

Title: Asymptotic Symmetries of Gauge and Gravity Theories in Minkowski Space

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Abstract: We argue that the infinite-dimensional BMS symmetry discovered by Bondi et. al in the 60s provides an exact symmetry of the quantum gravity S-matrix. The Ward identity of this symmetry is shown to be precisely Weinberg's soft graviton theorem, also discovered in the 60s. A parallel infinite-dimensional symmetry is found to be generated in nonabelian gauge theories by gauge transformations which go to an angle-dependent finite constant at null infinity. The Ward identity of this symmetry is shown to be the soft gluon theorem.

Lysov, M. tra, He

1. BMS symmetry of S-matrix
- 2.

Lysov, Mitra, He

1. BMS symmetry of S-matrix
2. Soft gravitons = goldstone bosons
- 3.

Lysov, Mitra, He

1. BMS symmetry of S-matrix
2. Soft gravitons = goldstone bosons
3. Cons. law = energy at every angle

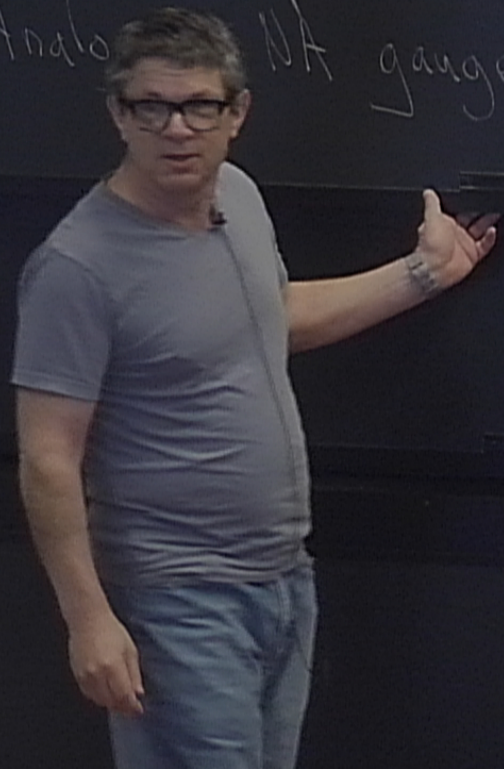
Lysov, Mitra, He

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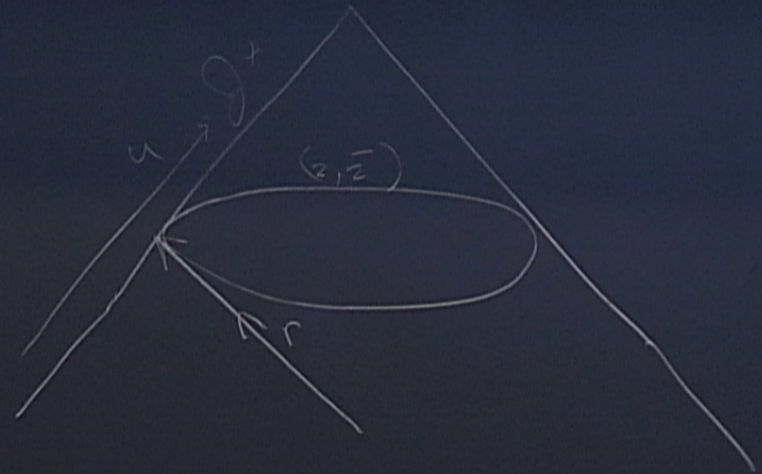
1. BMS symmetry of S-matrix
2. Soft gravitons = goldstone bosons
3. Cons. law = energy at every angle
4. Ward identity = soft graviton thm
5. Anal. NA gauge theories



