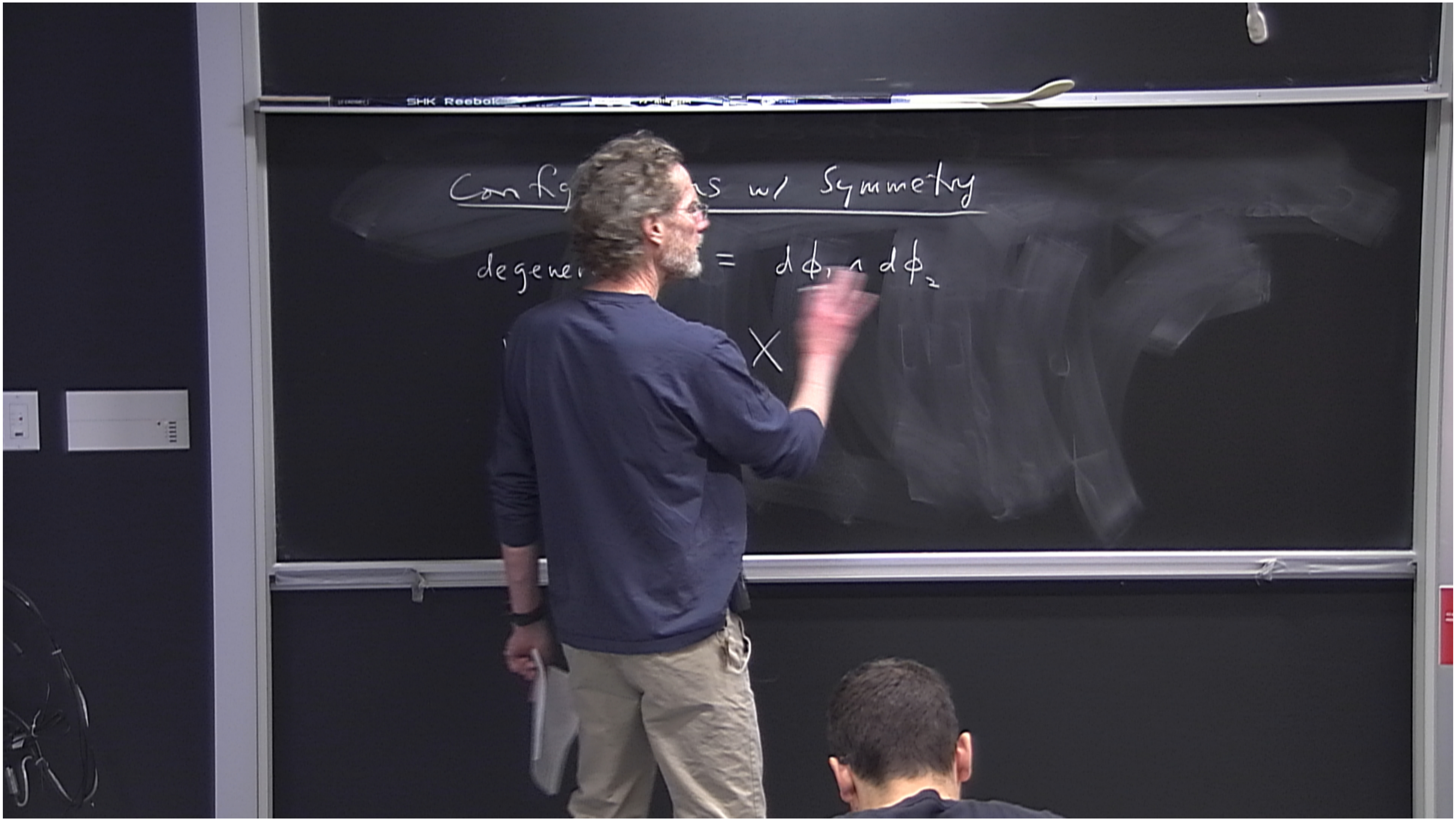
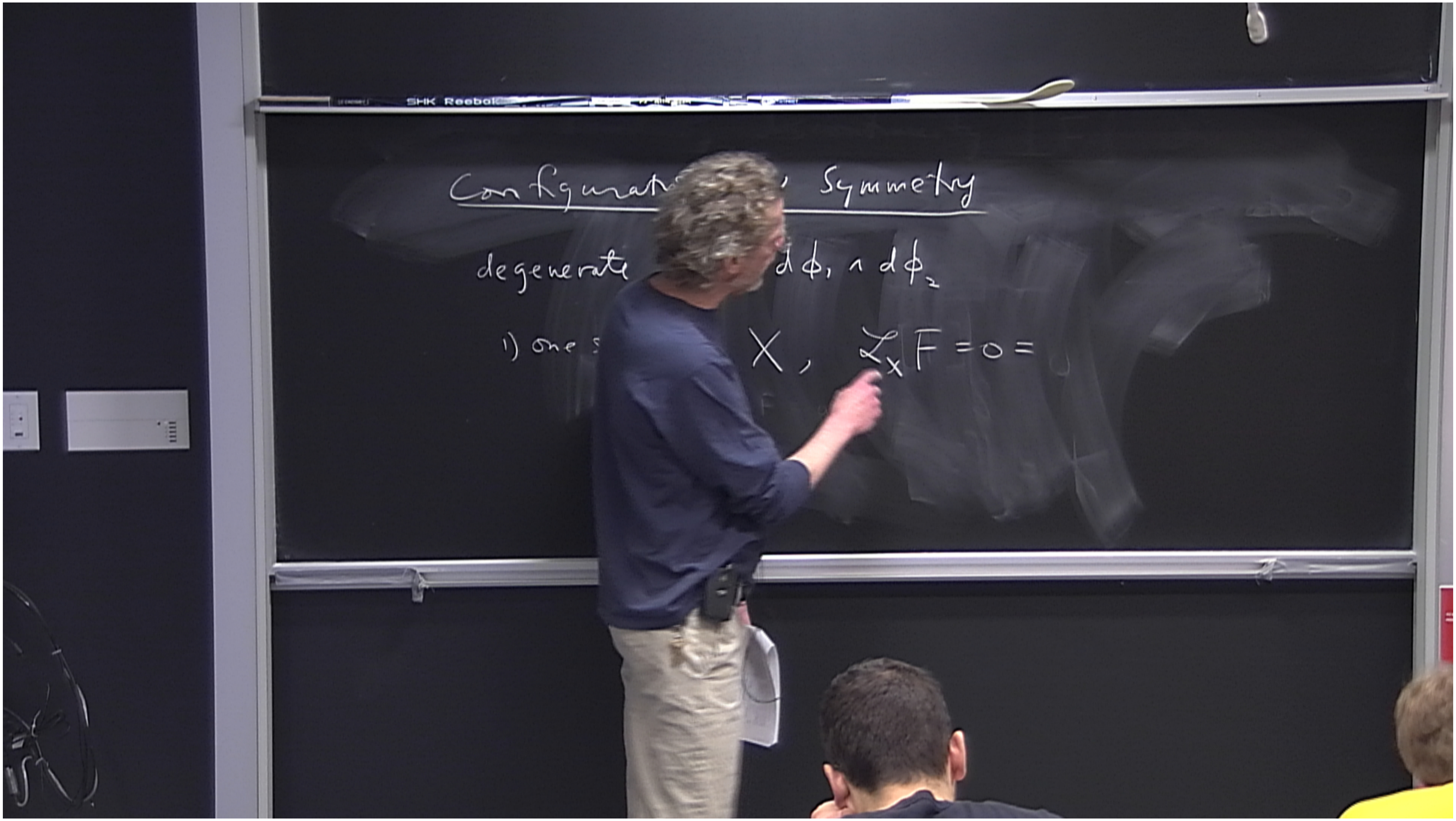
Title: Spacetime approach to force-free magnetospheres - Lecture 8

