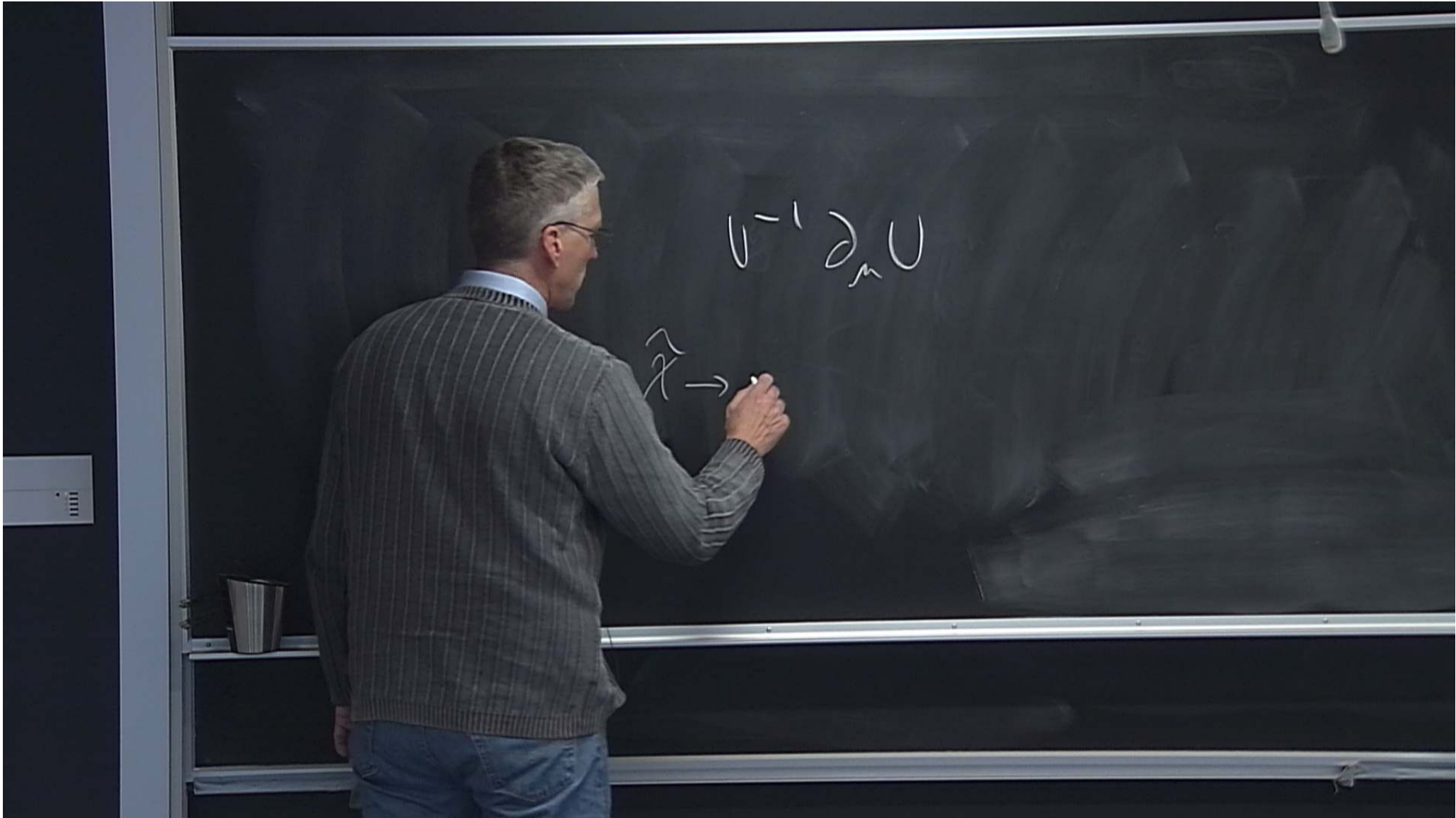


Title: Introduction to Effective Field Theories - Lecture 18

Date: Mar 14, 2014 02:30 PM

URL: <http://pirsa.org/14030079>

Abstract:



$$gU = \tilde{U}\gamma$$

$$U^{-1} \partial_m U$$

$$\tilde{\chi} \rightarrow \gamma \chi$$

$$\partial_m \tilde{\chi} \rightarrow \gamma \partial_m \chi + (\partial_m \gamma) \chi$$

$$gU = \tilde{U}\gamma \quad \Leftrightarrow U \rightarrow \tilde{U} = gU\gamma^{-1}$$

$$U^{-1} \partial_\mu U = -i A_\mu^i t_i + i e_\mu^\alpha X_\alpha$$

$$\rightarrow \gamma X$$

$$\rightarrow \gamma \partial_\mu X + (\partial_\mu \gamma) X$$

$$gU = \tilde{U}\gamma \quad \Leftrightarrow U \rightarrow \tilde{U} = gU\gamma^{-1}$$

$$U^{-1} \partial_\mu U = -i A_\mu^i t_i + i e_\mu^\alpha X_\alpha$$

$$\hat{\chi} \rightarrow \gamma \chi$$

$$\partial_\mu \tilde{\chi} \rightarrow \gamma \partial_\mu \chi + (\partial_\mu \gamma) \chi$$

$$U = e^{i\theta^\alpha X_\alpha} \\ = 1 + i\theta^\alpha X_\alpha$$

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Chiral Perturbation Theory

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$$

$$U = e^{\frac{i\omega_L^a \tau_a \gamma_L}{2} + \frac{i\omega_R^a \tau_a \gamma_R}{2}}$$

$$U = e^{\frac{i\omega_L^a \tau_a \gamma_L}{2} + \frac{i\omega_R^a \tau_a \gamma_R}{2}}$$

$$\gamma_L = \frac{1}{2}(1 + \gamma_5)$$

$$\gamma_R = \frac{1}{2}(1 - \gamma_5)$$

$$\gamma_5 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Chiral Perturbation Theory