... in the gauge theories

BMS⁺

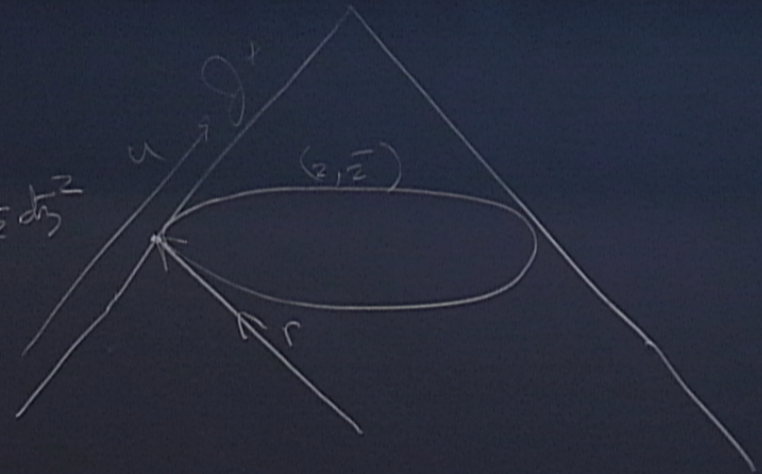
$$ds^2 = -du^2 - 2drdu + r^2 \gamma_{z\bar{z}} dz d\bar{z} \\ + \frac{2m_B(u, z, \bar{z})}{r} du^2 + r C_{z\bar{z}} dz^2$$



in the gauge theories

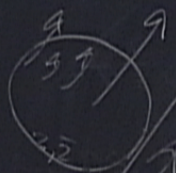
BMS⁺

$$ds^2 = -du^2 - 2drdu + r^2 \gamma_{z\bar{z}} dz d\bar{z} \\ + \frac{2m_B(u, z, \bar{z})}{r} du^2 + r(zz dz^2 + \bar{z}\bar{z} d\bar{z}^2) \\ + \dots$$



Supertranslations

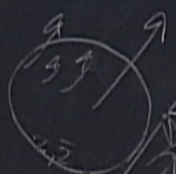
BMS symmetry of S-matrix



$$f(z, \bar{z}) \partial_u$$

Lesson 11: Supertranslations

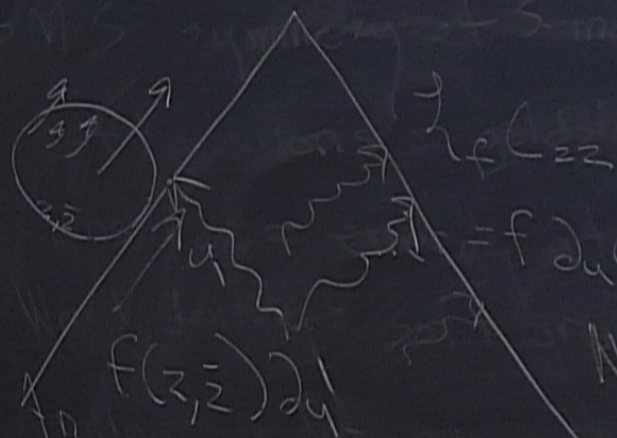
BMS symmetry of S-matrix



$f(z, \bar{z}) \partial_{\bar{z}}$

Supertranslations

BMS



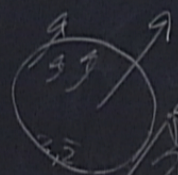
$$\delta g_{\mu\nu} = f \partial_\mu \partial_\nu \zeta - 2 D_\mu^2 f$$