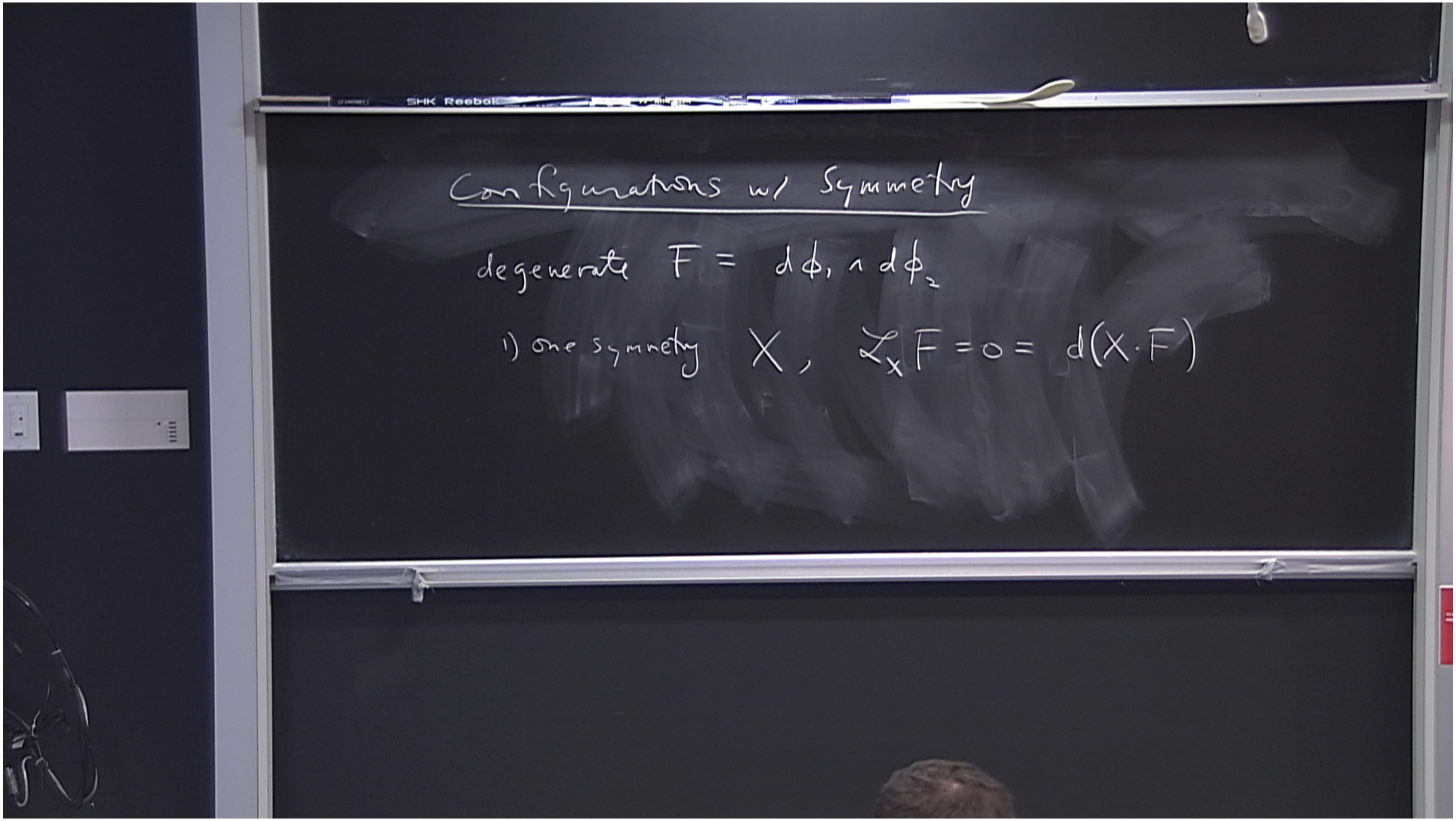
Date: Mar 05, 2014 10:30 AM

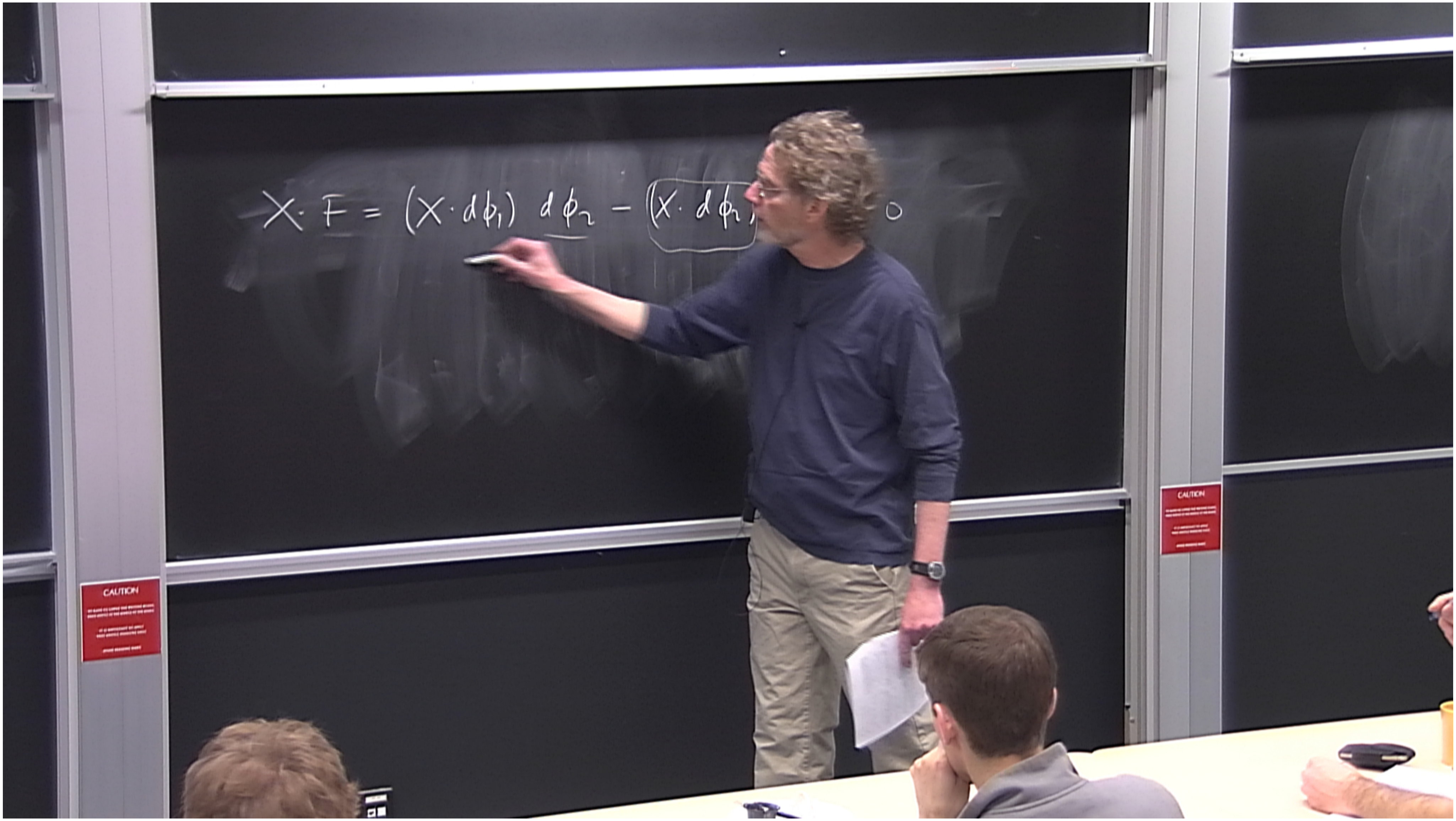
URL: http://pirsa.org/14030103

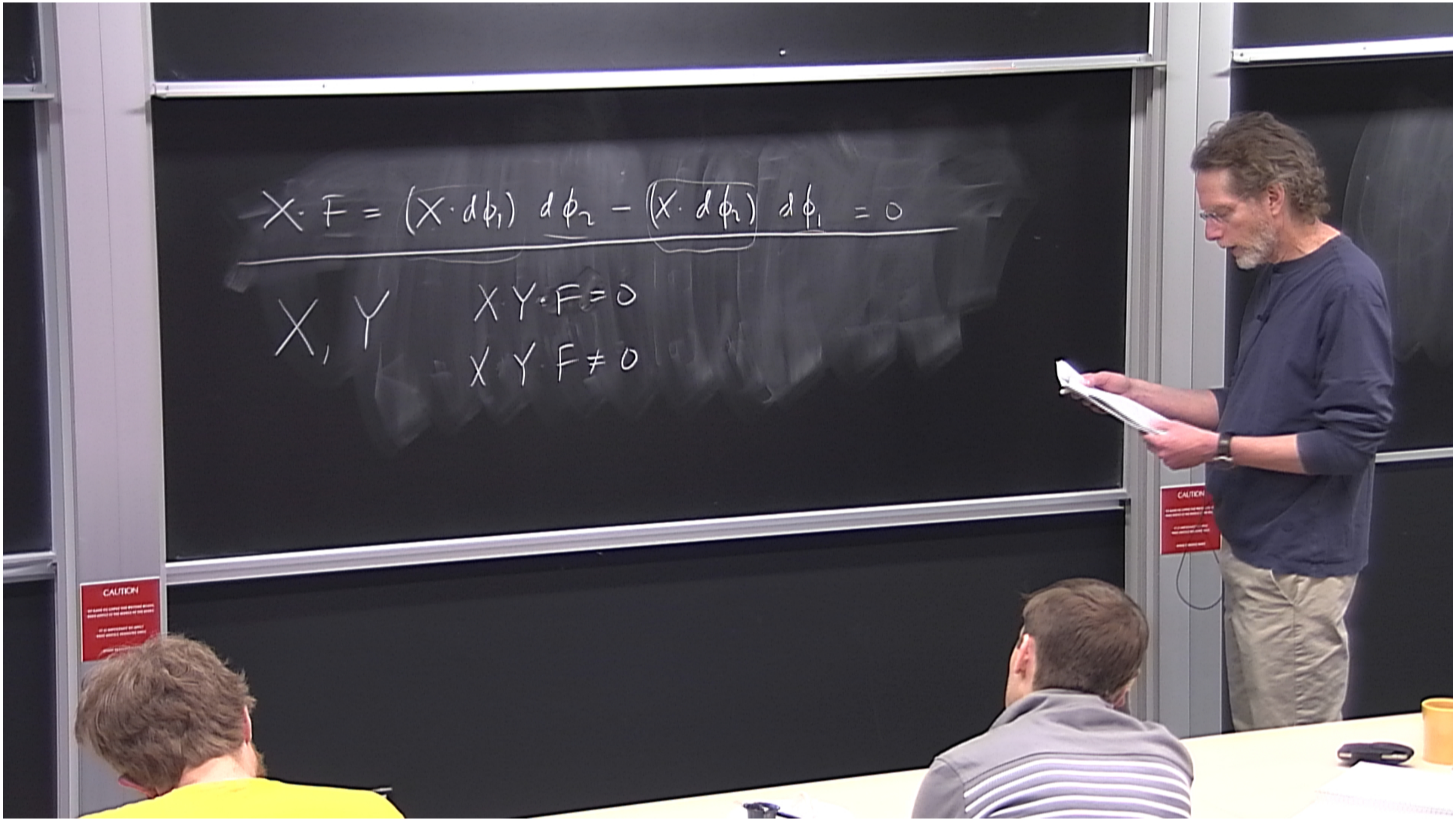
Abstract:

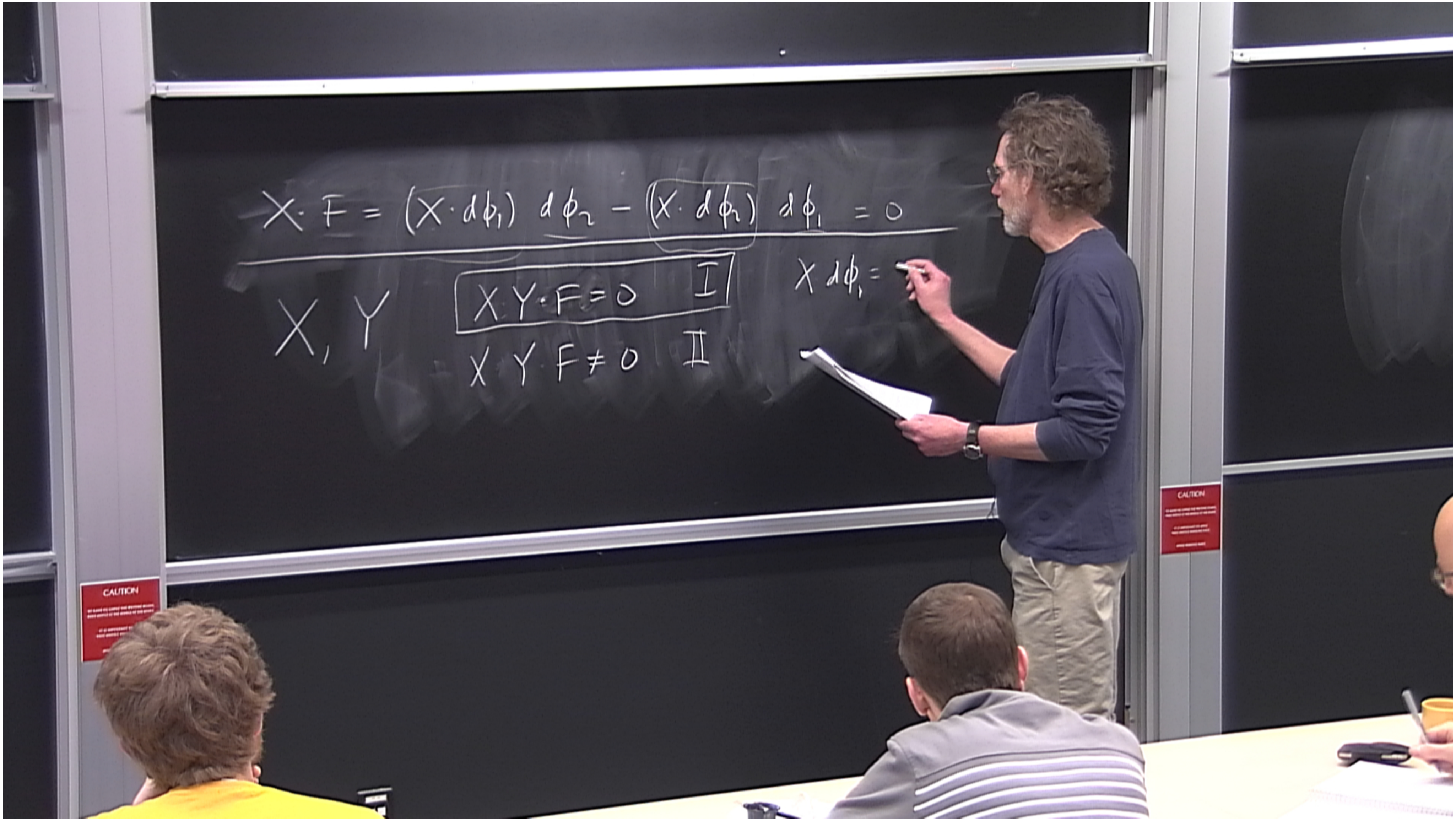

SHK Reebok
w/ Symmetry
= dϕ1 ∧ dϕ2

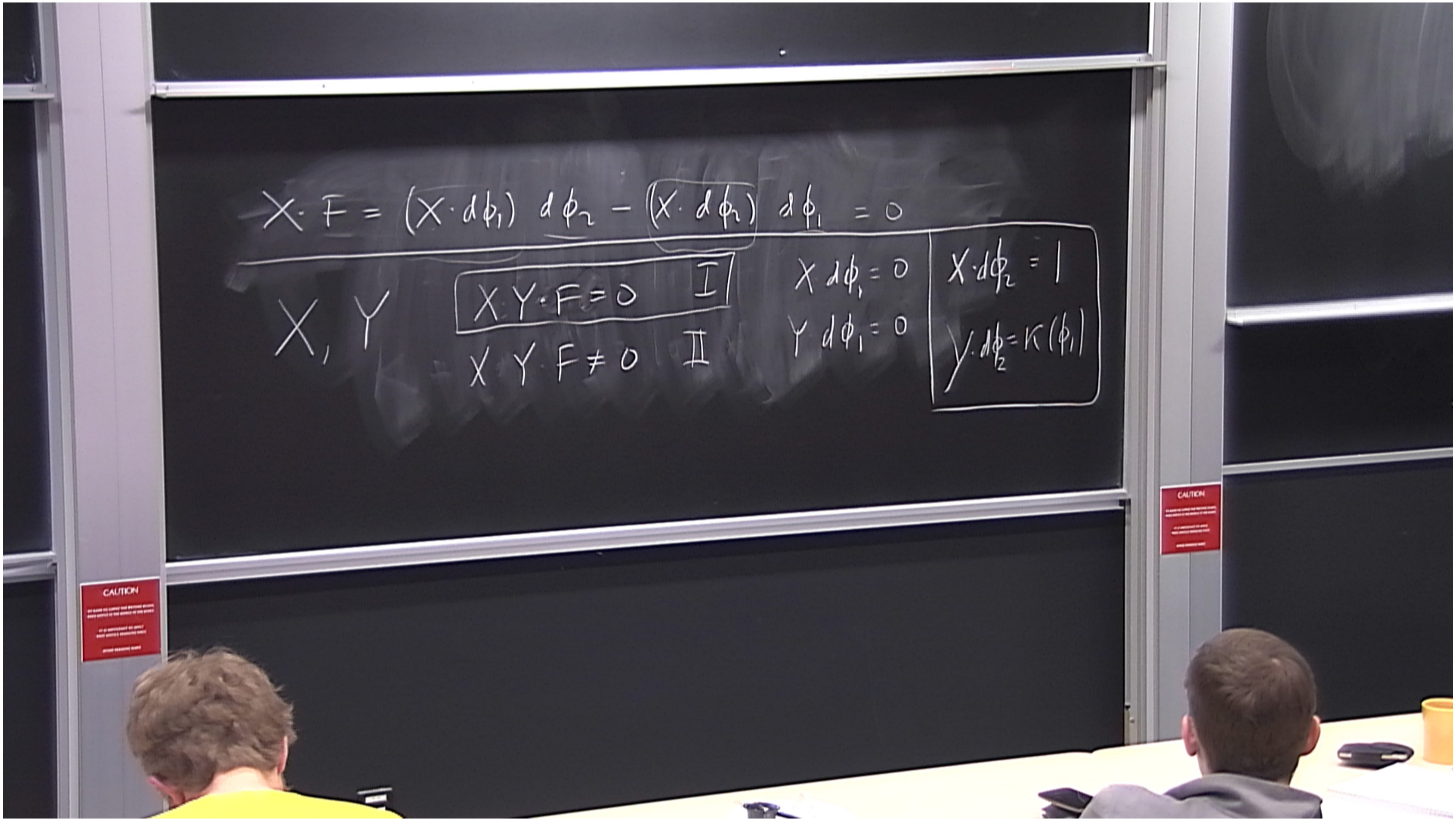

SHK Reebok
Symmetry
degenerate
$d\phi_1 \wedge d\phi_2$
1) one
X, $\mathcal{L}_X F = 0 =$