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix} \quad U = \frac{i\omega_L \tau_a \gamma_L + i\omega_R \tau_a \gamma_R}{2}$$

$$U = e^{\frac{i\omega_L \tau_a \gamma_L + i\omega_R \tau_a \gamma_R}{2}}$$

$$\{\gamma_5, \gamma^{\mu\nu}\} = 0$$

$$\gamma_L = \frac{1 - \gamma_5}{2}$$

$$\gamma = \frac{1 - \gamma_5}{2}$$

$$\gamma_R = \begin{pmatrix} 1 & | & \\ \hline & & \\ \hline & & \\ & & | & \\ & & & -1 \end{pmatrix}$$

$$\tau_1$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$g \rightarrow$$

$$\bar{q} \rightarrow \bar{q} e^{-i\alpha + i\beta \tau_3}$$

Chiral Perturbation Theory

$$\begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix} \quad U = \frac{i\omega_L^a \tau_a \gamma_L + i\omega_R^a \tau_a \gamma_R}{2}$$

$$U = e^{\frac{i\omega_L^a \tau_a \gamma_L + i\omega_R^a \tau_a \gamma_R}{2}}$$

$$\{\gamma_5, \gamma^{\mu\nu}\} = 0$$

$$\gamma_L = \frac{1}{2}(1 + \gamma_5)$$

$$\gamma_R = \frac{1}{2}(1 - \gamma_5)$$

$$\gamma_5 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= (0 \ 1 \ \bar{q} \ q)$$

$$q \rightarrow e^{i(\alpha + \beta \gamma_5)} q$$

$$\bar{q} = q^\dagger \gamma_0$$

$$\bar{q} \rightarrow \bar{q} e^{-i\alpha + i\beta \gamma_5}$$

$$G = SU_L(2) \times SU_R(2)$$

$$H = e^{\frac{i}{\hbar} \hat{\omega}_V T_a}$$

homogeneous

$$H = e^{\frac{i}{\hbar} \omega_v T_a}$$

To build the generator on Lagrangian

$$U = \left(\right)$$

CAUTION

$$U(\theta) = \begin{pmatrix} u(\theta) & 0 \\ 0 & u'(\theta) \end{pmatrix}$$

$$u(\theta) = e^{\frac{i}{2}\theta^2 \tau_c}$$

$$\vec{\alpha} \cdot \vec{\tau}$$

$$e^{i\alpha^a \tau_a} = \cos \alpha + i(\sin \alpha) \hat{\alpha}^a \tau_a$$

$$\alpha = \sqrt{(\alpha^1)^2 + (\alpha^2)^2 + (\alpha^3)^2}$$

$$\hat{\alpha}^a = \frac{\alpha^a}{\alpha}$$

transformations
homogeneous

$$U(\theta) = \begin{pmatrix} u(\theta) & 0 \\ 0 & u^{-1}(\theta) \end{pmatrix}$$

$$\vec{\alpha} \cdot \vec{\tau}$$

$$e^{i\alpha^a \tau_a} = \cos \alpha + i(\sin \alpha) \hat{\alpha}^a \tau_a$$

$$\alpha = \sqrt{(\alpha^1)^2 + (\alpha^2)^2 + (\alpha^3)^2}$$

$$\hat{\alpha}^a = \frac{\alpha^a}{\alpha}$$

$$u(\theta) = e^{\frac{i}{2}\theta^a \tau_a}$$

$$\tilde{\alpha} = \tilde{\alpha}(\theta, g)$$

$$gU = \tilde{U}\gamma$$

transforming

homogeneous

CAUTION

DO NOT TOUCH THE BOARD OR THE BOARDER. ALWAYS HANDLE THE BOARDER BY THE EDGES.

$$\theta^\alpha \rightarrow \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha$$

$$\langle \vec{\omega}_V + \frac{\theta}{2} \left(+\gamma_m \frac{\theta}{2} + \omega t \frac{\theta}{2} \right) \left[\vec{\omega}_A - \hat{\theta} (\hat{\theta} \cdot \vec{\omega}) \right]$$

