Title: Far from equilibrium energy flow in quantum critical systems

Date: Mar 25, 2014 02:00 PM

URL: http://pirsa.org/14030089

Abstract: We investigate far from equilibrium energy transport in strongly coupled quantum critical systems. Combining results from gauge-gravity duality, relativistic hydrodynamics, and quantum field theory, we argue that long-time energy transport after a local thermal quench occurs via a universal steady-state for any spatial dimensionality. This is described by a boosted thermal state. We determine the transport properties of this emergent steady state, including the average energy flow and its long-time fluctuations.

Far-from-equilibrium dynamics in CFTs and holography

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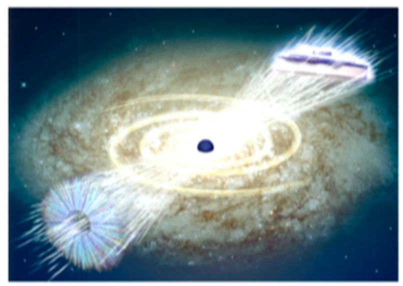


M.J. Bhaseen, B. Doyon, A. Lucas

Netherlands Organisation for Scientific Research

Perimeter Institute
Mar 2014

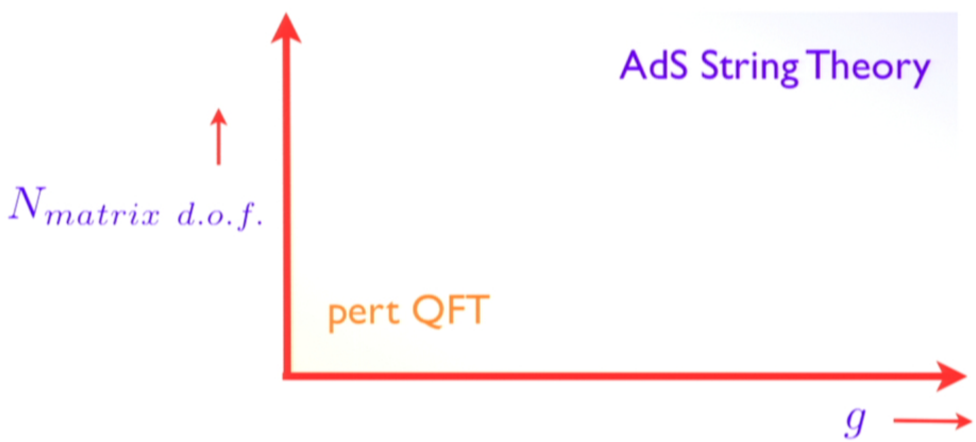
FOM

AdS/CFT

- The AdS/CFT correspondence

Witten;
Gubser, Klebanov, Polyakov

$$Z_{CFT}(J;g,N) = \exp i S_{AdS}^{\text{on-shell}}(\phi(\phi_{\partial AdS} = J))$$

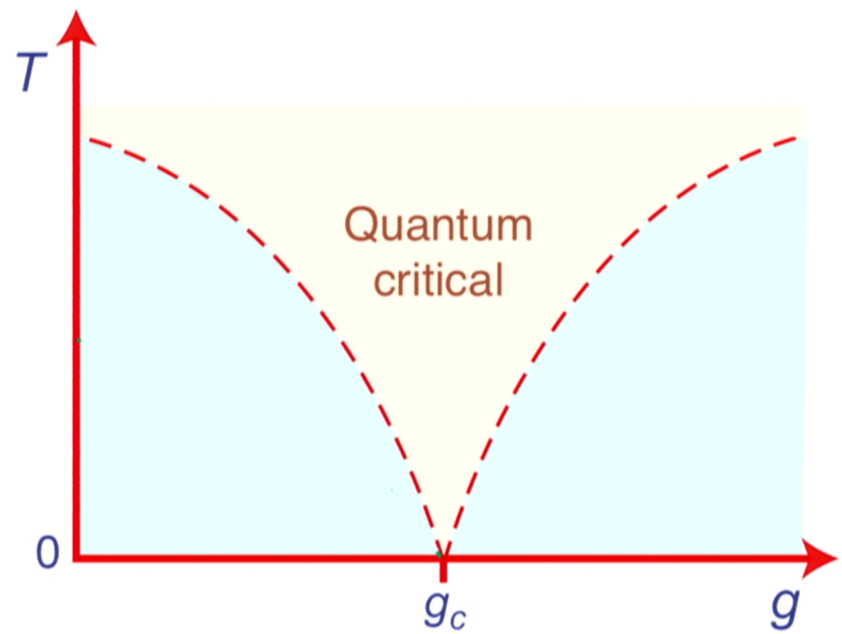


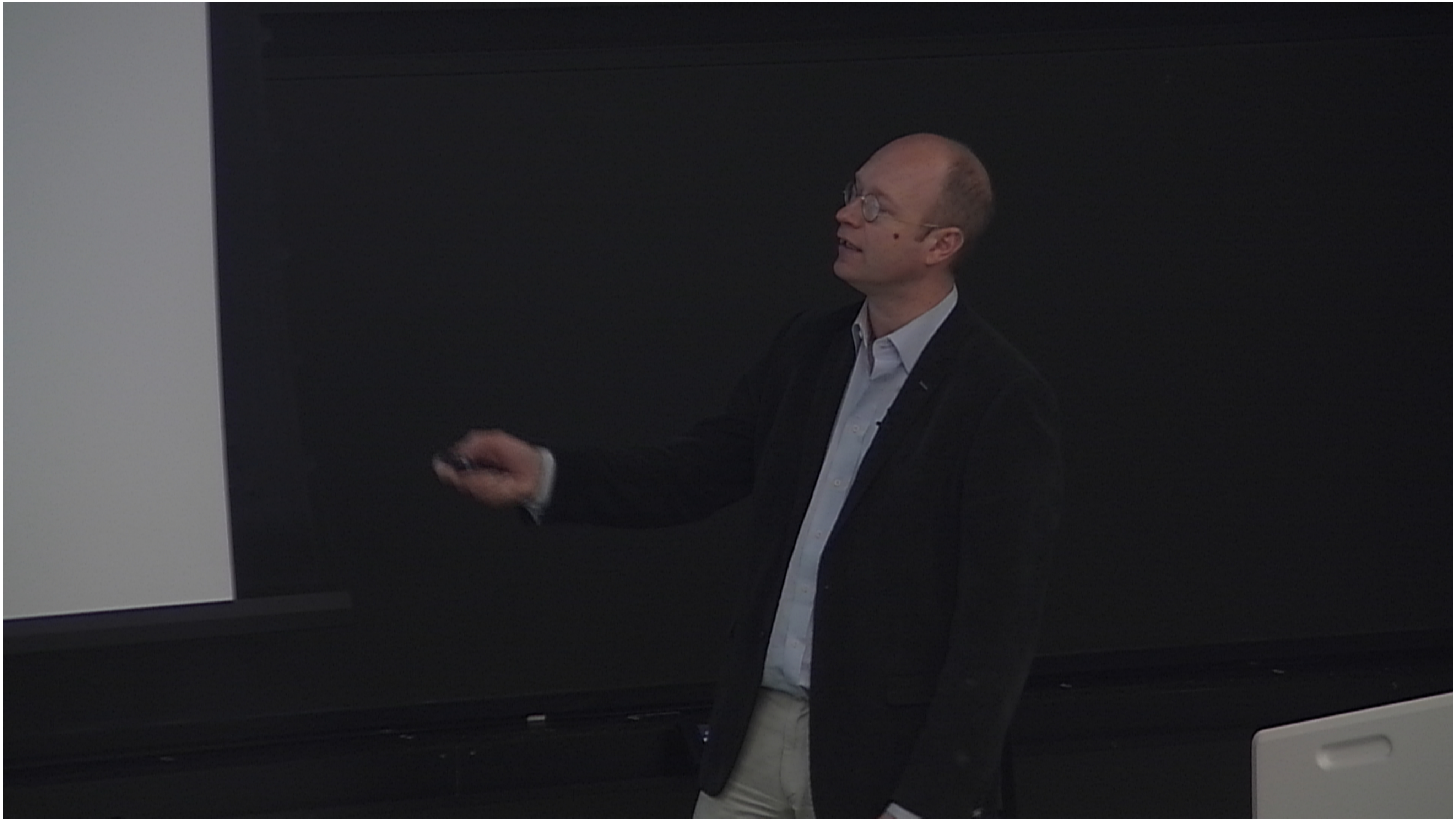
Use AdS/CFT as a tool to generate strongly coupled critical theories