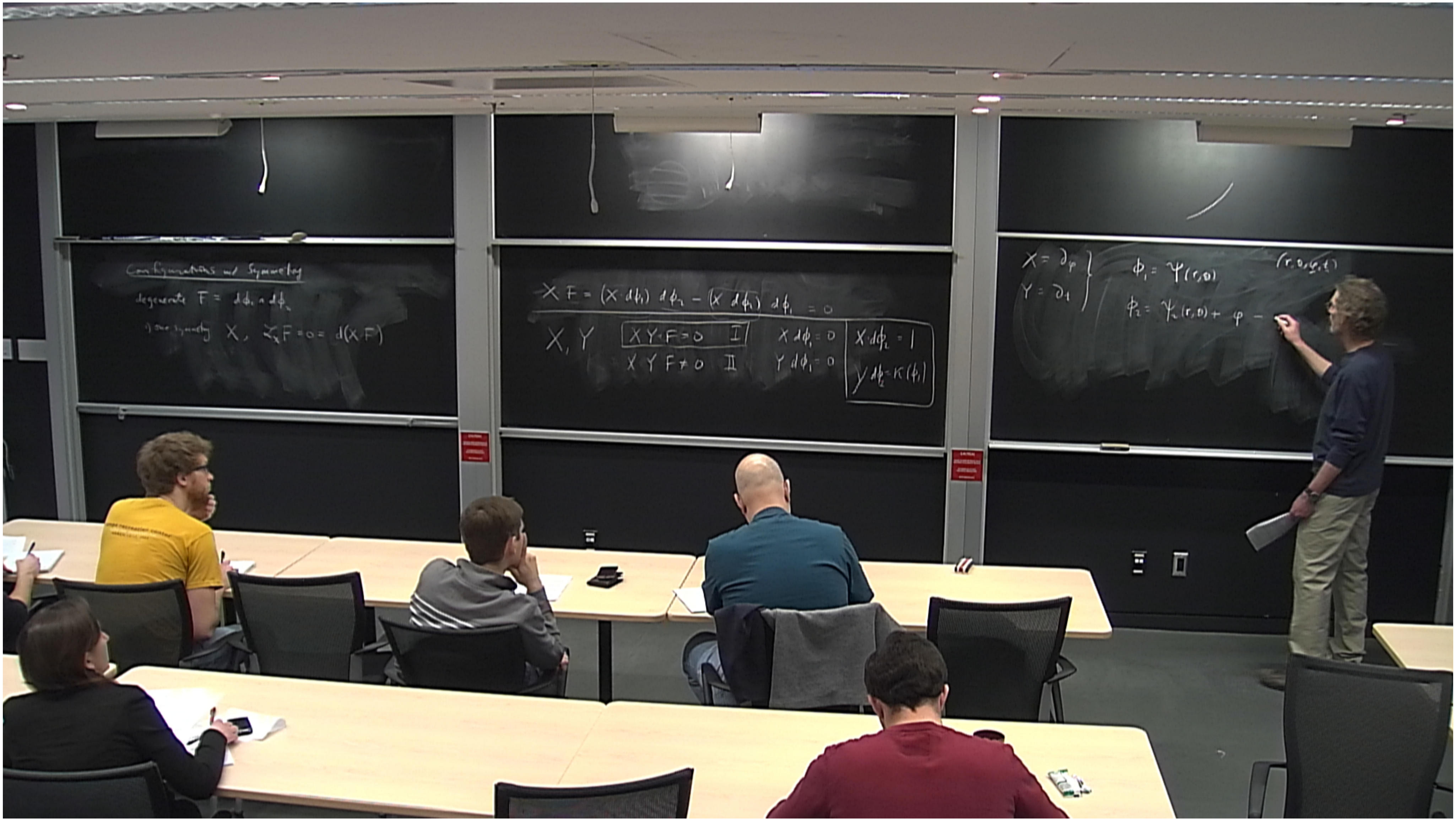

Configurations w/ Symmetry
degenerate $F = d\phi_1 \wedge d\phi_2$
1) one symmetry X, $\mathcal{L}_X F = 0 = d(X \cdot F)$

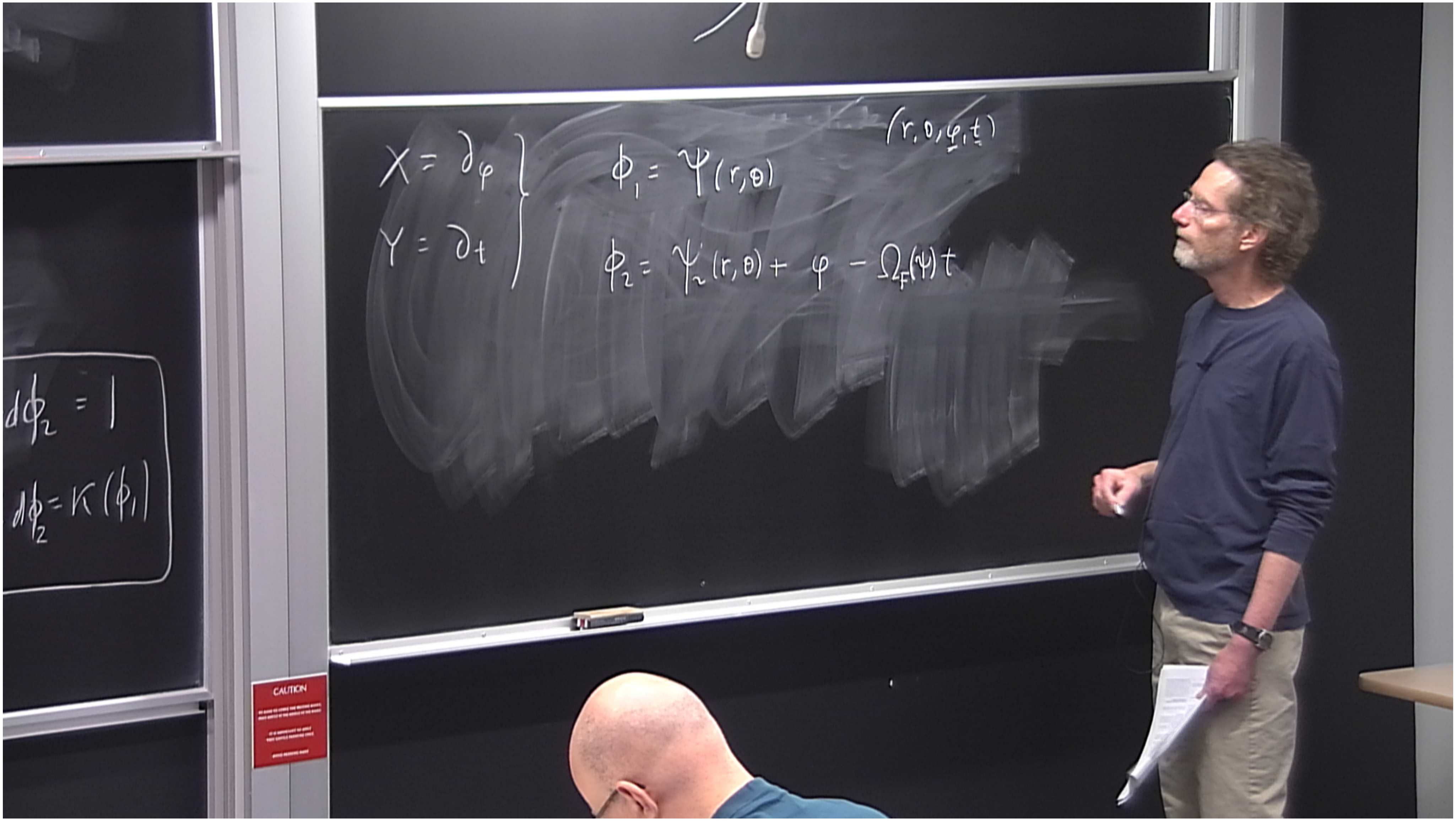

X · F = (X · dφ1) dφ2 − (X · dφ2
CAUTION
CAUTION

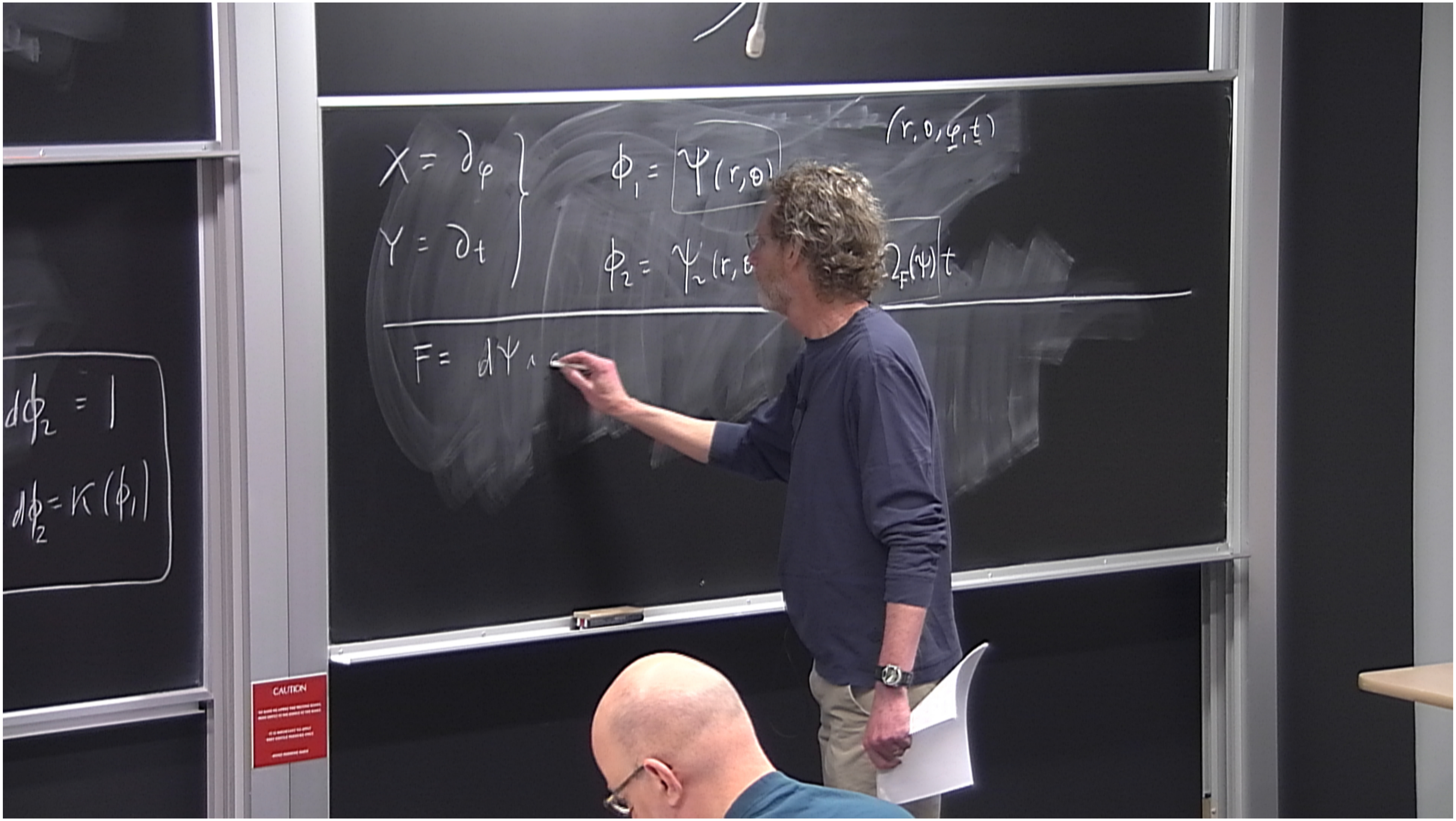

$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1 = 0$
X, Y
$X \cdot Y \cdot F = 0$
$X \cdot Y \cdot F \neq 0$

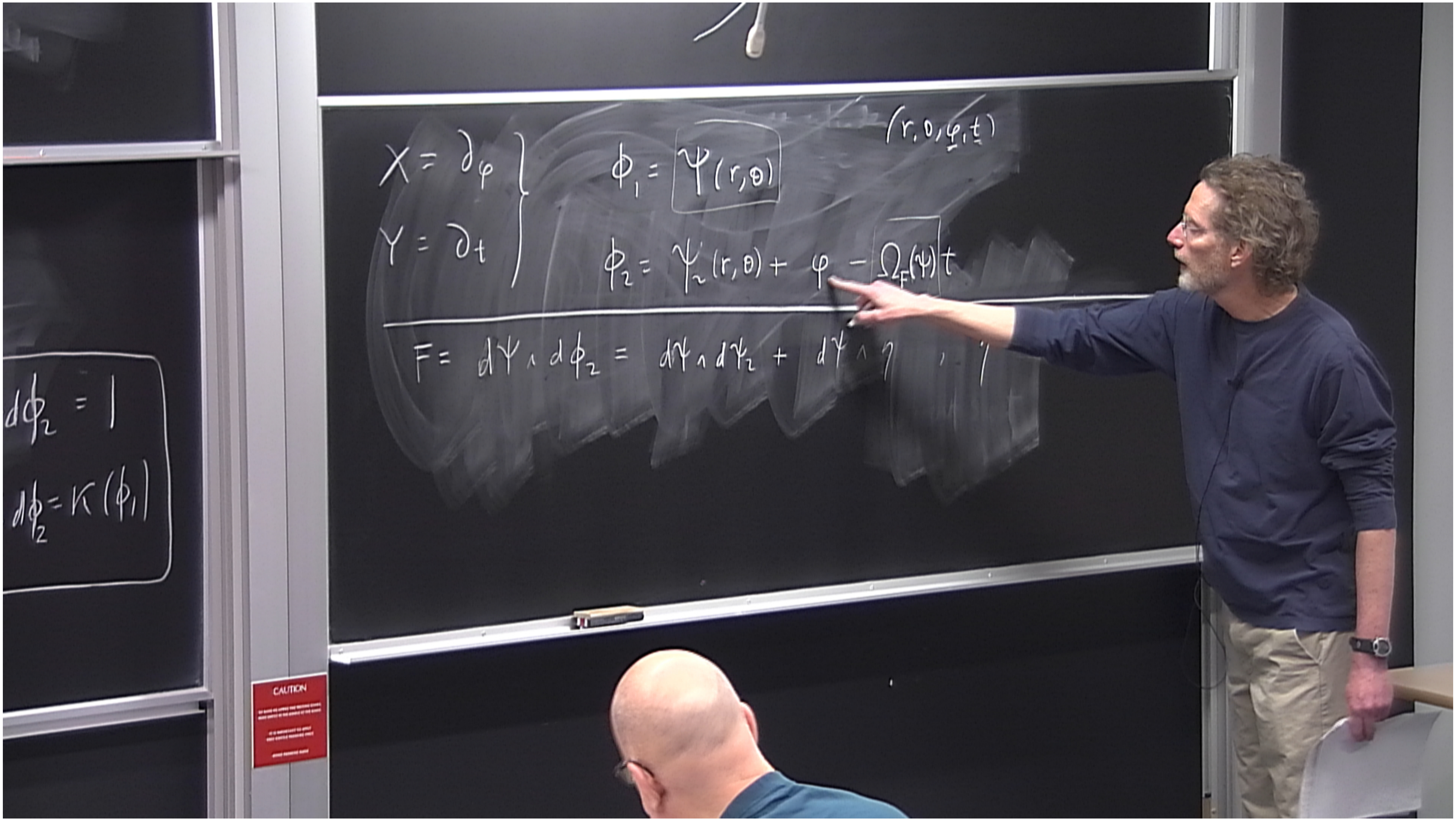

$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1 = 0$
X, Y
$X\,Y \cdot F = 0$ I
$X\,Y \cdot F \neq 0$ II
$X\, d\phi_1 =$
CAUTION