$$ds^2 = -dv^2 + 2dvdr + 2r^2 \gamma_{z\bar{z}} dz d\bar{z}$$

Supertranslations

BMS

BMS symmetry of S-matrix



$$\mathcal{L}_f C_{zz}$$

$$= f \partial_u C_{zz} - 2 D_z^2 f$$

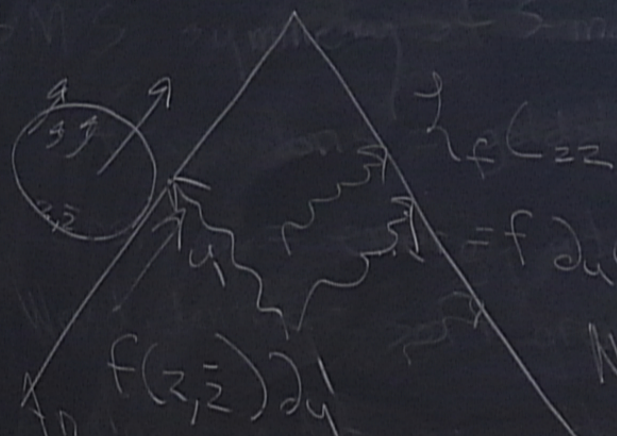
N_{zz}

$f(z, \bar{z})$

$$ds^2 = -dv^2 + 2dvdr + 2r^2 \gamma_{z\bar{z}} dzd\bar{z}$$

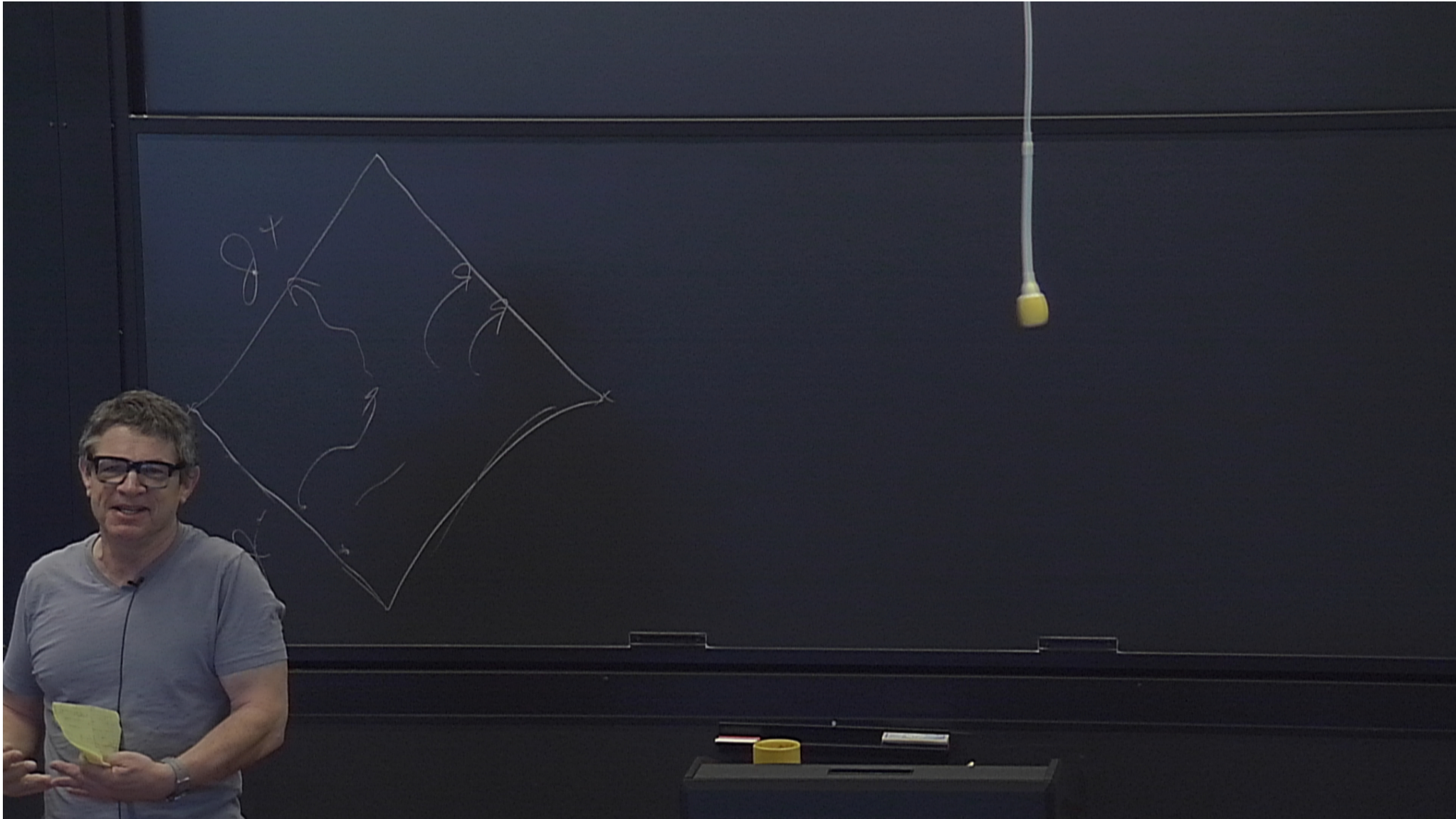
