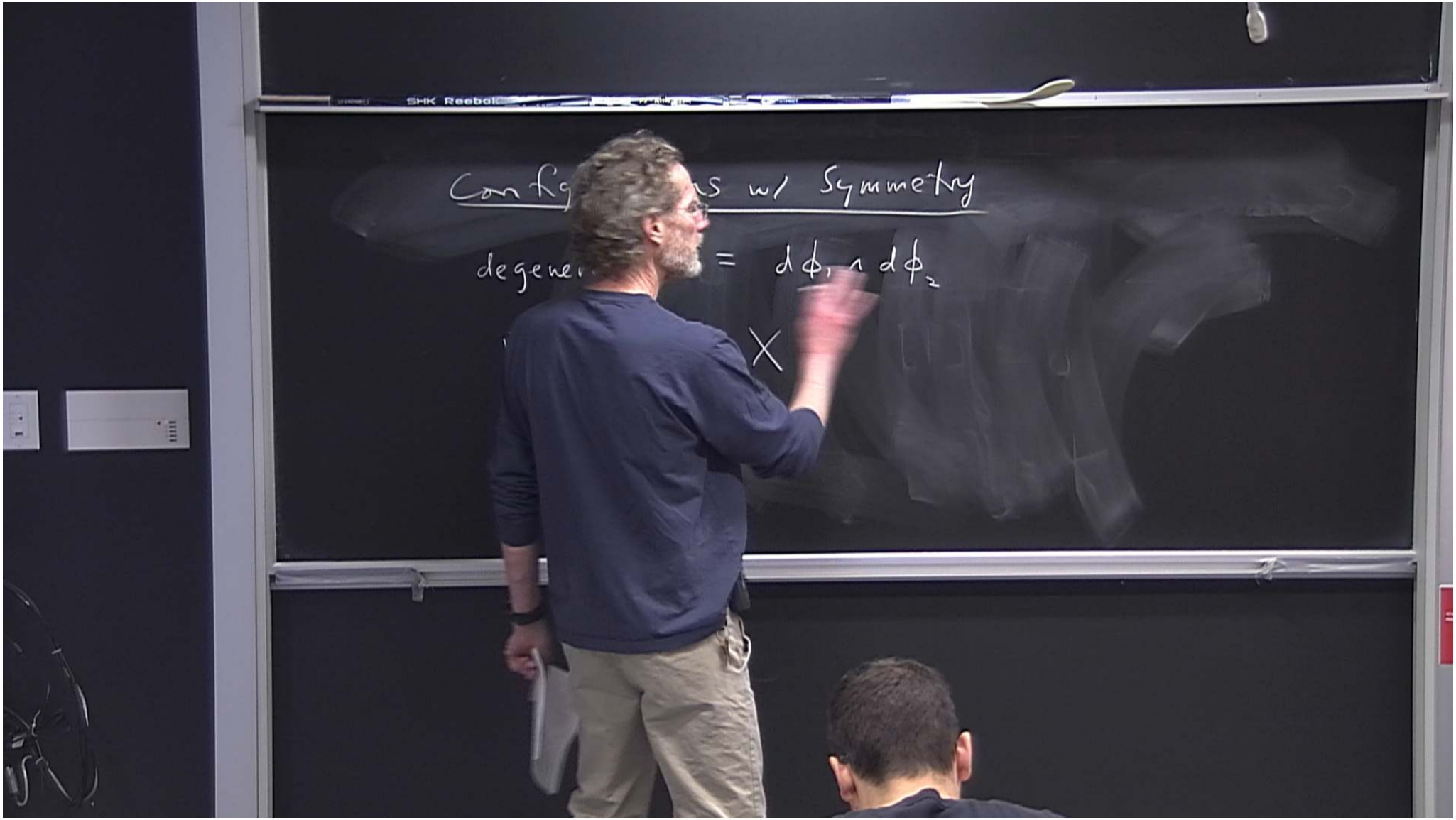
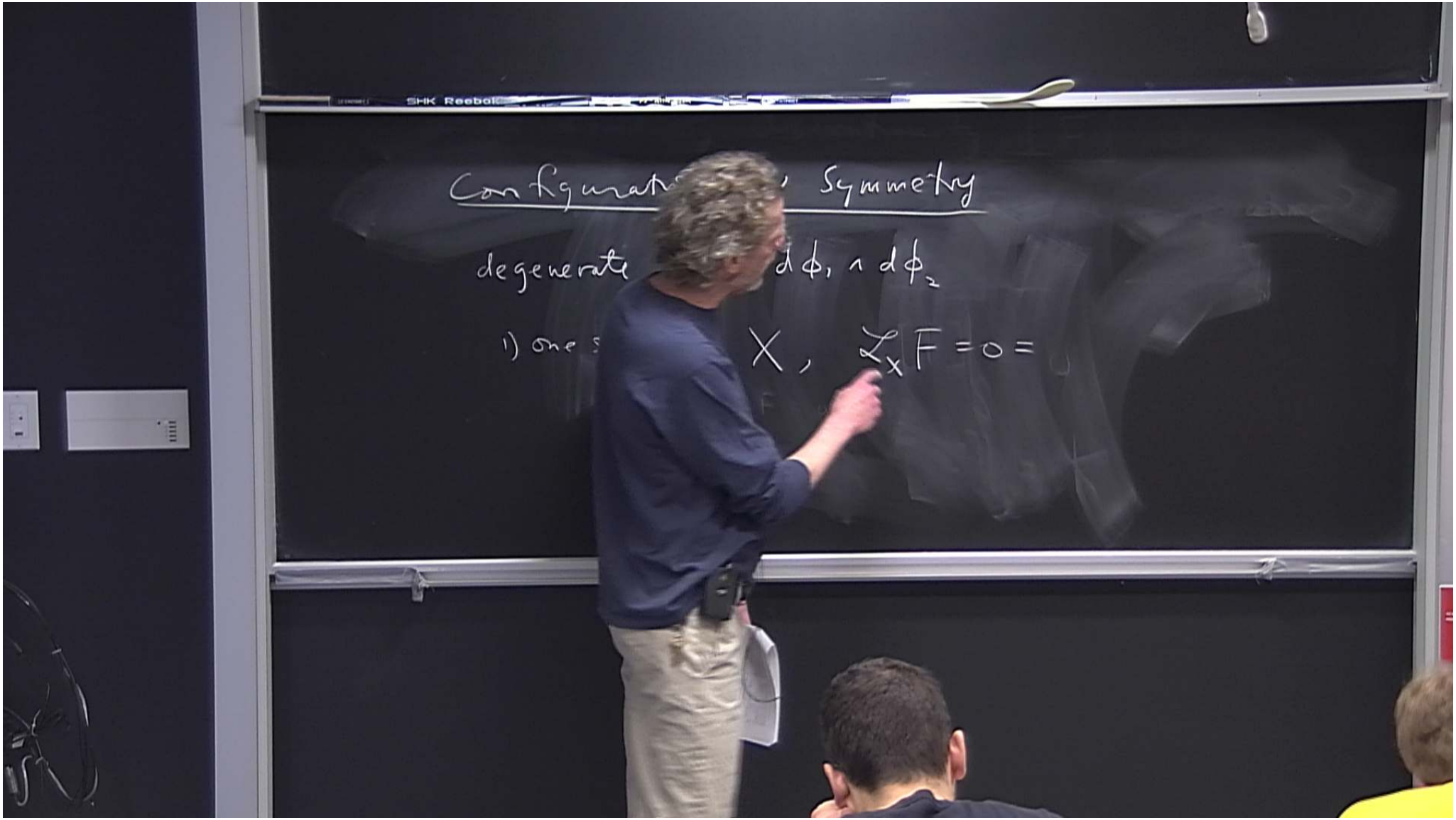
Title: Spacetime approach to force-free magnetospheres - Lecture 8