$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X \cdot d\phi_2)\, d\phi_1 = 0$
X, Y
$X \cdot Y \cdot F = 0$ I
$X \cdot Y \cdot F \neq 0$ II
$X\, d\phi_1 = 0$
$Y\, d\phi_1 = 0$
$X \cdot d\phi_2 = 1$
$Y \cdot d\phi_2 = K(\phi_1)$

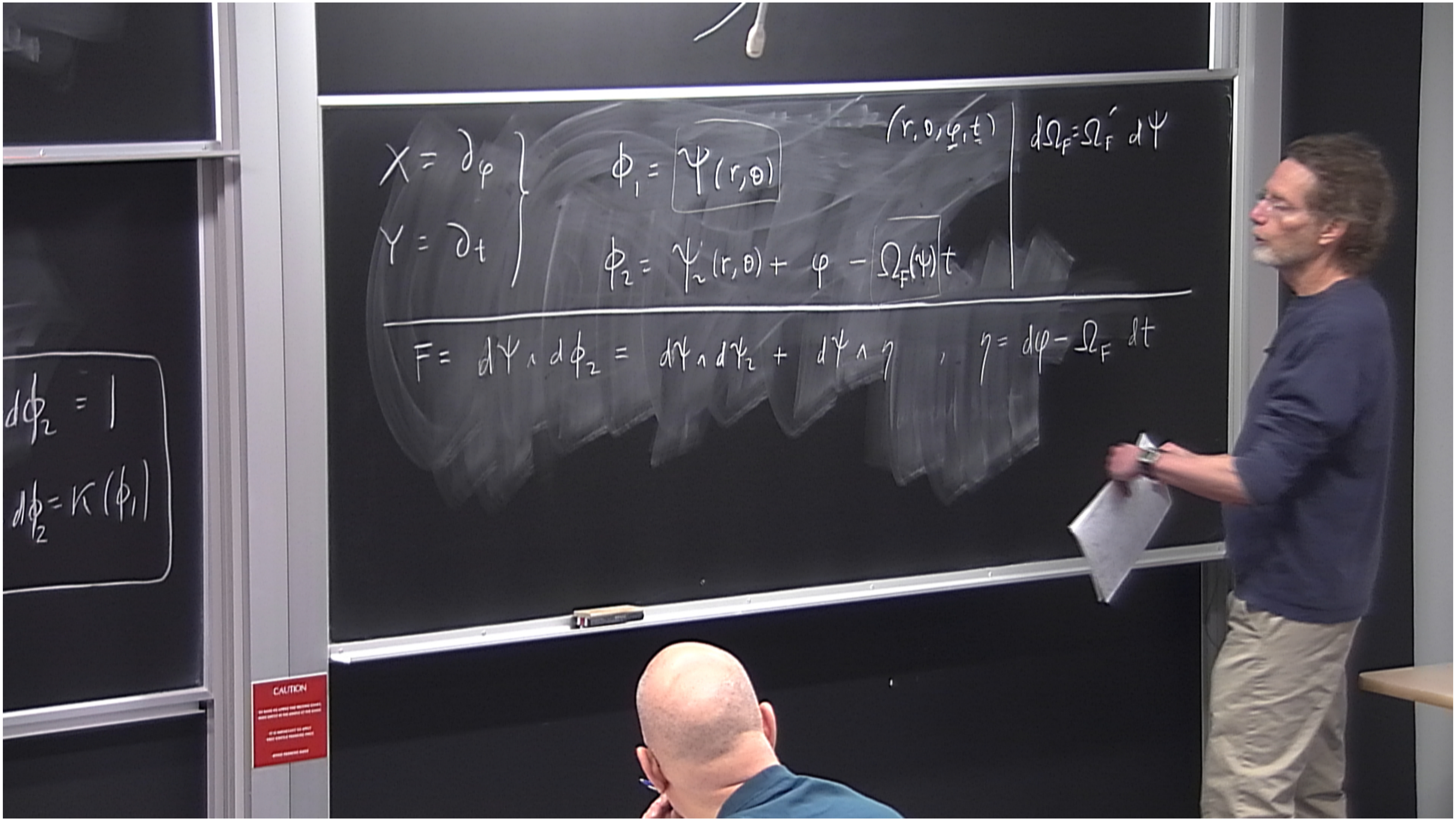

Configurations and Symmetry
degenerate $F = d\phi_1 \wedge d\phi_2$
$X \cdot F = (X \cdot d\phi_1)\, d\phi_2 - (X\, d\phi_2)\, d\phi_1 = 0$
X, Y
$X\, Y \cdot F = 0$ I
$X\, Y\, F \neq 0$ II
$X\, d\phi_1 = 0$
$Y\, d\phi_1 = 0$
$X \cdot d\phi_2 = 1$
$Y\, d\phi_2 = K(\phi_1)$
$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
$\phi_2 = \Psi_2(r,\theta) + \varphi -$
(r, θ, φ, t)


(r, θ, φ, t)
X = ∂_φ
Y = ∂_t
φ_1 = Ψ(r, θ)
φ_2 = ψ_2(r, θ) + φ − Ω_F(ψ) t
dφ_2 = 1
dφ_2 = κ(φ_1)

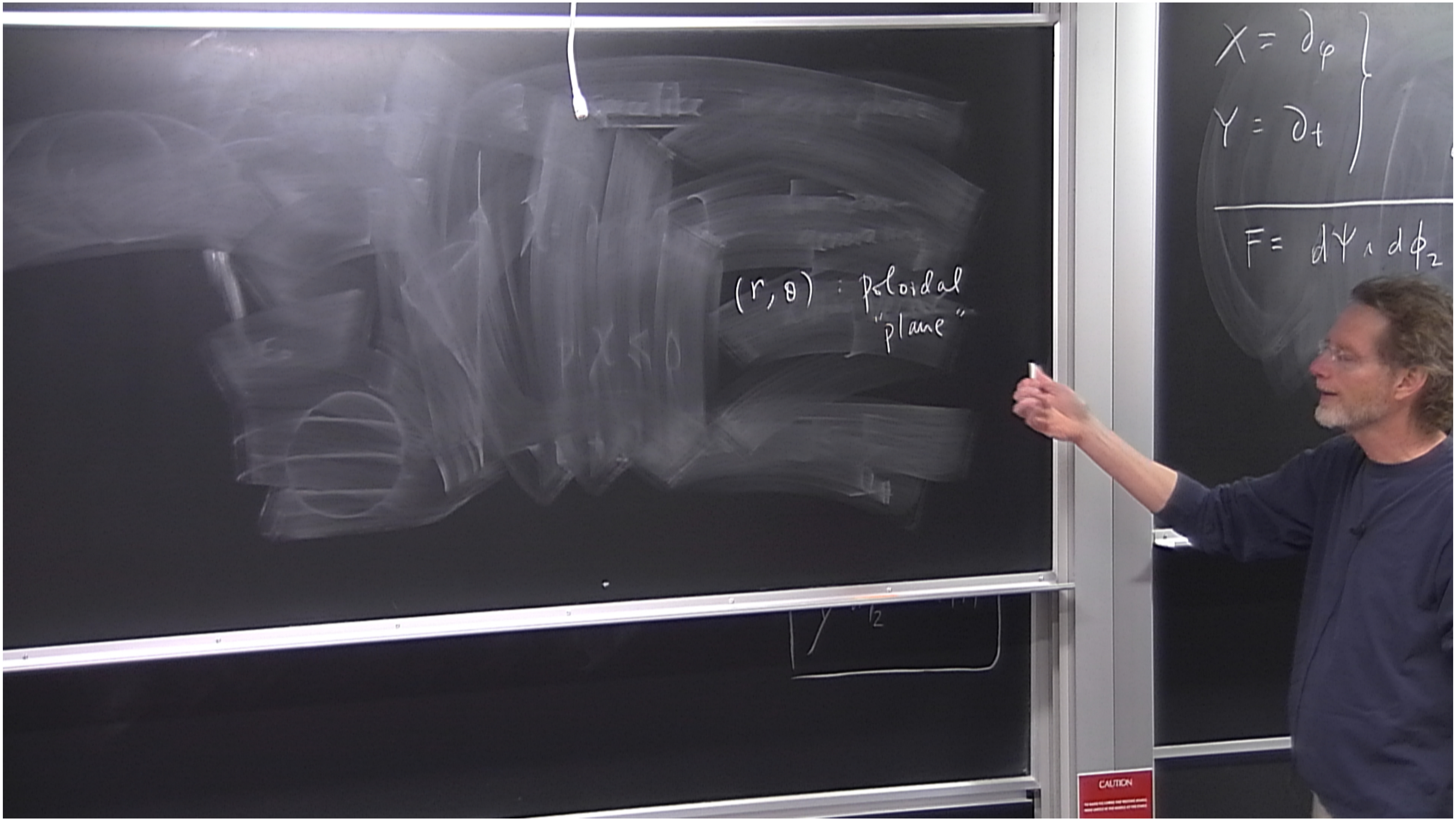

(r, θ, φ, t)
X = ∂_φ
Y = ∂_t
φ_1 = Ψ(r, θ)
φ_2 = Ψ_2(r, θ
2_F(Ψ) t
F = dΨ ∧
dφ_2 = 1
dφ_2 = K(φ_1)
CAUTION

