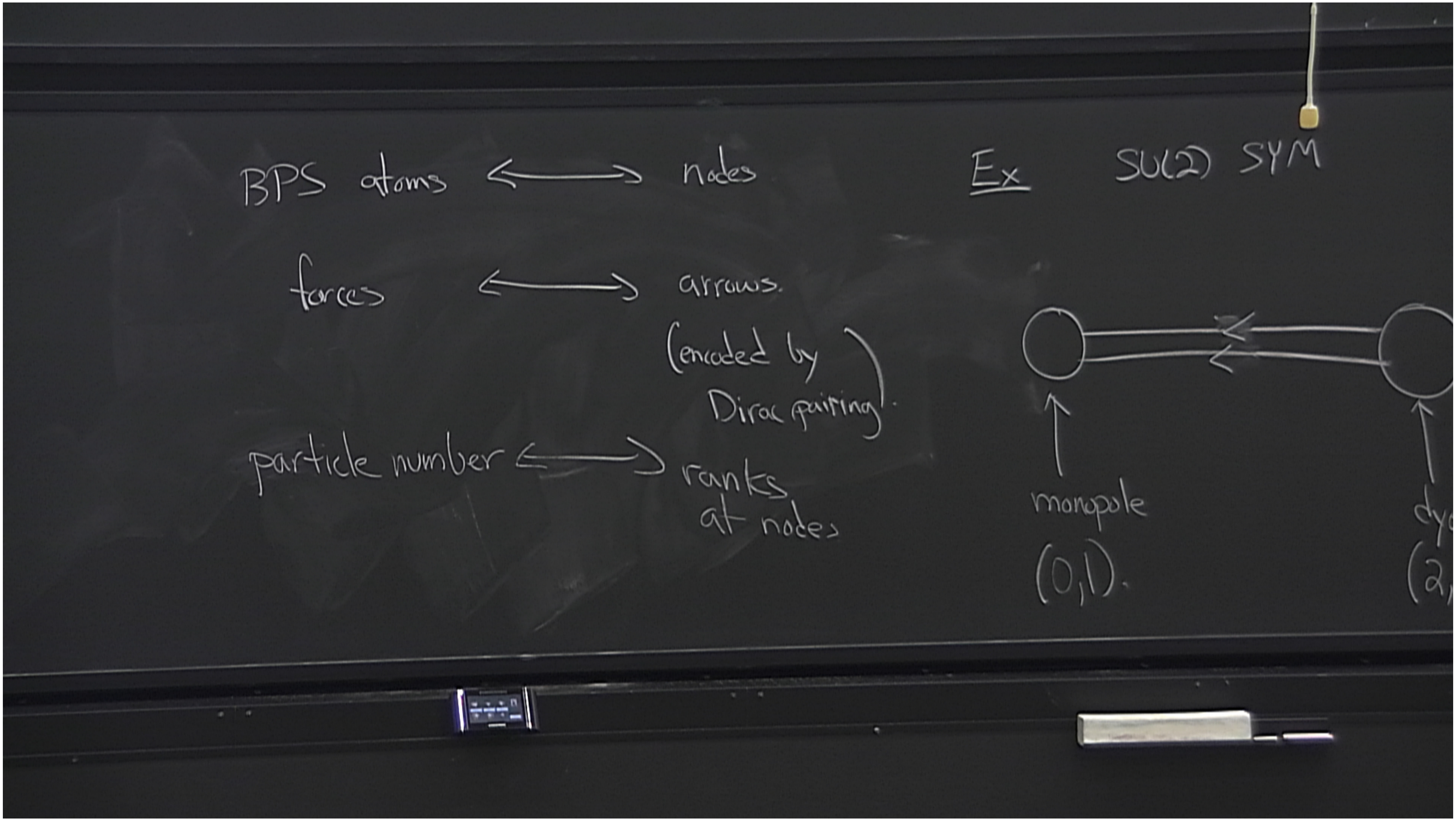
IR defects $\} = \Gamma$

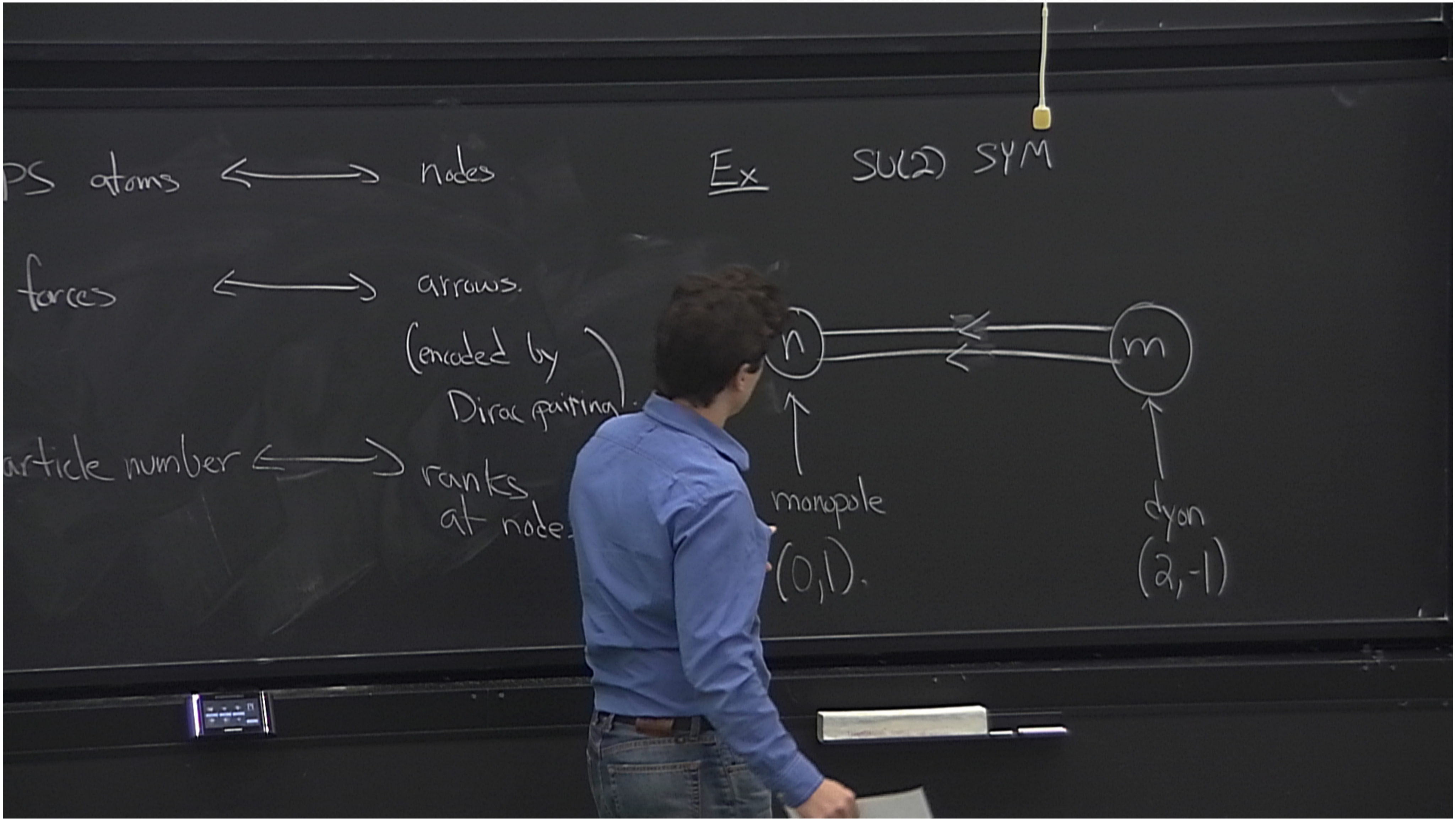
(true in theories defined by 2 M5-branes on Σ)

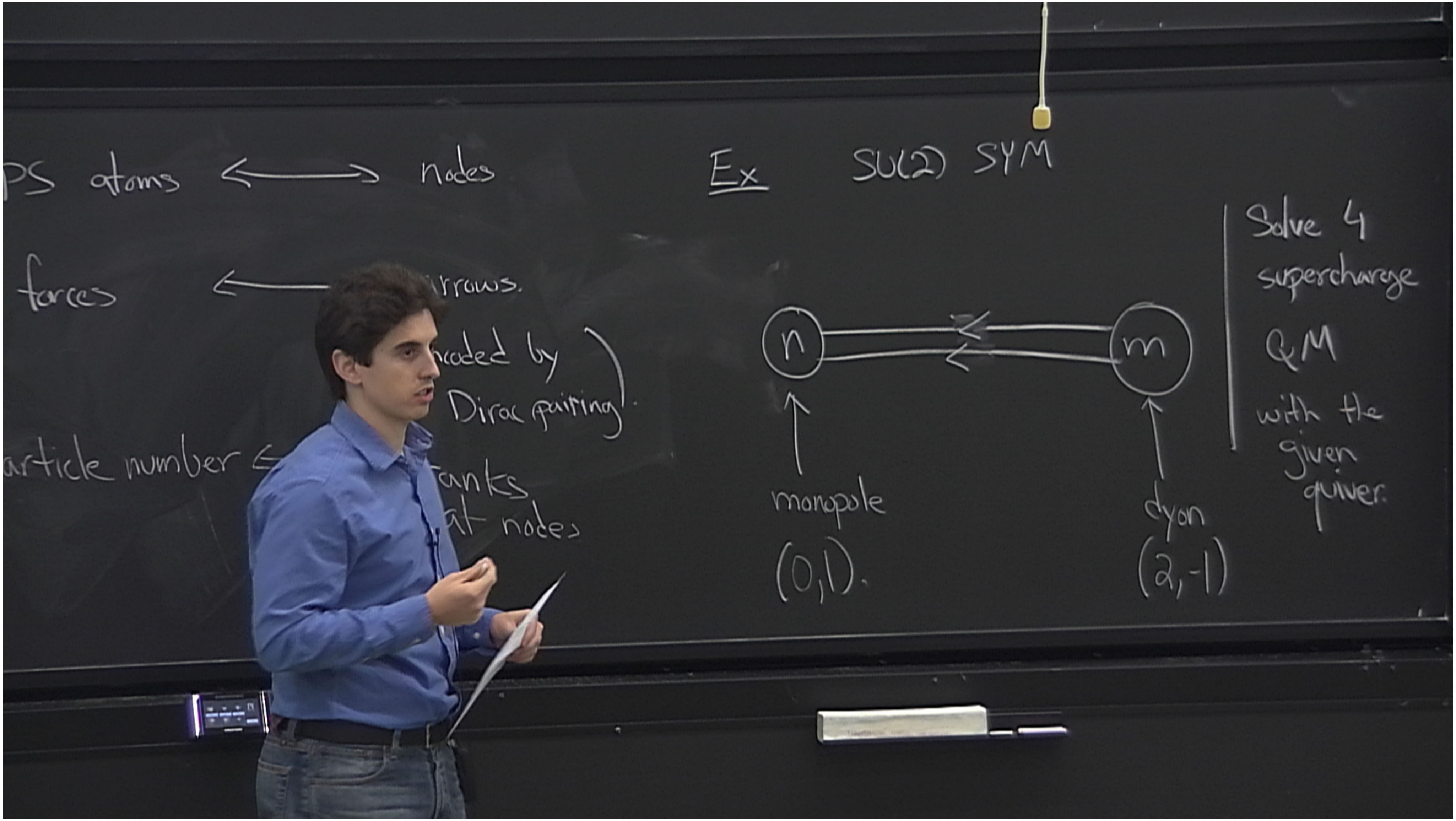

Quiver QM
non-rel description
of BPS state
BPS

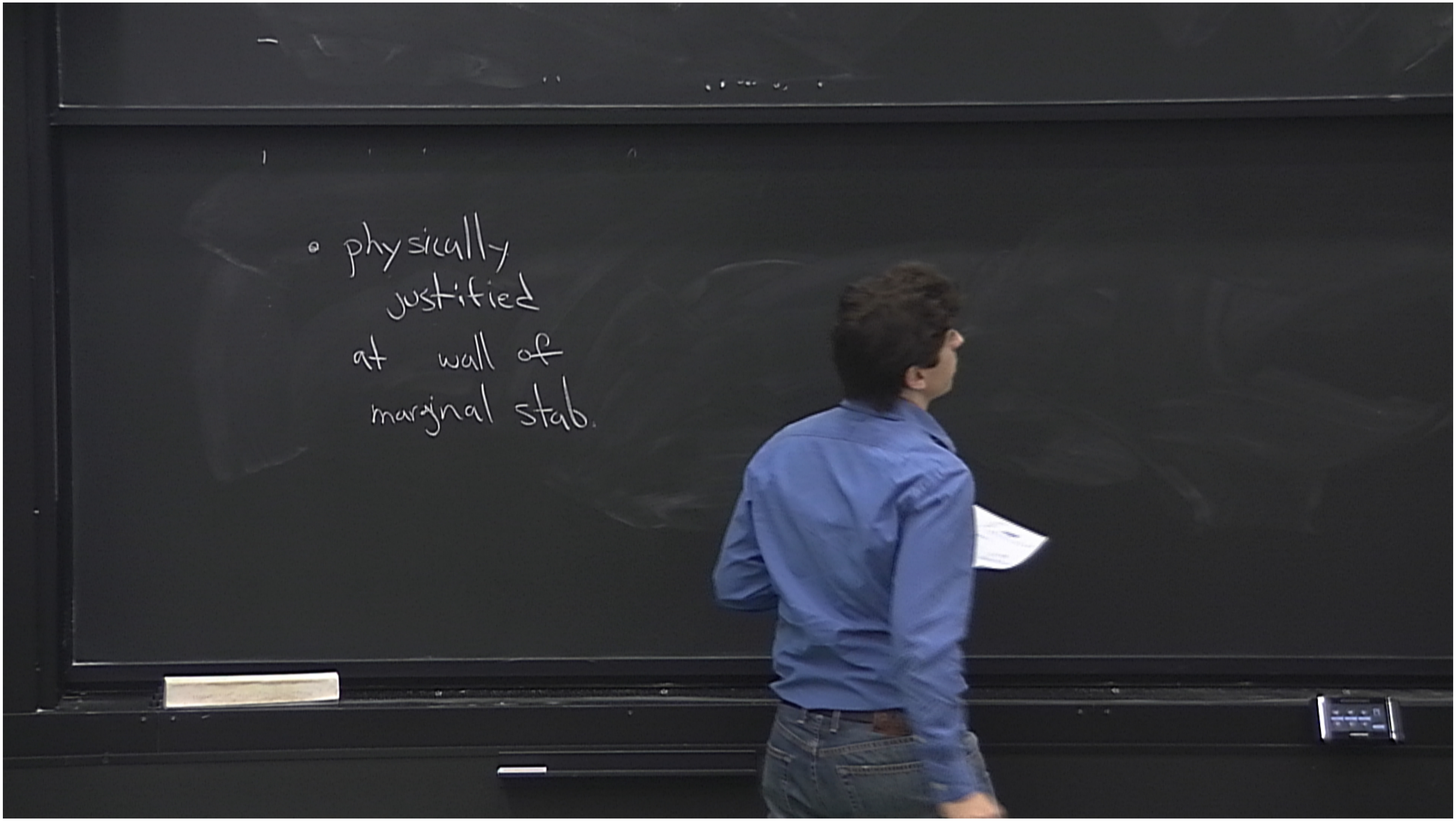

BPS atoms ⟷ nodes
forces ⟷ arrows.
Ex SU(2) SYM

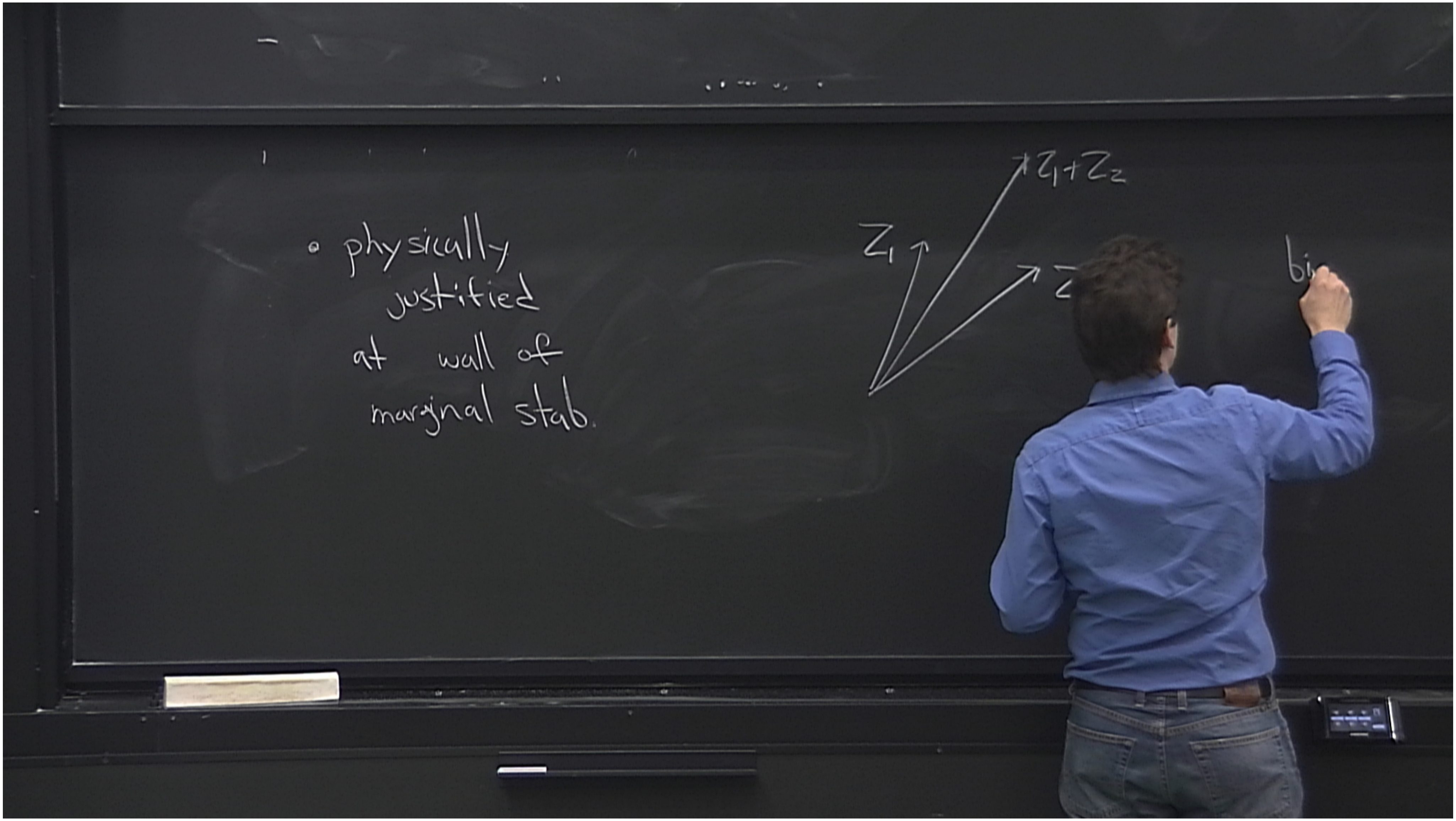

atoms ⟷ nodes
forces ⟷ arrows.
ed by
Dirac pairing
Ex SU(2) SYM
monopole
(0,1).
dyon
(2,-1)

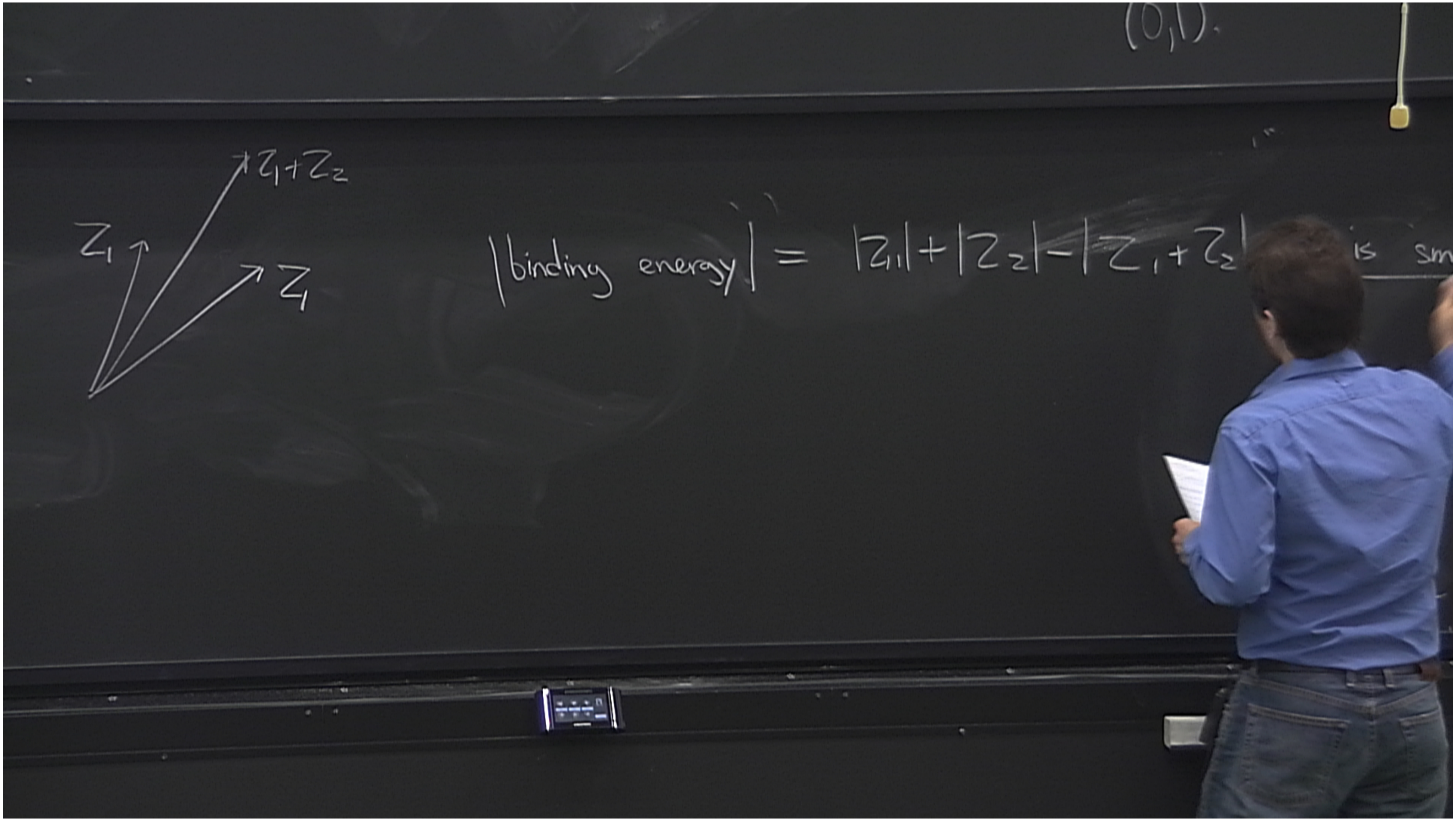

BPS atoms ⟷ nodes
forces ⟷ arrows.
(encoded by Dirac pairing).
particle number ⟷ ranks at nodes
Ex SU(2) SYM
monopole
(0,1).
dy
(2,


PS atoms
nodes
Ex
SU(2) SYM
forces
arrows.
(encoded by
Dirac pairing).
article number
ranks
at node
n
m
monopole
(0,1).
dyon
(2,-1)


PS atoms ⟷ nodes
forces ⟵ arrows.
coded by
Dirac pairing
article number
anks
at nodes
Ex SU(2) SYM
n
m
monopole
(0,1).
dyon
(2,-1)
Solve 4
supercharge
QM
with the
given
quiver.