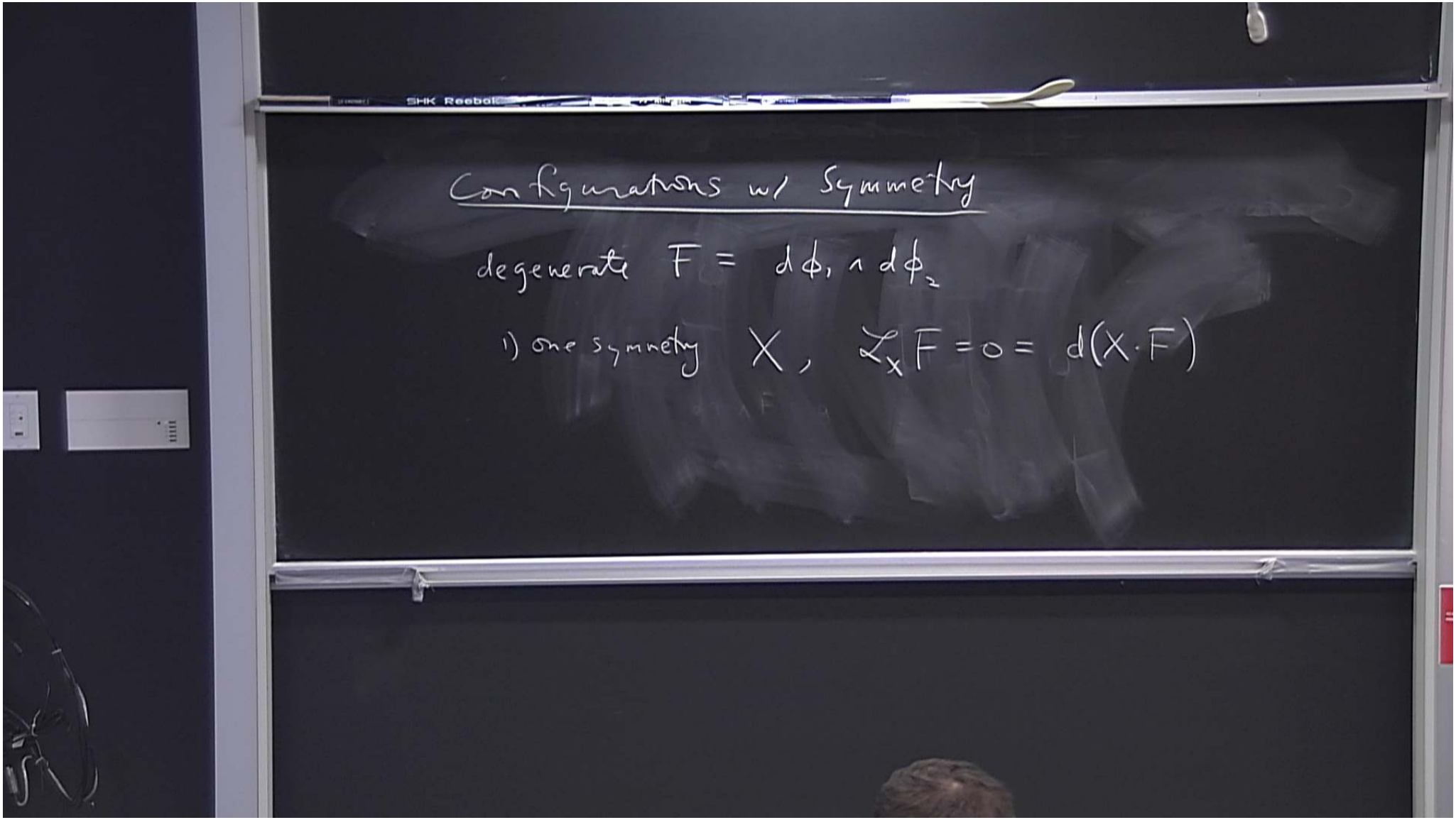
Date: Mar 05, 2014 10:30 AM

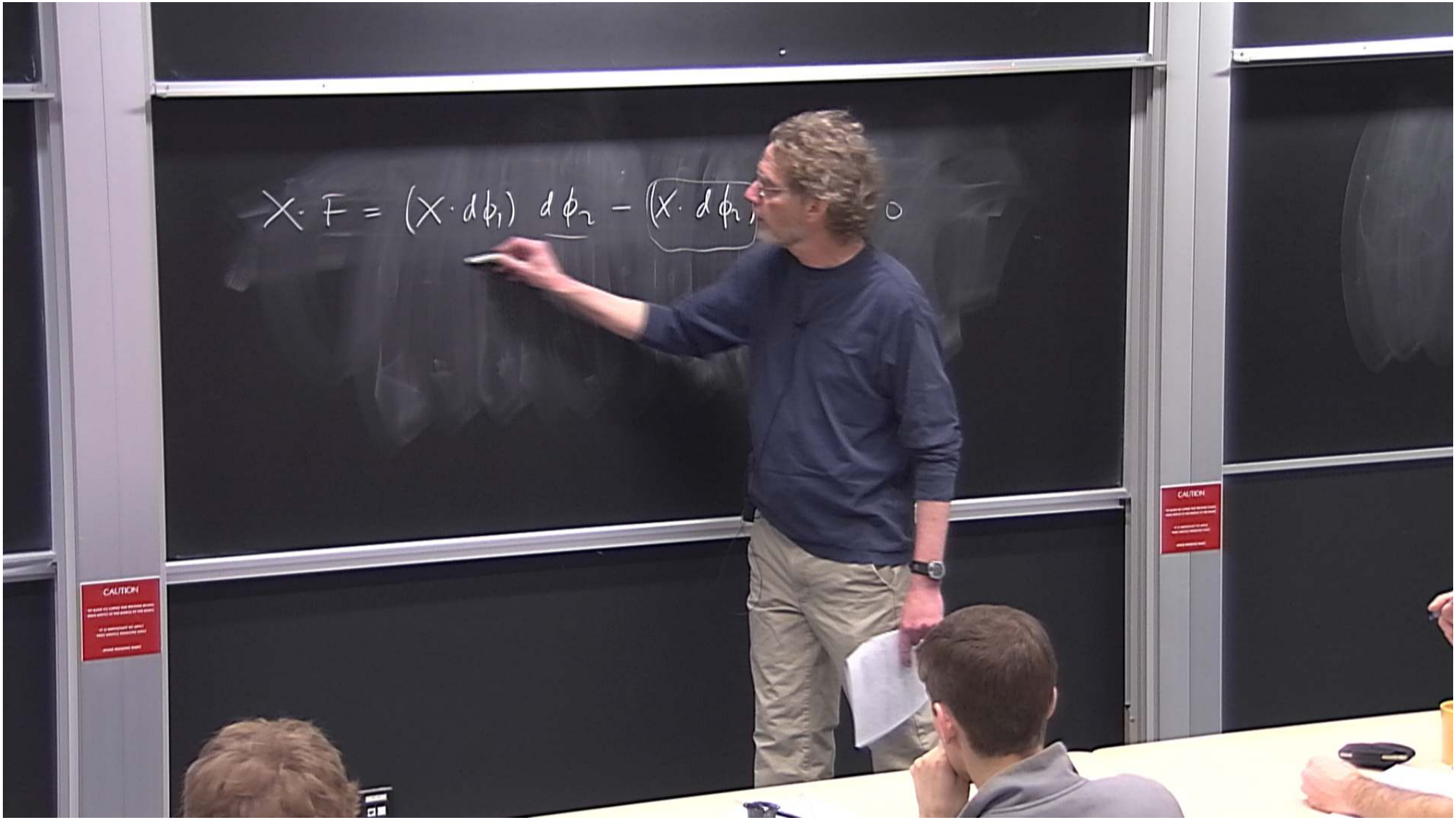
URL: http://pirsa.org/14030103

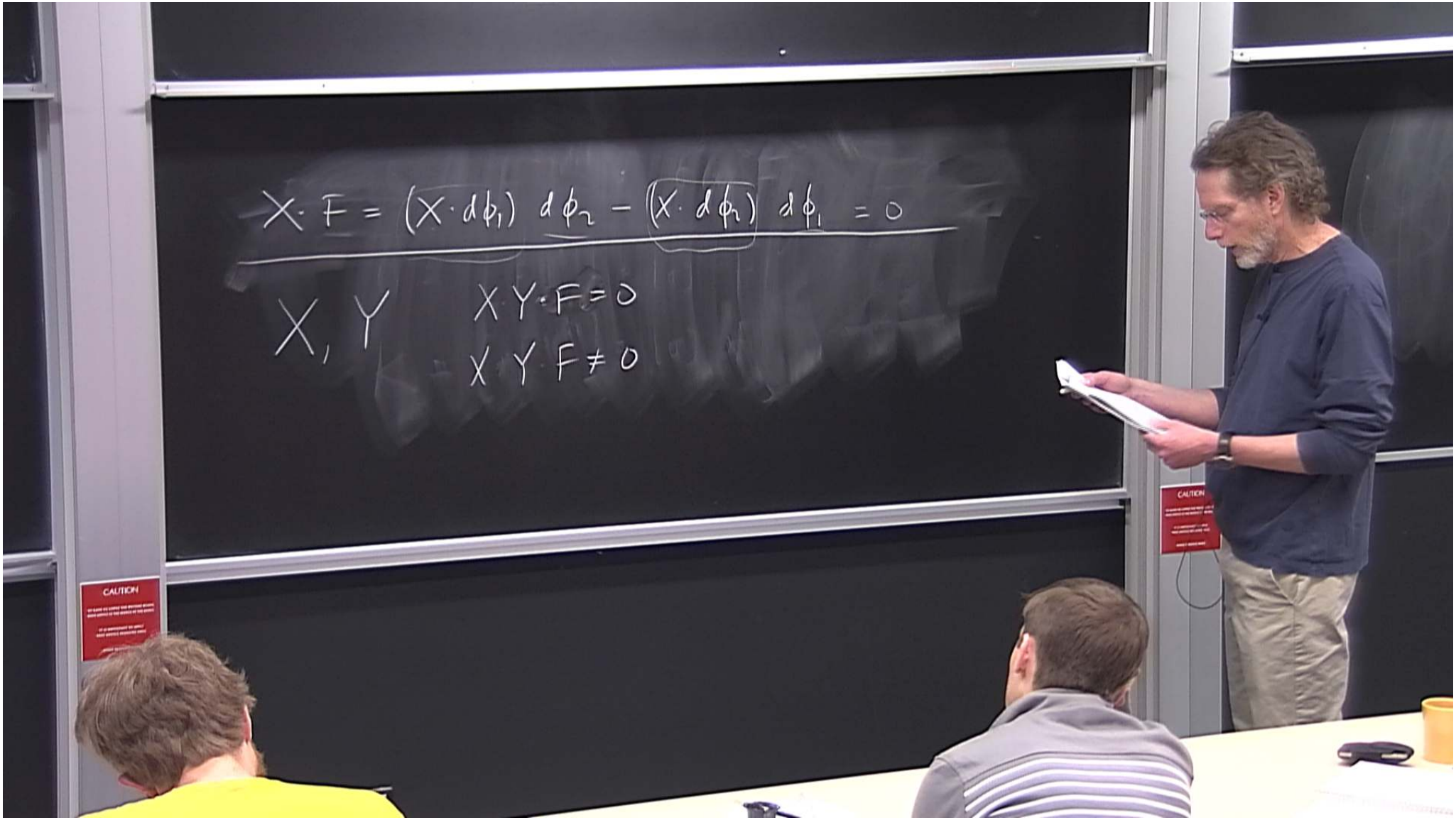
Abstract:

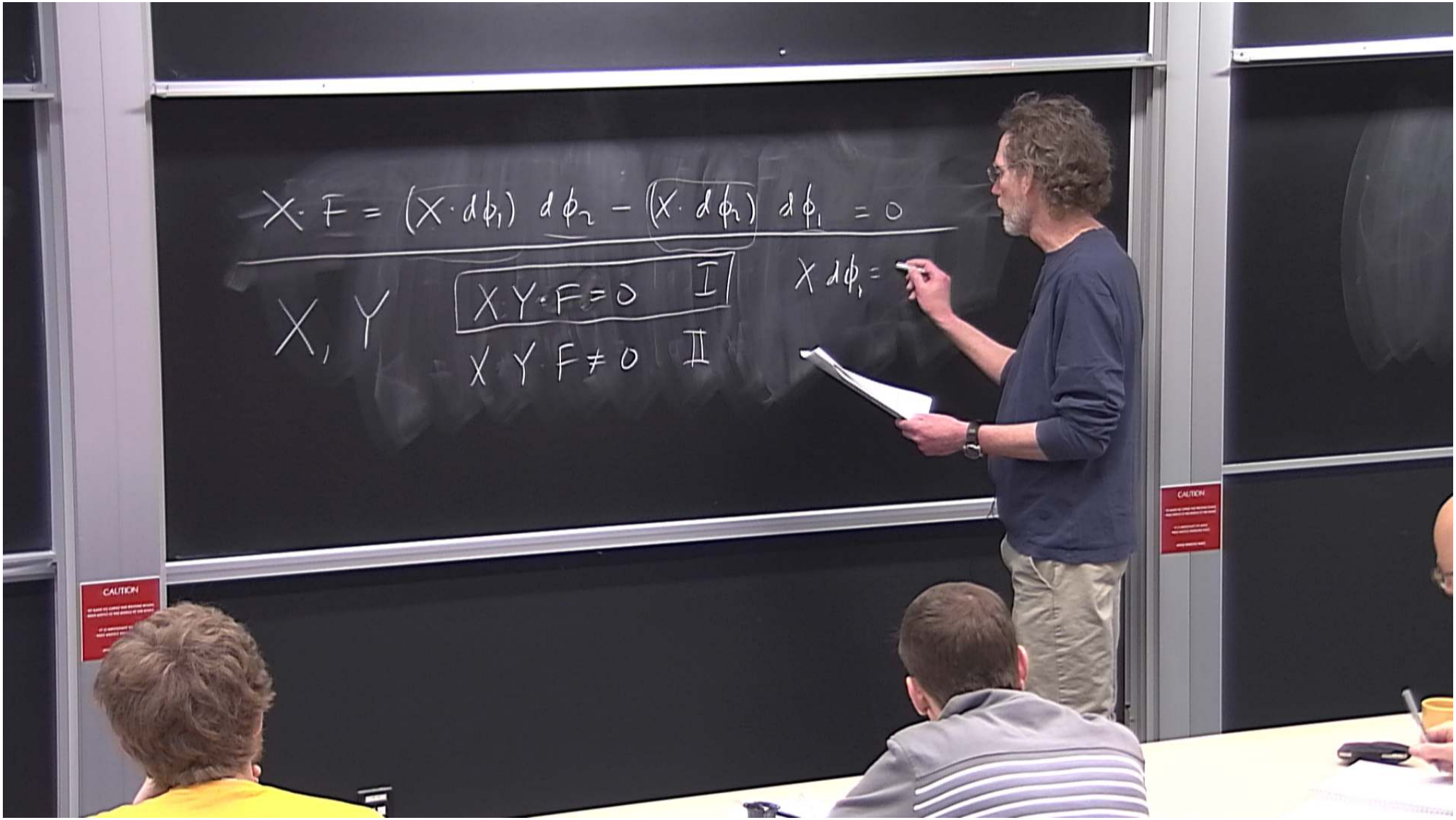

w/ Symmetry
= dφ1 ∧ dφ2
X

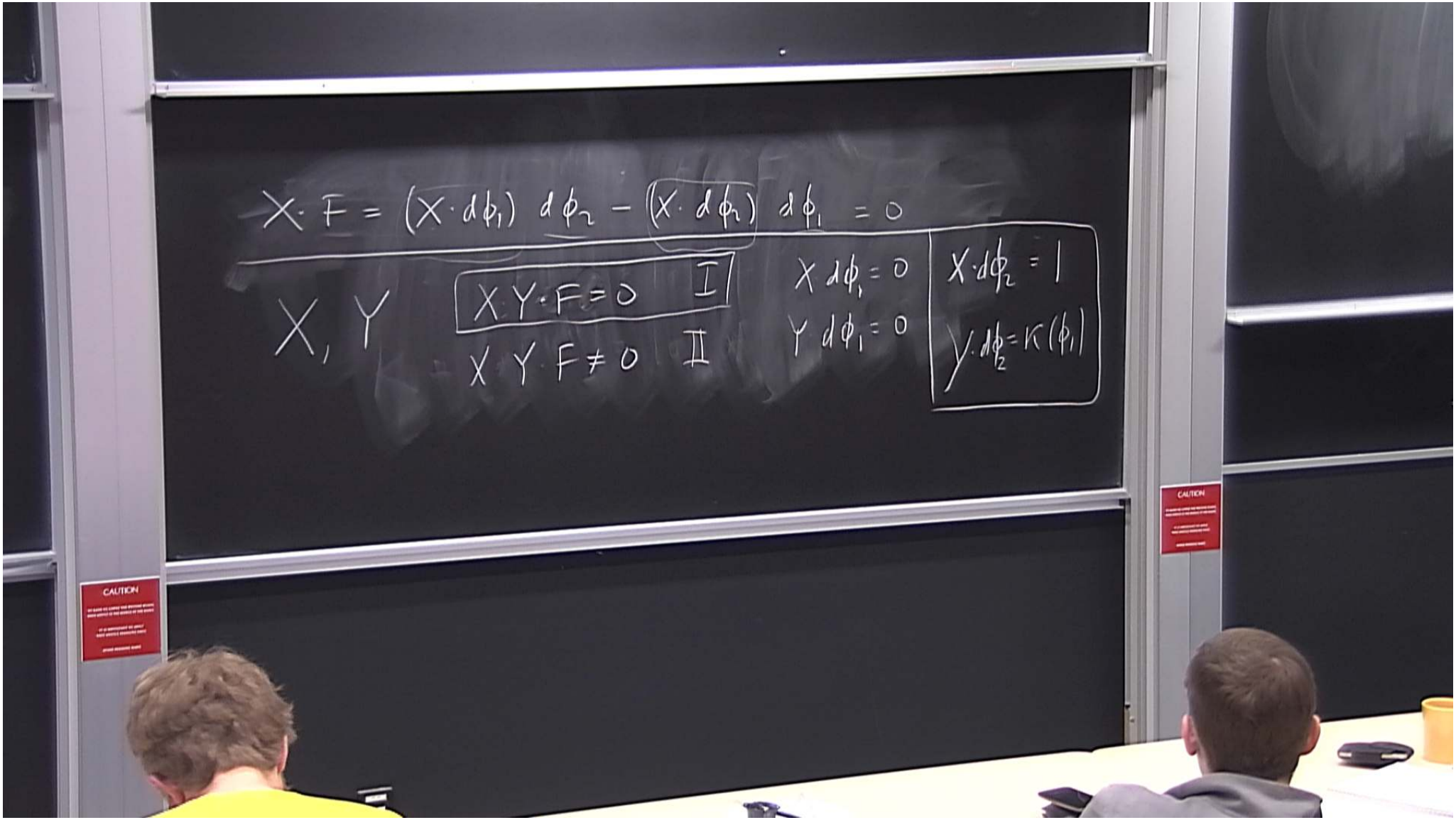

Symmetry
degenerate
$d\phi_1 \wedge d\phi_2$
1) one
X, $\mathcal{L}_X F = 0 =$

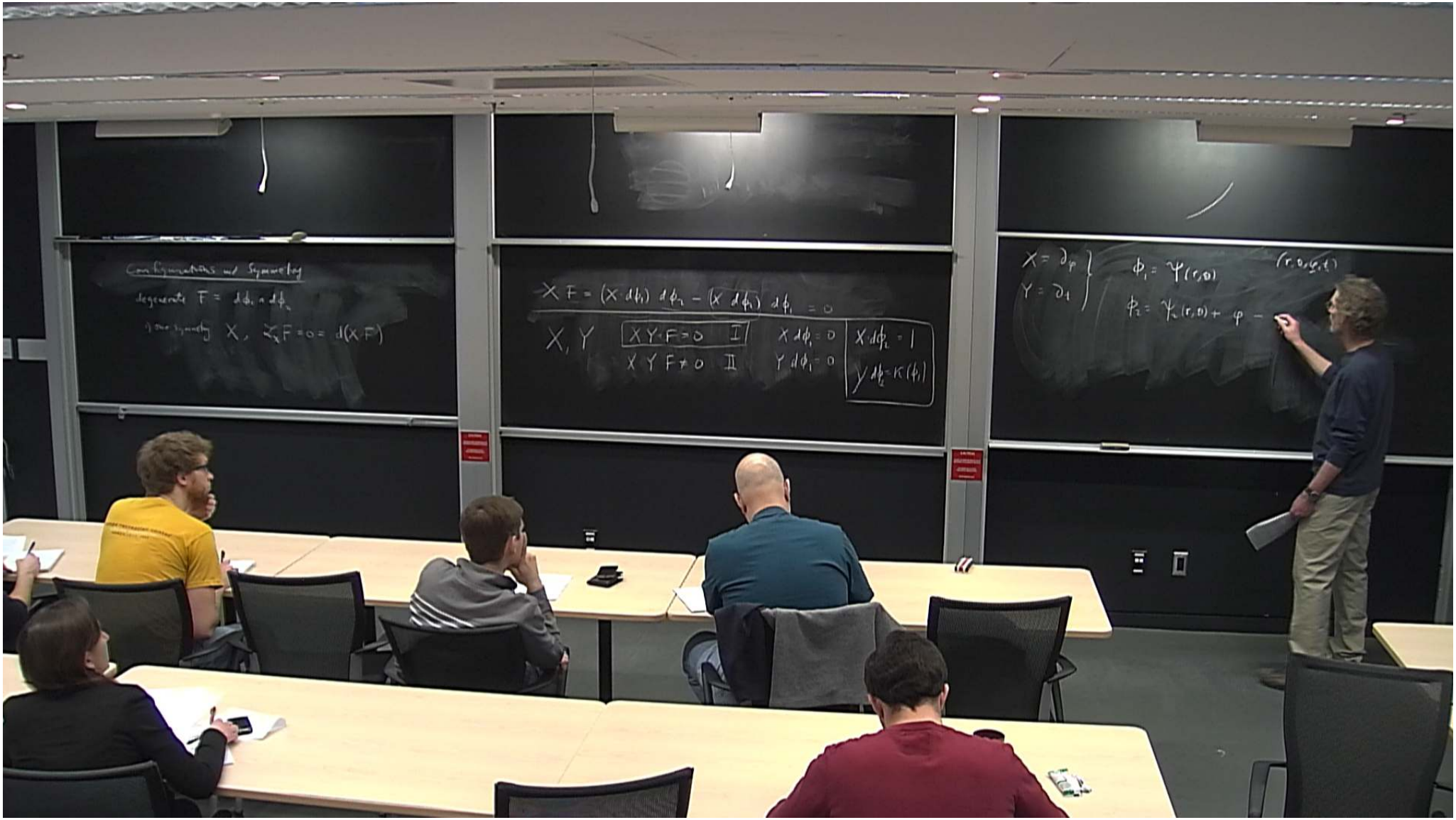

Configurations w/ Symmetry
degenerate $F = d\phi_1 \wedge d\phi_2$
1) one symmetry X, $\mathcal{L}_X F = 0 = d(X \cdot F)$

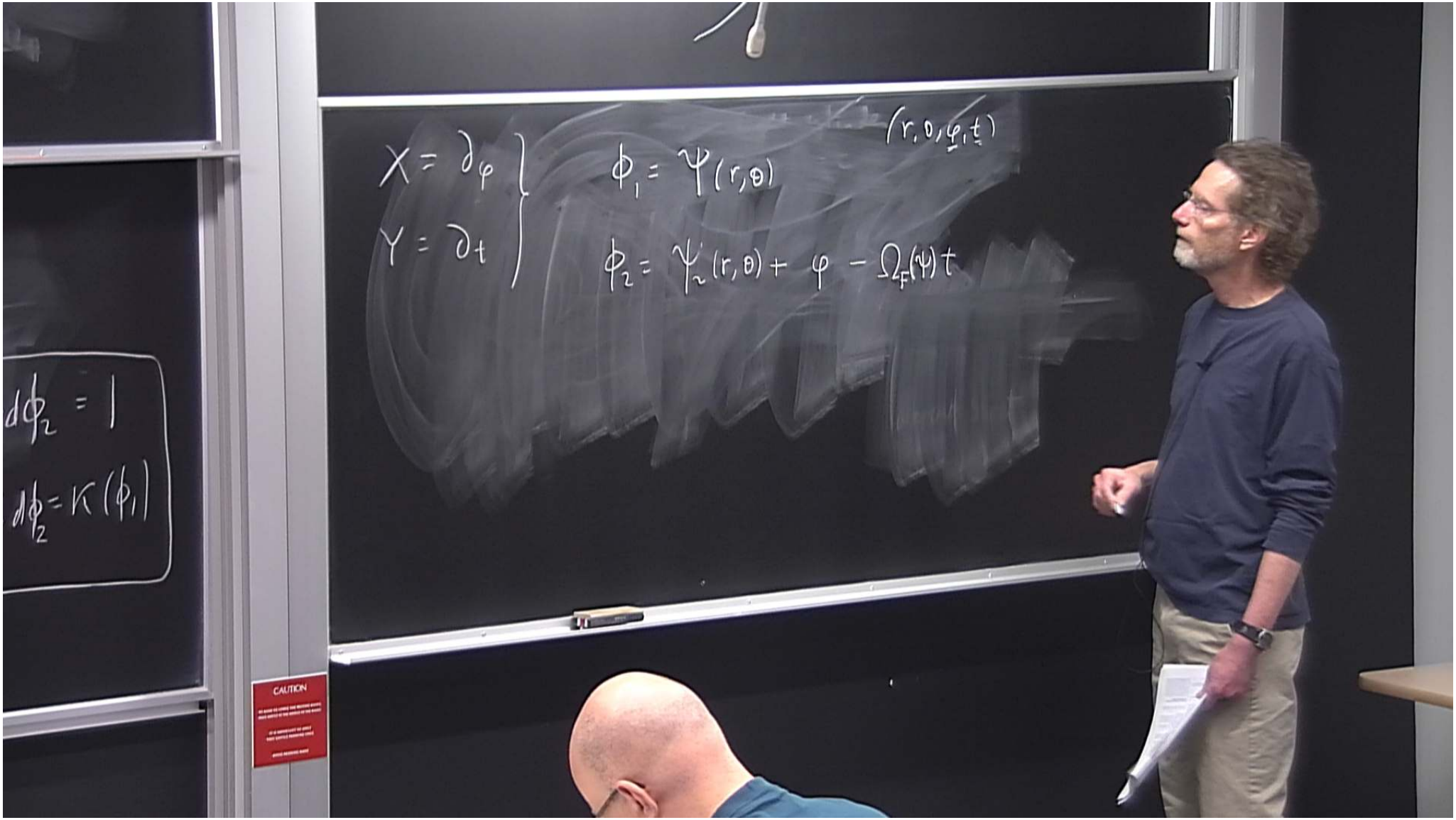

$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1$
CAUTION
CAUTION

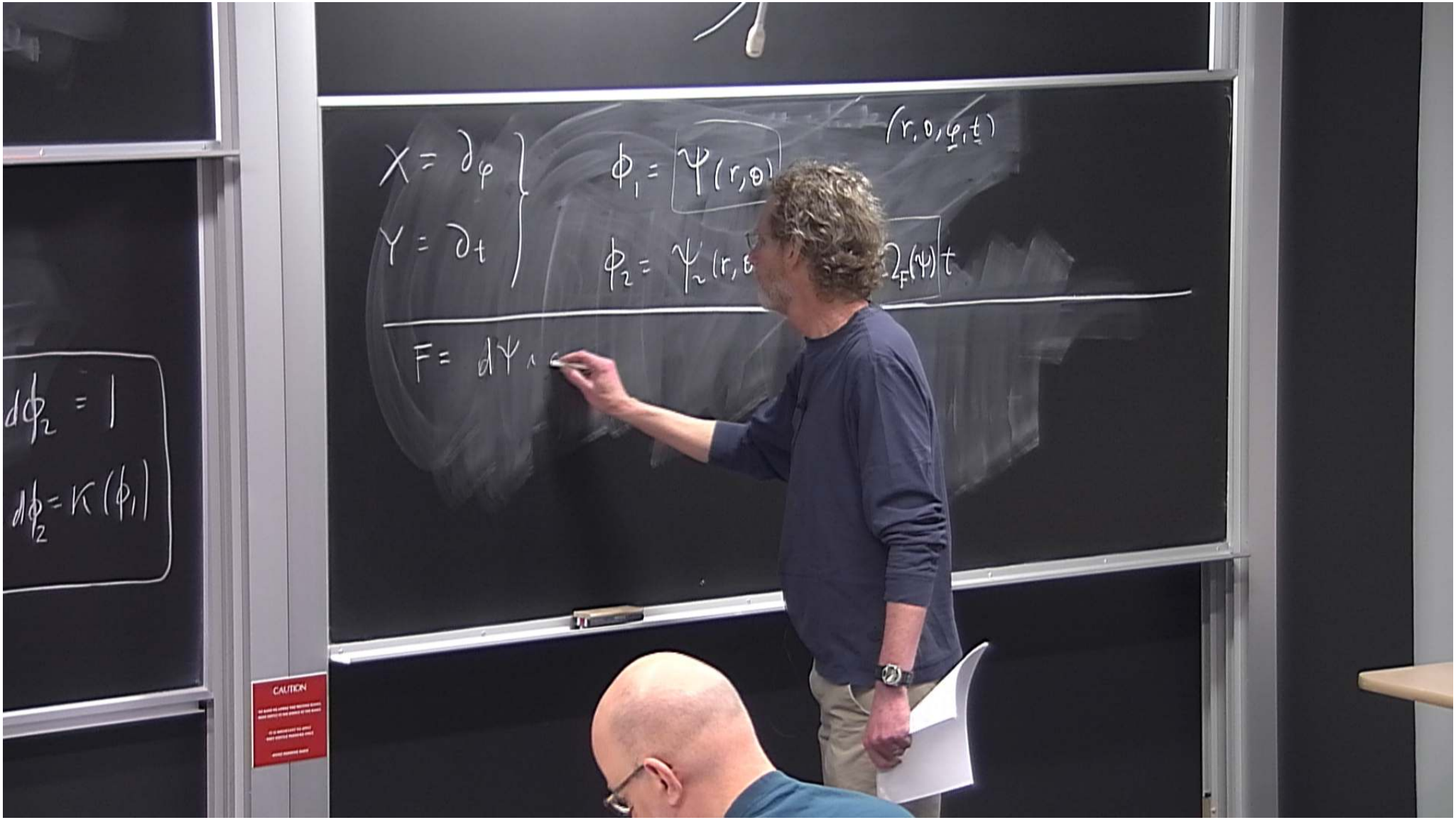

$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1 = 0$
X, Y
$XY \cdot F = 0$
$XYF \neq 0$
CAUTION
CAUTION

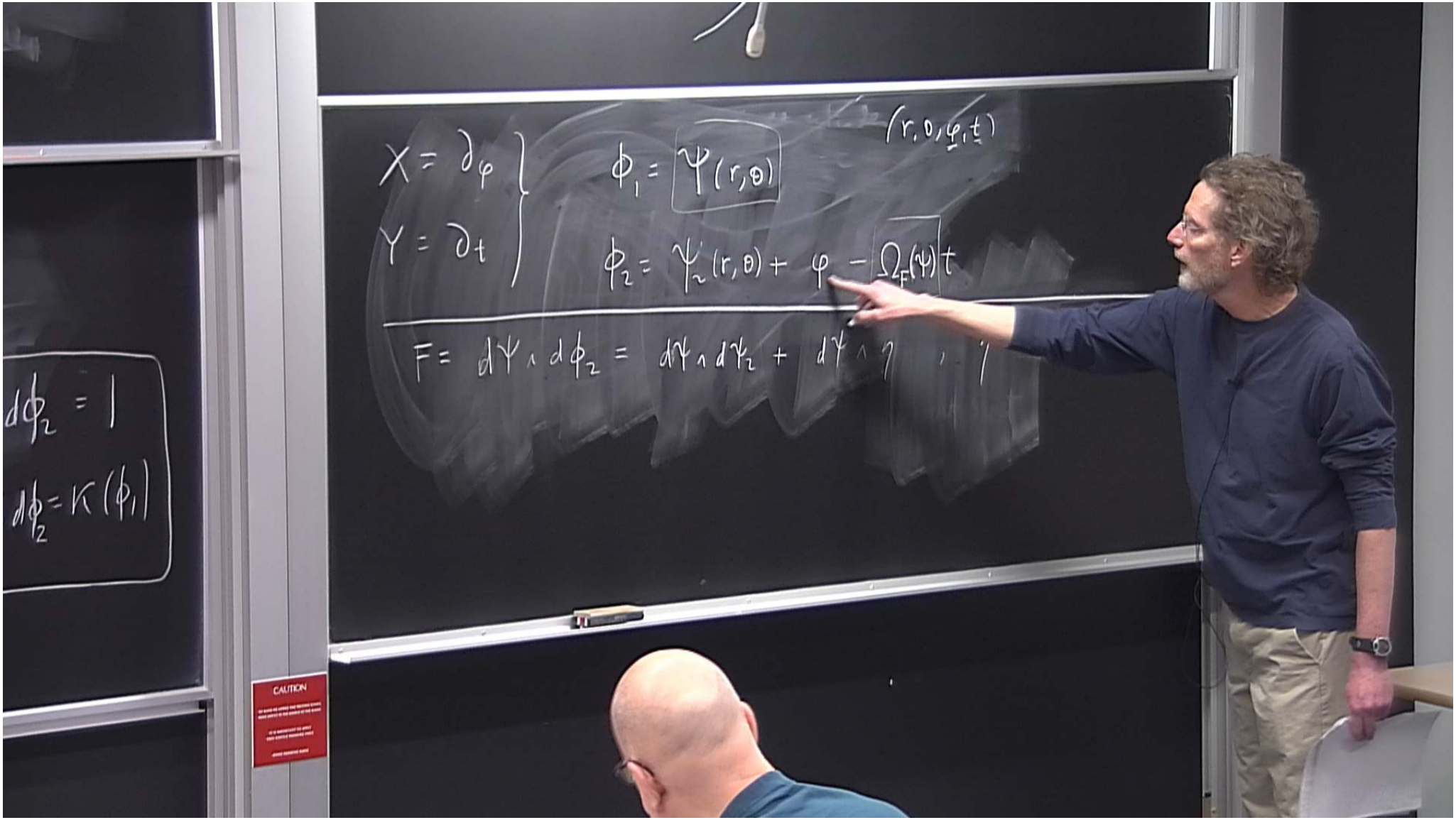

$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1 = 0$
X, Y
$XY \cdot F = 0$ I
$XY \cdot F \neq 0$ II
$X \, d\phi_1 =$
CAUTION
CAUTION

