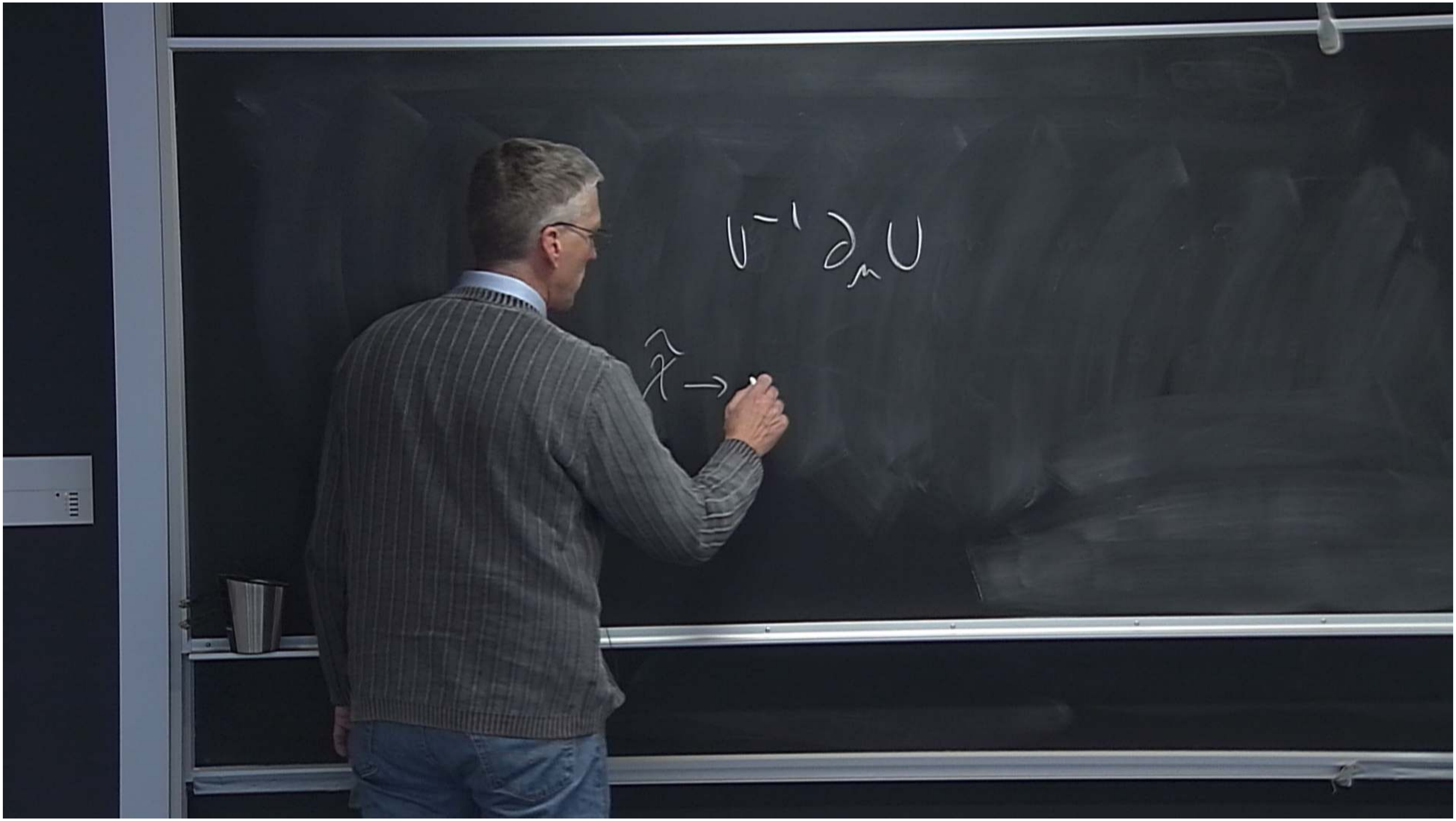
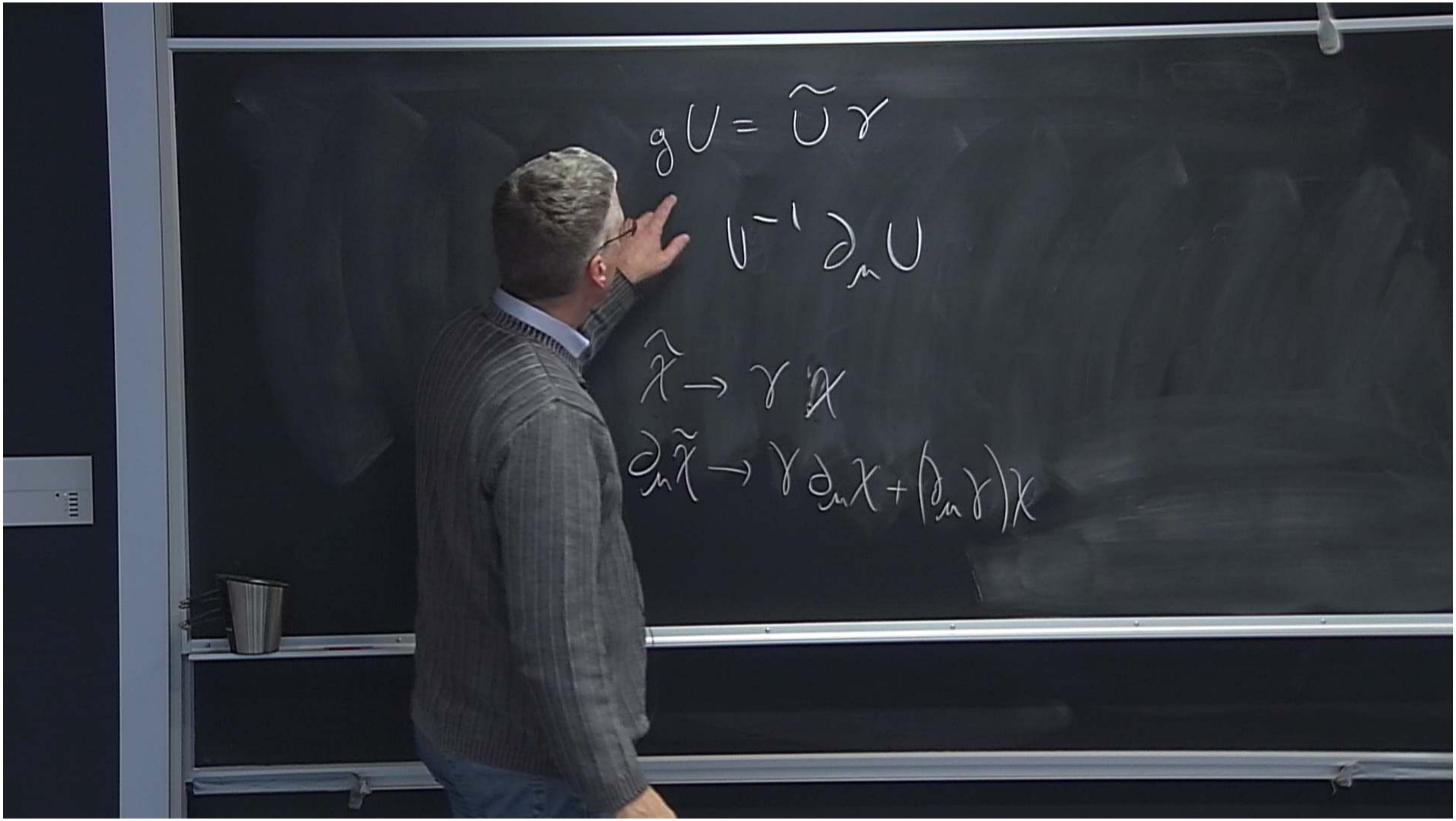
Title: Introduction to Effective Field Theories - Lecture 18