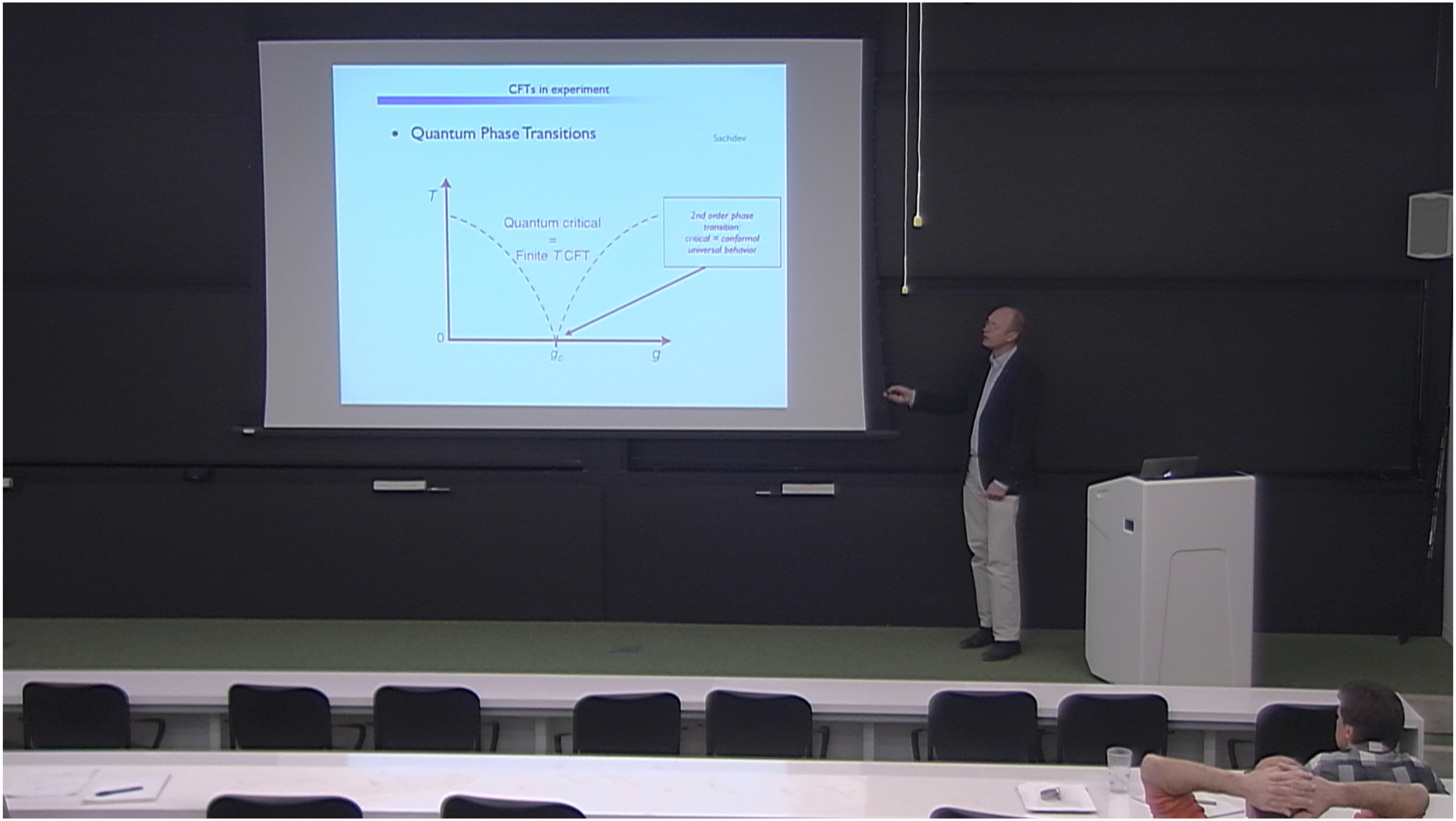
CFTs in experiment

- Quantum Phase Transitions

Sachdev



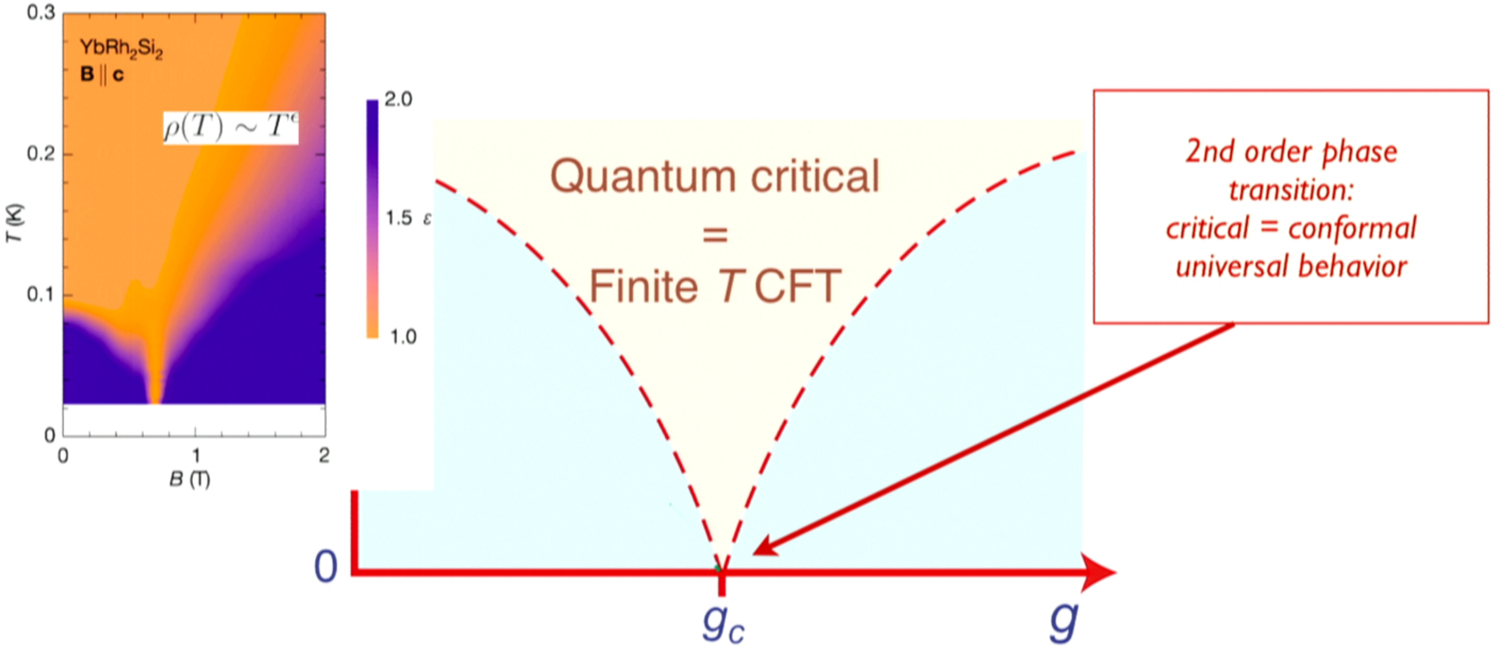



CFTs in experiment
• Quantum Phase Transitions
Sachdev
T
Quantum critical
=
Finite T CFT
2nd order phase transition: critical = conformal universal behavior
0
g_c
g

CFTs in experiment

- Quantum Phase Transitions

Sachdev



Custers et al, *Nature* **424**, 524 (2003)

AdS/CFT Dictionary

$$Z_{CFT}(J; g, N) = \exp iS^{\text{on-shell}}_{AdS}(\phi(\phi_{\partial AdS} = J))$$

CFT	AdS
$\mathcal{O}_\phi$	ϕ
J^μ_{global}	A^μ_{gauge}
Δ_ϕ	m_ϕ
$\ldots$	
finite$T$	BH
$\ldots$	

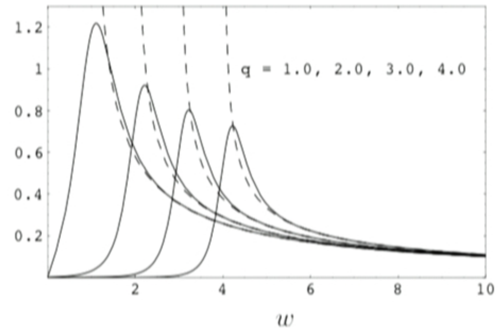
Witten; Gubser, Klebanov, Polyakov

Real time responses

- A remarkable ability of AdS/CFT
  - Direct crossover to hydrodynamics

Herzog, Kovtun, Sachdev, Son

$$\frac{1}{q^2}\mathrm{Im}\langle J^0 J^0 \rangle_R$$



$$q = 3k/4\pi T$$