$$+ \hat{\theta} (\hat{\theta} \cdot \vec{\omega}_A)$$

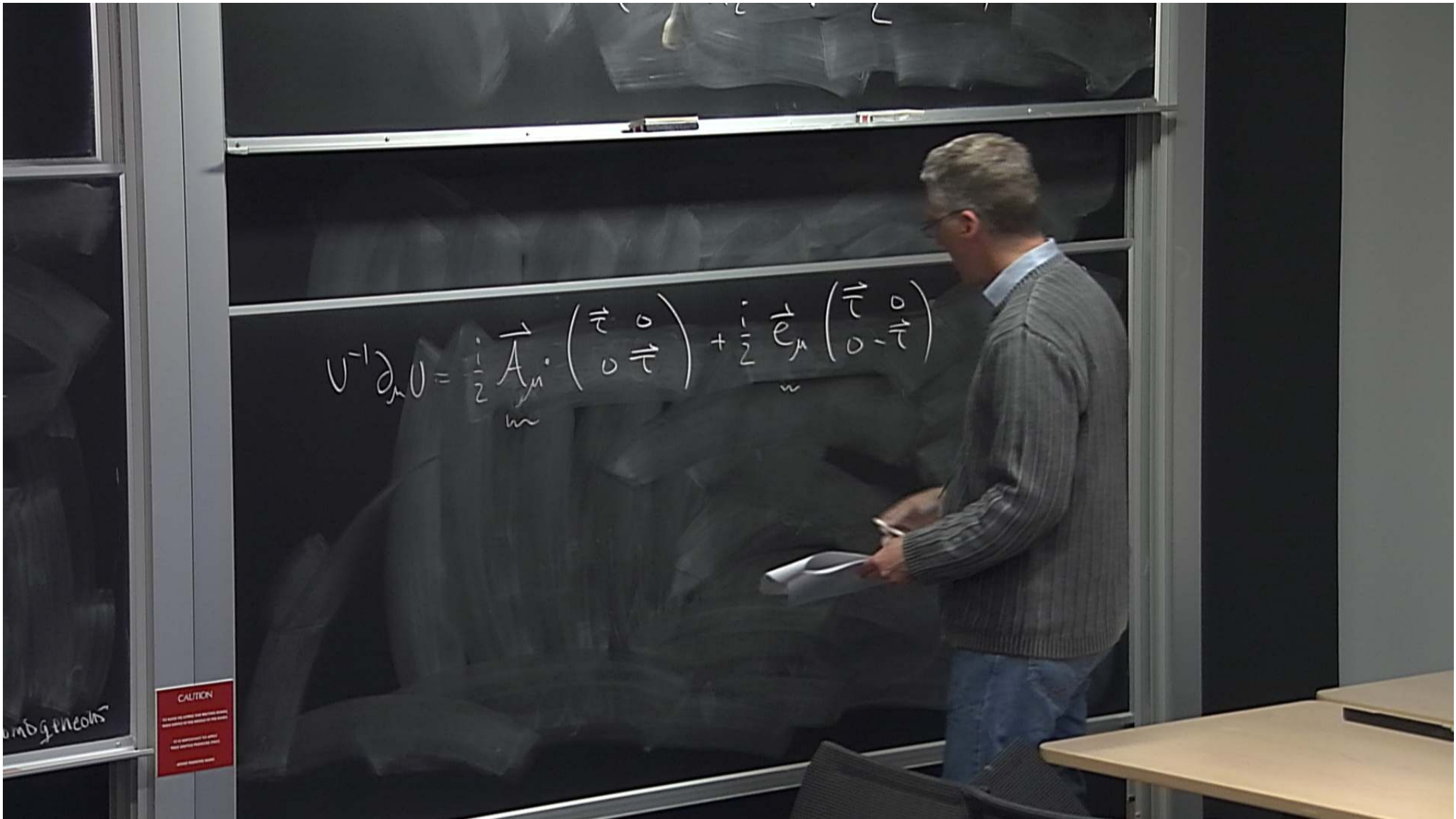
CAUTION
 DO NOT TOUCH THE BOARD OR THE BOARDER.
 IT IS DANGEROUS TO TOUCH THE BOARD OR THE BOARDER.
 PLEASE HOLD THE BOARDER.

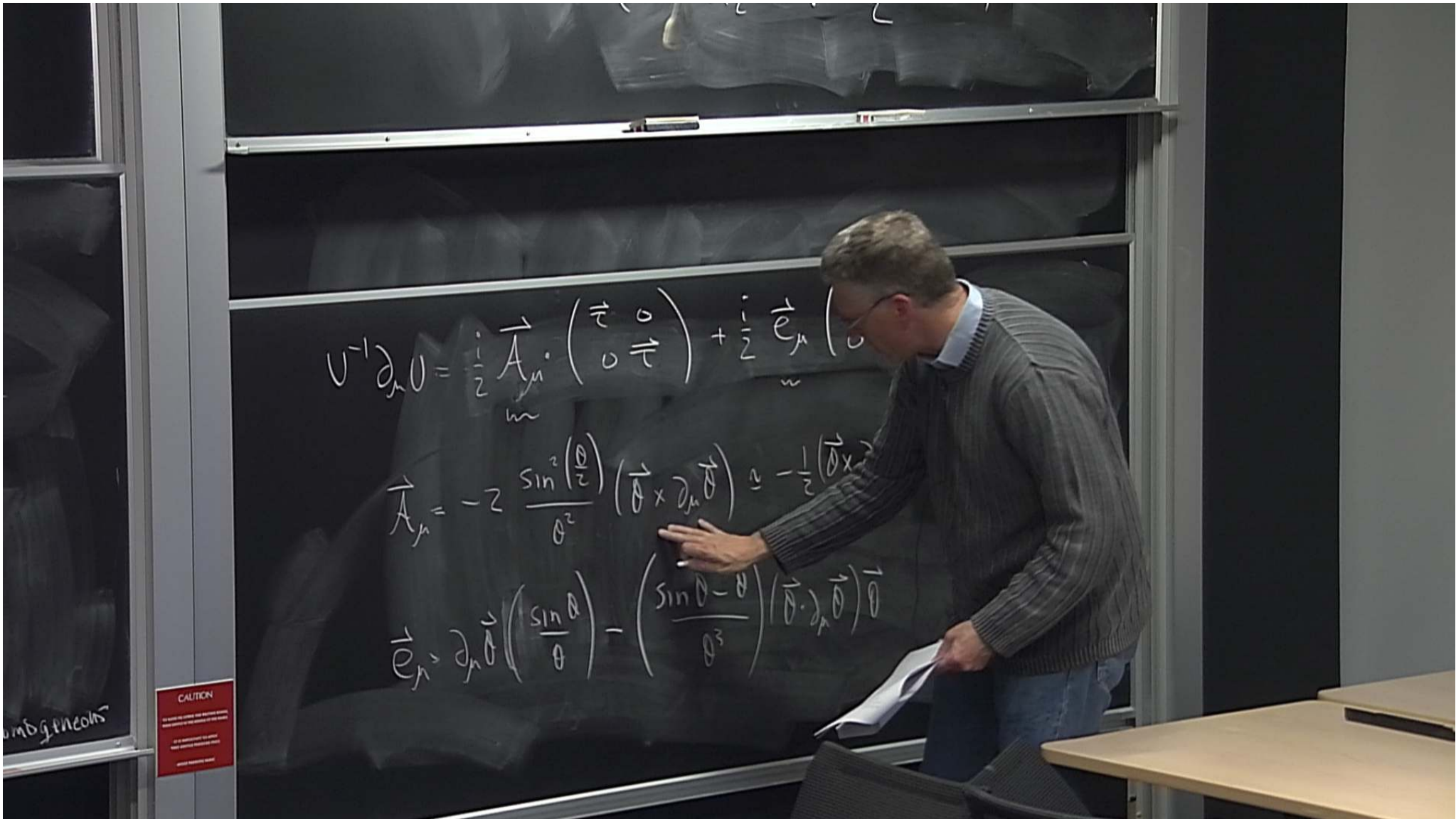
$$Q^\alpha \rightarrow \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha \quad \vec{\xi} = \hat{\theta} \times \vec{\omega}_V + \frac{\theta}{2} \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \left[\vec{\omega}_A - \hat{\theta} \right]$$