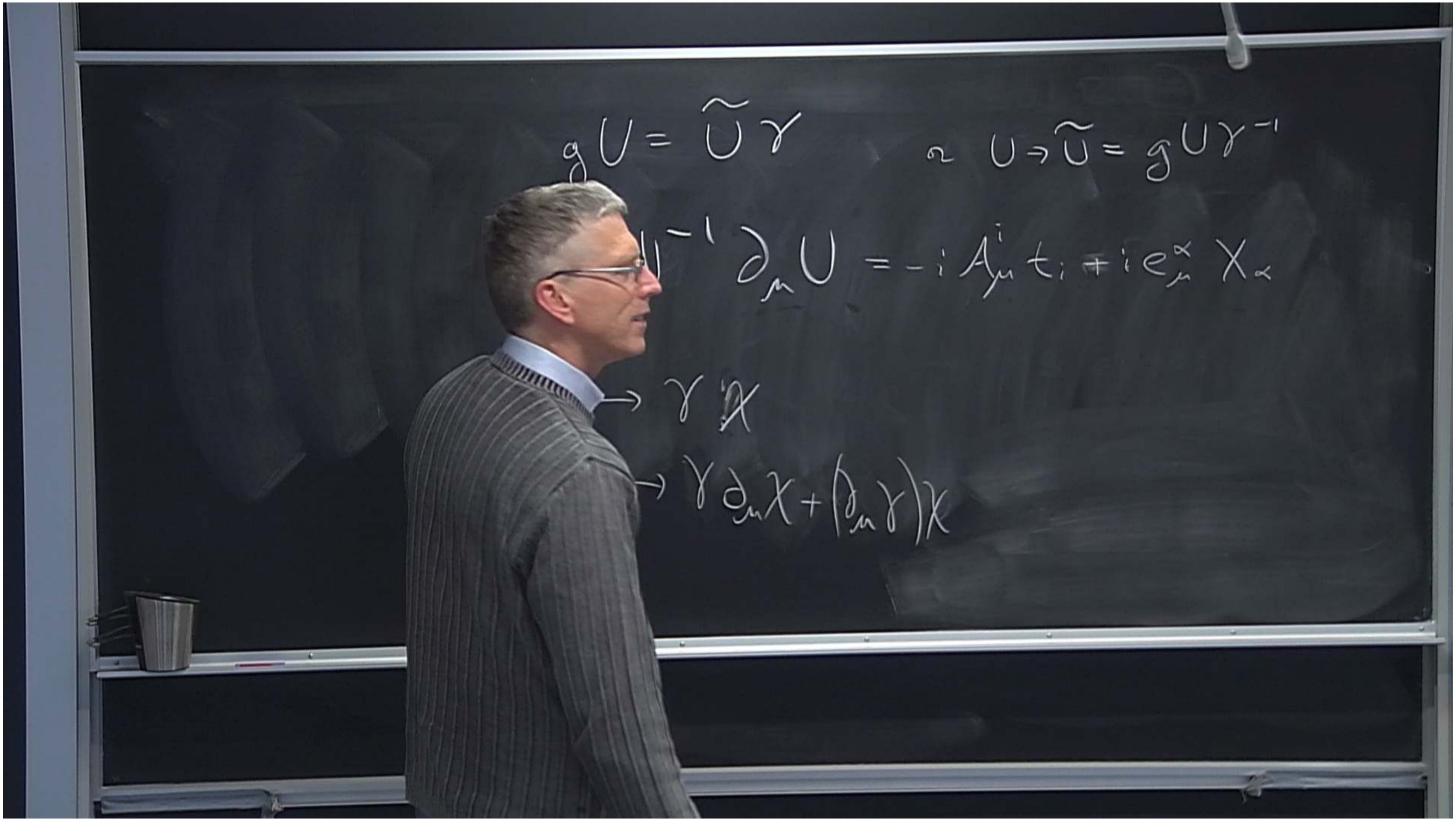
Date: Mar 14, 2014 02:30 PM

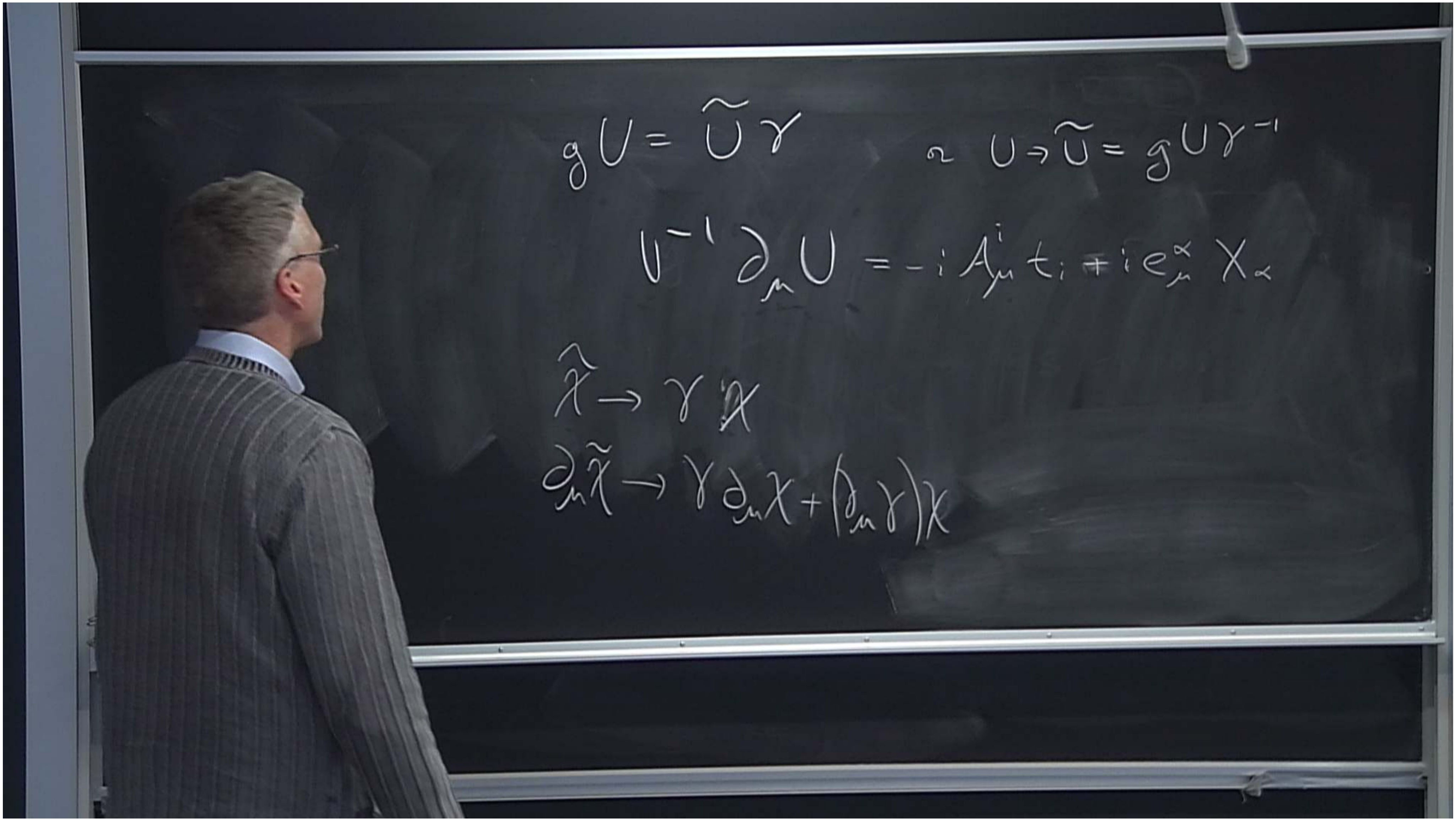
URL: http://pirsa.org/14030079

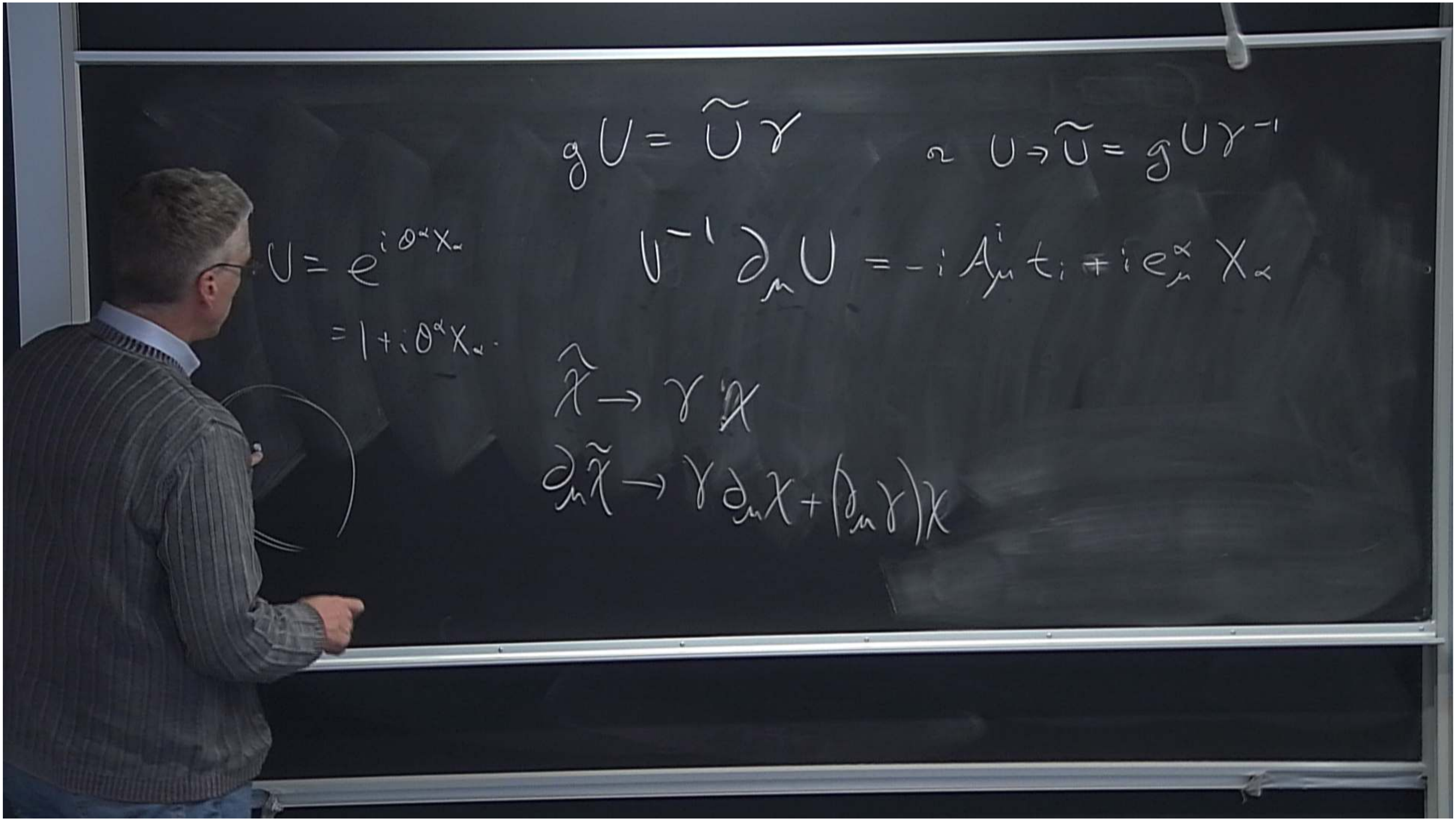
Abstract:


$U^{-1}\partial_\mu U$
$\tilde{x} \rightarrow$

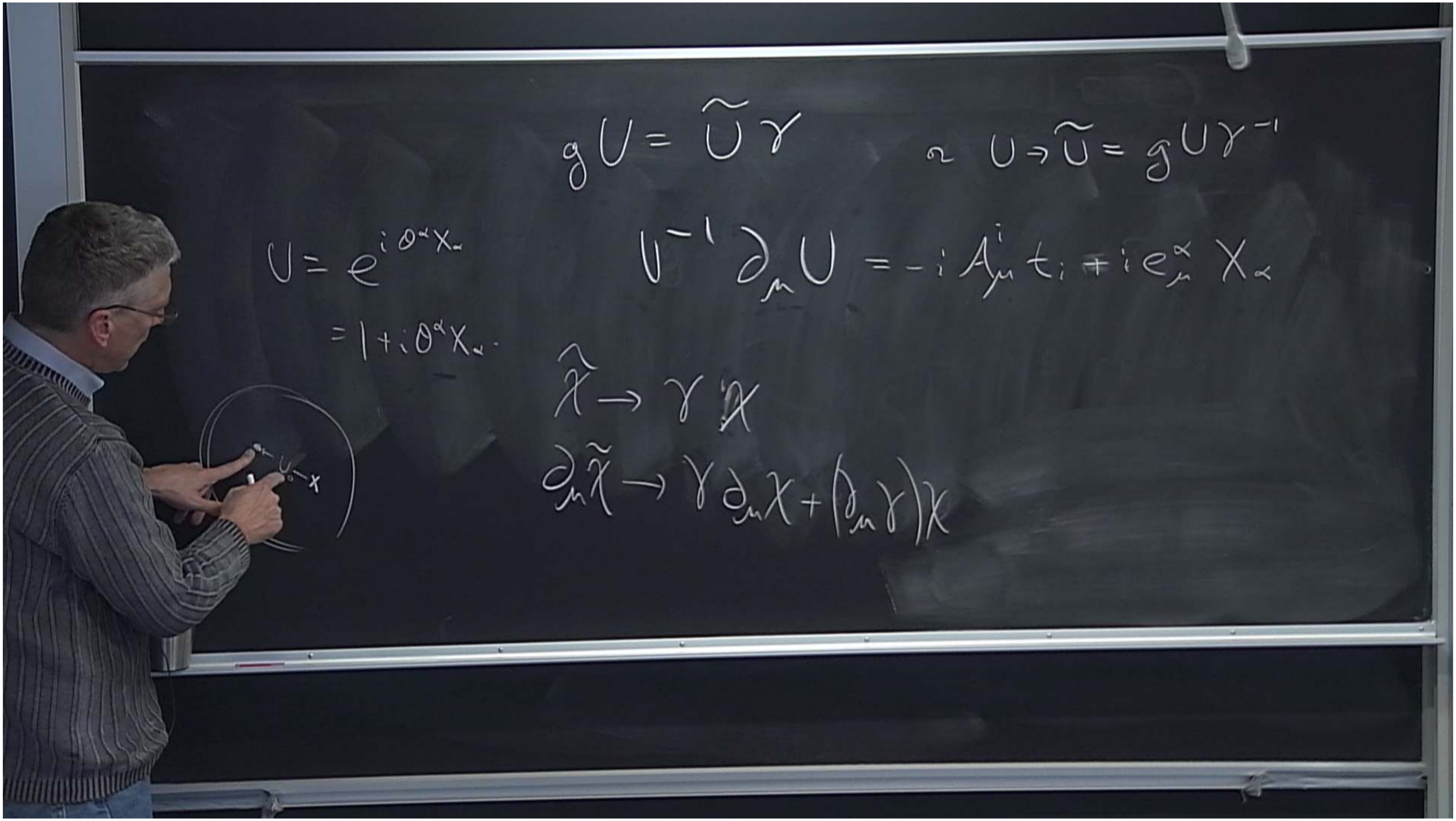

$gU = \tilde{U}\gamma$
$U^{-1}\partial_\mu U$
$\tilde{\chi} \to \gamma \chi$
$\partial_\mu \tilde{\chi} \to \gamma \partial_\mu \chi + (\partial_\mu \gamma)\chi$

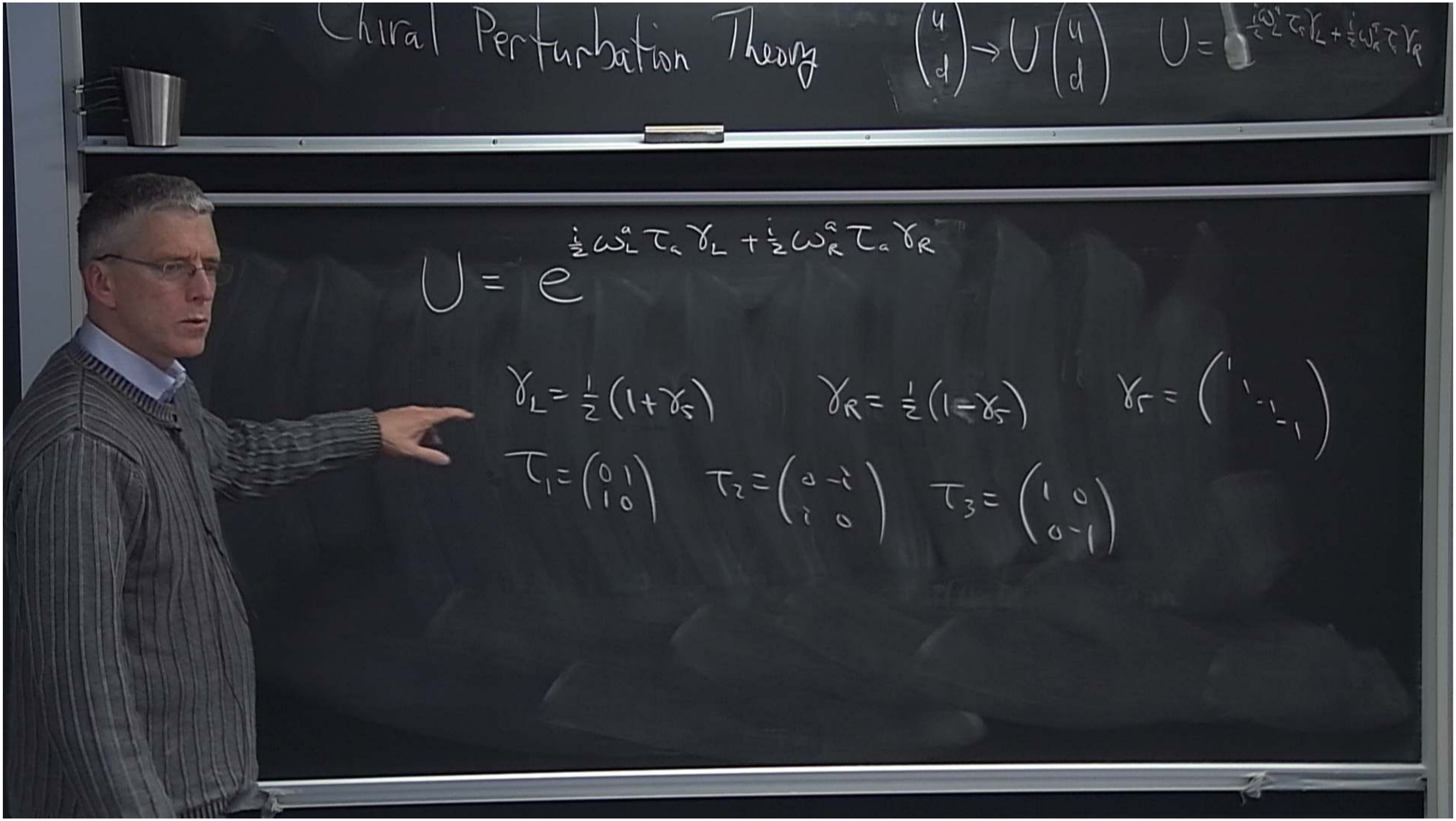

gU = Ũγ
or U → Ũ = gUγ^{-1}
U^{-1} ∂_μ U = -i A^i_μ t_i + i e^α_μ X_α
→ γ χ
→ γ ∂_μ χ + (∂_μ γ) χ