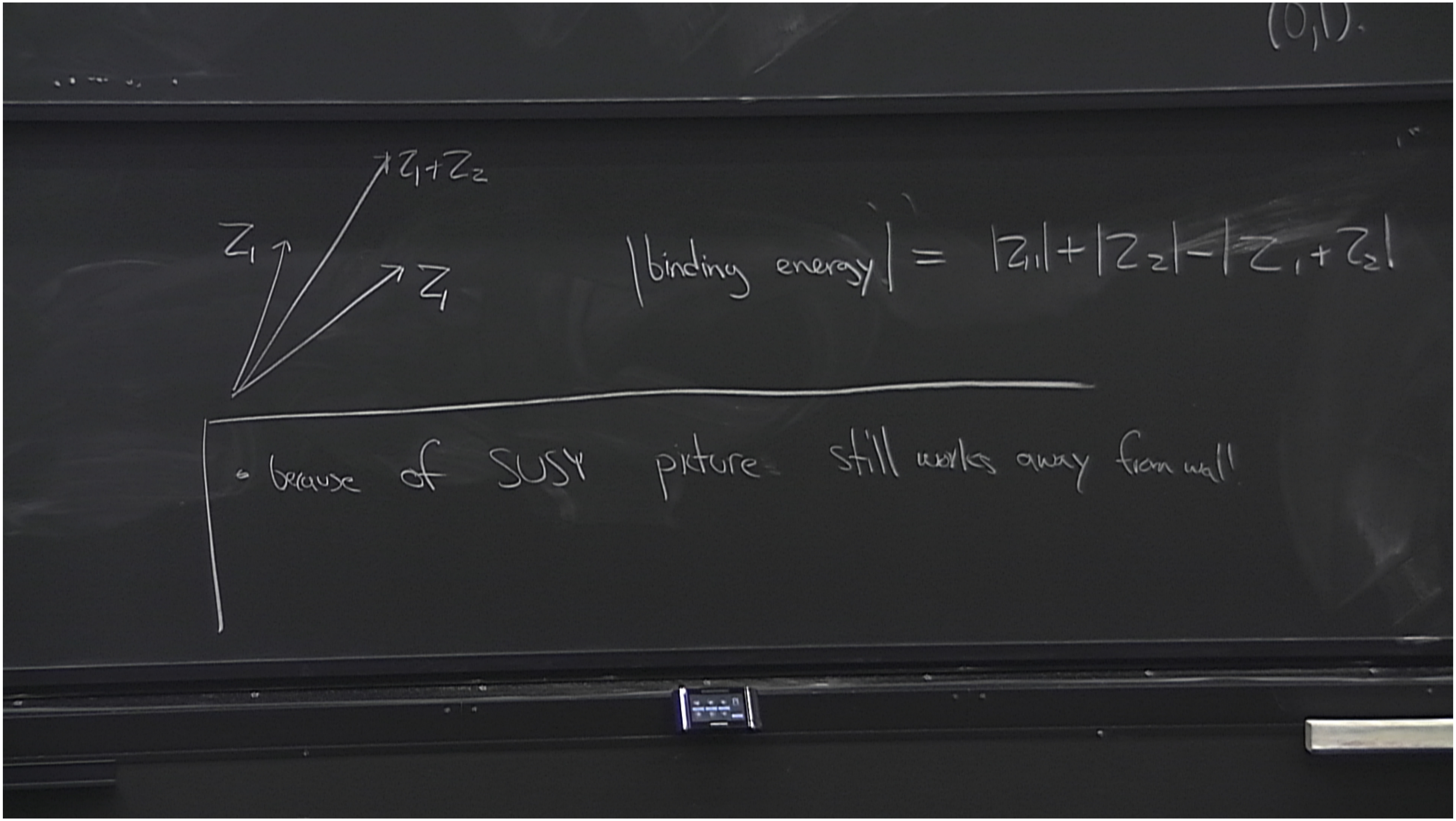

physically
justified
at wall of
marginal stab.

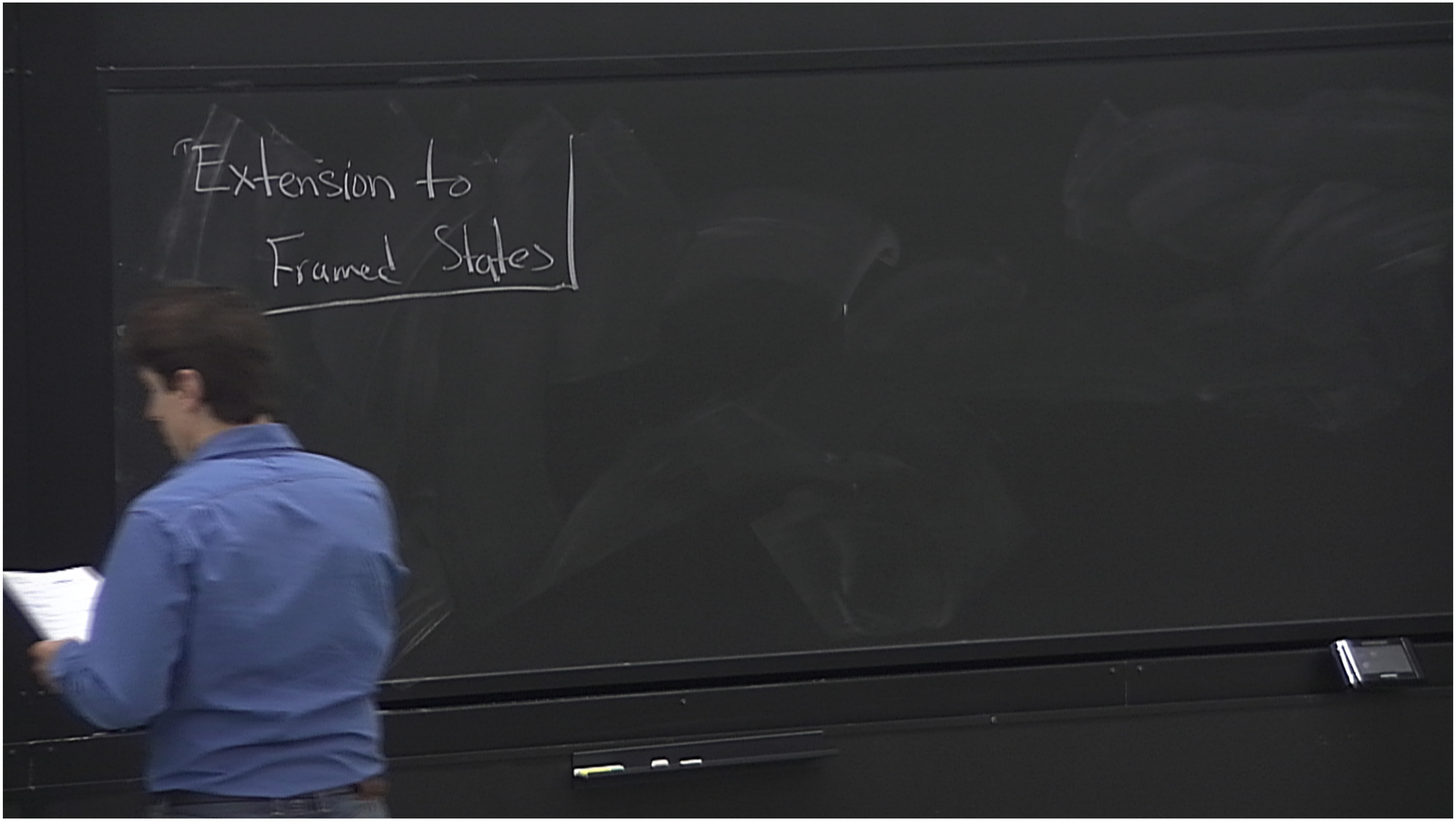

• physically
justified
at wall of
marginal stab.
Z1
Z1+Z2

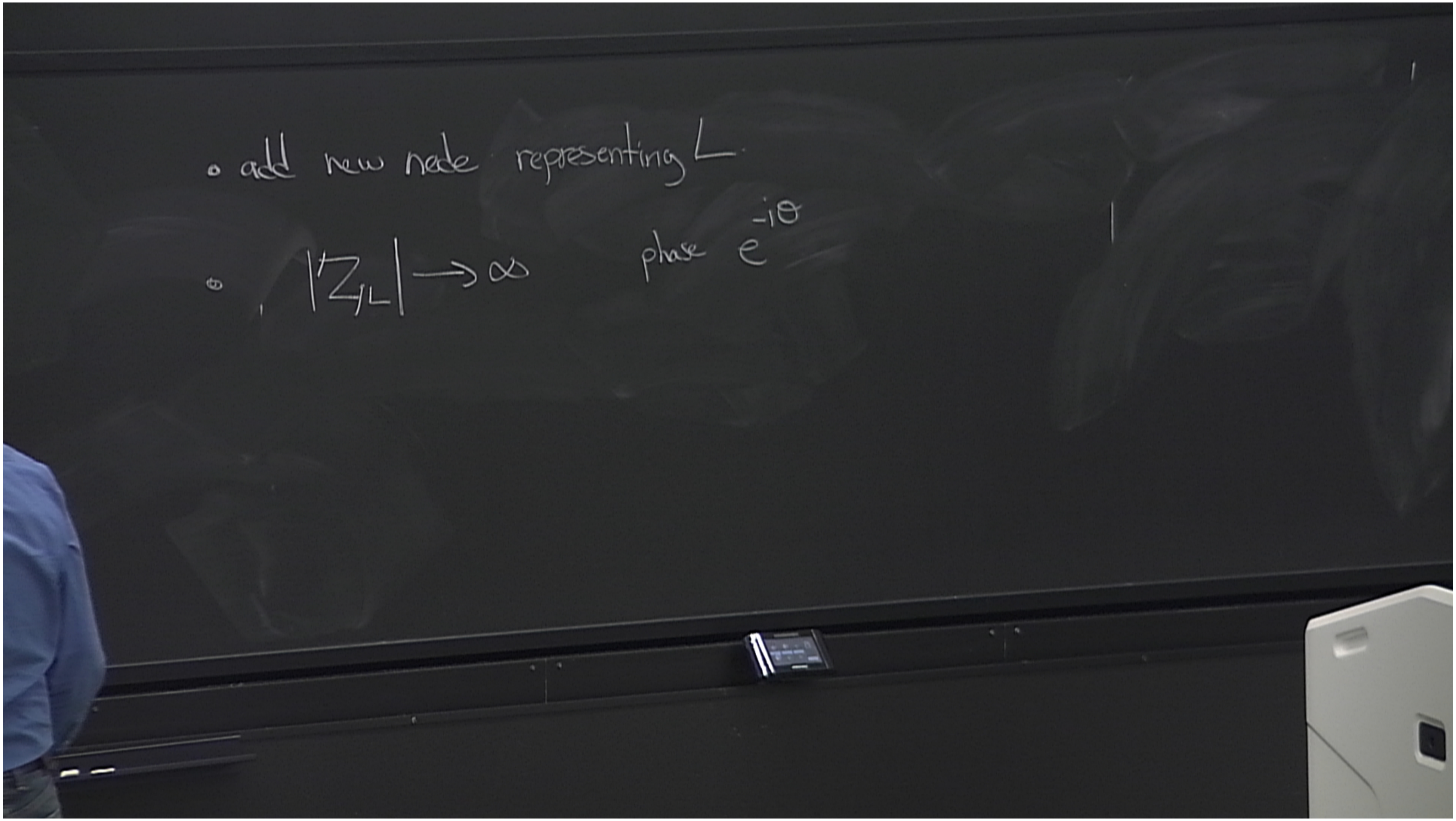

(0,1).
Z1+Z2
Z1
Z1
|binding energy| = |Z1|+|Z2|-|Z1+Z2|
is

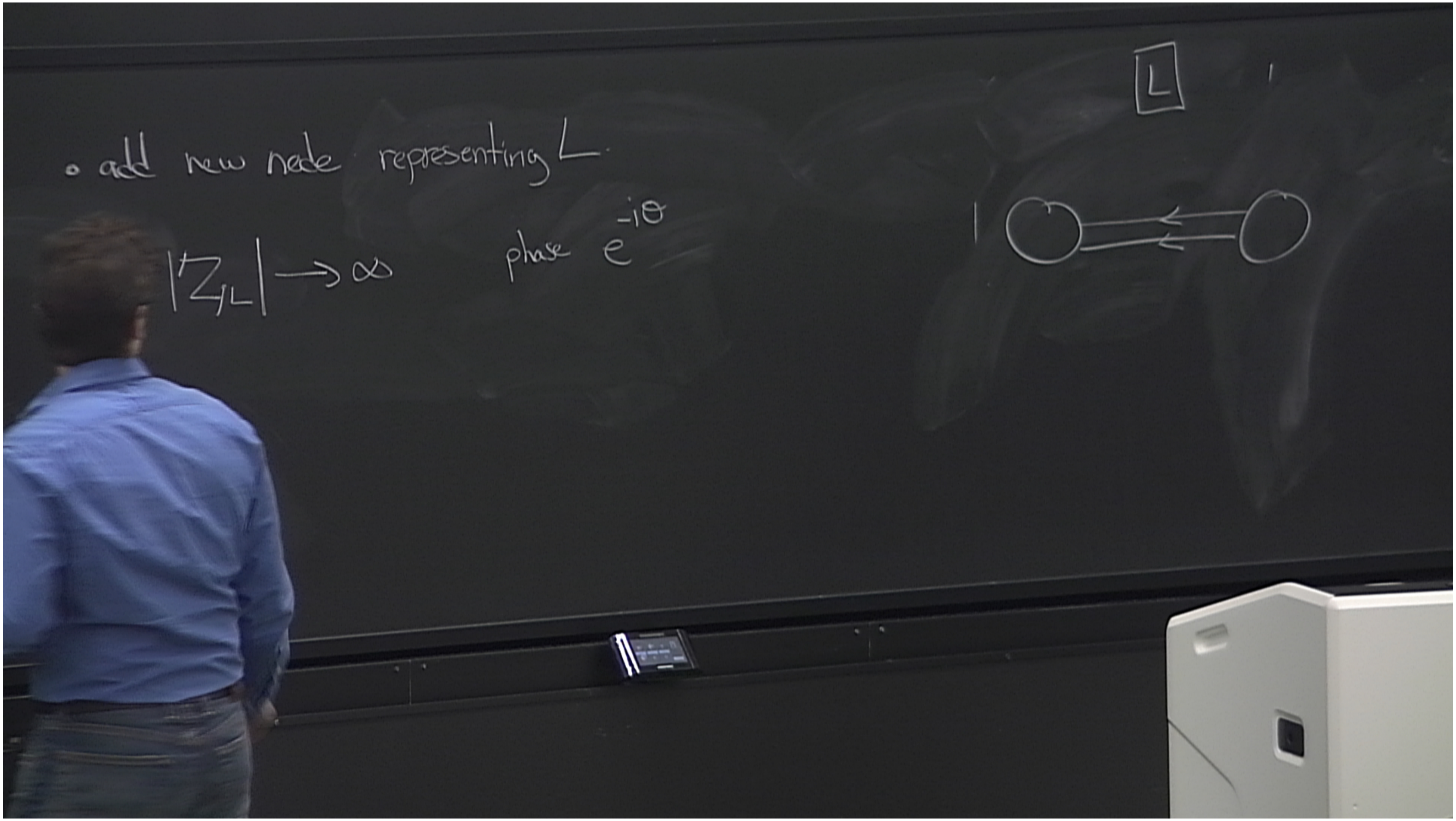
$(0,1)$. $(2,-1)$

$$|\text{binding energy}| = |Z_1| + |Z_2| - |Z_1 + Z_2| \quad \text{is \underline{small}}$$

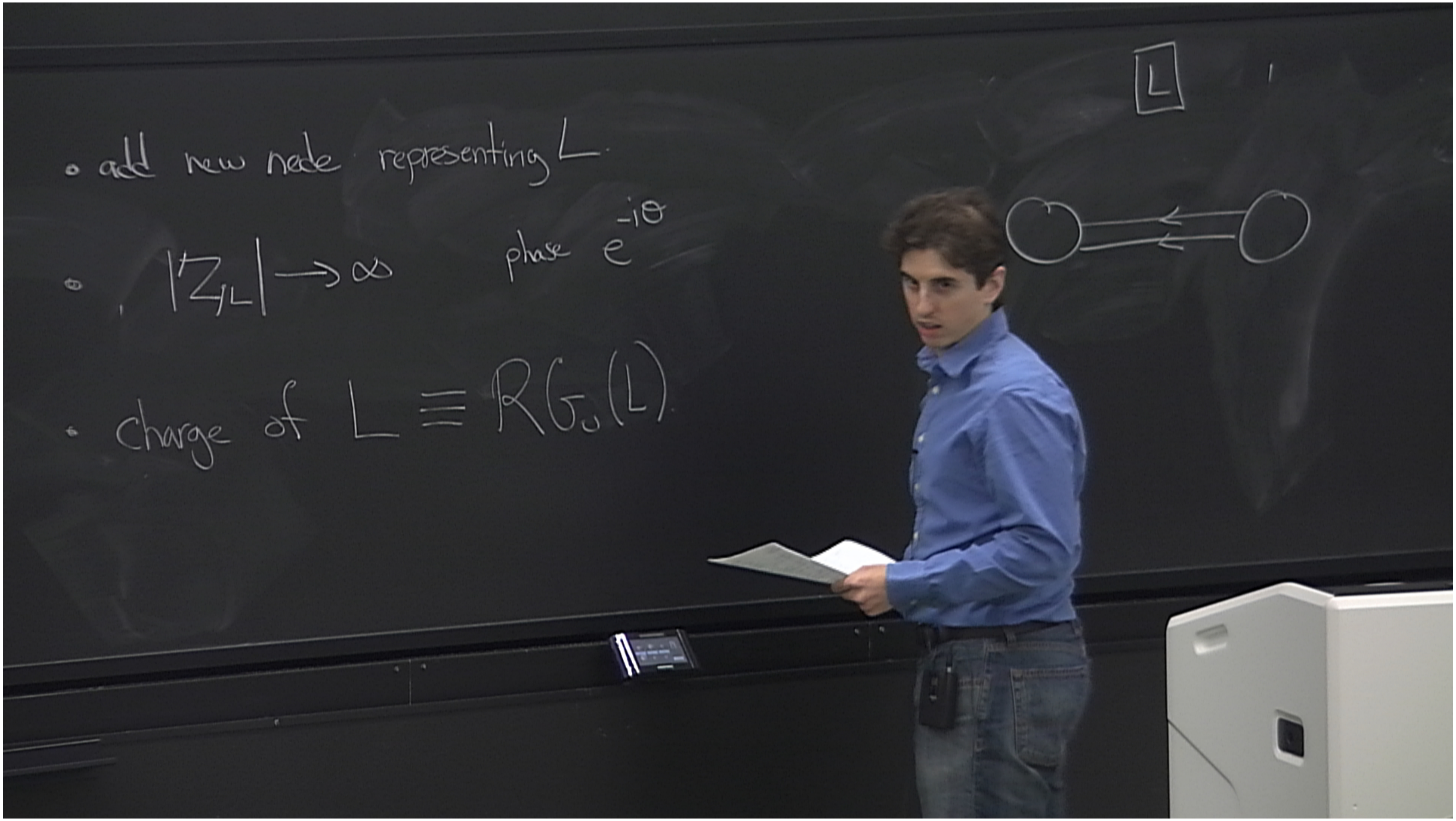

(0,1).
Z_1+Z_2
Z_1
Z_1
$|\text{binding energy}| = |Z_1|+|Z_2|-|Z_1+Z_2|$
• because of SUSY picture still works away from wall!