$$+ \hat{\theta} \left(\hat{\theta} \cdot \vec{\omega}_A \right) \\ = \vec{\omega}_A + \left(\hat{\theta} \times \vec{\omega}_V \right) + o(v)$$

$$\vec{u} = \vec{\omega}_V + \left(\hat{\theta} \times \vec{\omega}_A \right) \tan \frac{\theta}{2} \approx \vec{\omega}_V + \frac{\hat{\theta} \times \vec{\omega}_A}{2} + o(\theta^2)$$







Chiral Perturbation Theory

$$\begin{pmatrix} u \\ u \\ d \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ u \\ d \\ d \end{pmatrix}$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} \tau_1 + \tau_3 \\ \tau_2 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \tau_1 - \tau_3 \\ \tau_2 \end{pmatrix}$$

$$\begin{aligned} \delta_{mn} &= \delta_{mn} \left(1 - \frac{\theta^2}{3} \right) + \frac{1}{3} \partial_n \partial_n + o(\theta^4) \\ &= \delta_{mn} \left(\frac{\sin^2 \theta}{\theta^2} \right) + \partial_n \partial_n \left(\frac{\theta^2 - \sin^2 \theta}{\theta^4} \right) \end{aligned}$$

Chiral Perturbation Theory

$$\begin{pmatrix} u \\ u \\ d \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ u \\ d \\ d \end{pmatrix}$$

$$U = \frac{i\theta^2 \tau_3 \tau_8}{2} + \frac{i\omega^2 \tau_2 \tau_8}{2}$$

$$g_{mn} = \delta_{mn} \left(1 - \frac{\theta^2}{3} \right) + \frac{1}{3} \partial_m \partial_n + o(\theta^4)$$

$$= \delta_{mn} \left(\frac{\sin^2 \theta}{\theta^2} \right) + \partial_m \partial_n \left(\frac{\theta^2 - \sin^2 \theta}{\theta^4} \right)$$

$$\mathcal{L} = -\frac{F^2}{2} g_{mn}(\theta) \partial_m \theta^a \partial^n \theta^a + \dots$$

$$\gamma = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

Thomson

Canonical field: $\vec{\pi} = \vec{\partial} F$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} = \frac{1}{2F^2} (\vec{\pi} \cdot \partial_\mu \vec{\pi}) (\vec{\pi} \cdot \partial^\mu \vec{\pi}) + o(\pi^4)$$

Chiral Perturbation Theory

$$\begin{pmatrix} u \\ l \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ l \\ d \end{pmatrix}$$

$$U = \frac{i\sigma^1 \tau^2 \gamma_5}{2} + \frac{i\sigma^2 \tau^1 \gamma_5}{2}$$

$$g_{mn} = \delta_{mn} \left(1 - \frac{\theta^2}{3}\right) + \frac{1}{3} \partial_m \partial_n + o(\theta^4)$$

$$g_{mn} \left(\frac{\partial}{\partial x^m} \right) + \partial_n \partial_m \left(\frac{\theta^2 - \sin^2 \theta}{\theta^4} \right)$$

$$F^2 \quad g_{mn}(\theta) \partial_m \theta^a \partial^n \theta^b + \dots \quad \text{only parameter is } F.$$

homogeneous

$(\pi \partial \pi)$

How do we learn F ?

π^\pm decay because of a weak interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\psi} \gamma^\mu (1 + \gamma_5) \psi d W$$

CAUTION
Do not touch the screen and do not lean against it.
Do not touch the screen and do not lean against it.
Do not touch the screen and do not lean against it.

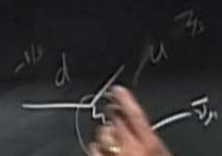
homogeneous

$\pi \partial \pi$

How do we learn F ?

π^\pm decay because of a weak interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{u} \gamma^\mu (1 + \gamma_5) d \bar{\mu} \gamma_\mu (1 + \gamma_5) \nu$$



CAUTION
Do not touch the board when the instructor is writing.
Do not touch the board when the instructor is writing.

homogeneous

$\pi^0 \rightarrow \pi^0$

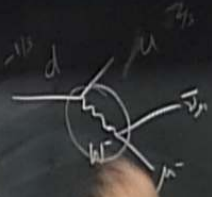
How do we learn F?

π^\pm decay because of a weak interaction

$$\pi^+ = u\bar{d}$$

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \underbrace{\bar{u}\gamma^\mu(1+\gamma_5)d}_{\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)} \bar{\mu}\gamma_\mu(1+\gamma_5)$$

$\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right) \left(\begin{smallmatrix} J_V^d & J_A^d \\ J_V^u & J_A^u \end{smallmatrix}\right)$ for the SU(2) x U(1) Sym



CAUTION
No open flames and smoking allowed
in this laboratory
It is prohibited to work
without proper supervision

homogeneous

How do we learn F ?