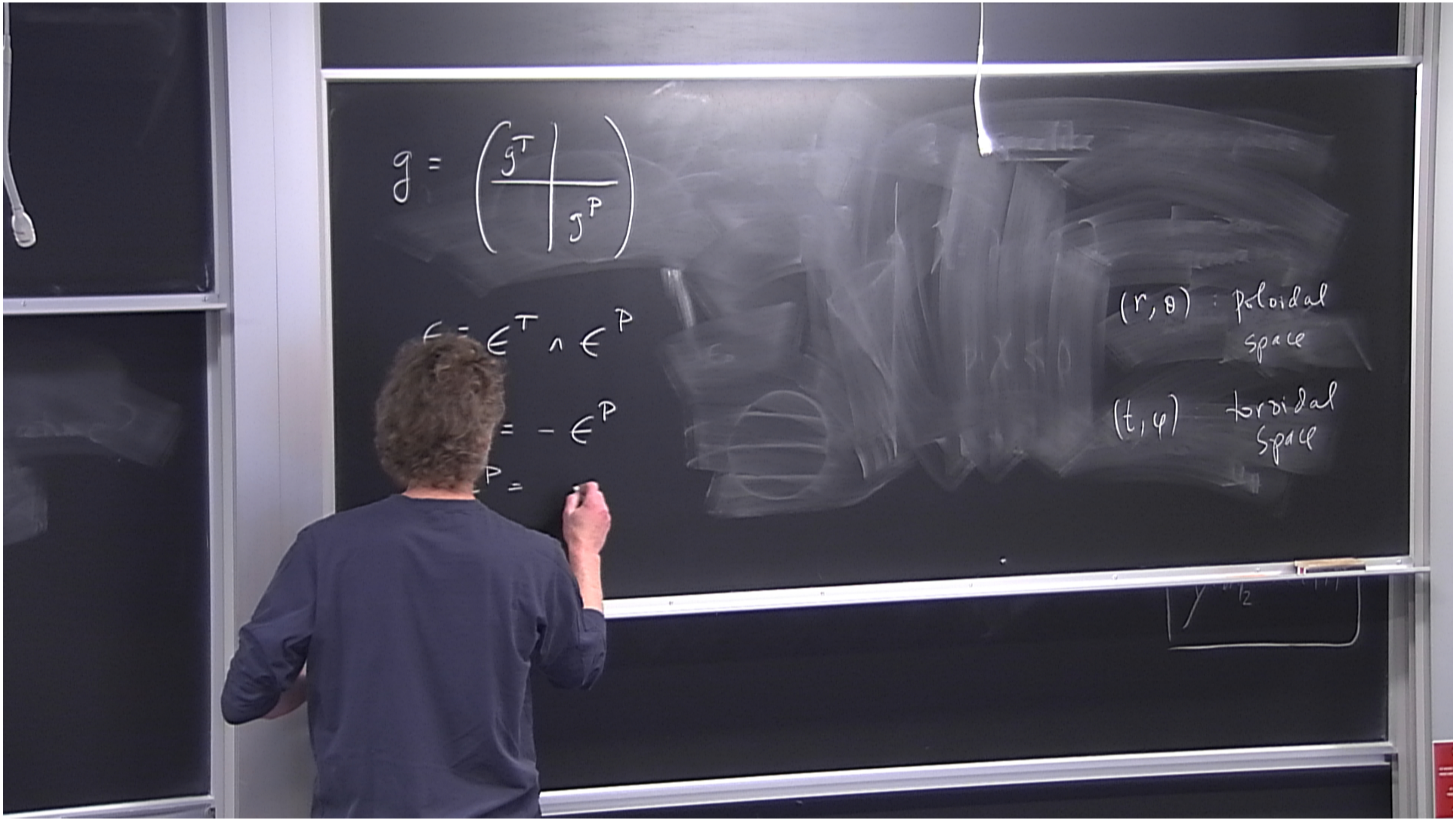


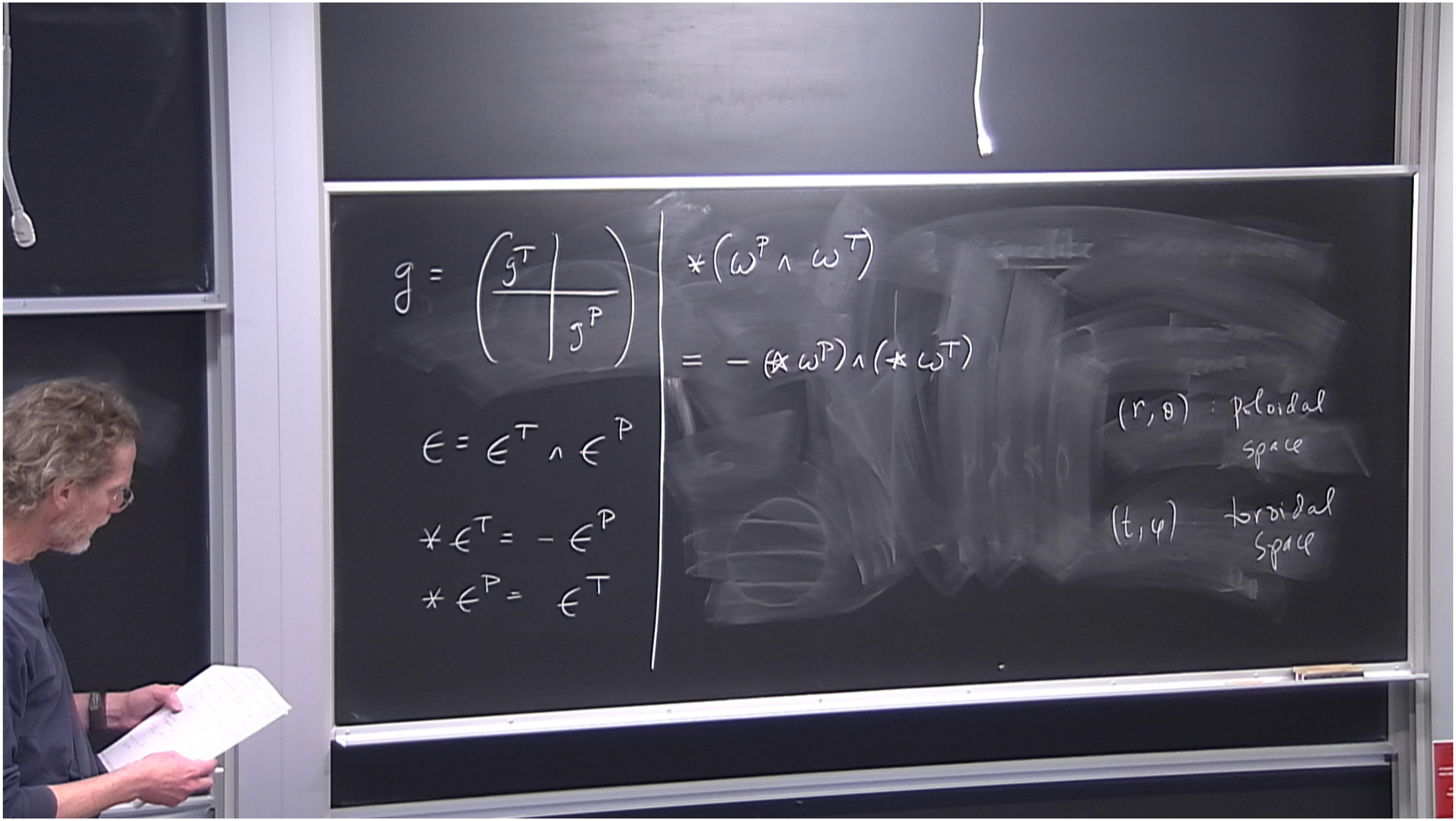
(r, θ, φ, t)
$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \Psi(r,\theta)$
$\phi_2 = \psi_2(r,\theta) + \varphi - \Omega_F(\psi)\, t$
$F = d\Psi \wedge d\phi_2 = d\Psi \wedge d\psi_2 + d\Psi \wedge$
$d\phi_2 = 1$
$d\phi_2 = K(\phi_1)$
CAUTION

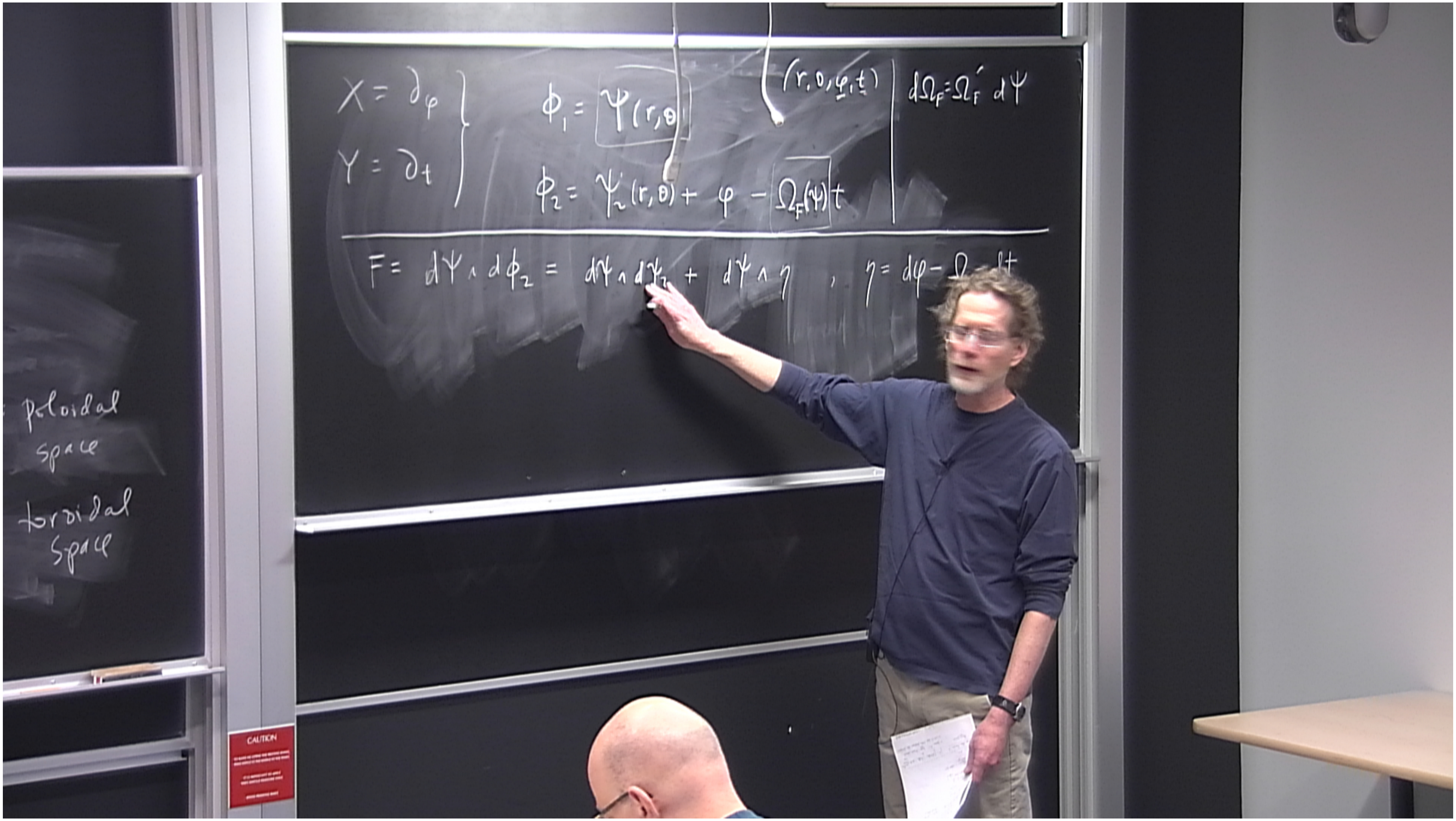


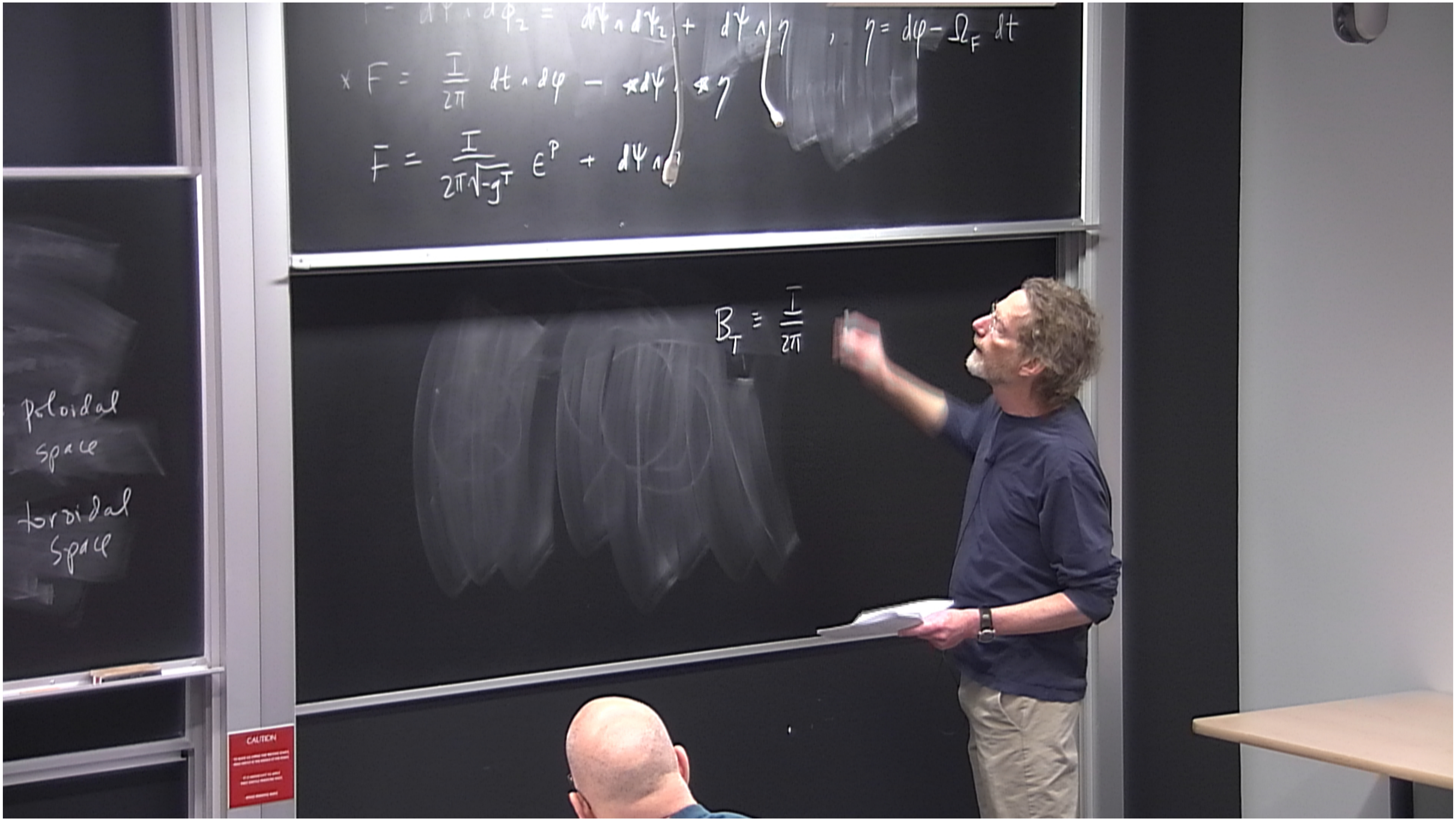
$X = \partial_\varphi$
$Y = \partial_t$
$\phi_1 = \boxed{\Psi(r,\theta)}$
$\phi_2 = \psi_2(r,\theta) + \varphi - \boxed{\Omega_F(\Psi)}\, t$
(r, θ, φ, t)
$d\Omega_F = \Omega_F' \, d\Psi$
$F = d\Psi \wedge d\phi_2 = d\Psi \wedge d\psi_2 + d\Psi \wedge \eta \quad , \quad \eta = d\varphi - \Omega_F \, dt$
$d\phi_2 = 1$
$d\phi_2 = K(\phi_1)$
CAUTION

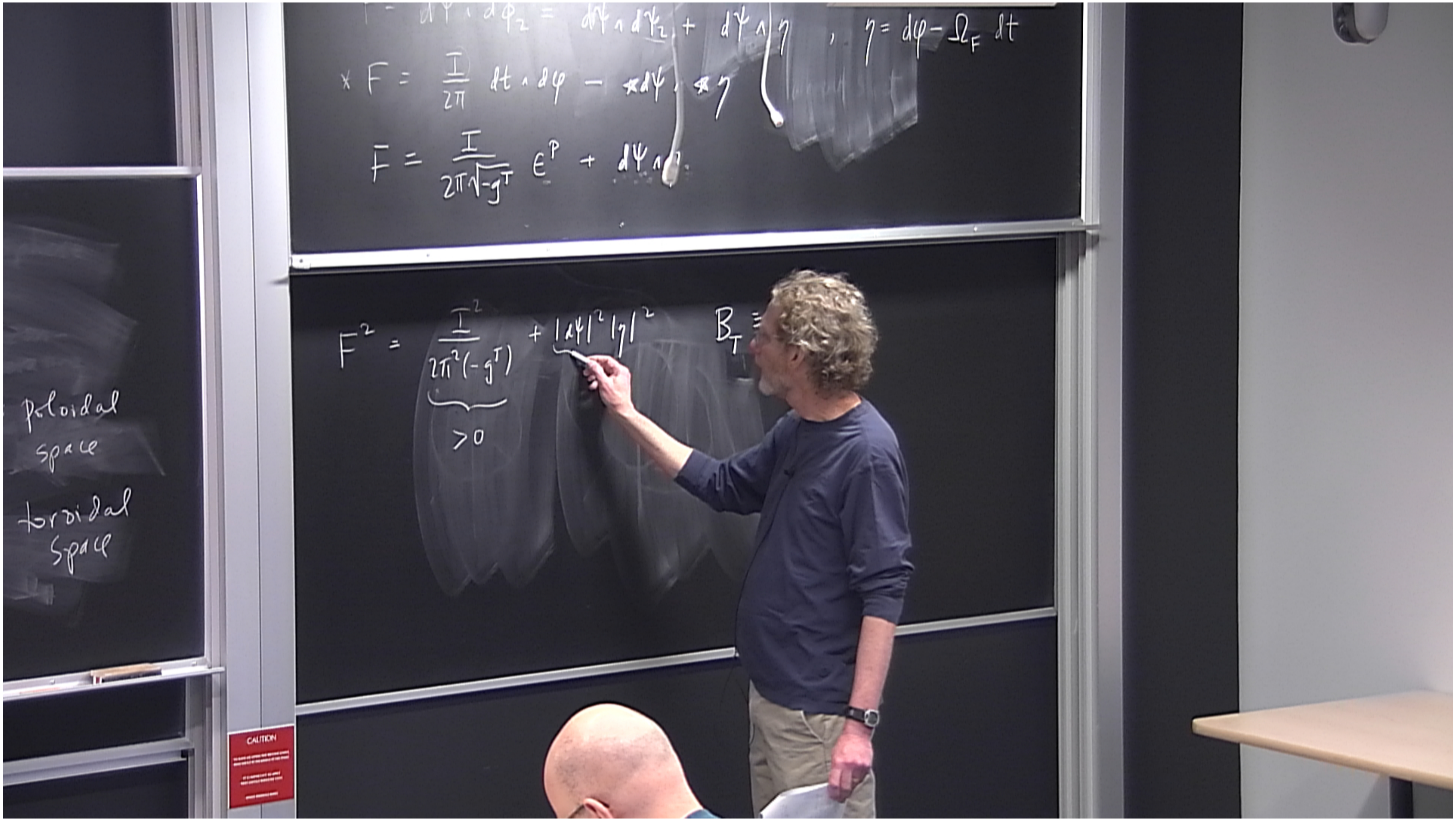

(r, θ) : poloidal
"plane"
X = ∂φ
Y = ∂t
F = dΨ ∧ dφ2


g = (g^T | g^P)
∈ = ∈^T ∧ ∈^P
= − ∈^P
P =
(r, θ) : poloidal space
(t, φ) toroidal space