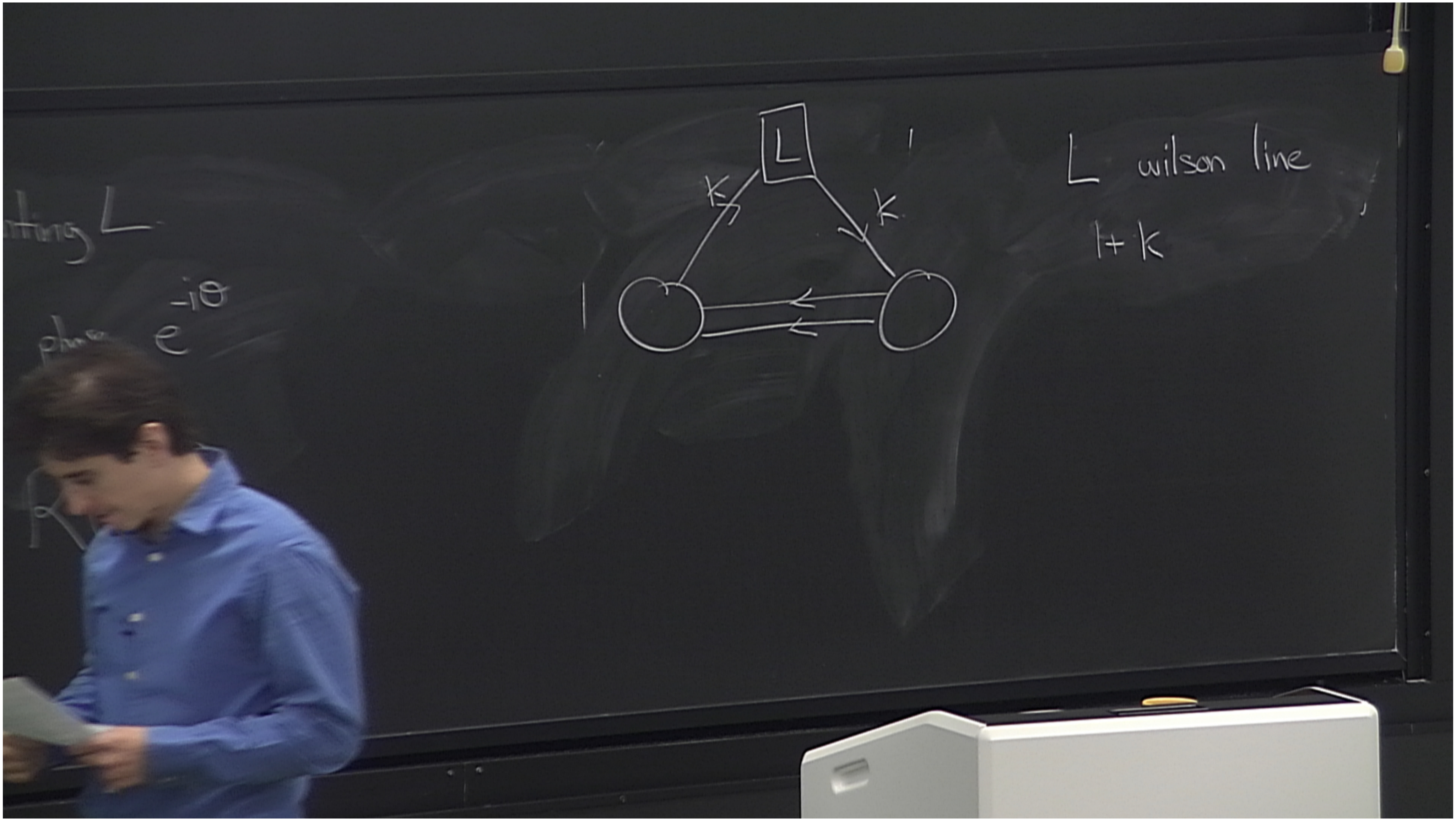

"Extension to
Framed States

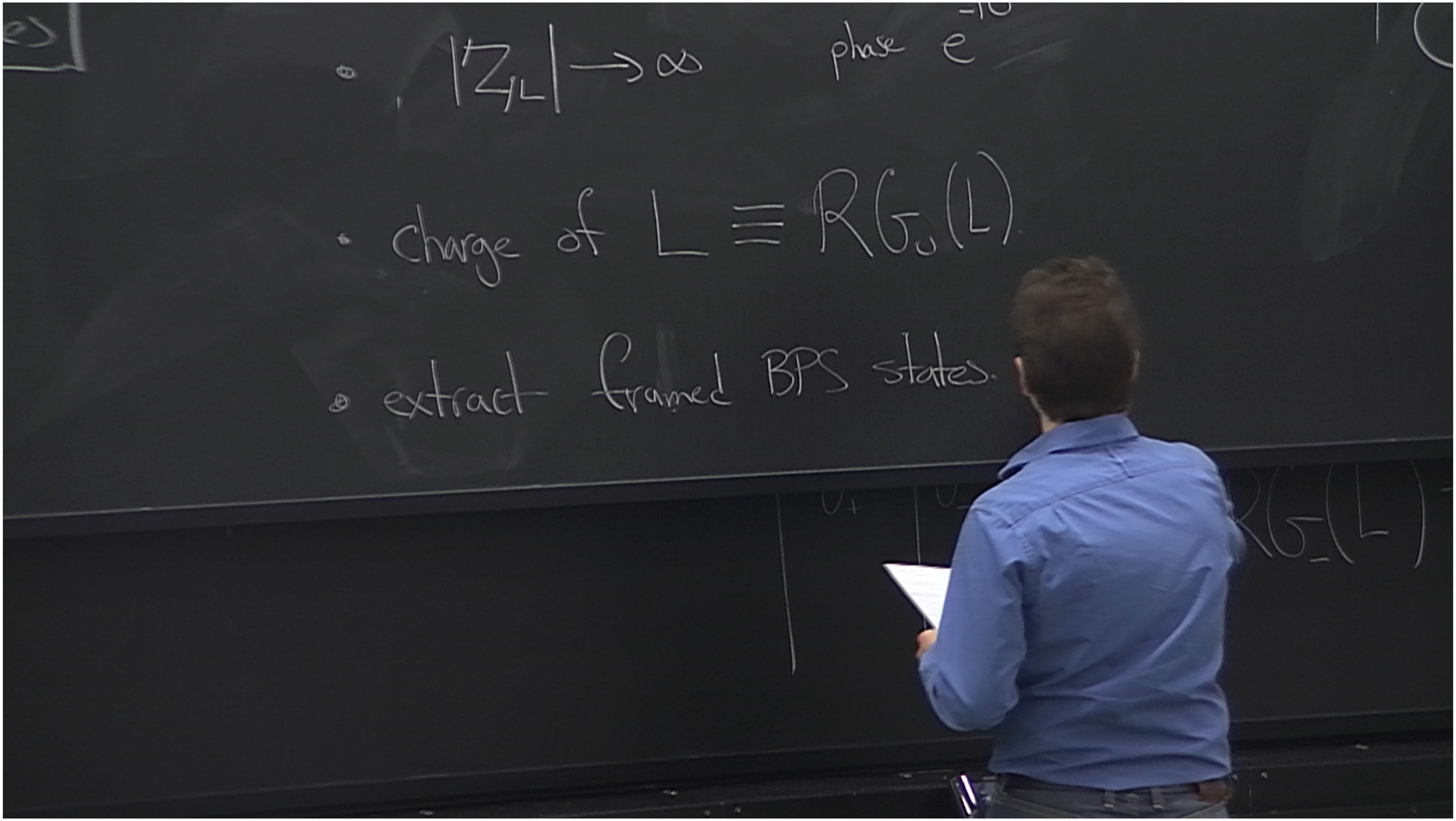

• add new node representing L
• $|Z_L| \to \infty$ phase $e^{-i\theta}$



• add new node representing L
|Z_L| → ∞
phase e^{-iθ}
L

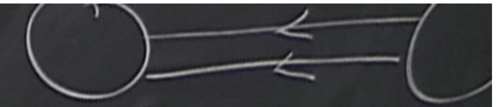

L
• add new node representing L
• |Z_L| → ∞ phase e^{-iθ}
• charge of L ≡ RG_0(L)


L
k
k
L wilson line
1+k



$|Z_L| \to \infty$
phase $e^{-i\theta}$
charge of $L \equiv RG_{\vartheta}(L)$.
extract framed BPS states.
$RG_{-}(L)$

- $|Z_{\gamma_L}| \to \infty$ phase $e^{-i\theta}$

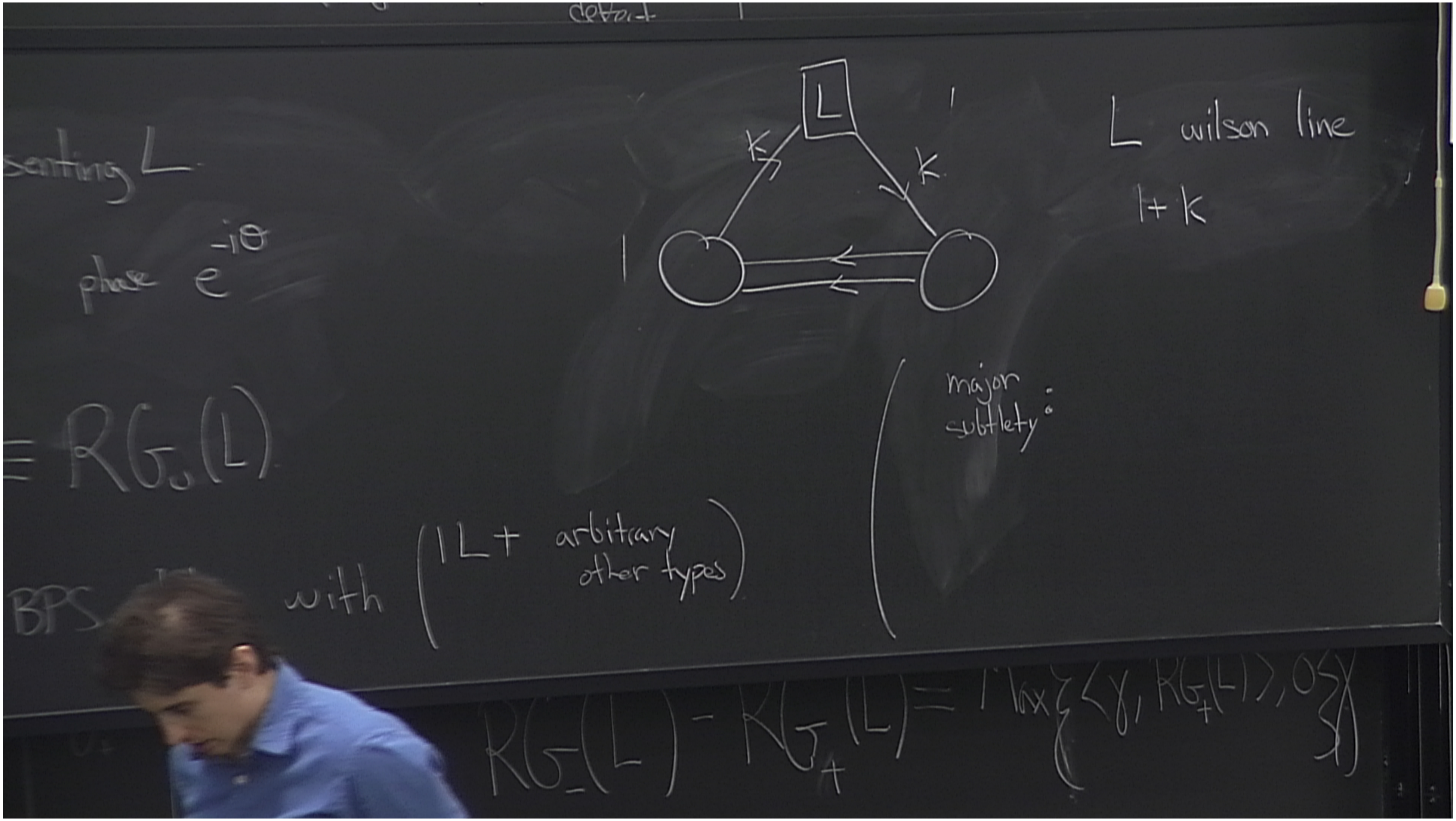


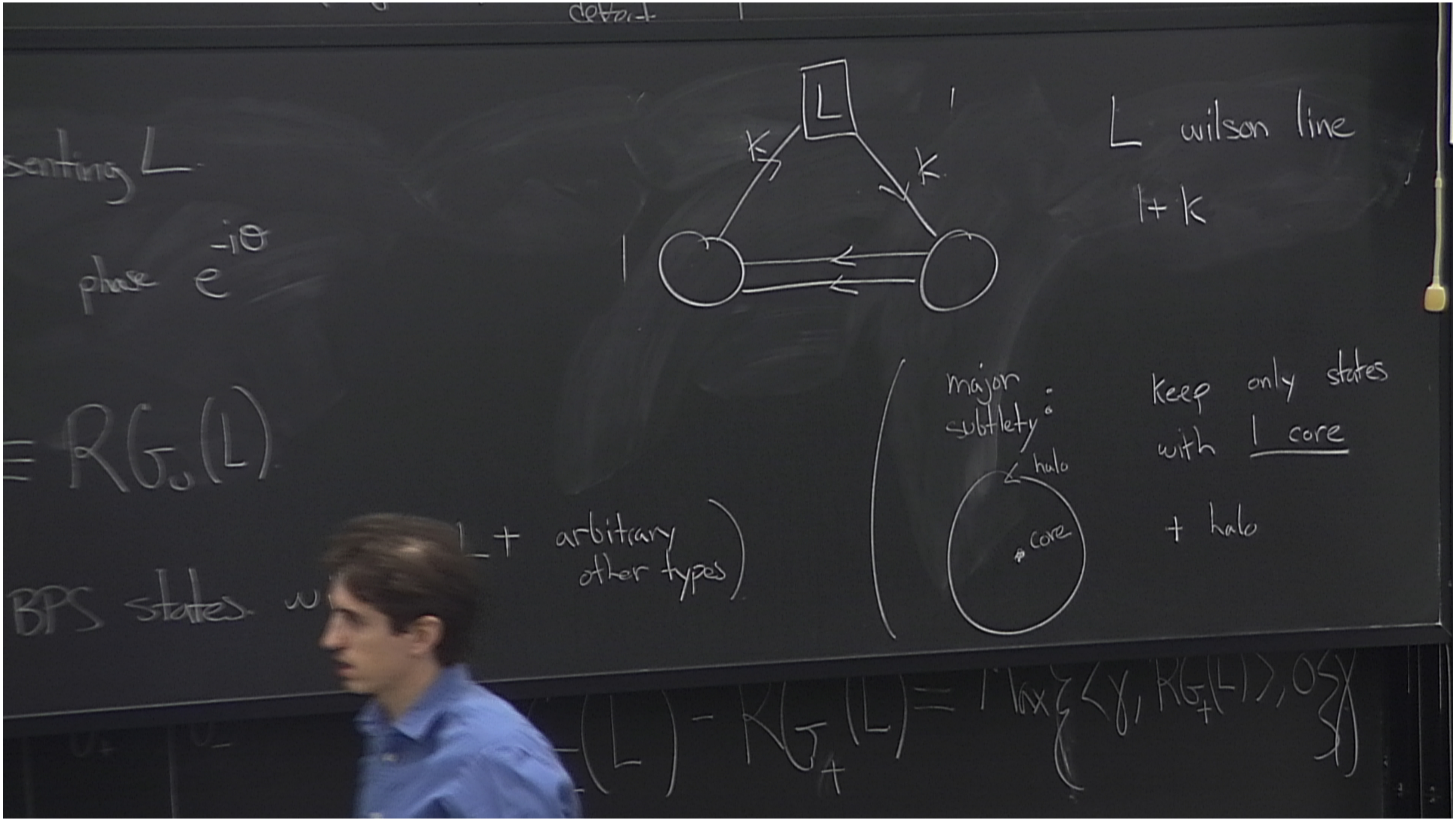
- charge of $L \equiv RG_{\vartheta}(L)$
- extract framed BPS states with $\left(1L + \text{arbitrary other types}\right)$

