Title: How Much Information Can SUSY QM Conserve About SUSY QFT?

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Abstract: In this presentation, evidence is given that supersymmetrical theories may be exceptional in their ability to conserve information about space-time representations under the impact of dimensional compactification. This is the essence of the concept of ``SUSY Holography."

How Much Information Can SUSY QM Conserve About SUSY QFT?

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4D Chiral Supermultiplet

$$(\boldsymbol{A},\ \boldsymbol{B},\ \boldsymbol{\psi_a},\ \boldsymbol{F},\ \boldsymbol{G})$$

$$\mathrm{D}_a A \;=\; \psi_a \quad ,$$

$$\mathrm{D}_a B \;=\; i\,(\gamma^5)_a{}^b\,\psi_b \quad ,$$

$$\mathrm{D}_a \psi_b \;=\; i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A \;-\; (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B \;-\; i\,C_{a\,b}\,F \;+\; (\gamma^5)_{a\,b} G \;,$$

$$\mathrm{D}_a F \;=\; (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad ,$$

$$\mathrm{D}_a G \;=\; i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad .$$

4D Vector Supermultiplet

$(A_\mu,\ \lambda_a,\ \mathrm{d})$

$$\mathrm{D}_a A_\mu = (\gamma_\mu)_a{}^b \lambda_b \ ,$$

$$\mathrm{D}_a \lambda_b = -\, i \tfrac{1}{4} ([\gamma^\mu, \gamma^\nu])_{ab} (\partial_\mu A_\nu - \partial_\nu A_\mu) + (\gamma^5)_{ab}\, \mathrm{d} \ ,$$

$$\mathrm{D}_a \mathrm{d} = i (\gamma^5 \gamma^\mu)_a{}^b\, \partial_\mu \lambda_b \ .$$

$\mathrm{SU}_\alpha(2)$

$$\Sigma^{ij} = i/4[\gamma^i , \gamma^j]$$

$$\Sigma^{12} = \frac{1}{2}(\sigma^2 \otimes \sigma^3) , \ \Sigma^{23} = \frac{1}{2}(\sigma^2 \otimes \sigma^1) , \ \Sigma^{31} = \frac{1}{2}(\mathrm{I}_2 \otimes \sigma^2) ,$$

$\mathrm{SU}_\beta(2)$

$$\frac{1}{2}(\sigma^3 \otimes \sigma^2) = -i\,\frac{1}{2}\gamma^0 , \ \frac{1}{2}(\sigma^1 \otimes \sigma^2) = -\frac{1}{2}\gamma^5 , \ \frac{1}{2}(\sigma^2 \otimes \mathrm{I}_2) = \frac{1}{2}\gamma^0\gamma^5 .$$

Due to the defining properties of the gamma matrices, these two SU(2) algebras commute. This implies that the complete set of sixteen elements in the covering algebra of the gamma matrices carry a representation of $\mathrm{SU}_\alpha(2) \otimes \mathrm{SU}_\beta(2)$.

$$\gamma_i = \epsilon_{ijk}\,\gamma^0\gamma^5\,\Sigma^{jk} ,$$

$SU_\alpha(2)$

$$\Sigma^{ij} = i/4[\gamma^i , \gamma^j]$$

$$\Sigma^{12} = \frac{1}{2}(\sigma^2 \otimes \sigma^3) , \quad \Sigma^{23} = \frac{1}{2}(\sigma^2 \otimes \sigma^1) , \quad \Sigma^{31} = \frac{1}{2}(I_2 \otimes \sigma^2) ,$$

$SU_\beta(2)$

$$\frac{1}{2}(\sigma^3 \otimes \sigma^2) = -i\,\frac{1}{2}\gamma^0 , \quad \frac{1}{2}(\sigma^1 \otimes \sigma^2) = -\frac{1}{2}\gamma^5 , \quad \frac{1}{2}(\sigma^2 \otimes \mathbf{I}_2) = \frac{1}{2}\gamma^0\gamma^5 .$$

Due to the defining properties of the gamma matrices, these two SU(2) algebras commute. This implies that the complete set of sixteen elements in the covering algebra of the gamma matrices carry a representation of $SU_\alpha(2) \otimes SU_\beta(2)$.

$$\gamma_i = \epsilon_{ijk}\,\gamma^0\gamma^5\,\Sigma^{jk} ,$$

$$\mathrm{D_I}\Phi_i \;=\; i\,(\mathrm{L_I})_{i\hat{k}}\Psi_{\hat{k}} \qquad \text{and} \qquad \mathrm{D_I}\Psi_{\hat{k}} \;=\; (\mathrm{R_I})_{\hat{k}i}\,(\partial_\tau\Phi_i) \quad ,$$

$$(\mathrm{L_I})_i{}^{\hat{j}}\,(\mathrm{R_J})_{\hat{j}}{}^{k} + (\mathrm{L_J})_i{}^{\hat{j}}\,(\mathrm{R_I})_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \quad ,$$

$$(\mathrm{R_J})_{\hat{i}}{}^{j}\,(\mathrm{L_I})_{j}{}^{\hat{k}} + (\mathrm{R_I})_{\hat{i}}{}^{j}\,(\mathrm{L_J})_{j}{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \quad .$$

$$(\mathrm{R_I})_{\hat{j}}{}^{k}\,\delta_{ik} = (\mathrm{L_I})_{i}{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}} \quad ,$$

$$\{\,\mathrm{D_I}\,,\;\mathrm{D_J}\,\}\Phi_i \;=\; i2\,\partial_\tau\Phi_i \quad , \quad \{\,\mathrm{D_I}\,,\;\mathrm{D_J}\,\}\Psi_{\hat{k}} \;=\; i2\,\partial_\tau\Psi_{\hat{k}}$$

