Title: The Unreasonable Effectiveness Of Quantum Physics in Modern Mathematics

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Abstract: Mathematics has proven to be "unreasonably effective" in understanding nature. The fundamental laws of physics can be captured in beautiful formulae. In this lecture I want to argue for the reverse effect: Nature is an important source of inspiration for mathematics, even of the purest kind. In recent years ideas from quantum field theory, elementary particles physics and string theory have completely transformed mathematics, leading to solutions of deep problems, suggesting new invariants in geometry and topology, and, perhaps most importantly, putting modern mathematical ideas in a `naturalâ€™ context.

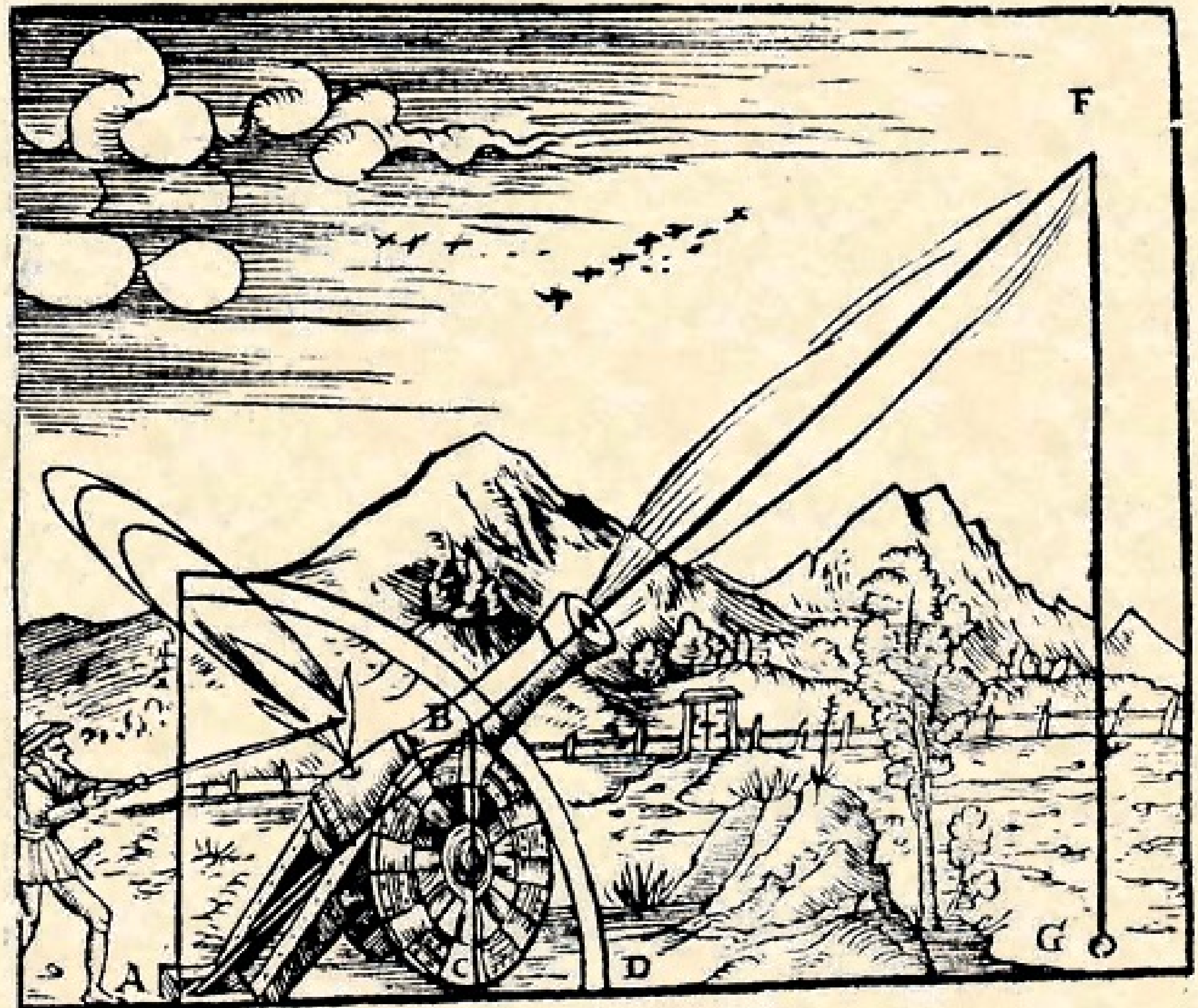
The Unreasonable Effectiveness of Quantum Physics in Modern Mathematics

Robbert Dijkgraaf

Institute for Advanced Study

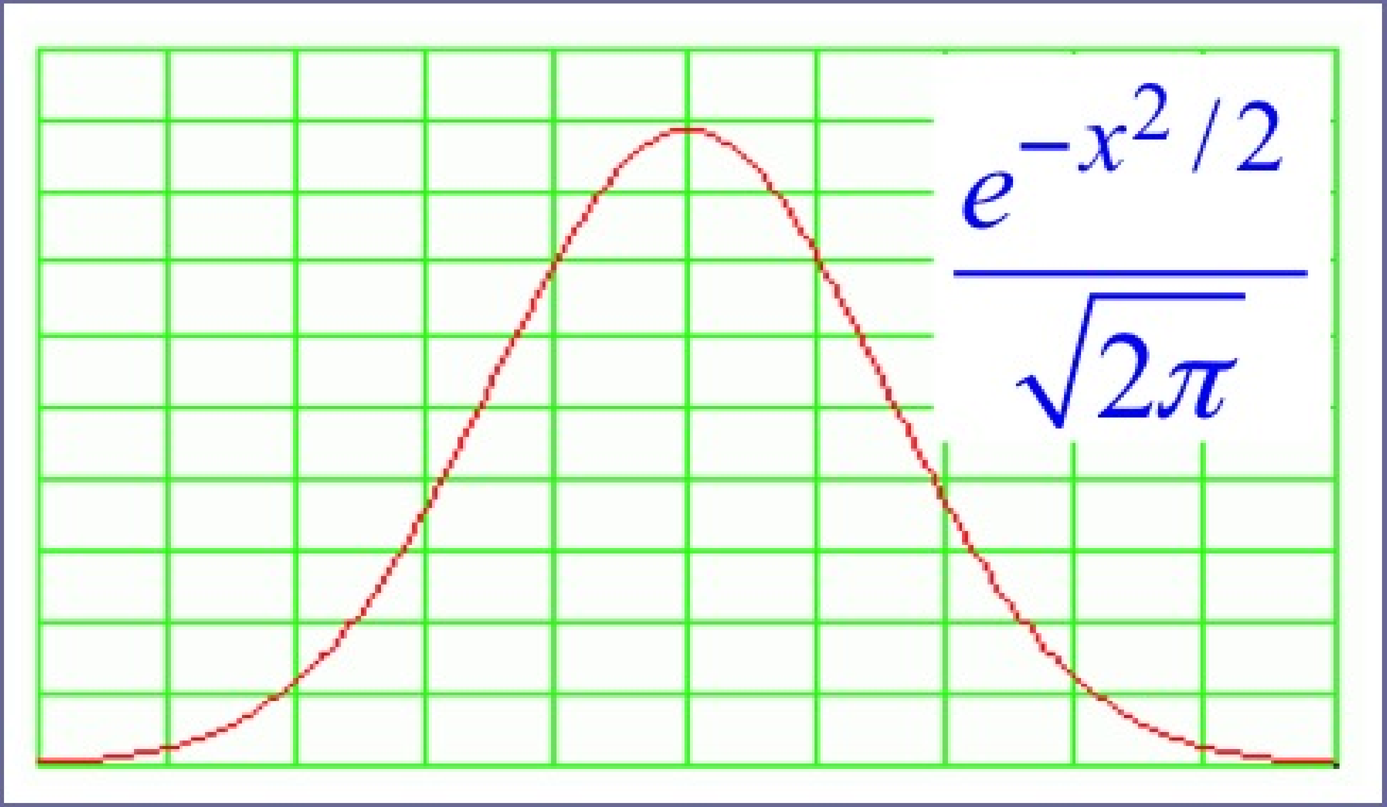
Public Lecture, Perimeter Institute, March 5, 2014

Mathematics & Physics

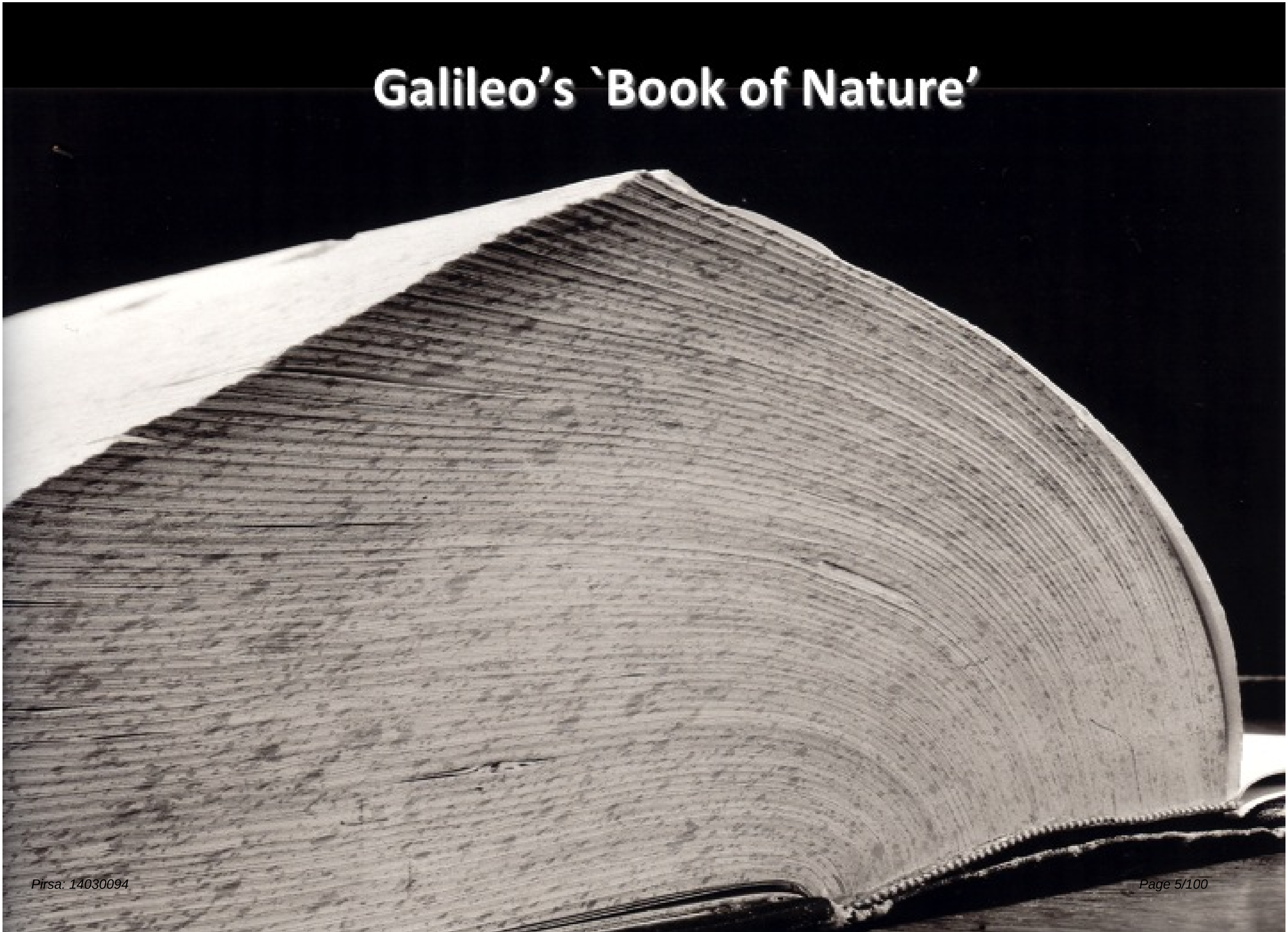


"The Unreasonable Effectiveness of Mathematics in the Natural Sciences."

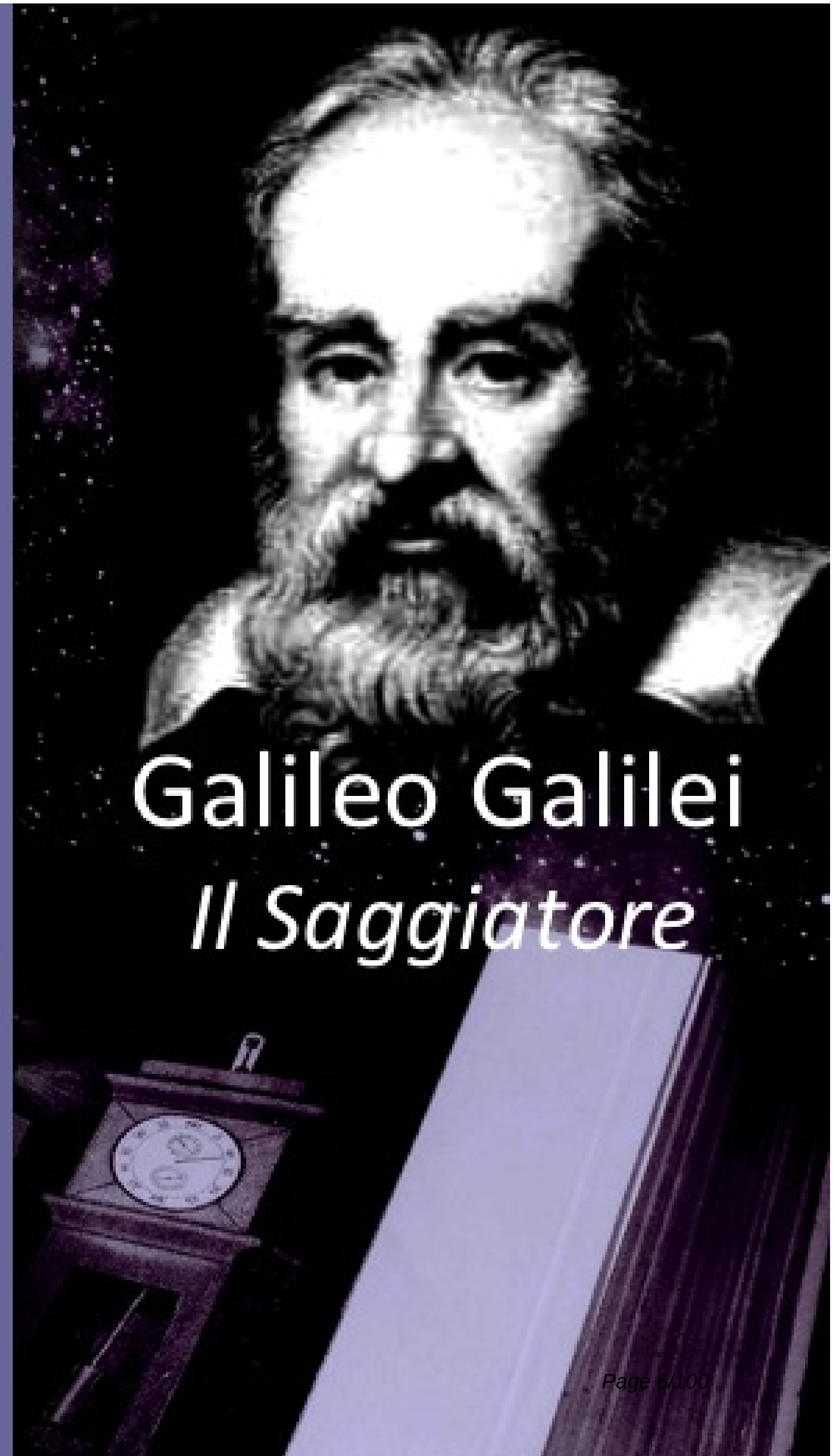
— Eugene Wigner (1960)



Galileo's `Book of Nature'



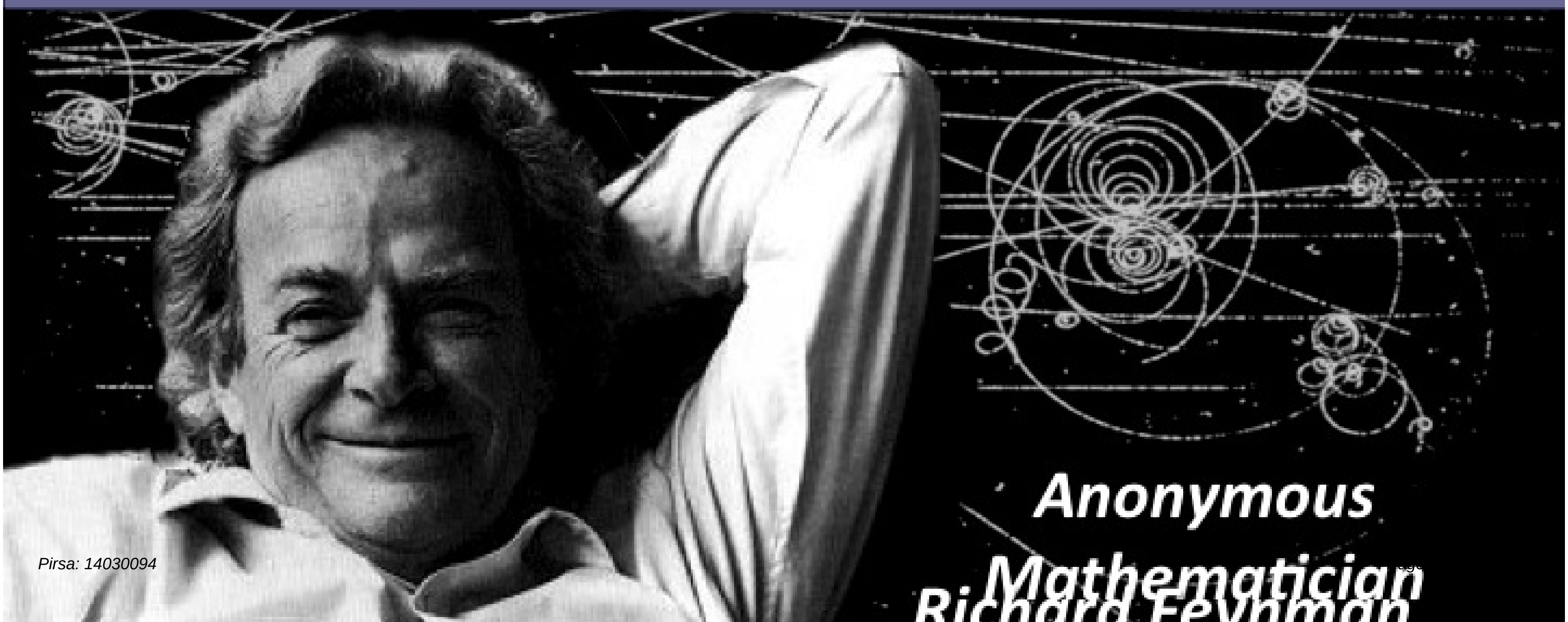
“Philosophy is written in this grand book — I mean the universe — which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. **It is written in the language of mathematics**, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth.”



“To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature … If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.”

“If all mathematics disappeared today, physics would be set back exactly one week.”

“That was the week that God created the world.”



#322, G. A. Plimpton Collection, Columbia University

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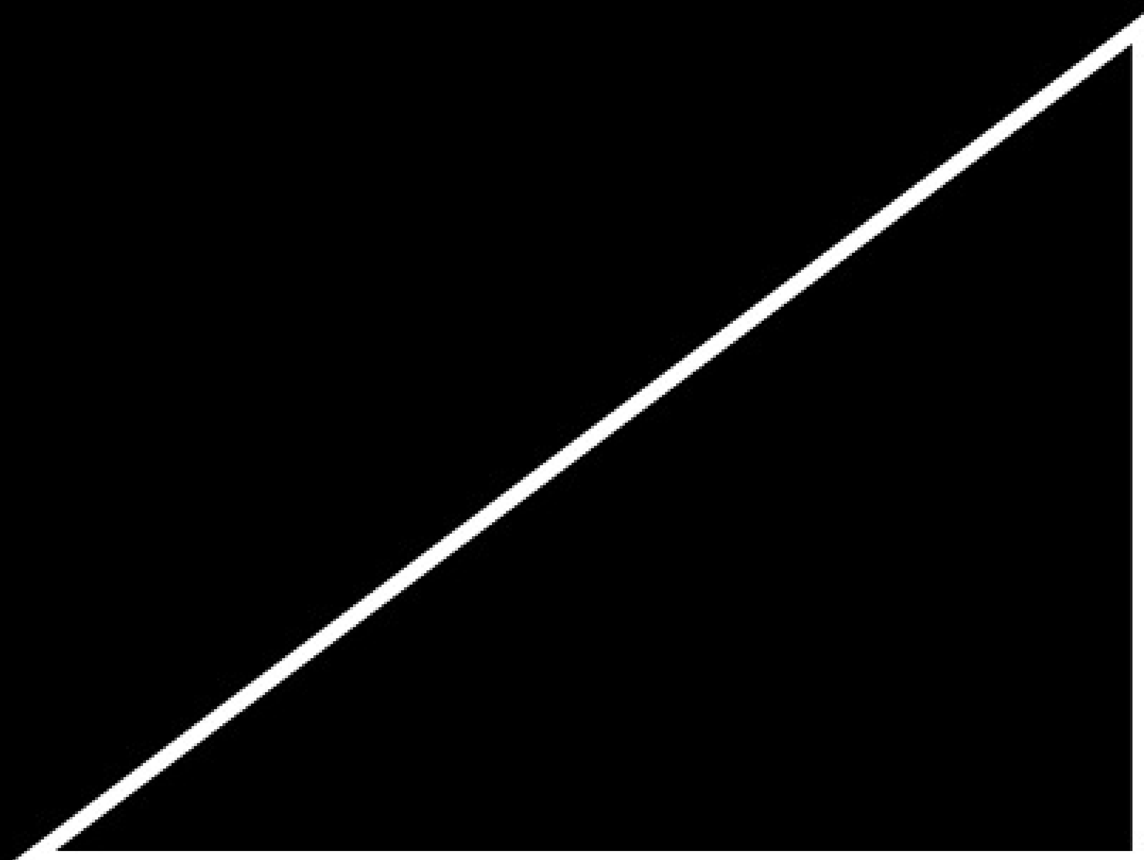
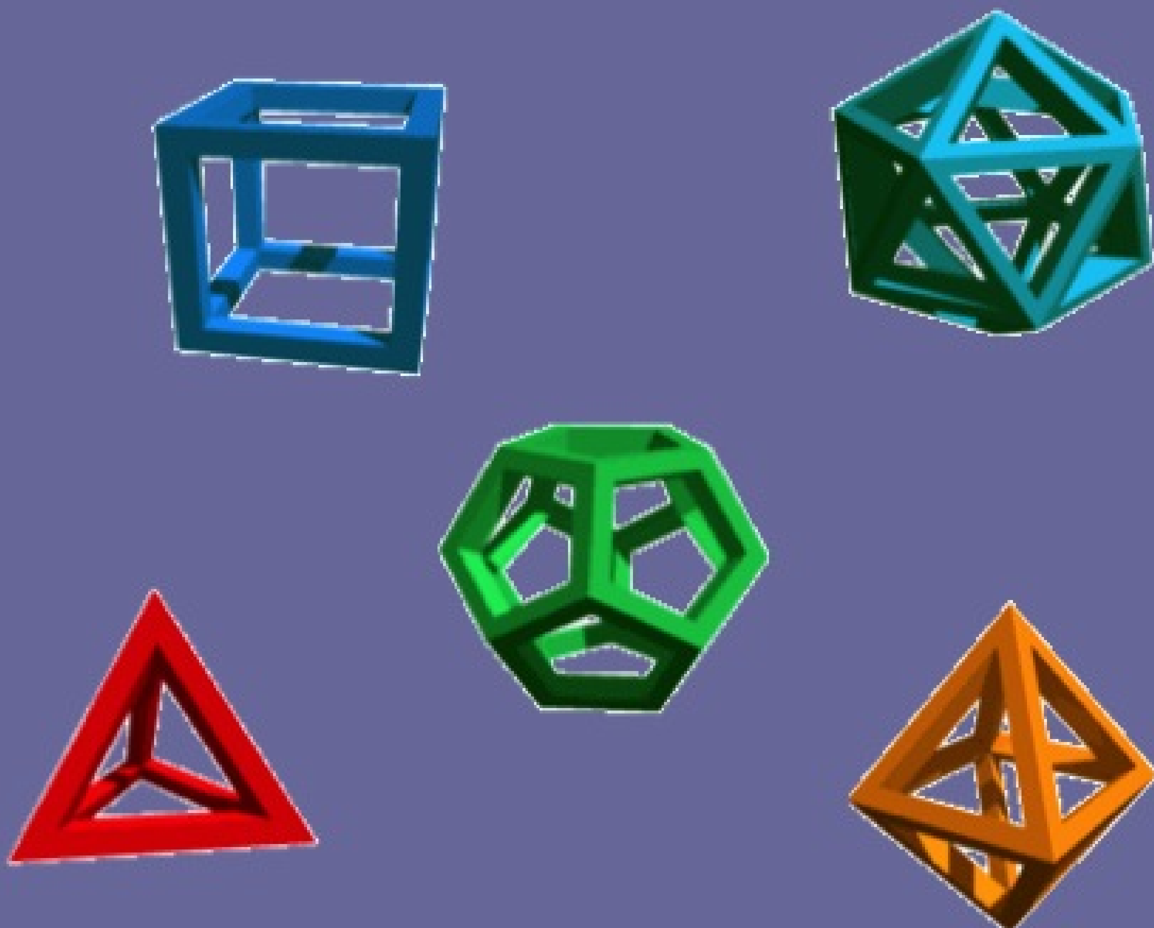
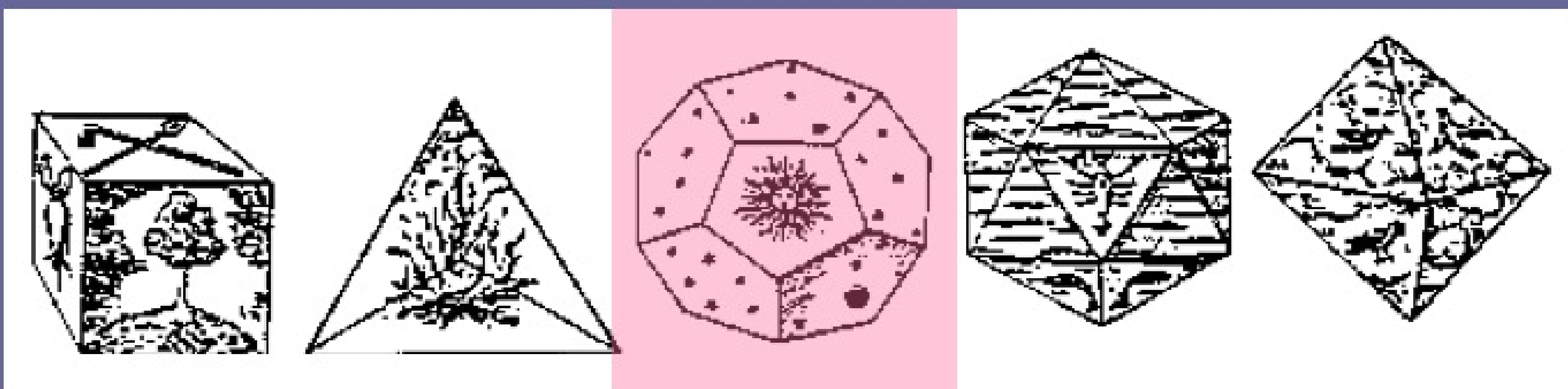
$$18541^2 = 13500^2 + 12709^2$$



Platonic Solids

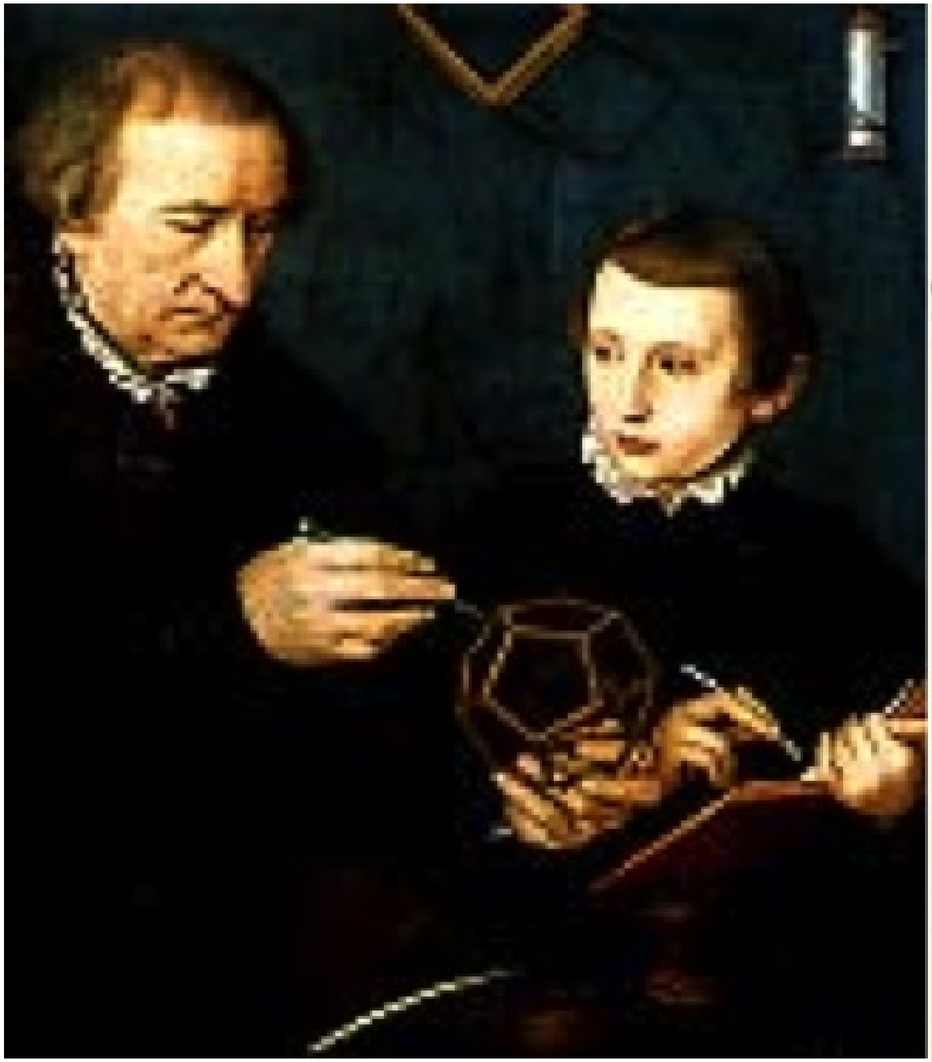


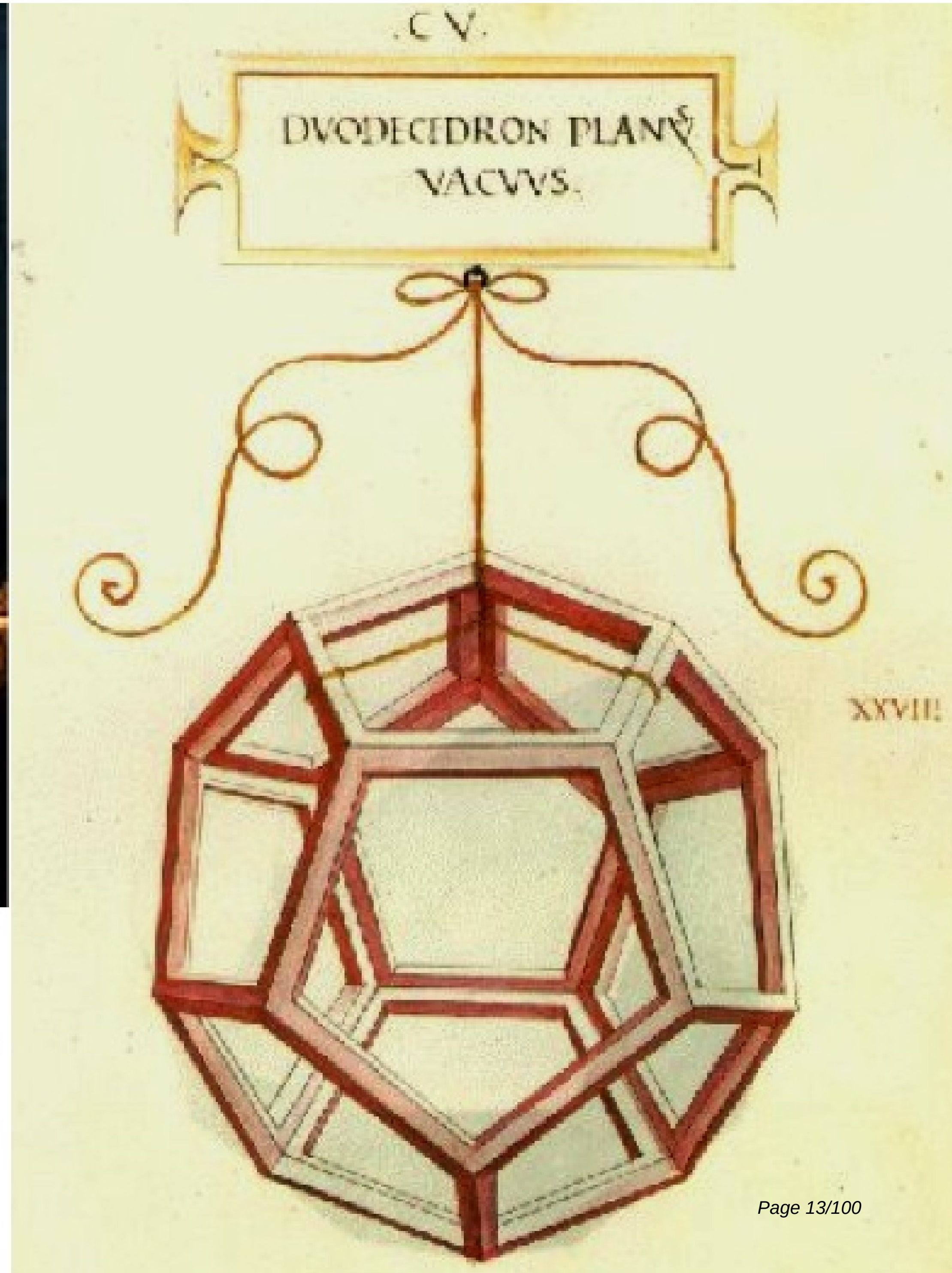
The Four Elements



Earth Fire Universe Water Air

Quintessence

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VACVVS.
XXVII.