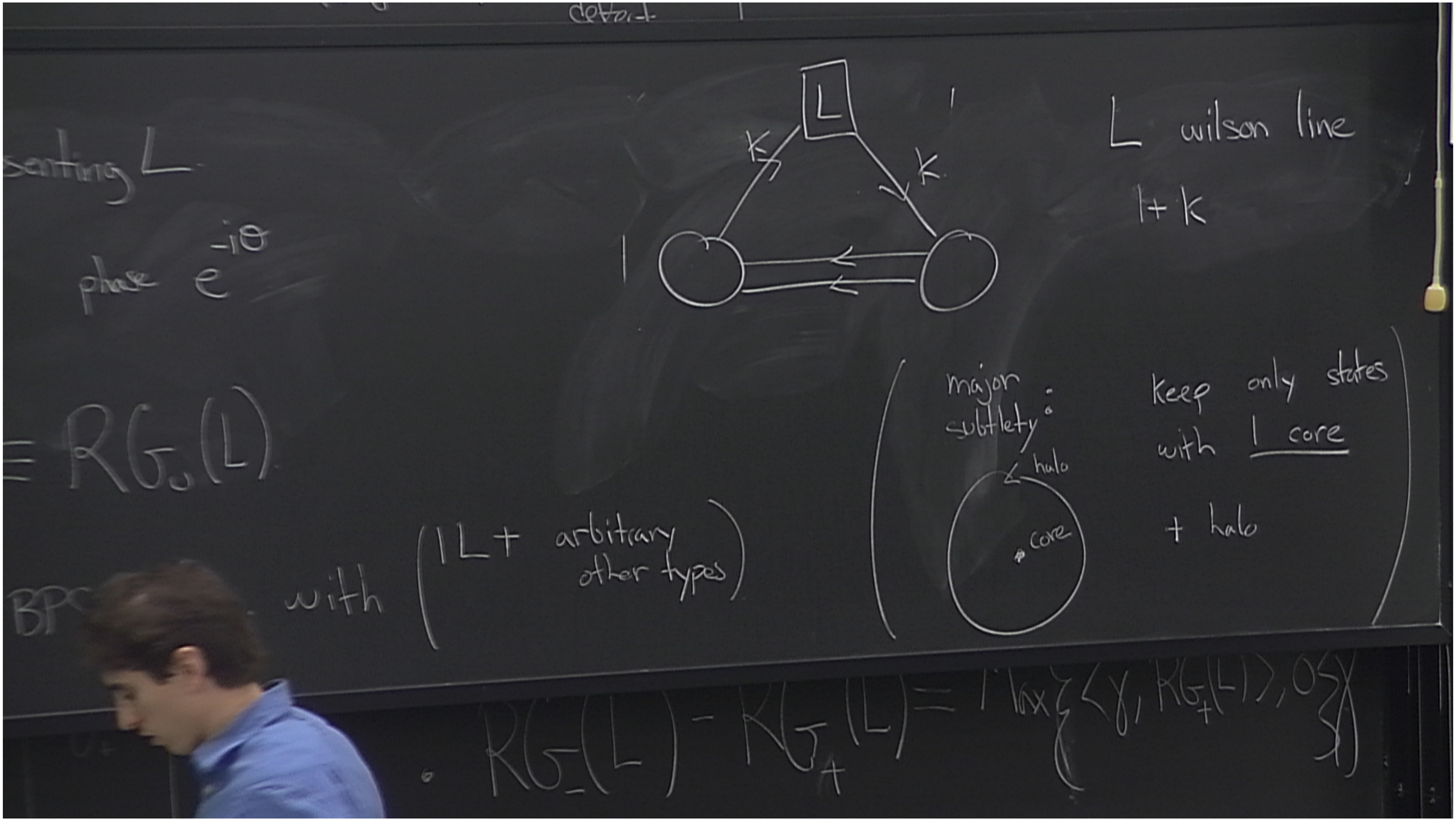
ϑ_+ ϑ_-

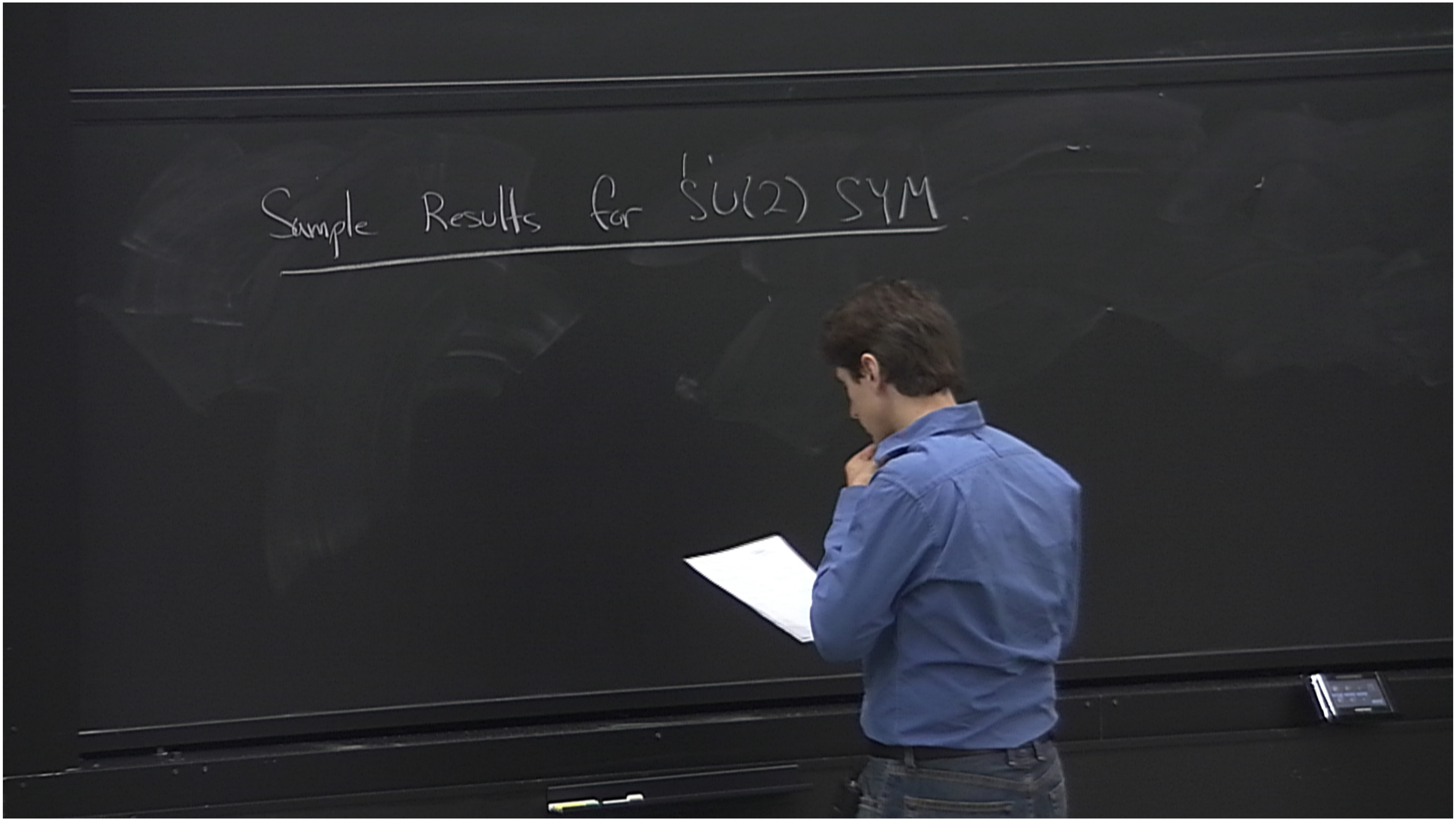
↖ locus where gs degen

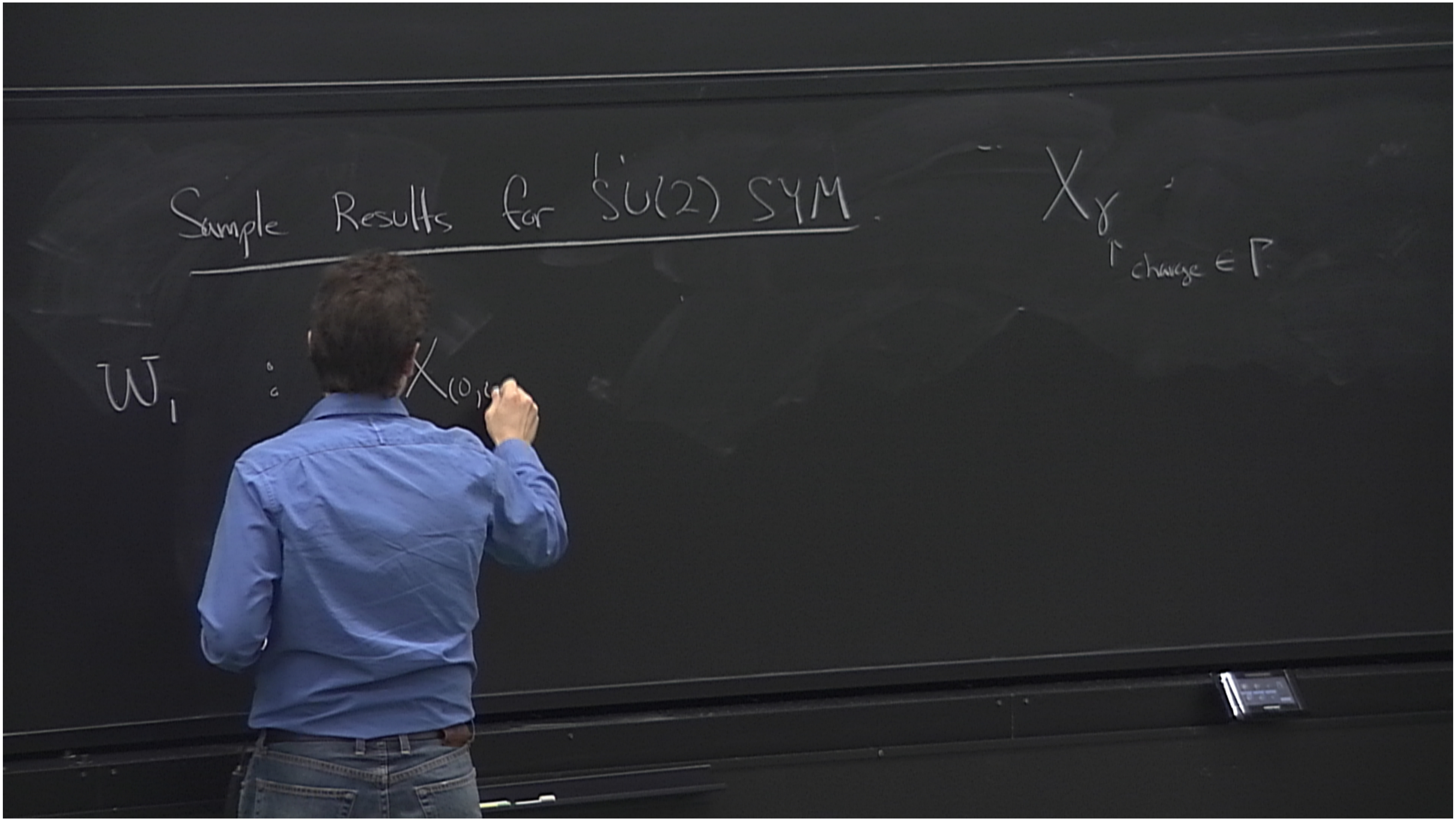
- $RG_{\vartheta_-}(L) - RG_{\vartheta_+}(L) =$

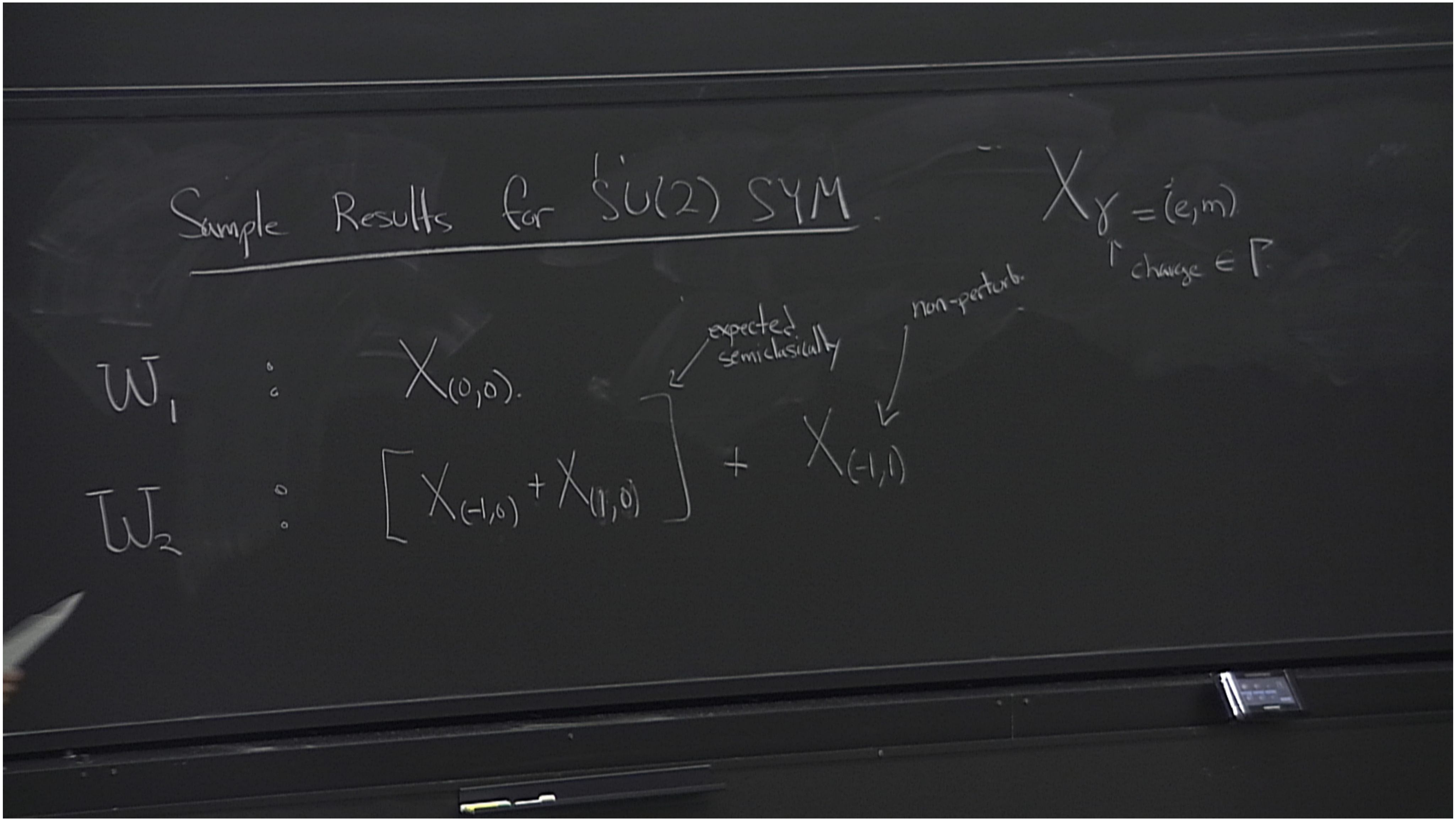

senting L
phase $e^{-i\theta}$
$\equiv RG_0(L)$
BPS
with (1L + arbitrary other types)
L
k
k
L wilson line
1+k
major subtlety:
$RG_-(L) - RG_+(L) = \mathrm{Max}\{\langle\gamma, RG_+(L)\rangle, 0\}\gamma$

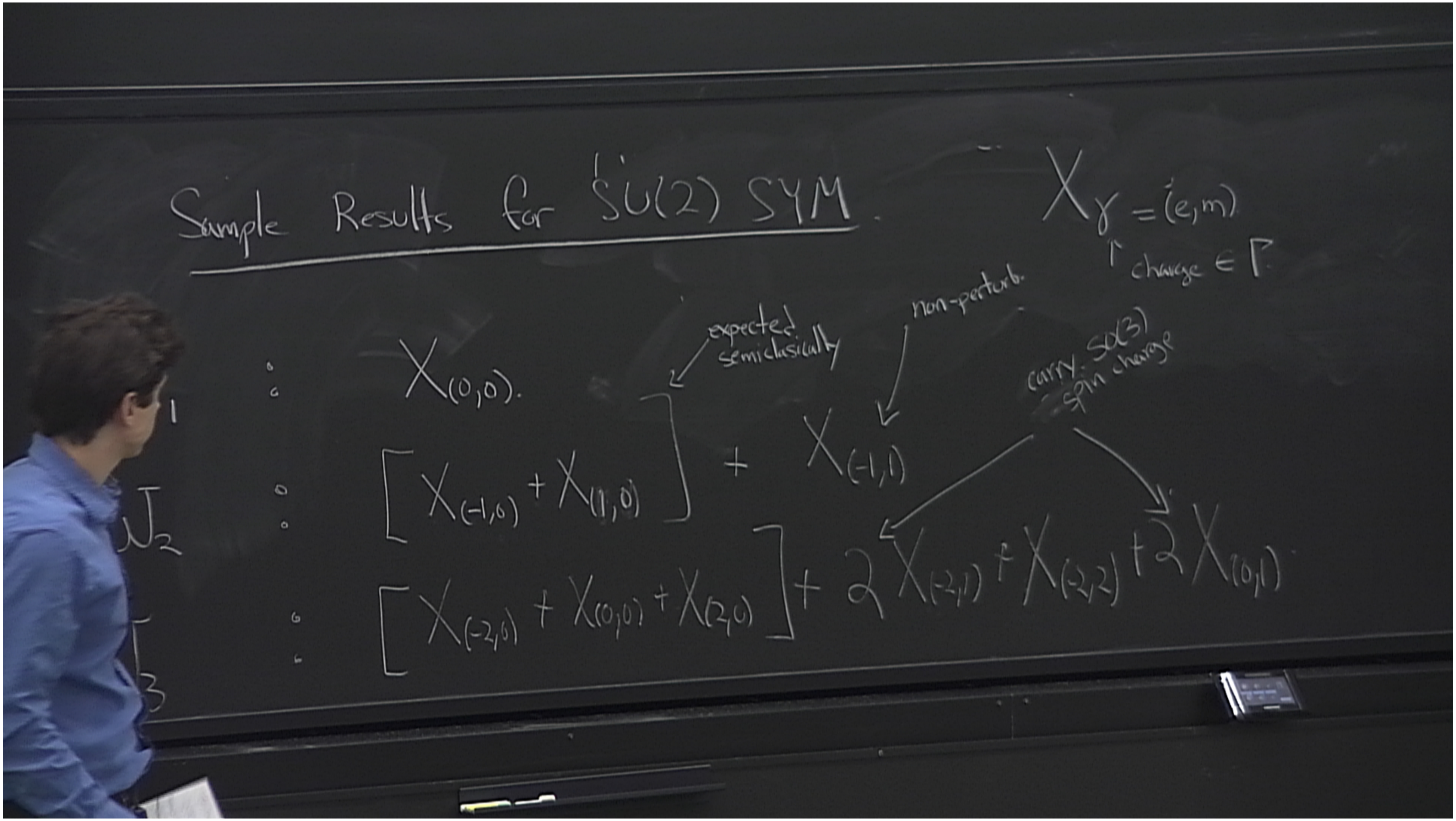

senting L
phase e^{-iθ}
≡ RG_0(L)
BPS states w
L + arbitrary other types
L
k
k
L wilson line
1+k
major subtlety:
halo
core
Keep only states with 1 core
+ halo
(L) - RG_+(L) = Max{⟨γ, RG_+(L)⟩, 0}

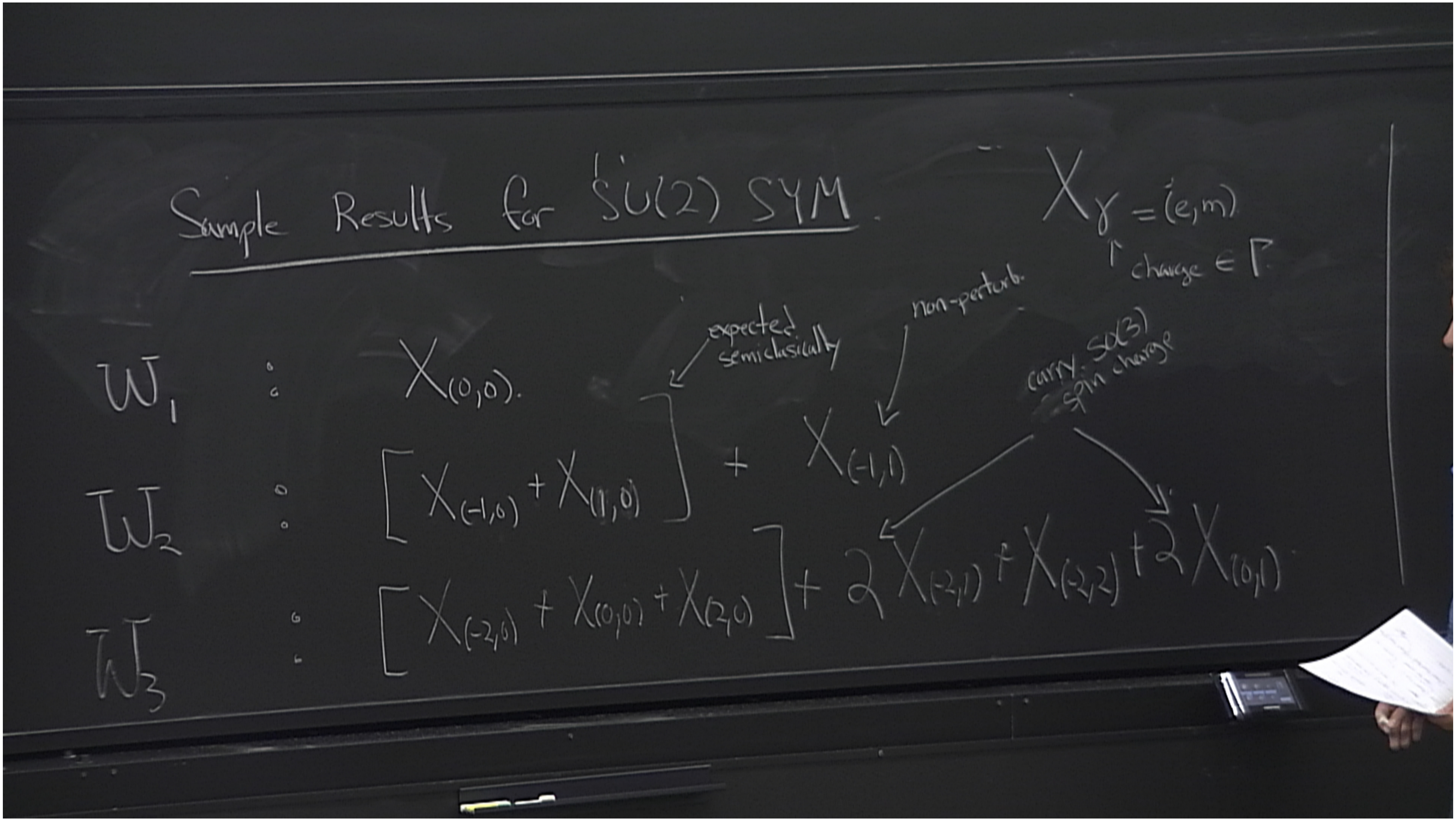

senting L
phase e^{-iθ}
≡ RG_0(L)
with (1L + arbitrary other types)
L
K
K
L wilson line
1+k
major subtlety:
halo
core
keep only states with 1 core + halo
RG_-(L) - RG_+(L) = Max{<γ, RG_+(L)>, 0}