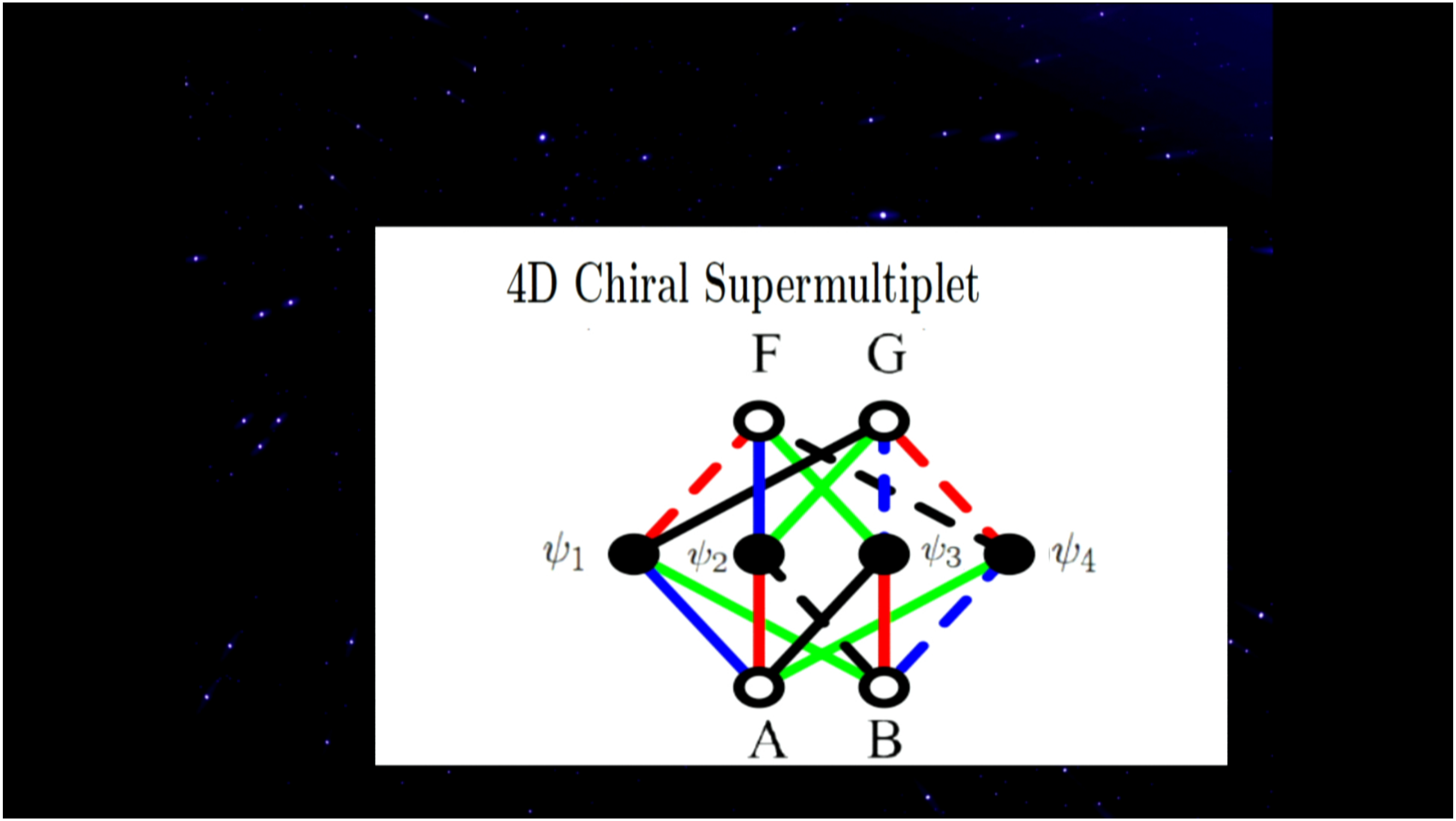
$$\partial_\mu = \mathcal{T}_\mu \frac{\partial}{\partial \tau} \quad , \quad \mathcal{T}_\mu \equiv (1, 0, 0, 0) \quad ,$$

$$\mathcal{L}_{Spin-0} = -\tfrac{1}{2}(\partial_\mu \phi^{\mathcal{I}})(\partial^\mu \phi^{\mathcal{I}}) \quad ,$$

$$\mathcal{L}_{Spin-0} = \tfrac{1}{2}(\partial_\tau \phi^1)(\partial_\tau \phi^1) + \tfrac{1}{2}(\partial_\tau \phi^2)(\partial_\tau \phi^2) + \tfrac{1}{2}(\partial_\tau \phi^3)(\partial_\tau \phi^3) \quad , \quad \text{and}$$