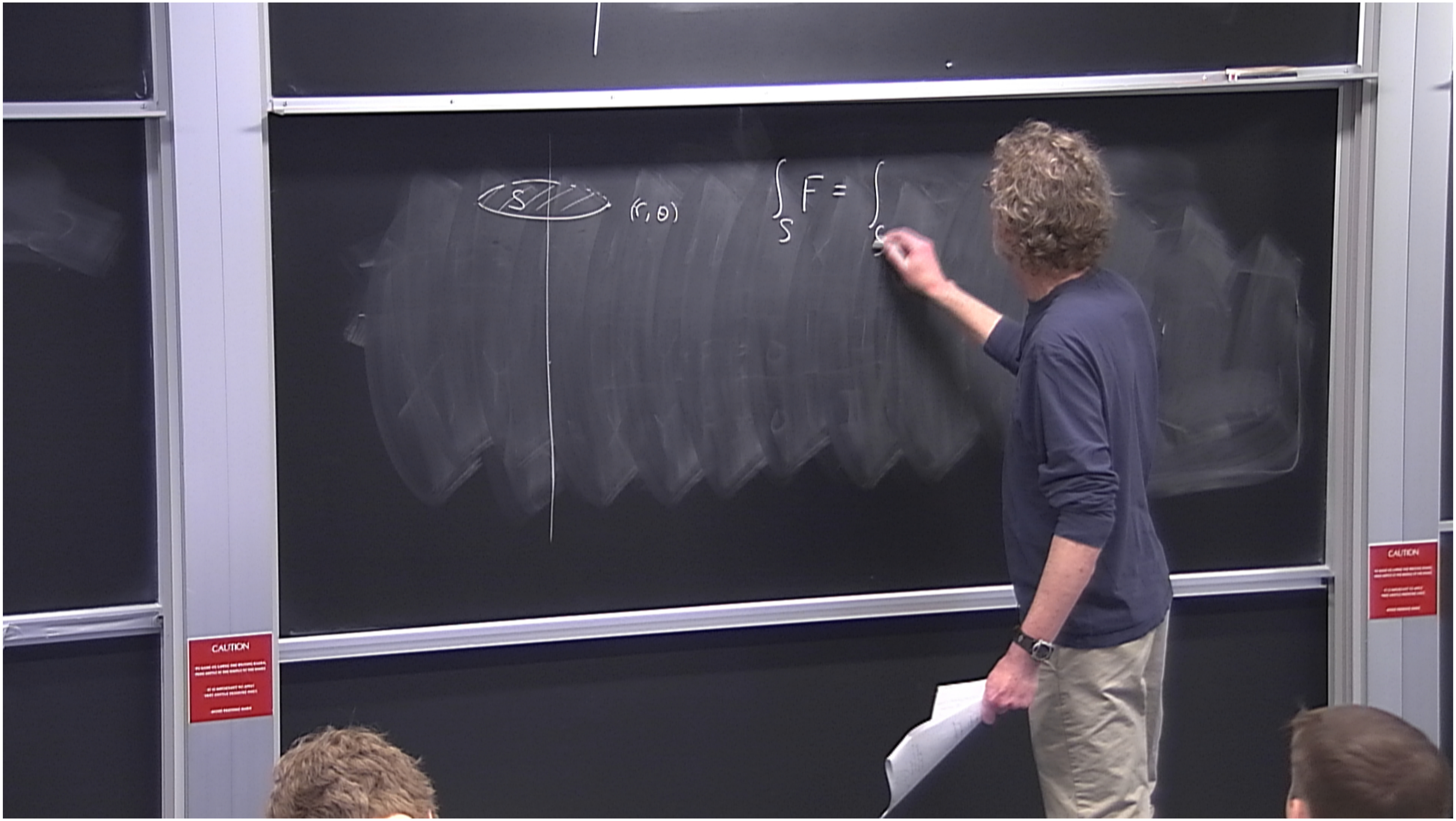

$g = \begin{pmatrix} g^T & \\ & g^P \end{pmatrix}$
$\epsilon = \epsilon^T \wedge \epsilon^P$
$* \epsilon^T = - \epsilon^P$
$* \epsilon^P = \epsilon^T$
$*(\omega^P \wedge \omega^T)$
$= -(*\omega^P) \wedge (*\omega^T)$
(r, θ) : poloidal space
(t, φ) toroidal space

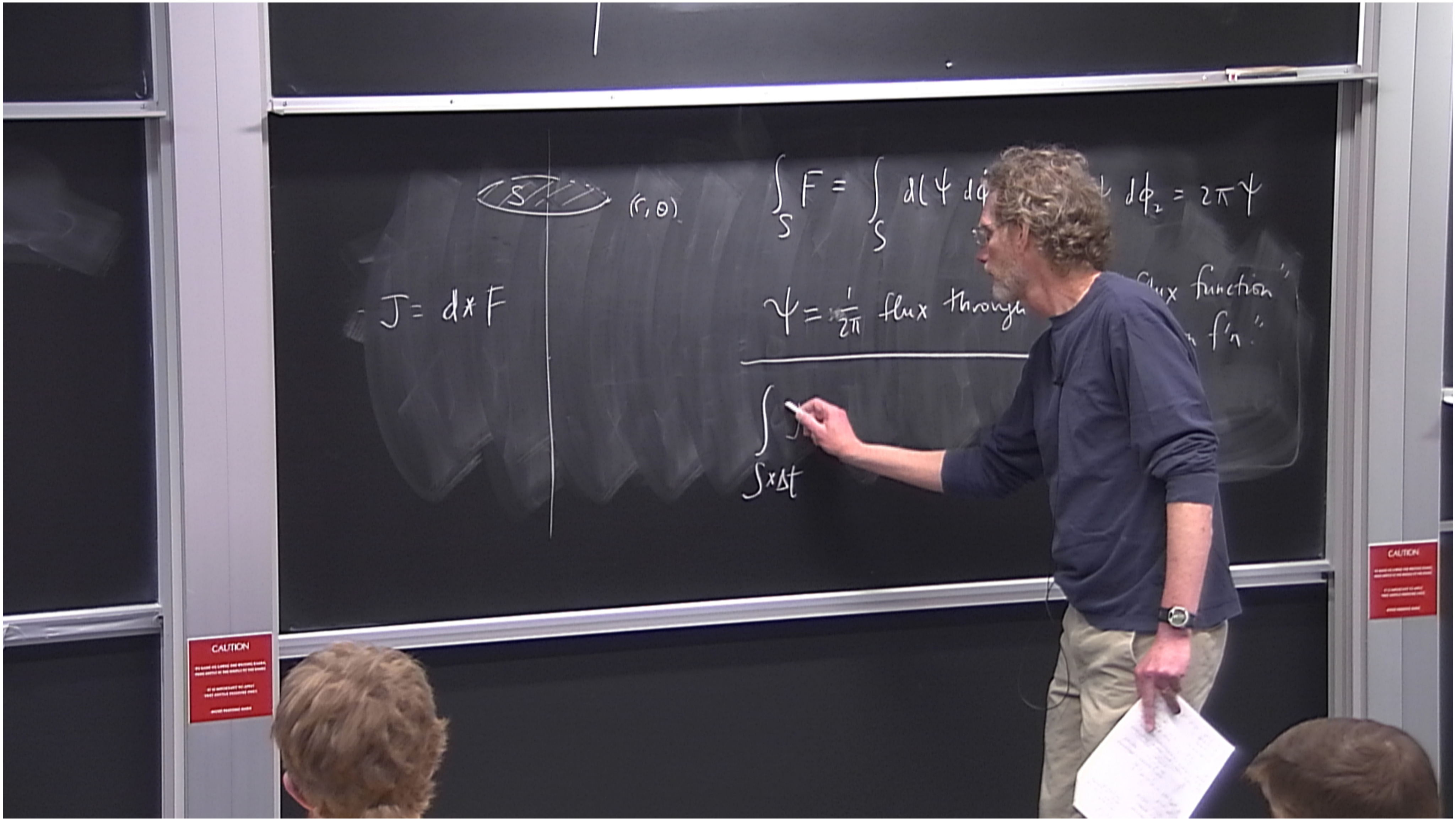

poloidal space
toroidal space
X = ∂_φ
Y = ∂_t
φ_1 = Ψ(r,θ)
φ_2 = ψ_2(r,θ) + φ − Ω_F(Ψ)t
(r,θ,φ,t)
dΩ_F = Ω_F′ dΨ
F = dΨ ∧ dφ_2 = dΨ ∧ dψ_2 + dΨ ∧ η , η = dφ − Ω_F dt
CAUTION

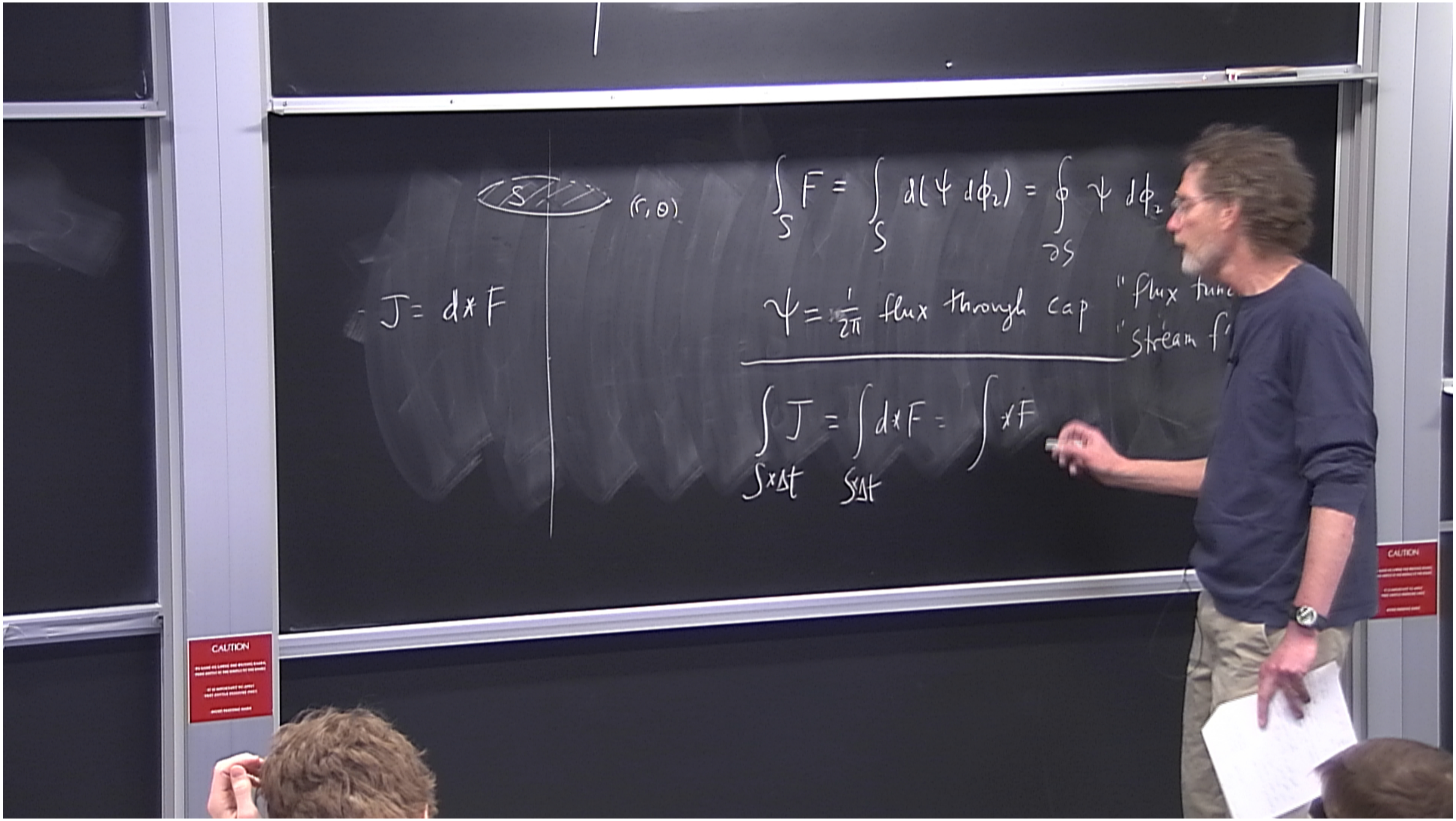

$F = \frac{I}{2\pi\sqrt{-g^T}}\epsilon^p + d\Psi \wedge \eta$
$\eta = d\varphi - \Omega_F\, dt$
$B_T \equiv \frac{I}{2\pi}$
poloidal space
toroidal space
CAUTION