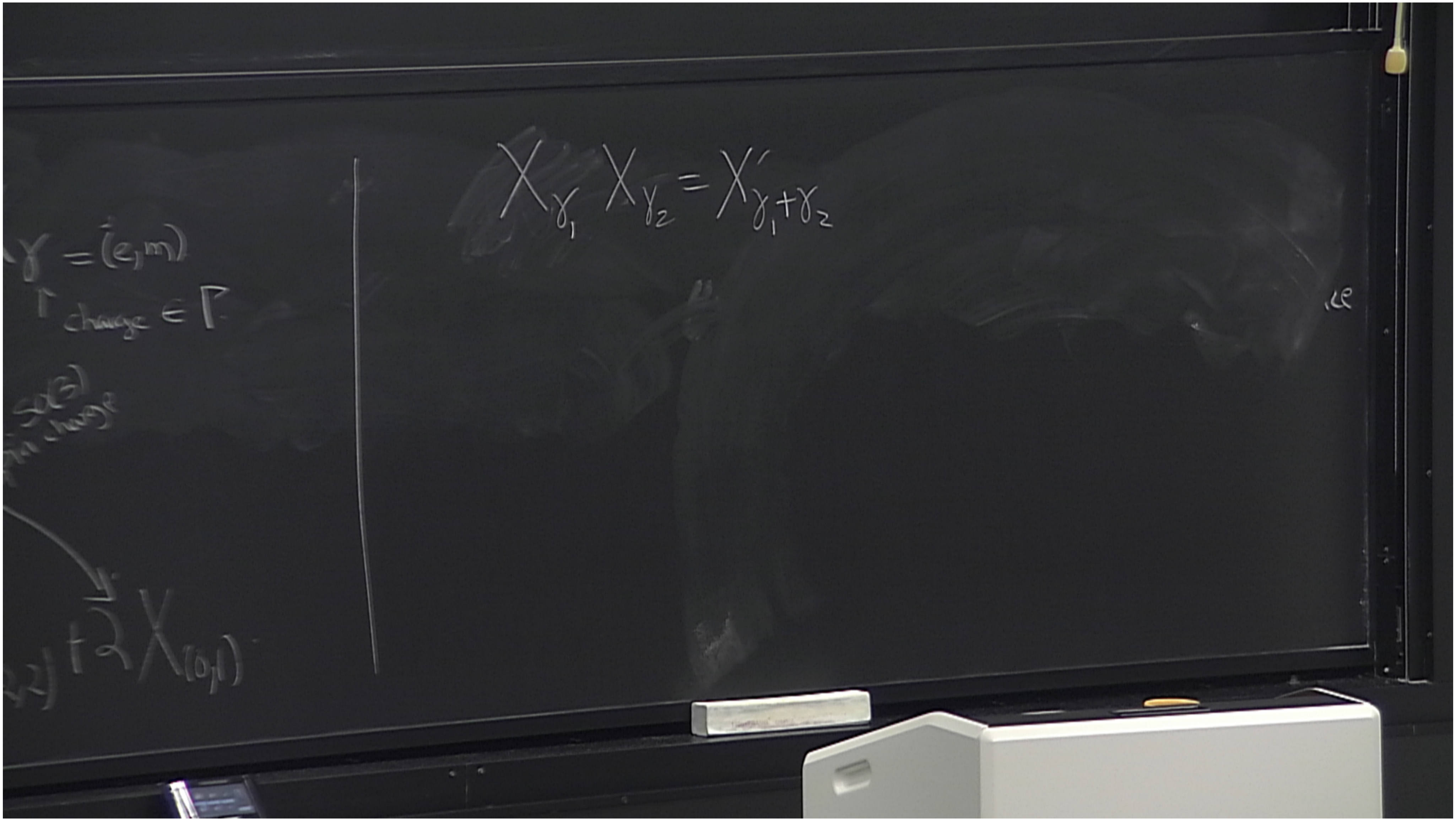

Sample Results for SU(2) SYM

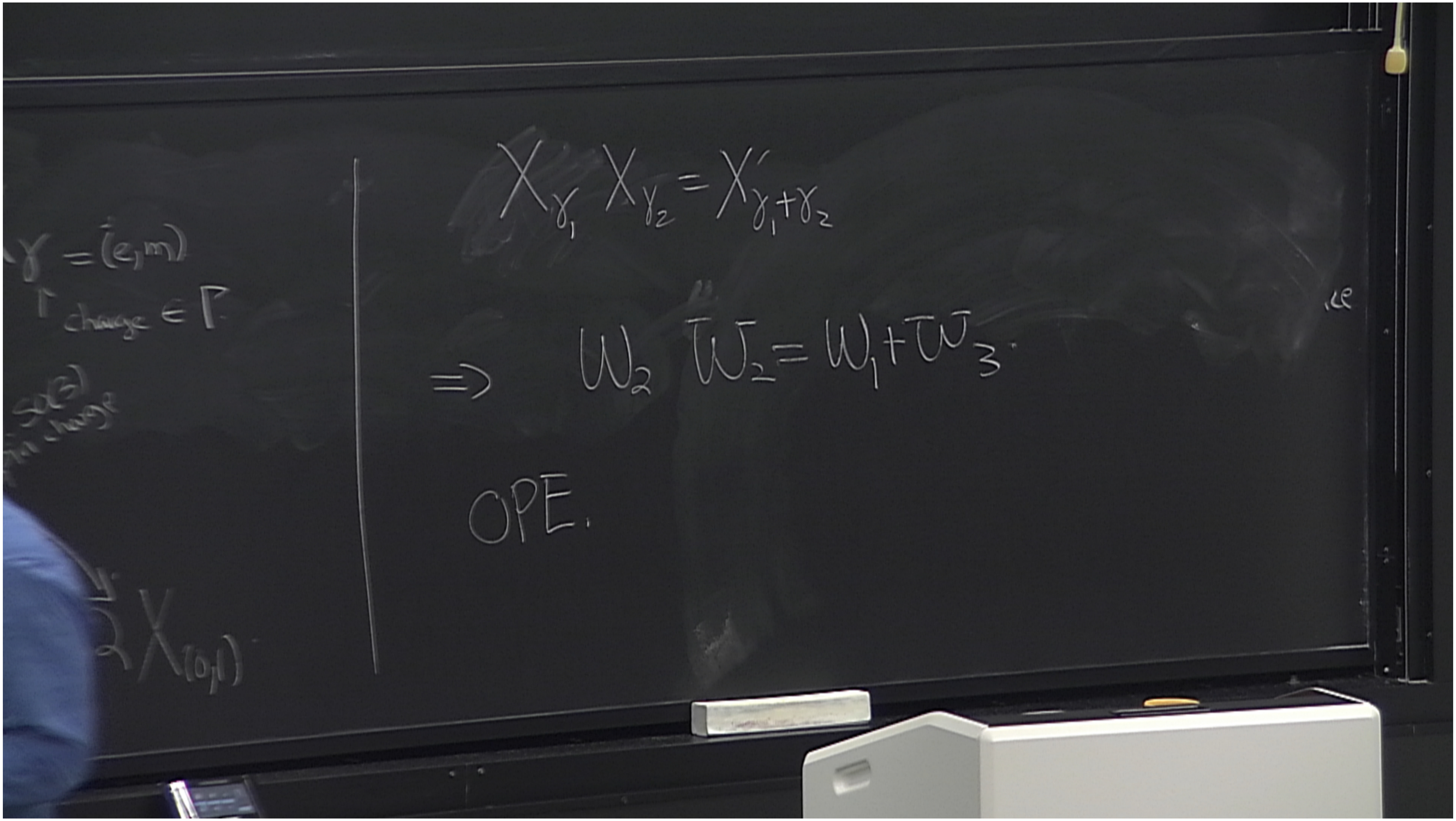

Sample Results for SU(2) SYM
X_γ
charge ∈ Γ
W_1 :

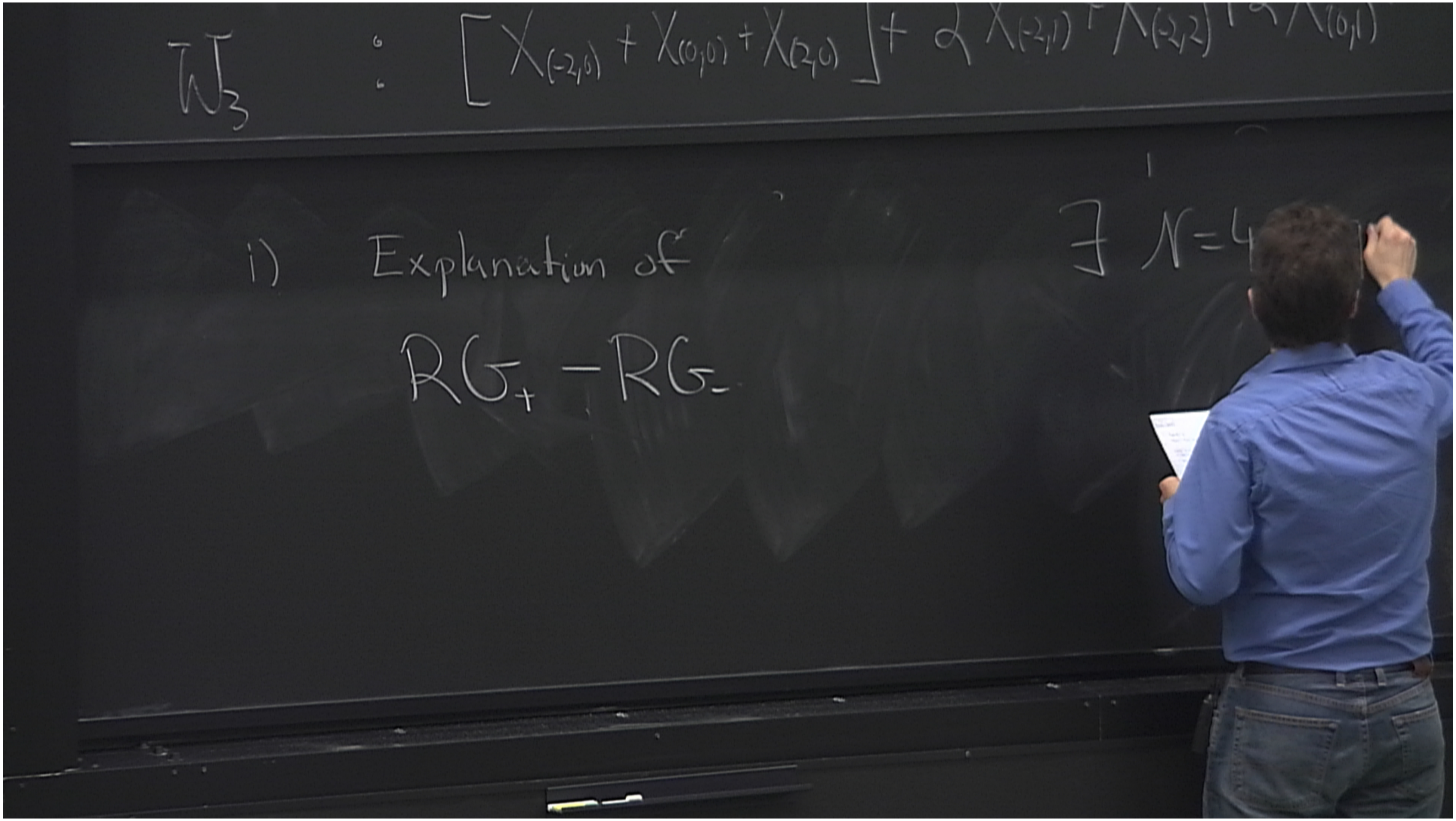

Sample Results for SU(2) SYM
X_γ $\gamma = (e,m)$
charge $\in \Gamma$
expected semiclassically
non-perturb.
$W_1 : X_{(0,0)}$.
$W_2 : \left[X_{(-1,0)} + X_{(1,0)} \right] + X_{(-1,1)}$


Sample Results for SU(2) SYM.
X_γ $\gamma = (e,m)$
charge $\in \Gamma$
expected semiclassically
non-perturb.
carry SO(3) spin charge
J_1 : $X_{(0,0)}$.
J_2 : $[X_{(-1,0)} + X_{(1,0)}] + X_{(-1,1)}$
J_3 : $[X_{(-2,0)} + X_{(0,0)} + X_{(2,0)}] + 2X_{(-2,1)} + X_{(-2,2)} + 2X_{(0,1)}$

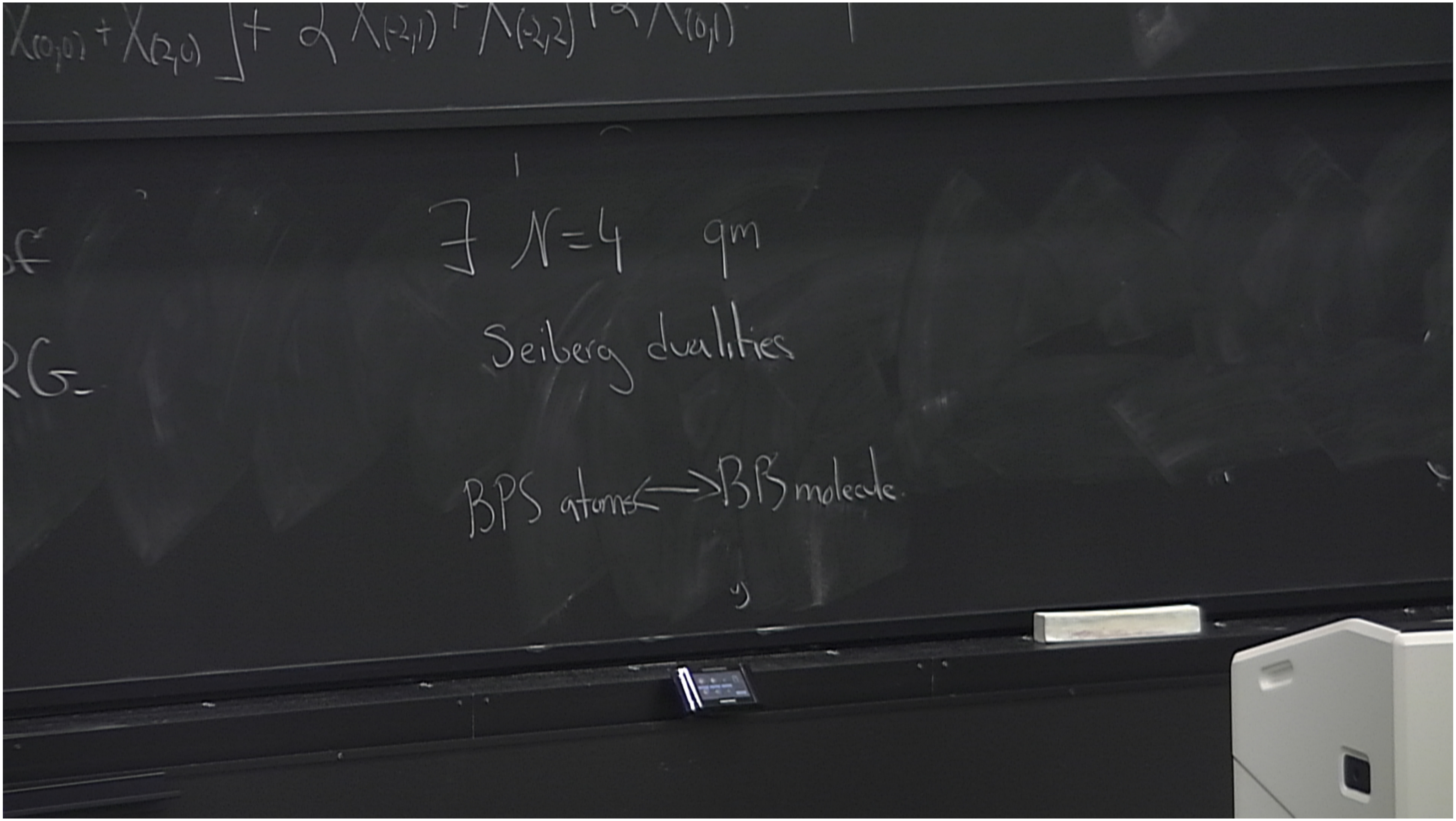

Sample Results for SU(2) SYM
X_γ $\gamma = (e,m)$
charge $\in \Gamma$
expected semiclassically
non-perturb.
carry SO(3) spin charge
W_1 : $X_{(0,0)}$.
W_2 : $[X_{(-1,0)} + X_{(1,0)}] + X_{(-1,1)}$
W_3 : $[X_{(-2,0)} + X_{(0,0)} + X_{(2,0)}] + 2X_{(-2,1)} + X_{(-2,2)} + 2X_{(0,1)}$

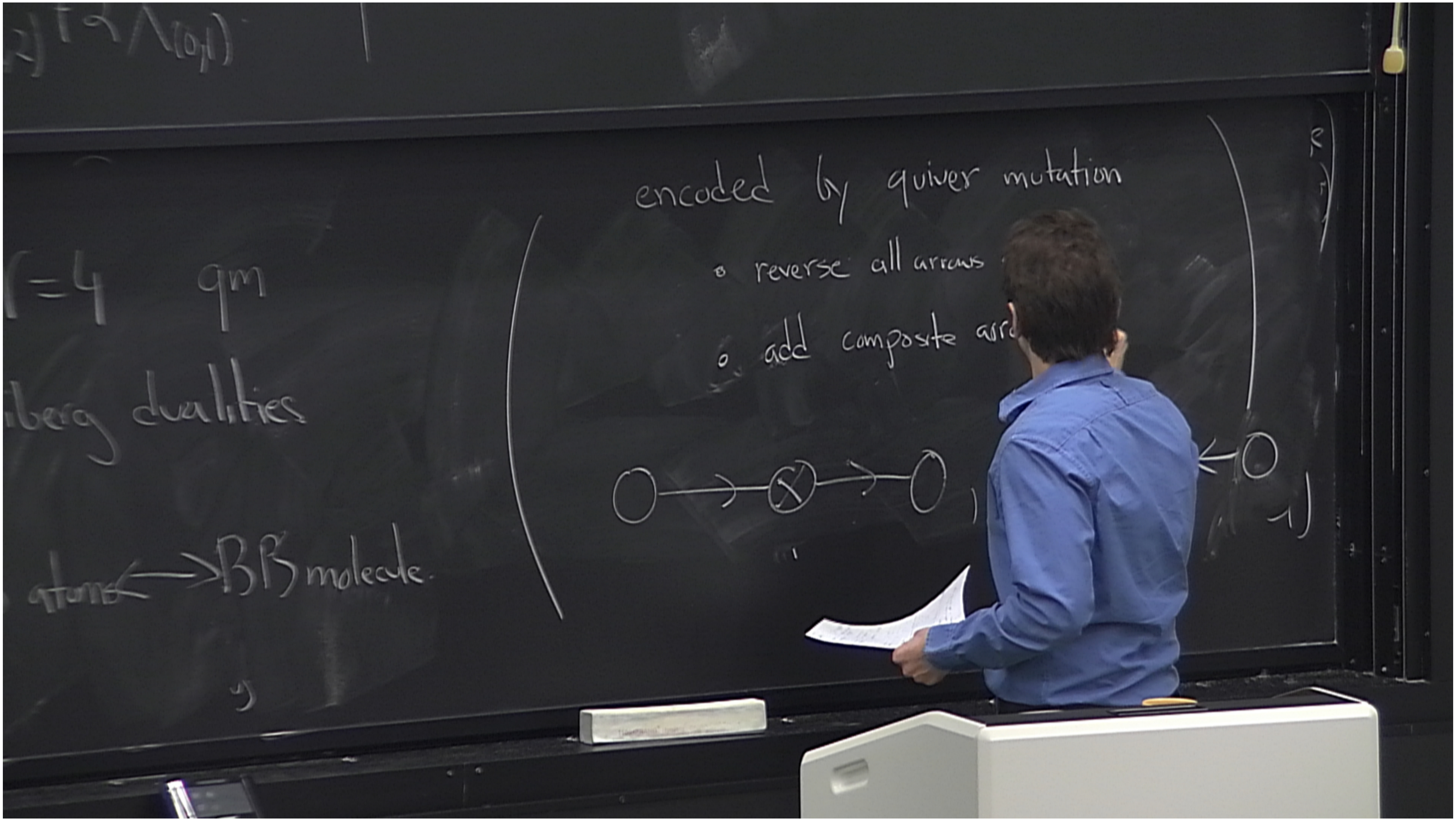

$X_{\gamma_1} X_{\gamma_2} = X'_{\gamma_1+\gamma_2}$
$\gamma = (e,m)$
$\Gamma_{\text{charge}} \in \Gamma$
$2X_{(0,1)}$