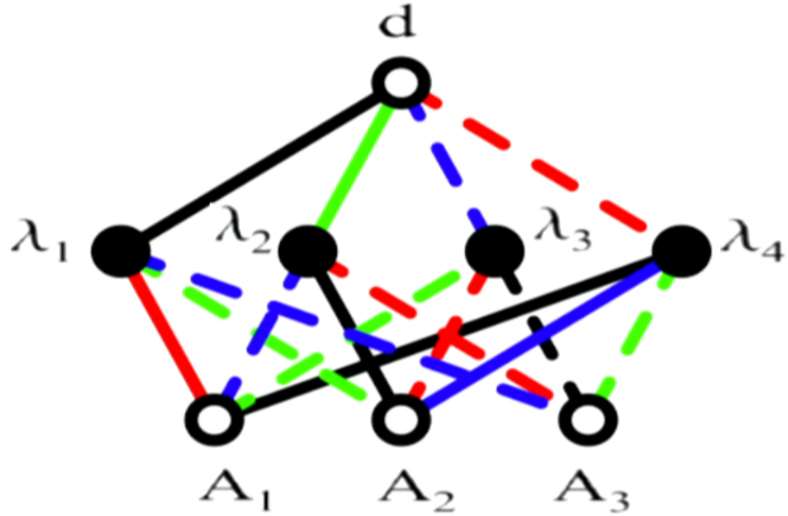
$$\mathcal{L}_{Spin-1} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\tfrac{1}{2} F_{0i} F^{0i} \quad .$$

$$\mathcal{L}_{Spin-1} = \tfrac{1}{2} \left[(\partial_\tau A_1)^2 + (\partial_\tau A_2)^2 + (\partial_\tau A_3)^2 \right] \quad .$$


4D Chiral Supermultiplet
F
G
ψ_1
ψ_2
ψ_3
ψ_4
A
B

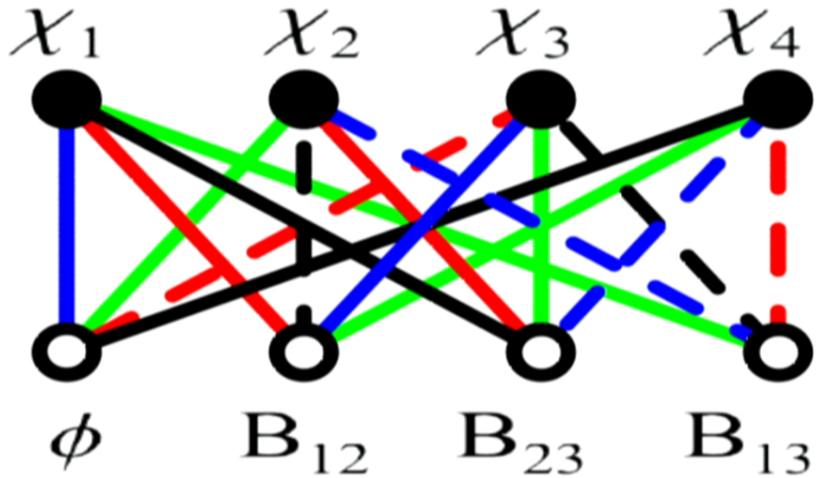
4D Vector Supermultiplet

$(A_\mu,\ \lambda_a,\ \mathrm{d})$



4D Tensor Supermultiplet

$$(\varphi,\ B_{\mu\nu},\ \chi_a)$$



Node Lowering

$$F \rightarrow \partial_\tau F \ , \ G \rightarrow \partial_\tau G \ , \ \mathrm{d} \rightarrow \partial_\tau \mathrm{d} \ ,$$

$$\begin{aligned} \mathrm{D}_a A &= \psi_a \ , & \mathrm{D}_a B &= i\,(\gamma^5)_a{}^b \psi_b \ , \\ \mathrm{D}_a F &= (\gamma \cdot \mathcal{T})_a{}^b \psi_b \ , & \mathrm{D}_a G &= i\,(\gamma^5 \gamma \cdot \mathcal{T})_a{}^b \psi_b \ , \end{aligned}$$

$$\mathrm{D}_a \psi_b = i\,(\gamma \cdot \mathcal{T})_{ab}\,(\partial_\tau A) - (\gamma^5 \gamma \cdot \mathcal{T})_{ab}\,(\partial_\tau B) - iC_{ab}\,(\partial_\tau F) + (\gamma^5)_{ab}\,(\partial_\tau G) \ ,$$