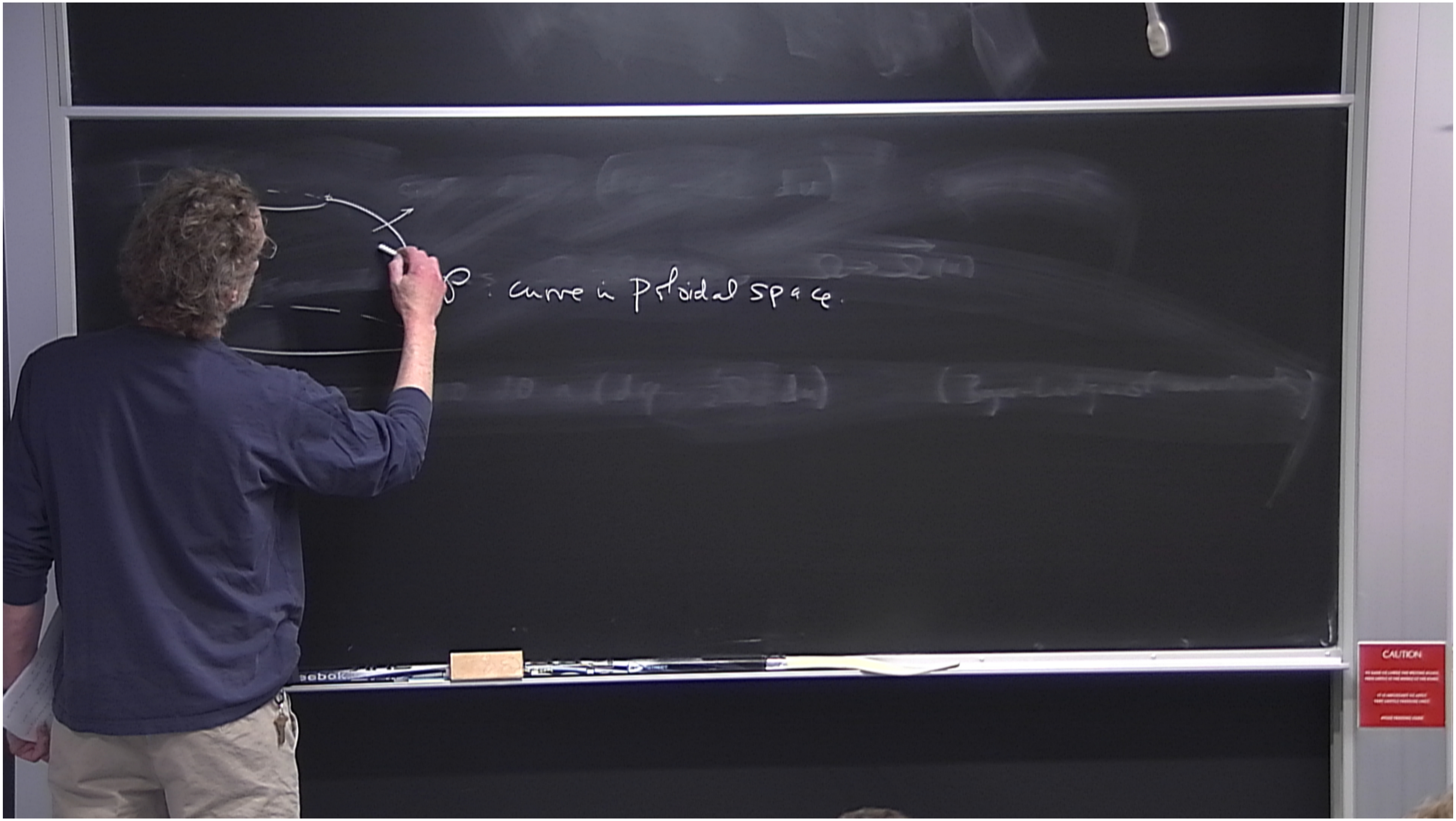
η = dφ − Ω_F dt
*F = I/2π dt ∧ dφ − *dΨ ∧ *η
F = I/(2π√(−g^T)) ε^P + dΨ ∧ η
F² = I²/(2π²(−g^T)) + |dΨ|²|η|²
> 0
B_T ≡
poloidal space
toroidal space
CAUTION

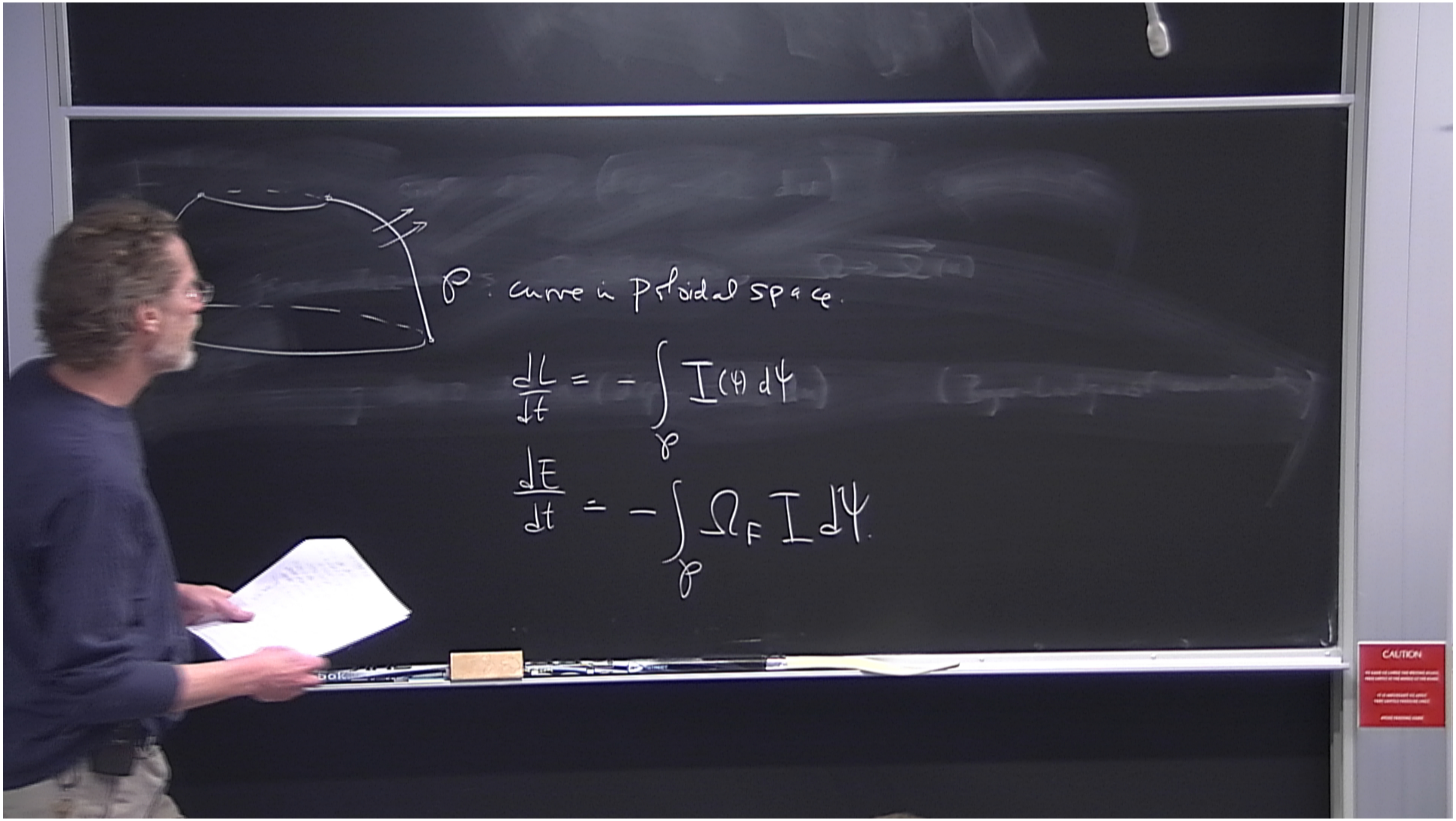

S
(r, θ)
∫_S F = ∫
CAUTION
CAUTION


S
(r, θ)
J = d*F
∫ F = ∫ d(ψ dφ
dφ₂ = 2πψ
ψ = 1/2π flux through
flux function
∫×Δt

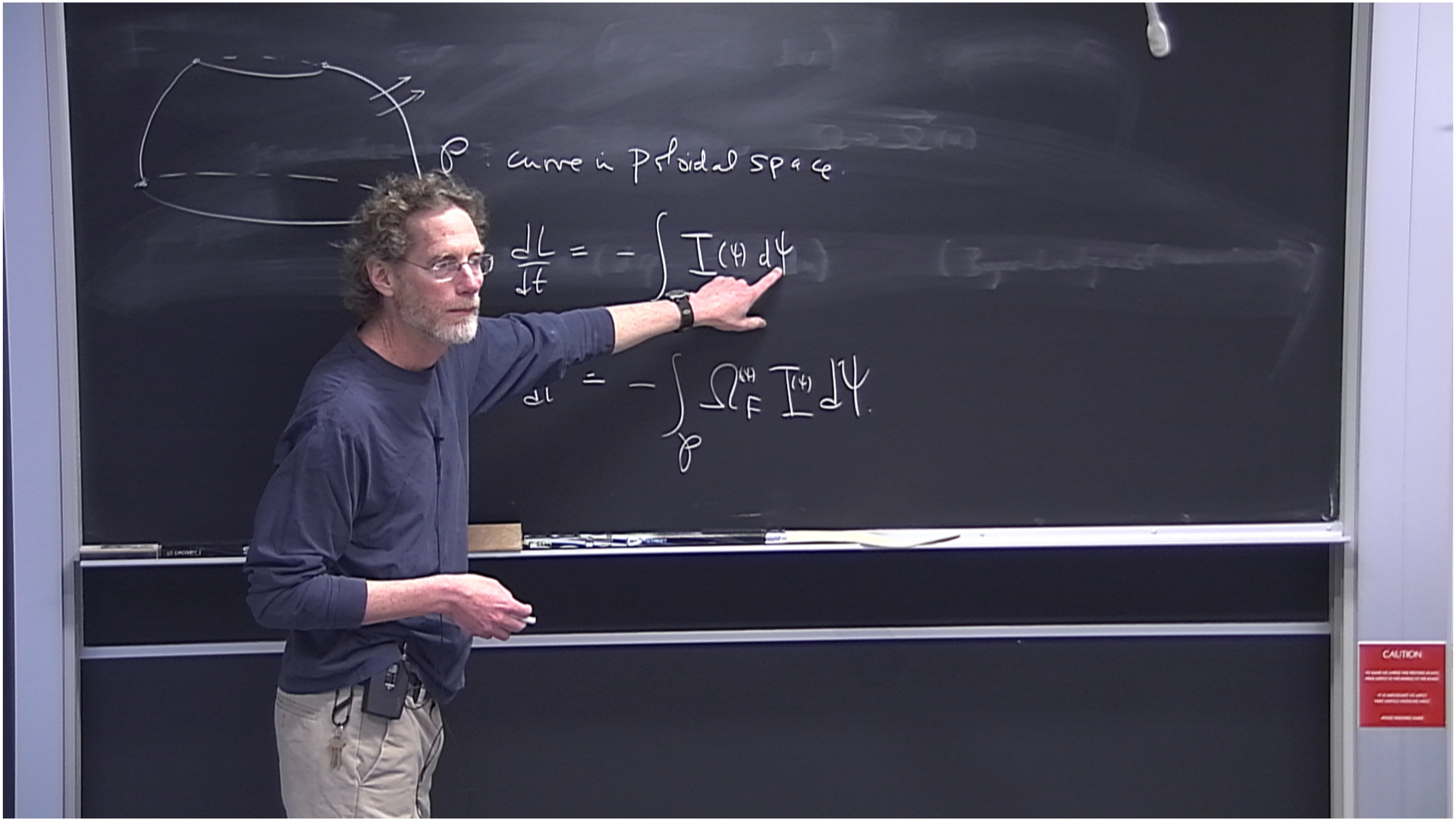


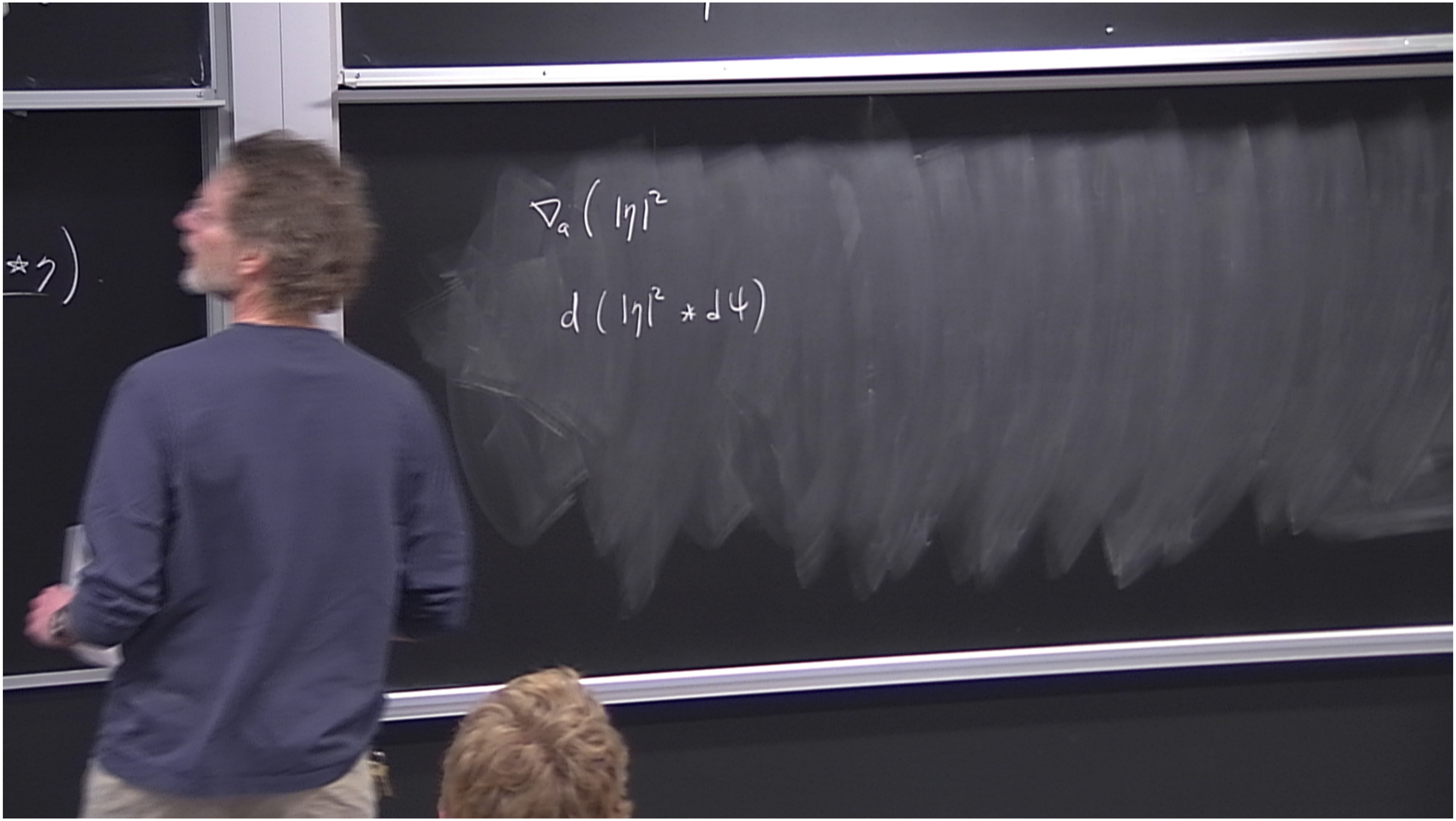
S
(r, θ)
J = d*F
∫_S F = ∫_S d(ψ dφ₂) = ∮_∂S ψ dφ₂
ψ = 1/2π flux through cap
"flux func
"stream f
∫_{S×Δt} J = ∫_{S×Δt} d*F = ∫ *F