$$w = 3\omega/4\pi T$$

Far-from-equilibrium dynamics

- Driven Steady State?
  - Non-thermal distributions

- Universality in Non-equilibrium dynamics?
  - Kibble-Zurek scaling
  - Kolmogorov scaling

Chesler, Yaffe; de Boer, Kesko-Vakkuri +9; Bhaseen, Gauntlett, Simons, Sonner, Wiseman; Basu, Das, Nishioka Takanayagi; Albash, Johnson; Abajo-Arrastia, Aparicio, Lopez; Ebrahim, Headrick; Bhattacharyya, Minwalla;.... Buchel, Lehner, Myers, van Niekerk; Das, Galante, Myers, ...

Motivation: unique ability of holography

Actual: Combination of holography, hydrodynamics and QFT

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- Thermal Quench in 1+1 CFTs

Bernard, Doyon

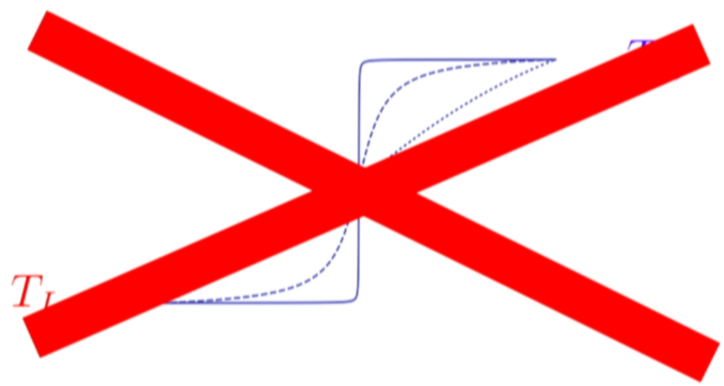
- Thermal Quench in 1+1 CFTs

Bernard, Doyon

T_L	T_R

$$t = 0$$

- Intuitive expectation



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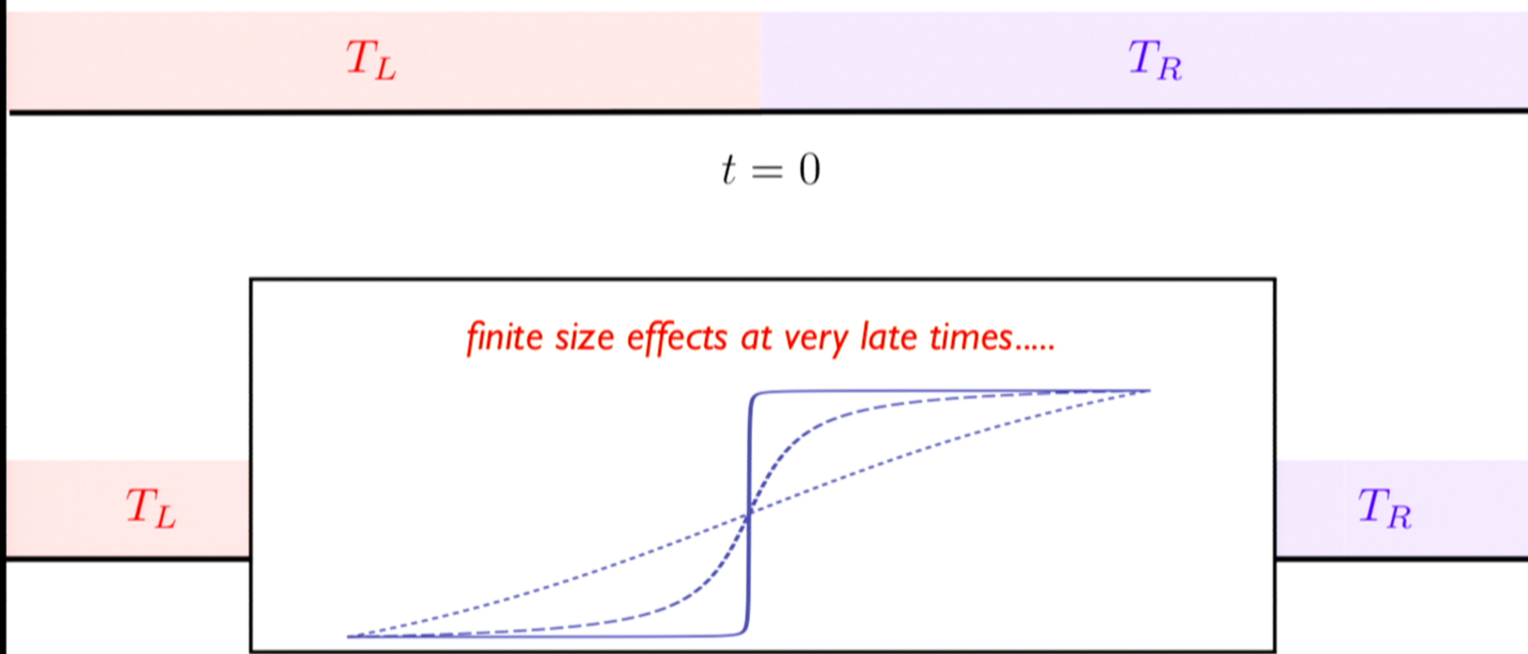