L-matrices and R-matrices

$$(\mathrm{L}_1)_{ik} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \ , \quad (\mathrm{L}_2)_{ik} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \ ,$$

$$(\mathrm{L}_3)_{ik} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \ , \quad (\mathrm{L}_4)_{ik} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \ .$$

and

$$(\mathrm{R}_1)_{ik} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix} \ , \quad (\mathrm{R}_2)_{ik} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \ ,$$

$$(\mathrm{R}_3)_{ik} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \ , \quad (\mathrm{R}_4)_{ik} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \ .$$

Valise Formulation

$$\mathrm{D}_a \rightarrow \mathrm{D}_{\mathrm{I}}$$

$$\Phi_1 = A \ , \ \Phi_2 = B \ , \ \Phi_3 = \widehat{F} \ , \ \Phi_4 = \widehat{G} \ ,$$

$$\Psi_1 = -i\psi_1 \ , \ \Psi_2 = -i\psi_2 \ , \ \Psi_3 = -i\psi_3 \ , \ \Psi_4 = -i\psi_4 \ ,$$

$$\mathrm{D}_{\mathrm{I}}\Phi_i = i\,(\mathrm{L}_{\mathrm{I}})_{i\hat{k}}\,\Psi_{\hat{k}} \ , \ \mathrm{D}_{\mathrm{I}}\Psi_{\hat{k}} = (\mathrm{R}_{\mathrm{I}})_{\hat{k}i}\,\frac{d}{d\tau}\Phi_i \ .$$

$N = 4$ "Adinkra/γ-matrix Holography Equation"

$$(\mathrm{R}_{\mathrm{I}}^{(\mathcal{R})})_i{}^j\,(\mathrm{L}_{\mathrm{J}}^{(\mathcal{R})})_j{}^{\hat{k}} - (\mathrm{R}_{\mathrm{J}}^{(\mathcal{R})})_i{}^j\,(\mathrm{L}_{\mathrm{I}}^{(\mathcal{R})})_j{}^{\hat{k}} = 2\Big[\ell_{\mathrm{IJ}}^{(\mathcal{R})1}(\gamma^2\gamma^3)_i{}^{\hat{k}} + \ell_{\mathrm{IJ}}^{(\mathcal{R})2}(\gamma^3\gamma^1)_i{}^{\hat{k}} + \ell_{\mathrm{IJ}}^{(\mathcal{R})3}(\gamma^1\gamma^2)_i{}^{\hat{k}} + i\,\widehat{\ell}_{\mathrm{IJ}}^{(\mathcal{R})1}(\gamma^0)_i{}^{\hat{k}} + i\,\widehat{\ell}_{\mathrm{IJ}}^{(\mathcal{R})2}(\gamma^5)_i{}^{\hat{k}} + \widehat{\ell}_{\mathrm{IJ}}^{(\mathcal{R})3}(\gamma^0\gamma^5)_i{}^{\hat{k}}\Big]$$

Boolean Factor/Permutation Group

$$(\mathrm{L}_{\mathrm{I}})_i{}^{\hat{k}} = (\mathcal{S}^{(\mathrm{I})})_i{}^{\hat{\ell}}\,(\mathcal{P}_{(\mathrm{I})})_{\hat{\ell}}{}^{\hat{k}}, \qquad \text{for each fixed I} = 1, 2, \dots, N.$$

$$(\mathcal{S}^{(\mathrm{I})})_i{}^{\hat{\ell}} = \begin{bmatrix} (-1)^{b_1} & 0 & 0 & \cdots \\ 0 & (-1)^{b_2} & 0 & \cdots \\ 0 & 0 & (-1)^{b_3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \leftrightarrow \quad \left(\boldsymbol{\mathcal{R}}_{\mathrm{I}} = \sum_{i=1}^{\mathrm{d}} b_i\, 2^{i-1}\right)_b$$

$$\mathrm{D_I}\Phi_i \;=\; i\,(\mathrm{L_I})_{i\hat{k}}\Psi_{\hat{k}} \qquad \text{and} \qquad \mathrm{D_I}\Psi_{\hat{k}} \;=\; (\mathrm{R_I})_{\hat{k}i}\,(\partial_\tau\Phi_i) \quad ,$$

$$(\,\mathrm{L_I}\,)_i{}^{\hat{j}}\,(\,\mathrm{R_J}\,)_{\hat{j}}{}^{k} + (\,\mathrm{L_J}\,)_i{}^{\hat{j}}\,(\,\mathrm{R_I}\,)_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \quad ,$$

$$(\,\mathrm{R_J}\,)_{\hat{i}}{}^{j}\,(\,\mathrm{L_I}\,)_j{}^{\hat{k}} + (\,\mathrm{R_I}\,)_{\hat{i}}{}^{j}\,(\,\mathrm{L_J}\,)_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \quad .$$

$$(\,\mathrm{R_I}\,)_{\hat{j}}{}^{k}\,\delta_{ik} = (\,\mathrm{L_I}\,)_i{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}} \quad ,$$

$$\{\,\mathrm{D_I}\;,\;\mathrm{D_J}\,\}\Phi_i \;=\; i2\,\partial_\tau\Phi_i \quad , \quad \{\,\mathrm{D_I}\;,\;\mathrm{D_J}\,\}\Psi_{\hat{k}} \;=\; i2\,\partial_\tau\Psi_{\hat{k}}$$