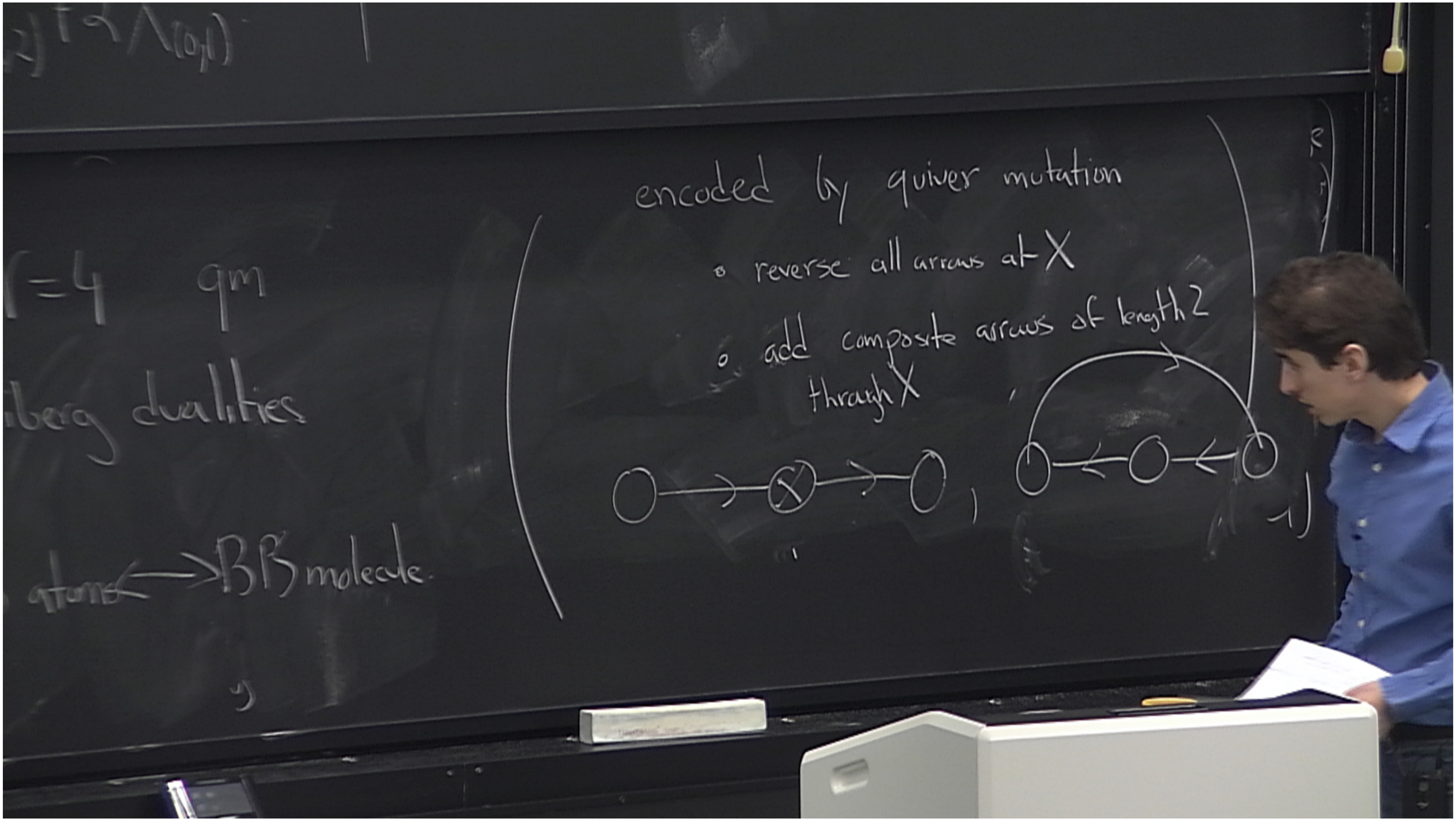

$\gamma = (e,m)$
$\Gamma_{charge} \in \Gamma$
$X_{(0,1)}$
$X_{\gamma_1} X_{\gamma_2} = X'_{\gamma_1+\gamma_2}$
$\Rightarrow \quad W_2 \; \bar{W}_2 = W_1 + \bar{W}_3.$
OPE.

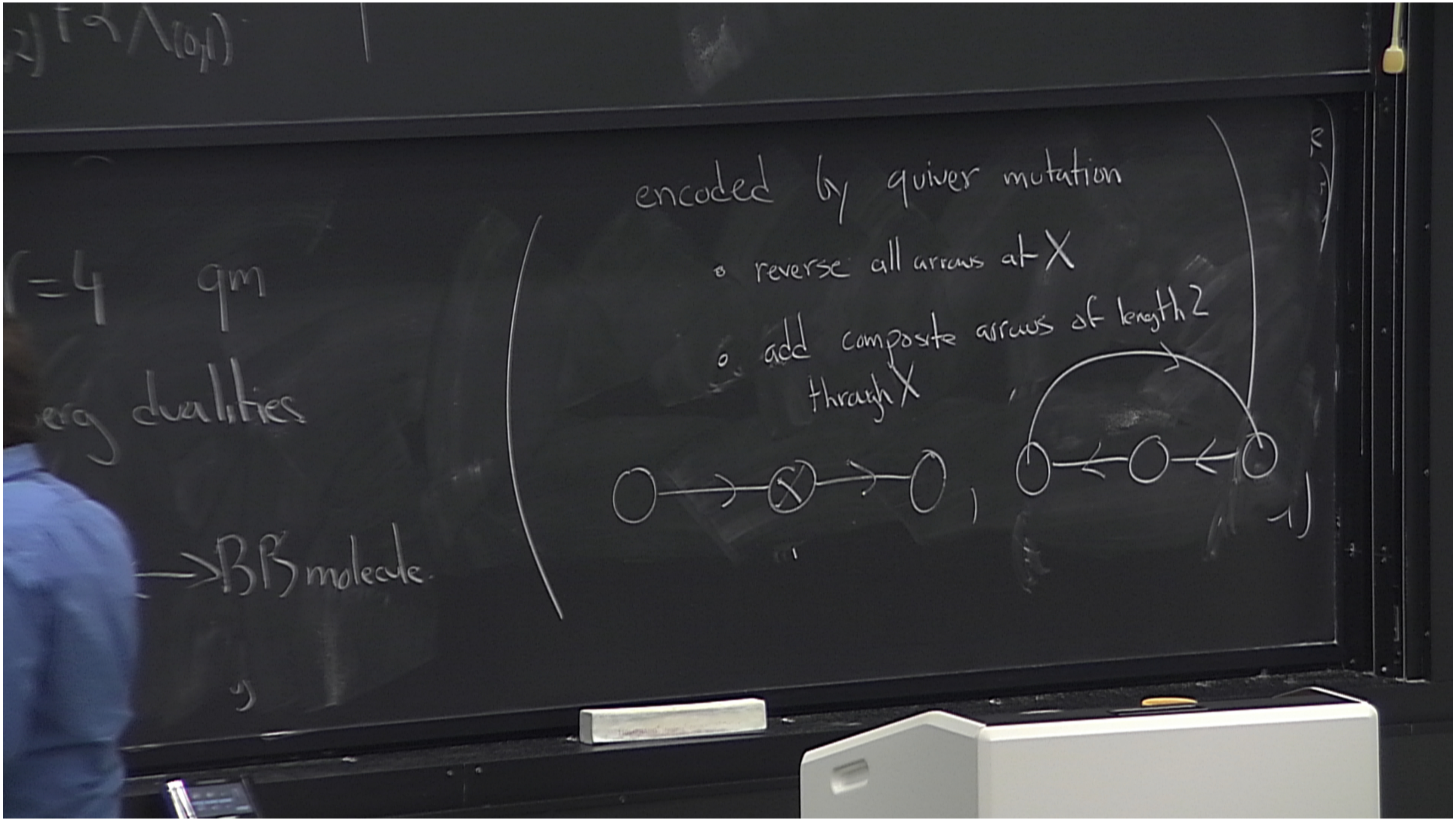


W_3 : $[X_{(-2,0)} + X_{(0,0)} + X_{(2,0)}] + 2X_{(-2,1)} + X_{(-2,2)} + 2X_{(0,1)}$
i) Explanation of
$RG_+ - RG_-$
$\exists\ N=4$


$X_{(0,0)} + X_{(2,0)}] + 2 X_{(-2,1)} + X_{(-2,2)} + 2 X_{(0,1)}$
∃ N=4 qm
Seiberg dualities
BPS atoms ⟷ BB molecule


encoded by quiver mutation
reverse all arrows
add composite
qm
dualities
BB molecule


encoded by quiver mutation
reverse all arrows at X
add composite arrows of length 2 through X
dualities
BB molecule



encoded by quiver mutation
reverse all arrows at X
add composite arrows of length 2
through X
dualities
BB molecule

$$W_3 : \left[X_{(-2,0)} + X_{(0,0)} + X_{(2,0)} \right] + 2\, X_{(-2,1)} + X_{(-2,2)} + 2\, X_{(0,1)}$$

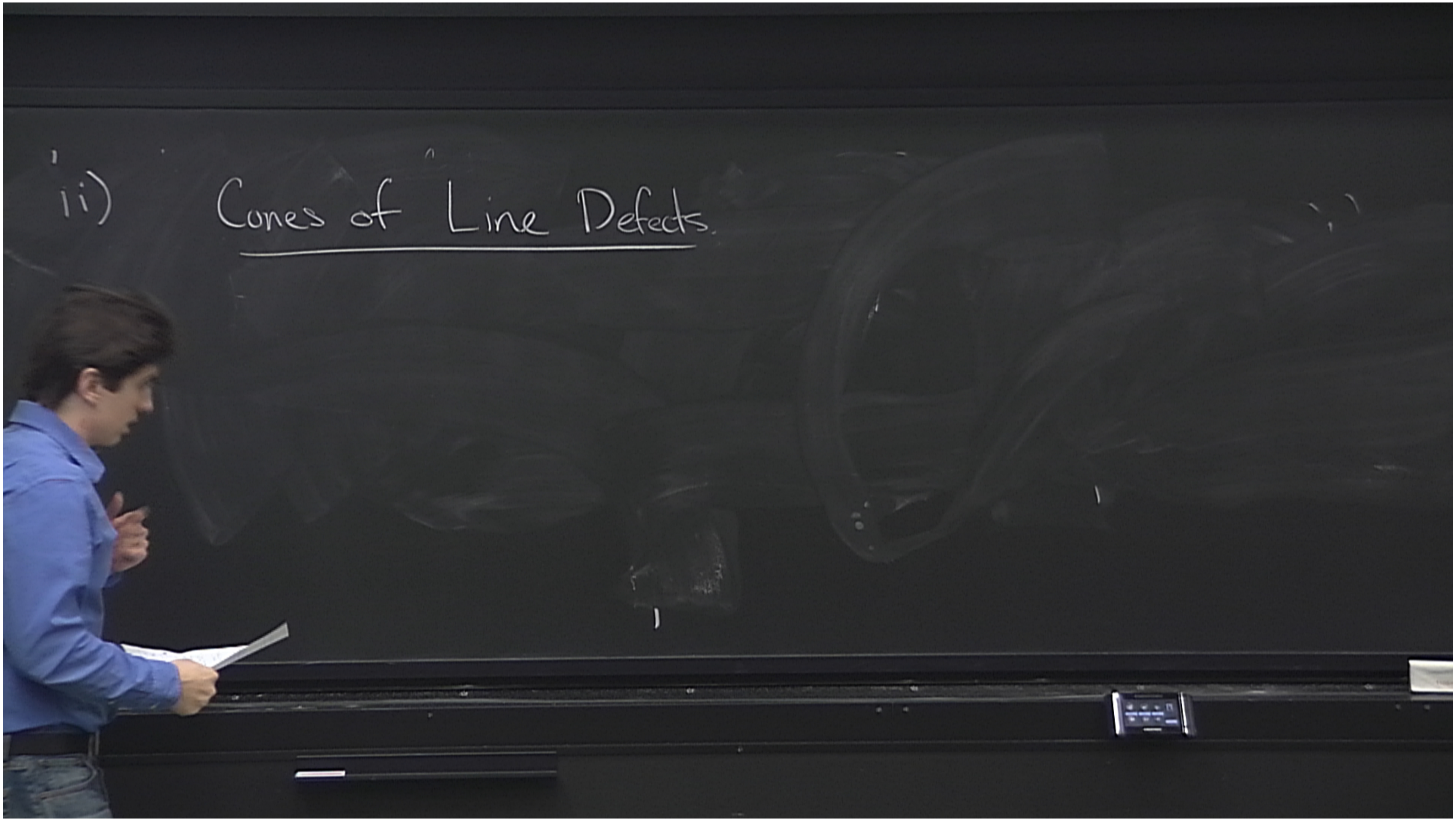
i) Explanation of

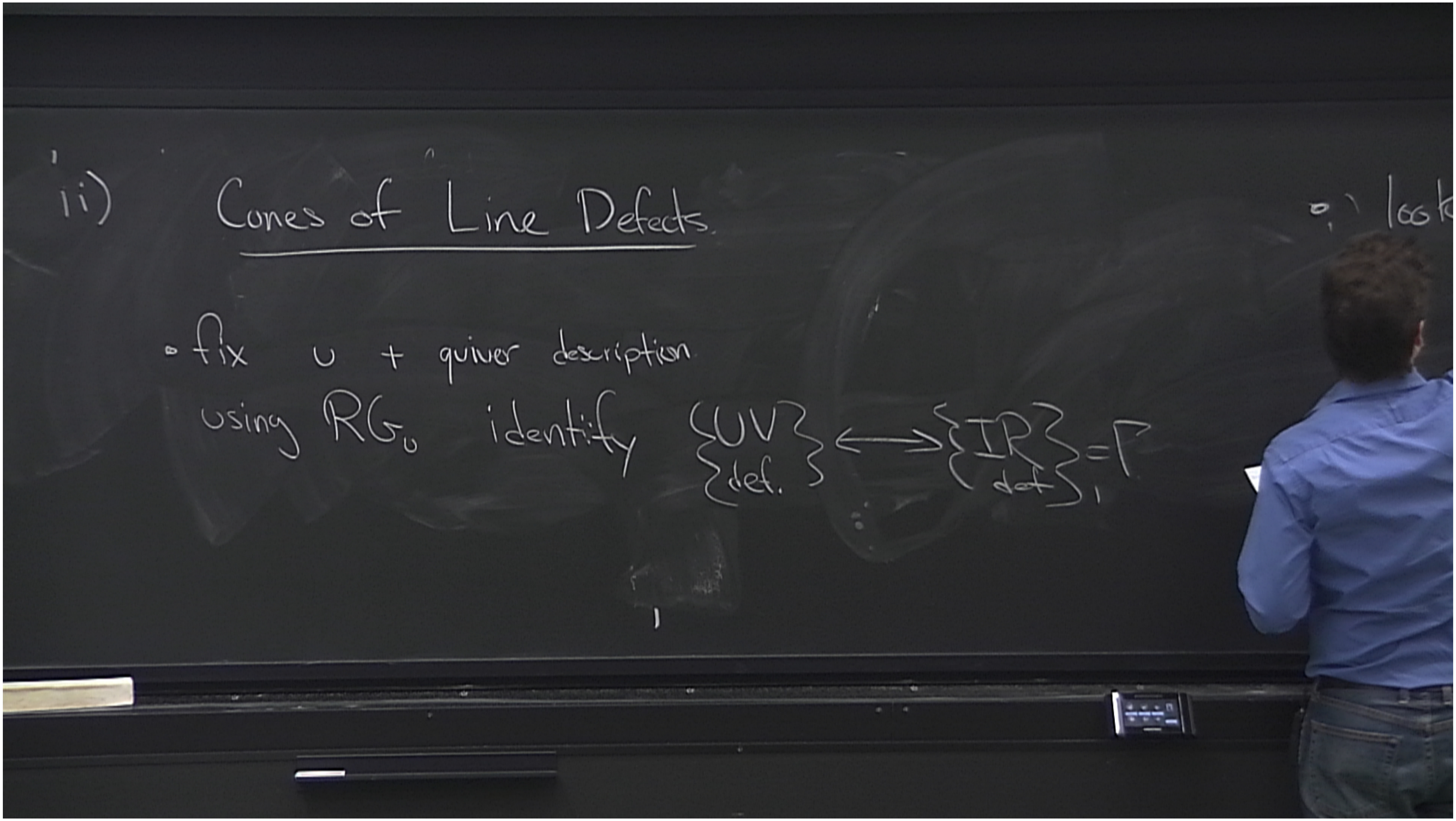
$$RG_+ - RG_- = (\cdots)$$

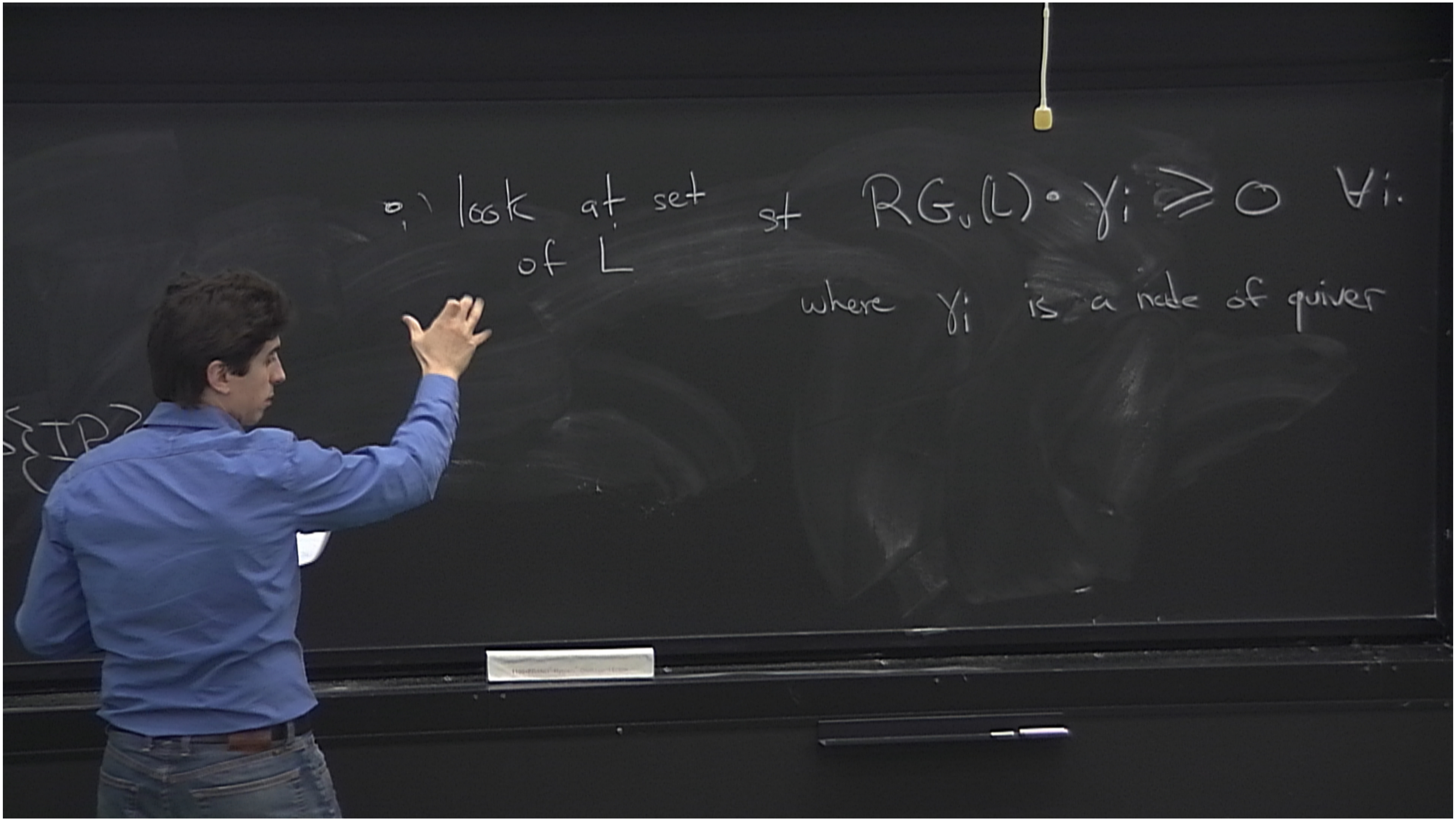
$\exists\ \mathcal{N}=4$ qm

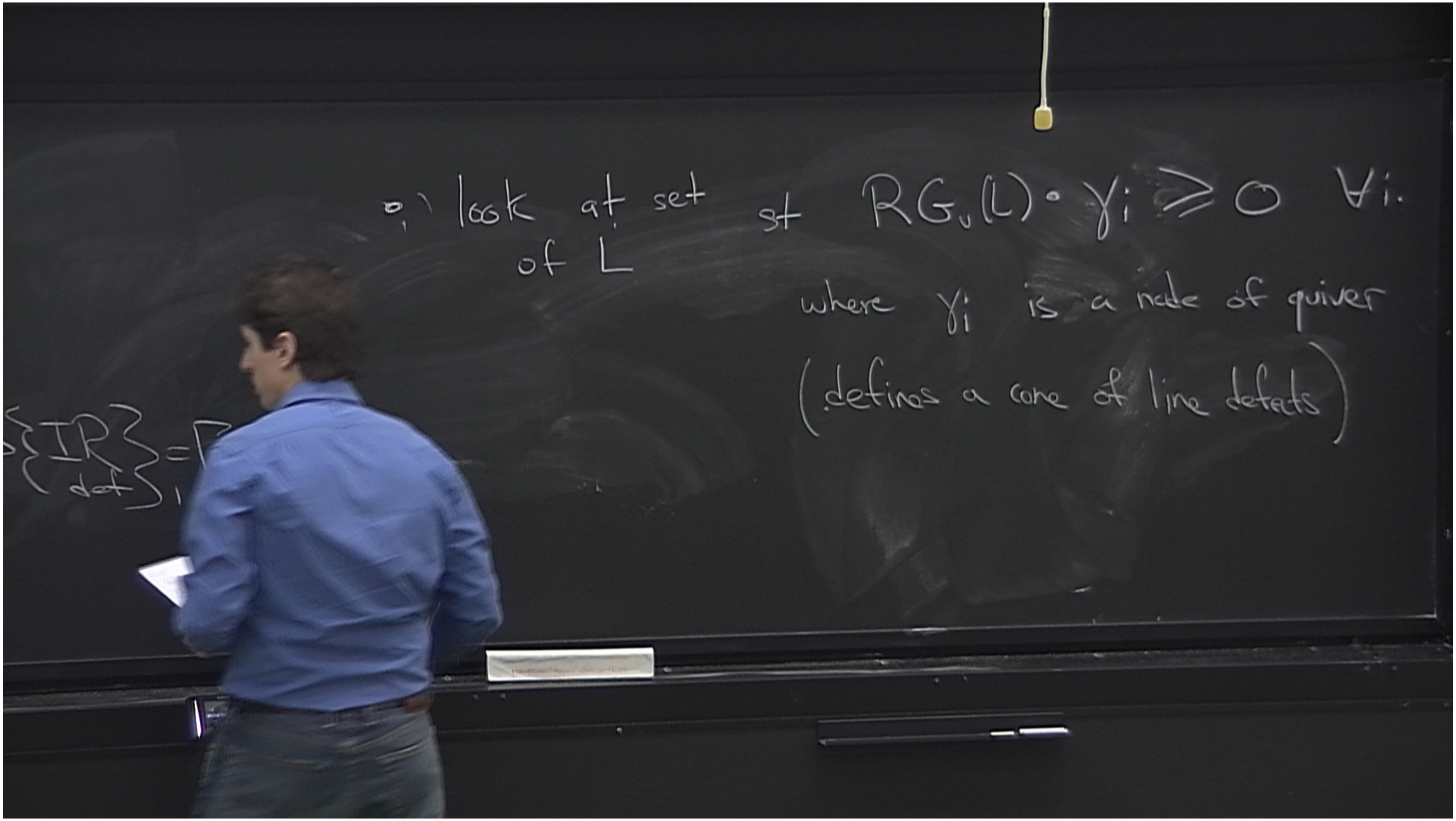
Seiberg dualities

BPS atoms $\longleftrightarrow$ BB m


ii) Cones of Line Defects


ii) Cones of Line Defects
• fix u + quiver description
using RG_u identify {UV def.} ⟷ {IR def.} = Γ
•) look


look at set of L st RG(L) · γi ≥ 0 ∀i.
where γi is a node of quiver



∘) look at set of L st $RG_v(L) \cdot \gamma_i \geq 0 \ \forall i$.
where γ_i is a node of quiver
(defines a cone of line defects)