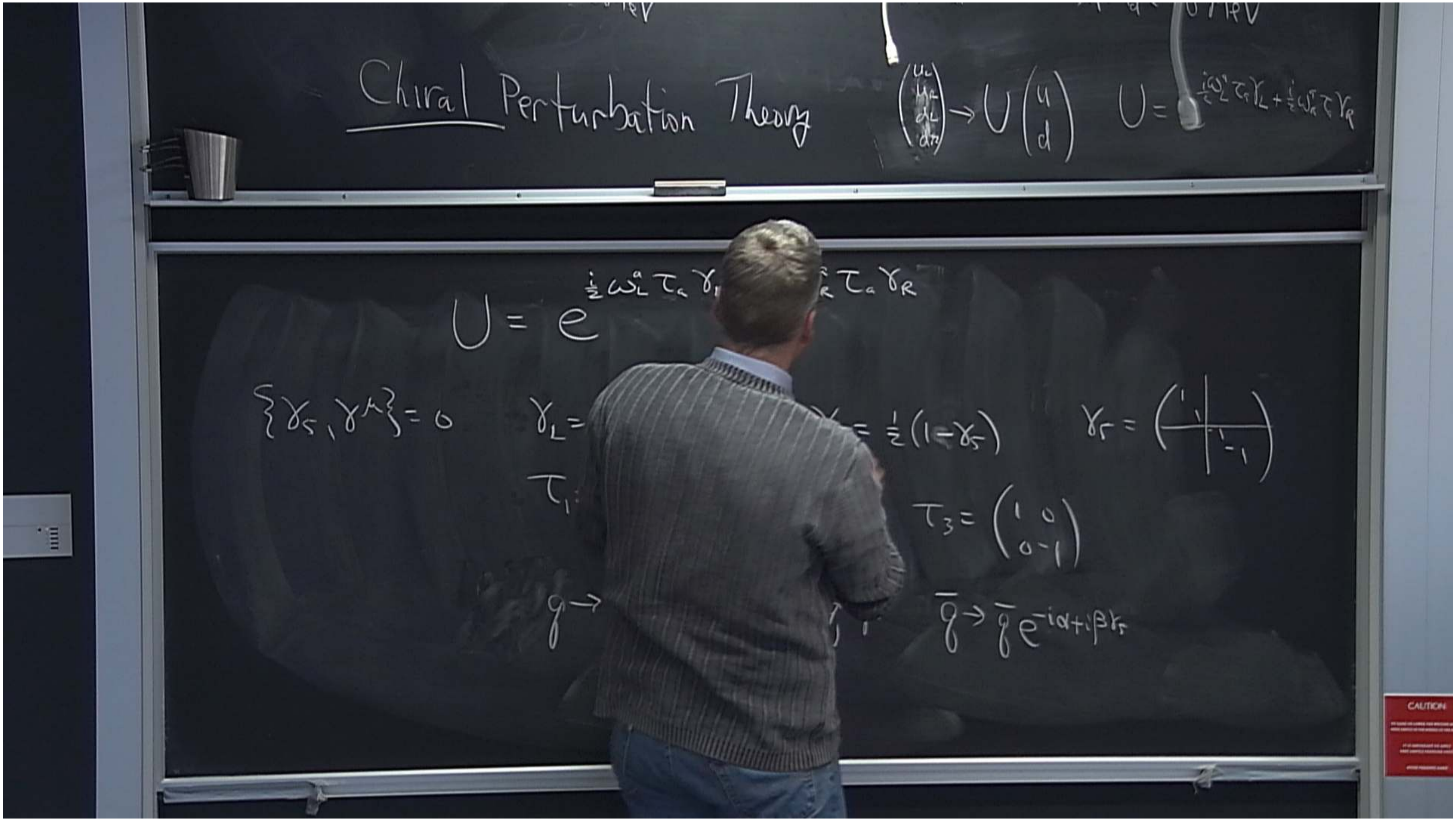

$gU = \widetilde{U}\gamma$
$\sim$ $U \to \widetilde{U} = gU\gamma^{-1}$
$U^{-1}\partial_\mu U = -iA^i_\mu t_i + ie^\alpha_\mu X_\alpha$
$\tilde{\chi} \to \gamma\chi$
$\partial_\mu\tilde{\chi} \to \gamma\partial_\mu\chi + (\partial_\mu\gamma)\chi$

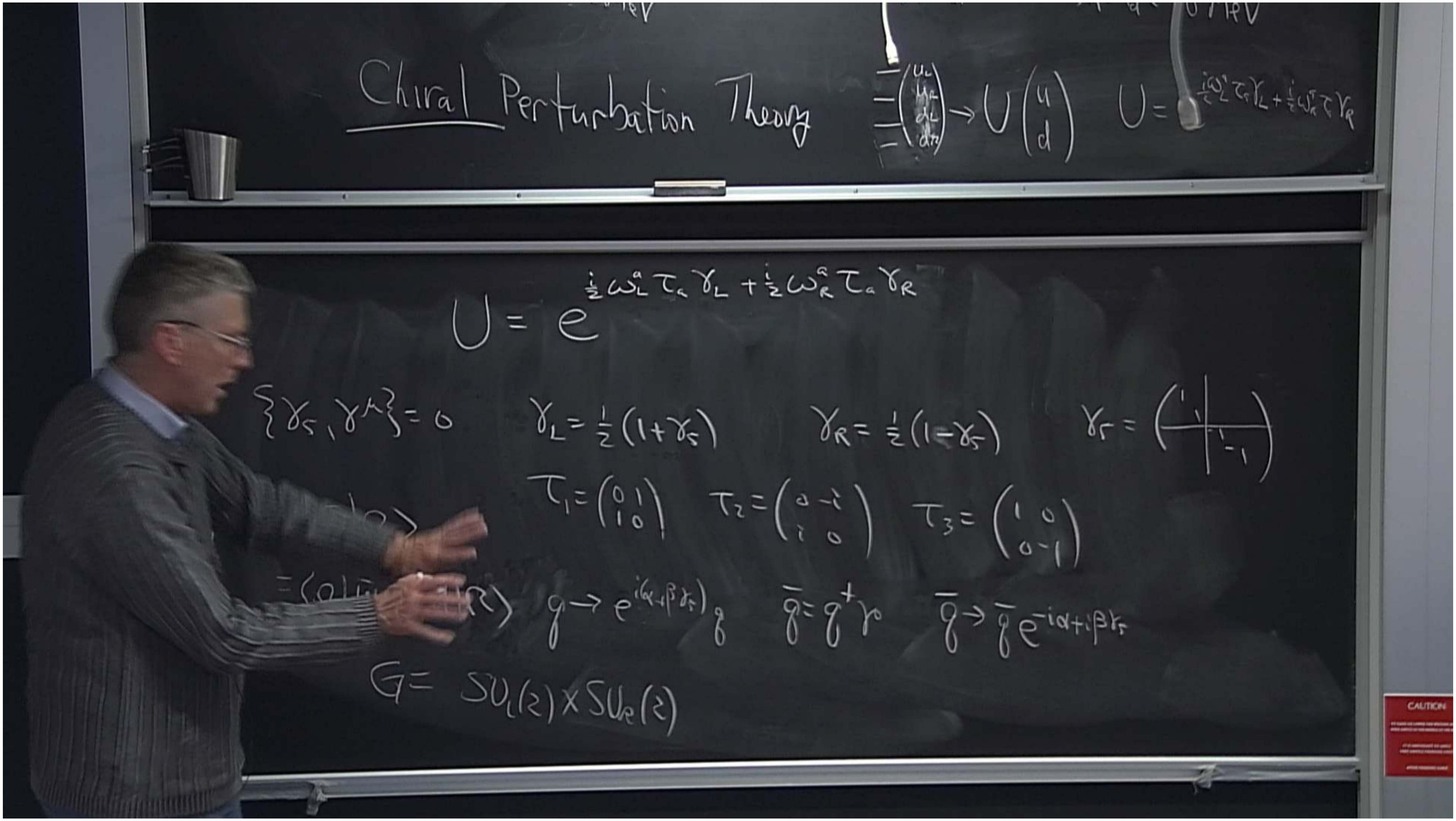


$gU = \tilde{U}\gamma$ or $U \to \tilde{U} = gU\gamma^{-1}$
$U = e^{i\theta^\alpha X_\alpha}$
$= 1 + i\theta^\alpha X_\alpha$
$U^{-1}\partial_\mu U = -iA^i_\mu t_i + ie^\alpha_\mu X_\alpha$
$\tilde{\chi} \to \gamma\chi$
$\partial_\mu\tilde{\chi} \to \gamma\partial_\mu\chi + (\partial_\mu\gamma)\chi$