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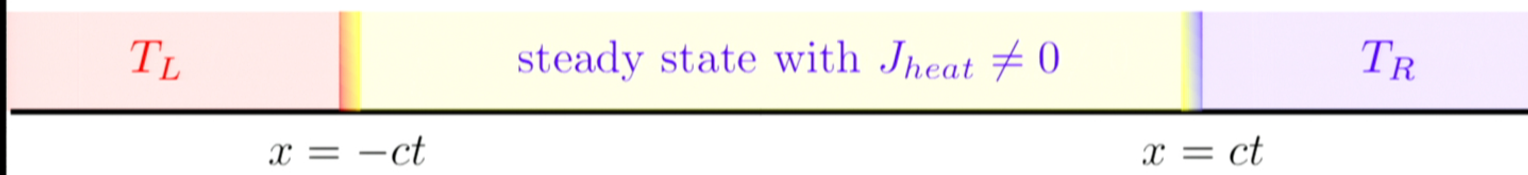

$$\langle J \rangle = \frac{c\pi}{12}(T_L^2 - T_R^2)$$

- Call $\langle J \rangle = J(\beta_L, \beta_R)$ with $\beta_L = 1/T_L, \;\; \beta_R = 1/T_R$

$$\langle J^{n+1} \rangle = \frac{d^n}{d\mu^n} \, J(\beta_L - \mu, \beta_R + \mu)\Big|_{\mu=0}$$

- Thermal Quench in 1+1 CFTs

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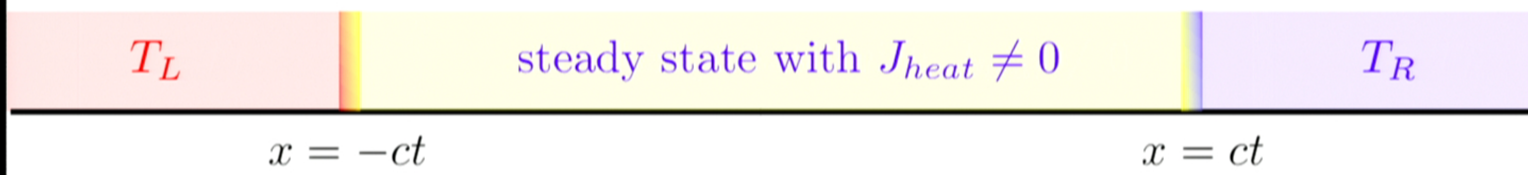

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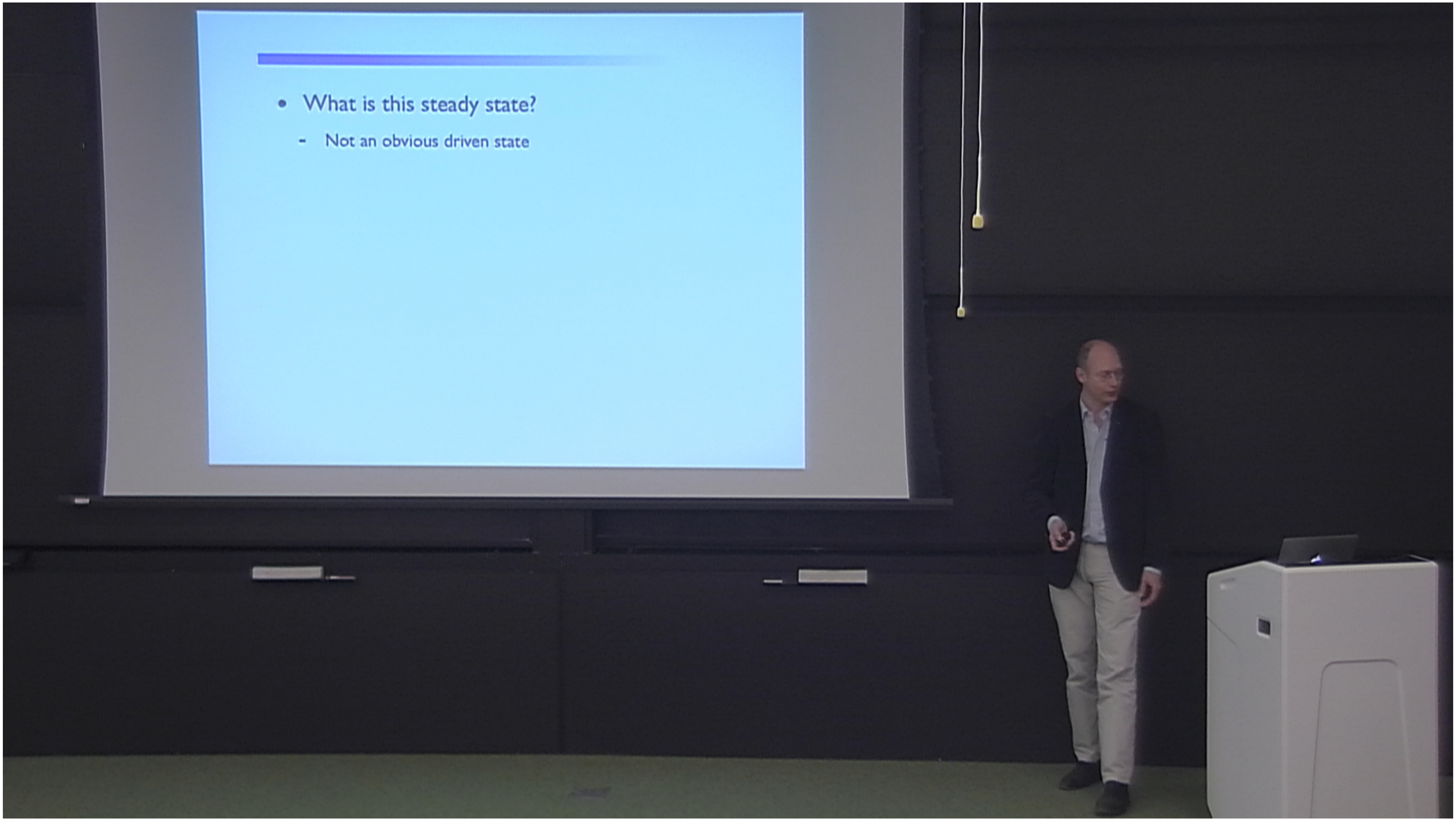
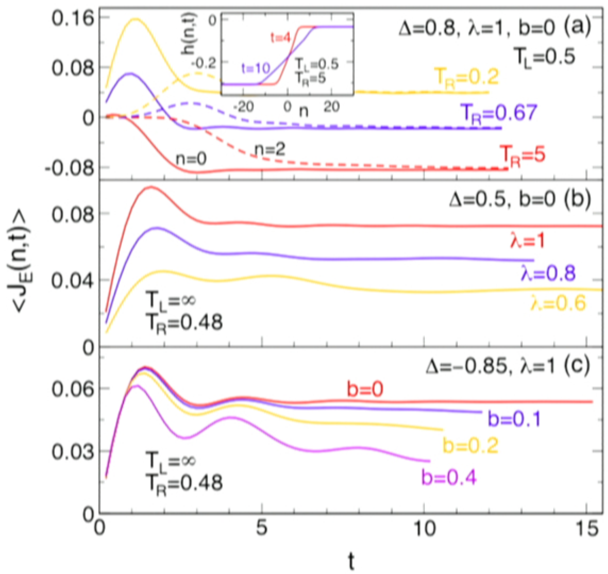
- What is this steady state?
  - Not an obvious driven state