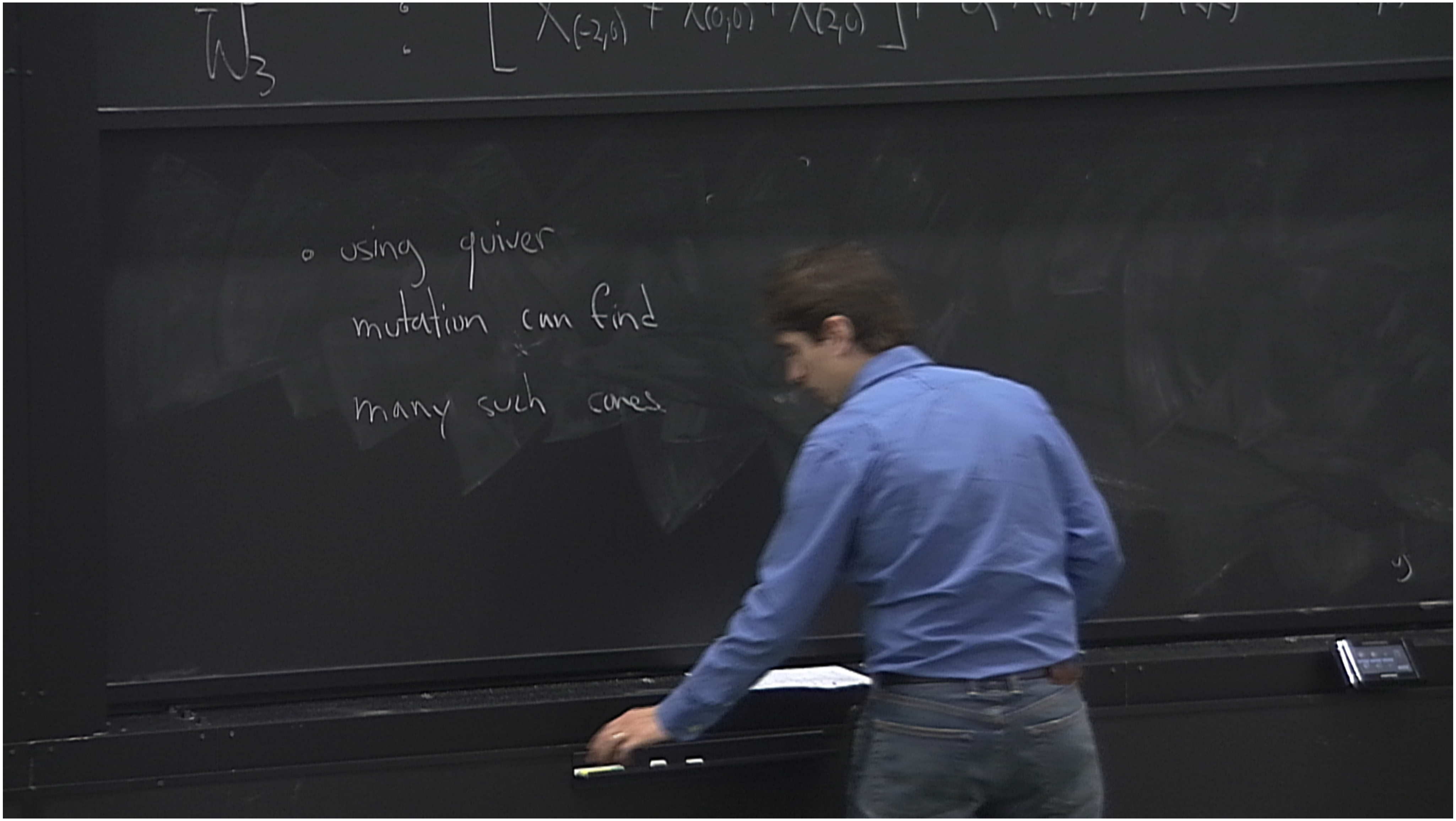
$\{\text{IR dof}\}_i = \Gamma$

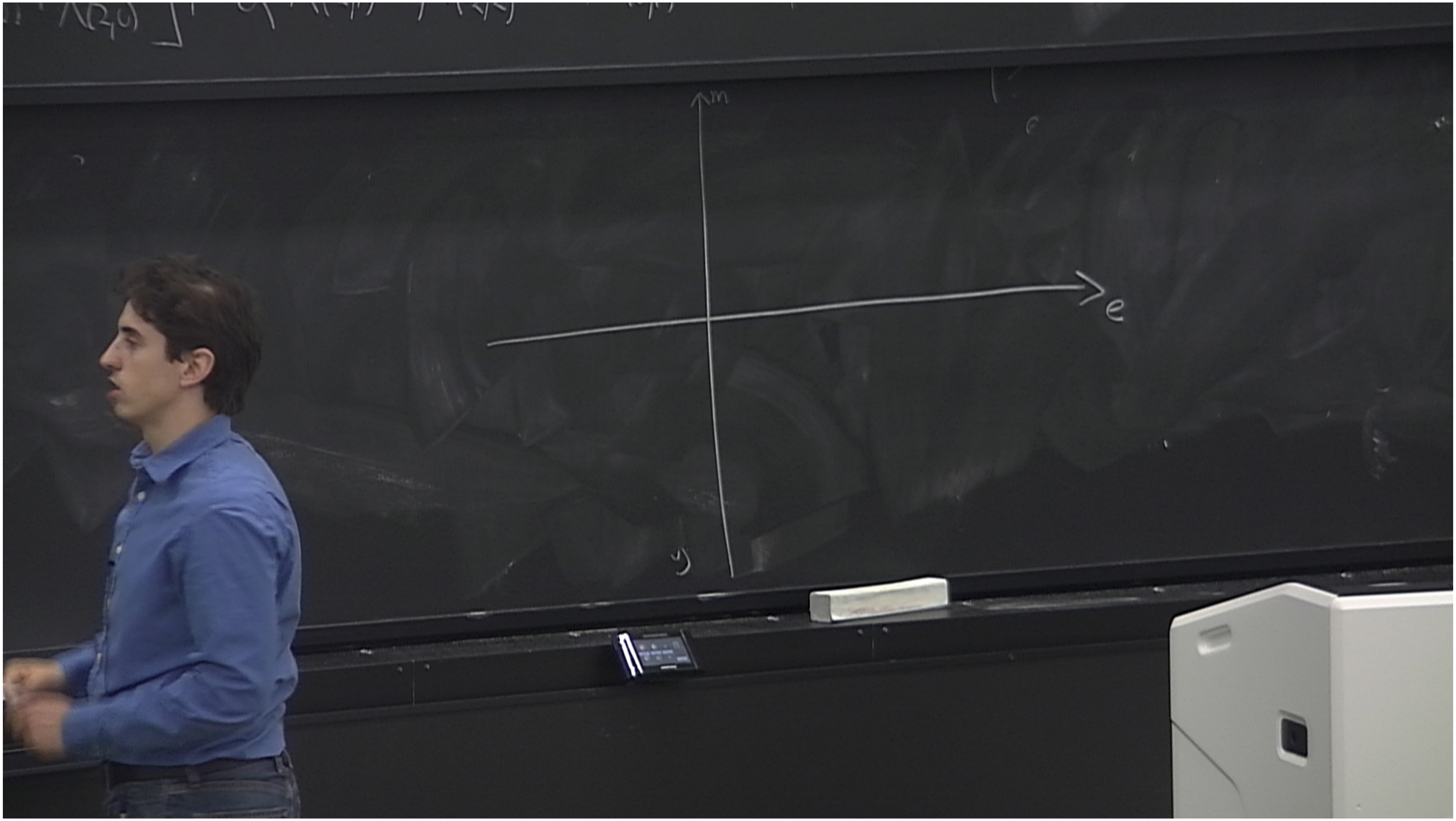
- look at set of L st $RG_v(L)\cdot\gamma_i \geq 0 \ \forall i$.

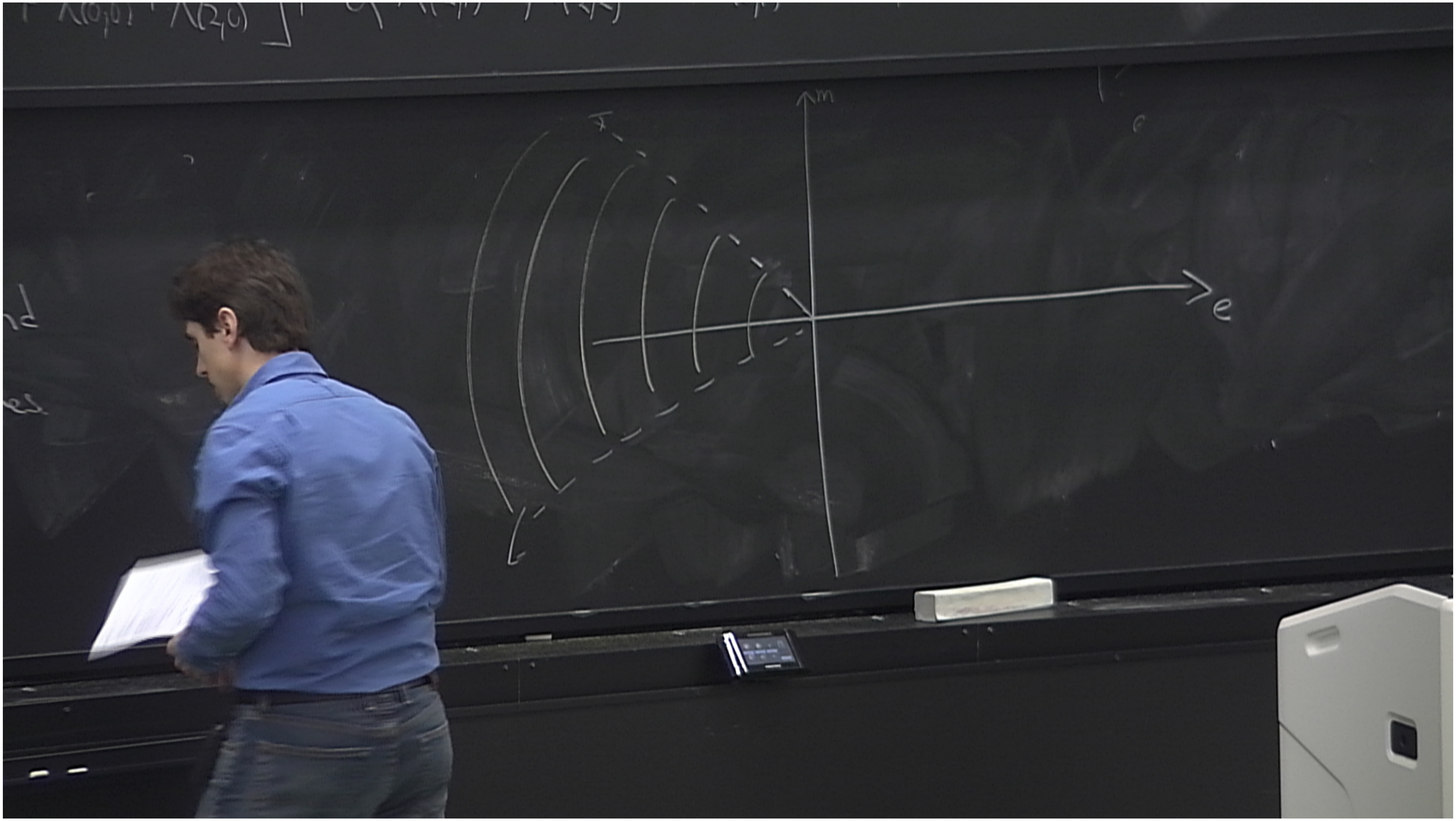
  where γ_i is a node of quiver

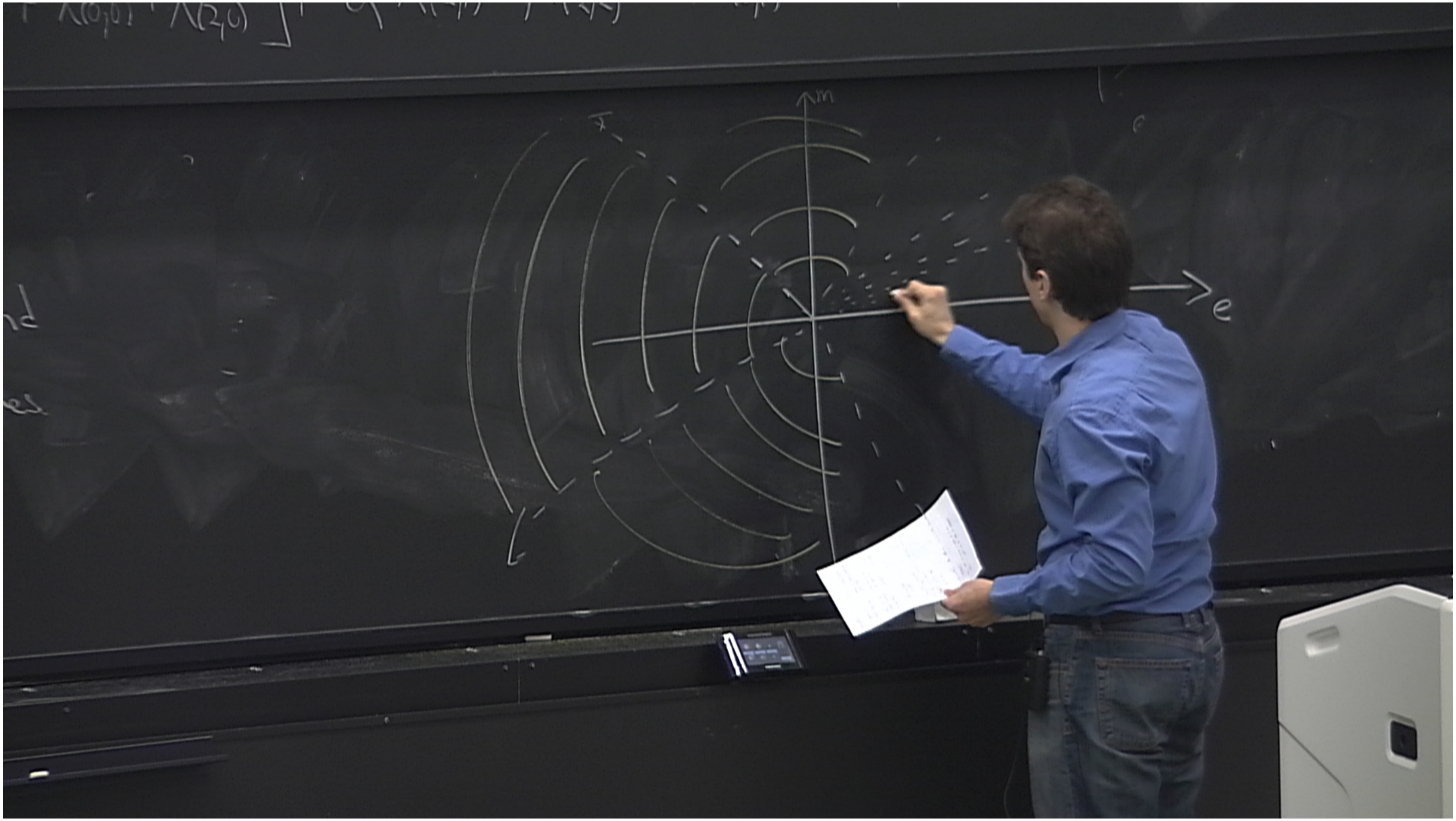
  (defines a cone of line defects $\mathcal{C}$)

- $L_1, L_2 \in \mathcal{C}$, $\quad L_1 L_2 = RG_v^{-1}\left(RG_v(L_1) + RG_v(L_2)\right)$

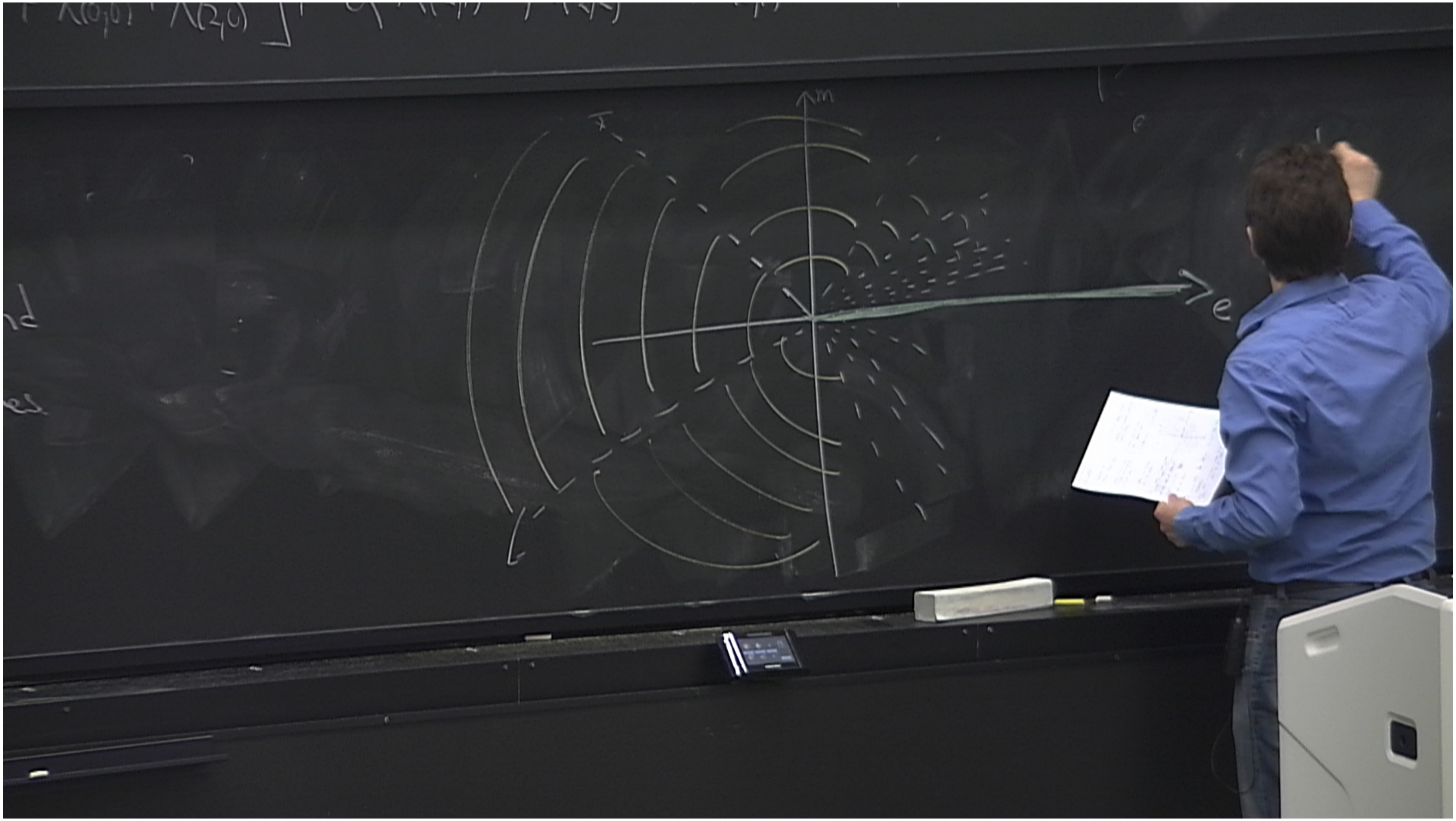

∘ using quiver
mutation can find
many such cones.