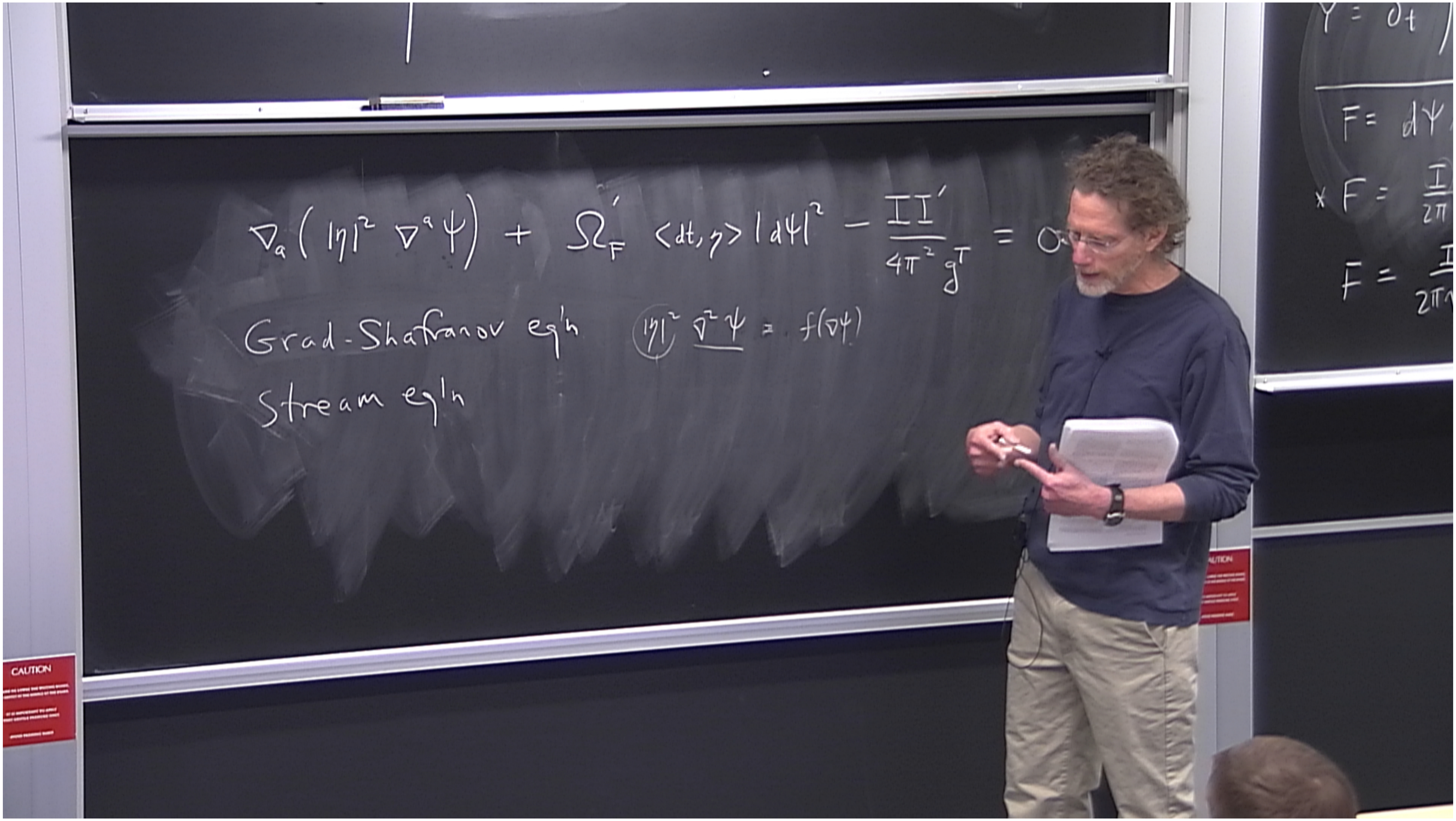

curve in poloidal space.

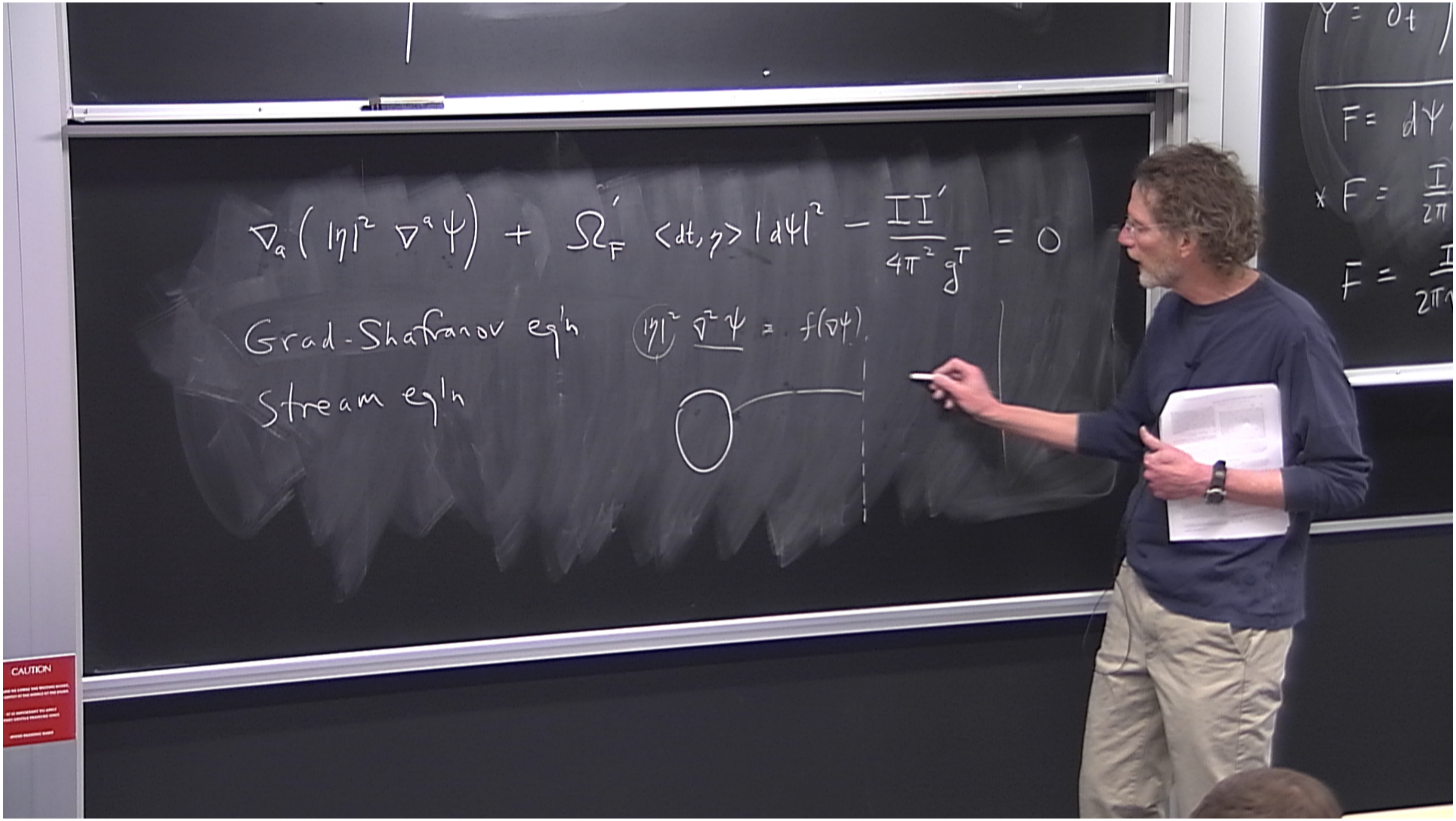


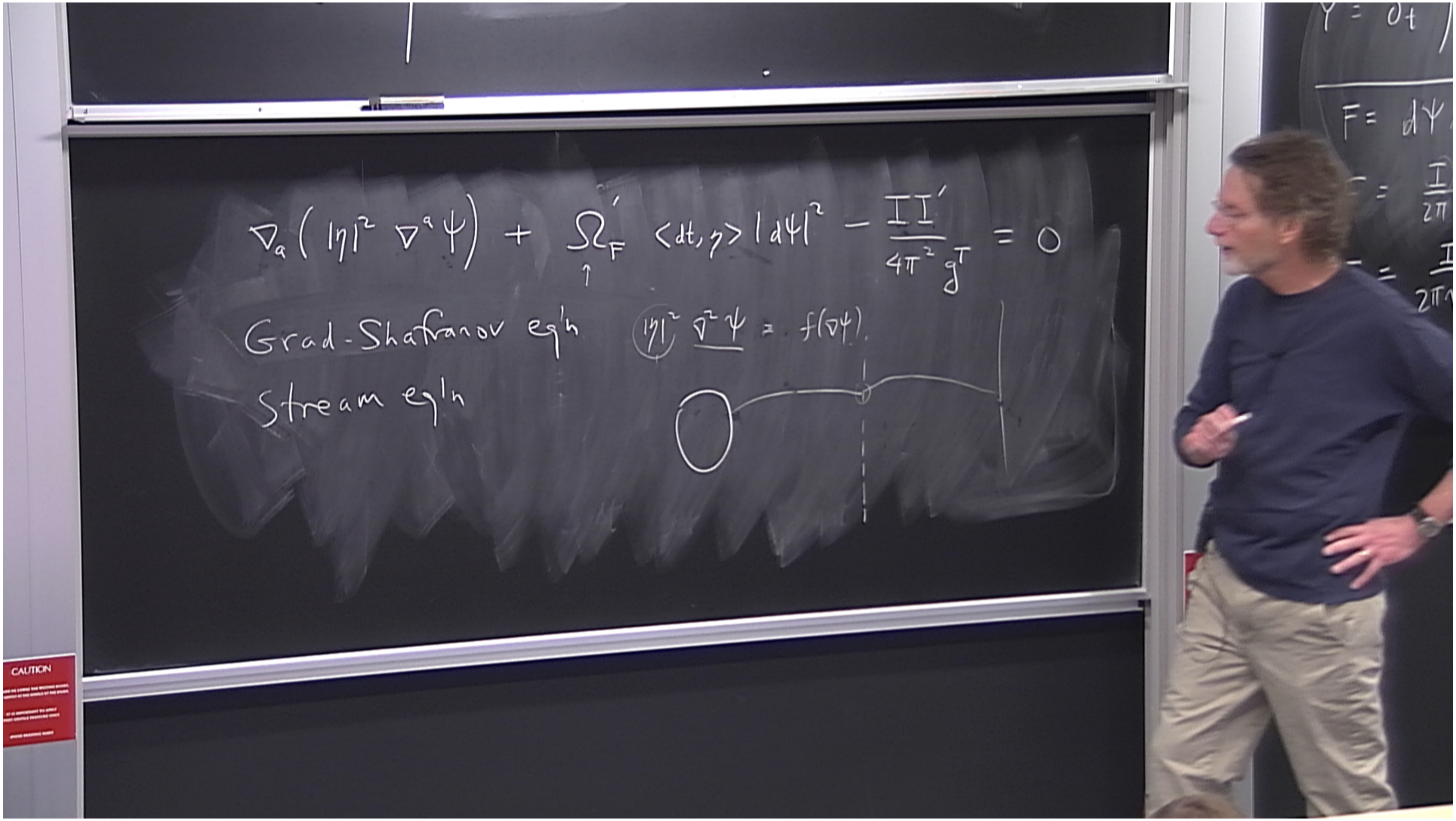
$\mathcal{P}$: curve in poloidal space.
$\frac{dL}{dt} = -\int_{\mathcal{P}} I(\psi)\, d\psi$
$\frac{dE}{dt} = -\int_{\mathcal{P}} \Omega_F I\, d\psi$


ρ : curve in poloidal space
$\frac{dL}{dt} = -\int I(\psi)\, d\psi$
$dL = -\int_\rho \Omega_F^{(\psi)} I^{(\psi)}\, d\psi$


$\star \eta)$
$\nabla_a (|\eta|^2$
$d (|\eta|^2 * d\psi)$


$\nabla_a \left(|\eta|^2 \nabla^a \psi \right) + \Omega_F' \langle dt, \eta \rangle |d\psi|^2 - \frac{I I'}{4\pi^2 g^T} = 0$
Grad-Shafranov eq'n $|\eta|^2 \nabla^2 \psi = f(\nabla\psi)$
Stream eq'n
CAUTION


∇_a (|η|² ∇^a ψ) + Ω_F' ⟨dt, η⟩ |dψ|² − II' / (4π² g^T) = 0
Grad-Shafranov eq'n
|η|² ∇²ψ = f(∇ψ)
Stream eq'n
F = dψ
CAUTION


∇_a (|η|² ∇^a ψ) + Ω_F′ ⟨dt, η⟩ |dψ|² − II′ / (4π² g^T) = 0
Grad-Shafranov eq'n
|η|² ∇²ψ = f(∇ψ)
Stream eq'n
F = dψ
CAUTION