π^\pm decay because of a weak interaction $\bar{u} + d \rightarrow \mu + \bar{\nu}$



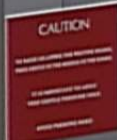
$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \underbrace{\bar{u} \gamma^\mu (1 + \gamma_5) d}_{\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix} \right)} \bar{\nu} \gamma_\mu (1 + \gamma_5) \nu + c.c.$$

$\pi^+ = u \bar{d}$



$\langle \Omega | \bar{u} \gamma^\mu d | \pi^+ \rangle$

$\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix} \right) \left(\underline{\underline{J_V^d + J_A^d}} \right)$ for the $SU(2) \times SU(2)$ symmetry



$\partial \rho^{i4\pi}$
homogeneous

$\vec{\pi}^a + O(\pi^4)\pi^a\partial\pi$
sectors at
orders')

How do we learn F ?

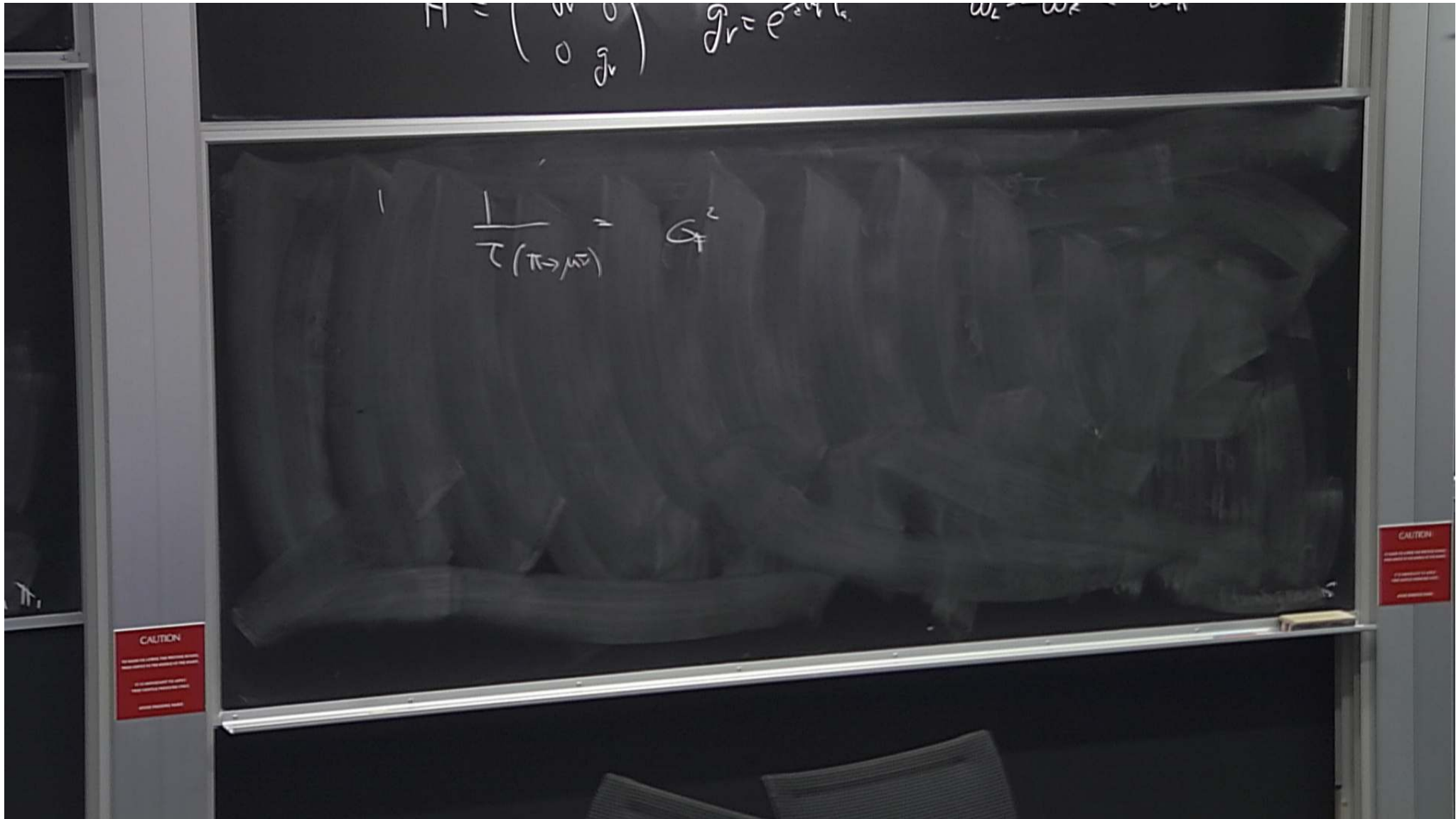
Need current for $SU(2) \times SU(2)$ using
procedure in $\mathcal{L}(\pi)$