What is this steady state?
Not an obvious driven state

Karrasch, Ilan, Moore

- Time dependent DMRG (density matrix renormalization group)
  - XXZ Hamiltonian



$$h_n = J_n\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta_n S_n^z S_{n+1}^z\right) + b_n\left(S_n^z - S_{n+1}^z\right)$$

$$J_n = \begin{cases} 1 & n \text{ odd} \\ \lambda & n \text{ even} \end{cases}, \quad \Delta_n = \Delta\ , \quad b_n = \frac{(-1)^n b}{2}$$

Karrasch, Ilan, Moore

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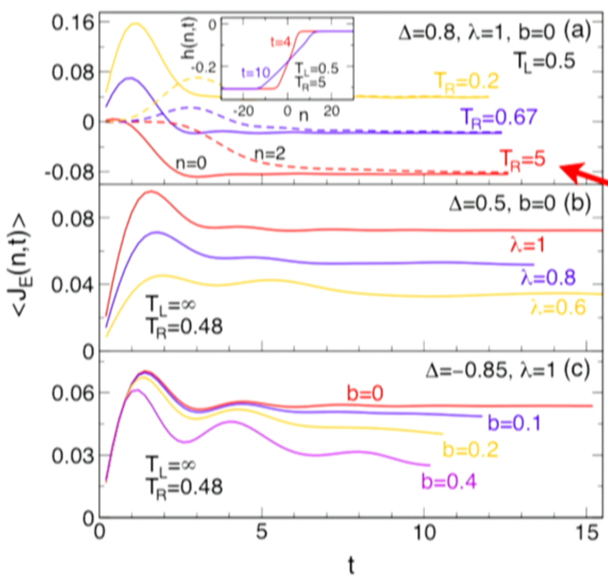
  - XXZ Hamiltonian



$$h_n = J_n\left(S^x_n S^x_{n+1} + S^y_n S^y_{n+1} + \Delta_n S^z_n S^z_{n+1}\right) + b_n\left(S^z_n - S^z_{n+1}\right)$$

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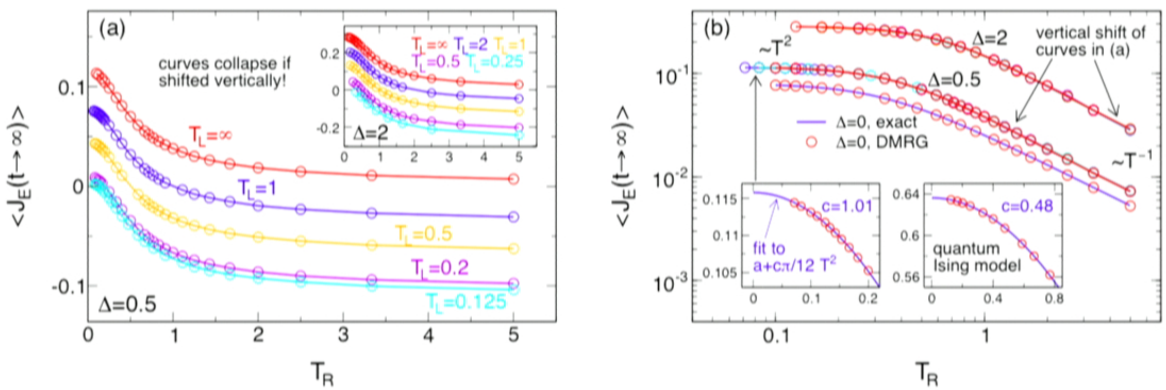
homogeneous constant heat flow

Karrasch, Ilan, Moore

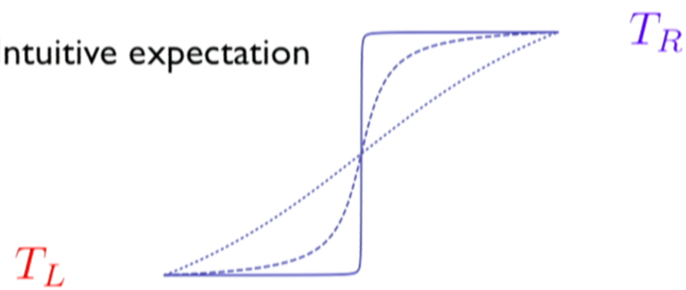
- Time dependent DMRG
  - XXZ Hamiltonian