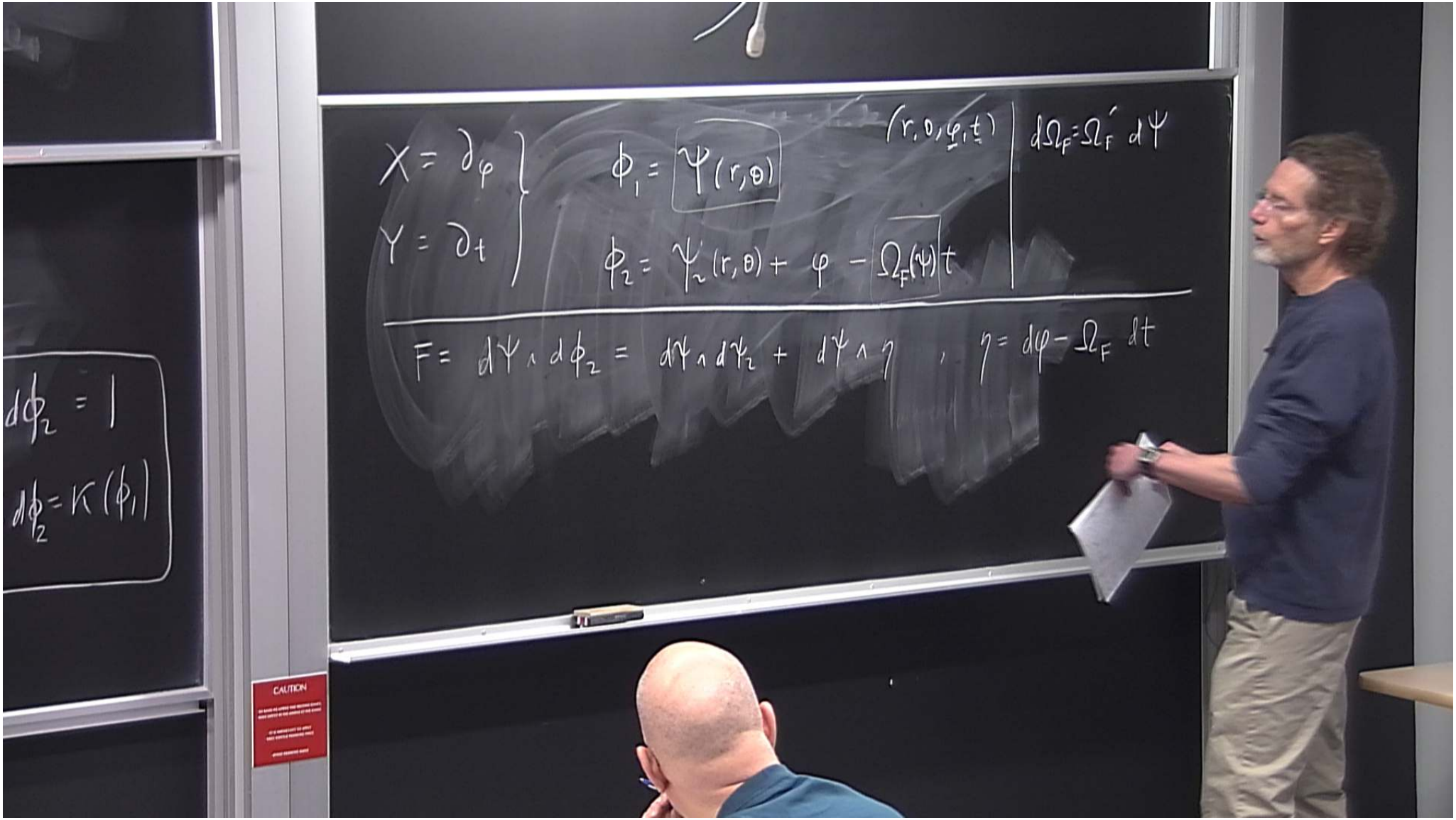

$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1 = 0$
X, Y
$XY \cdot F = 0$ I
$XY \cdot F \neq 0$ II
$X\, d\phi_1 = 0$
$Y\, d\phi_1 = 0$
$X \cdot d\phi_2 = 1$
$Y \cdot d\phi_2 = K(\phi_1)$
CAUTION
CAUTION

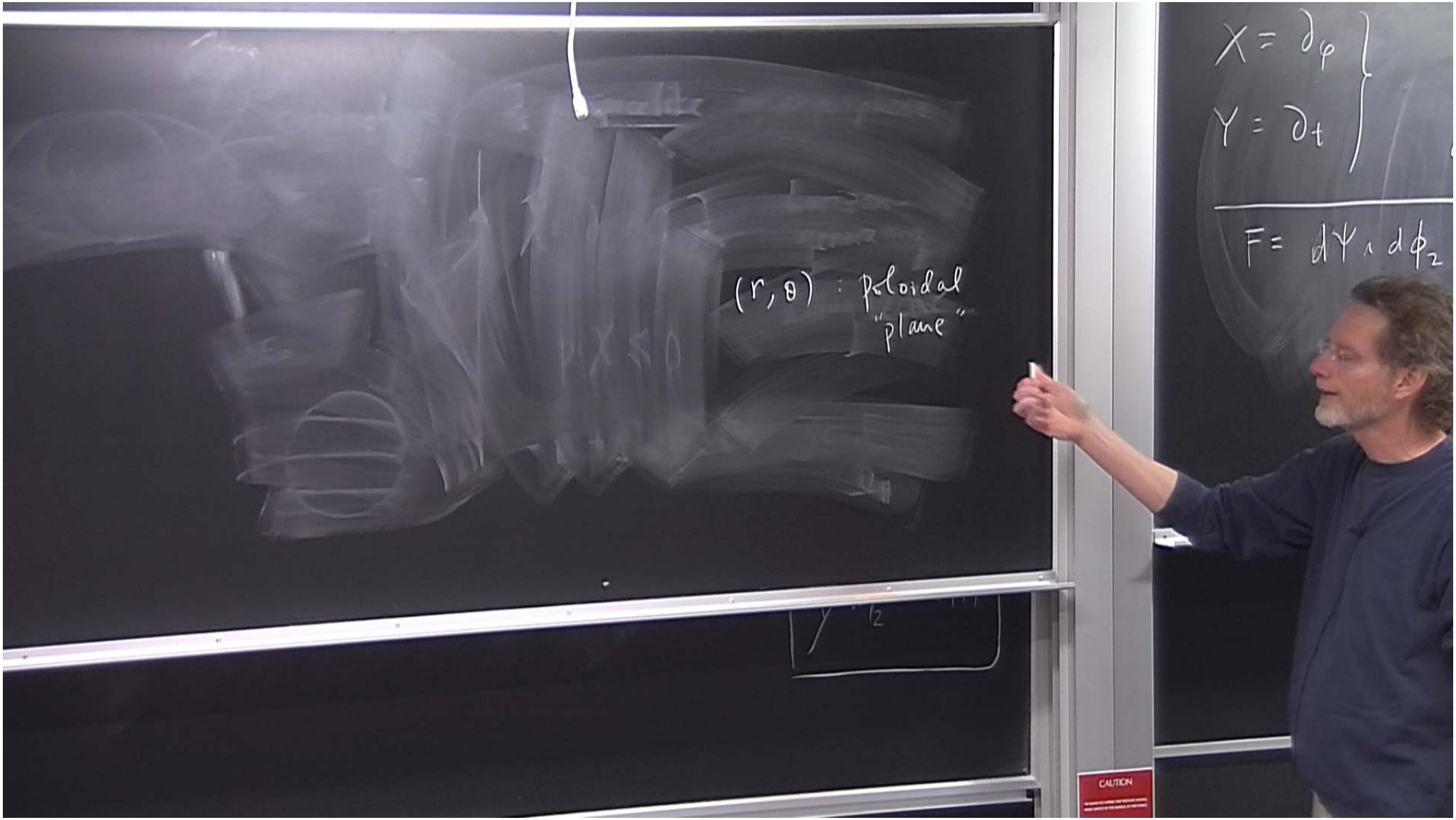

Conservation laws and Symmetry
degenerate F = dφ1 ∧ dφ2
X, Y
XY·F=0 I
XYF≠0 II
X dφ1 = 0
Y dφ1 = 0
X dφ2 = 1
Y dφ2 = K(φ1)
X = ∂φ
Y = ∂t
φ1 = ψ(r,0)
φ2 = ψ2(r,0) + φ

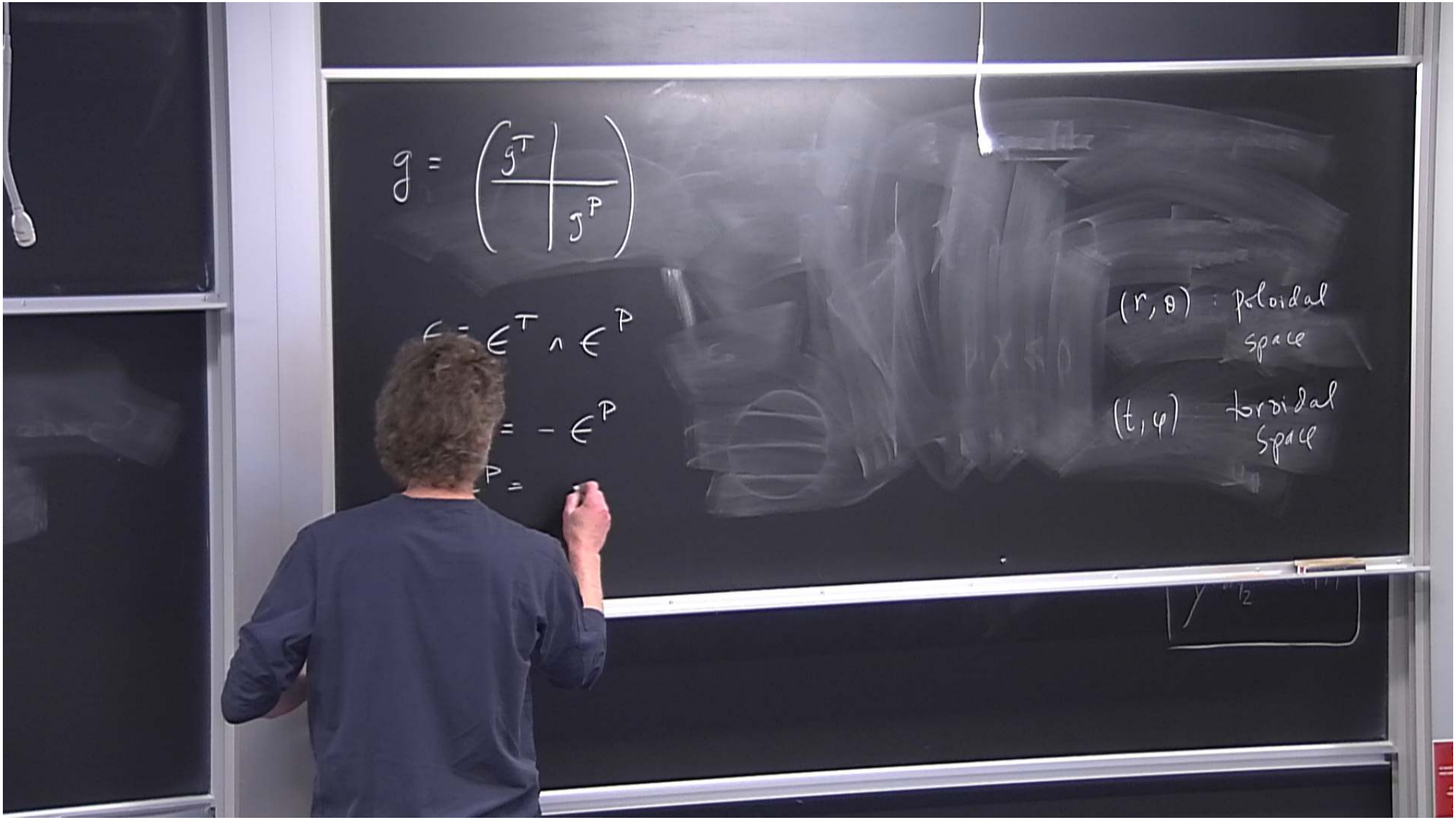

$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
(r, θ, φ, t)
$\phi_2 = \Psi_2(r,\theta) + \varphi - \Omega_F(\Psi)t$
$d\phi_2 = 1$
$d\phi_2 = K(\phi_1)$
CAUTION