$N = 4$ "Adinkra/$\boldsymbol{\gamma}$-matrix Holography Equation"

$$(\mathrm{R}_{\mathrm{I}}^{(\mathcal{R})})_i{}^j \, (\mathrm{L}_{\mathrm{J}}^{(\mathcal{R})})_j{}^{\hat k} - (\mathrm{R}_{\mathrm{J}}^{(\mathcal{R})})_i{}^j \, (\mathrm{L}_{\mathrm{I}}^{(\mathcal{R})})_j{}^{\hat k} = 2\Big[\ell_{\mathrm{IJ}}^{(\mathcal{R})1}(\gamma^2\gamma^3)_i{}^{\hat k} + \ell_{\mathrm{IJ}}^{(\mathcal{R})2}(\gamma^3\gamma^1)_i{}^{\hat k}$$

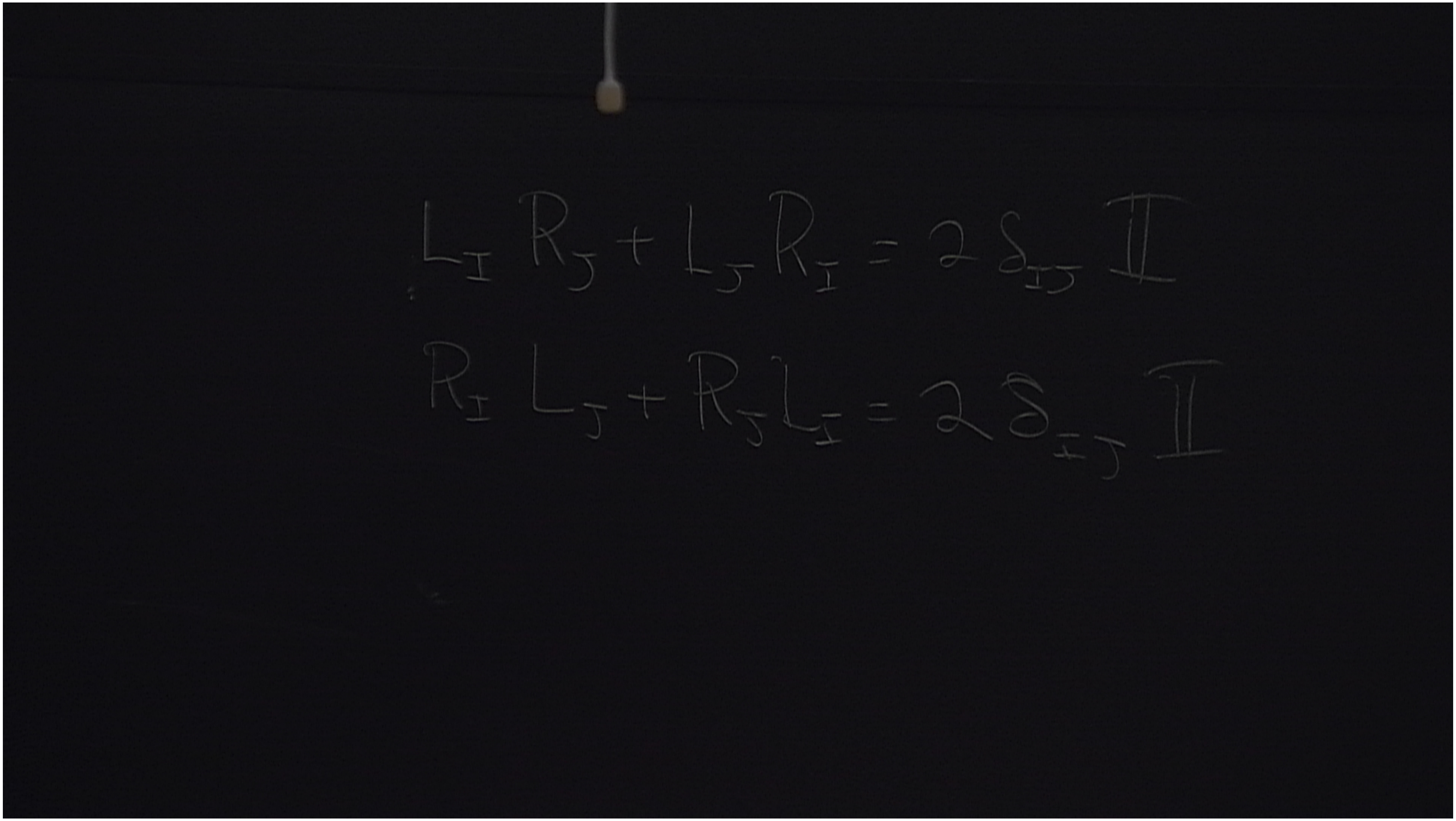
$$+ \ell_{\mathrm{IJ}}^{(\mathcal{R})3}(\gamma^1\gamma^2)_i{}^{\hat k} + i\,\widehat{\ell}_{\mathrm{IJ}}^{(\mathcal{R})1}(\gamma^0)_i{}^{\hat k}$$

$$+ i\,\widehat{\ell}_{\mathrm{IJ}}^{(\mathcal{R})2}(\gamma^5)_i{}^{\hat k} + \widehat{\ell}_{\mathrm{IJ}}^{(\mathcal{R})3}(\gamma^0\gamma^5)_i{}^{\hat k}\Big]$$