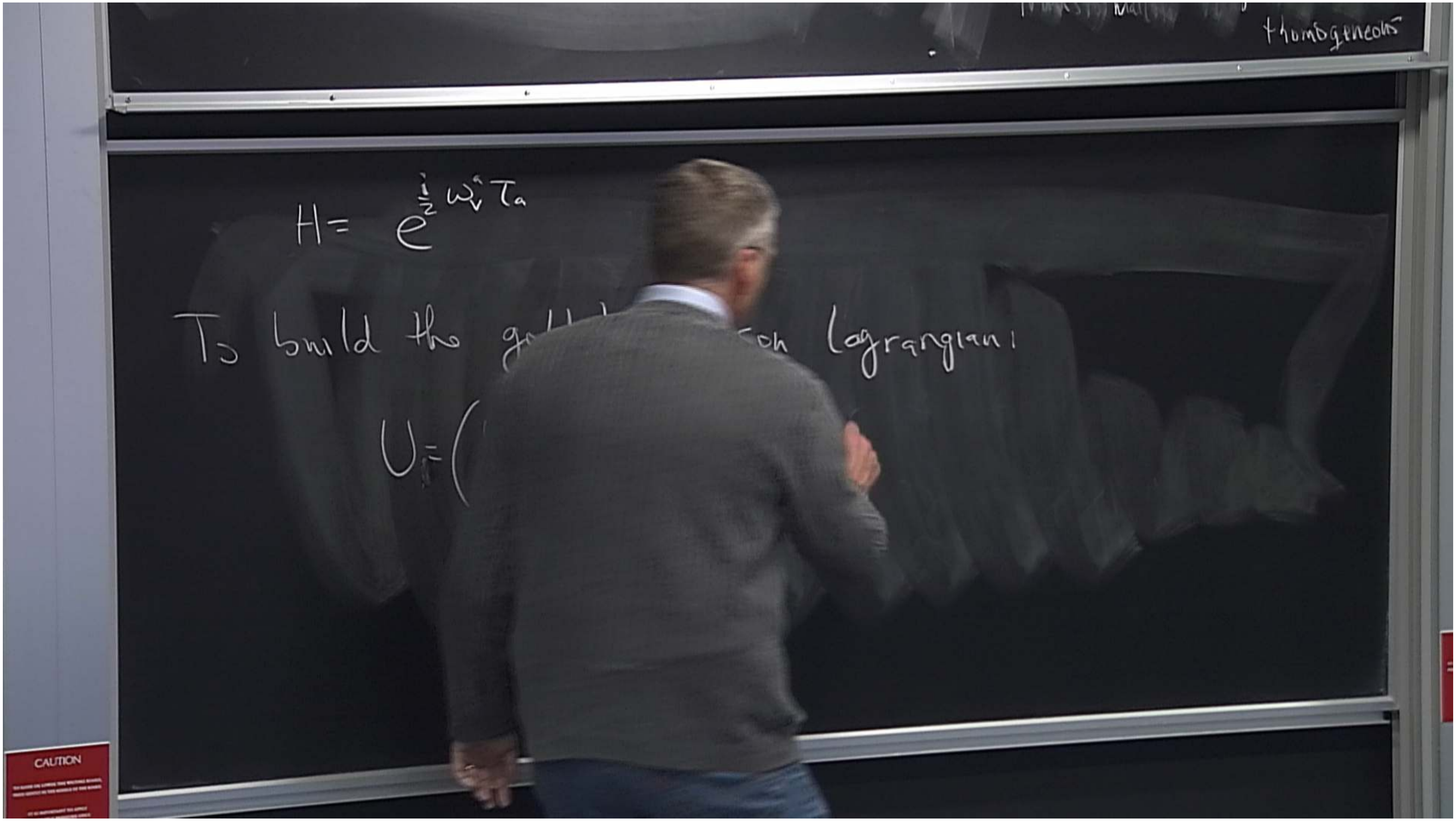

$gU = \tilde{U}\gamma$
or $U \to \tilde{U} = gU\gamma^{-1}$
$U = e^{i\theta^\alpha X_\alpha}$
$= 1 + i\theta^\alpha X_\alpha \cdot$
$U^{-1}\partial_\mu U = -iA_\mu^i t_i + ie_\mu^\alpha X_\alpha$
$\hat{\chi} \to \gamma\chi$
$\partial_\mu\tilde{\chi} \to \gamma\partial_\mu\chi + (\partial_\mu\gamma)\chi$

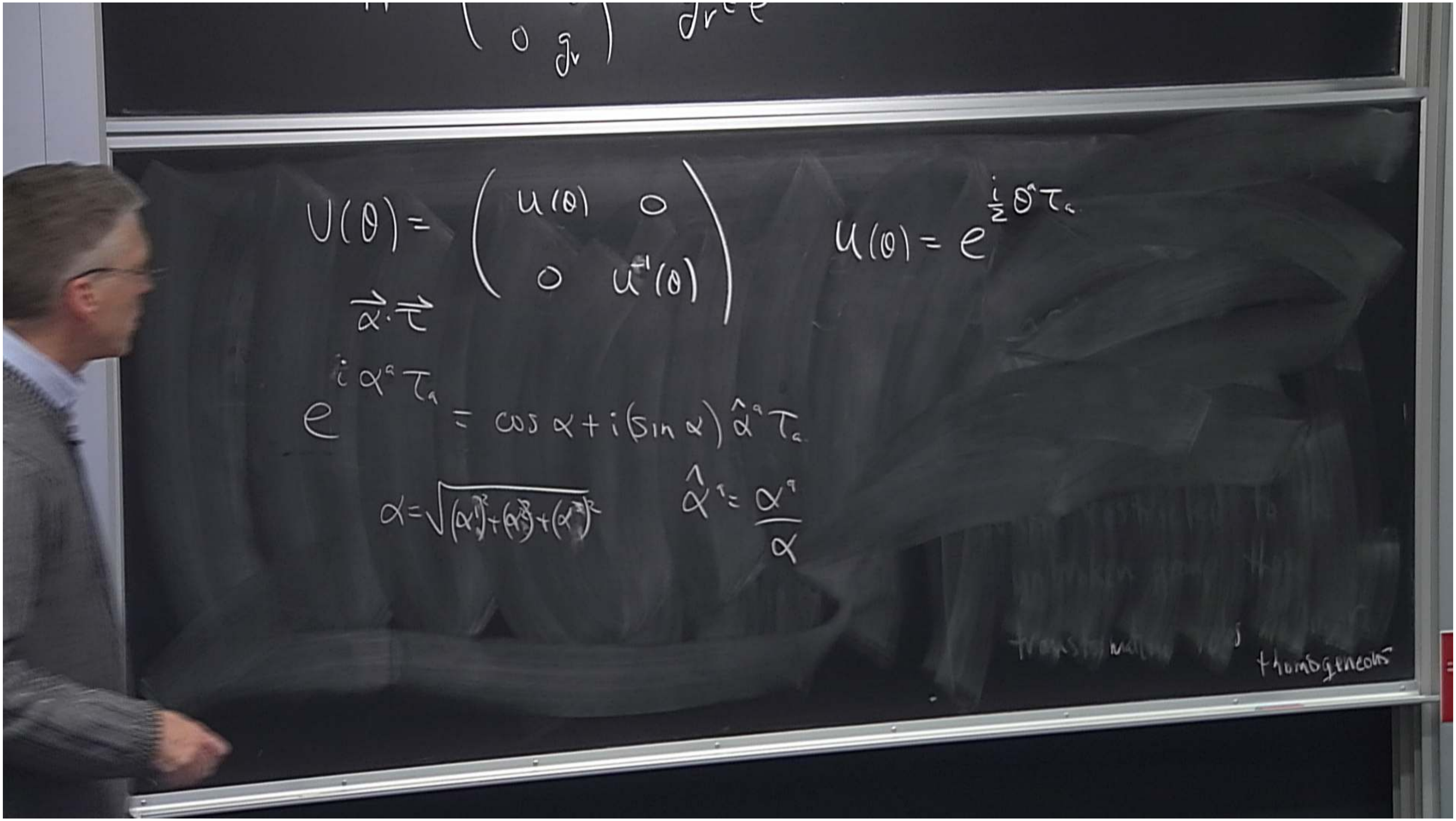

Chiral Perturbation Theory
$\begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix}$
$U = e^{\frac{i}{2}\omega_L^a \tau_a \gamma_L + \frac{i}{2}\omega_R^a \tau_a \gamma_R}$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$
$\gamma_5 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$


Chiral Perturbation Theory
CAUTION


Chiral Perturbation Theory
$U = e^{\frac{i}{2}\omega_L^a \tau_a \gamma_L + \frac{i}{2}\omega_R^a \tau_a \gamma_R}$
$\{\gamma_5, \gamma^\mu\} = 0$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$q \to e^{i(\alpha + \beta\gamma_5)} q$
$\bar{q} = q^\dagger \gamma_0$
$\bar{q} \to \bar{q} e^{-i\alpha + i\beta\gamma_5}$
$G = SU_L(2) \times SU_R(2)$
CAUTION

$$H = e^{\frac{i}{2}\omega_\nu^a \tau_a}$$


+homogeneous
H = $e^{\frac{i}{2}\omega_v^a T_a}$
To build the g... ...on Lagrangian:
U = (
CAUTION


$U(\theta) = \begin{pmatrix} u(\theta) & 0 \\ 0 & u^{-1}(\theta) \end{pmatrix}$
$u(\theta) = e^{\frac{i}{2}\theta^a \tau_a}$
$\vec{\alpha}\cdot\vec{\tau}$
$e^{i\alpha^a \tau_a} = \cos\alpha + i(\sin\alpha)\hat{\alpha}^a \tau_a$
$\alpha = \sqrt{(\alpha^1)^2 + (\alpha^2)^2 + (\alpha^3)^2}$
$\hat{\alpha}^a = \frac{\alpha^a}{\alpha}$

$$U(\theta) = \begin{pmatrix} u(\theta) & 0 \\ 0 & u^{-1}(\theta) \end{pmatrix}$$