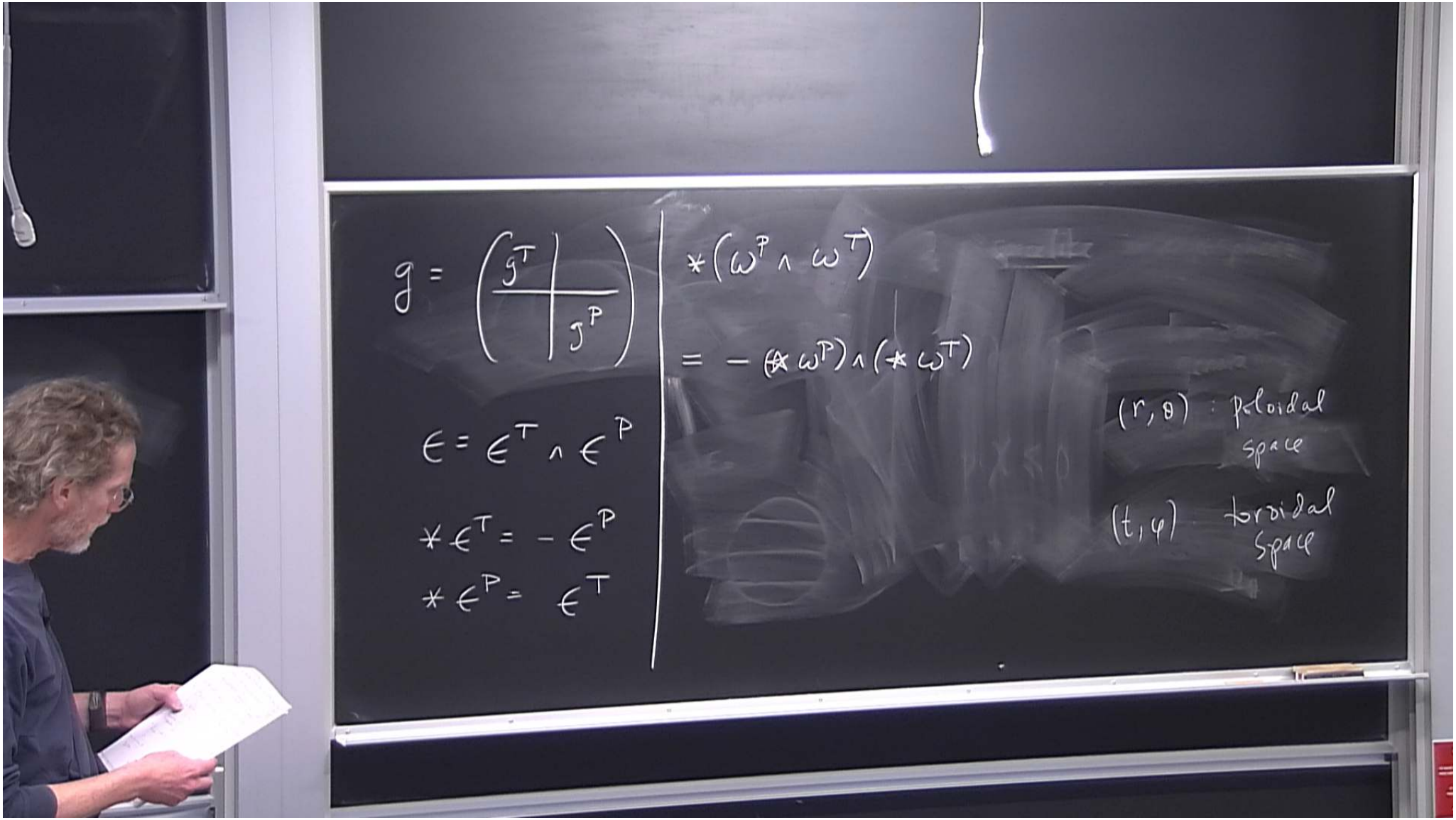

$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
(r, θ, φ, t)
$\phi_2 = \Psi_2(r,$
$-2_F(\Psi)t$
$F = d\Psi \wedge$
$d\phi_2 = 1$
$d\phi_2 = K(\phi_1)$
CAUTION

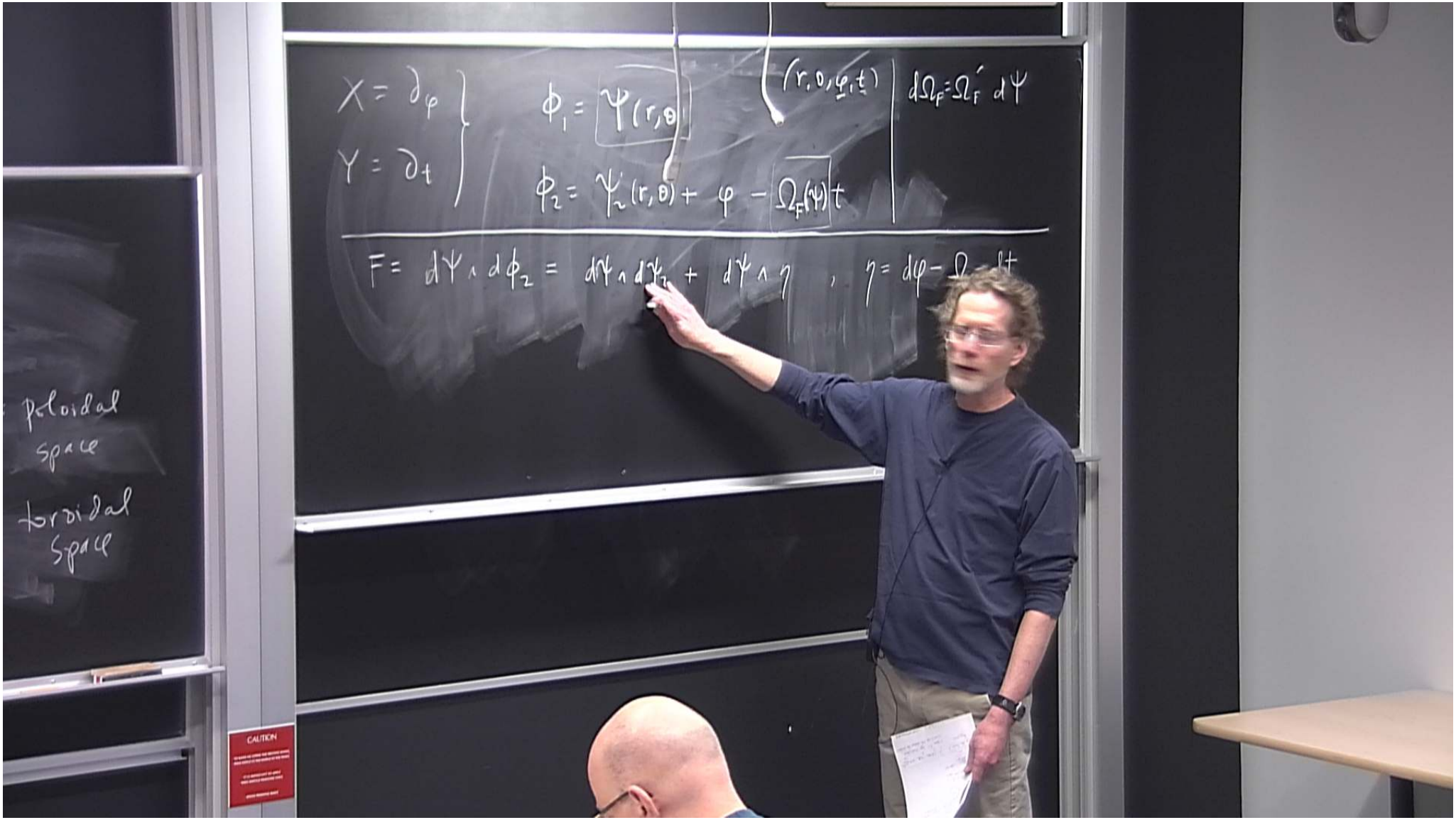

$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
(r, θ, φ, t)
$\phi_2 = \Psi_2(r,\theta) + \varphi - \Omega_F(\Psi) t$
$F = d\Psi \wedge d\phi_2 = d\Psi \wedge d\Psi_2 + d\Psi \wedge$
$d\phi_2 = 1$
$d\phi_2 = \kappa(\phi_1)$
CAUTION

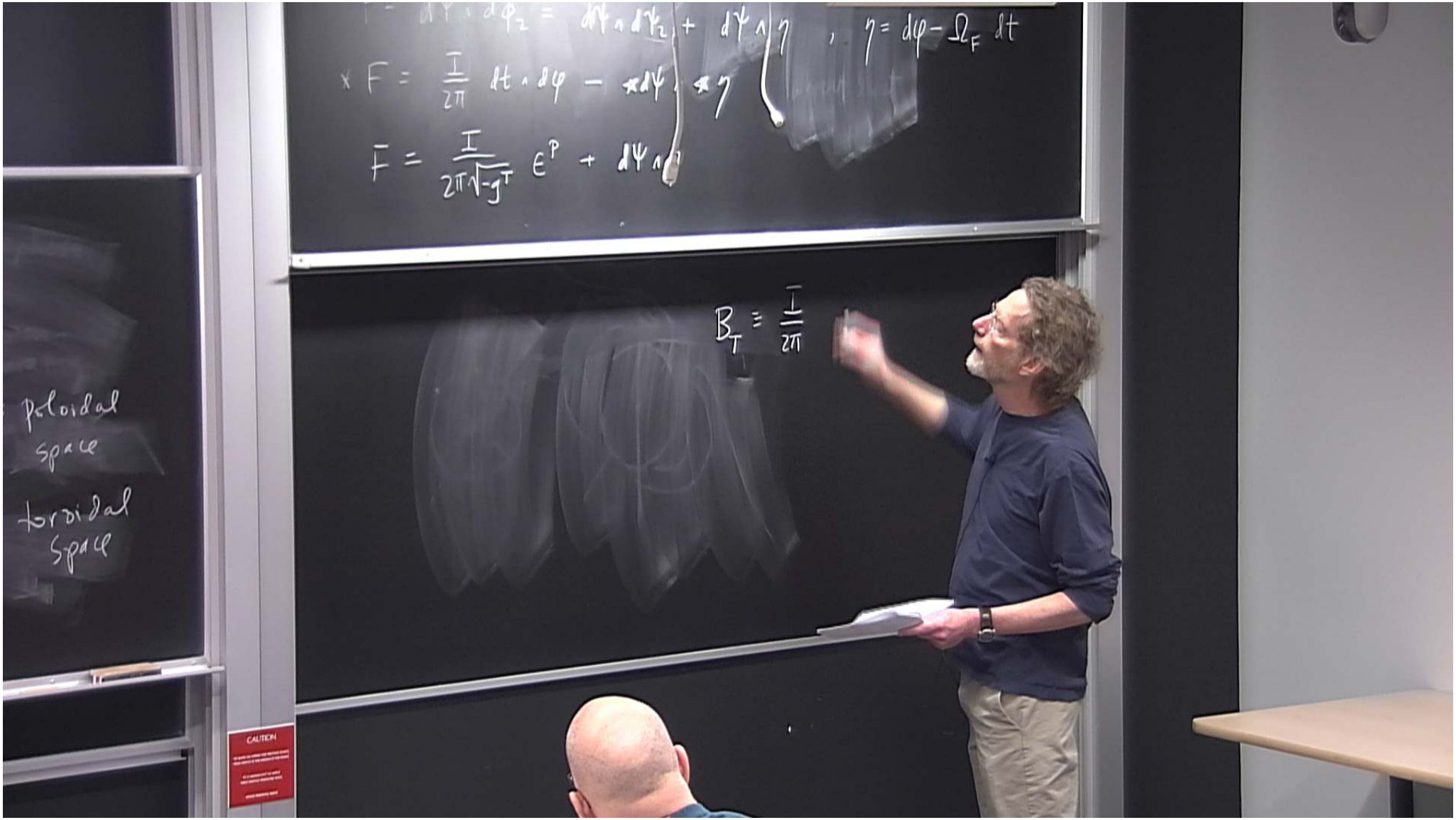

$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
$\phi_2 = \Psi_2(r,\theta) + \varphi - \Omega_F(\Psi)t$
(r,θ,φ,t)
$d\Omega_F = \Omega_F' \, d\Psi$
$F = d\Psi \wedge d\phi_2 = d\Psi \wedge d\Psi_2 + d\Psi \wedge \eta \quad , \quad \eta = d\varphi - \Omega_F \, dt$
$d\phi_2 = 1$
$d\phi_2 = K(\phi_1)$
CAUTION