Luca Pacioli

Old Hall of the House of Representatives, The Hague

Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background

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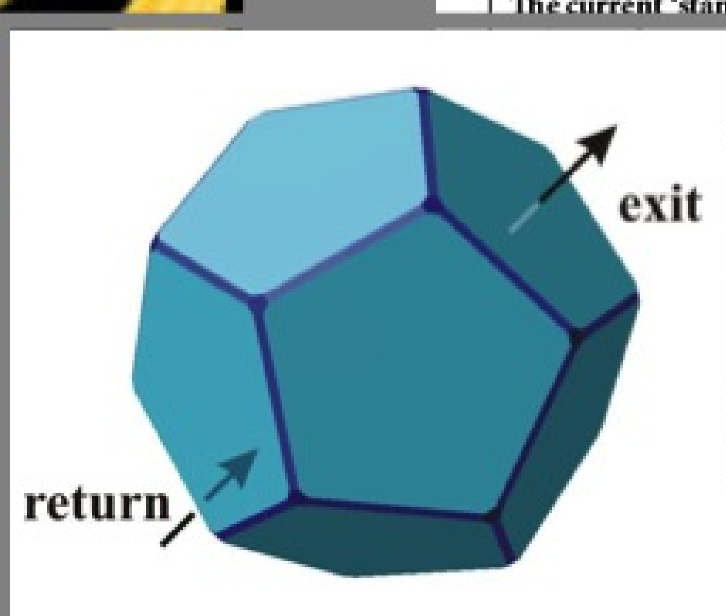
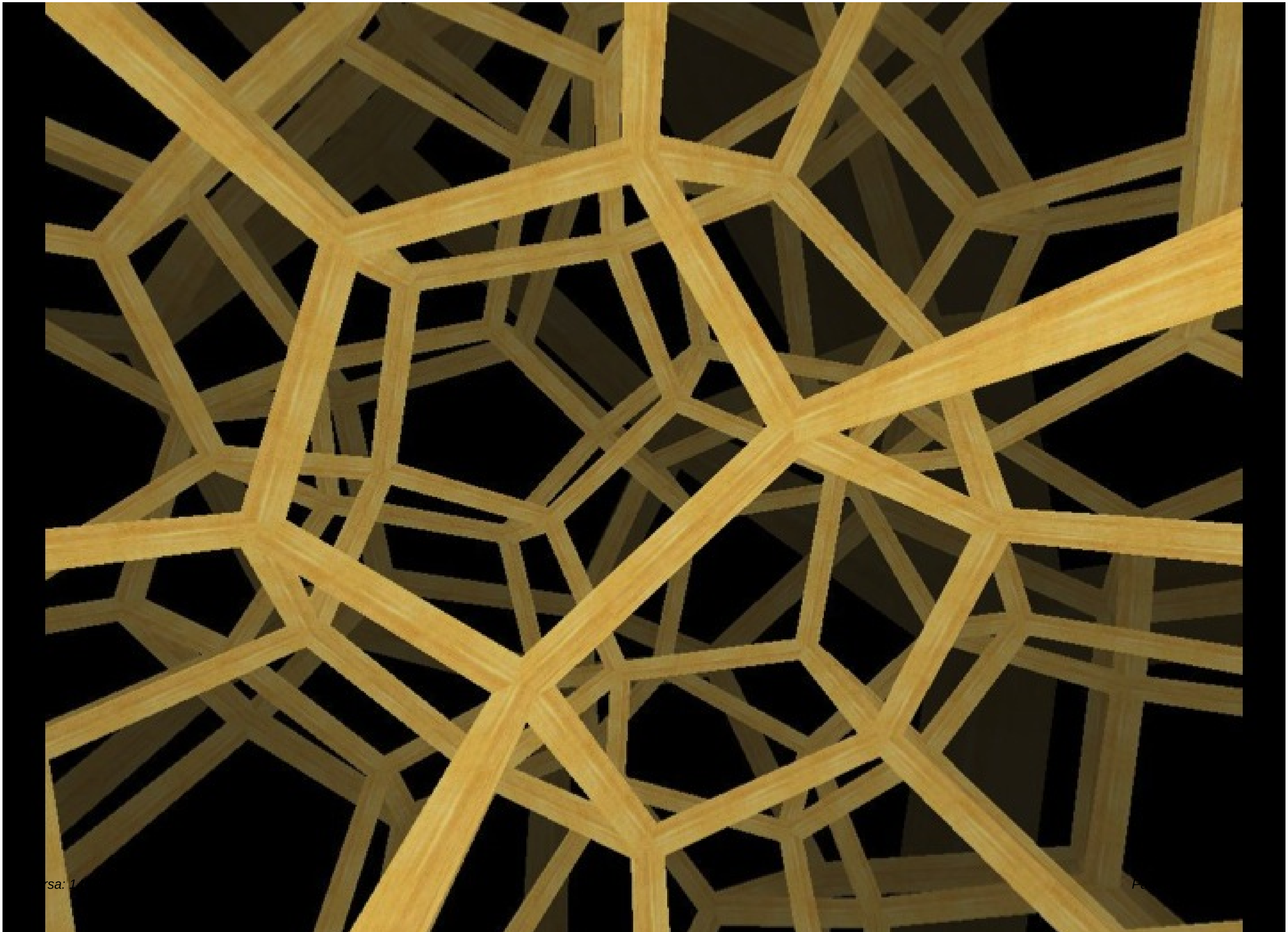
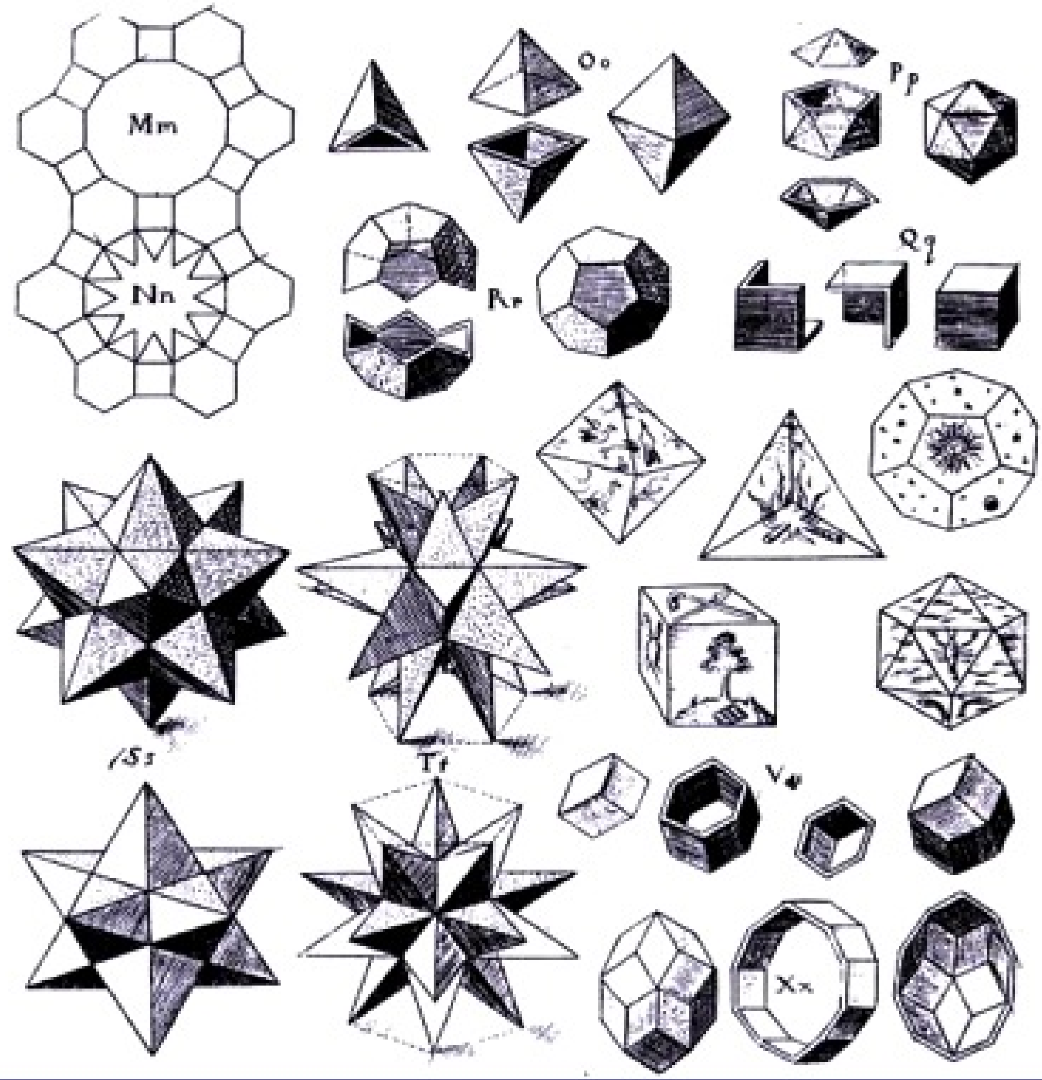
The current 'standard model' of cosmology posits an infinite flat
xpanding under the pressure of dark energy.
n the Wilkinson Microwave Anisotropy Probe
this model to spectacular precision on all but
. Temperature correlations across the micro-
ectations on angular scales narrower than 60°
redictions, vanish on scales wider than 60°.
ons have been proposed[3,4]. One natural
s the underlying geometry of space—namely,
opology[6]. In an infinite flat space, waves from
ld fill the universe on all length scales. The
mperature correlations on scales beyond 60°
oadest waves are missing, perhaps because
ig enough to support them. Here we present a

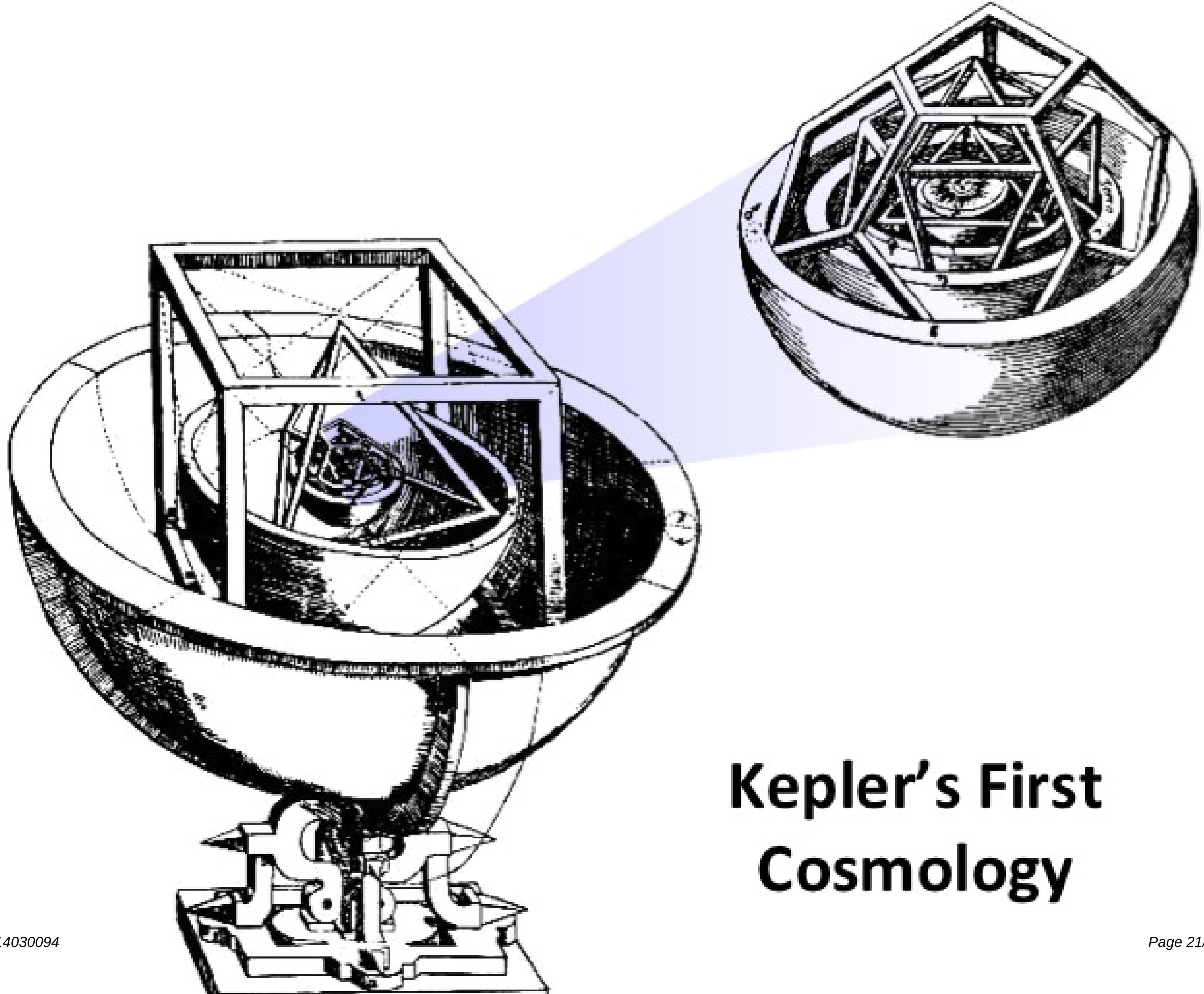






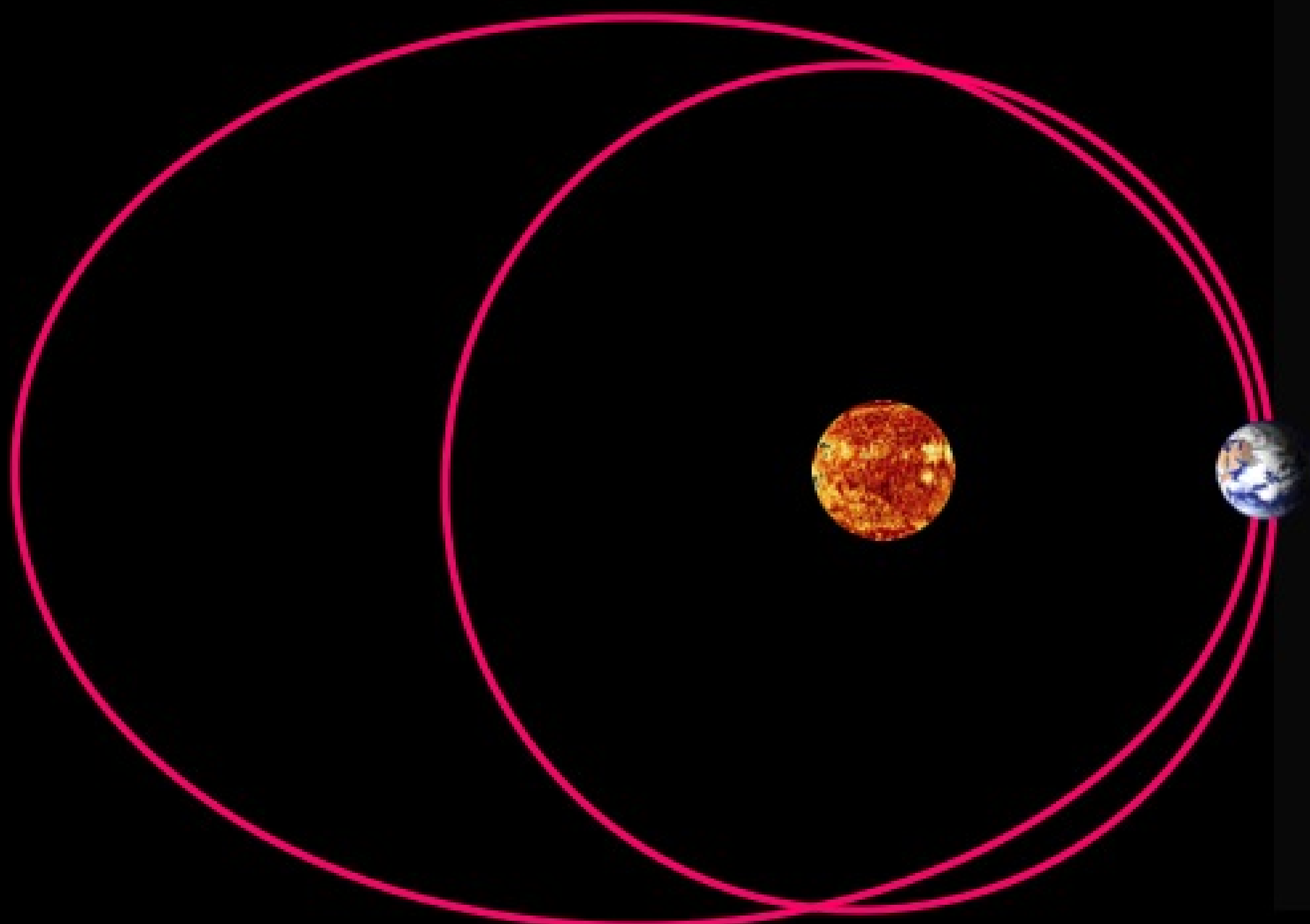
Johannes Kepler (1571-1630)

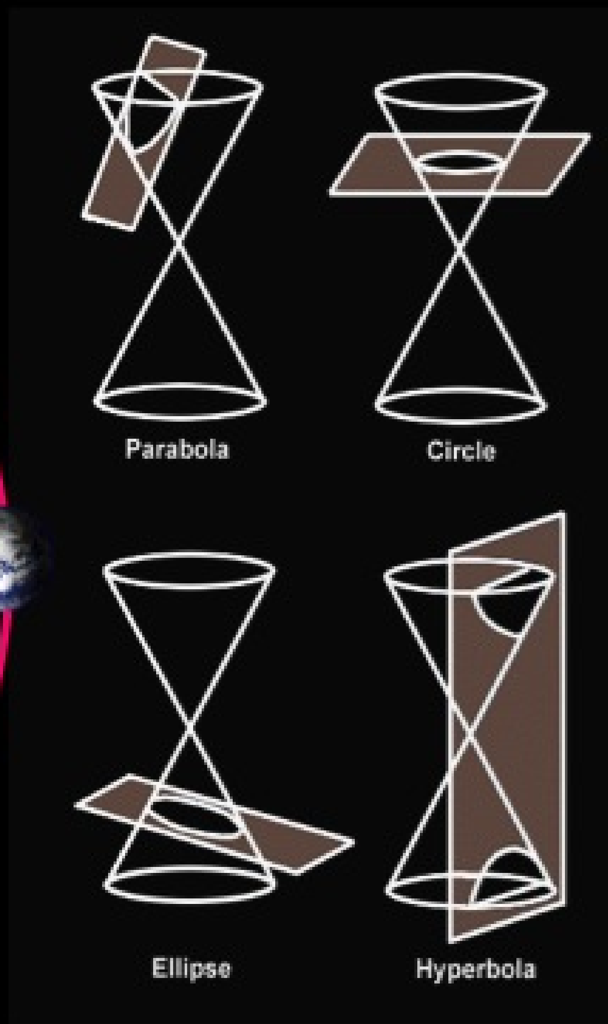
Mysterium Cosmographicum (1596)



Kepler’s First Cosmology

Kepler’s First Law





$$A = B$$

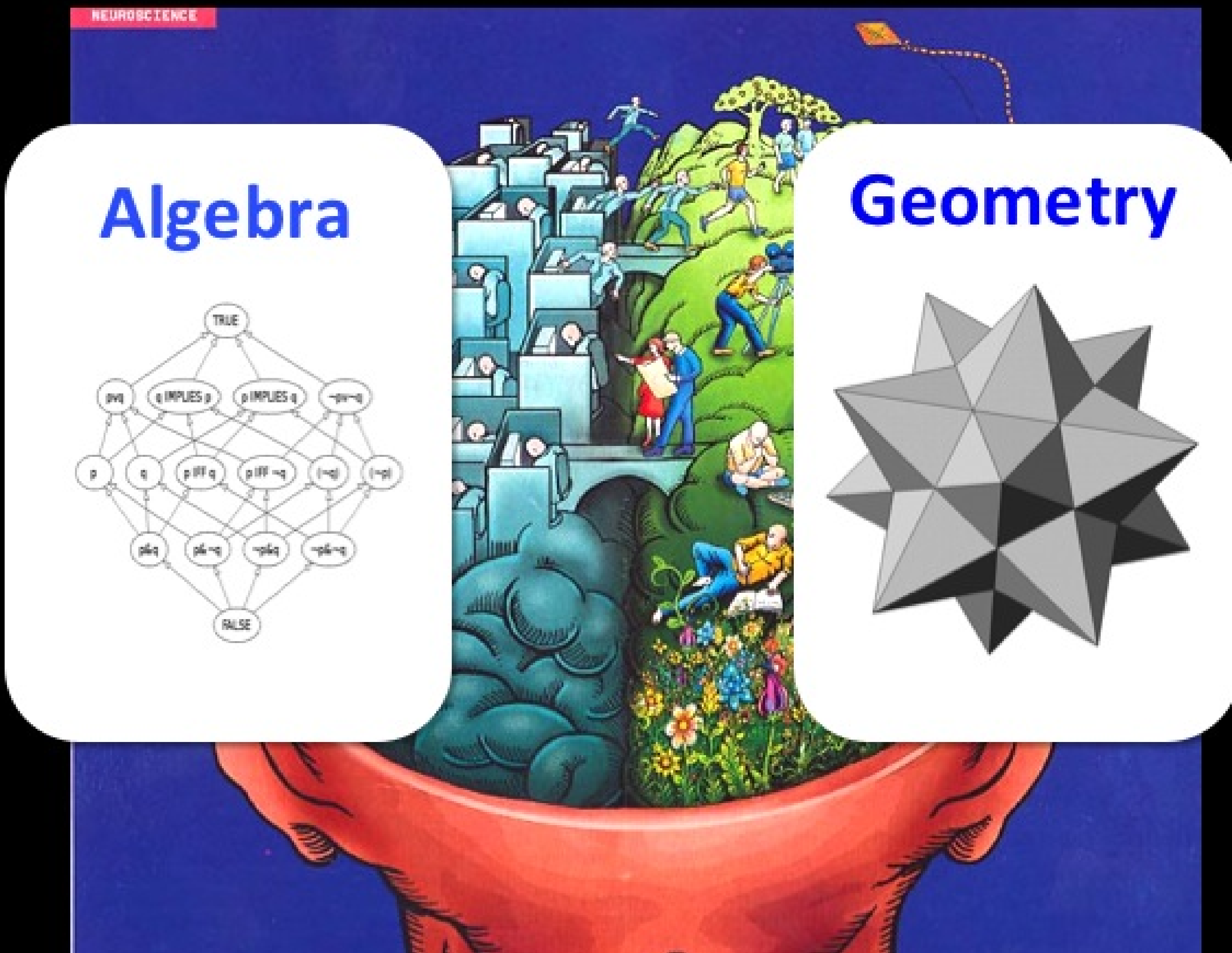
Clinton's Principle

$$A = B$$

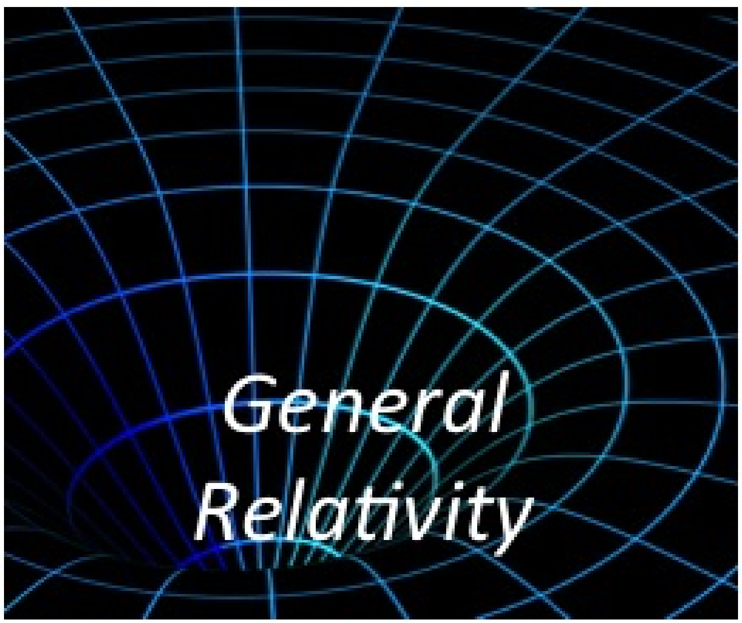
It depends on what the meaning of `is' is.