Boolean Factor/Permutation Group

$$(\mathrm{L}_{\mathrm{I}})_i{}^{\hat k} = (\mathcal{S}^{(\mathrm{I})})_i{}^{\hat\ell}\,(\mathcal{P}_{(\mathrm{I})})_{\hat\ell}{}^{\hat k}, \qquad \text{for each fixed } \mathrm{I} = 1, 2, \dots, N.$$

$$(\mathcal{S}^{(\mathrm{I})})_i{}^{\hat\ell} = \begin{bmatrix} (-1)^{b_1} & 0 & 0 & \cdots \\ 0 & (-1)^{b_2} & 0 & \cdots \\ 0 & 0 & (-1)^{b_3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \leftrightarrow \quad \left(\boldsymbol{\mathcal{R}}_{\mathrm{I}} = \sum_{i=1}^{\mathrm{d}} b_i \, 2^{i-1} \right)_b$$


$L_I R_J + L_J R_I = 2\delta_{IJ} \mathbb{1}$
$R_I L_J + R_J L_I = 2\delta_{IJ} \mathbb{1}$

$$(\mathrm{L}_1)_{i\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$(\mathrm{L}_1)_{i\hat{k}} = (10)_b \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} ,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 2 \\ 3 \end{bmatrix}$$

implies

$$1 \to 1 \ , \ 2 \to 4 \ , \ 3 \to 2 \ , \ 4 \to 3 \ ,$$

reveals the cycle $(2\,4\,3)$. We then can write $\mathrm{L}_1 = (10)_b (2\,4\,3)$.

Taking A Second Look

1 Chiral Multiplet Matrices

$$L_1 = (10)_b(2\,4\,3)\ ,\ L_2 = (6)_b(1\,3\,4)\ ,\ L_3 = (0)_b(1\,4\,2)\ ,\ L_4 = (12)_b(1\,2\,3)$$

$$R_1 = (12)_b(2\,3\,4)\ ,\ R_2 = (10)_b(1\,4\,3)\ ,\ R_3 = (0)_b(1\,2\,4)\ ,\ R_4 = (9)_b(1\,3\,2)$$

2 Vector Multiplet Matrices

$$L_1 = (10)_b(1\,2\,4\,3)\ ,\ L_2 = (12)_b(2\,3)\ ,\ L_3 = (0)_b(1\,4)\ ,\ L_4 = (6)_b(1\,3\,4\,2)$$

$$R_1 = (12)_b(1\,3\,4\,2)\ ,\ R_2 = (10)_b(2\,3)\ ,\ R_3 = (0)_b(1\,4)\ ,\ R_4 = (13)_b(1\,2\,4\,3)$$

3 Tensor Multiplet Matrices

$$L_1 = (14)_b(2\,3\,4)\ ,\ L_2 = (2)_b(1\,4\,3)\ ,\ L_3 = (4)_b(1\,2\,4)\ ,\ L_4 = (8)_b(1\,3\,2)$$

$$R_1 = (14)_b(2\,4\,3)\ ,\ R_2 = (2)_b(1\,3\,4)\ ,\ R_3 = (4)_b(1\,4\,2)\ ,\ R_4 = (8)_b(1\,2\,3)$$

SUSY Holography Conjecture:

There is sufficient information within 1d SUSY QM systems to re-construct the kinematic structure of all higher dimensional representations of SUSY.

S.J. Gates, Jr. & L. Rana,
''Tuning the RADIO to the off-shell 2-D Fayet hypermultiplet problem,''
hep-th/9602072

S. J. Gates, Jr., W. D. Linch, III, J. Phillips,
''When superspace is not enough,''
hep-th/0211034

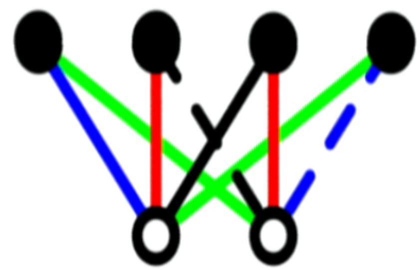


Figure # 1: Adinkra for On-shell Chiral Supermultiplet

$$\mathrm{D}_a A = \psi_a \quad , \quad \mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \quad ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma \cdot \mathcal{T})_{a\,b}\,\partial_\tau A - (\gamma^5 \gamma \cdot \mathcal{T})_{a\,b}\,\partial_\tau B \quad .$$