$$u(\theta) = e^{\frac{i}{2}\theta^a \tau_a}$$

$$\vec{\alpha}\cdot\vec{\tau}$$

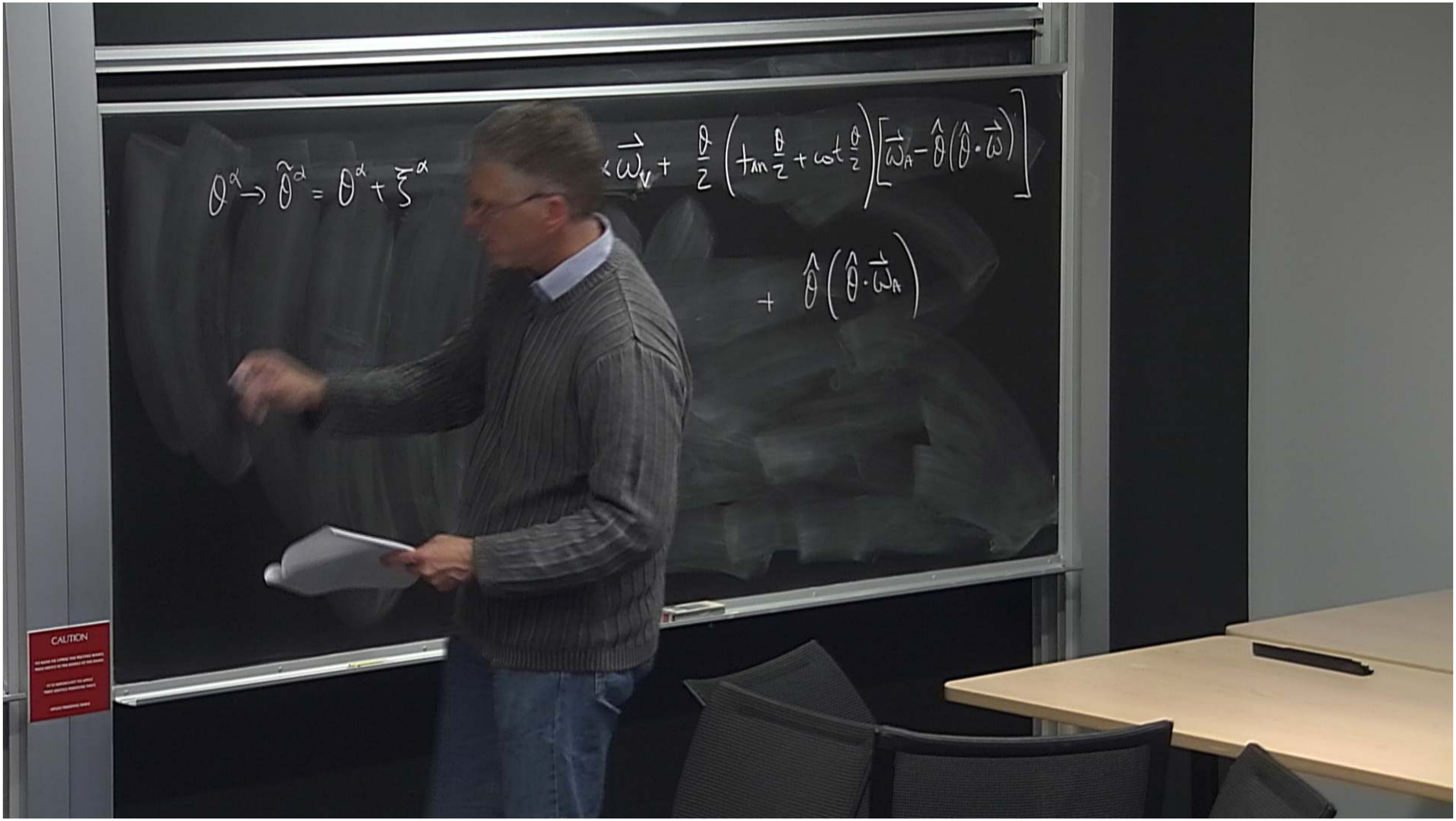
$$e^{i\alpha^a \tau_a} = \cos\alpha + i(\sin\alpha)\,\hat{\alpha}^a \tau_a$$

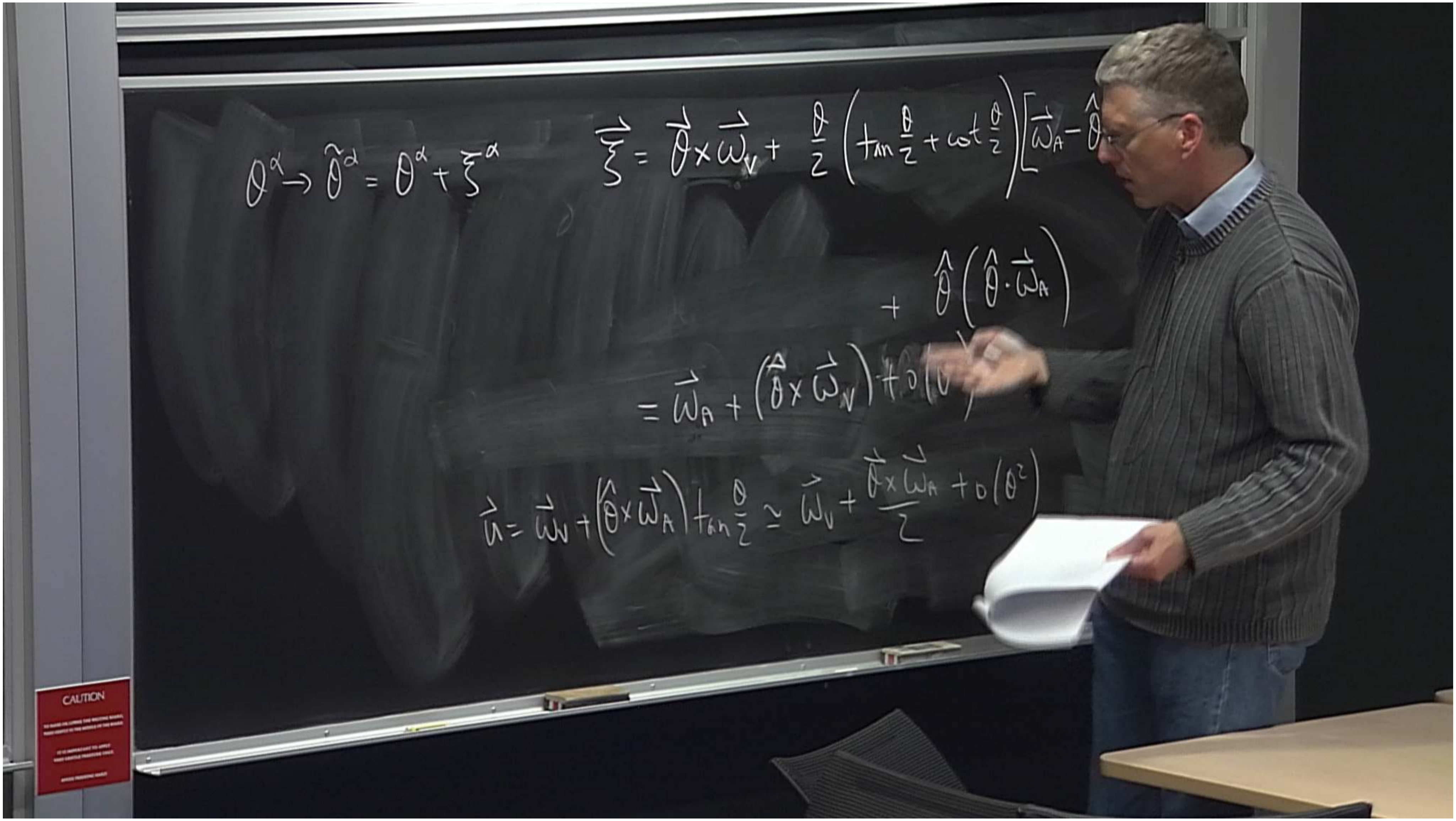
$$\alpha = \sqrt{(\alpha^1)^2 + (\alpha^2)^2 + (\alpha^3)^2} \qquad \hat{\alpha}^a = \frac{\alpha^a}{\alpha}$$

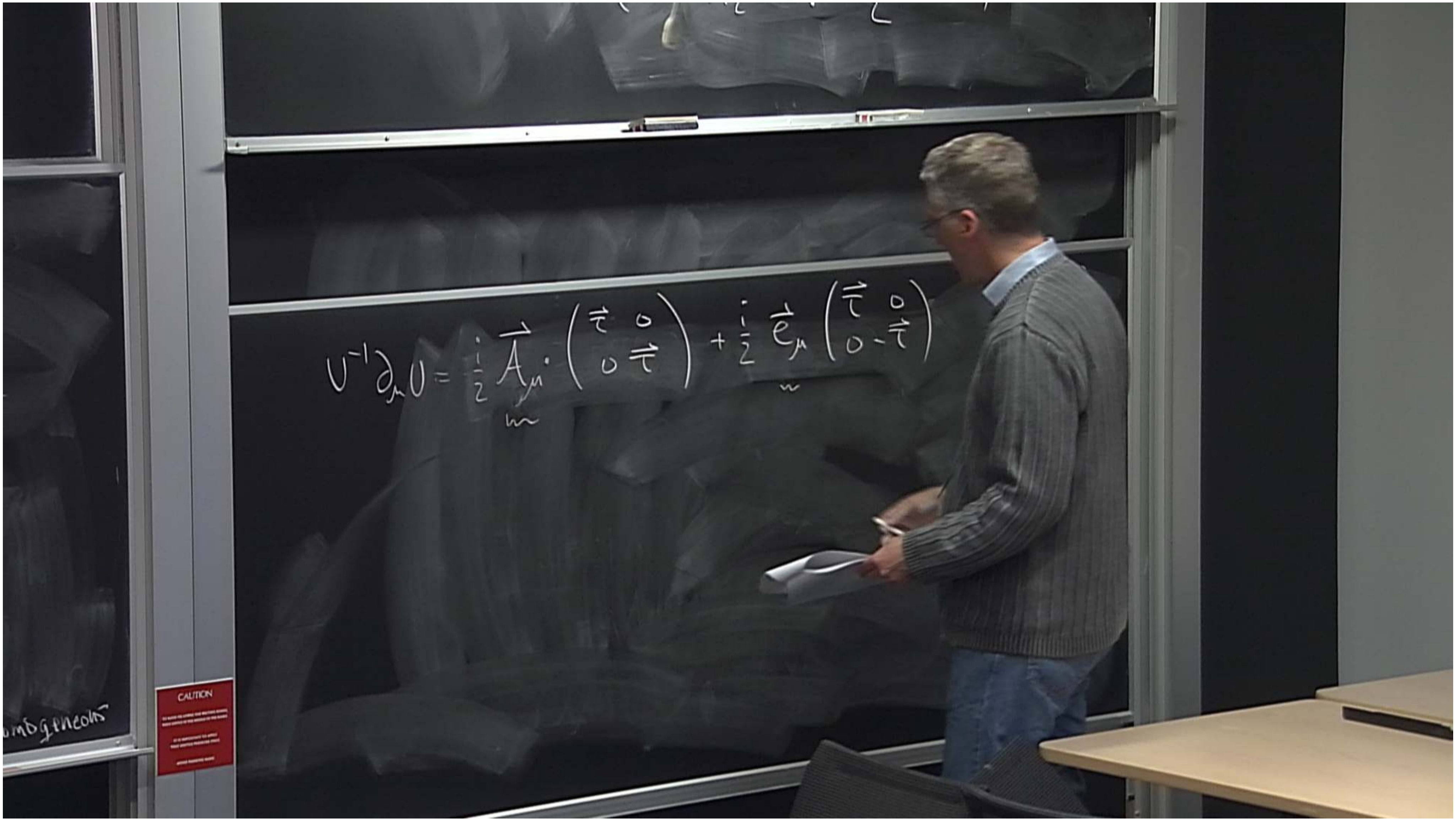
$$\tilde{\theta} = \tilde{\theta}(\theta, g)$$