$$\langle J \rangle \sim f_L(T_L) + f(T_R)$$

- How to understand this state?
  - Constant Heat flow vs Temperature relaxation

Intuitive expectation



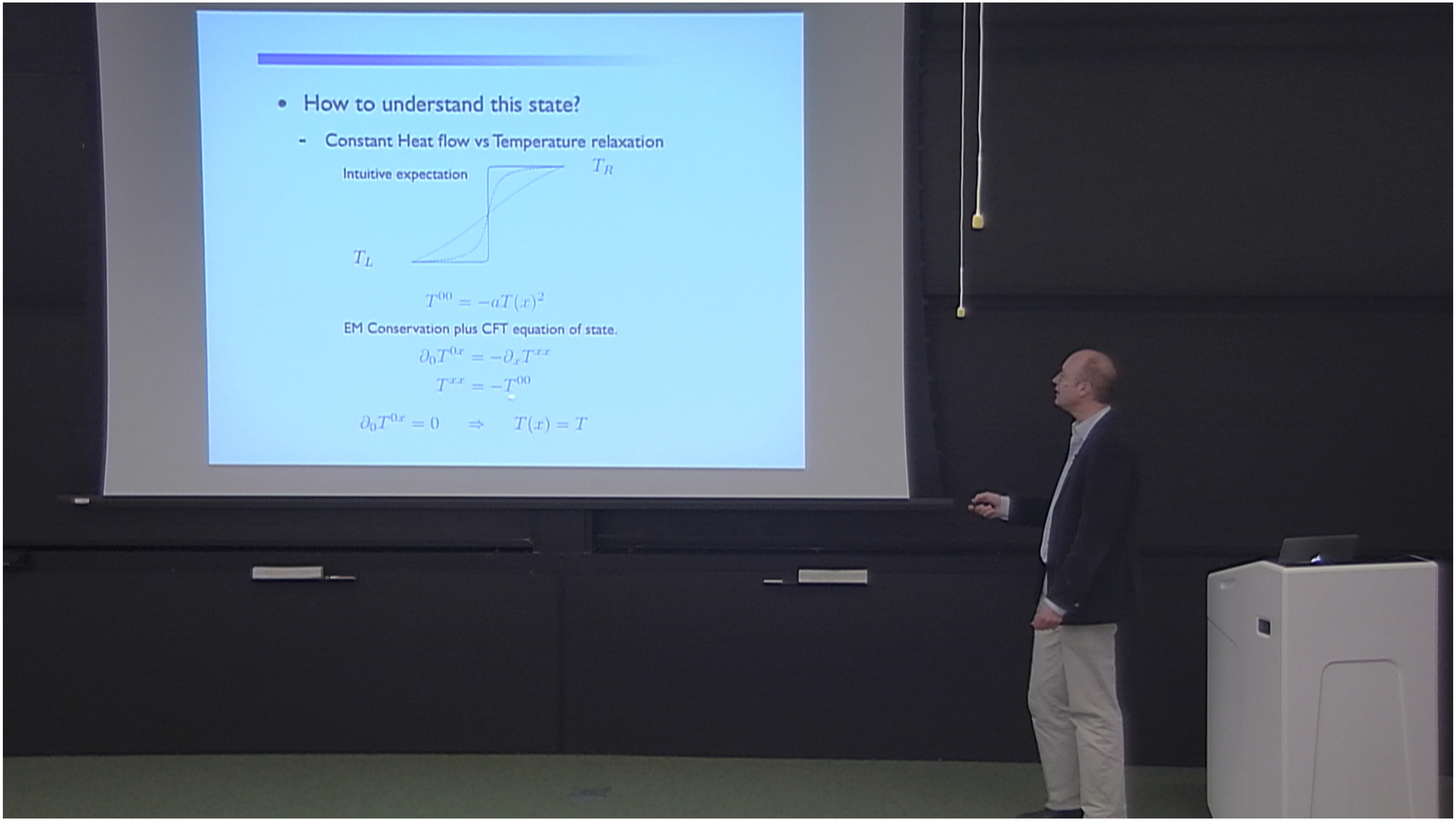
$$T^{00} = -aT(x)^2$$

EM Conservation plus CFT equation of state.

$$\partial_0 T^{0x} = -\partial_x T^{xx}$$

$$T^{xx} = -T^{00}$$

$$\partial_0 T^{0x} = 0 \quad \Rightarrow \quad T(x) = T$$



How to understand this state?
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Intuitive expectation
T_R
T_L
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- How to understand this state?
  - holomorphic factorization (integrability)
  - In a 1+1 dim CFT left and right movers do not interact

Left of the interface

$$J_{p>0} \sim T_L^2 \ , \quad J_{p<0} \sim T_L^2$$

Right of the interface

$$J_{p>0} \sim T_R^2 \ , \quad J_{p<0} \sim T_R^2$$

At the interface $t = 0$

$$J_{p>0} \sim T_L^2 \ , \quad J_{p<0} \sim T_R^2$$

very special to 1+1 D

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very special to 1+1 D

- Holography
  - Only Heat, i.e. pure AdS-gravity

$$S = \int \sqrt{-g}(R - 2\Lambda)$$

  - aAdS Solution to Einstein with a constant unsourced Heat current

$$ds^2 = \frac{L^2}{r^2} dr^2 + g_{ij}^{(0)}(r) dx^i dx^j$$

$$g_{ij}^{(0)} = \frac{r^2}{L^2} + \ldots + \frac{1}{r^d} \langle T_{ij} \rangle + \ldots$$

$$\langle T_{ij} \rangle = \begin{pmatrix} -\rho & J \\ J & \rho \end{pmatrix}$$

- Holography
  - Unique solution: boosted BTZ black hole

$$ds^2 = -\frac{r^2}{L^2}(1 - \frac{M^2\cosh^2(\eta)}{r^2})dt^2 + \frac{r^2 + M^2\sinh^2(\eta)}{L^2}dx^2 + \frac{M^2}{L^2}\sinh(2\eta)dxdt + \frac{L^2}{r^2}\frac{dr^2}{(1-\frac{M^2}{r^2})}$$

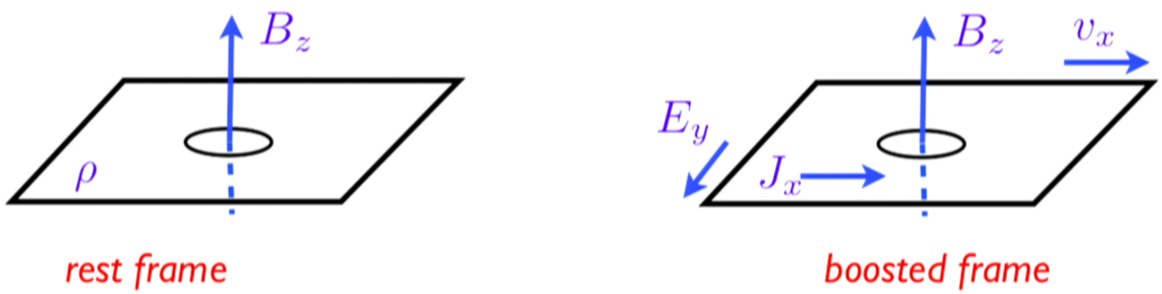
  - This is dual to a state with constant heat current

$$\langle T^{0x}\rangle = \frac{c\pi}{6}T_{BH}\sinh(2\eta)$$

...,
...,
eg Figueras, Wiseman
Fischetti, Marolf

- "Boosts" to understand real transport
  - Classical Hall effect

Bhaseen, Green, Sondhi; Hartnoll, Kovtun



$E = 0$

$J = 0$

$E = -v \times B$

$J = \rho v$

$$J_i = \sigma_{ij} E_j \quad \Rightarrow \quad \sigma_{xy} = \frac{\rho}{B_z}$$

- Holography
  - Unique solution: boosted BTZ black hole

$$ds^2 = -\frac{r^2}{L^2}(1 - \frac{M^2\cosh^2(\eta)}{r^2})dt^2 + \frac{r^2 + M^2\sinh^2(\eta)}{L^2}dx^2 + \frac{M^2}{L^2}\sinh(2\eta)dxdt + \frac{L^2}{r^2}\frac{dr^2}{(1-\frac{M^2}{r^2})}$$