Image vs Text

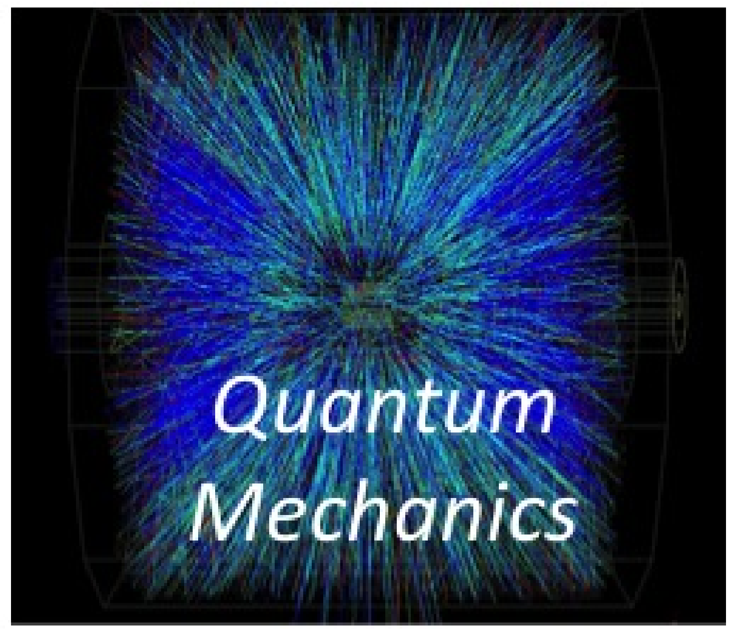
Mathematical Brain



Geometry



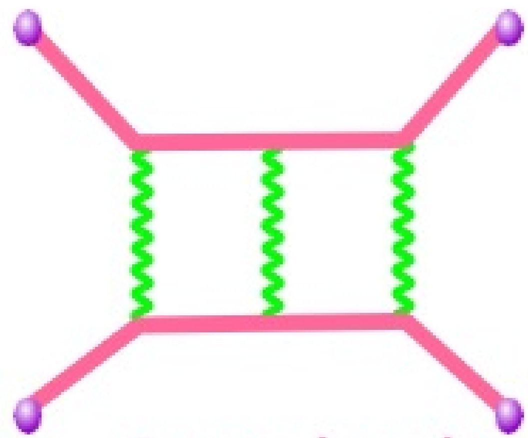
Algebra



Reduction

Macrophysics

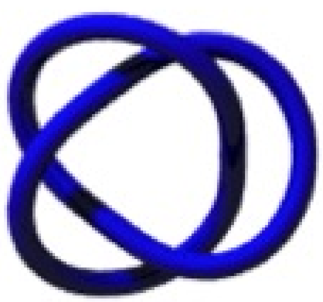
Microphysics



Standard Model

Quantization

Geometry



K

geometric object

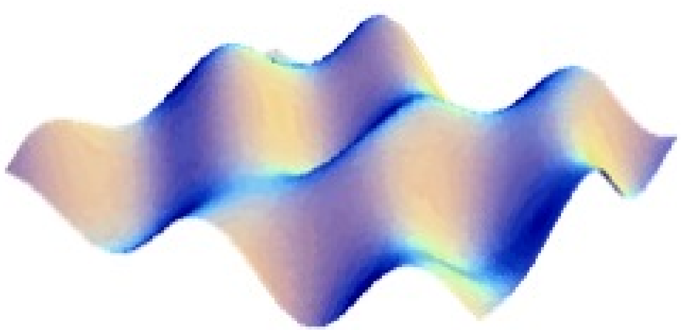
Algebra

$$Z(K) \in C$$

quantum invariant

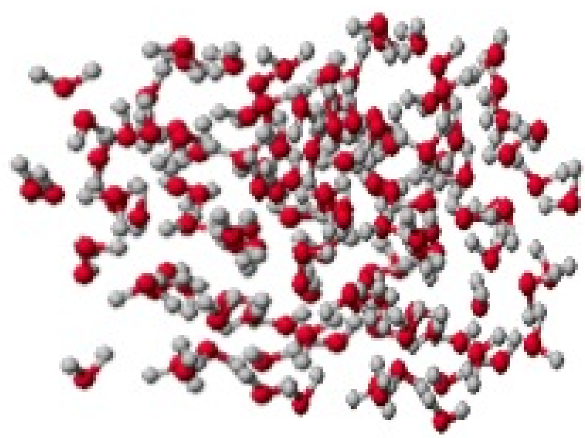
Emergence ←

Macrophysics



hydrodynamics
thermodynamics

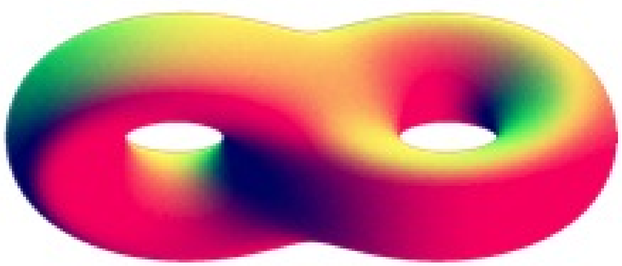
Microphysics



molecules
statistical mechanics

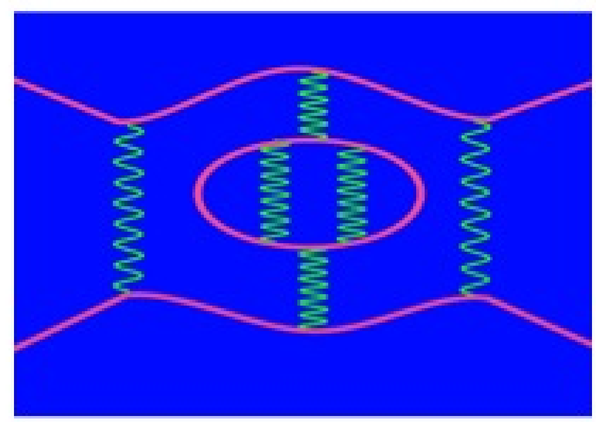
Emergence

Geometry

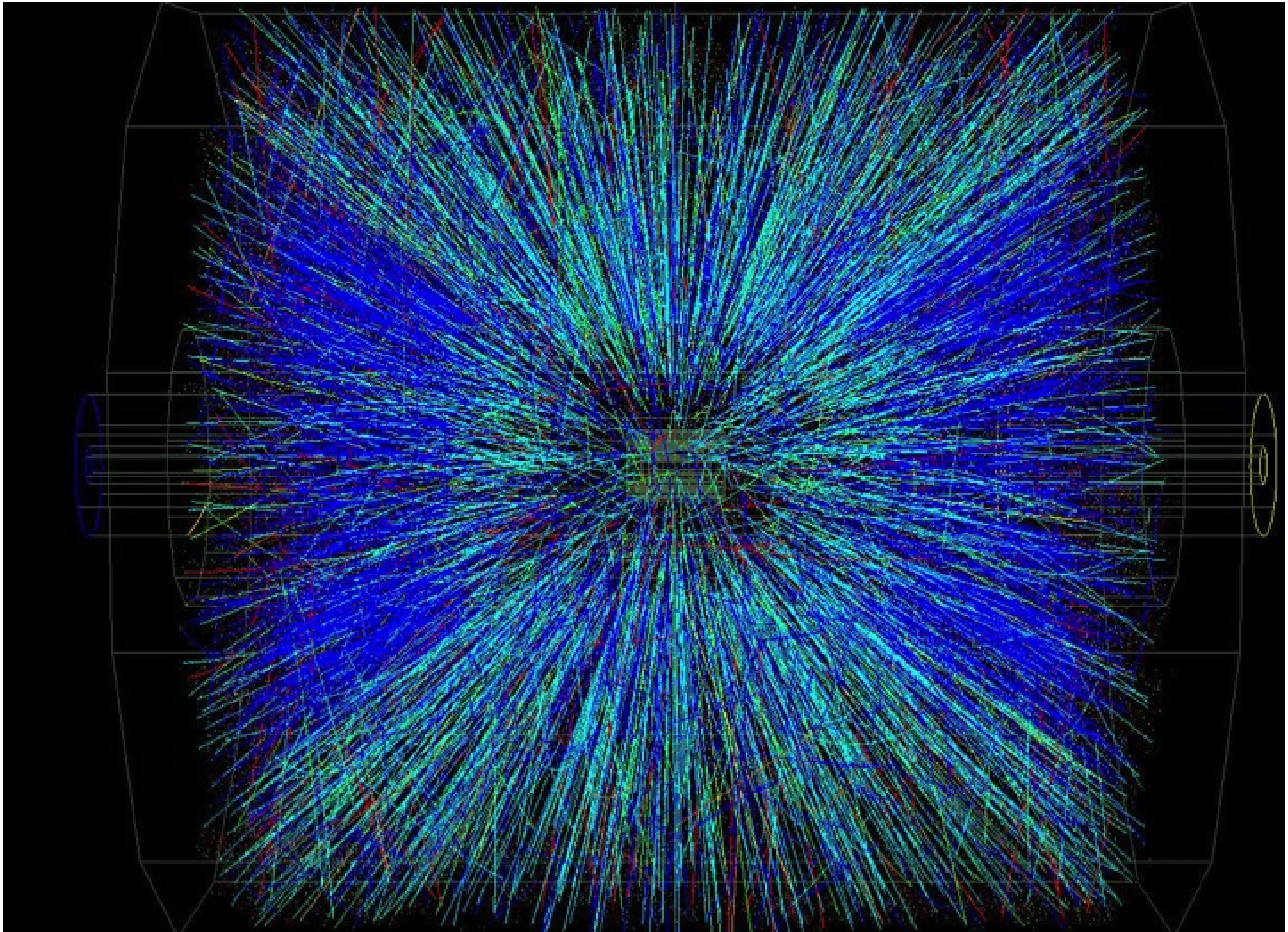


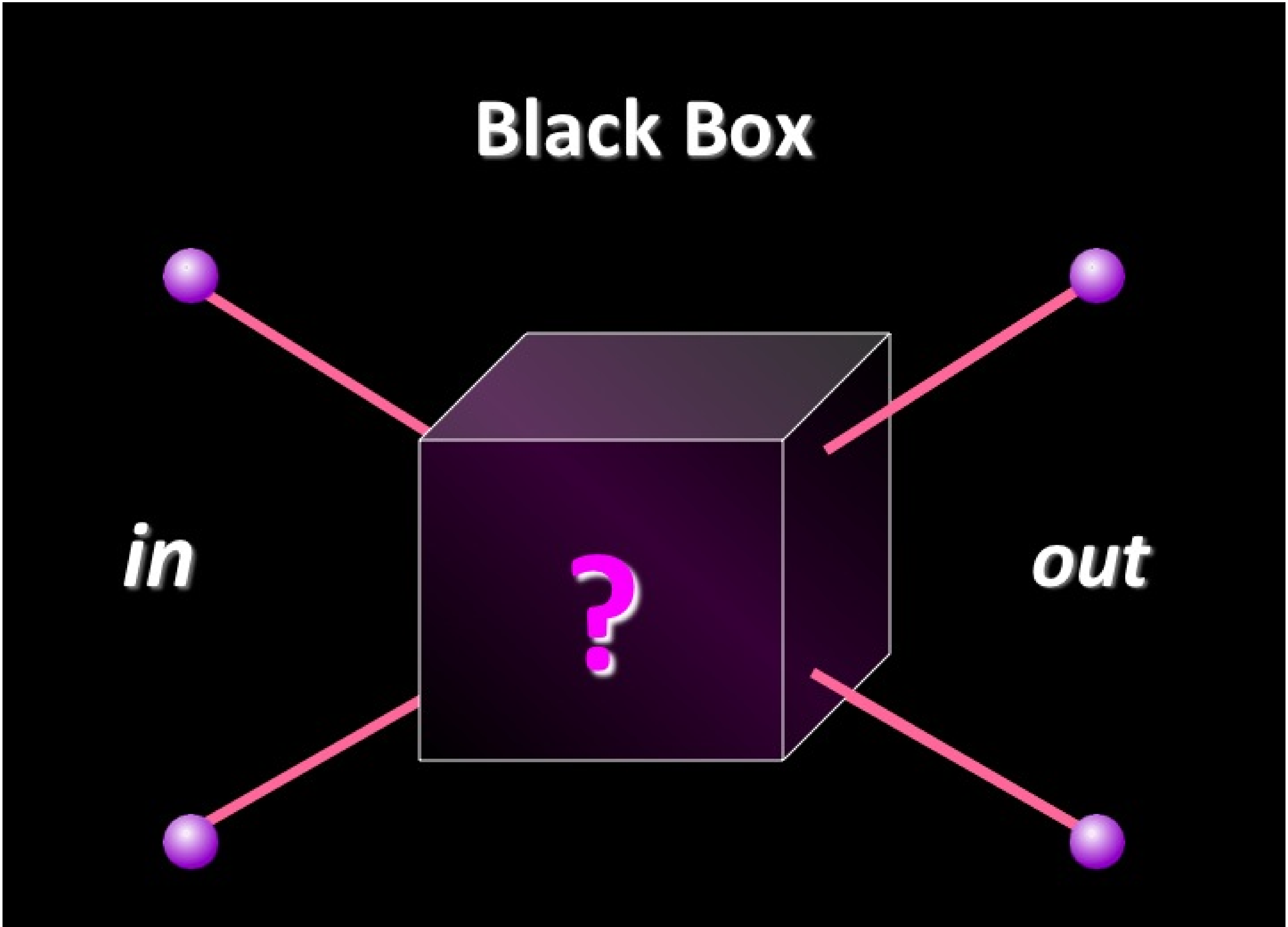
effective geometry

Algebra



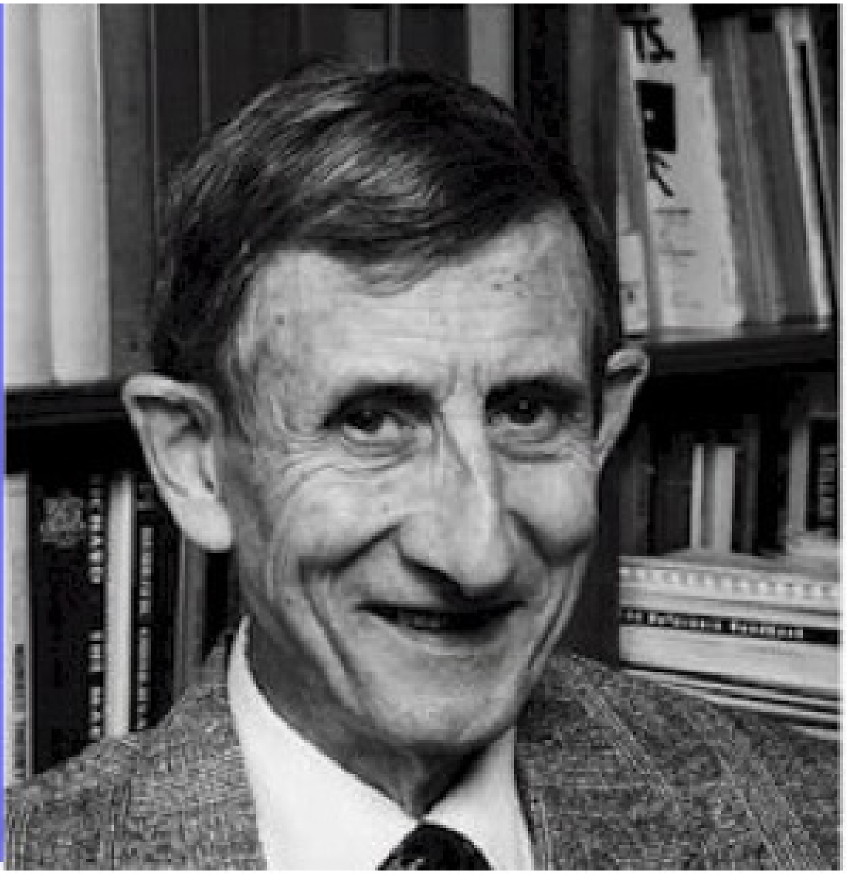
quantum system




Black Box
in
?
out

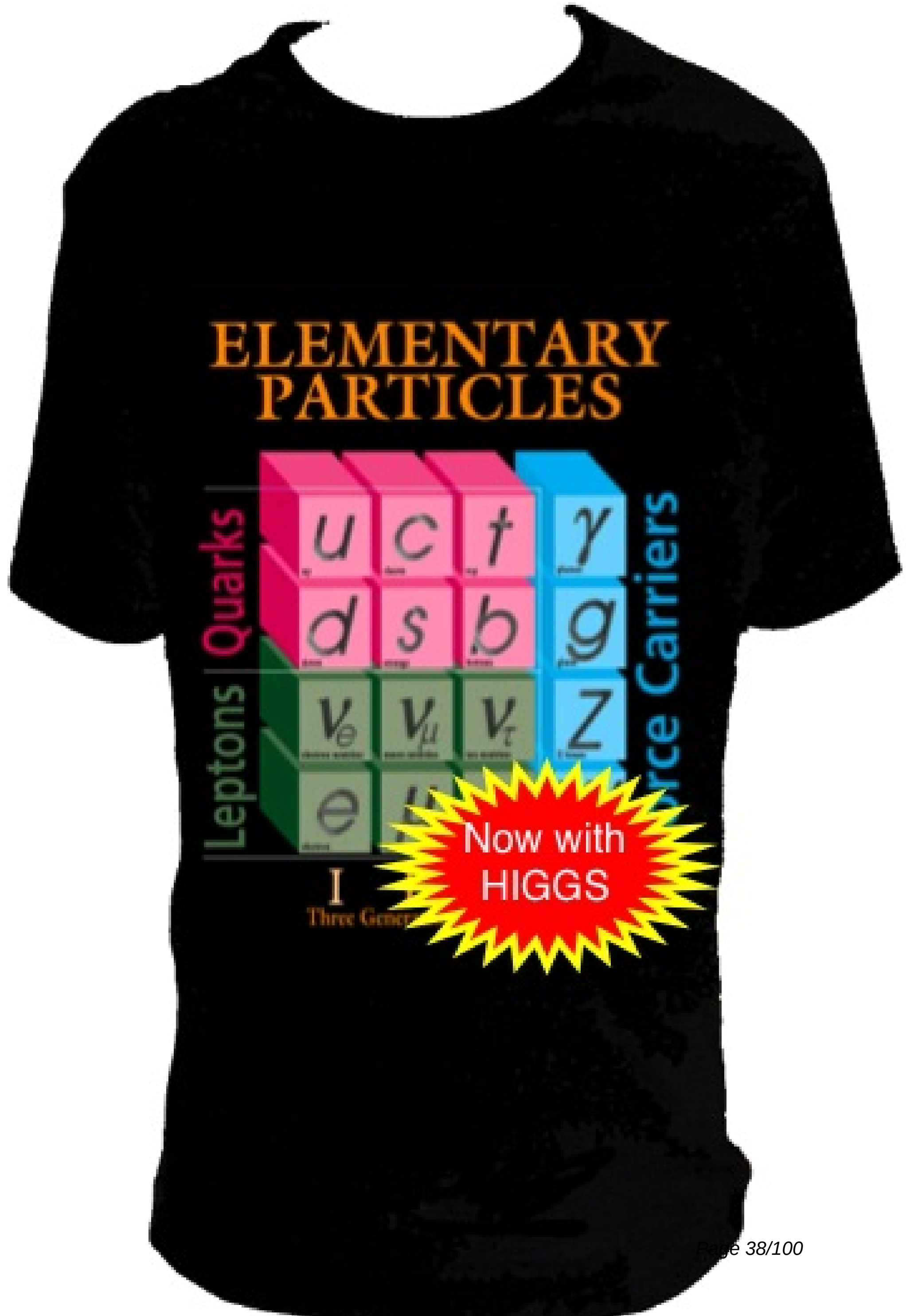
Freeman Dyson

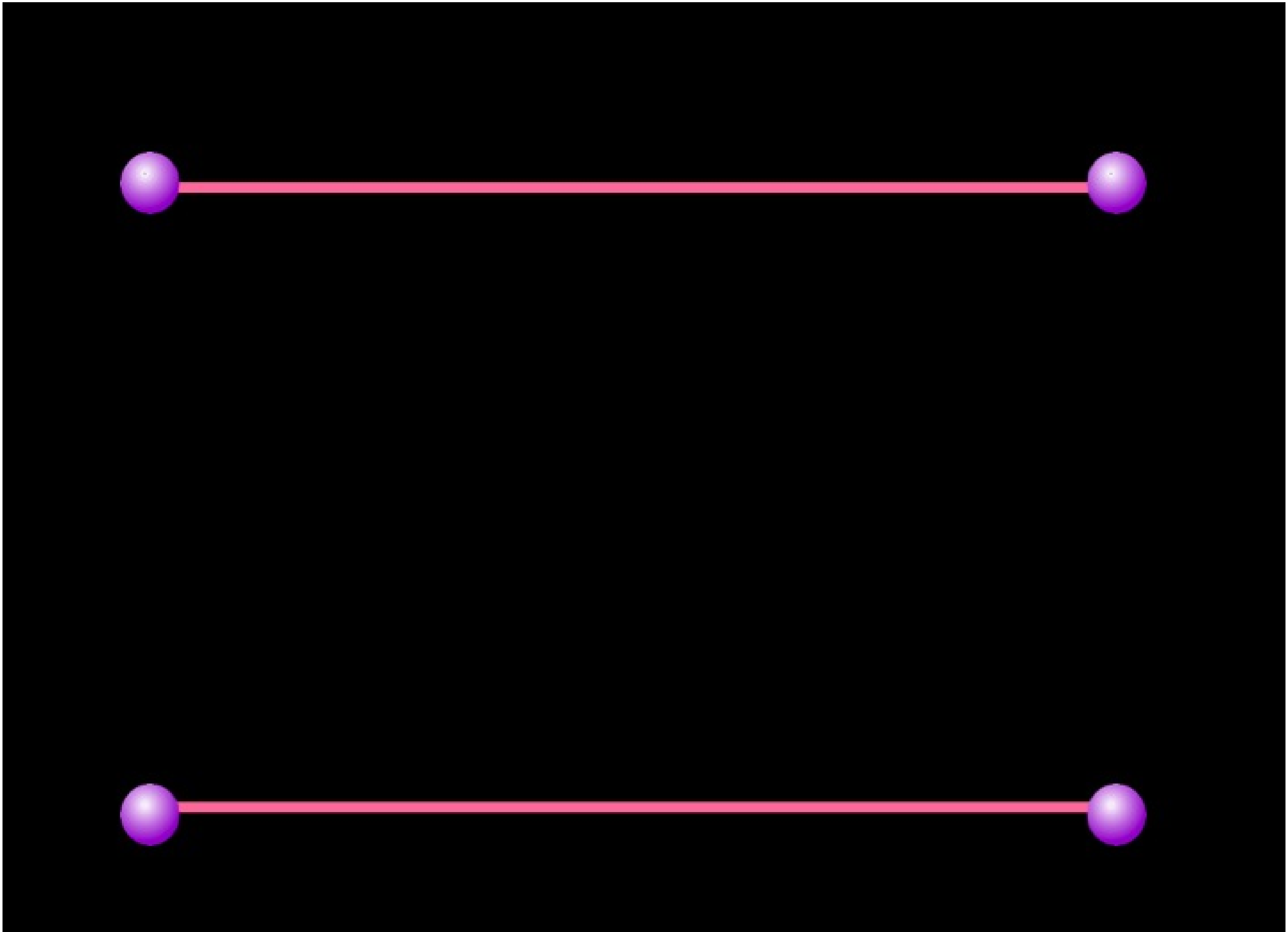
(*Gibbs Lecture*, 1972)

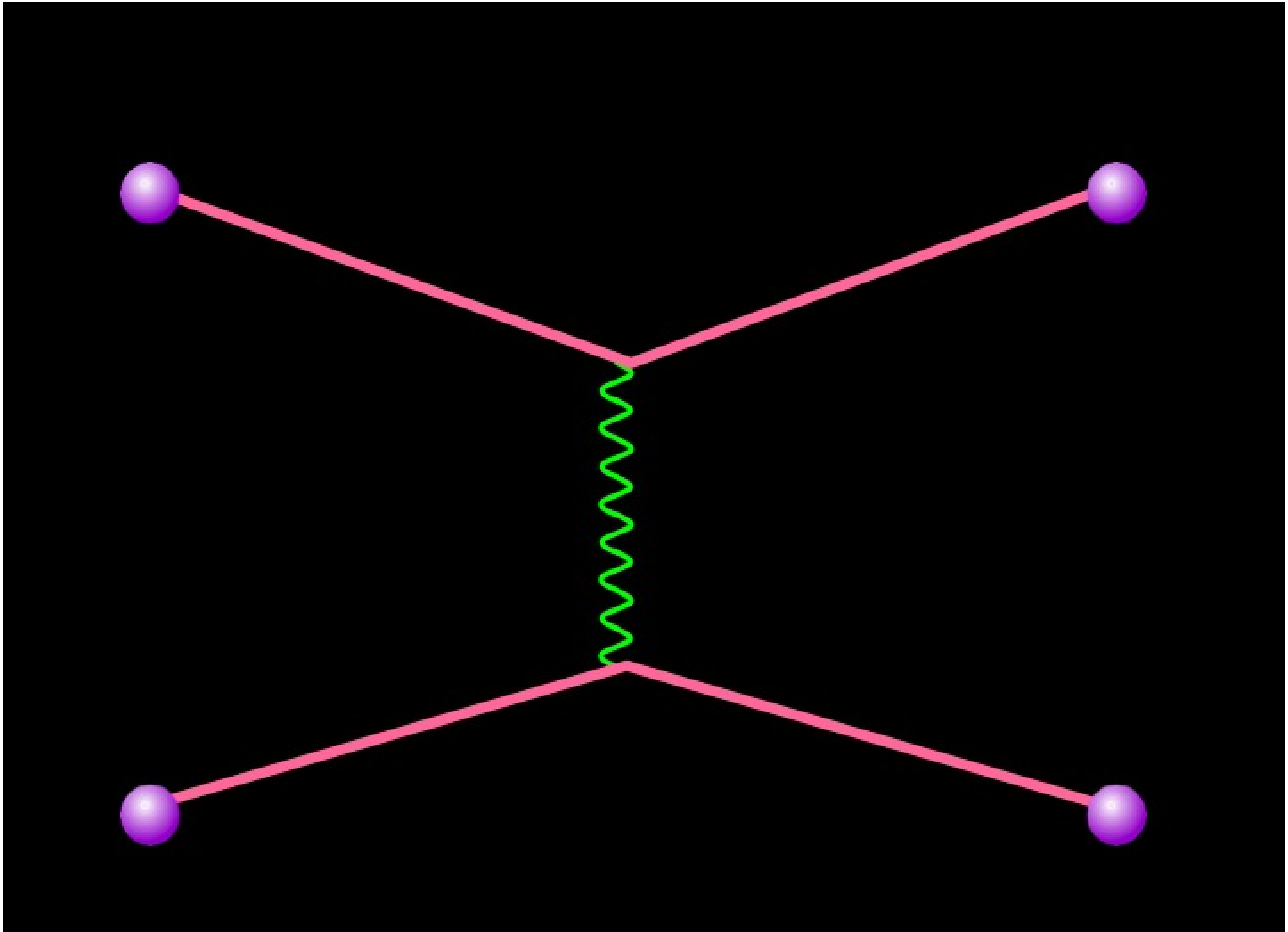


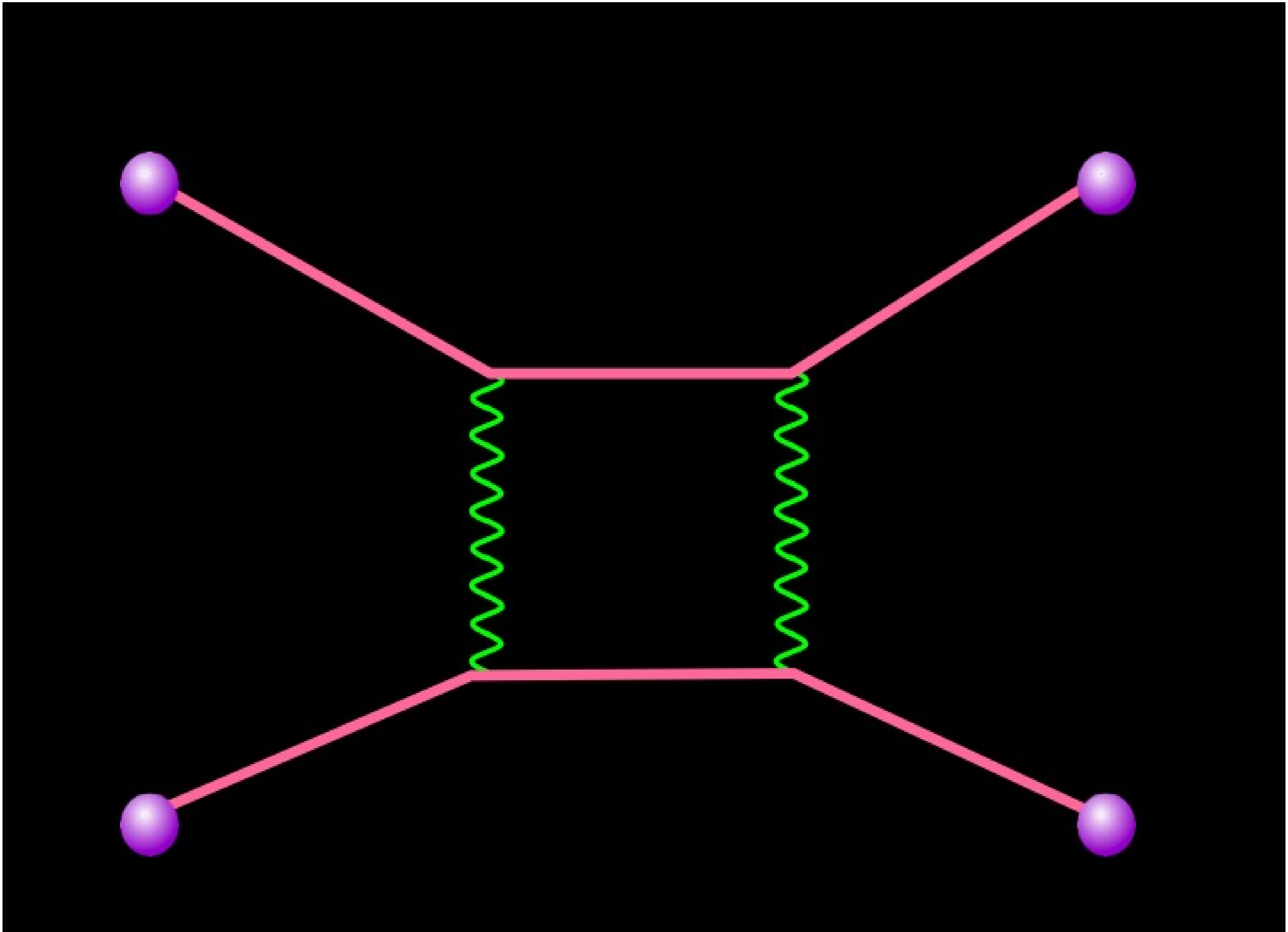
"I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce."

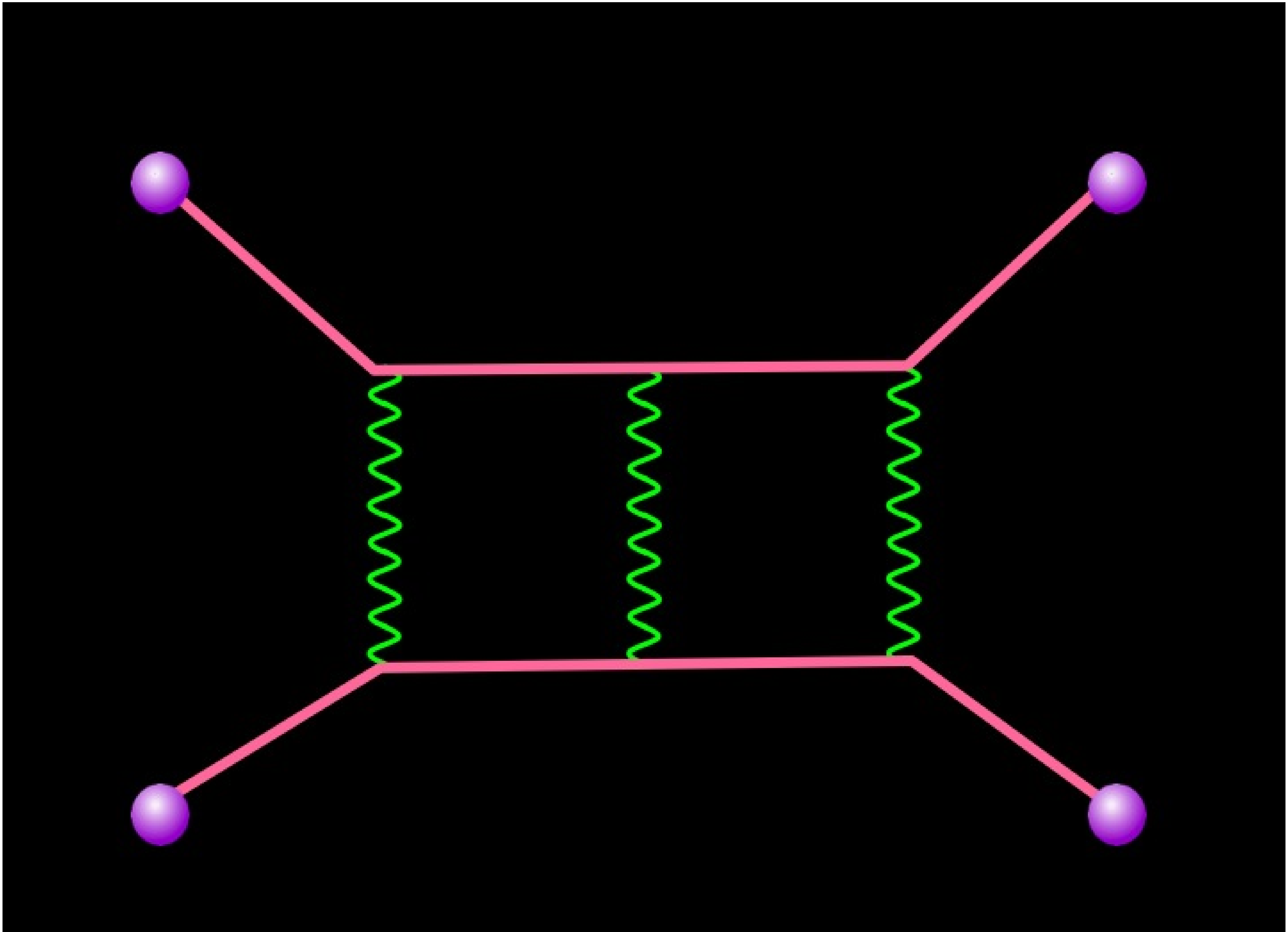
$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$
$\frac{1}{2}ig_s^2(\bar q^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$
$M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H -$
$\frac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} +$
$\frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$
$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$
$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$
$W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2}g^2W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$
$\frac{1}{2}g^2W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$
$g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2s_wc_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$
$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$
$\frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$
$gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$
$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$
$\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$
$igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$
$igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$
$\frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$
$W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- +$
$W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- -$
$g^2s_w^2A_\mu A_\mu\phi^+\phi^- - \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u_j^\lambda(\gamma\partial + m_u^\lambda)u_j^\lambda -$
$\bar d_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda + igs_wA_\mu[-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar u_j^\lambda\gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar d_j^\lambda\gamma^\mu d_j^\lambda)] +$
$\frac{ig}{4c_w}Z^0_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar u_j^\lambda\gamma^\mu(\frac{4}{3}s_w^2 -$
$1 - \gamma^5)u_j^\lambda) + (\bar d_j^\lambda\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)] + \frac{ig}{2\sqrt2}W^+_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$
$(\bar u_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)] + \frac{ig}{2\sqrt2}W^-_\mu[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d_j^\kappa C^\dagger_{\lambda\kappa}\gamma^\mu(1 +$
$\gamma^5)u_j^\lambda)] + \frac{ig}{2\sqrt2}\frac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] -$
$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) +$
$m_u^\lambda(\bar u_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa)] + \frac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa) - m_u^\kappa(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1 -$
$\gamma^5)u_j^\kappa)] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar u_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar d_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u_j^\lambda\gamma^5 u_j^\lambda) -$
$\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d_j^\lambda\gamma^5 d_j^\lambda) + \bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2 - M^2)X^- + \bar X^0(\partial^2 -$
$\frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0X^- - \partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar YX^- -$
$\partial_\mu\bar X^+Y) + igc_wW^-_\mu(\partial_\mu\bar X^-X^0 - \partial_\mu\bar X^0X^+) + igs_wW^-_\mu(\partial_\mu\bar X^-Y -$
$\partial_\mu\bar YX^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ -$
$\partial_\mu\bar X^-X^-) - \frac{1}{2}gM[\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H] +$
$\frac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] +$
$igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \frac{1}{2}igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]$