Supertranslations

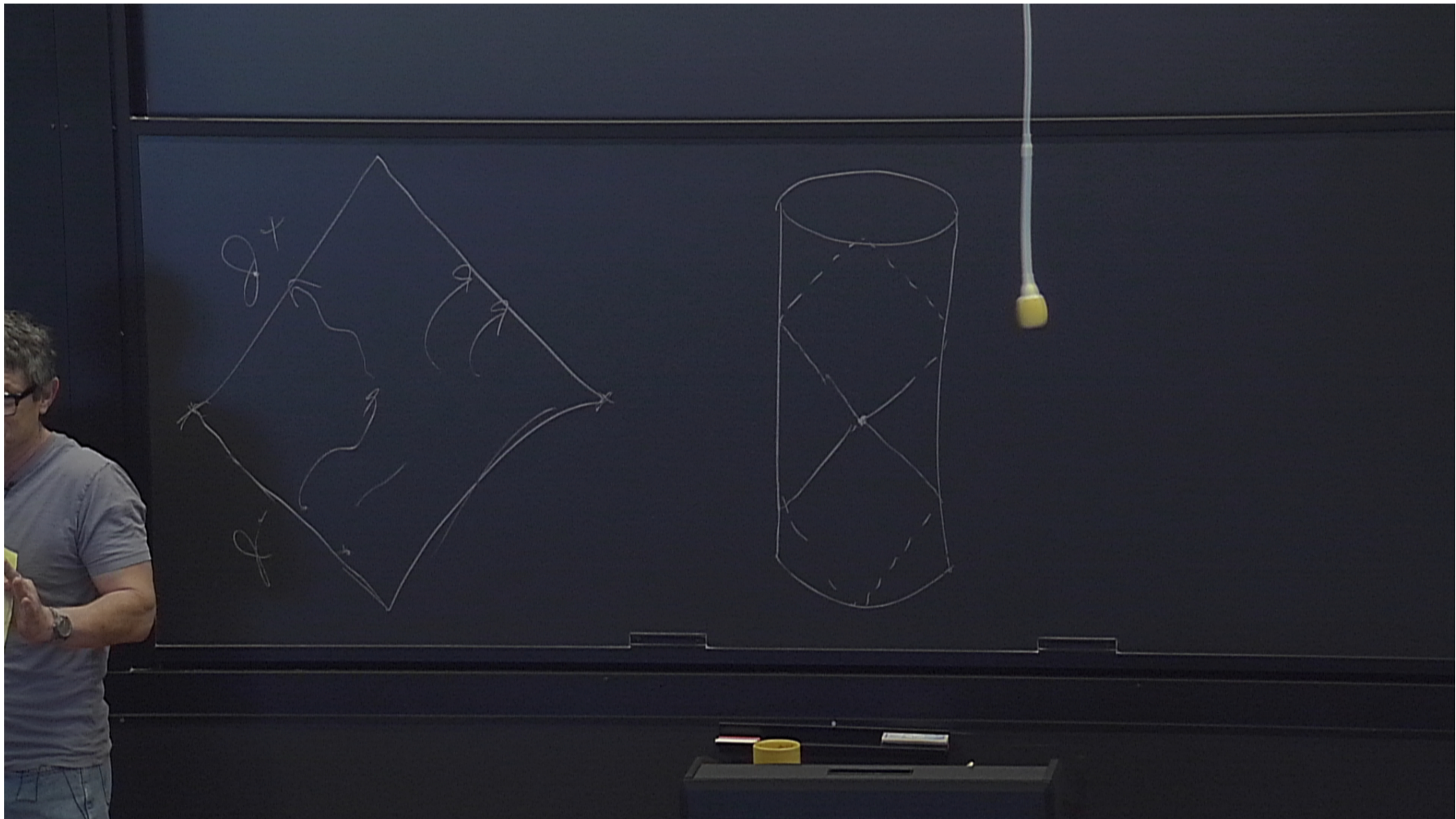
BMS

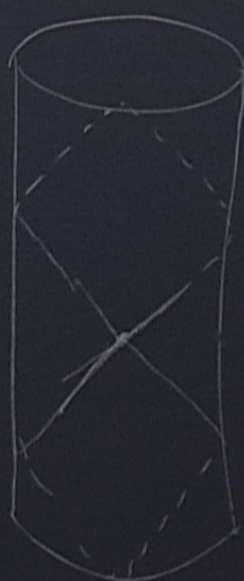
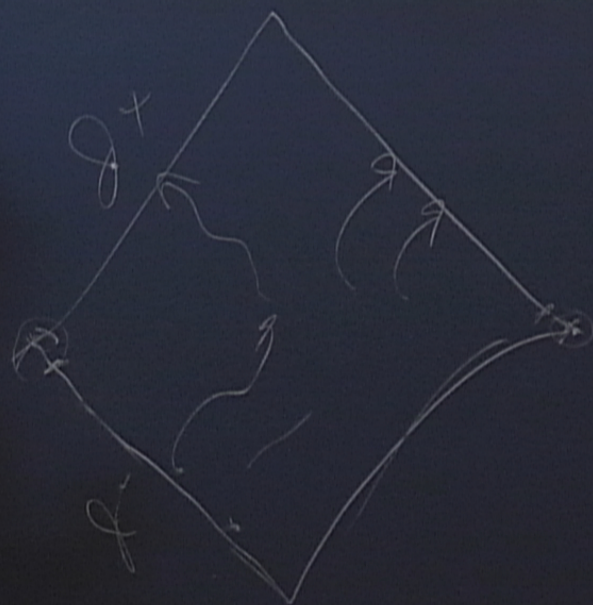