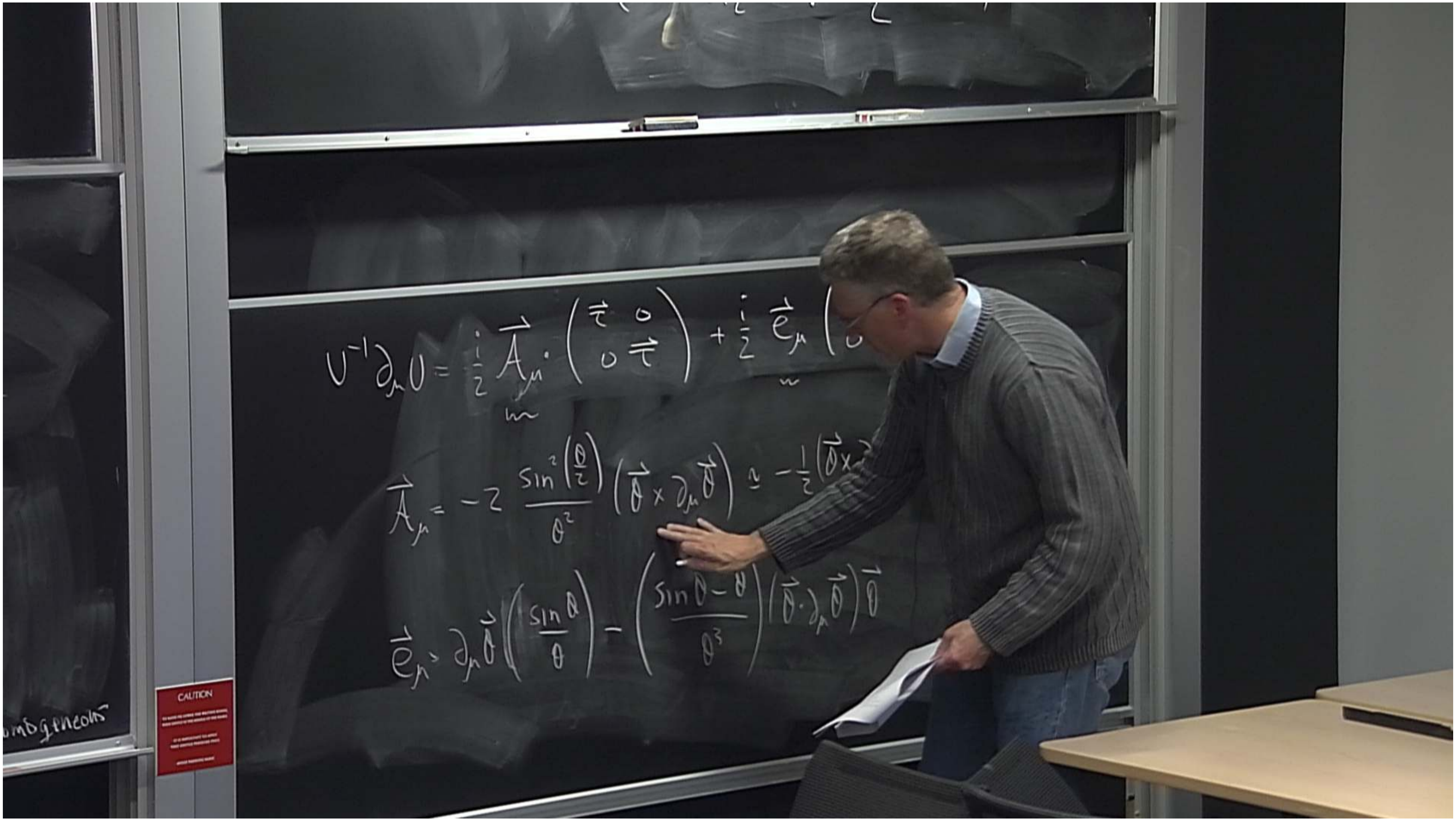
$$gU = \tilde{U}\gamma$$

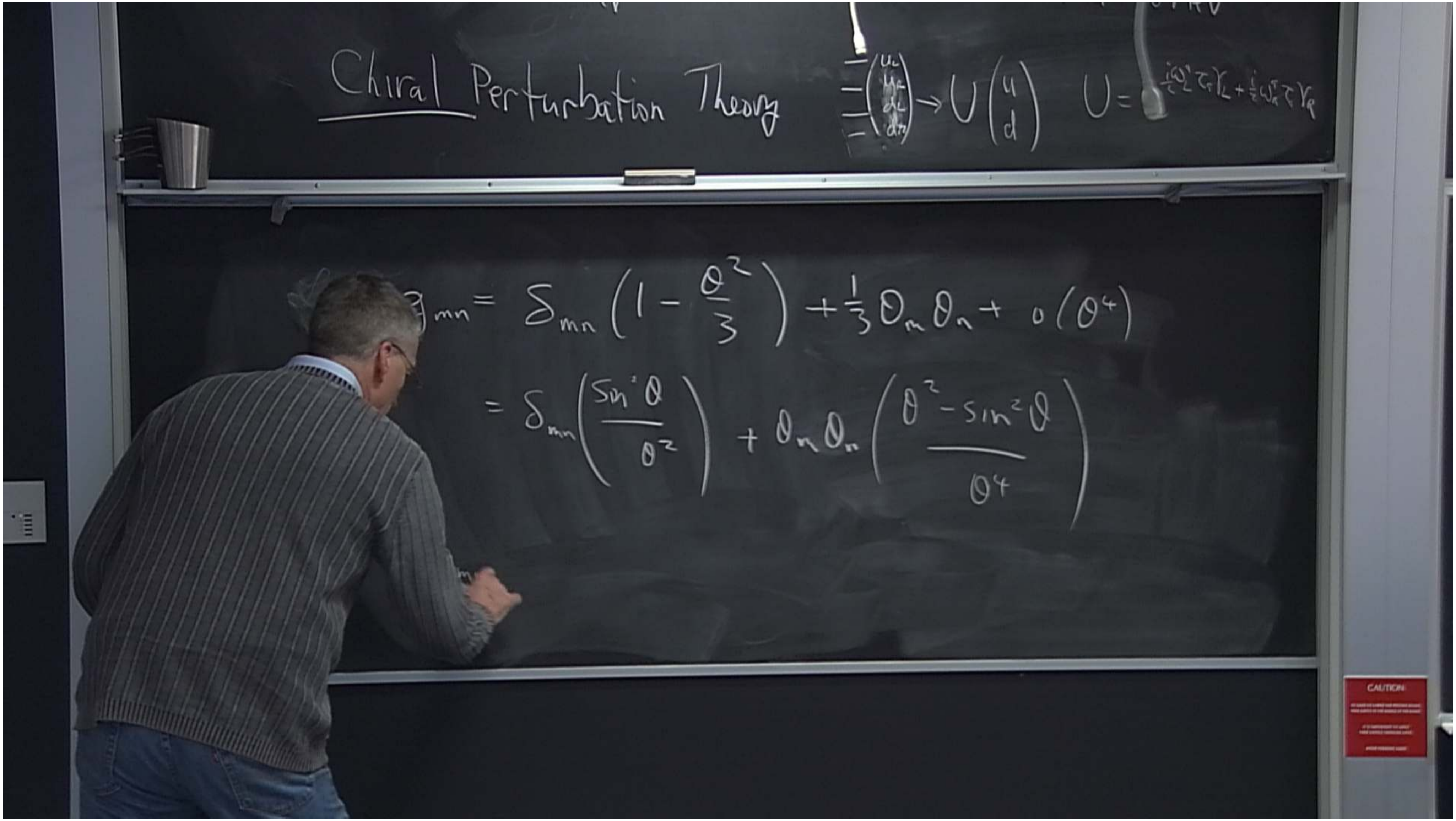
+homogeneous

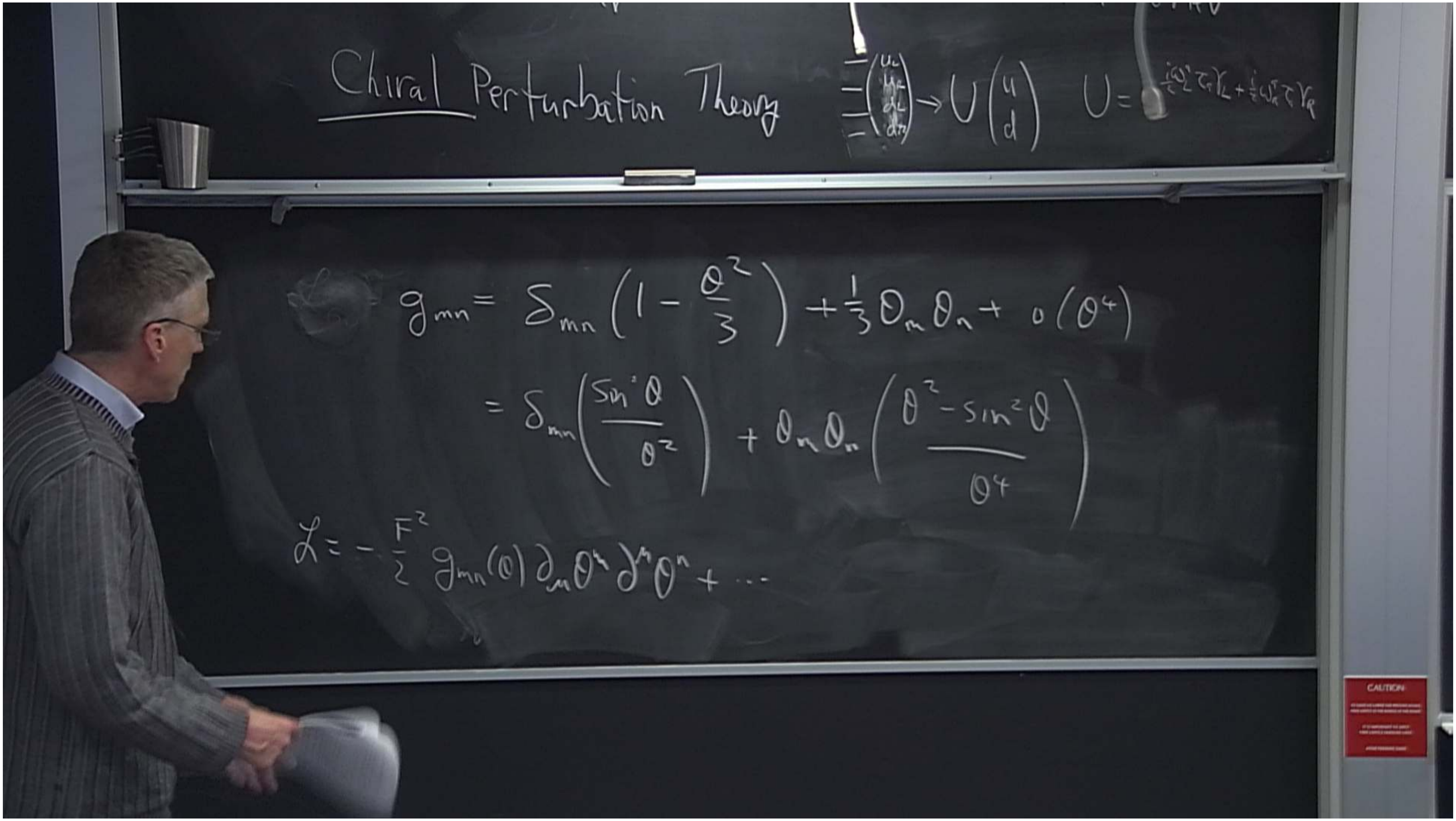

$\theta^\alpha \to \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha$
$\vec{\omega}_V + \frac{\theta}{2}\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\left[\vec{\omega}_A - \hat{\theta}\left(\hat{\theta}\cdot\vec{\omega}\right)\right]$
$+ \hat{\theta}\left(\hat{\theta}\cdot\vec{\omega}_A\right)$
CAUTION


$\theta^\alpha \to \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha$
$\vec{\xi} = \vec{\theta} \times \vec{\omega}_V + \frac{\theta}{2}\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\left[\vec{\omega}_A - \hat{\theta}\right.$