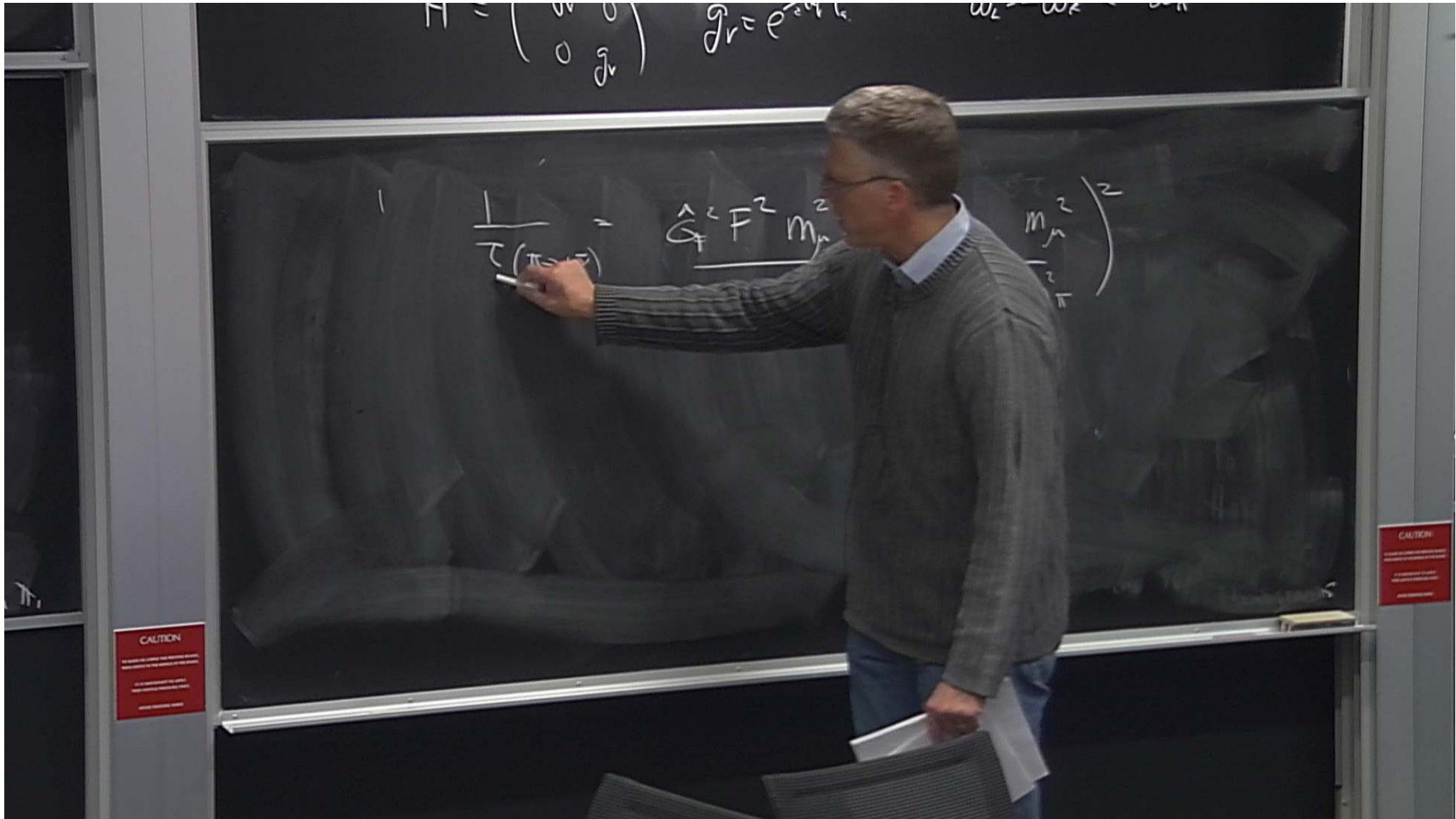
$$\vec{j}_V^a = -(\vec{\pi} \times \partial^a \vec{\pi}) + O(\pi^4)$$

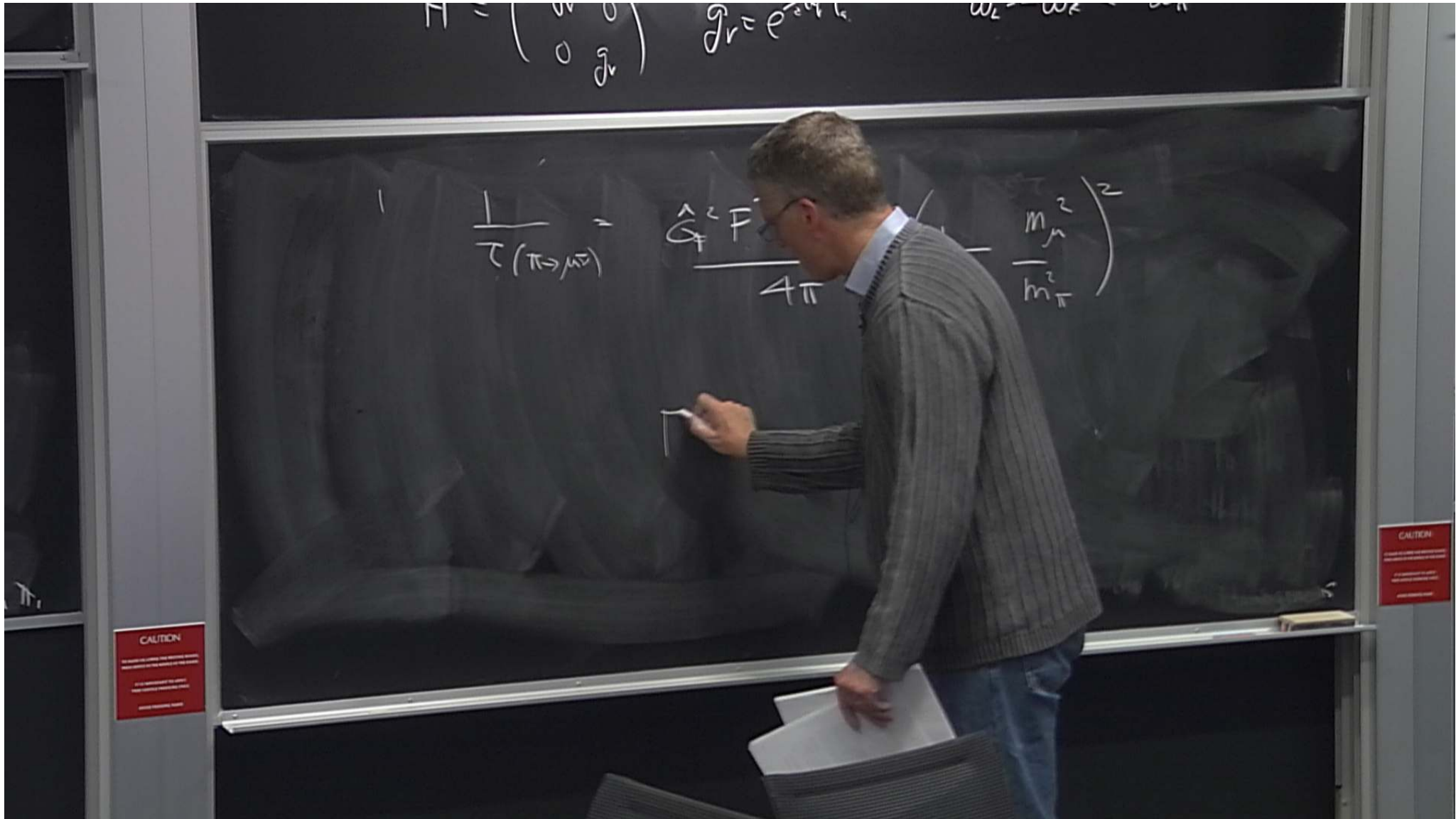
$$\vec{j}_A^a = F \partial^a \vec{\pi} +$$