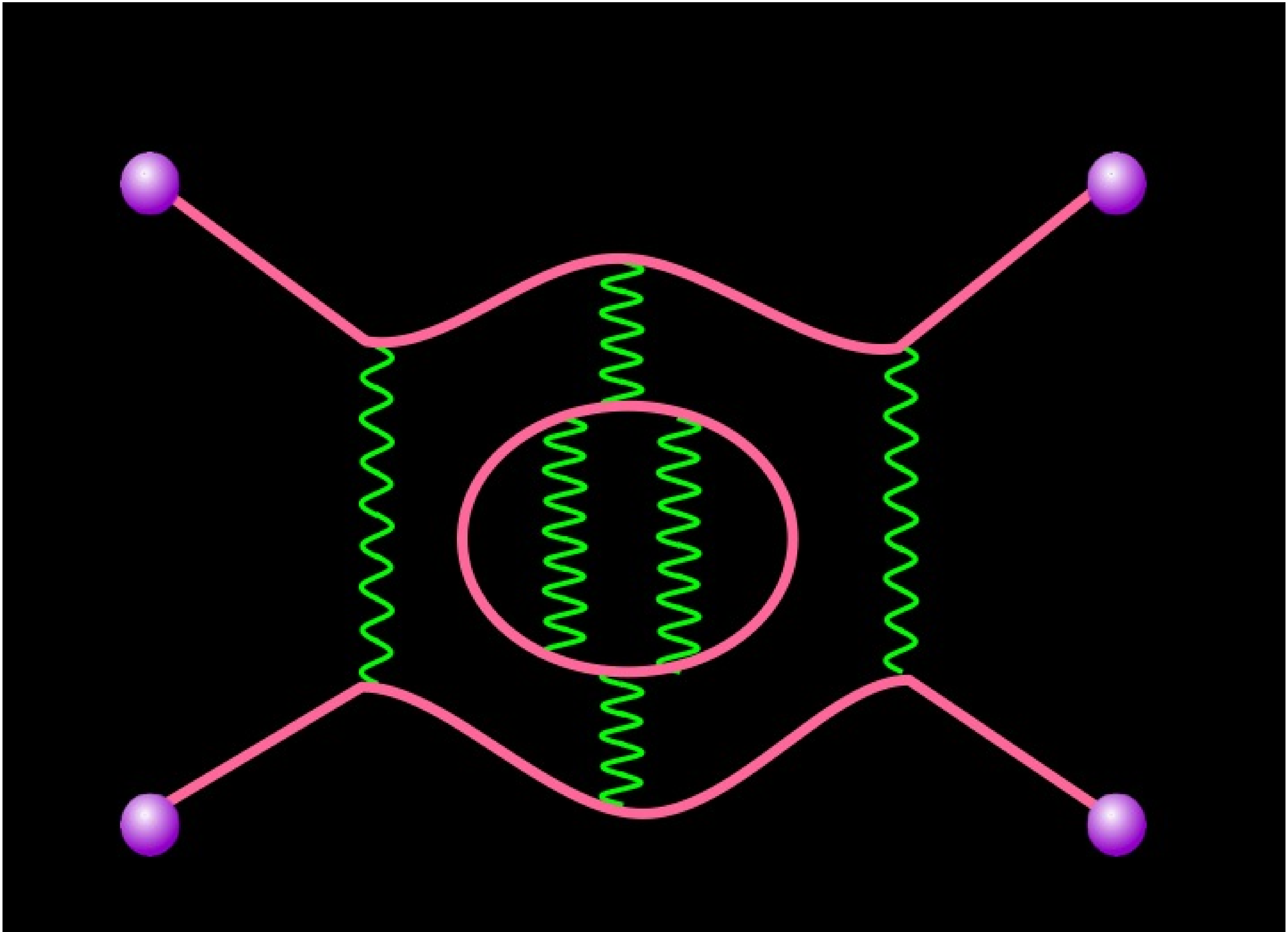


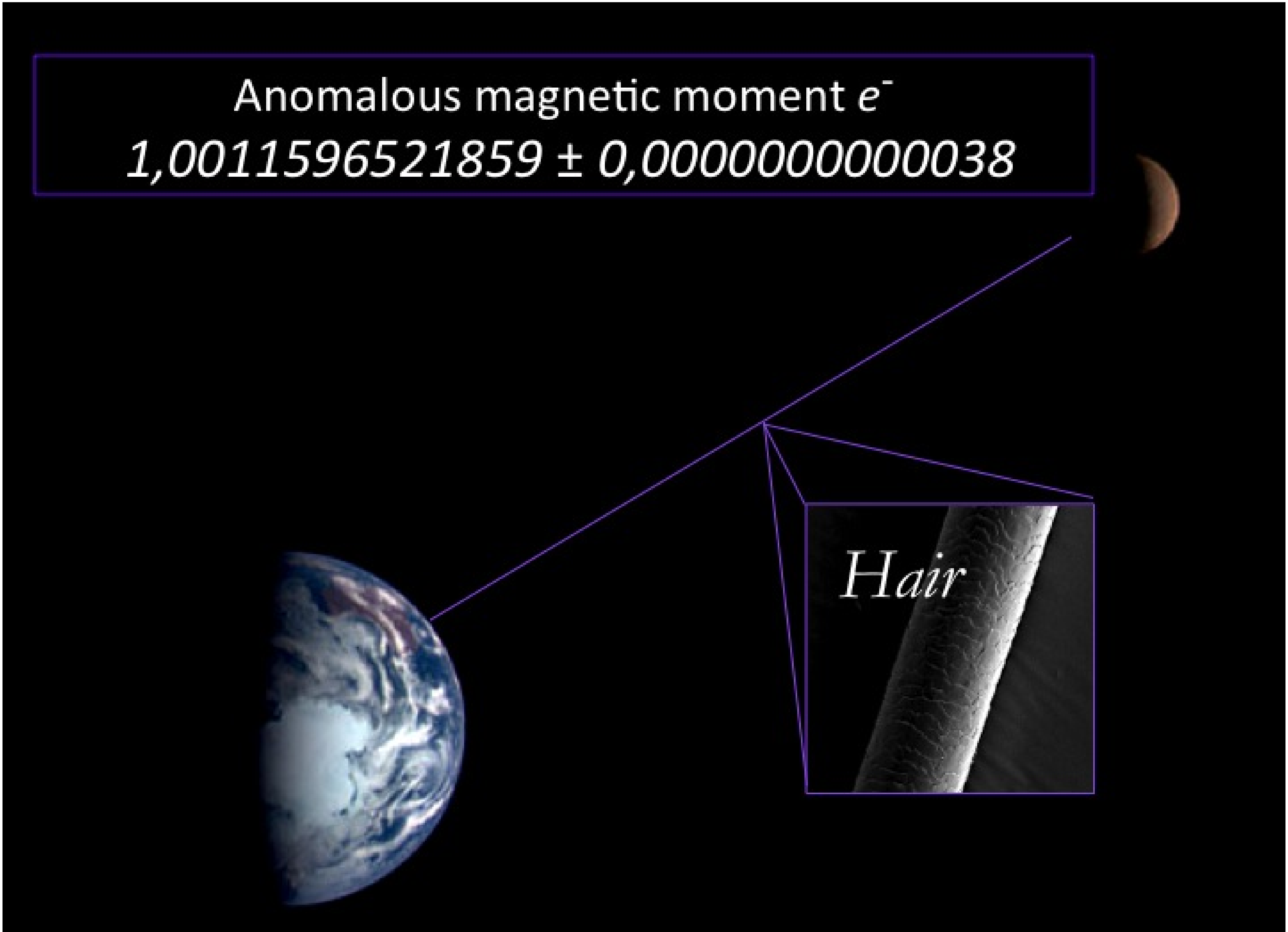






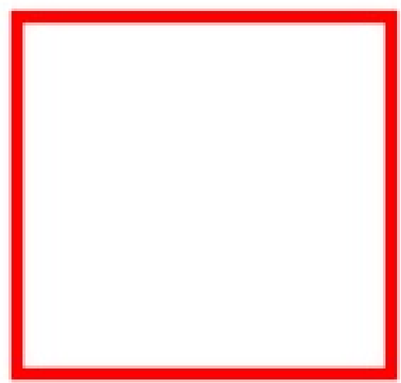




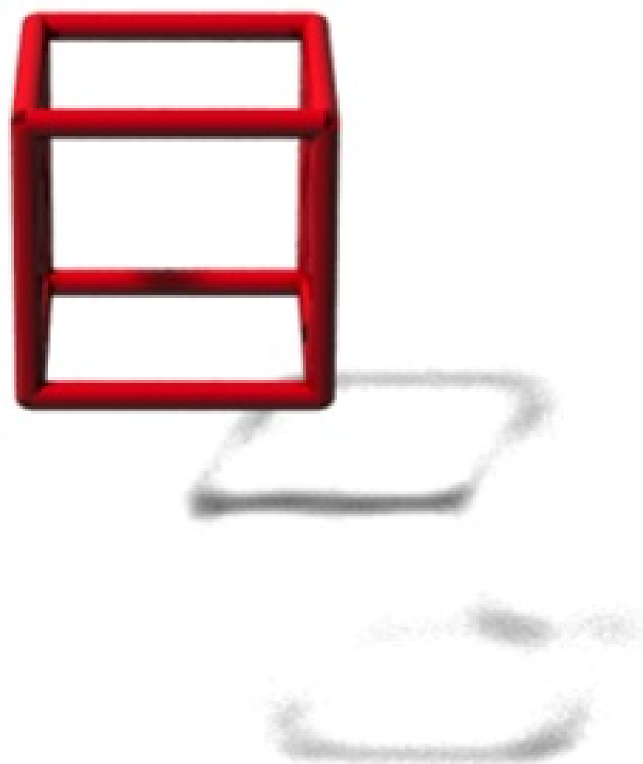

Anomalous magnetic moment e^-
1,0011596521859 ± 0,0000000000038
Hair

“Time is the fourth dimension”

Who's Afraid
Of Extra Dimensions?

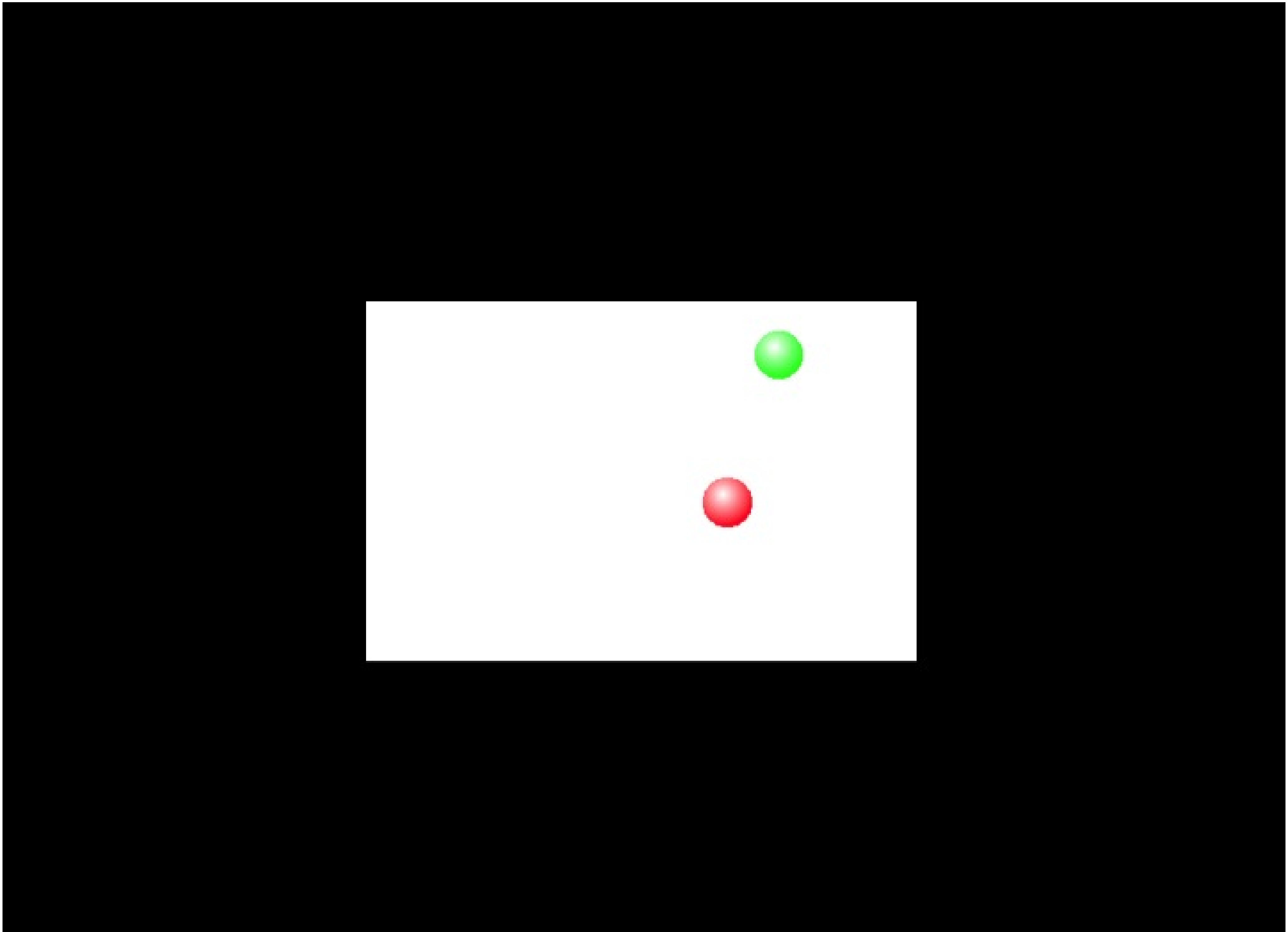


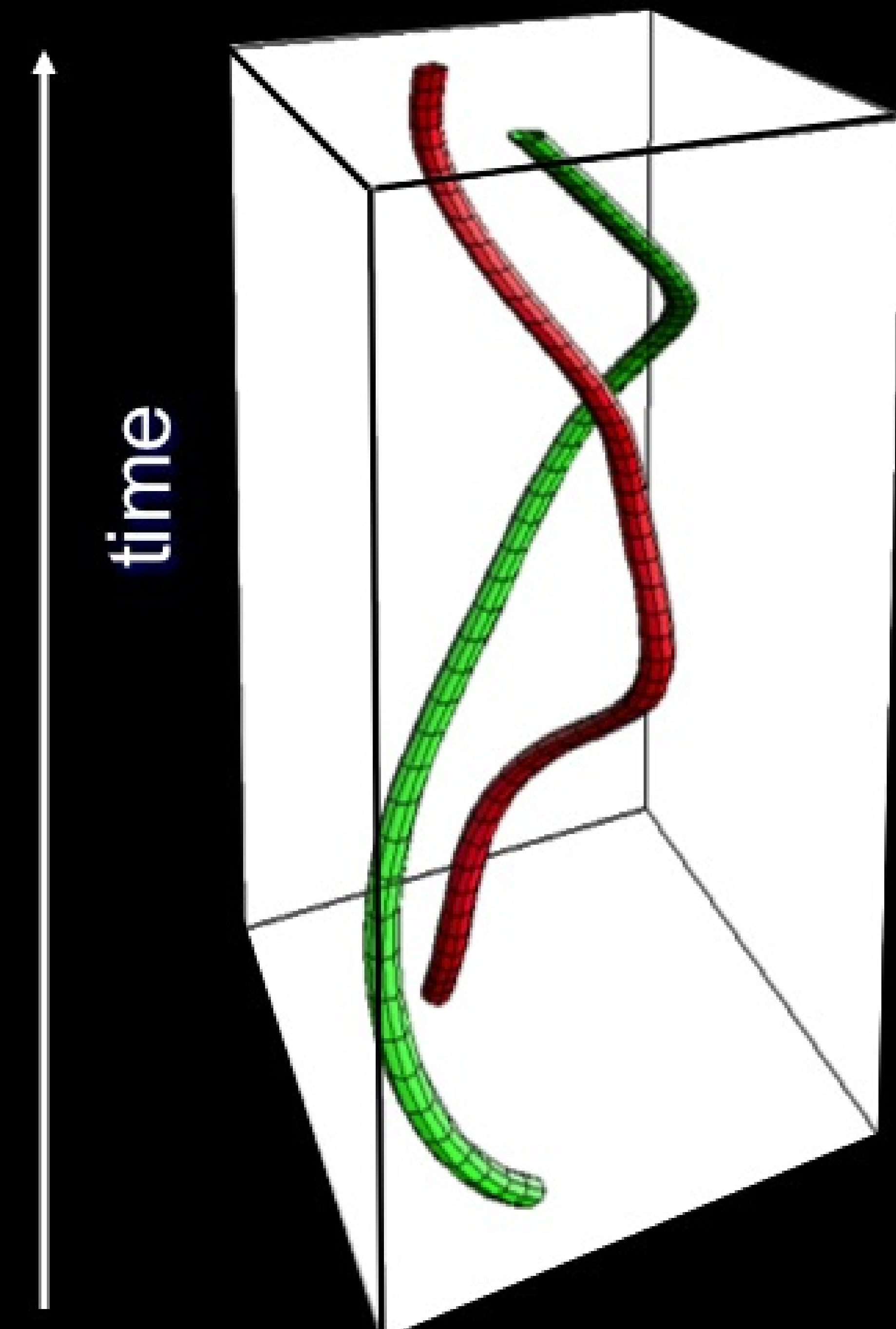
Two Dimensions



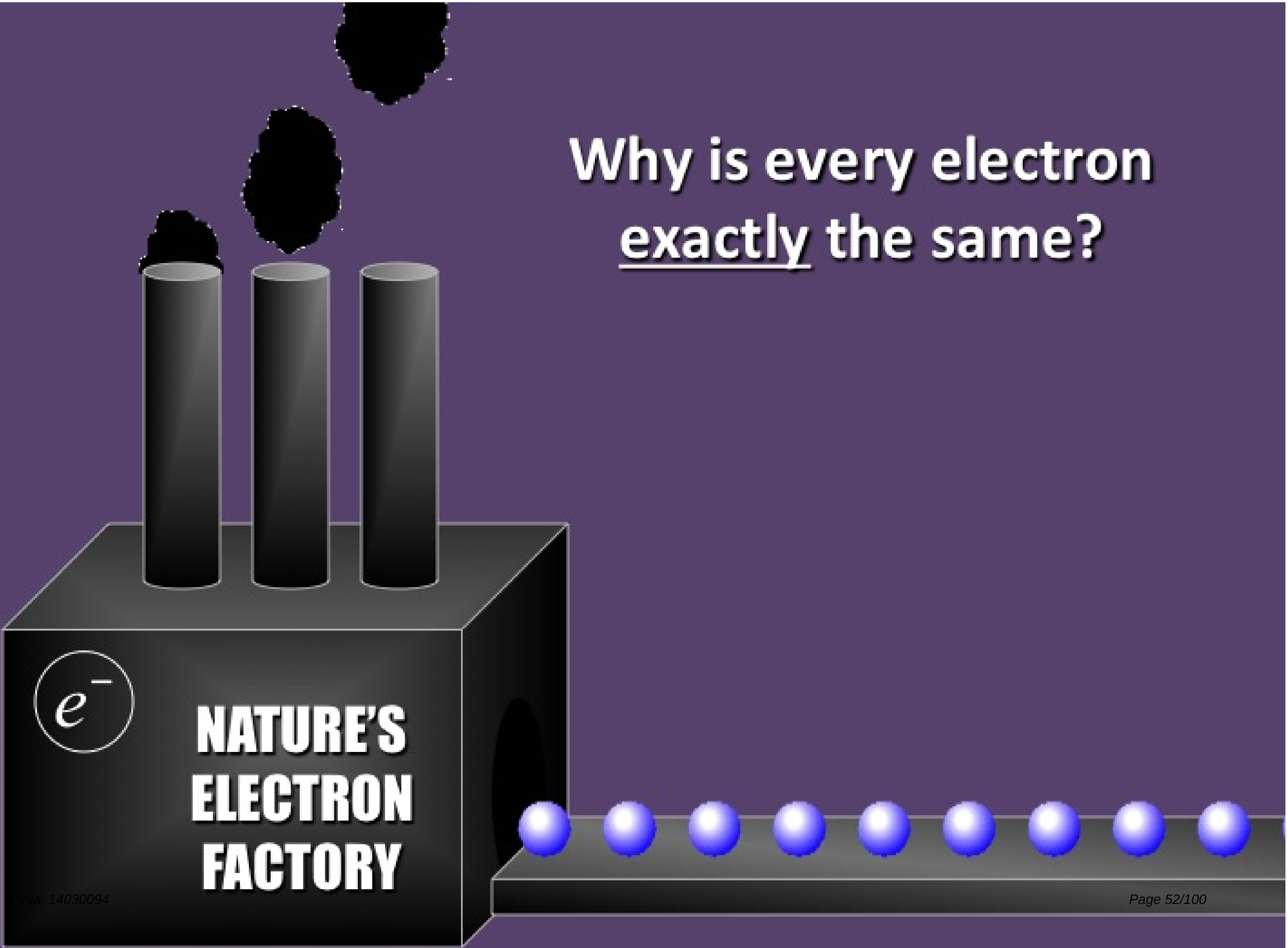
Three Dimensions

Four Dimensions




time

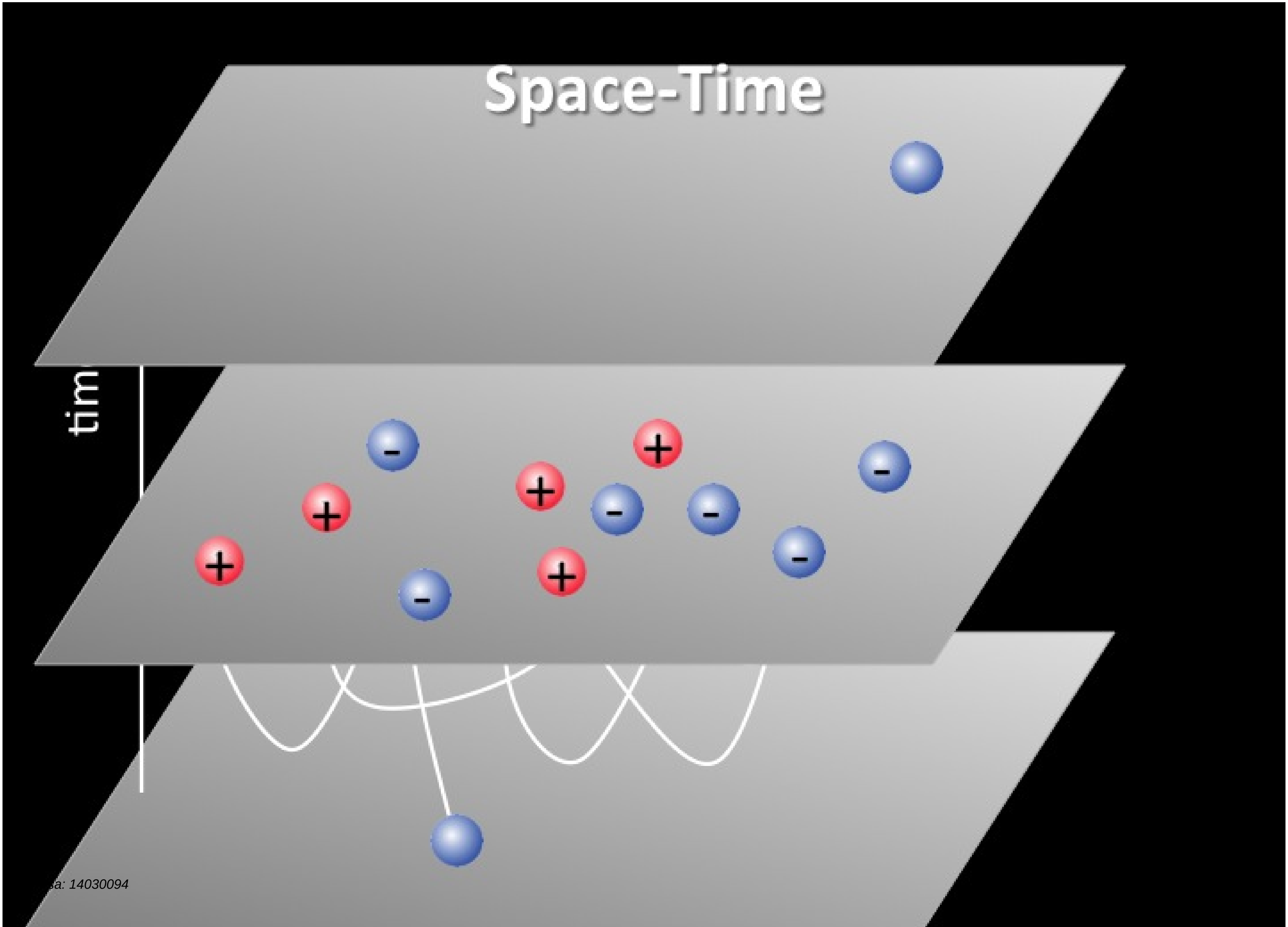
Why is every electron exactly the same?



There is only one electron in the universe!

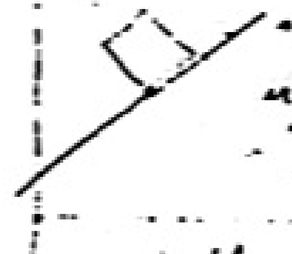
John Wheeler

Richard Feynman


Space-Time
+
-

62

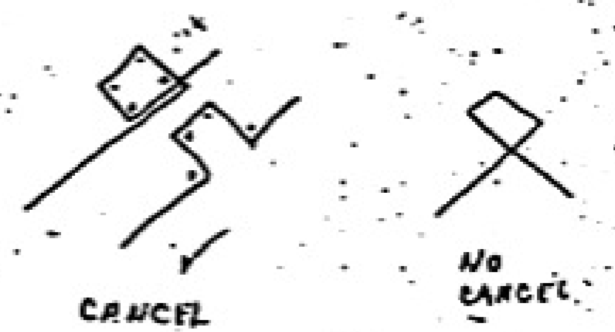
Each turn to + turn gives $+i\epsilon$
" " " − " " $-i\epsilon$

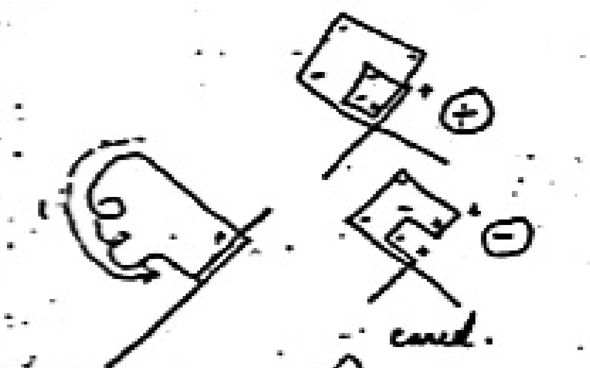


turn left $+i(-i)(-i)(+i)$
+ turn right $(-i)(+i)(-i)(+i)$

cancels

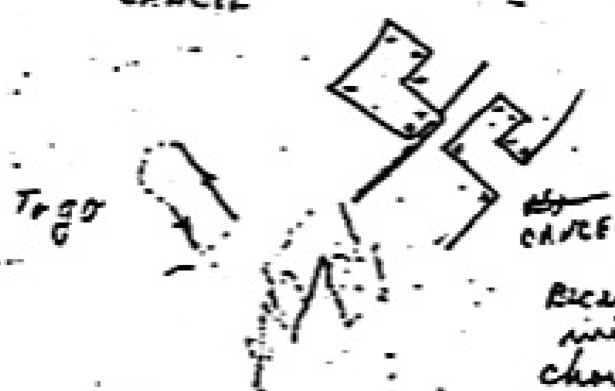
∴ any ~~closed~~ completely closed loop cancels





Rule.

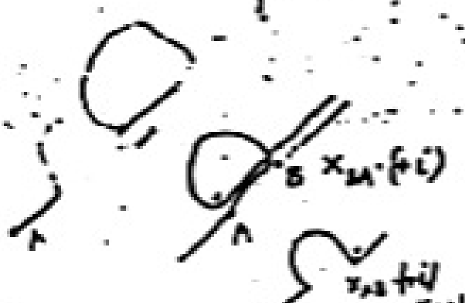
∴ if a path from A to B is pursued in one direction A→B the amp is X_{AB}. If pursued in other direction B→A it is $X_{BA} = -X^*_{AB}$

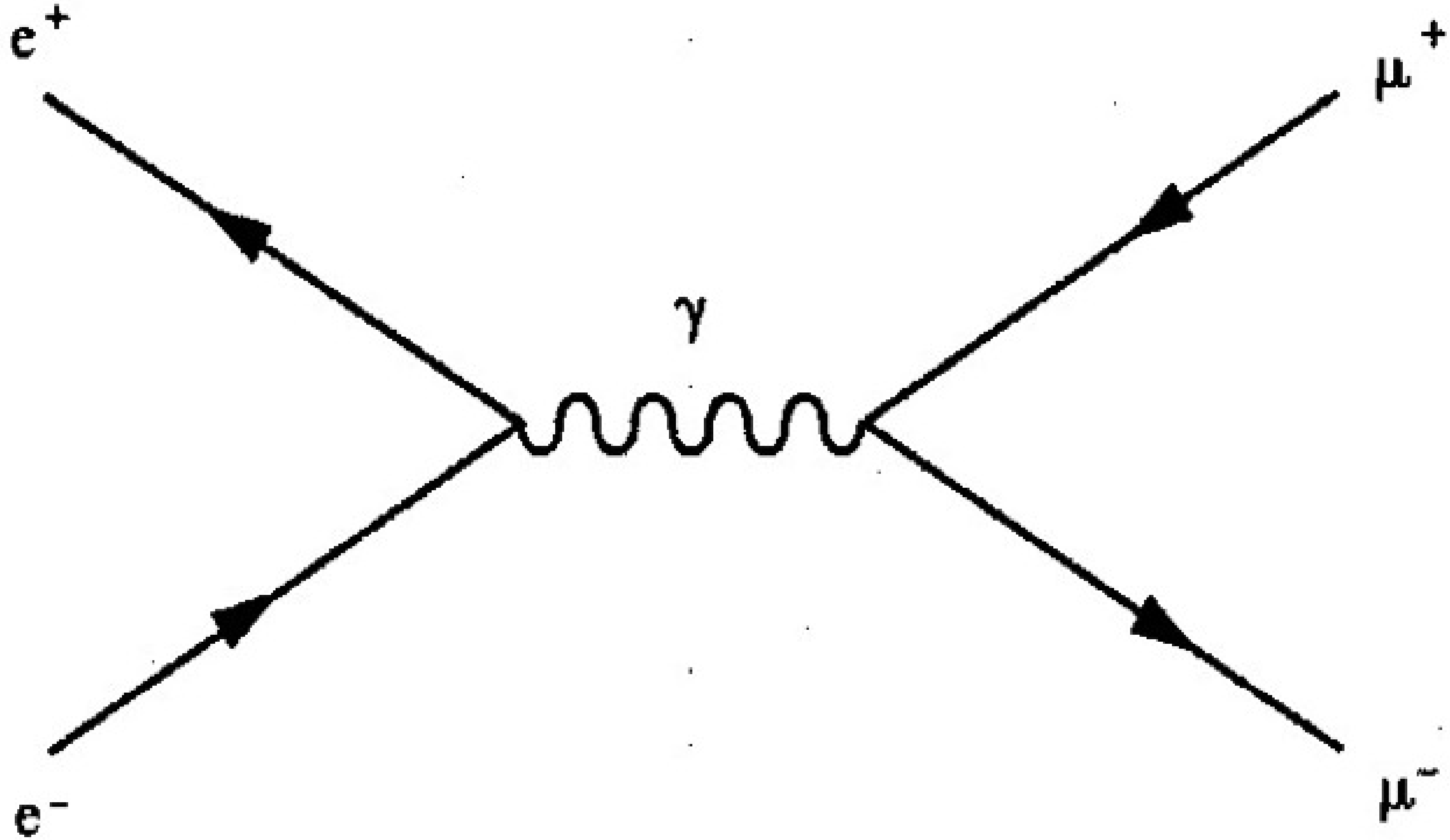


Because each turn for + is a turn for minus in reverse (∴ *) except when it changes, at a max or min of path. But the no. MAX + MIN is odd, hence the sign changes.