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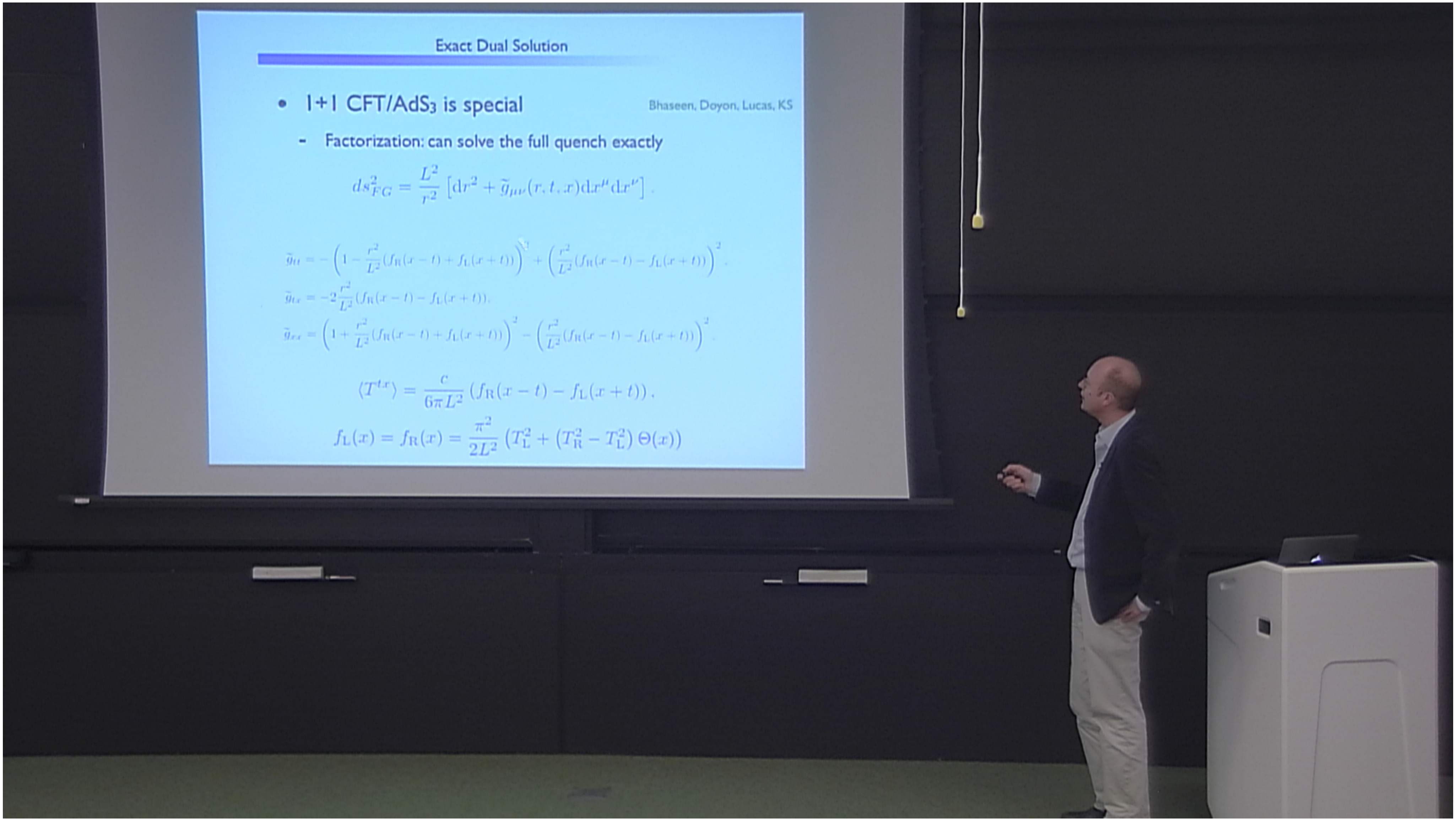
The novel steady state coincides with the boosted equilibrium state identifying

$$T_L = Te^{\eta},$$
$$T_R = Te^{-\eta}$$

Bhaseen, Doyon, Lucas, KS

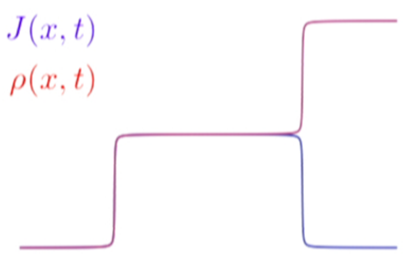

Exact Dual Solution
1+1 CFT/AdS3 is special
Bhaseen, Doyon, Lucas, KS
Factorization: can solve the full quench exactly
$ds^2_{FG} = \frac{L^2}{r^2}\left[\mathrm{d}r^2 + \tilde{g}_{\mu\nu}(r,t,x)\mathrm{d}x^\mu \mathrm{d}x^\nu\right]$.
$\tilde{g}_{tt} = -\left(1 - \frac{r^2}{L^2}(f_R(x-t) + f_L(x+t))\right)^2 + \left(\frac{r^2}{L^2}(f_R(x-t) - f_L(x+t))\right)^2$,
$\tilde{g}_{tx} = -2\frac{r^2}{L^2}(f_R(x-t) - f_L(x+t))$,
$\tilde{g}_{xx} = \left(1 + \frac{r^2}{L^2}(f_R(x-t) + f_L(x+t))\right)^2 - \left(\frac{r^2}{L^2}(f_R(x-t) - f_L(x+t))\right)^2$.
$\langle T^{tx}\rangle = \frac{c}{6\pi L^2}\left(f_R(x-t) - f_L(x+t)\right)$.
$f_L(x) = f_R(x) = \frac{\pi^2}{2L^2}\left(T_L^2 + (T_R^2 - T_L^2)\,\Theta(x)\right)$

- Boosted equilibrium suggests hydro applies
  - d+1 Conformal Hydro for a thermal quench

    Effective dimensional reduction to 1+1 dimension

    d+1 Conformal $\neq$ Integrable = dissipation

$$J(x,t) = \theta(t+x) + \theta(t-x) - 1$$



sound fronts

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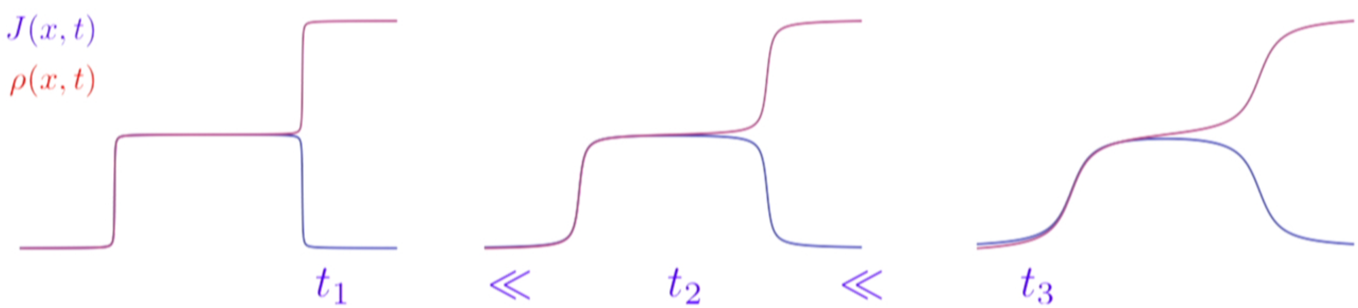
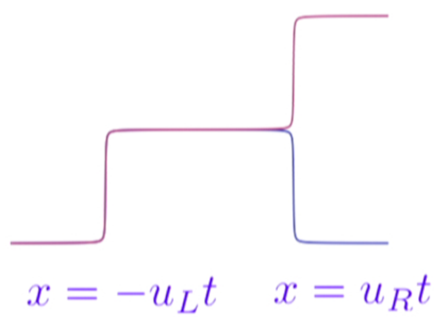


- Matching across shocks

$$T_{\mu\nu} = T_{\mu\nu}(x + u_L t) + T_{\mu\nu}(x - u_R t)$$

$$\int_{shock} \partial_\mu T^{\mu\nu} = \int_{shock} \partial_x T^{x\mu} + u_{shock} \partial_x T^{0\mu} = 0$$