$$\mathcal{L}_f C_{zz} = f \partial_u C_{zz} - 2D_z^2 f$$

$$ds^2 = -dv^2 + 2dvdr + 2r^2 \gamma_{z\bar{z}} dz d\bar{z}$$



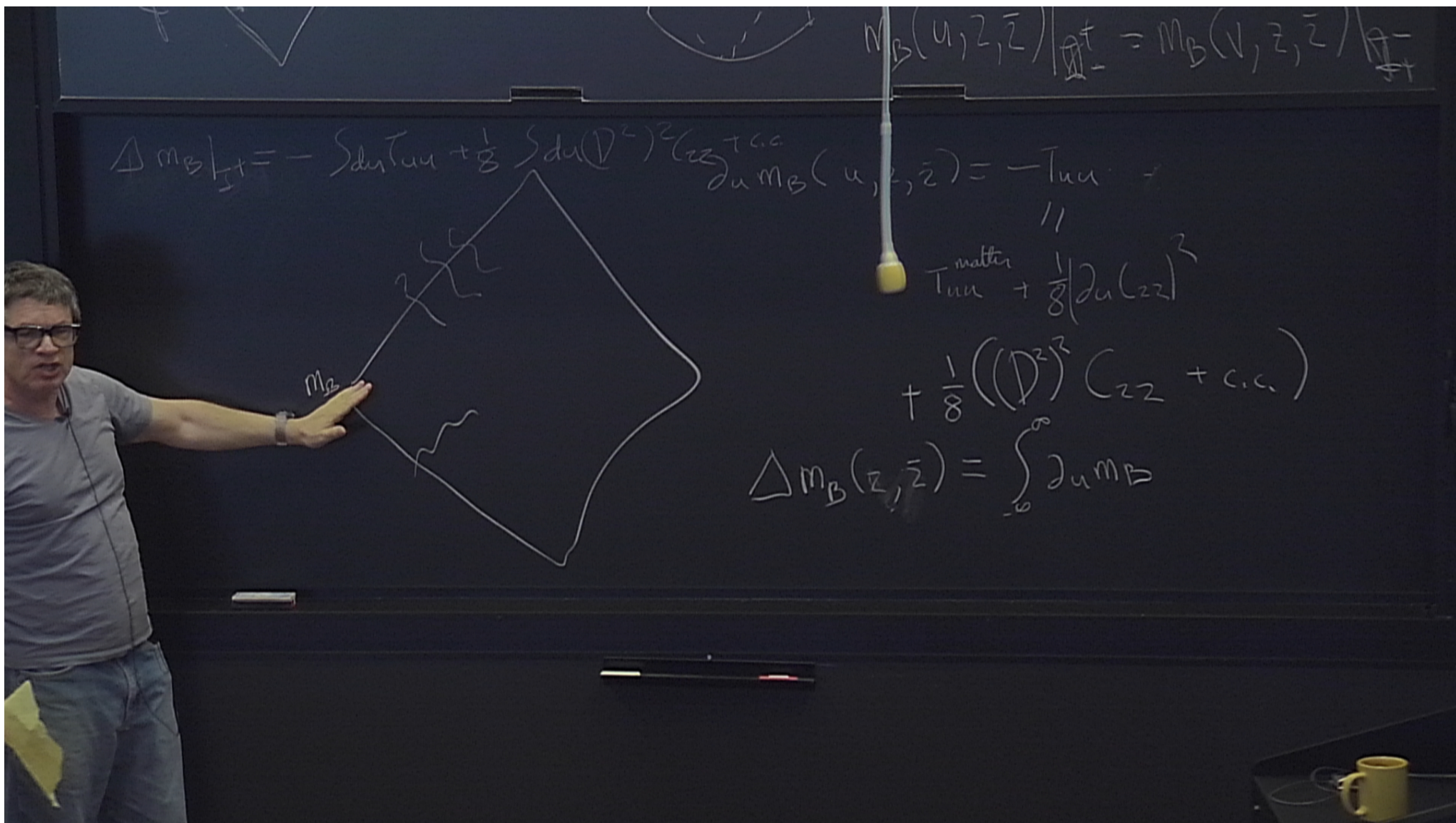




Christodoulou
Klaernerman
90's

$$g^+(z) = g^-(z^A)$$

$$f^+(z, \bar{z}) = f^-(z_A, \bar{z}_A)$$



$$M_B(u, z, \bar{z})|_{\mathcal{H}^+} = M_B(v, z, \bar{z})|_{\mathcal{H}^+}$$

$$\Delta m_B|_{\mathcal{H}^+} = - \int du T_{uu} + \frac{1}{8} \int du (D^2)^2 (z\bar{z} + c.c.)$$