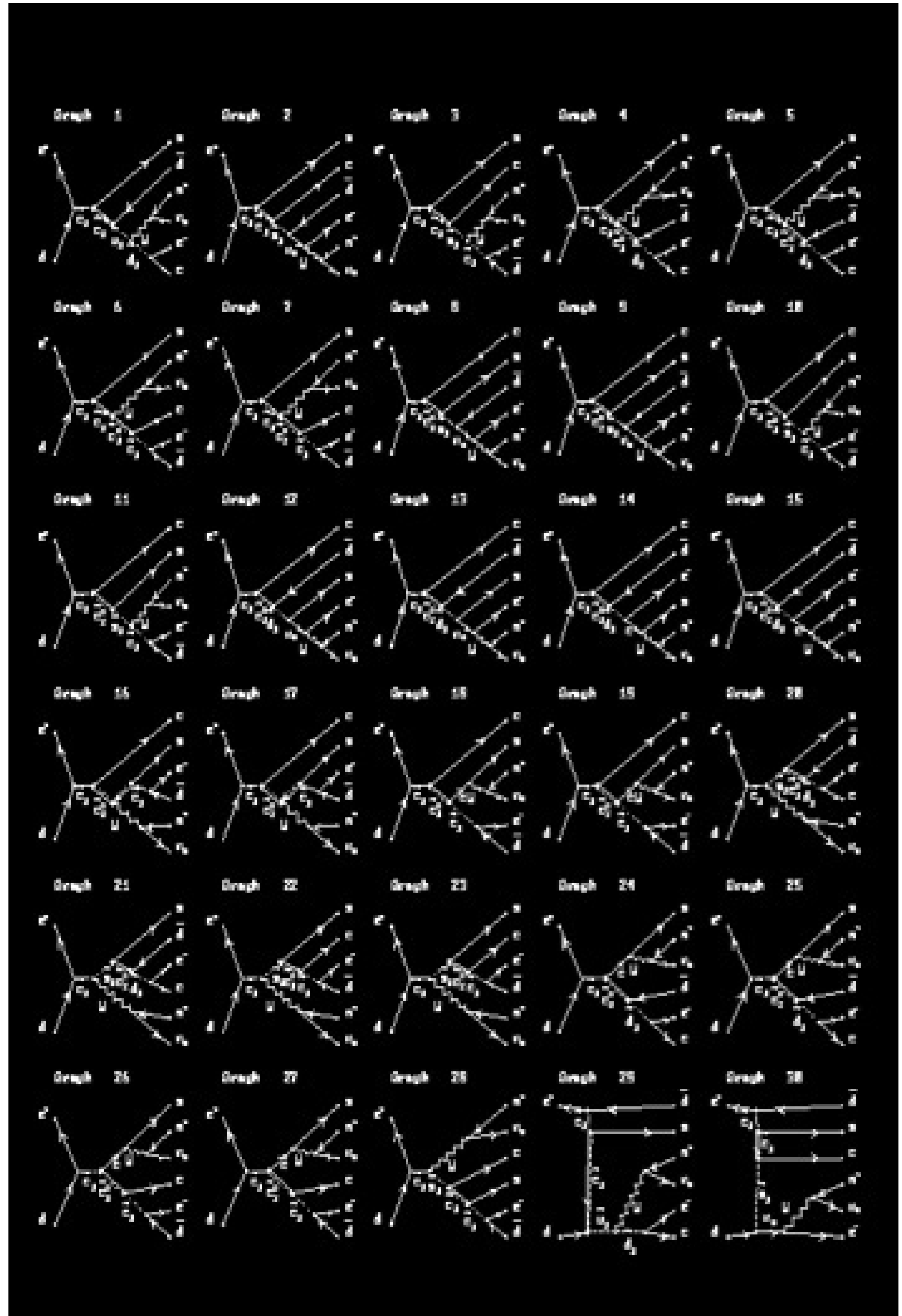
If we start + stop going in same direction, then $X_{AB} = X^*_{BA}$ because the no. of turns is even.

∴ If there is an over lapping section

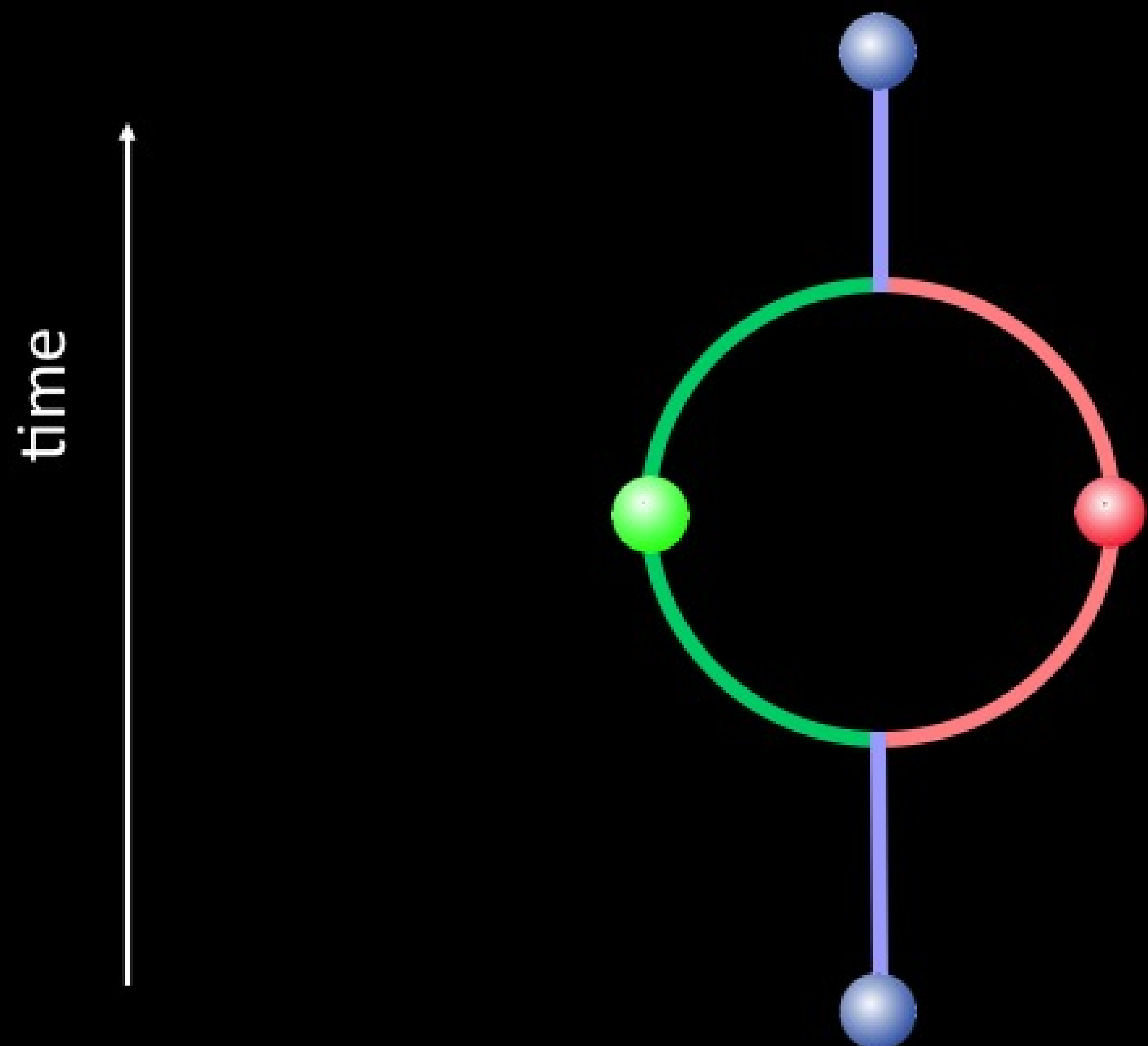



e⁺
μ⁺
γ
e⁻
μ⁻

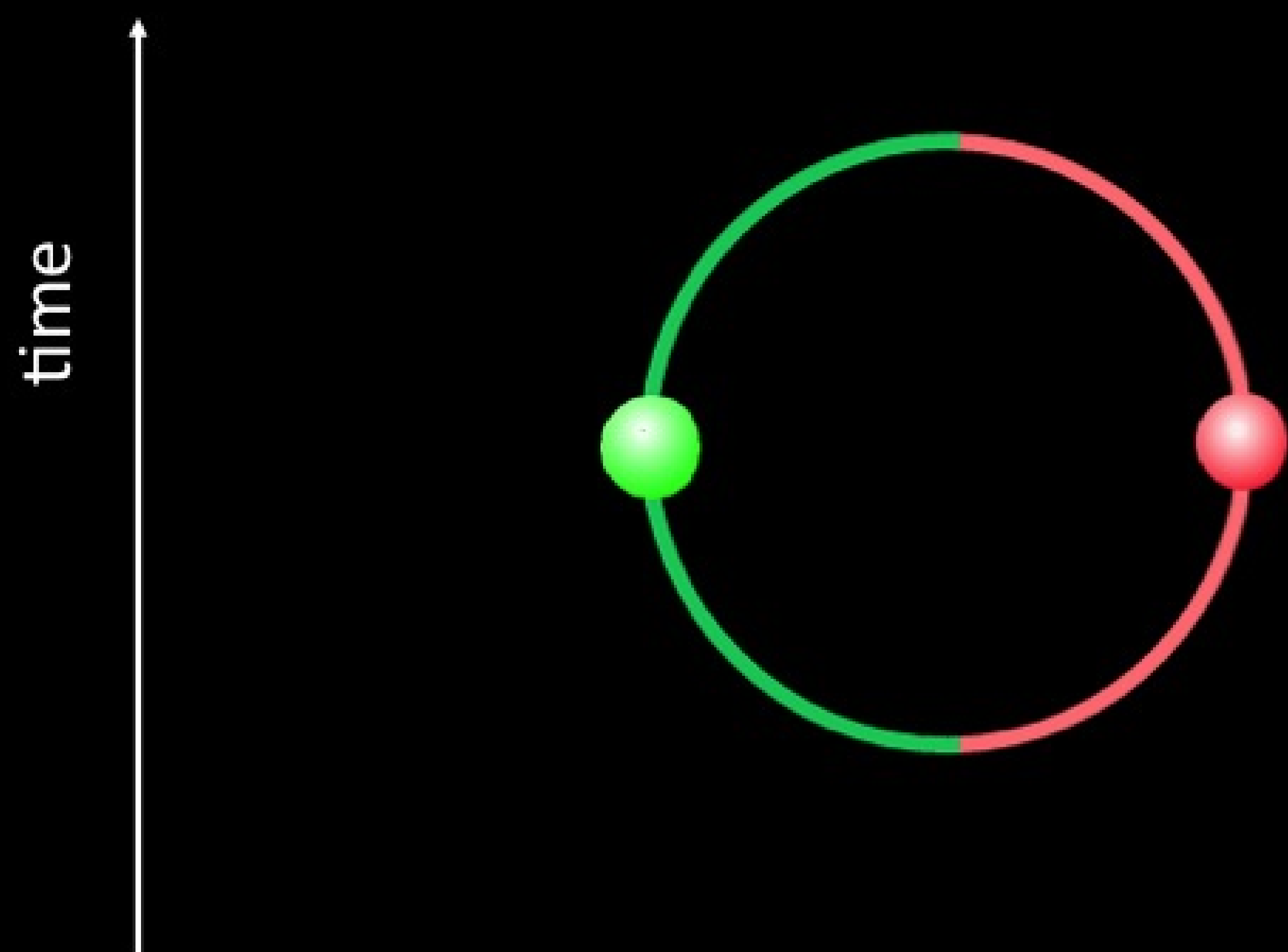
$$
\begin{aligned}
&-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
&\tfrac{1}{2}ig_s^2(\bar q^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
&M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu\partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H\partial_\mu H - \\
&\tfrac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} + \\
&\tfrac{2M}{g}H + \tfrac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
&W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
&W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
&\tfrac{1}{2}g^2W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
&g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
&\tfrac{1}{8}g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2] - \\
&gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
&W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
&\phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
&igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
&igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-] - \\
&\tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
&W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
&W^-_\mu\phi^+) + \tfrac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - \\
&g^2s_w^2A_\mu A_\mu\phi^+\phi^- - \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j - \\
&\bar d^\lambda_j(\gamma\partial+m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar e^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar d^\lambda_j\gamma^\mu d^\lambda_j)] + \\
&\tfrac{ig}{4c_w}Z^0_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - \\
&1-\gamma^5)u^\lambda_j) + (\bar d^\lambda_j\gamma^\mu(1-\tfrac{8}{3}s_w^2-\gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt2}W^+_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + \\
&(\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt2}W^-_\mu[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+ \\
&\gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt2}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] - \\
&\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + \\
&m_u^\lambda(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)] + \tfrac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1- \\
&\gamma^5)u^\kappa_j)] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar u^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar d^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \\
&\tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j) + \bar X^+(\partial^2-M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0(\partial^2 - \\
&\tfrac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0X^- - \partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar YX^- - \\
&\partial_\mu\bar X^+Y) + igc_wW^-_\mu(\partial_\mu\bar X^-X^0 - \partial_\mu\bar X^0X^+) + igs_wW^-_\mu(\partial_\mu\bar X^-Y - \\
&\partial_\mu\bar YX^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ - \\
&\partial_\mu\bar X^-X^-) - \tfrac{1}{2}gM[\bar X^+X^+H + \bar X^-X^-H + \tfrac{1}{c_w^2}\bar X^0X^0H] + \\
&\tfrac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \tfrac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \\
&igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \tfrac{1}{2}igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]
\end{aligned}
$$