$$\{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, A = i\,2\,(\gamma \cdot \mathcal{T})_{a\,b}\,\partial_\tau\, A \quad , \quad \{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, B = i\,2\,(\gamma \cdot \mathcal{T})_{a\,b}\,\partial_\tau\, B \quad ,$$

$$\{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, \psi_c = i\,2\,(\gamma \cdot \mathcal{T})_{a\,b}\,\partial_\tau\, \psi_c - i\,(\gamma^\mu)_{a\,b}\,(\gamma_\mu \gamma \cdot \mathcal{T})_c{}^d \partial_\tau\, \psi_d \quad .$$

$$\{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, \psi_c = i\,2\,(\gamma \cdot \mathcal{T})_{a\,b}\,\partial_\tau\, \psi_c + i\,2\,(\gamma^\mu)_{a\,b}\,(\gamma_\mu)_c{}^d \mathcal{K}_d(\psi) \quad ,$$

$$\mathcal{K}_c(\psi) = -\,\frac{1}{2}\,(\gamma \cdot \mathcal{T})_c{}^d \partial_\tau\, \psi_d \quad ,$$

$$\Phi_i = (A,\, B) \quad , \quad \Psi_{\hat{k}} = -i\,(\psi_1,\, \psi_2,\, \psi_3,\, \psi_4) \ ,$$

$$(\mathrm{L}_1)_{i\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \ , \quad (\mathrm{L}_2)_{i\hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \ ,$$

$$(\mathrm{L}_3)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \ , \quad (\mathrm{L}_4)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \ ,$$

$$(\mathrm{R}_1)_{i\hat{k}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \ , \quad (\mathrm{R}_2)_{i\hat{k}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \ ,$$

$$(\mathrm{R}_3)_{i\hat{k}} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \ , \quad (\mathrm{R}_4)_{i\hat{k}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \ .$$

$$(\mathrm{L}_{\mathrm{I}})_i{}^{\hat{j}}\,(\mathrm{R}_{\mathrm{J}})_{\hat{j}}{}^{k} + (\mathrm{L}_{\mathrm{J}})_i{}^{\hat{j}}\,(\mathrm{R}_{\mathrm{I}})_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \ ,$$

$$(\mathrm{R}_{\mathrm{J}})_{\hat{i}}{}^{j}\,(\mathrm{L}_{\mathrm{I}})_{j}{}^{\hat{k}} + (\mathrm{R}_{\mathrm{I}})_{\hat{i}}{}^{j}\,(\mathrm{L}_{\mathrm{J}})_{j}{}^{\hat{k}} = \delta_{\mathrm{IJ}}\,(\mathrm{I})_{\hat{i}}{}^{\hat{k}} + [\vec{\alpha}\beta^1]_{\mathrm{IJ}} \cdot (\vec{\alpha}\beta^1)_{\hat{i}}{}^{\hat{k}}$$

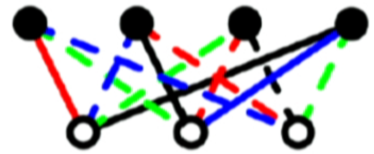


Figure # 2: Adinkra for On-shell Chiral Supermultiplet

$$\mathrm{D}_a A_i \;=\; (\gamma_i)_a{}^b \lambda_b \quad ,$$
$$\mathrm{D}_a \lambda_b \;=\; -\, i \tfrac{1}{2}([\, \gamma \cdot \mathcal{T} \,,\, \gamma^i \,])_{ab} \, (\, \partial_\tau A_i \,) \quad .$$

Define the (3×1) bosonic "field vector" and (4×1) fermionic "field vector"

$$\Phi_i \;=\; (A_1,\, A_2,\, A_3\,) \quad , \quad \Psi_{\hat{k}} \;=\; -i\,(\lambda_1,\, \lambda_2,\, \lambda_3,\, \lambda_4) \;\; ,$$

$$\{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, A_i \;=\; i\, 2\, (\gamma \cdot \mathcal{T})_{a\,b}\, \partial_\tau\, A_i \;\; ,$$
$$\{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, \lambda_c \;=\; i\, 2\, (\gamma \cdot \mathcal{T})_{a\,b}\, \partial_\tau\, \lambda_c \;-\; i\, \tfrac{1}{2}\, (\gamma^\mu)_{a\,b}\, (\gamma_\mu \gamma \cdot \mathcal{T})_c{}^d\, \partial_\tau\, \lambda_d$$
$$+\; i\, \tfrac{1}{16}\, ([\, \gamma^\alpha \,,\, \gamma^\beta \,])_{a\,b}\, ([\, \gamma_\alpha \,,\, \gamma_\beta \,] \gamma \cdot \mathcal{T})_c{}^d\, \partial_\tau\, \lambda_d \quad .$$