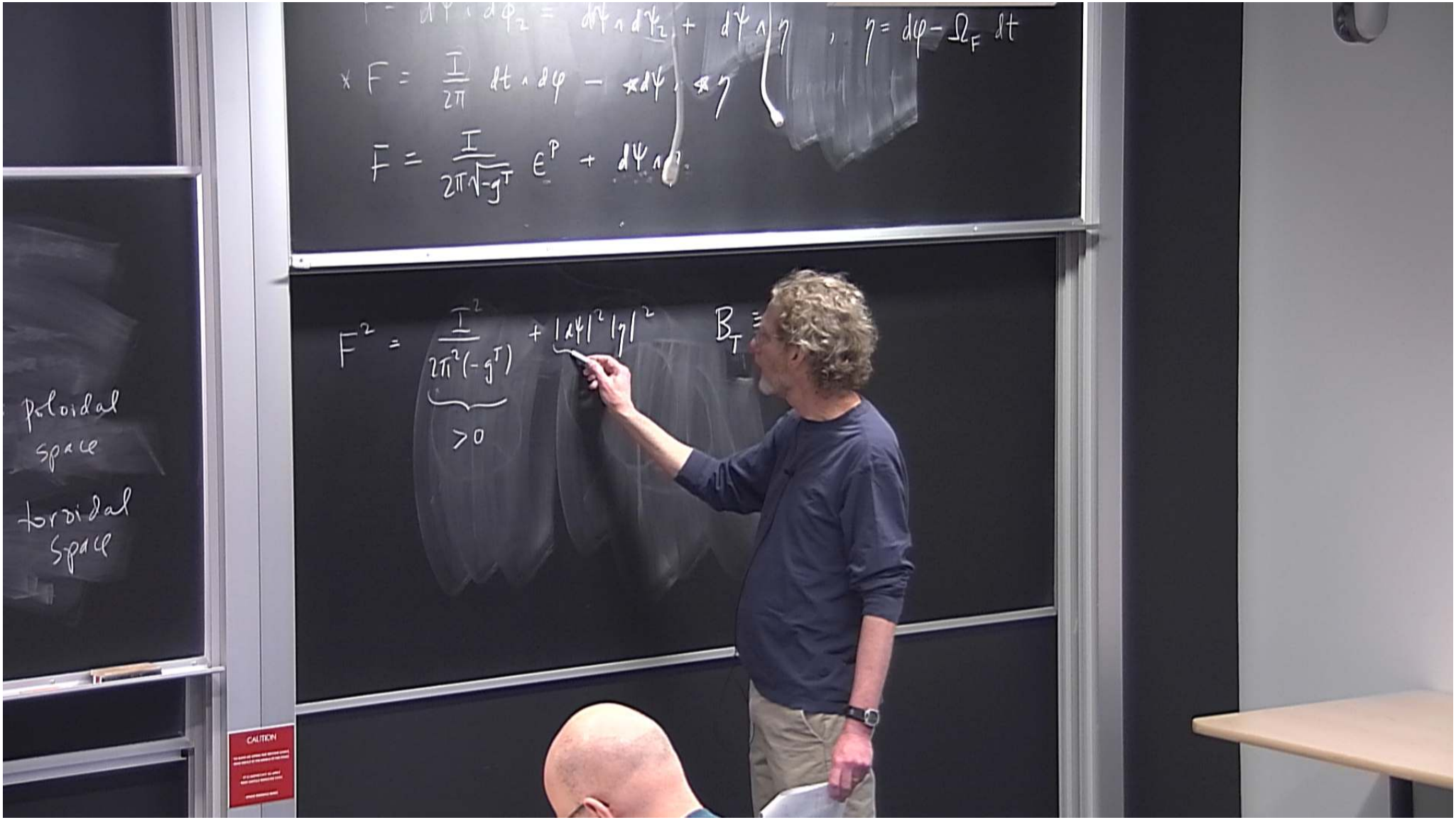

(r, θ) : poloidal "plane"
X = ∂φ
Y = ∂t
F = dΨ ∧ dφ
CAUTION

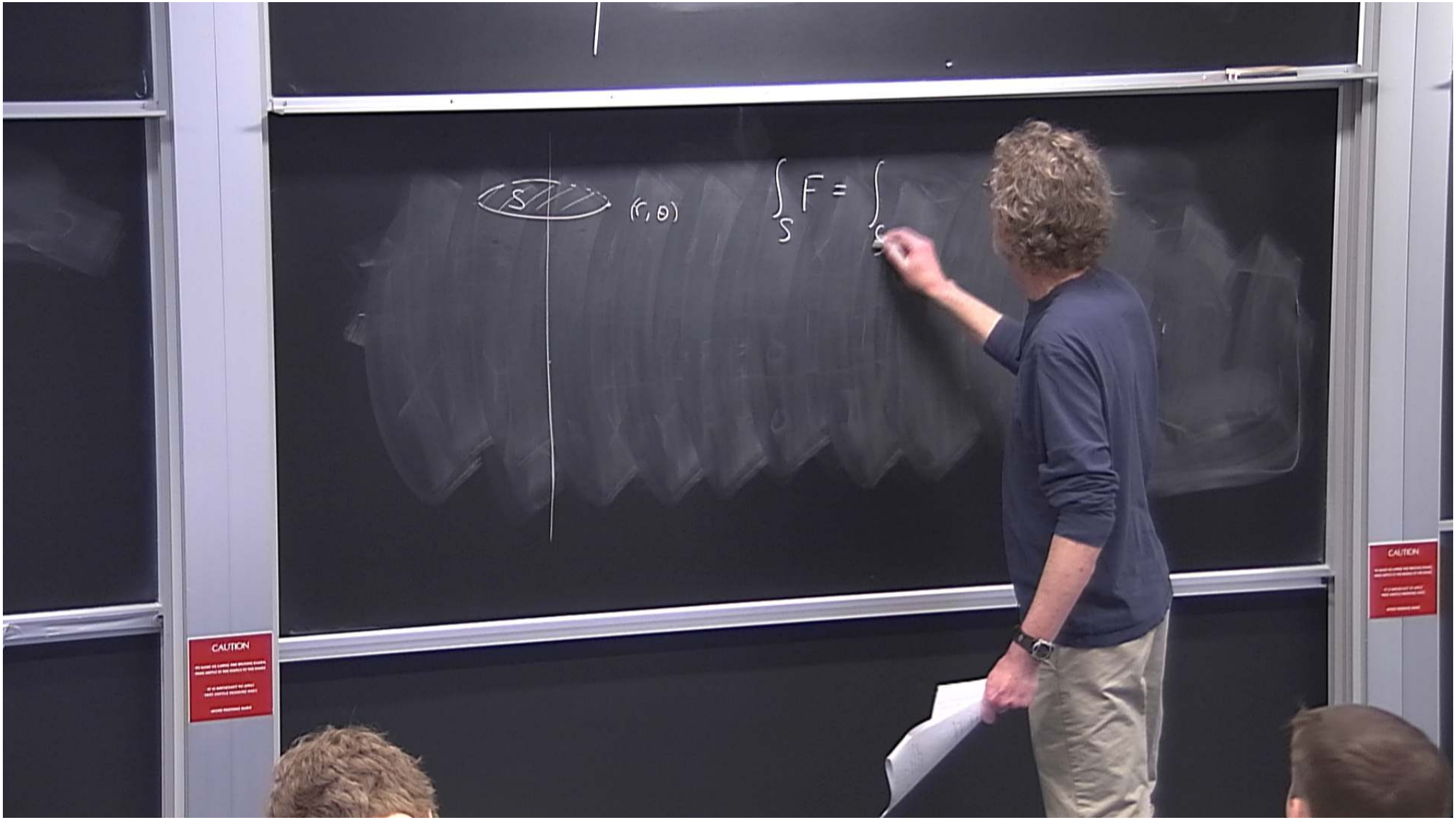

$g = \begin{pmatrix} g^T & \\ & g^P \end{pmatrix}$
$\epsilon = \epsilon^T \wedge \epsilon^P$
$= -\epsilon^P$
(r, θ) poloidal space
(t, φ) toroidal space

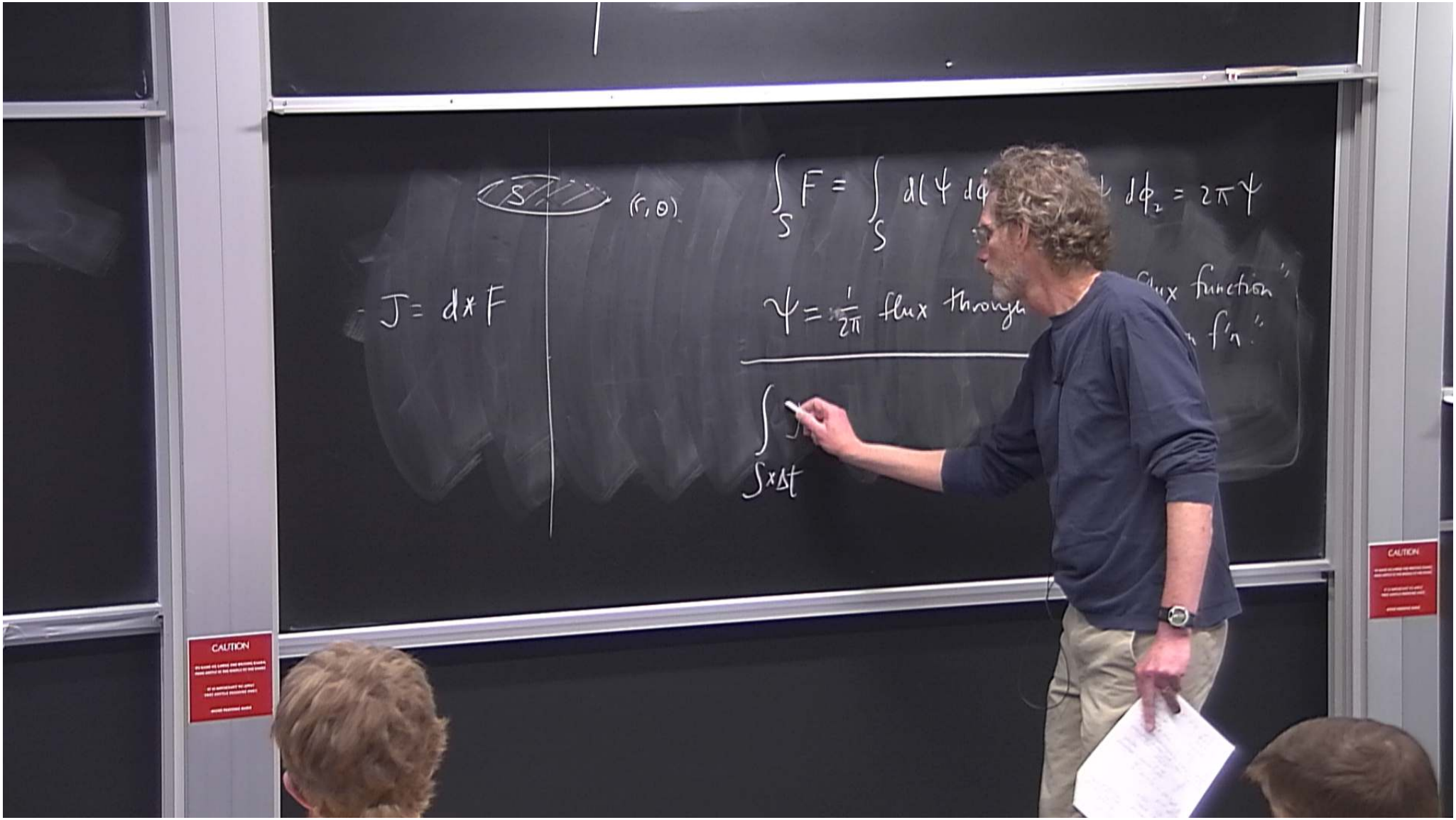

$g = \left(\begin{array}{c|c} g^T & \\ \hline & g^P \end{array}\right)$
$\epsilon = \epsilon^T \wedge \epsilon^P$
$*\epsilon^T = -\epsilon^P$
$*\epsilon^P = \epsilon^T$
$*(\omega^P \wedge \omega^T)$
$= -(*\omega^P) \wedge (*\omega^T)$
(r,θ) : poloidal space
(t,φ) toroidal space

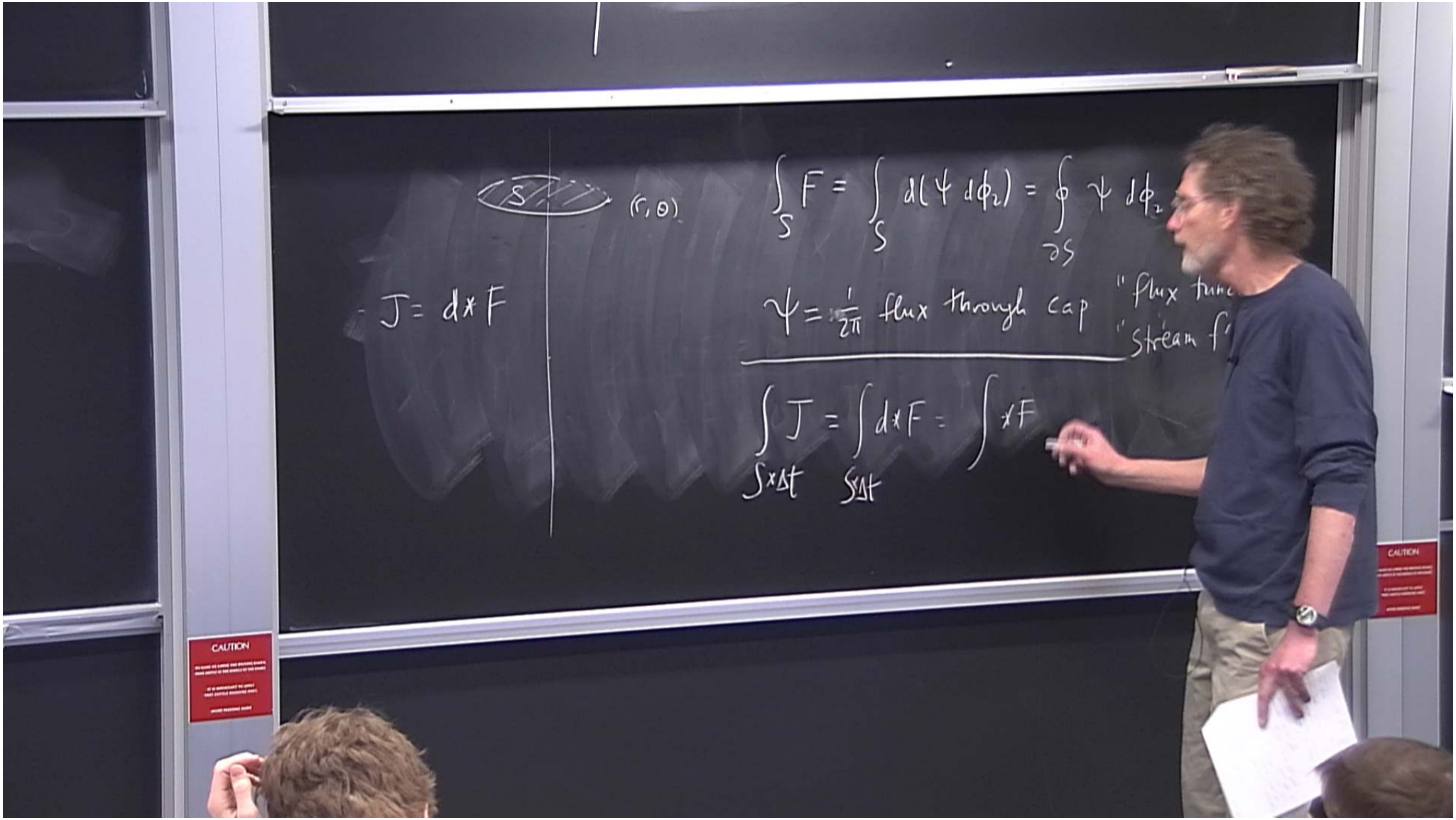

$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
$\phi_2 = \Psi_2(r,\theta) + \varphi - \Omega_F(\Psi)t$
(r,θ,φ,t)
$d\Omega_F = \Omega_F' \, d\Psi$
$F = d\Psi \wedge d\phi_2 = d\Psi \wedge d\Psi_2 + d\Psi \wedge \eta$, $\eta = d\varphi - \Omega_F dt$
poloidal space
toroidal space
CAUTION