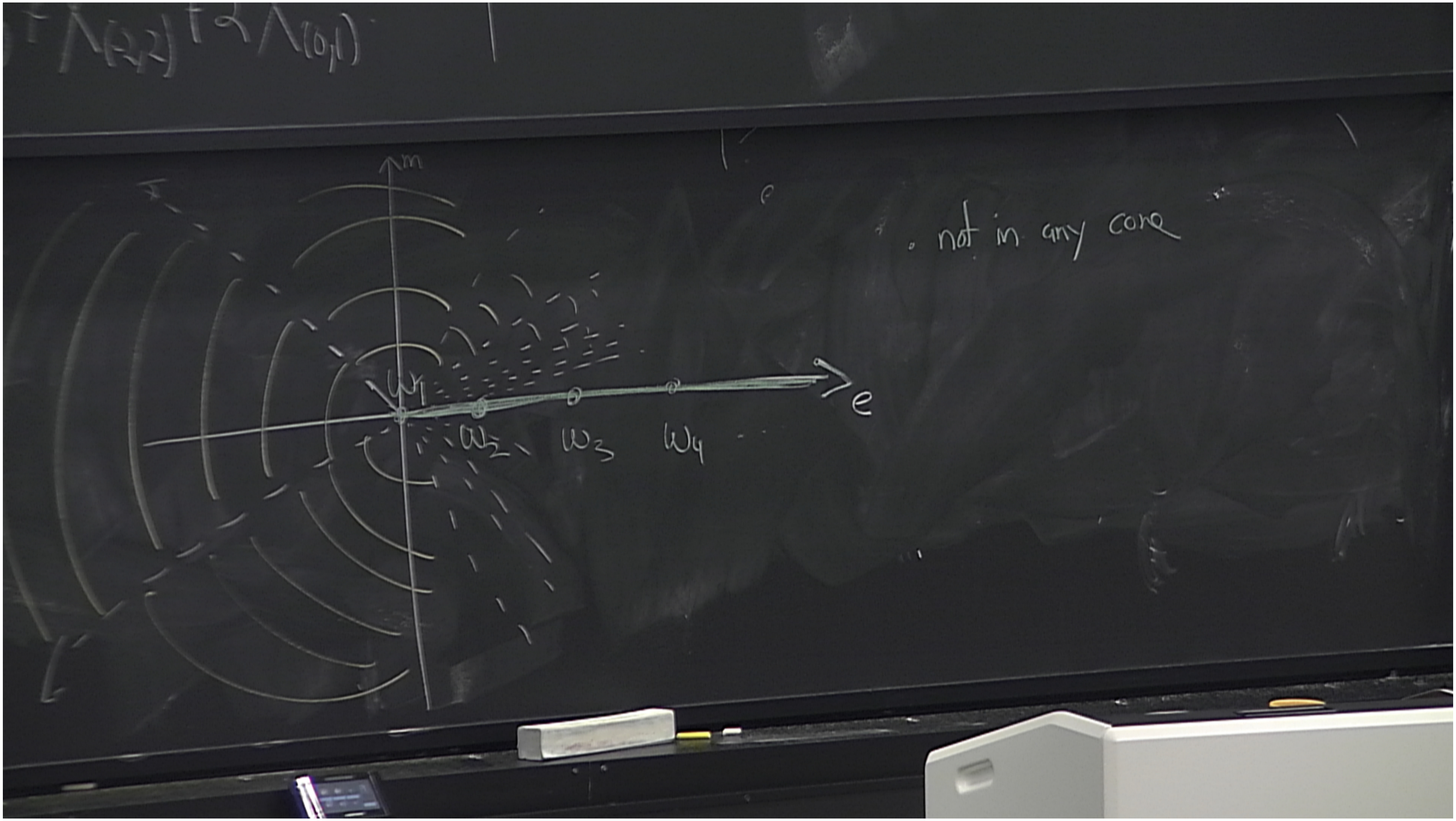

m
e
y


m
e

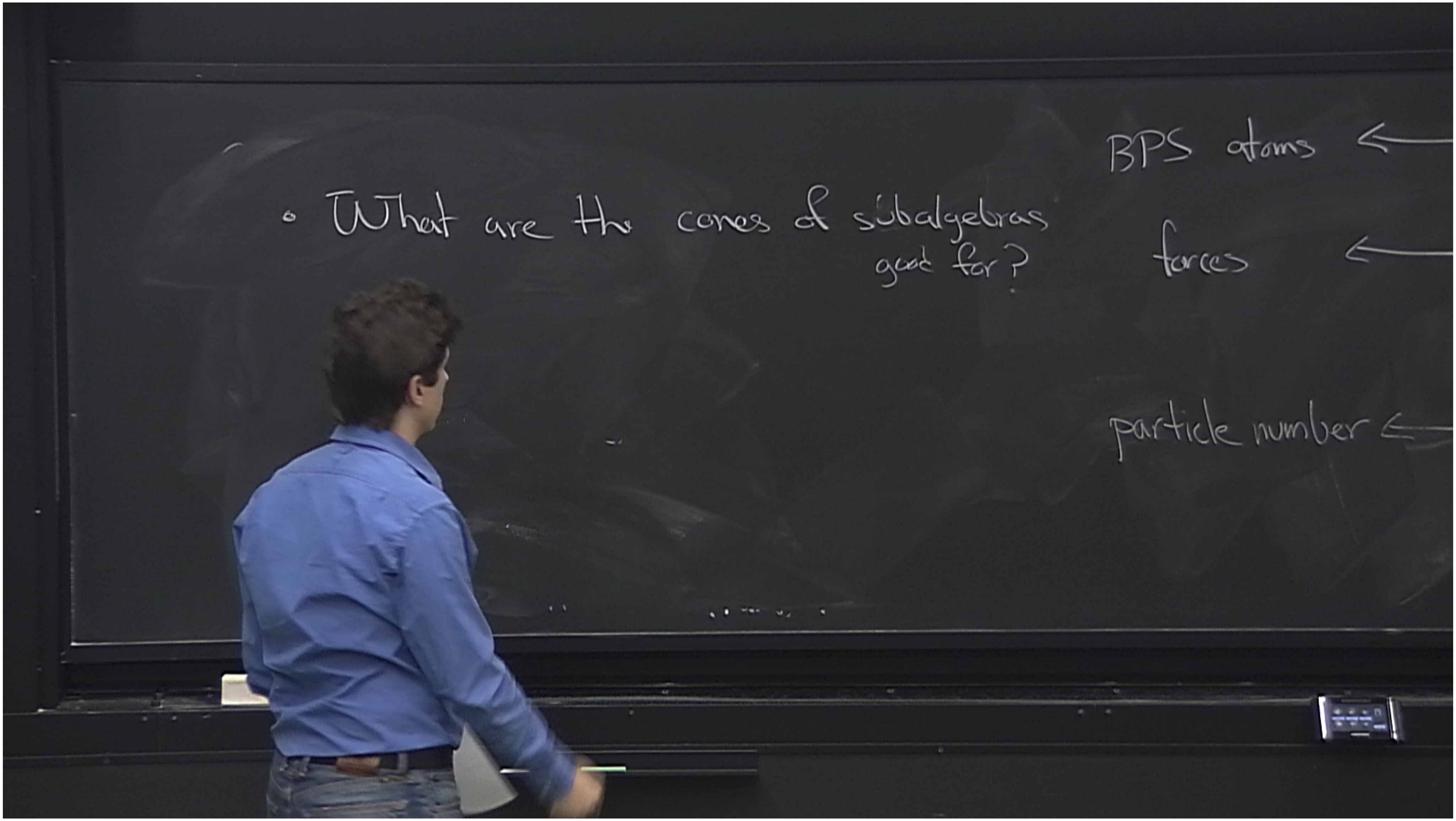


m
e

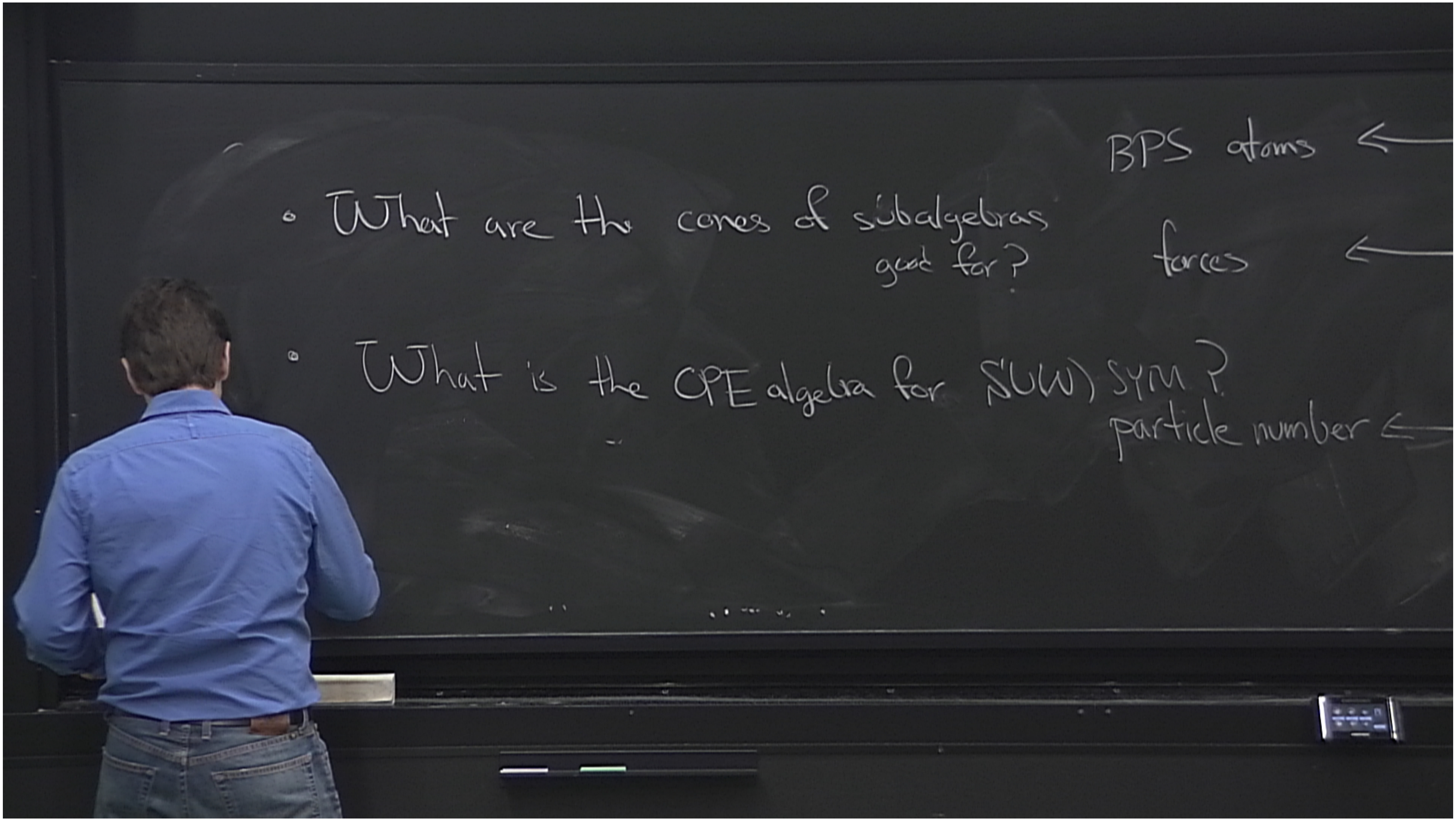

m
e

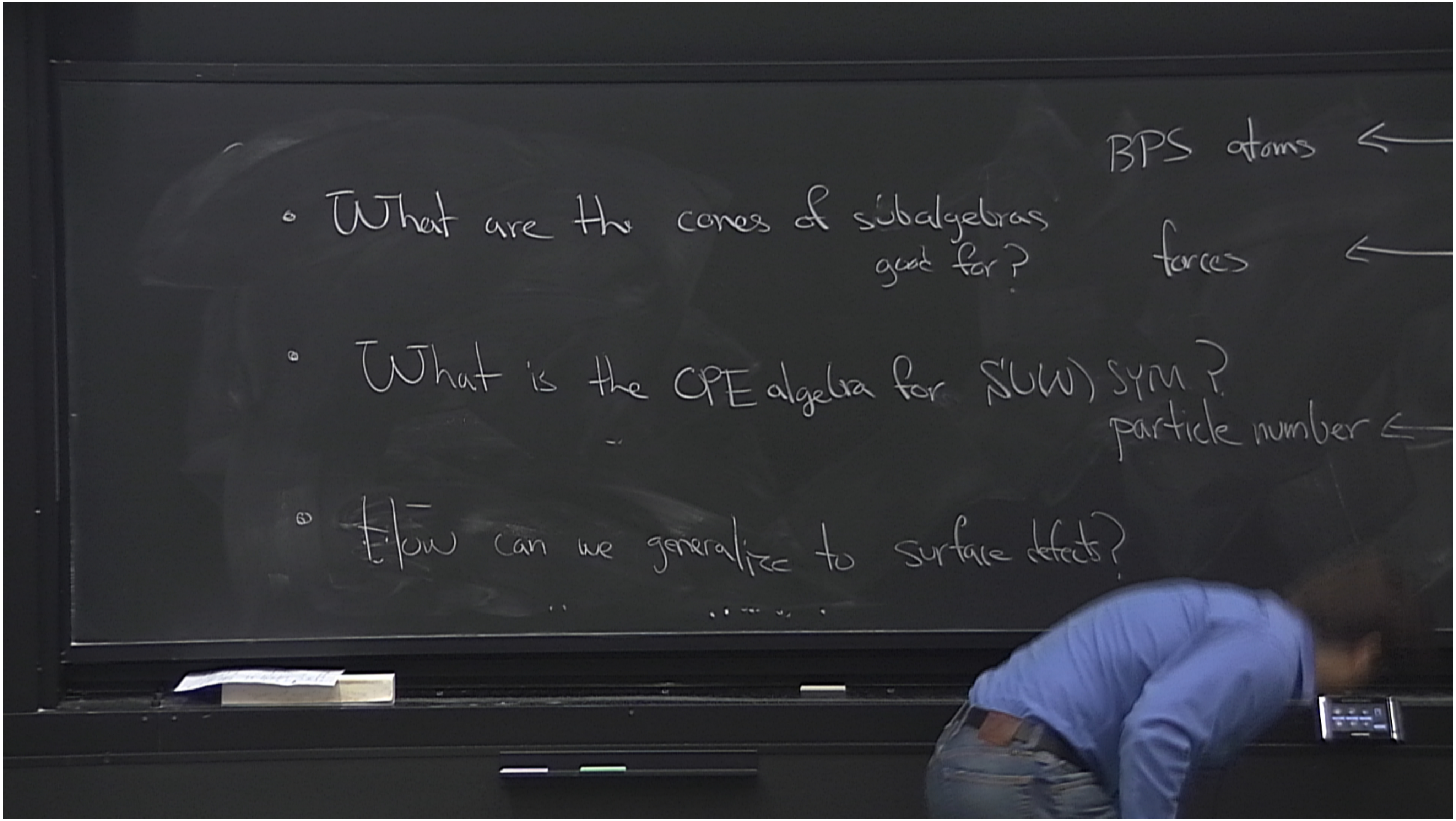


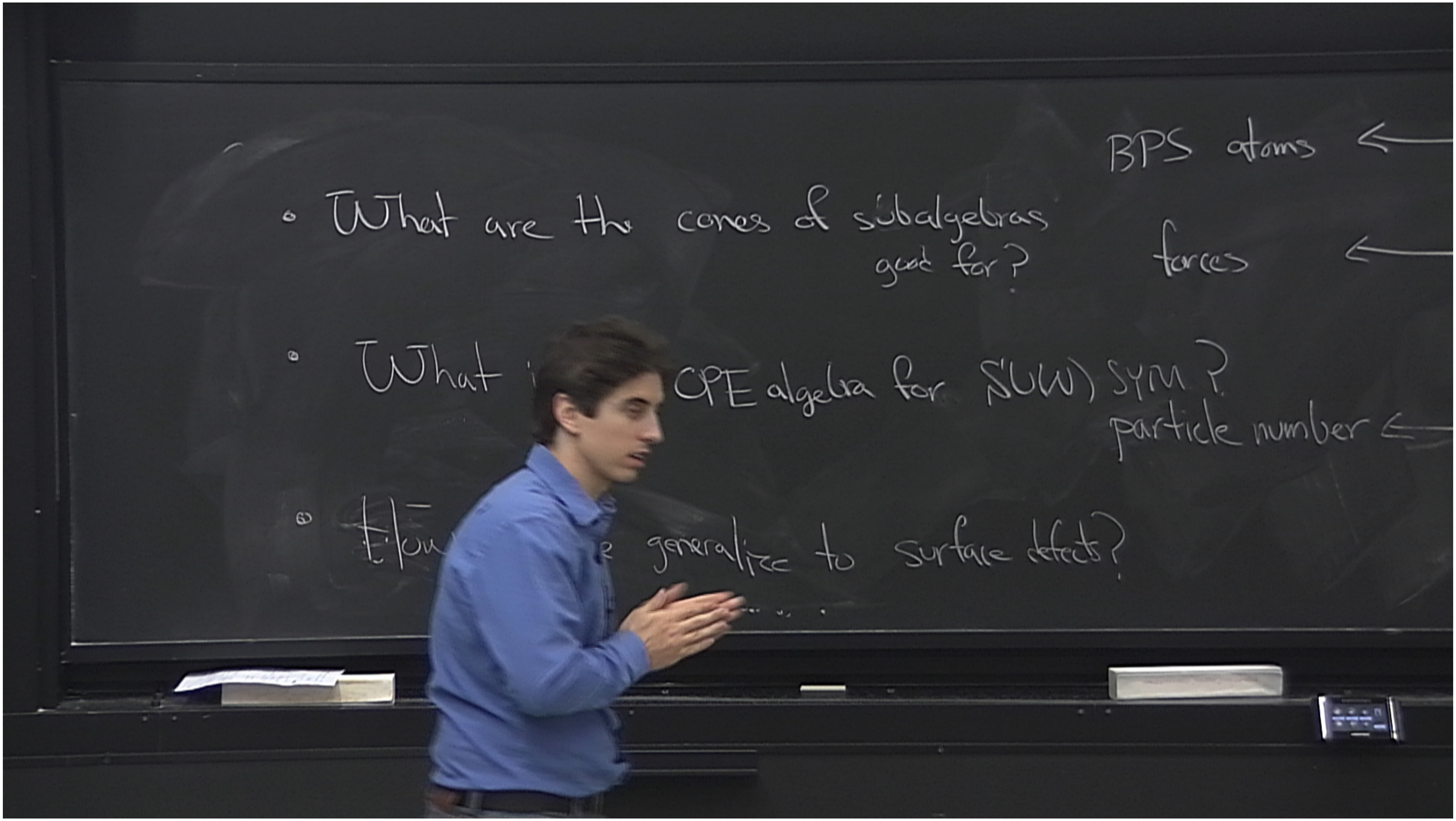
· not in any cone
m
e
ω_1
ω_2
ω_3
ω_4



• What are the cones of subalgebras good for?
BPS atoms ⟸
forces ⟸
particle number ⟸


BPS atoms
What are the cones of subalgebras good for?
forces
What is the OPE algebra for SU(N) SYM?
particle number


BPS atoms ⇐
• What are the cones of subalgebras good for?
forces ⇐
• What is the OPE algebra for SU(N) SYM?
particle number ⇐
• How can we generalize to surface defects?


BPS atoms
What are the cones of subalgebras good for?
forces
What
OPE algebra for SU(N) SYM?
particle number
generalize to surface defects?