↑

CAUTION





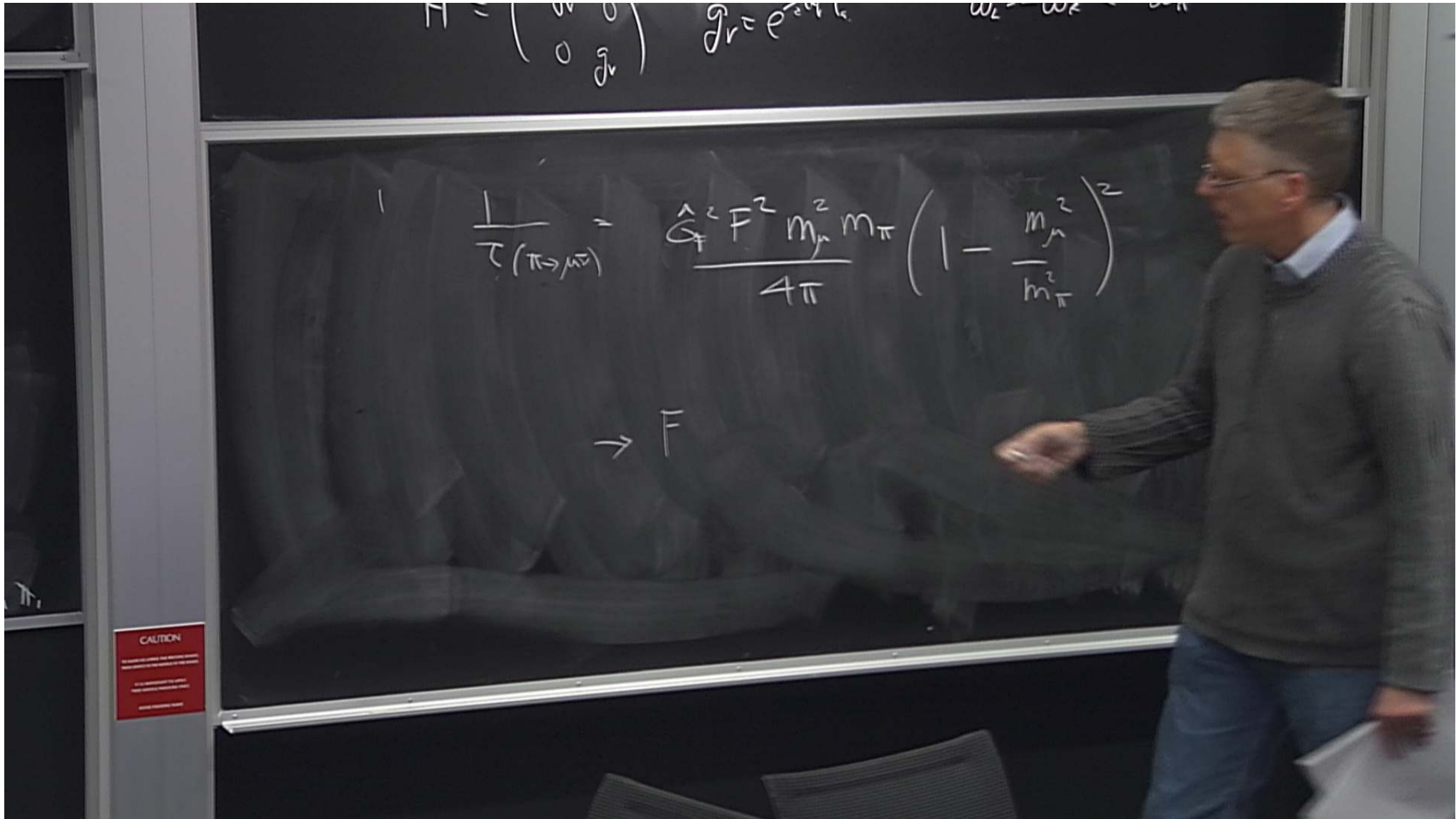


$$H = \begin{pmatrix} \omega & 0 \\ 0 & \tilde{\omega} \end{pmatrix} \quad \tilde{\omega} = e^{-2\gamma t} \quad \omega_L = \omega_R = \dots$$

$$\frac{1}{\tau(\pi \rightarrow \mu^+ \nu)}$$

$$\frac{\hat{G}_F^2 F^2}{4\pi}$$

$$\left(\frac{m_\mu^2}{m_\pi^2} \right)^2$$



Canonical field: $\vec{\pi} = \partial F$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} = \frac{1}{2F^2} (\vec{\pi} \cdot \partial_\mu \vec{\pi}) (\vec{\pi} \cdot \partial^\mu \vec{\pi}) + o(\pi^4)$$

Given F can predict all scattering cross sections at low energy ("soft pion theorems")