$$\{\, \mathrm{D}_a \,,\, \mathrm{D}_b \,\}\, \lambda_c \;=\; i\, 2\, (\gamma \cdot \mathcal{T})_{a\,b}\, \partial_\tau\, \lambda_c \;+\; i 2\, (\gamma^\mu)_{a\,b}\, (\gamma_\mu)_c{}^d\, \widehat{K}_d(\lambda)$$
$$-\; i\, \tfrac{1}{4}\, ([\, \gamma^\alpha \,,\, \gamma^\beta \,])_{a\,b}\, ([\, \gamma_\alpha \,,\, \gamma_\beta \,])_c{}^d\, \widehat{K}_d(\lambda) \;\; ,$$
$$\widehat{K}_c(\lambda) \;\equiv\; -\, \tfrac{1}{4} (\gamma \cdot \mathcal{T})_c{}^d\, \partial_\tau\, \lambda_d \;\; ,$$

$$(\mathrm{L}_1)_{i\hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad , \quad (\mathrm{L}_2)_{i\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad ,$$

$$(\mathrm{L}_3)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad , \quad (\mathrm{L}_4)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad ,$$

arXiv.org > hep-th > arXiv:1311.3736

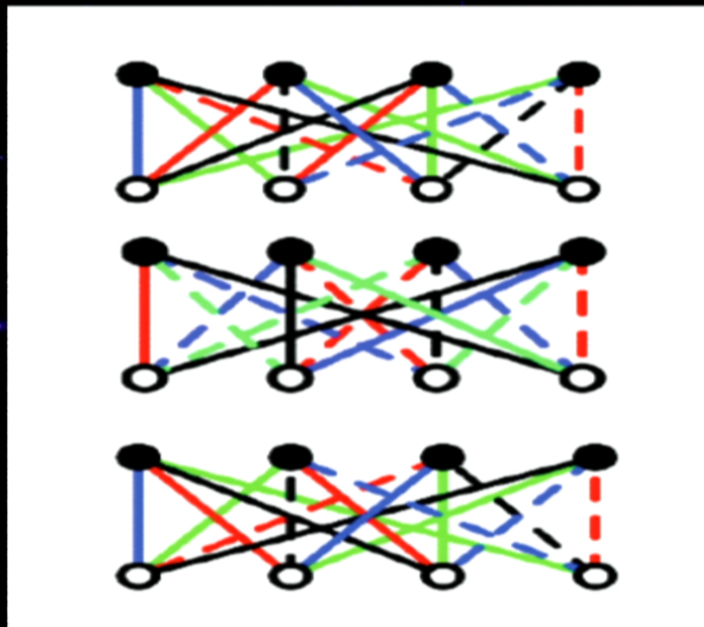
High Energy Physics - Theory

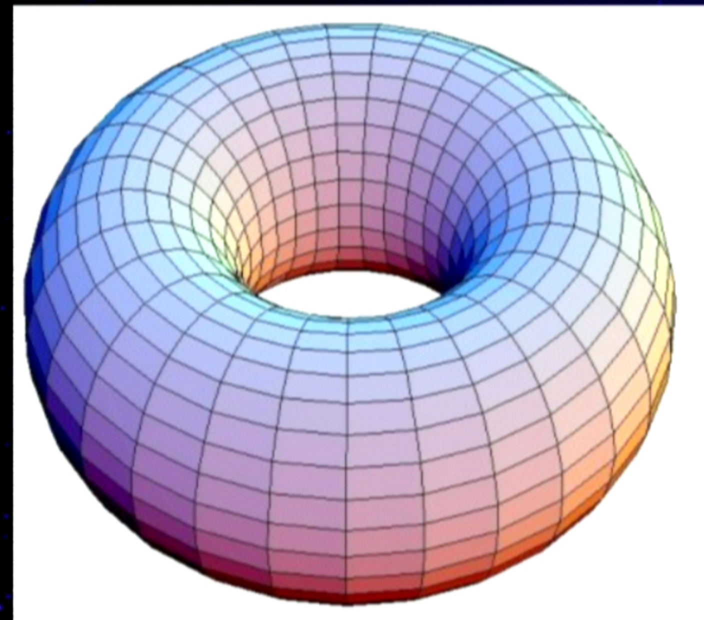
Geometrization of N-Extended 1-Dimensional Supersymmetry Algebras

Charles Doran, Kevin Iga, Greg Landweber, Stefan Mendez-Diez

(Submitted on 15 Nov 2013)

The problem of classifying off-shell representations of the N-extended one-dimensional super Poincar\'{e} algebra is closely related to the study of a class of decorated graphs known as Adinkras. We show that these combinatorial objects possess a form of emergent supergeometry: Adinkras are equivalent to very special super Riemann surfaces with divisors. The method of proof critically involves Grothendieck's theory of "dessins d'enfants", work of Cimasoni-Reshetikhin expressing spin structures on Riemann surfaces via dimer models, and an observation of Donagi-Witten on parabolic structure from ramified coverings of super Riemann surfaces.





A Beyli pair, $(\mathcal{X}, \beta)$ is a closed Riemann surface X equipped a Belyi map, $\beta : \mathcal{X} \to CP^1$ that is ramified at most over $\{0, 1, \infty\}$. Adinkras induce an integer-valued Morse function on the Riemann surface which is also a divisor.

$$g = 1 + 2^{N-k-3}(N - 4)$$