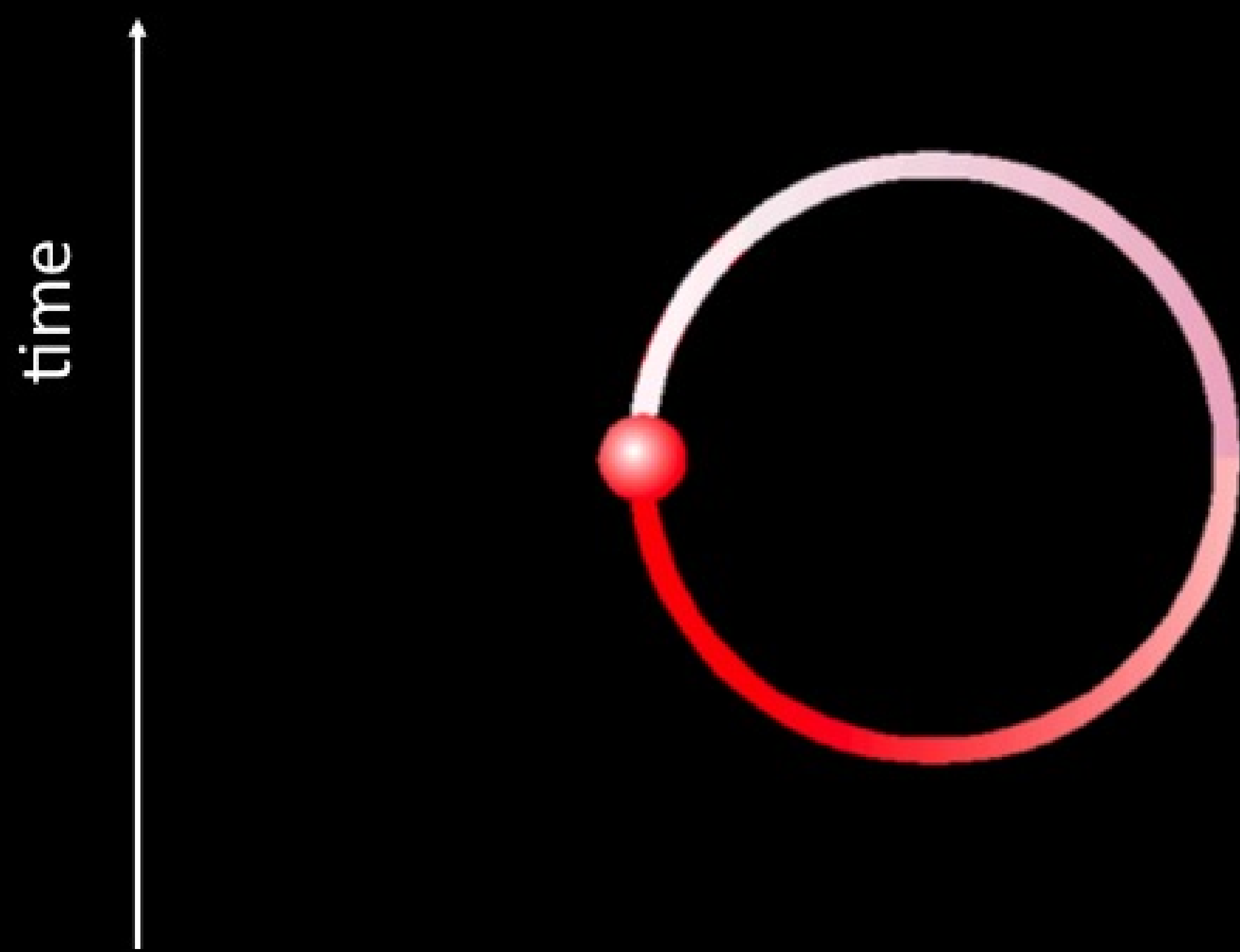
Virtual Particles

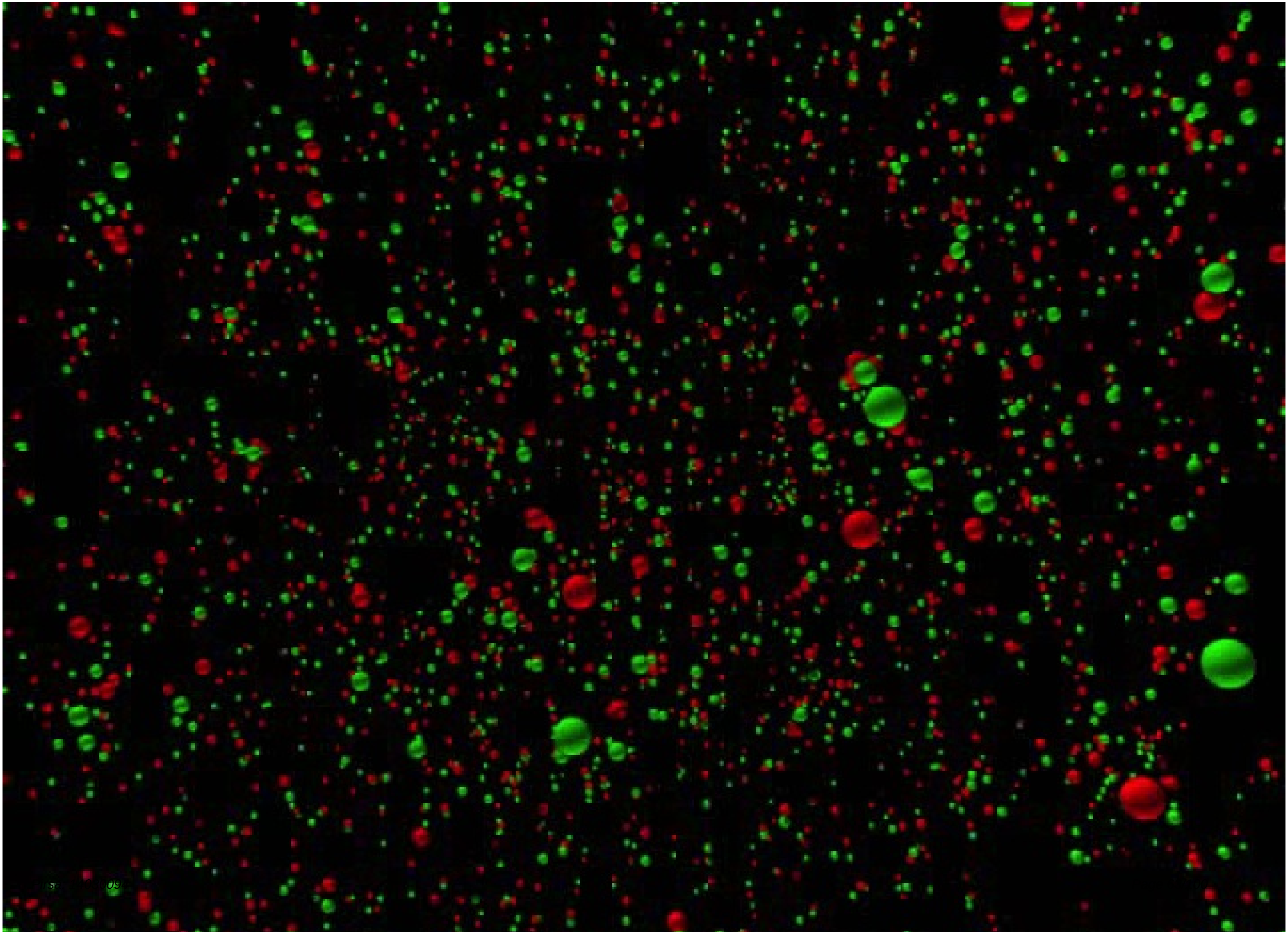


Vacuum Fluctuations

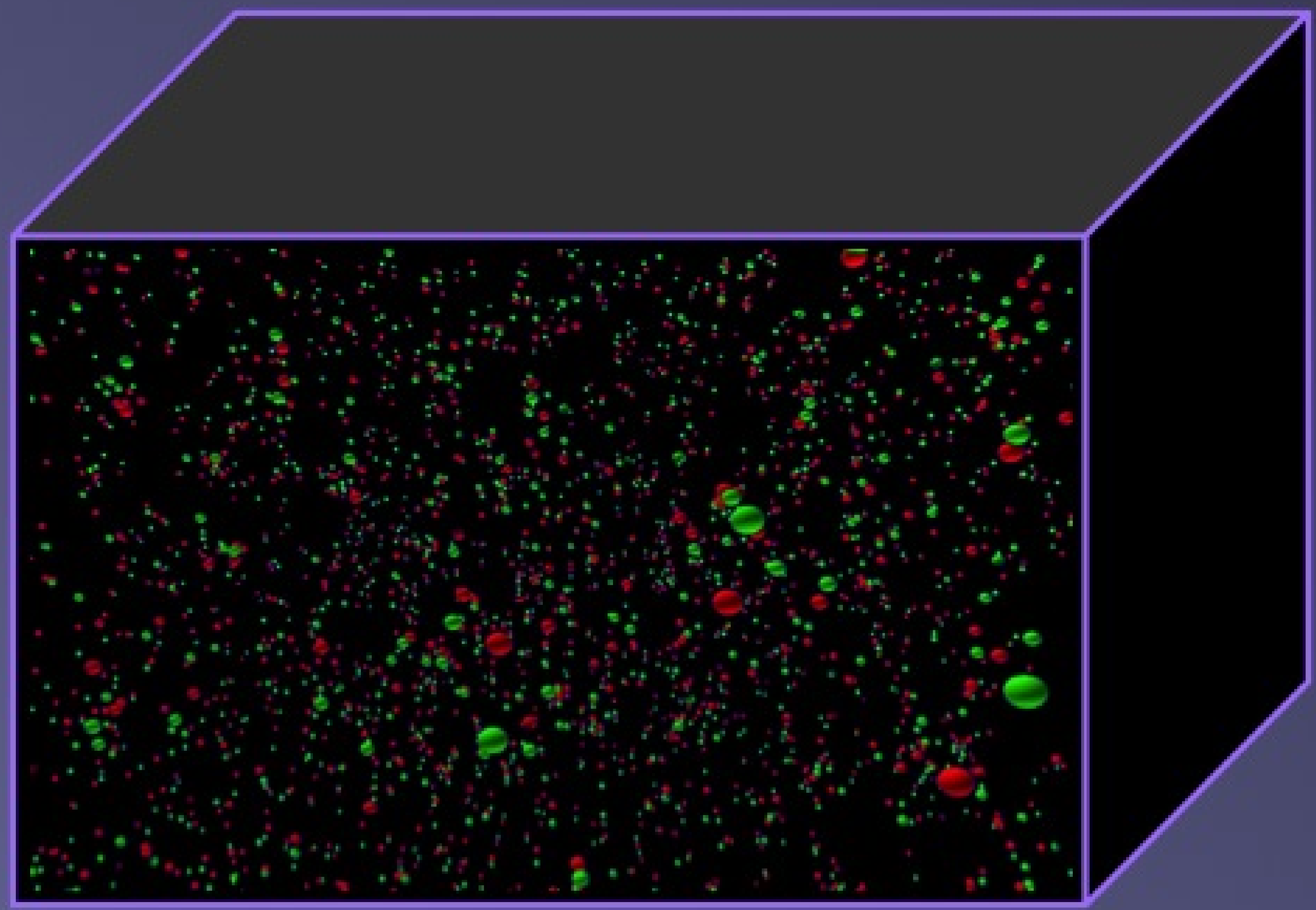


Vacuum Fluctuations

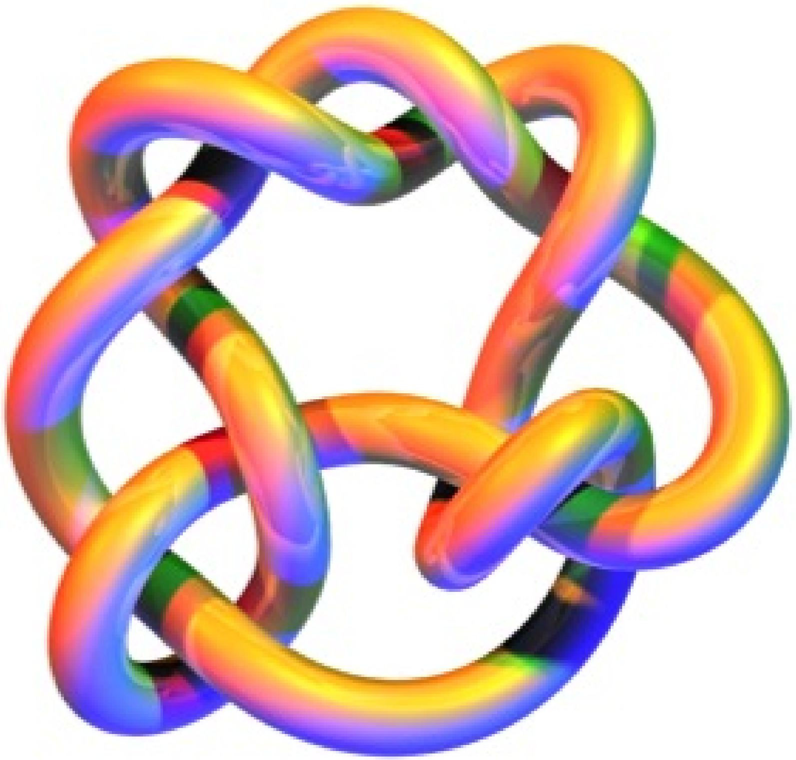




Dark Energy



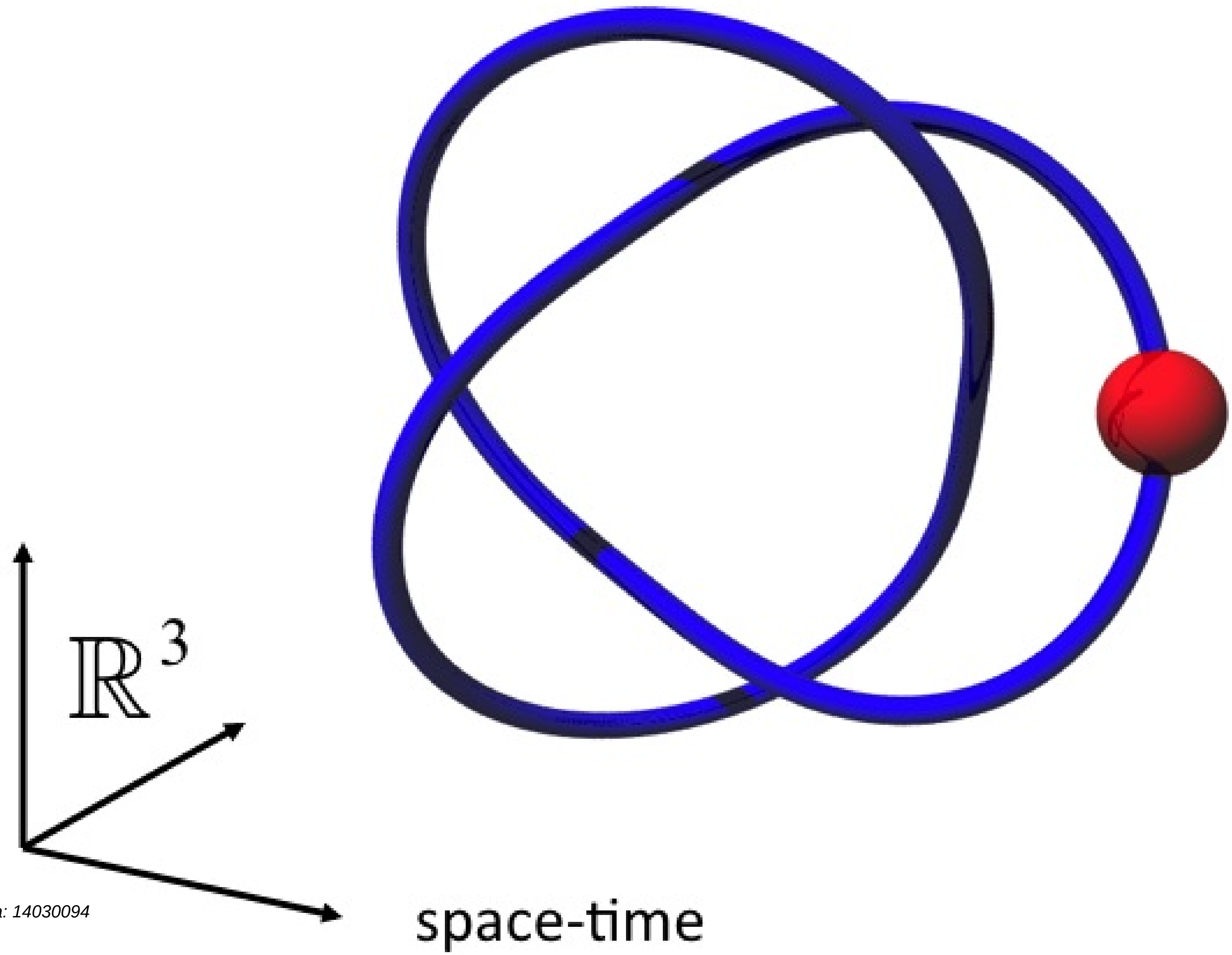
Quantum Space-Time



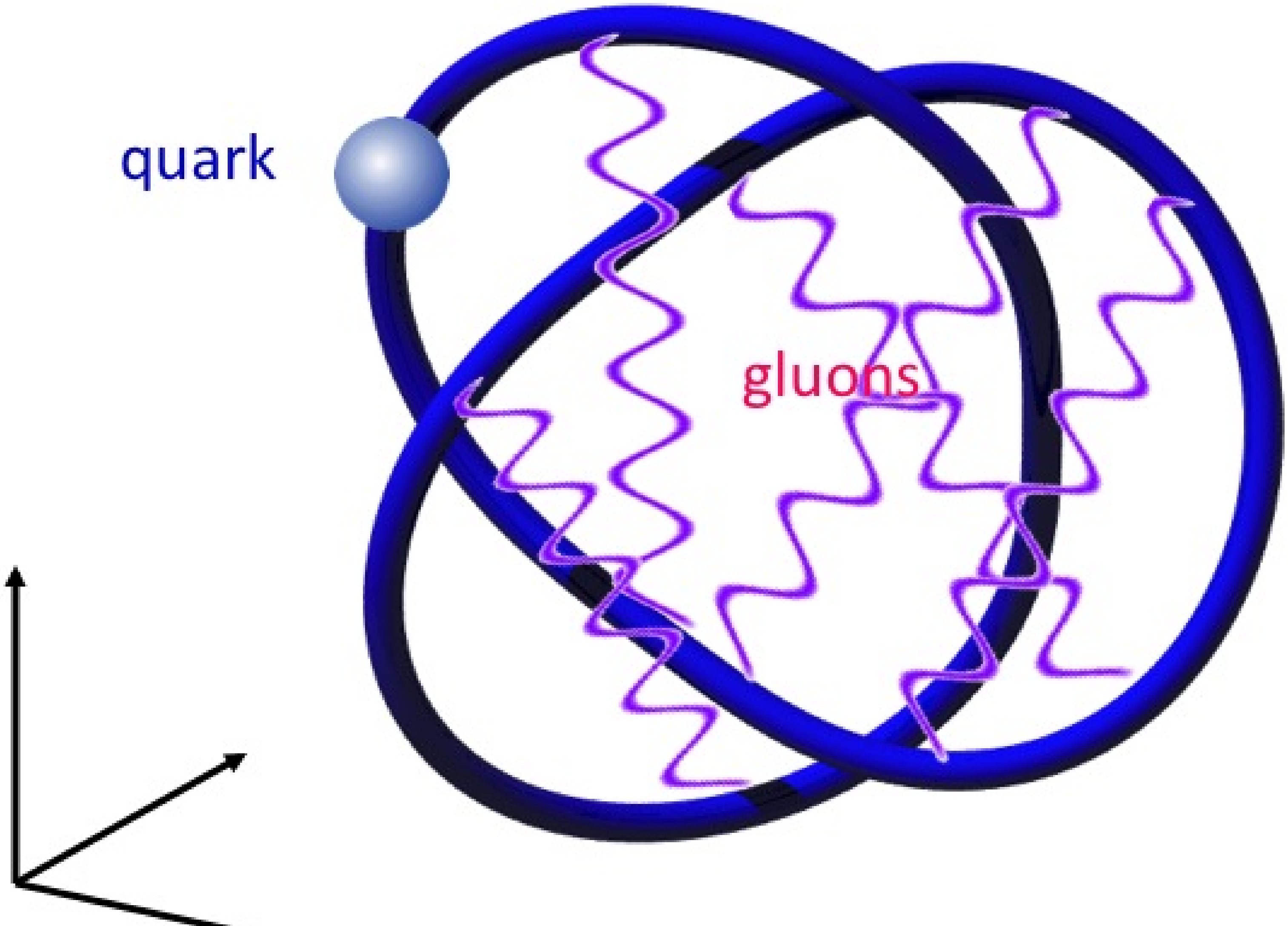
Knot Theory


The Book
Of Knots

Chern-Simons Gauge Theory

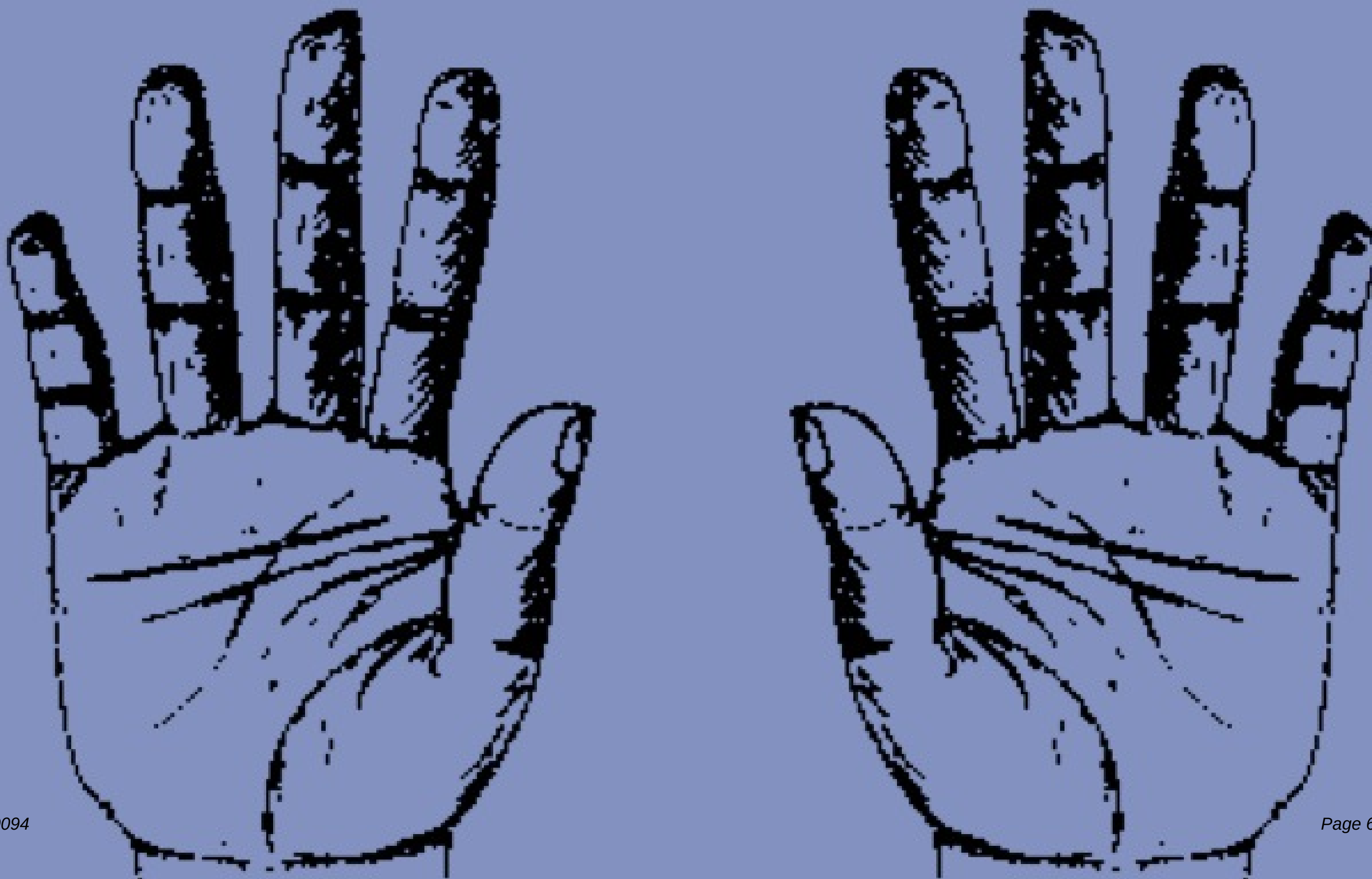


Quantum Amplitude

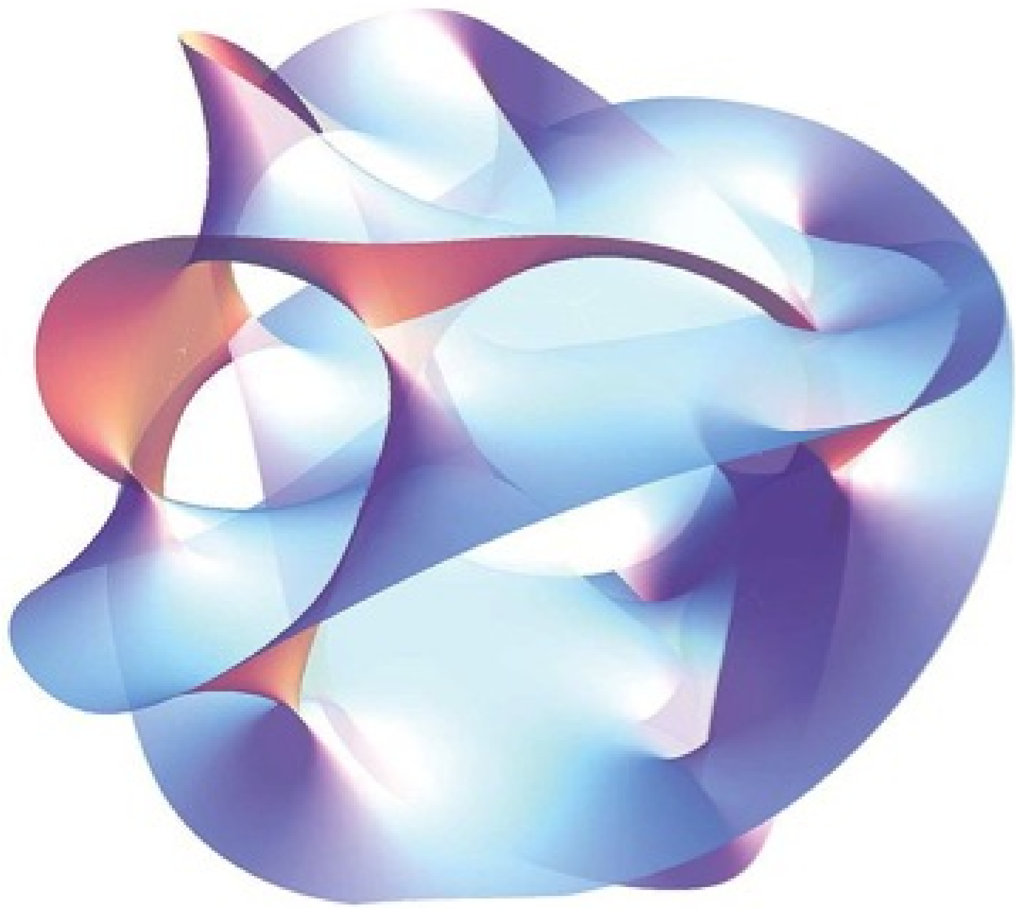


Counting

Enumerative Geometry



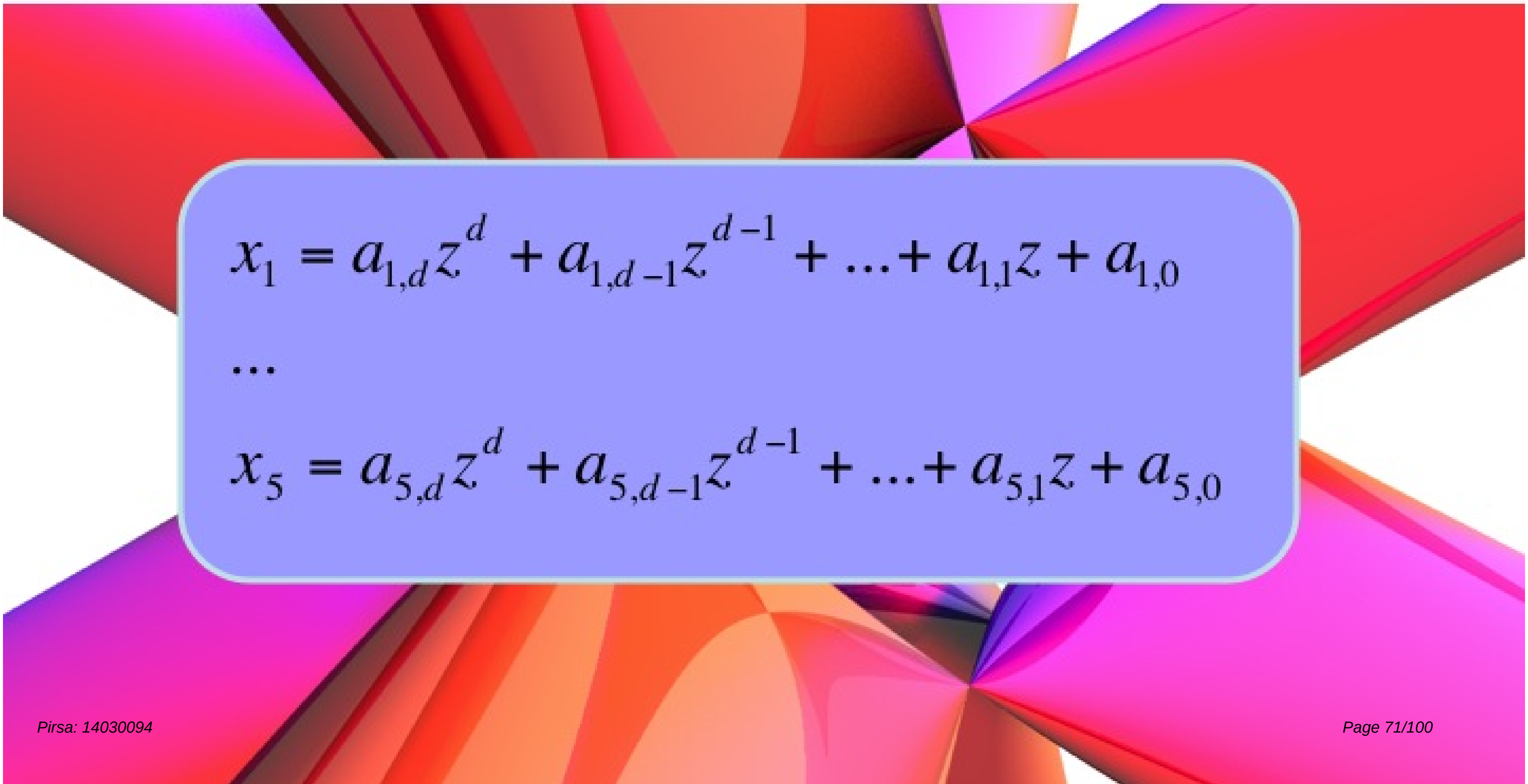
The Quintic



$$x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0$$

Gromov-Witten theory

$$N_d = \#\, curves\ of\ degree\ d$$



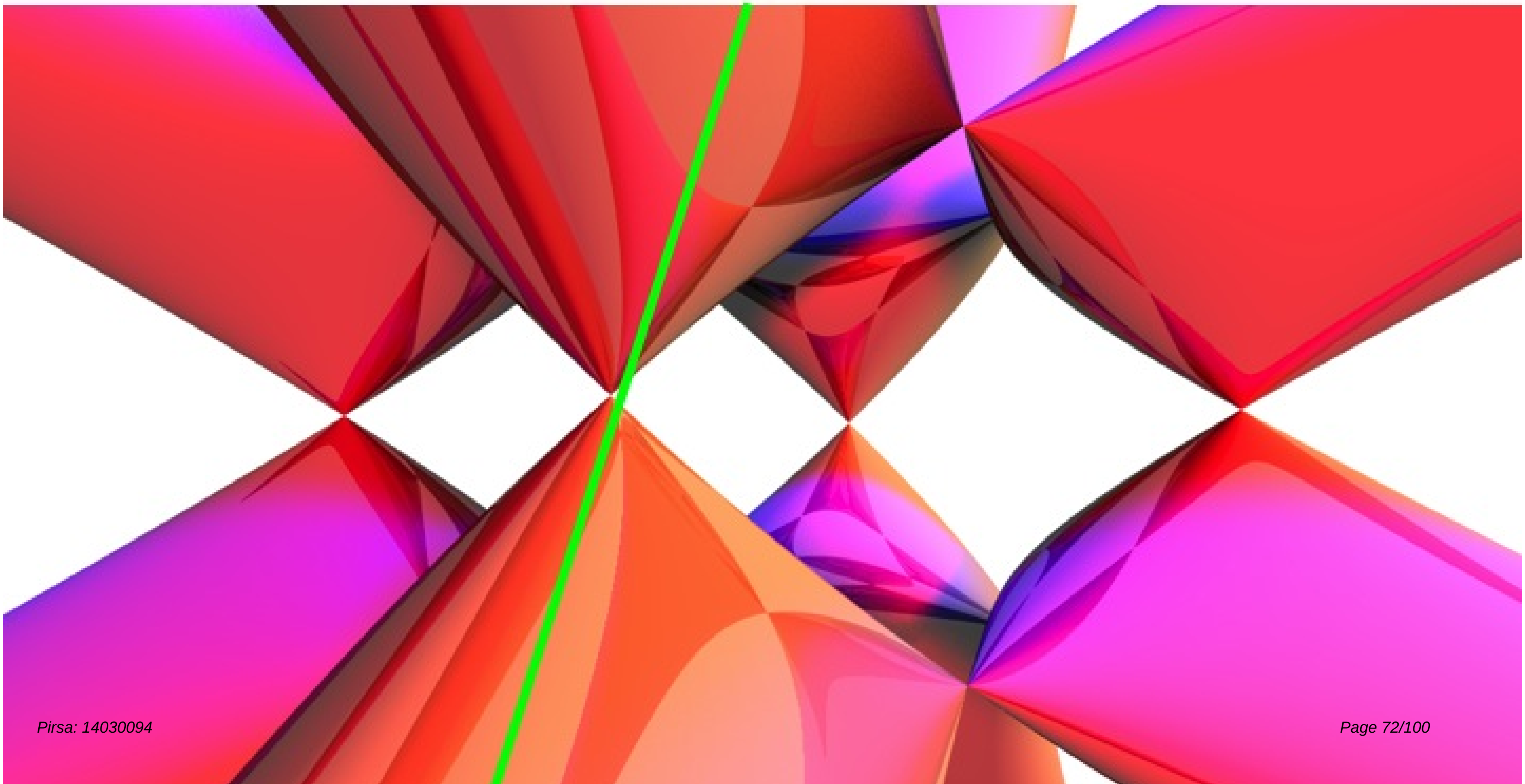
$$x_1 = a_{1,d} z^d + a_{1,d-1} z^{d-1} + \ldots + a_{1,1} z + a_{1,0}$$

$$\ldots$$

$$x_5 = a_{5,d} z^d + a_{5,d-1} z^{d-1} + \ldots + a_{5,1} z + a_{5,0}$$

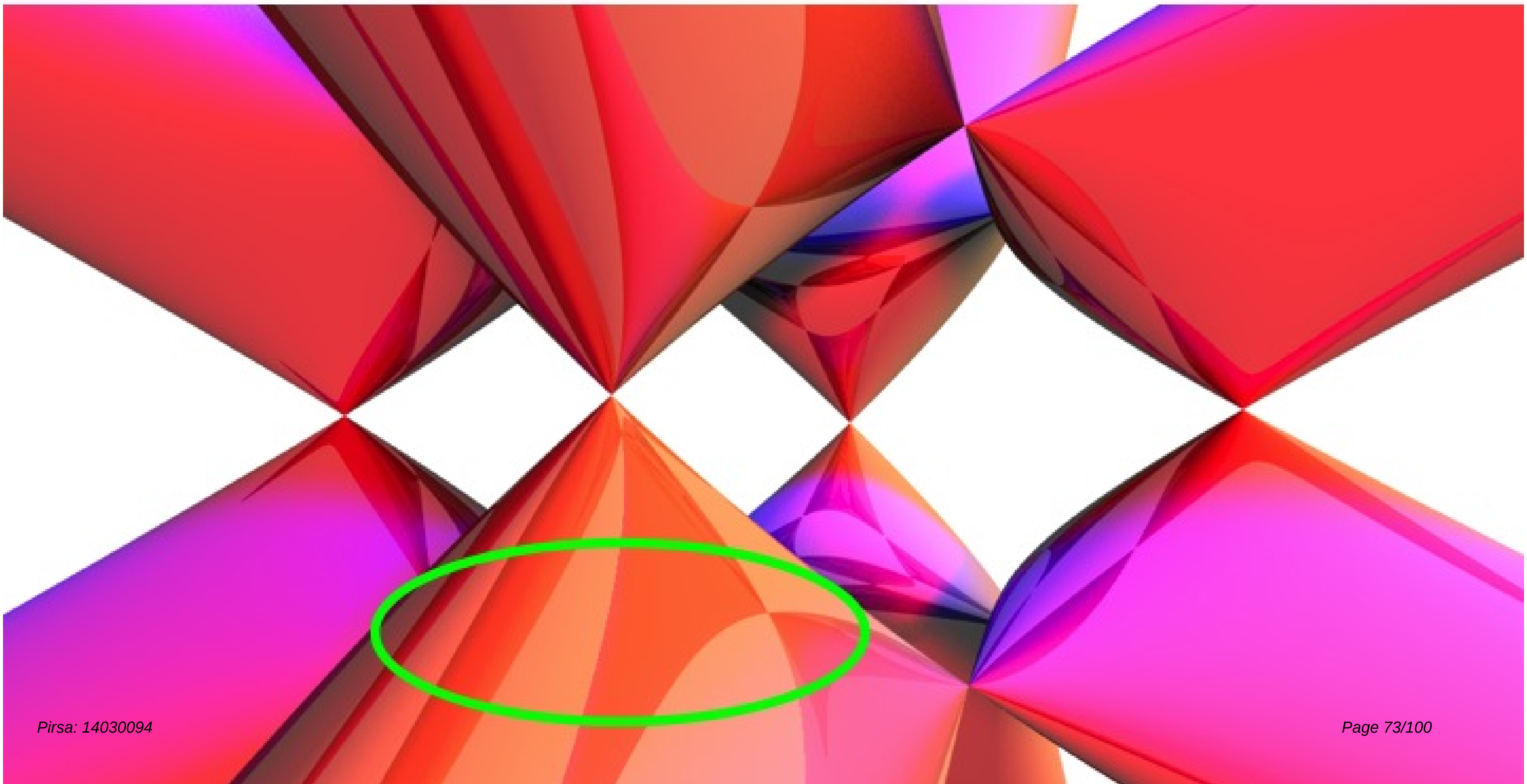
d=1 Lines

$N_1 = 2{,}875$



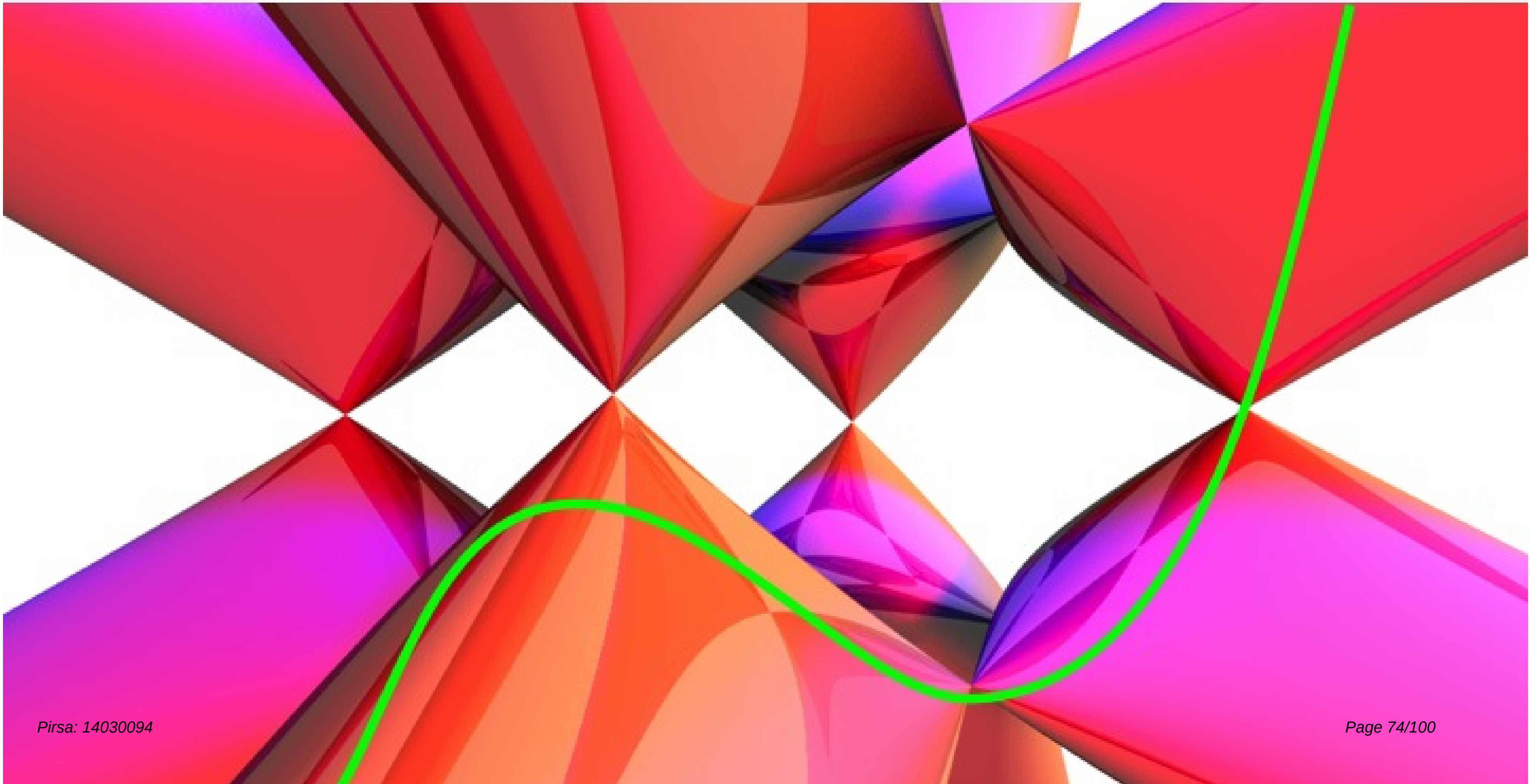
d=2 Conics

$N_2 = 609{,}250$



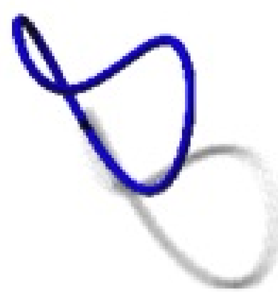
d=3 Cubics

N_3=317,206,375

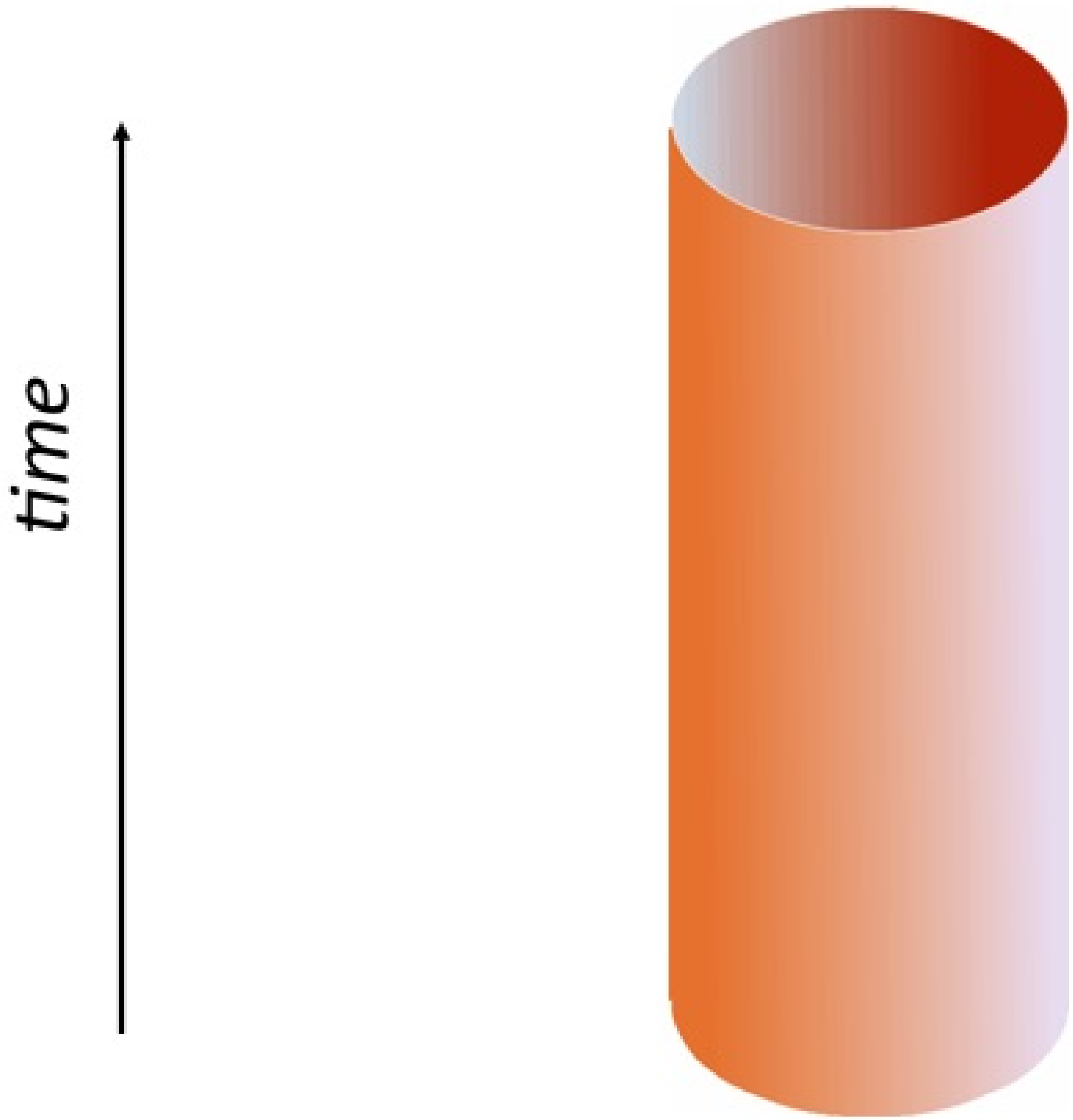


$$N_1 = 2875$$
$$N_2 = 609250$$
$$N_3 = 317206375$$
$$N_4 = 242467530000$$
$$N_5 = 229305888887625$$
$$N_6 = 248249742118022000$$
$$N_7 = 295091050570845659250$$
$$N_8 = 375632160937476603550000$$
$$N_9 = 503840510416985243645106250$$
$$N_{10} = 704288164978454686113488249750$$