4 equations for 4 unknowns



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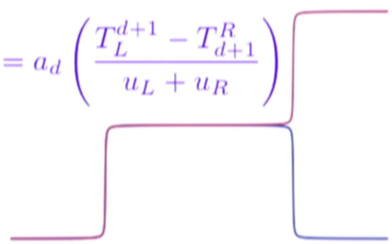
4 equations for 4 unknowns

$$T_{ss} = \sqrt{T_L T_R} \qquad u_L = \frac{1}{d}\sqrt{\frac{\chi + d}{\chi + d^{-1}}} \qquad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}} \qquad \eta_{ss} = \frac{\chi - 1}{\sqrt{(\chi + d^{-1})(\chi + d)}}$$

$$\langle T^{tx} \rangle = a_d \left(\frac{T_L^{d+1} - T_{d+1}^R}{u_L + u_R} \right)$$

$$\chi = \left(\frac{T_L}{T_R} \right)^{\frac{d+1}{2}}$$



$$x = -u_L t \qquad x = u_R t$$

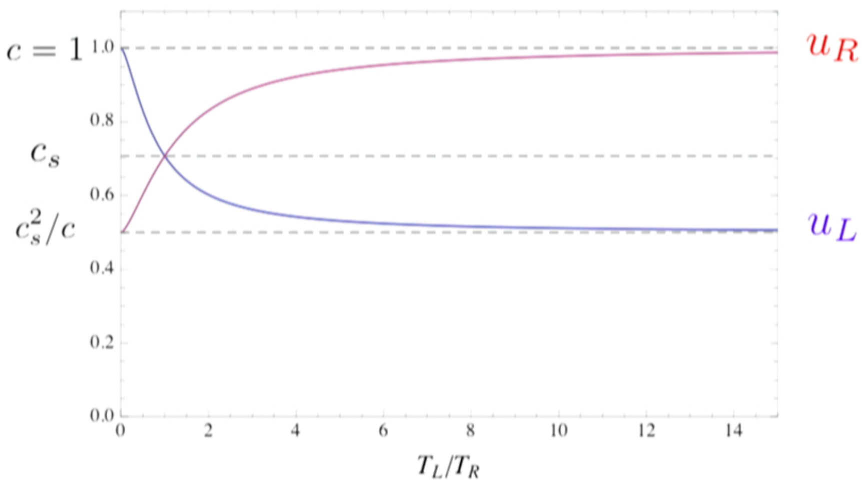
- Shocks are non-linear sound waves

$$u_L = \frac{1}{d}\sqrt{\frac{\chi + d}{\chi + d^{-1}}} \qquad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}}$$

asymmetric

$$\chi = \left(\frac{T_L}{T_R}\right)^{\frac{d+1}{2}}$$

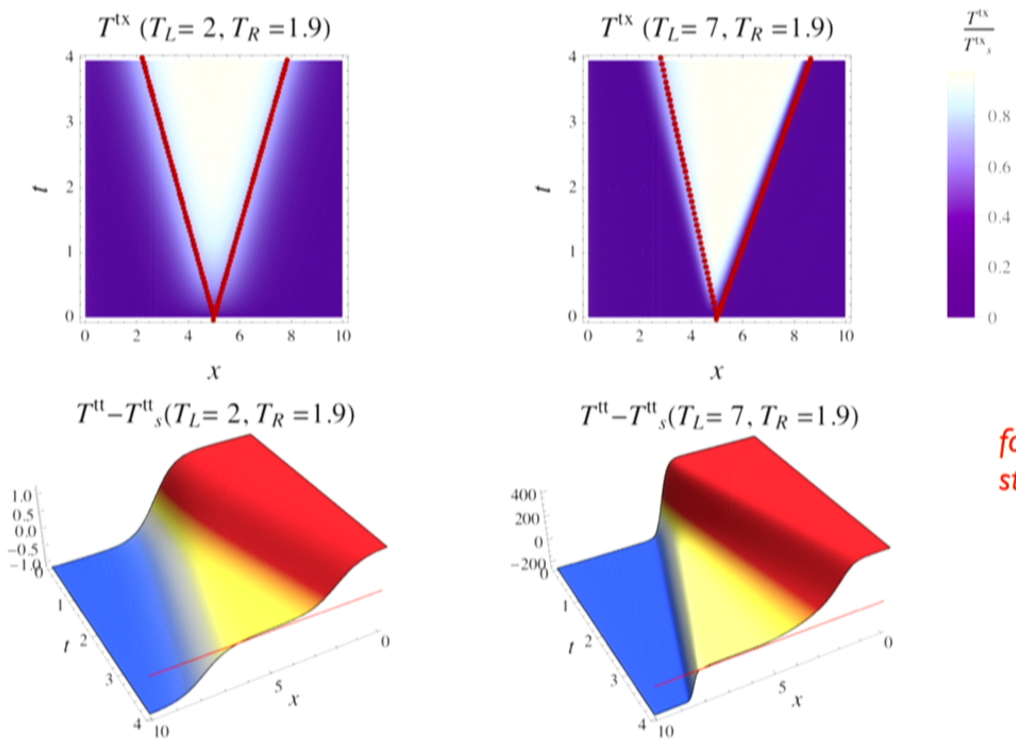
$$u_L u_R = c_s^2 = \frac{1}{d}$$



- Confirming with numerical (ideal) hydro

$$u_L = \frac{1}{d}\sqrt{\frac{\chi + d}{\chi + d^{-1}}} \qquad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}}$$

asymmetric



formation of steady state

- Dissipative corrections should not change this
  - Small shocks can be traced in linear response

$$T(x,t) = T_L + \frac{T_R - T_L}{4}\left[2 + \mathrm{erf}\frac{x - t/\sqrt{d}}{4D_{\|}t} + \mathrm{erf}\frac{x + t/\sqrt{d}}{4D_{\|}t}\right]$$

width $\sqrt{D_{\|}t}$ *is smaller than the distance* $t/\sqrt{d}$

Numerically confirmed by Chang, Karch, Yarom

- Turbulence?
  - Assumption: completely smooth *T* discontinuity
  - Allows reduction to eff 1+1 dim system

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  - Small shocks can be traced in linear response

$$T(x,t) = T_L + \frac{T_R - T_L}{4}\left[2 + \mathrm{erf}\frac{x - t/\sqrt{d}}{4D_{\|}t} + \mathrm{erf}\frac{x + t/\sqrt{d}}{4D_{\|}t}\right]$$

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- Turbulence?
  - Assumption: completely smooth *T* discontinuity
  - Allows reduction to eff 1+1 dim system

- The Fluctuation Spectrum
  - So far we have looked at xpv $\langle T_{\mu\nu} \rangle$
  - Cumulants of the current at the interface

$$c_n \equiv \langle J^n(x=0) \rangle$$

Extended Fluctuation Relation

$$\langle J^{n+1} \rangle = \frac{d^n}{d\mu^n} J(\beta_L - \mu, \beta_R + \mu)\Big|_{\mu=0}$$

Non-Thermal distributions

- The Fluctuation Spectrum
  - So far we have looked at xpv $\langle T_{\mu\nu} \rangle$ (=hydro)
  - Cumulants of the current at the interface

$$c_n \equiv \langle J^n(x=0) \rangle$$

Extended Fluctuation Relation

$$F(z) = \sum \frac{1}{n!} z^n c_n$$

$$\frac{dF(z)}{dz} = J(\beta_L - z, \beta_R + z)$$

holds in any d !

- Outlook
  - Direct Momentum relaxation vs dissipation.
  - Including charge discontinuity in the quench.
  - Effects of turbulence.
  - Direct connection to cold atom experimental set-ups.