$$\Delta m_B|_{\mathcal{H}^-} = - \int dv T_{vv}$$

$$+ \frac{1}{8} \int dv (D^2)^2 (z\bar{z} + c.c.)$$

soft + graviton contribution

$$T_{uu}^{\text{matter}} + \frac{1}{8} \int du (D^2)^2 (z\bar{z} + c.c.)$$

$$+ \frac{1}{8} \int du (D^2)^2 (z\bar{z} + c.c.)$$

$$\Delta m_B(z, \bar{z}) = \int_{-\infty}^{\infty} du m_B$$

$\Delta m_B|_{I^+} = - \int du T_{uu} + \frac{1}{8} \int du (D^2)^2 (z\bar{z} + c.c.)$
 $\Delta m_B|_{I^-} = - \int dv T_{vv}$
 $+ \frac{1}{8} \int dv (D^2)^2 (\bar{z}z + c.c.)$

$m_B(u, z, \bar{z})|_{I^+} = m_B(v, \bar{z}, \bar{z})|_{I^-}$
 $\partial_u m_B(u, z, \bar{z}) = -T_{uu} -$
 $\quad \quad \quad \uparrow$
 $\quad \quad \quad \text{soft + grav. tor contribution}$
 $T_{uu}^{\text{matter}} + \frac{1}{8} (\partial_u (z\bar{z}))^2$
 $+ \frac{1}{8} ((D^2)^2 (z\bar{z} + c.c.))$
 $\Delta m_B(z, \bar{z}) = \int_{-\infty}^{\infty} du m_B$

$$\begin{aligned}
 T^+[f] &= \int_{I^+} d^2z \gamma_{z\bar{z}} \dot{f}(z, \bar{z}) m_B \\
 &= \int_{I^+} d\lambda d^2z \gamma_{z\bar{z}} f T_{\lambda\lambda} \\
 &= \int_{I^+} d\lambda d^2z f (D^z)^2 (z\bar{z} + c.c.) \\
 &= F^+
 \end{aligned}
 \qquad
 \begin{aligned}
 &= T^-(f) \\
 \{T^+[f], G_{zz}\} &= \int_{I^-} d\lambda d^2z T_{\lambda\lambda} \\
 &= F \partial_\lambda (z\bar{z} - 2D_z^2 f) + F^-
 \end{aligned}$$

$$\begin{aligned}
T^+[f] &= \int_{I^+} d^2z \gamma_{z\bar{z}} \hat{f}(z, \bar{z}) m_B \\
&= \int_{I^+} d\eta d^2z \gamma_{z\bar{z}} f T_{\eta\eta} \\
&= \int_{I^+} d\eta d^2z f (D^z)^2 (z\bar{z} + \epsilon\epsilon) \\
&= F^+
\end{aligned}$$

$$\begin{aligned}
&= T^-(f) \\
&= \int_{I^-} d\eta d^2z T_{\eta\eta} \\
&= F^-
\end{aligned}$$

$$\{T^+[f], G_{zz}\} = F \partial_\eta (z\bar{z} - 2D_z^2 f) + F^-$$

$$M_B(u, z, \bar{z})|_{\mathcal{H}_-^+} = M_B(v, z, \bar{z})|_{\mathcal{H}_-^+}$$

$$|out\rangle = S |in\rangle$$

$$T^+(f)S - ST^-(f) = 0$$

$$T(f) |z_1^m, z_2^m, \dots\rangle$$

↓
 E_1^m

$$= \sum_k E_k^m f(z_k^m) |z_1^m, \dots\rangle$$

$$= M_B + F^- |z_1^m, \dots\rangle$$

$$\langle z_1^{\text{out}} | T^\dagger S - S T | z_1^{\text{in}} \rangle = 0 \quad \text{BMS}$$

$$\begin{aligned} & \sum_k (E_k^{\text{in}} f(z_k^{\text{in}}) - E_k^{\text{out}} f(z_k^{\text{out}})) \langle z_1^{\text{out}} | S | z_1^{\text{in}} \rangle \\ & = \langle z_1^{\text{out}} | F^\dagger S - S F | z_1^{\text{in}} \rangle \end{aligned}$$