Strings

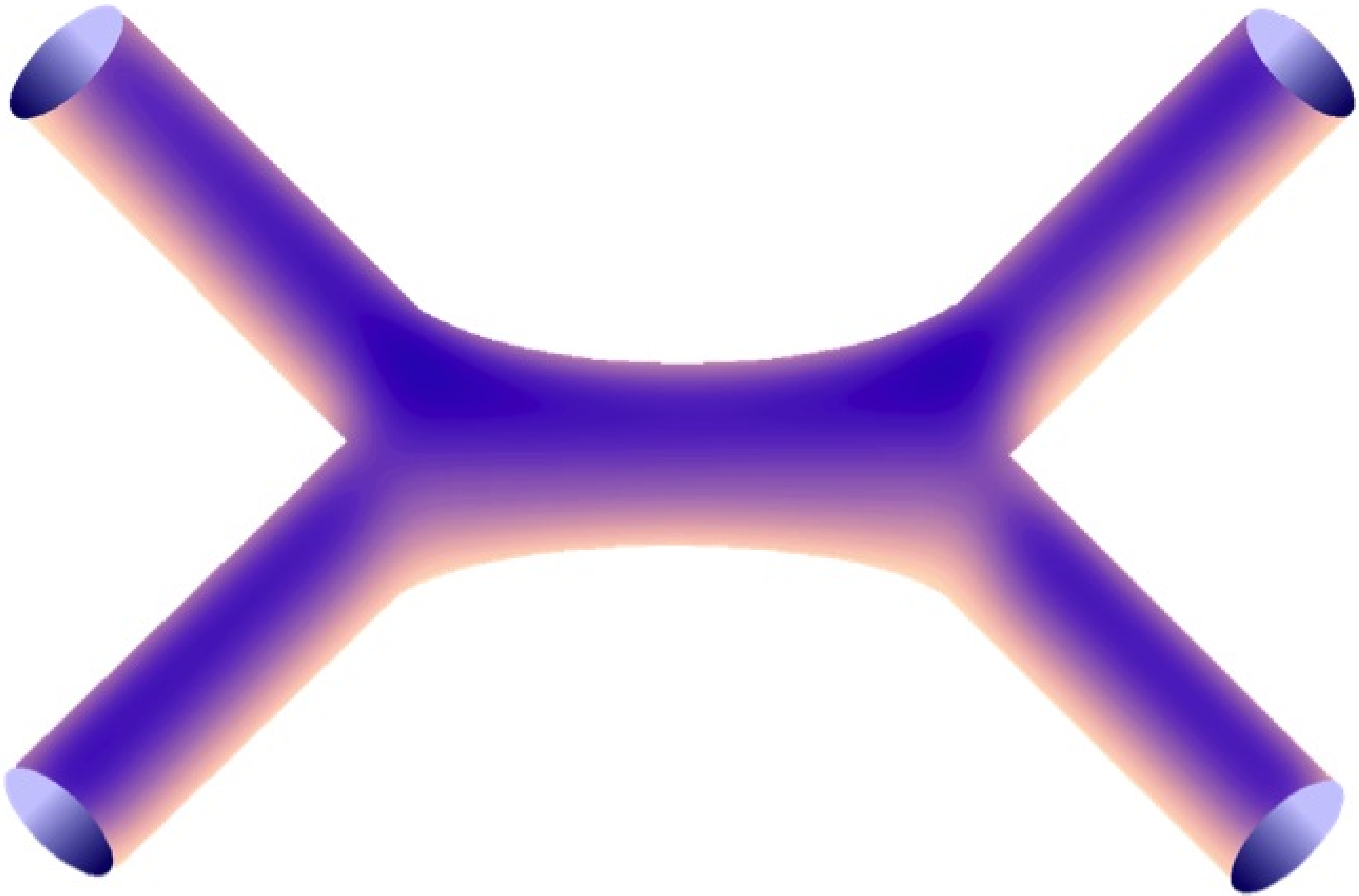


Worldsheet

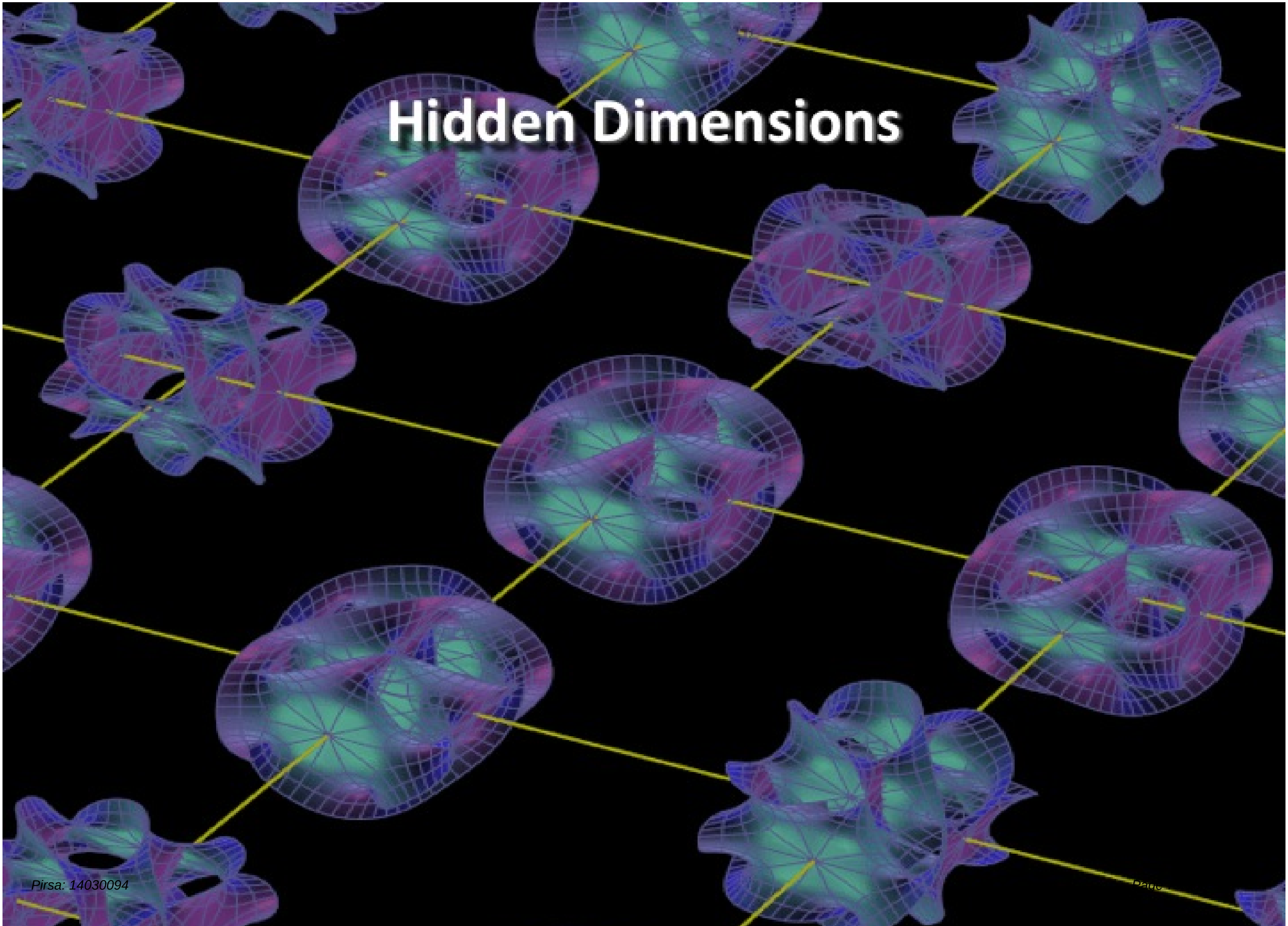


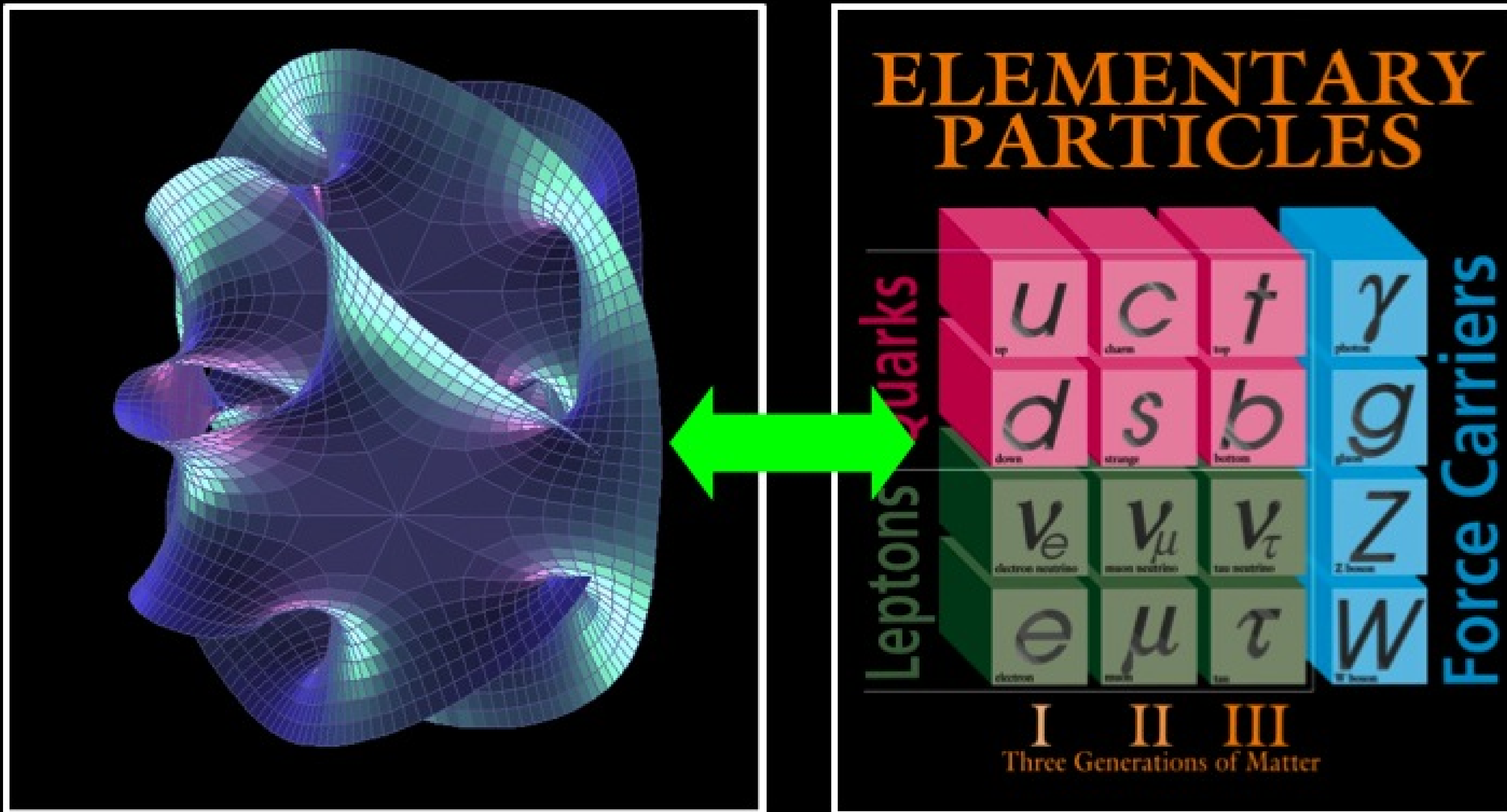
Riemann surface
algebraic curve

string



Hidden Dimensions

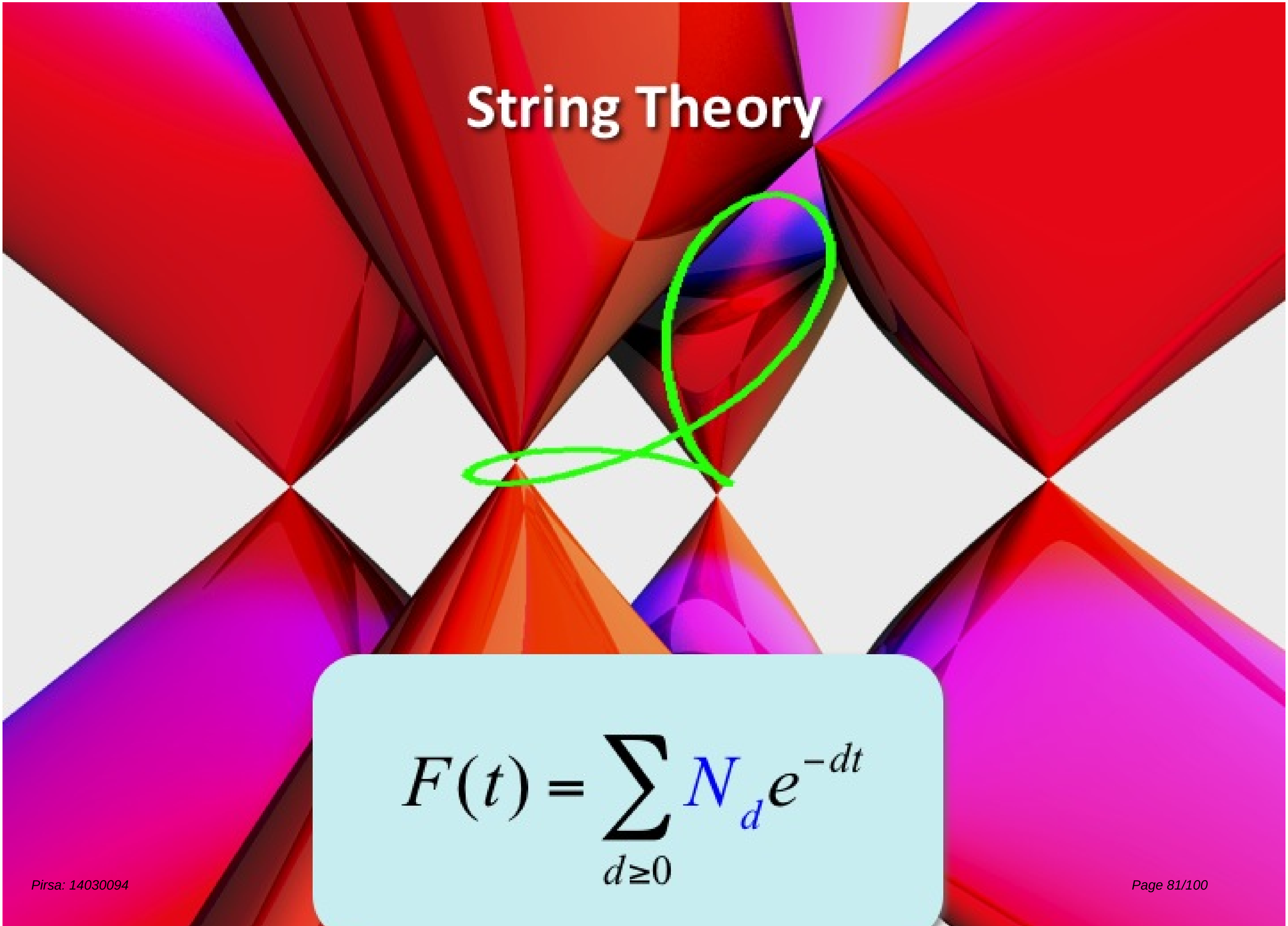




Hidden Dimensions

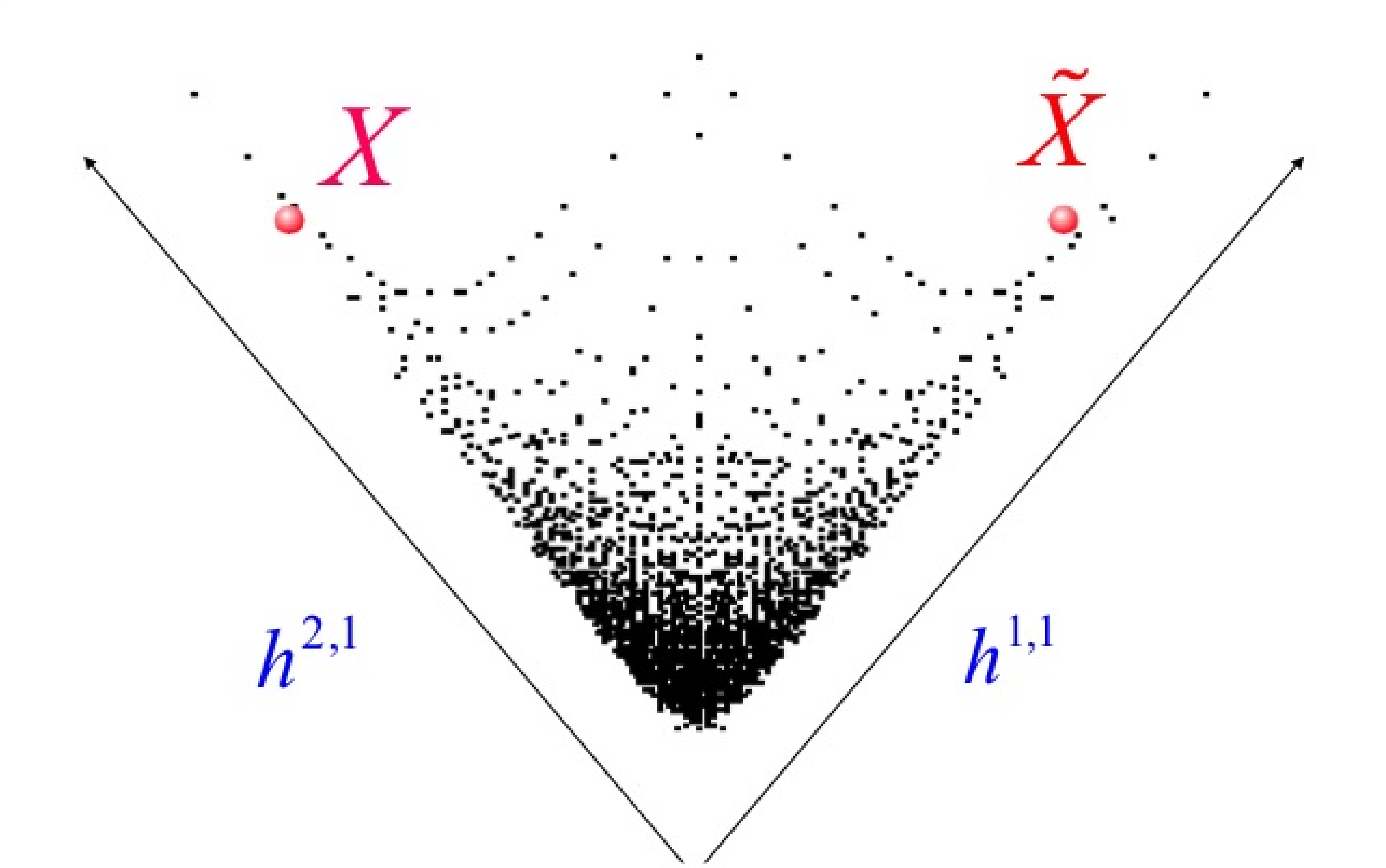
Particles Forces

String Theory

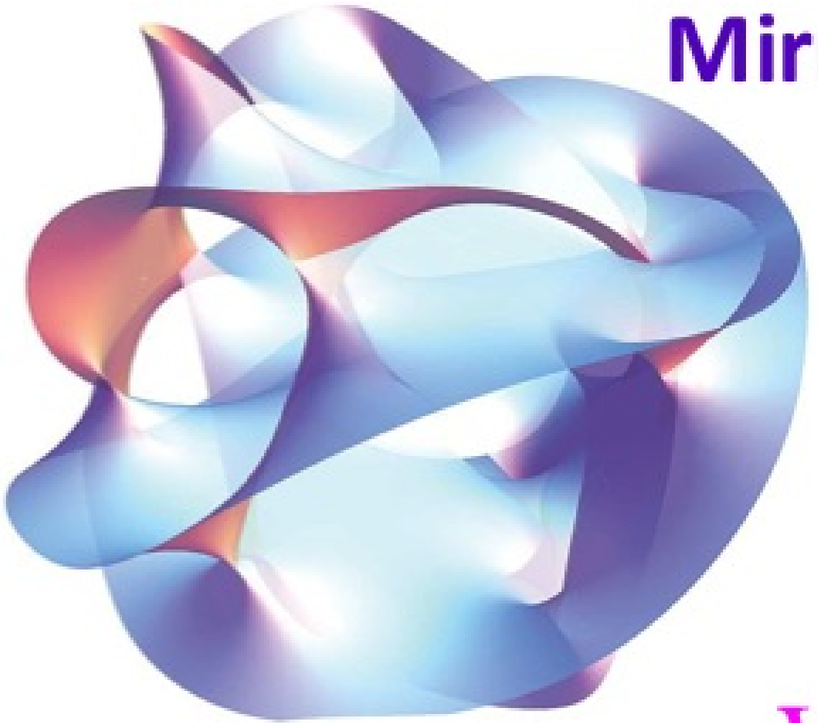


$$F(t) = \sum_{d \geq 0} N_d e^{-dt}$$

Calabi-Yau Spaces



Mirror Symmetry



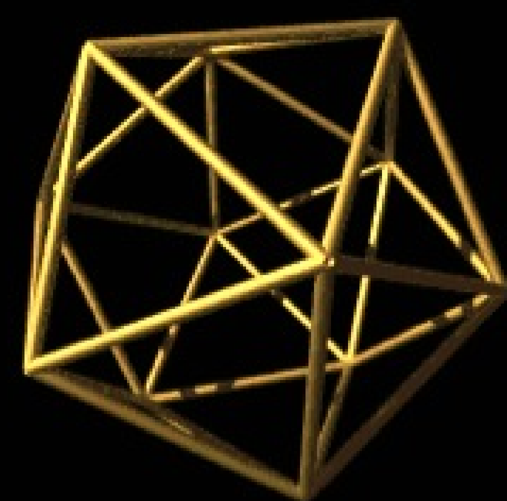
X $\tilde{X}$

$$F(t) = \sum_{d \geq 0} N_d e^{-td}$$

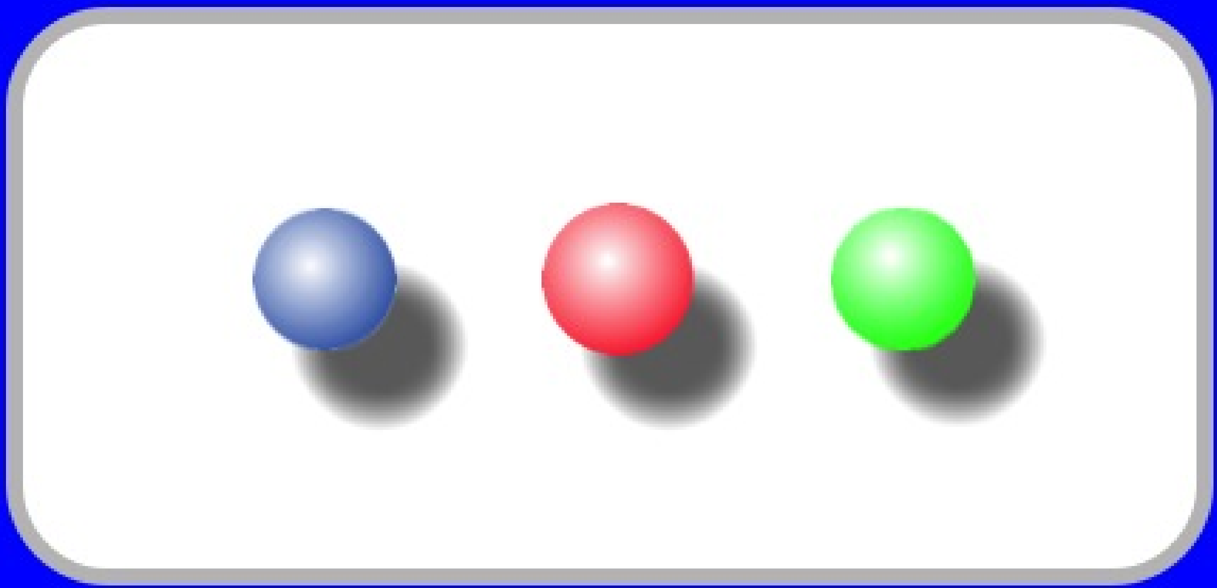
$$F(t) = \oint \Omega$$

quantum classical

Symmetry

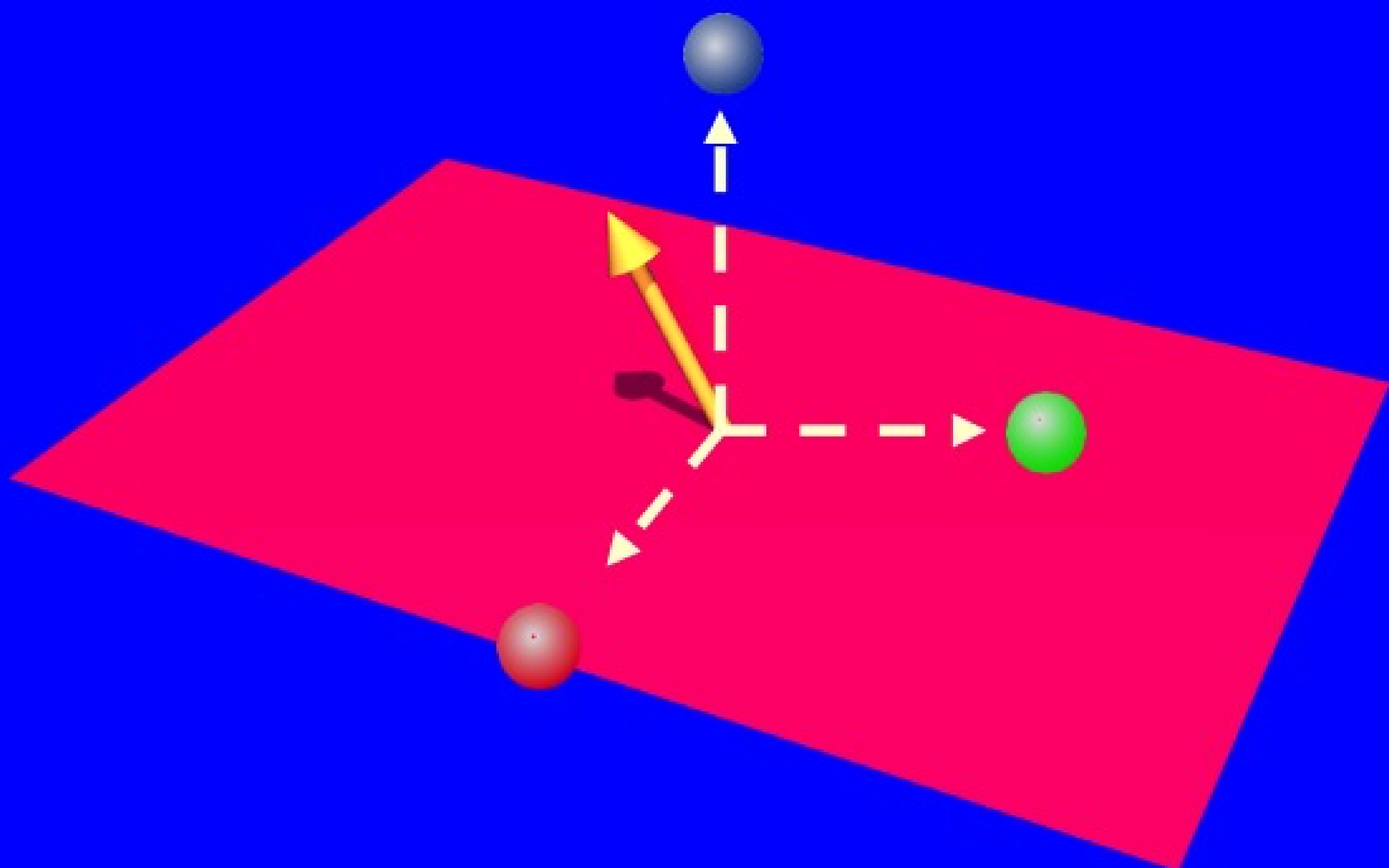


Strong Force (QCD)