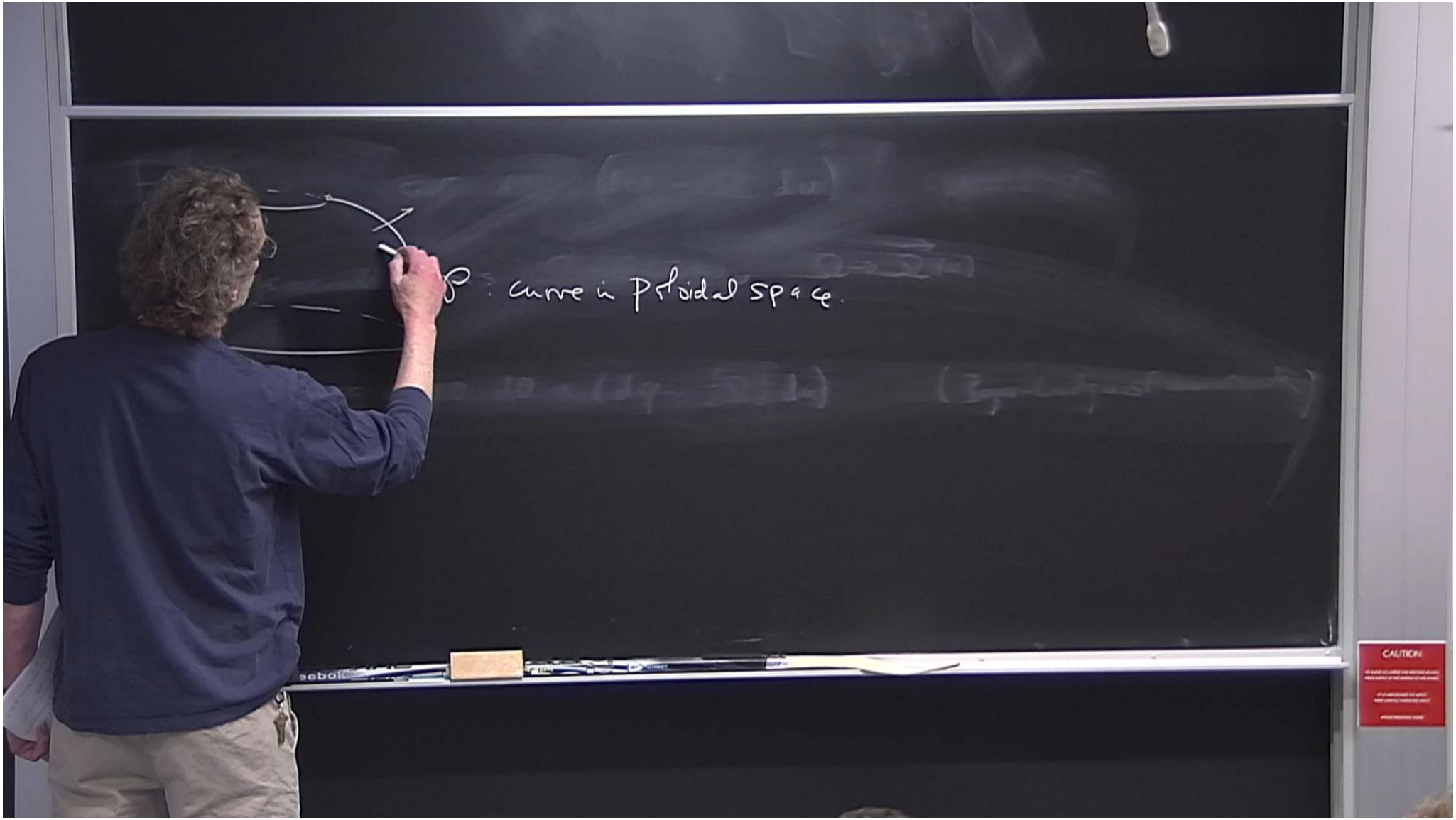

poloidal space
toroidal space
CAUTION

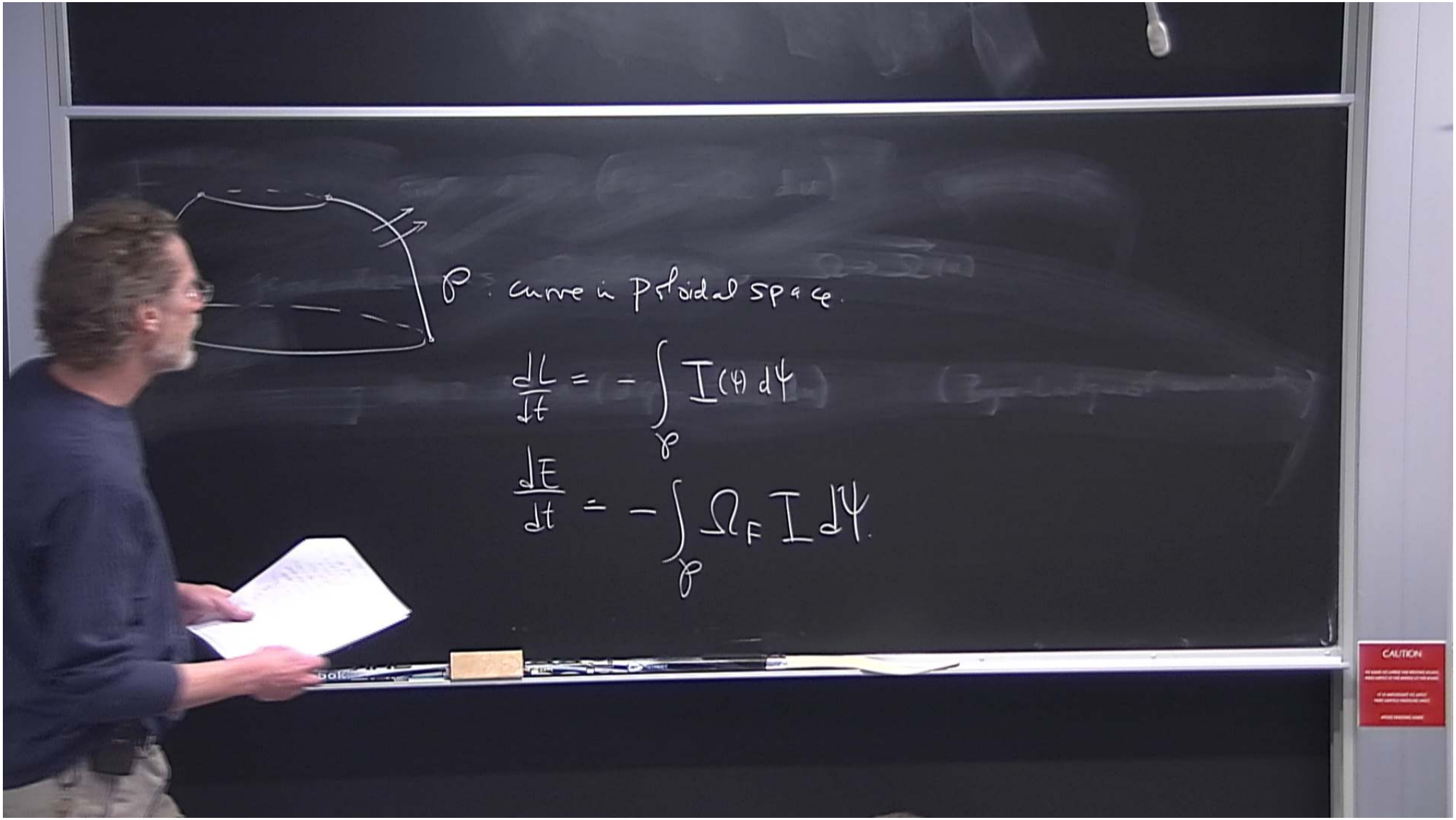

poloidal space
toroidal space
CAUTION

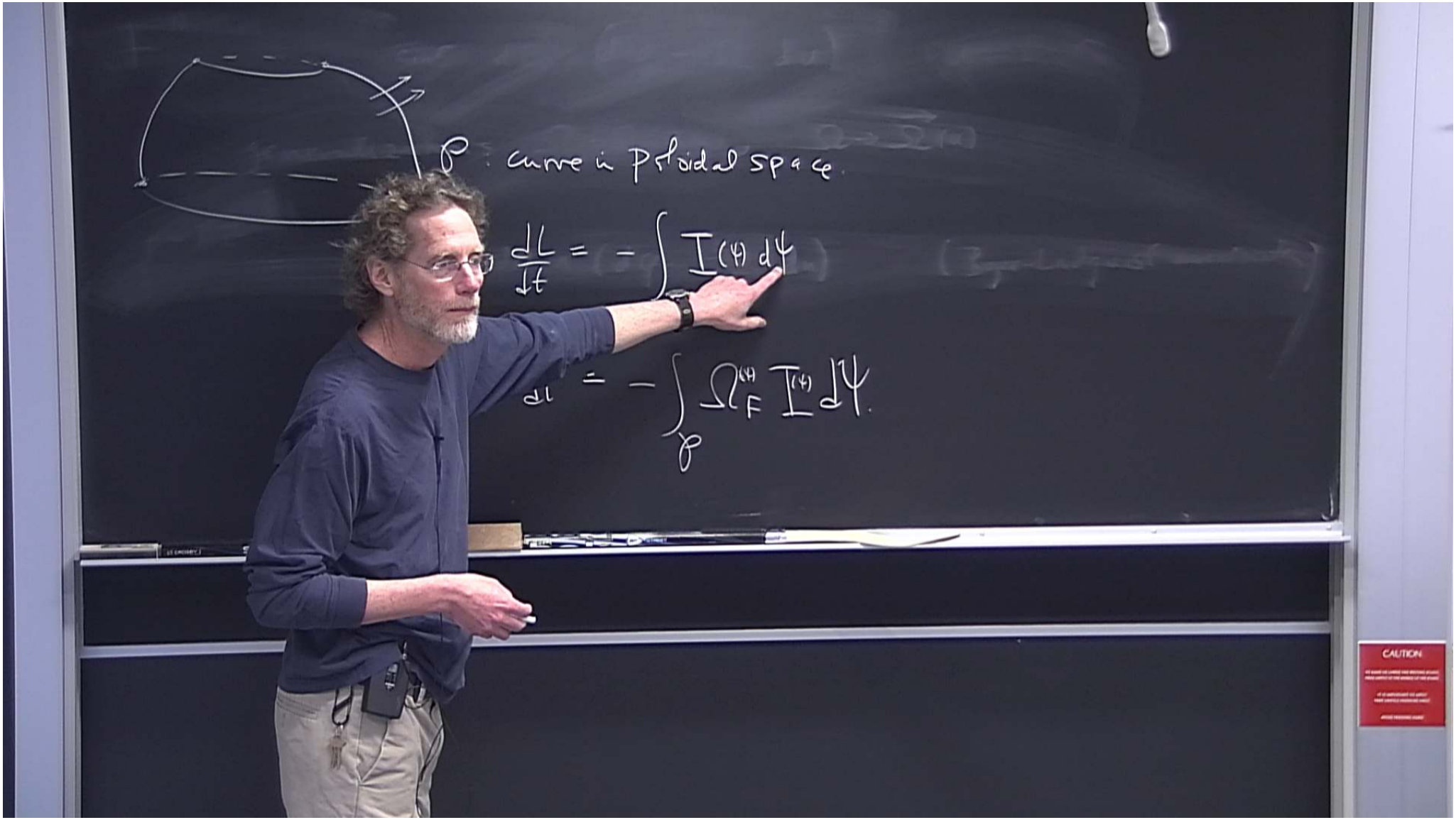

S
(r, θ)
∫S F = ∫
CAUTION
CAUTION

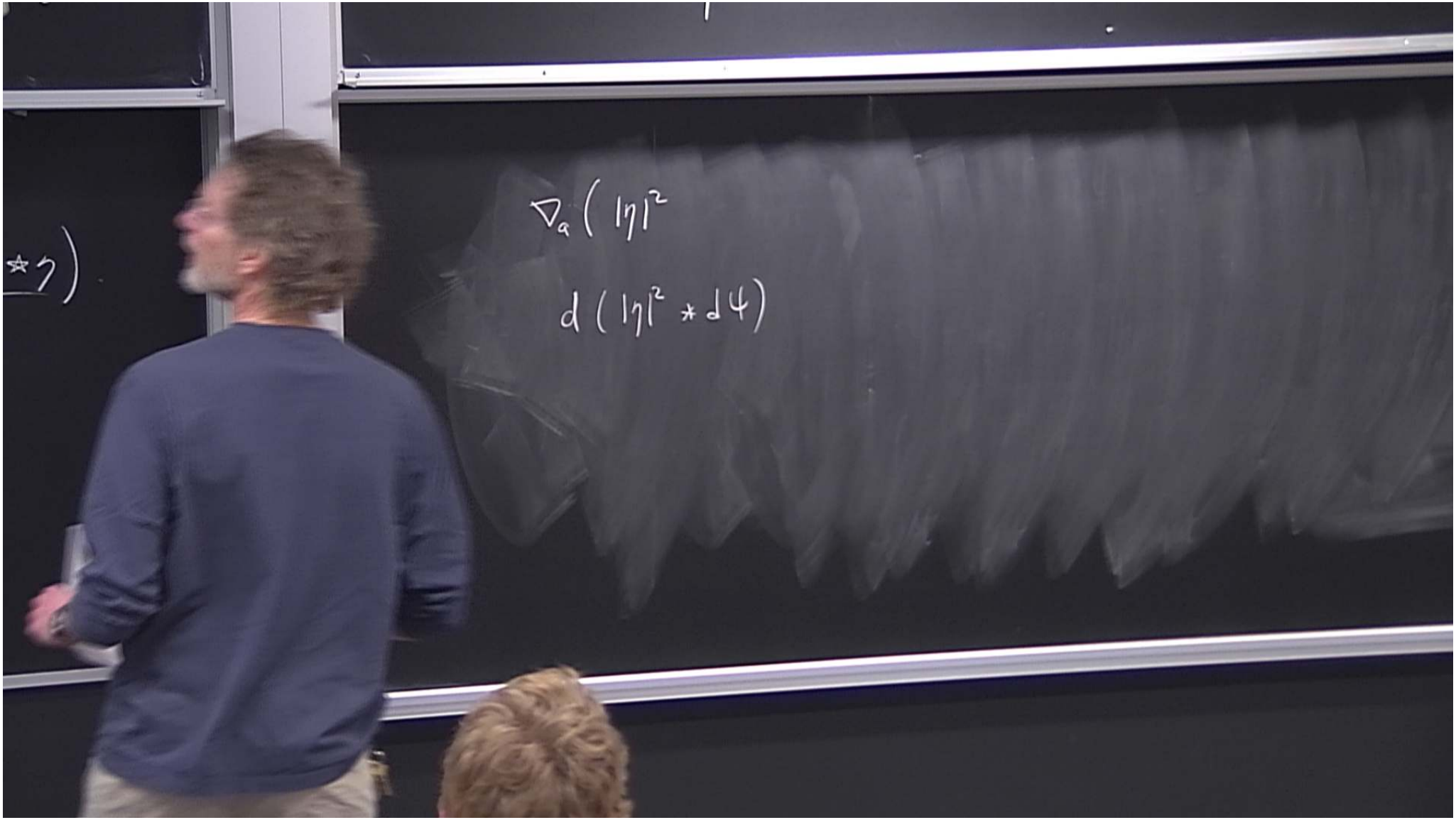

S
(r, θ)
$\int_S F = \int_S d(\psi\, d\phi$
$d\phi_2 = 2\pi\psi$
$J = d{*}F$
$\psi = \frac{1}{2\pi}$ flux through
flux function
$\int_{S\times\Delta t}$
CAUTION
CAUTION