CAUTION

Canonical field: $\vec{\pi} = \partial F$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} = \frac{1}{2F^2} (\vec{\pi} \cdot \partial_\mu \vec{\pi}) (\vec{\pi} \cdot \partial^\mu \vec{\pi}) + \mathcal{O}(\pi^4) \partial\pi\partial\pi$$

Given F can predict all scattering cross sections at low energy ("soft pion theorems")

CAUTION

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Given F can predict all scattering cross sections at low energy ("soft pion theorems")

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Given F can predict all scattering cross sections at low energy ("soft pion theorems")

Canonical field: $\vec{\pi} = \partial F$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} = \frac{1}{2F^2} (\vec{\pi} \cdot \partial_\mu \vec{\pi}) (\vec{\pi} \cdot \partial^\mu \vec{\pi}) + o(\pi^4) \partial \pi \partial \pi$$

Given F can predict all scattering cross sections at low energy ("soft pion theorems")

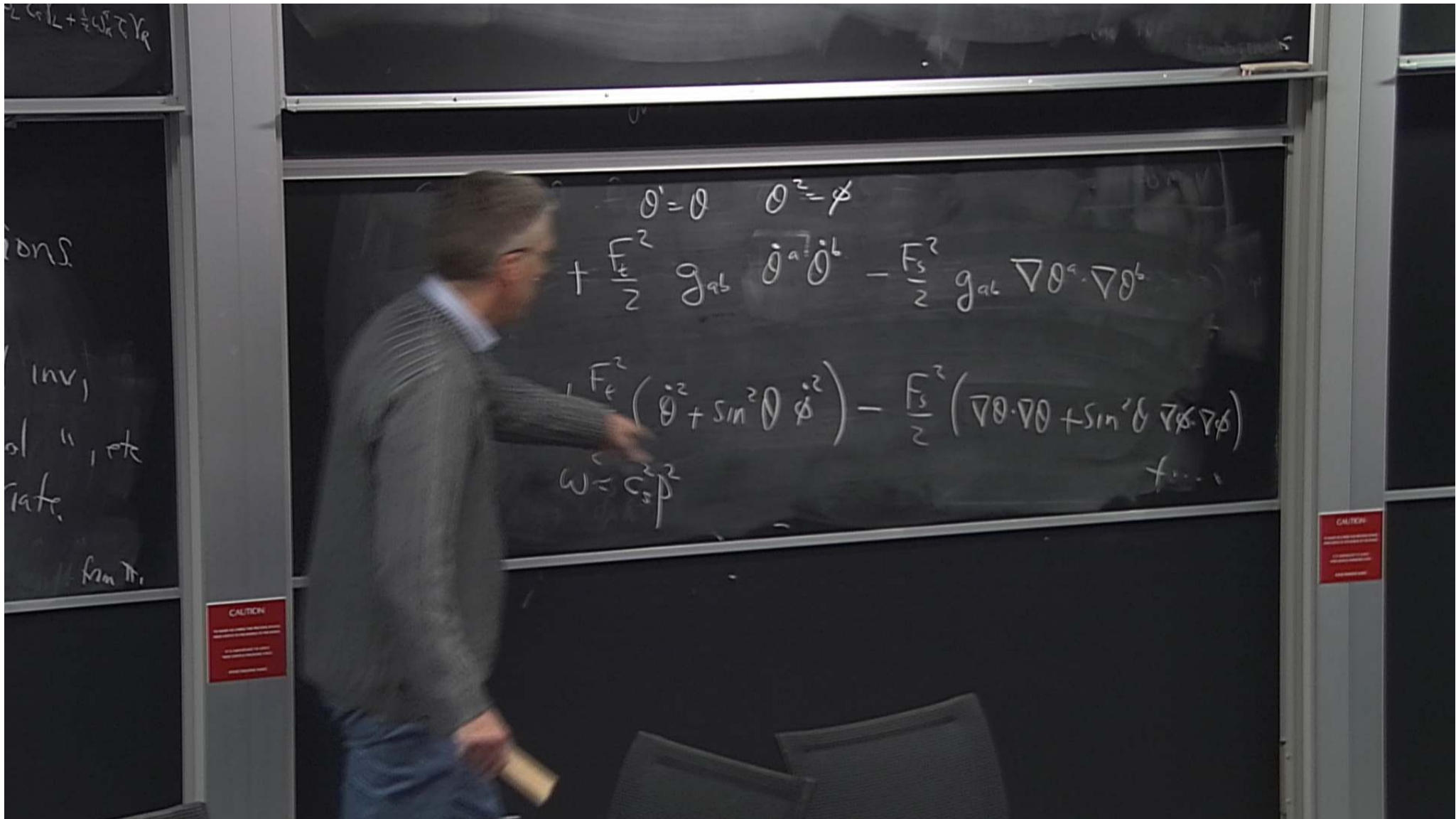
Chiral Perturbation Theory

$$= \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow U \begin{pmatrix} u \\ d \end{pmatrix} \quad U = \frac{1}{2} \sigma_2 G \sigma_2 + \frac{1}{2} \sigma_3 \tau_3$$

How things differ in nonrelativistic situations

Spacetime symmetries of \mathcal{L} replaced by rotational inv,
translational " , etc
as appropriate.

from II.



$$\theta^1 = 0 \quad \theta^2 = \phi$$

$$+ \frac{F_e^2}{2} g_{ab} \dot{\theta}^a \dot{\theta}^b - \frac{F_s^2}{2} g_{ab} \nabla \theta^a \cdot \nabla \theta^b$$

$$+ \frac{F_e^2}{2} (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) - \frac{F_s^2}{2} (\nabla \theta \cdot \nabla \theta + \sin^2 \theta \nabla \phi \cdot \nabla \phi) + \dots$$

$$\omega = c_s^2 p^2$$

Ask for: $F_{ab} = \partial_a A_b - \partial_b A_a$ to be invariant

$F_{ab} \sim \epsilon_{ab} \Rightarrow$ non-zero

CAUTION