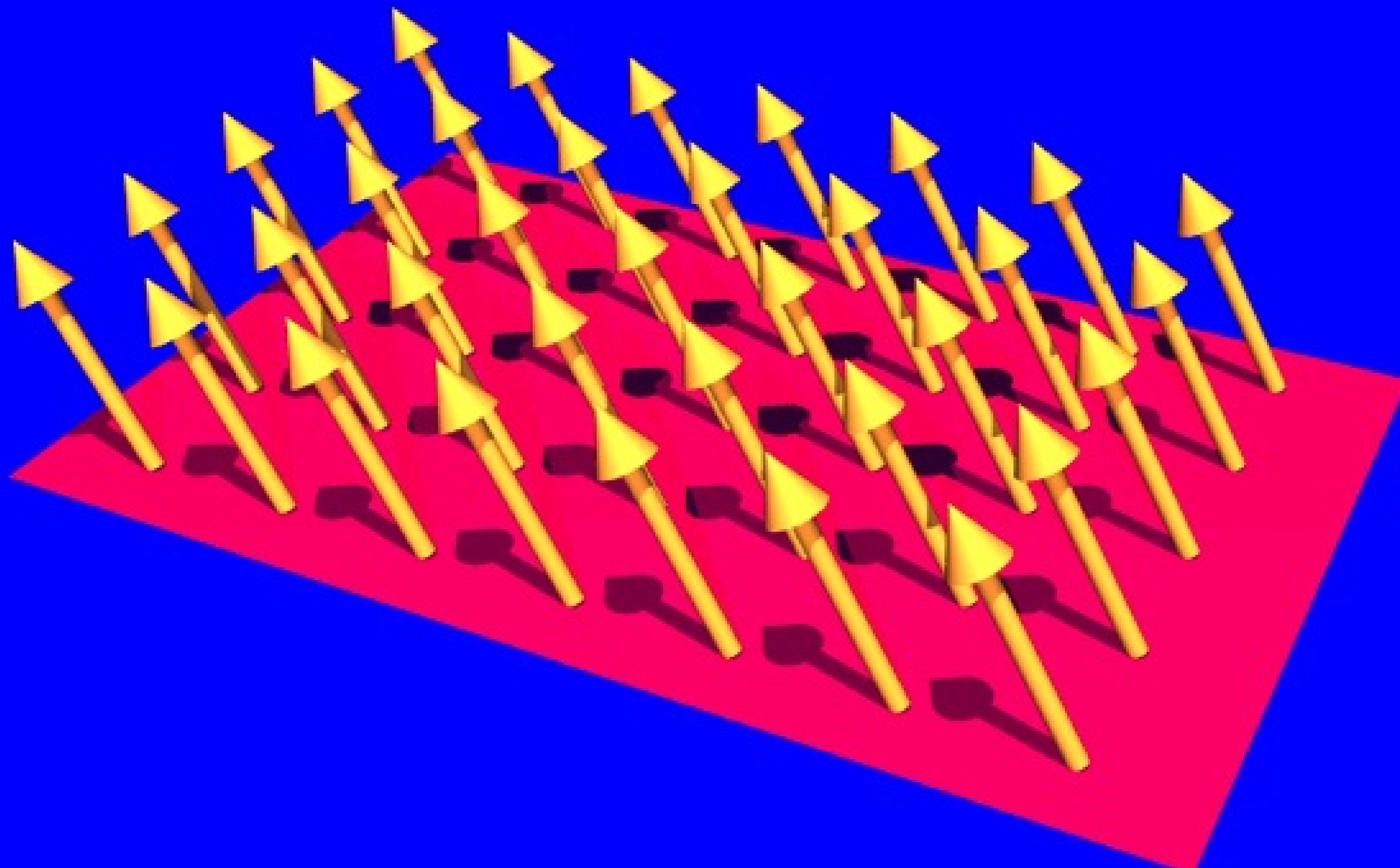


3 colors of quarks

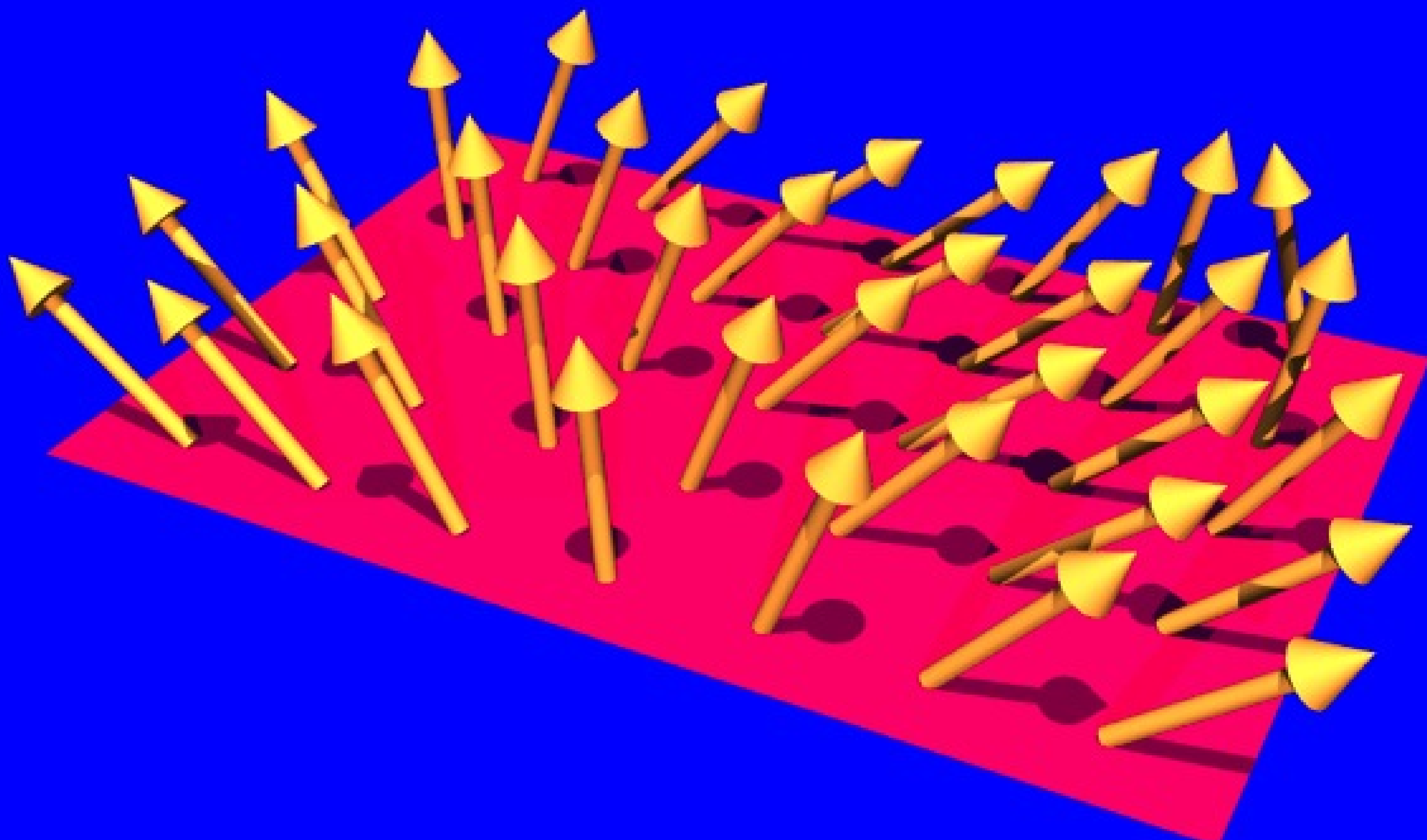
Symmetry



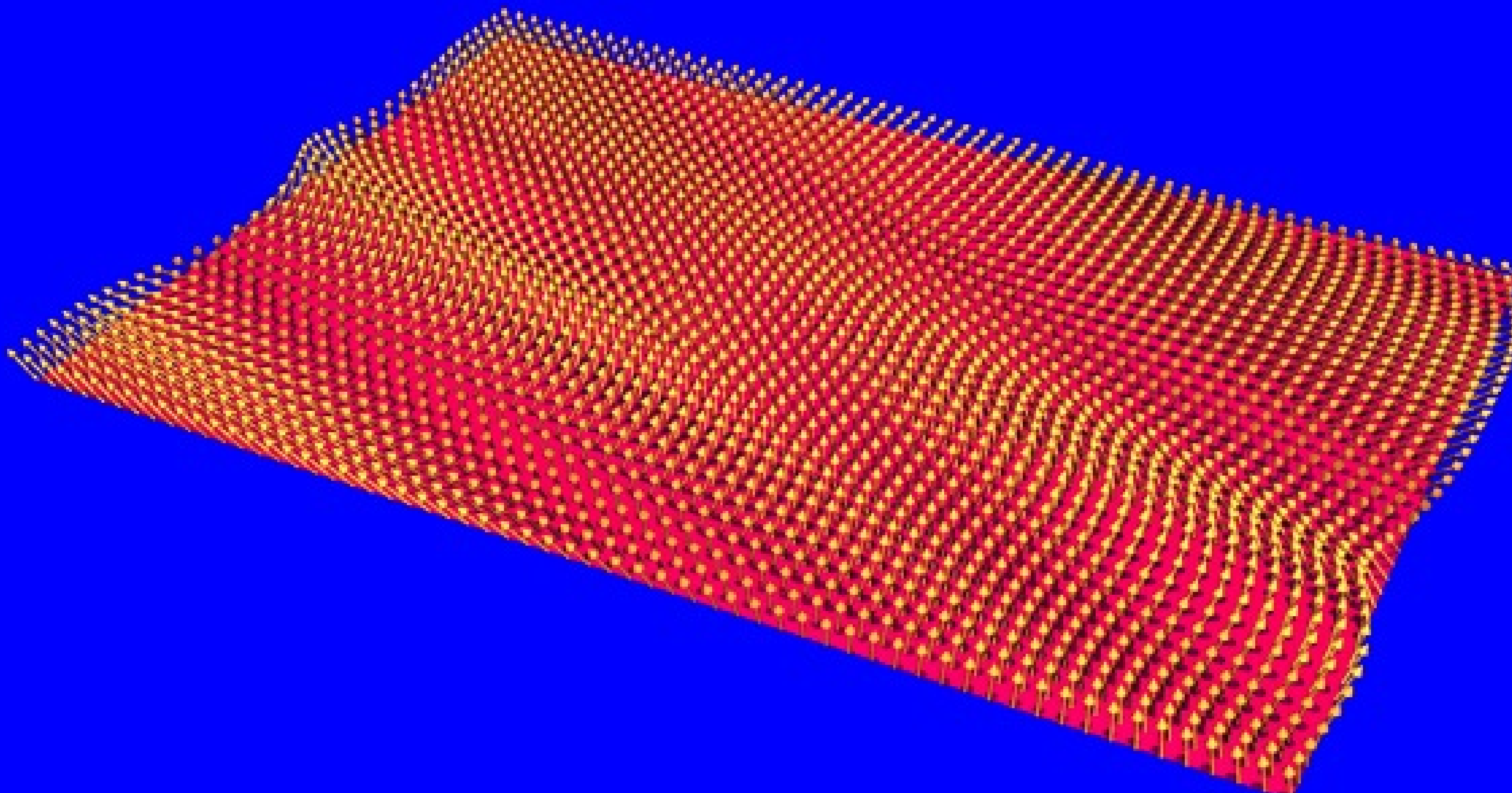
Global Symmetry



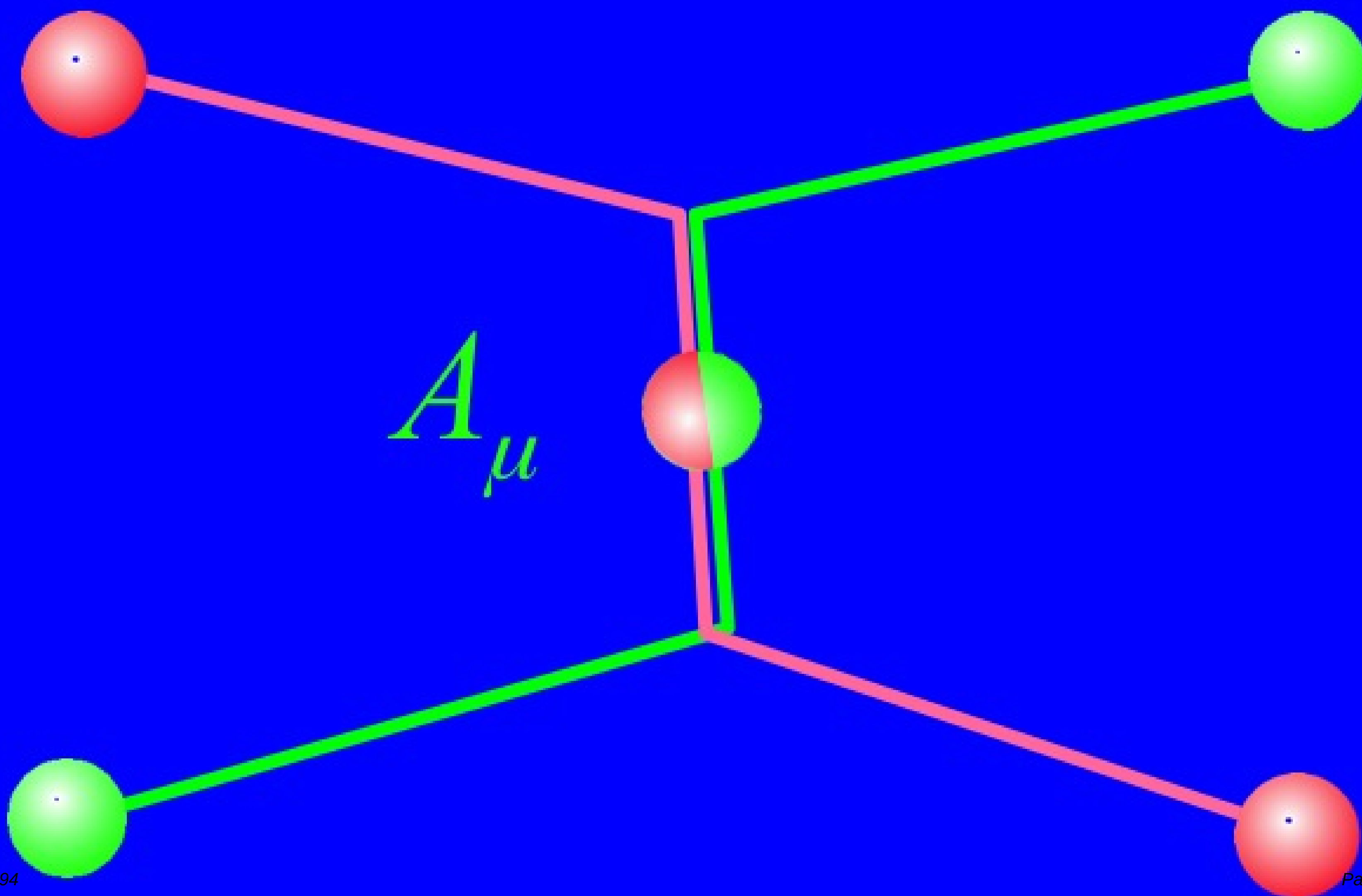
Local Gauge Symmetry



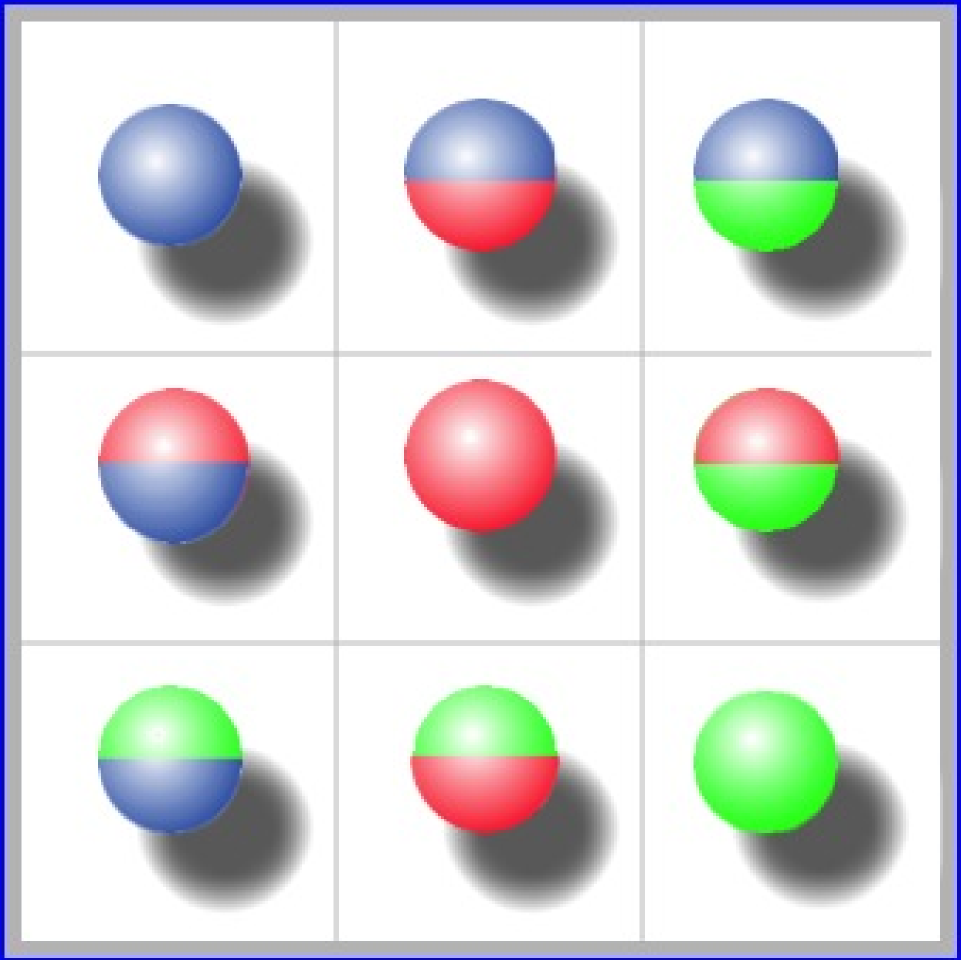
Gauge Fields



Intermediate Gauge Bosons



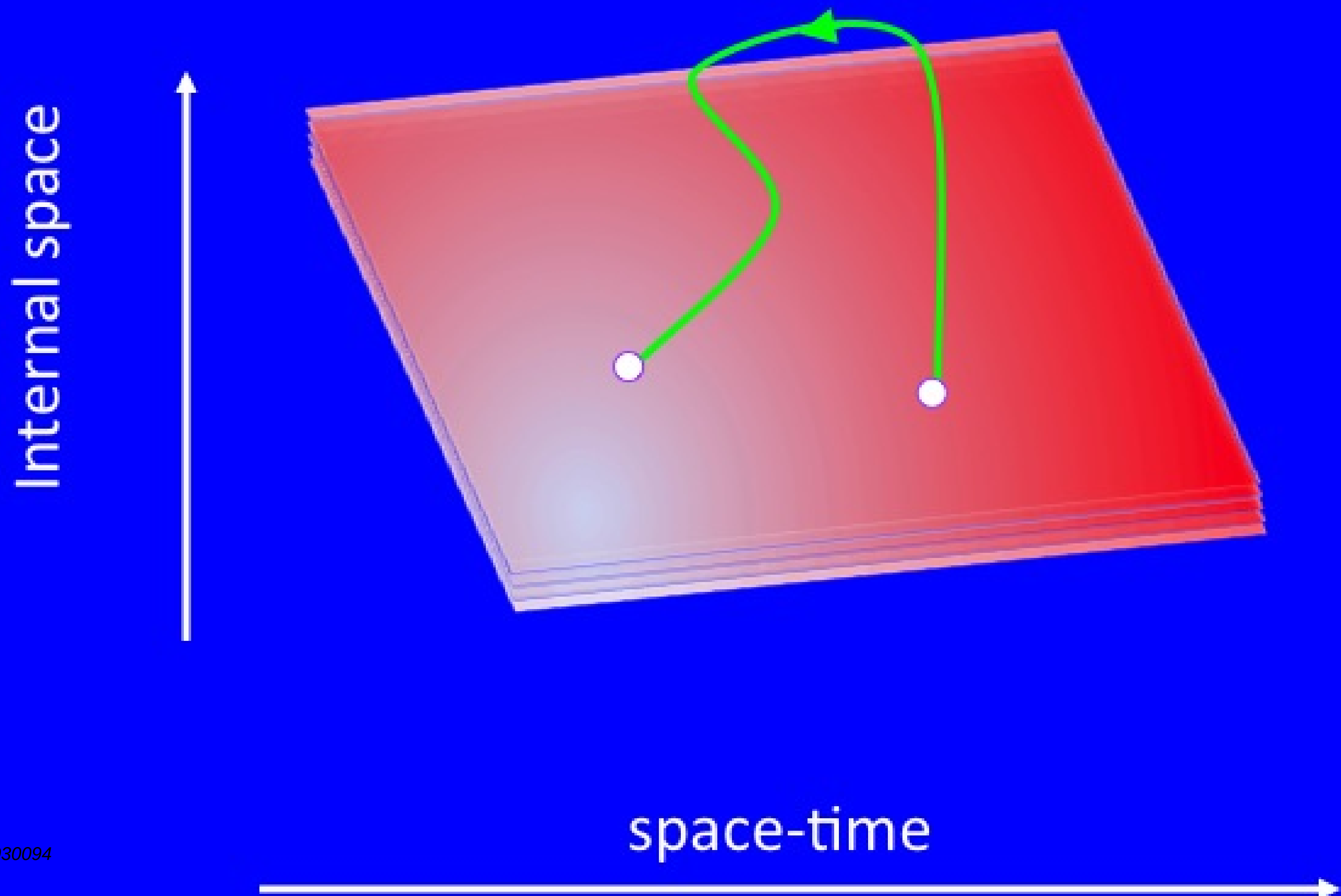
Gluons



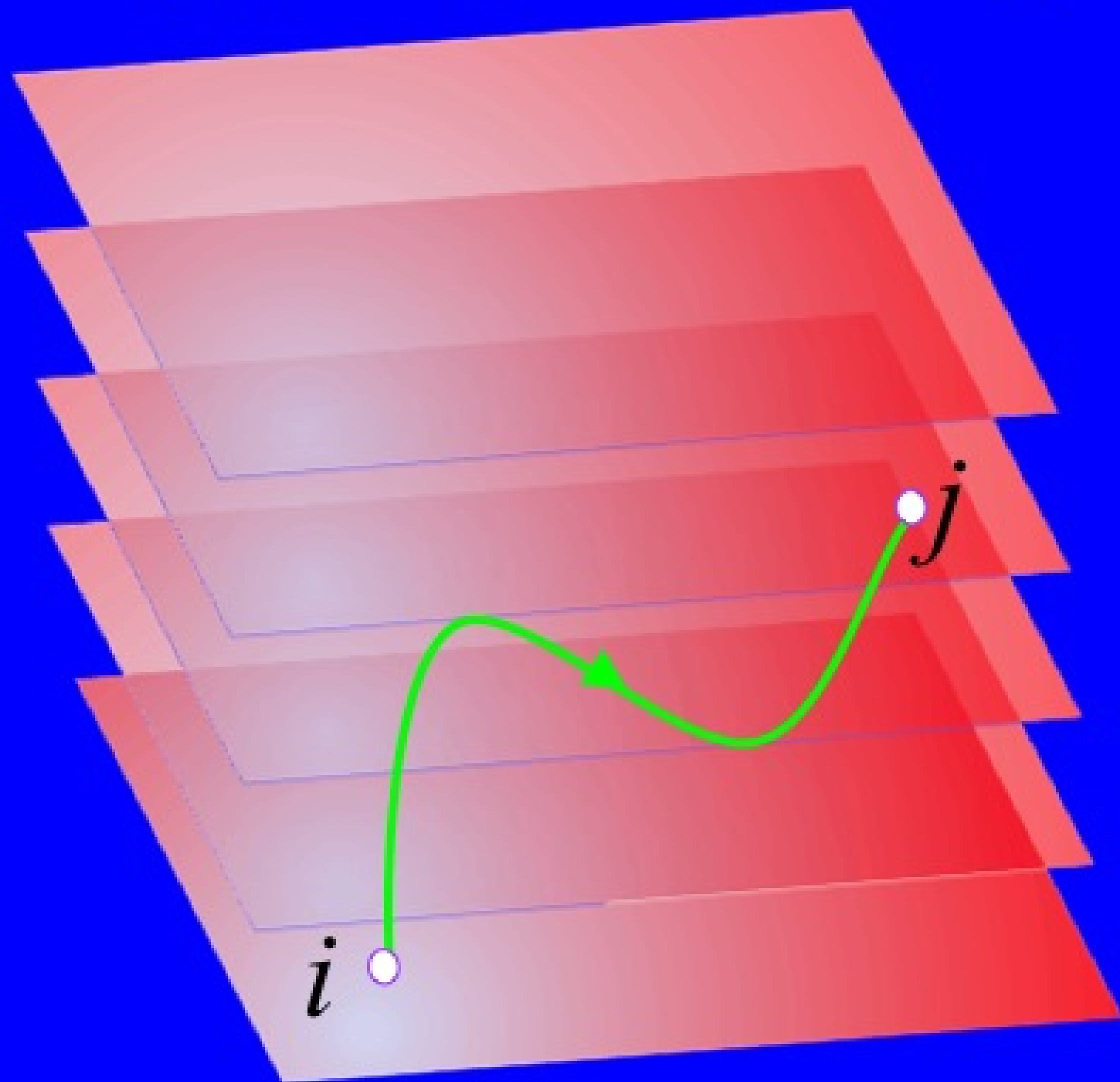
$N \times N$ matrix

D-branes

multiplicity N

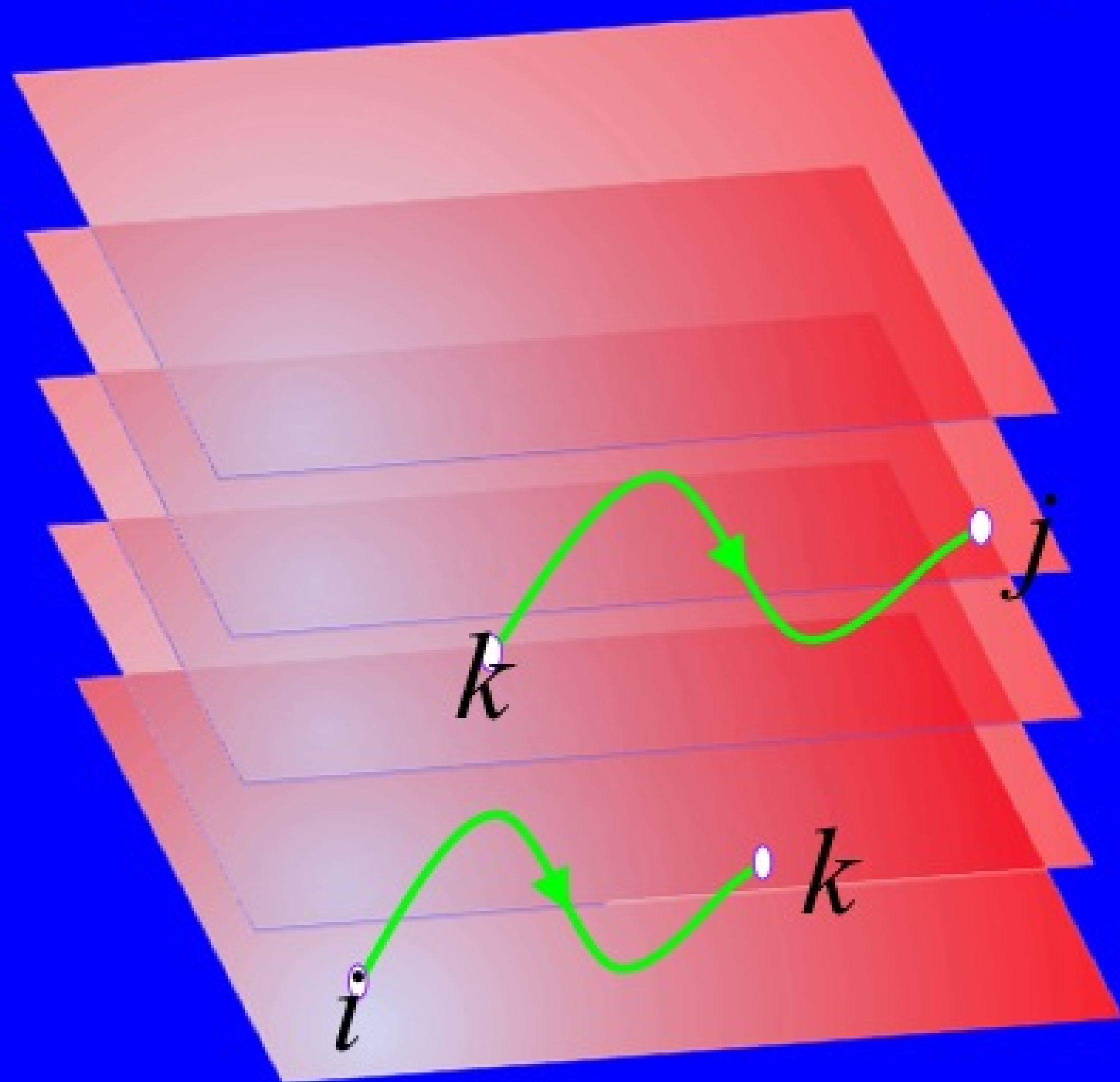


U(N) Yang-Mills Theory

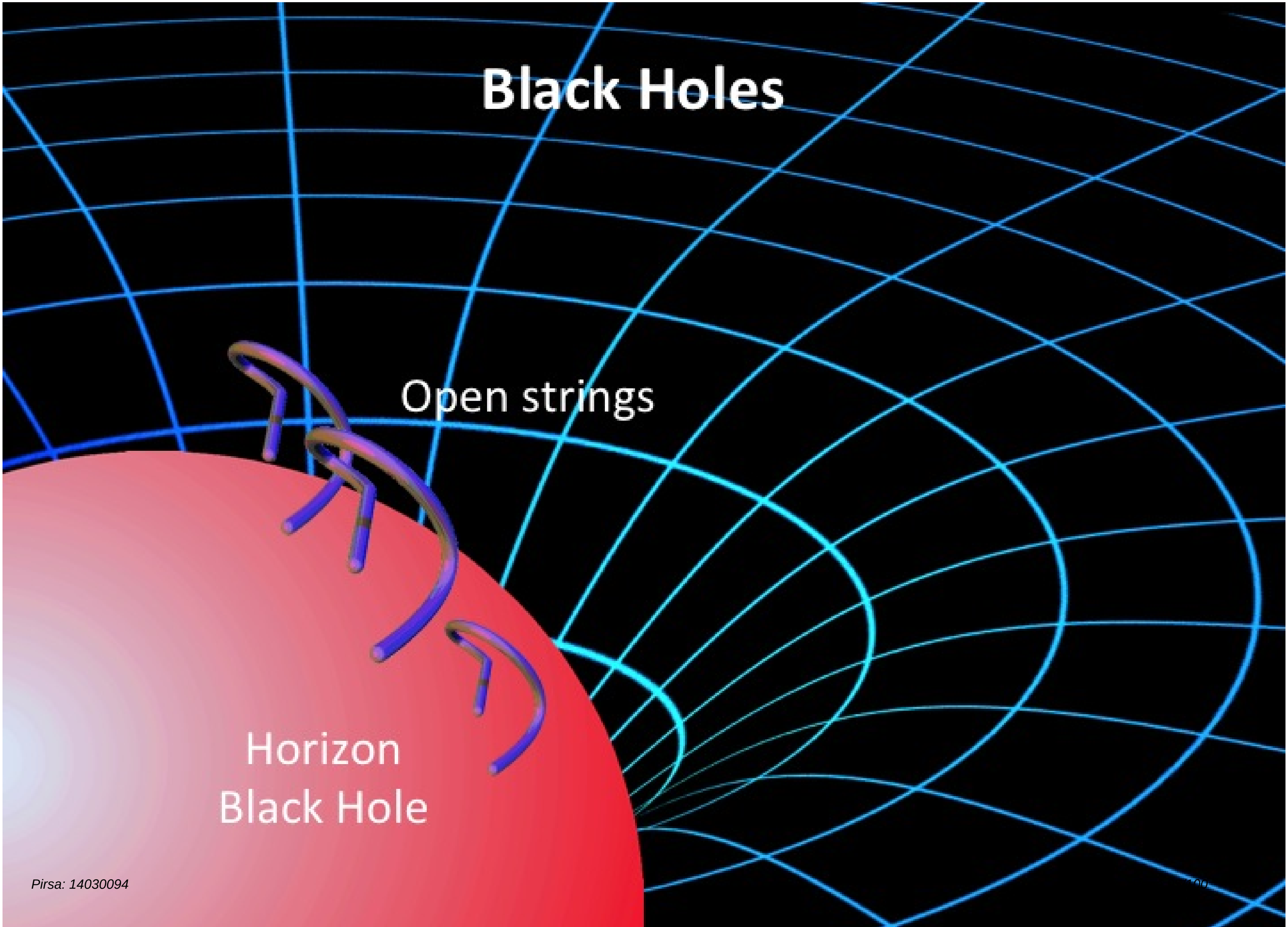


$N \times N$ *matrix of open strings* A_{ij}

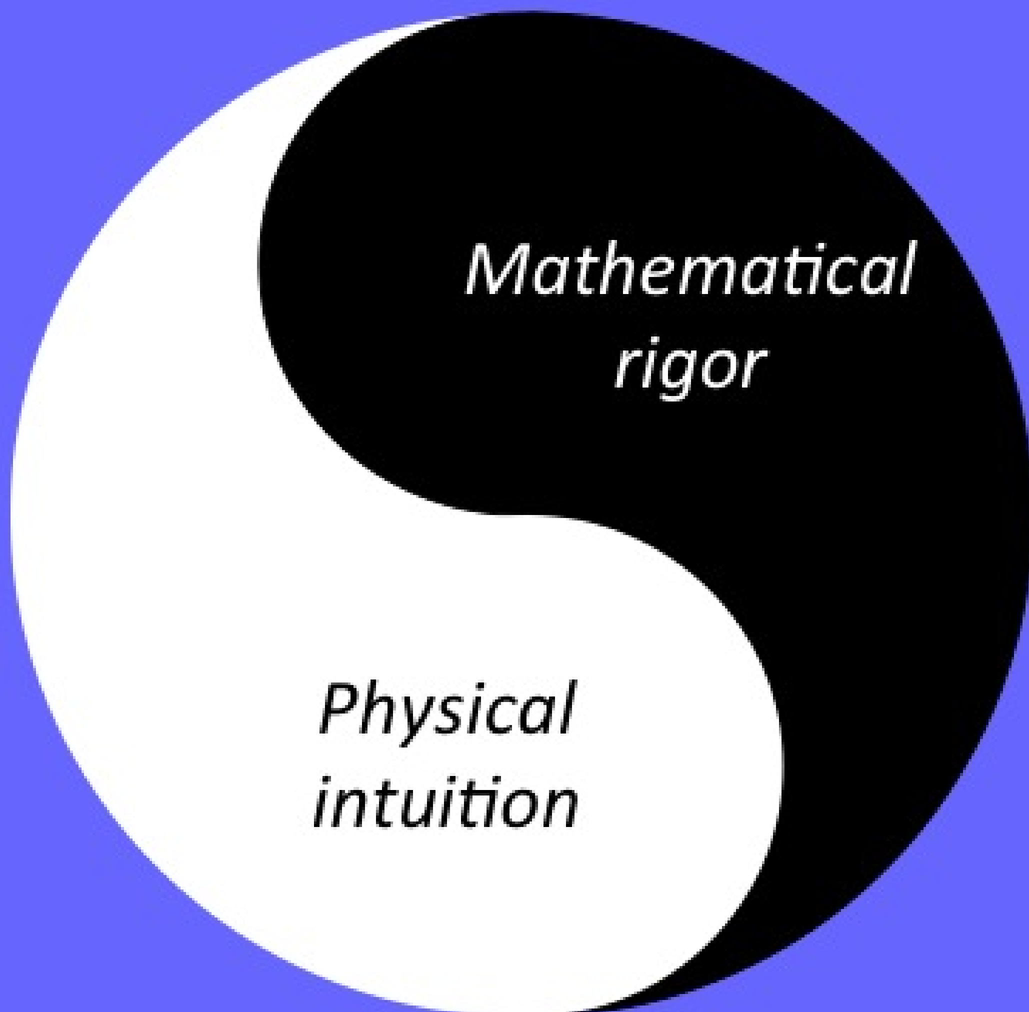
U(N) Yang-Mills Theory



matrix multiplication $\Sigma_k A_{ik} A_{kj}$


Black Holes
Open strings
Horizon
Black Hole

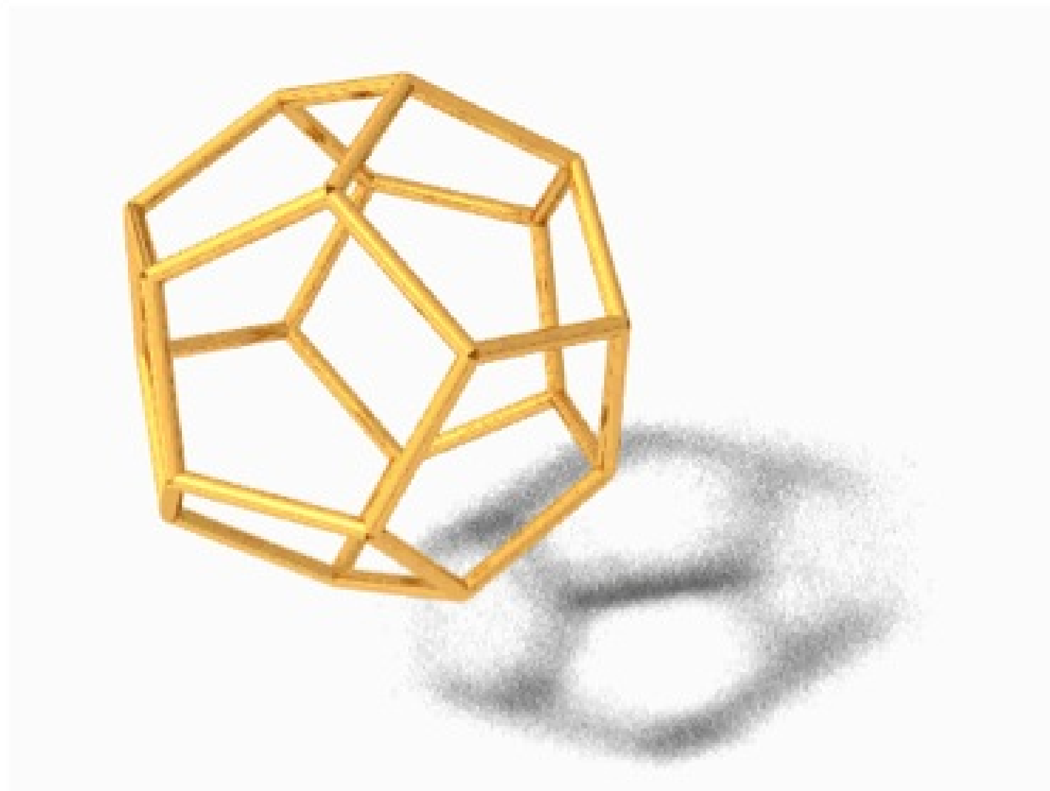
Mathematics/Physics Complementarity



The Temptations of Mathematics

Plato's Cave

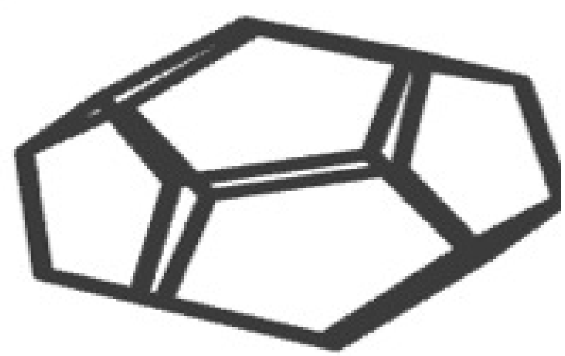
Mathematical Dream



Physical Reality

Quantum Cave

Physical Dream



Mathematical Reality

[redacted]