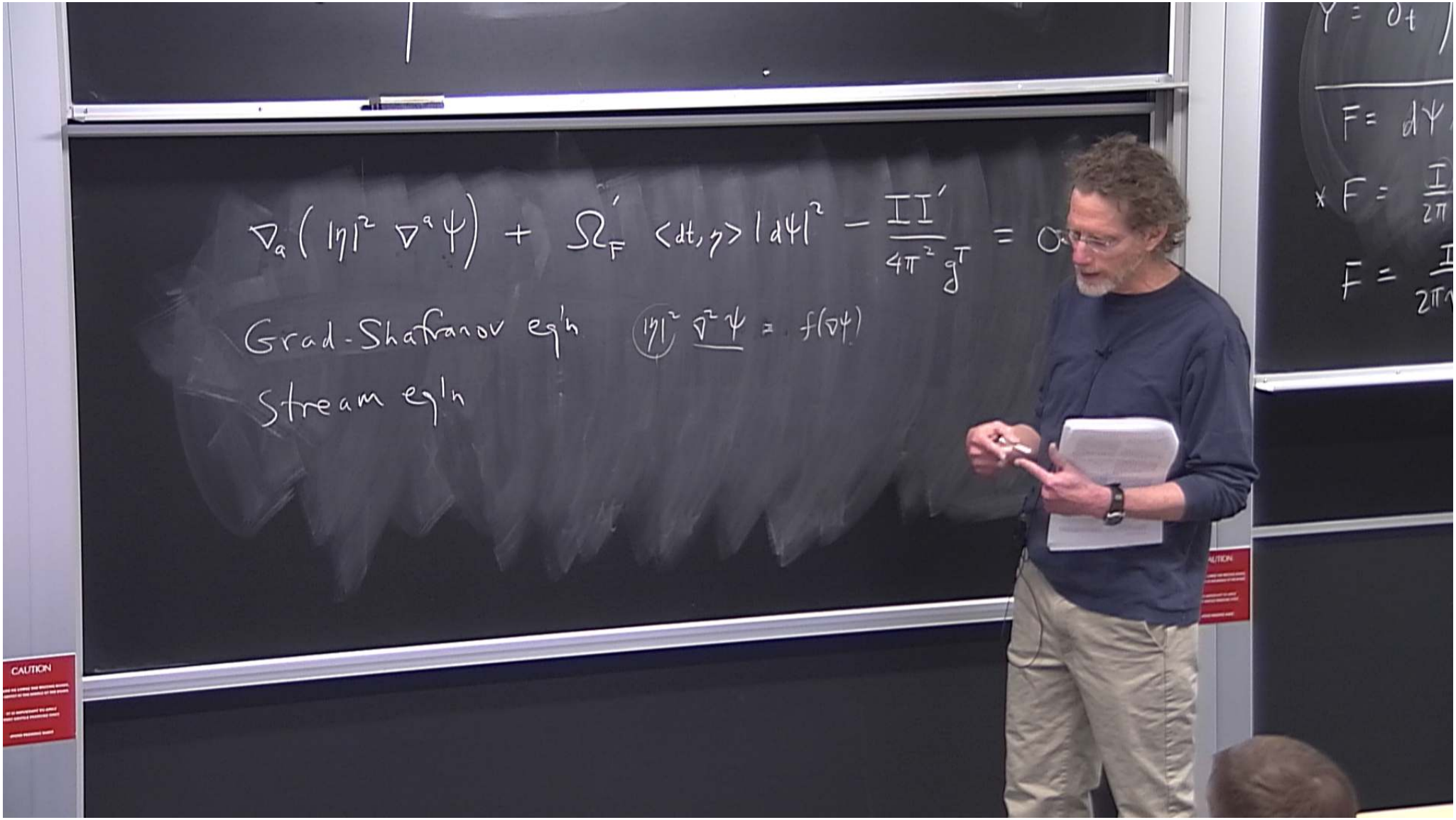

S
(r, θ)
$\int_S F = \int_S d(\psi\, d\phi_2) = \oint_{\partial S} \psi\, d\phi_2$
$J = d{*}F$
$\psi = \frac{1}{2\pi}$ flux through cap
"flux
"stream
$\int_{S\times\Delta t} J = \int_{S\Delta t} d{*}F = \int {*}F$
CAUTION

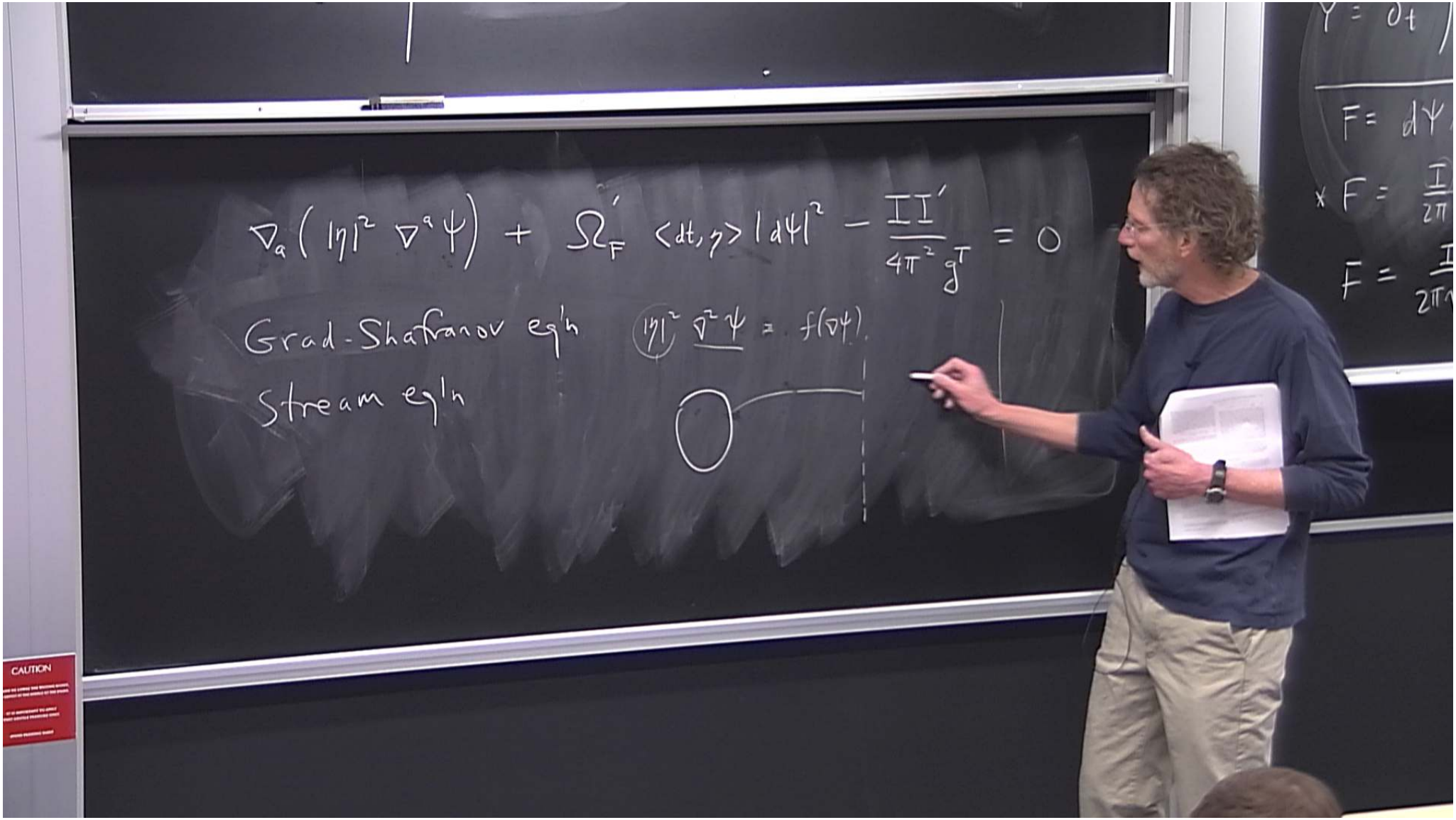

ρ : curve in ptoidal space.
CAUTION


ρ : curve in poloidal space.
$\frac{dL}{dt} = -\int_{\rho} I(\psi)\, d\psi$
$\frac{dE}{dt} = -\int_{\rho} \Omega_F I\, d\psi$.
CAUTION


ρ : curve in poloidal space
$\frac{dL}{dt} = -\int I(\psi)\, d\psi$
$dL = -\int_{\rho} \Omega_F^{(\psi)} I^{(\psi)} d\psi$
CAUTION


$\nabla_a(|\eta|^2$
$d(|\eta|^2 * d\psi)$


$\nabla_a (|\eta|^2 \nabla^a \psi) + \Omega_F' \langle dt, \eta \rangle |d\psi|^2 - \frac{II'}{4\pi^2 g^T} = 0$
Grad-Shafranov eq'n
$|\eta|^2 \nabla^2 \psi = f(\nabla\psi)$
Stream eq'n
CAUTION


$\nabla_a\left(|\eta|^2\nabla^a\psi\right) + \Omega_F' \langle dt, \eta\rangle |d\psi|^2 - \frac{II'}{4\pi^2 g^T} = 0$
Grad-Shafranov eq'n
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Stream eq'n
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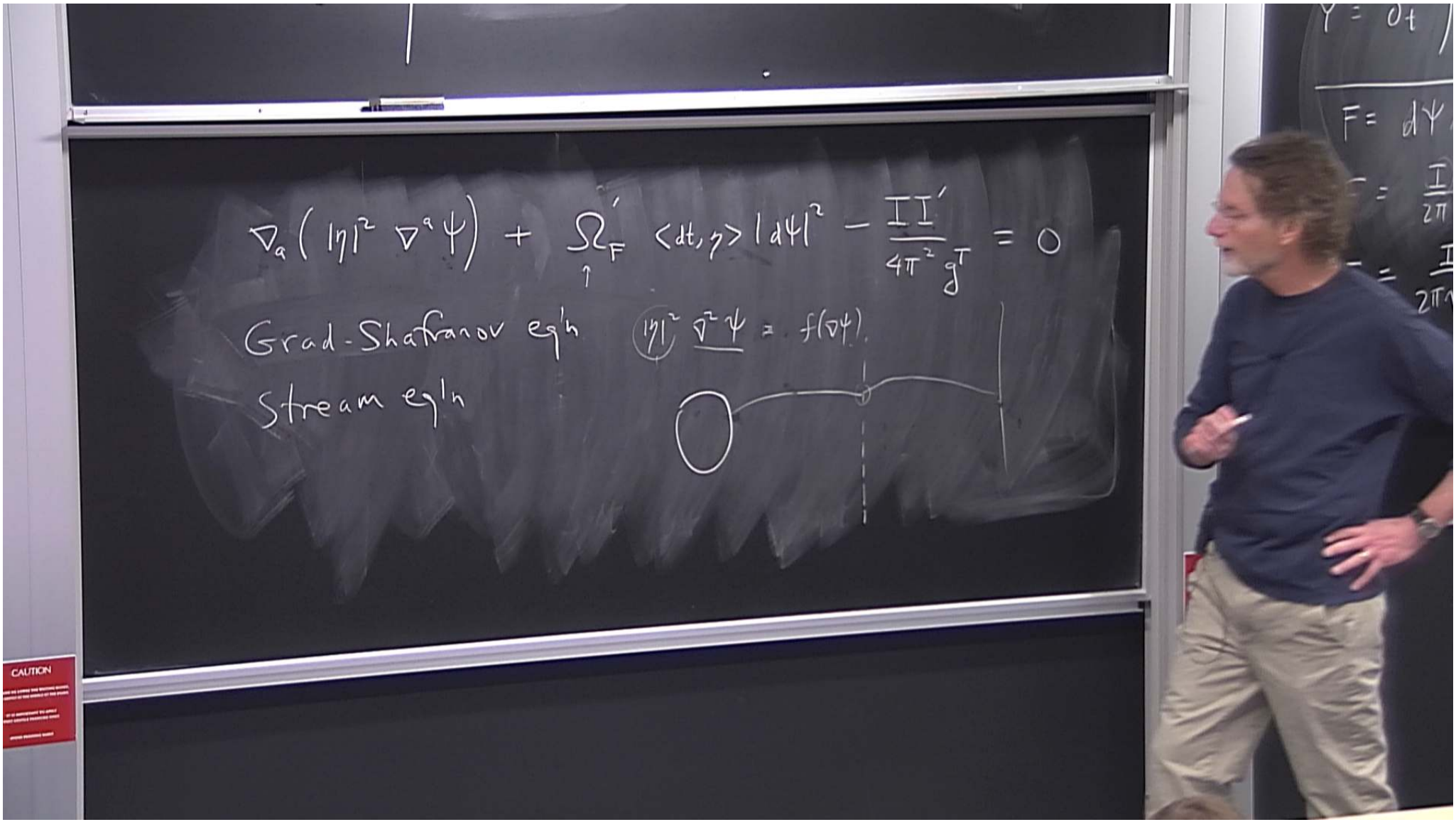
$\nabla_a\left(|\eta|^2\nabla^a\psi\right) + \Omega_F' \langle dt, \eta\rangle |d\psi|^2 - \frac{II'}{4\pi^2 g^T} = 0$
Grad-Shafranov eq'n
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