$+ \hat{\theta}\left(\hat{\theta}\cdot\vec{\omega}_A\right)$
$= \vec{\omega}_A + \left(\hat{\theta} \times \vec{\omega}_V\right) + O(\theta)$
$\vec{u} = \vec{\omega}_V + \left(\hat{\theta} \times \vec{\omega}_A\right)\tan\frac{\theta}{2} \simeq \vec{\omega}_V + \frac{\vec{\theta} \times \vec{\omega}_A}{2} + O(\theta^2)$
CAUTION

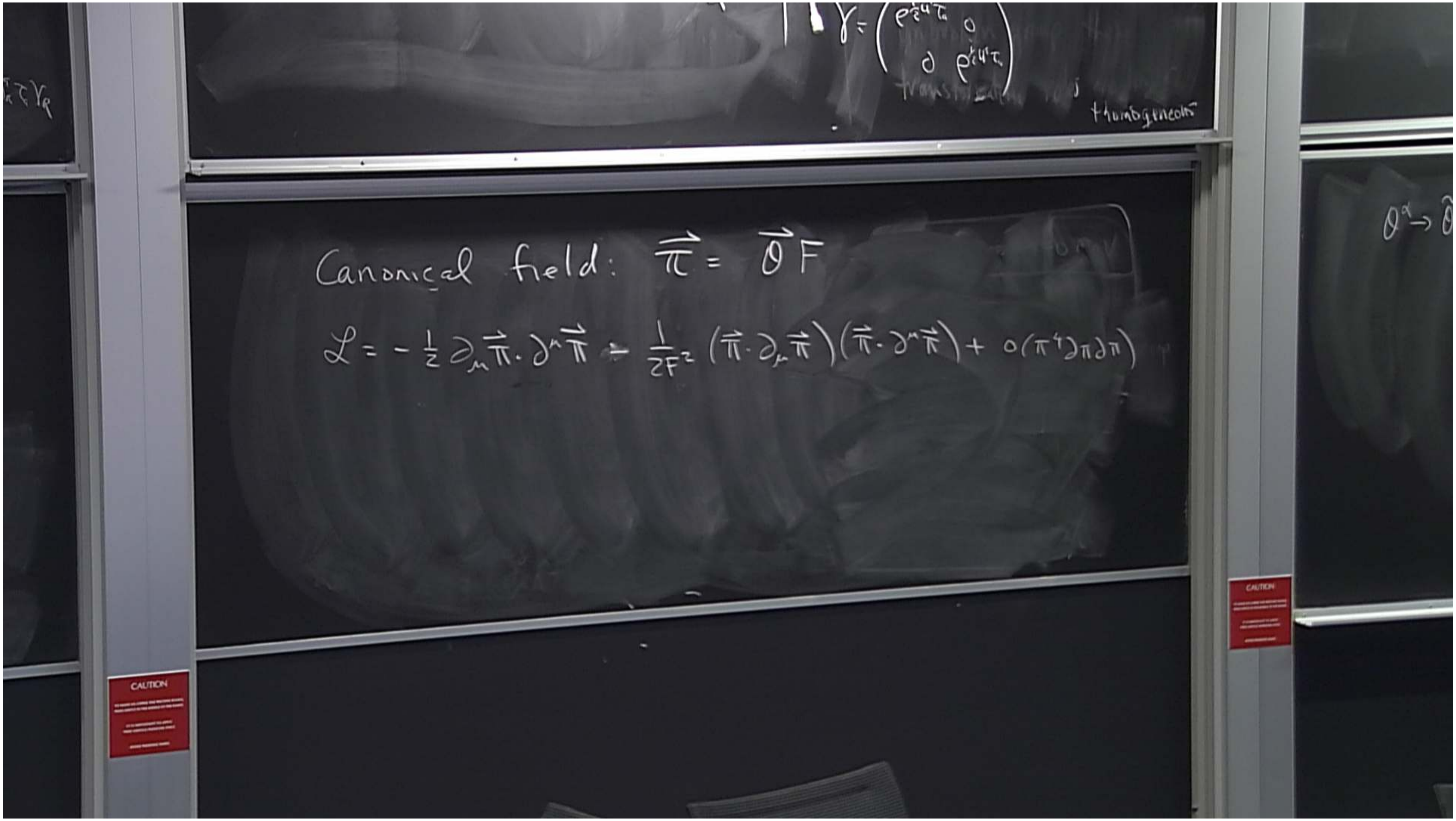

$U^{-1}\partial_\mu U = \frac{i}{2}\vec{A}_\mu \cdot \begin{pmatrix} \vec{\tau} & 0 \\ 0 & \vec{\tau} \end{pmatrix} + \frac{i}{2}\vec{e}_\mu \begin{pmatrix} \vec{\tau} & 0 \\ 0 & -\vec{\tau} \end{pmatrix}$
CAUTION

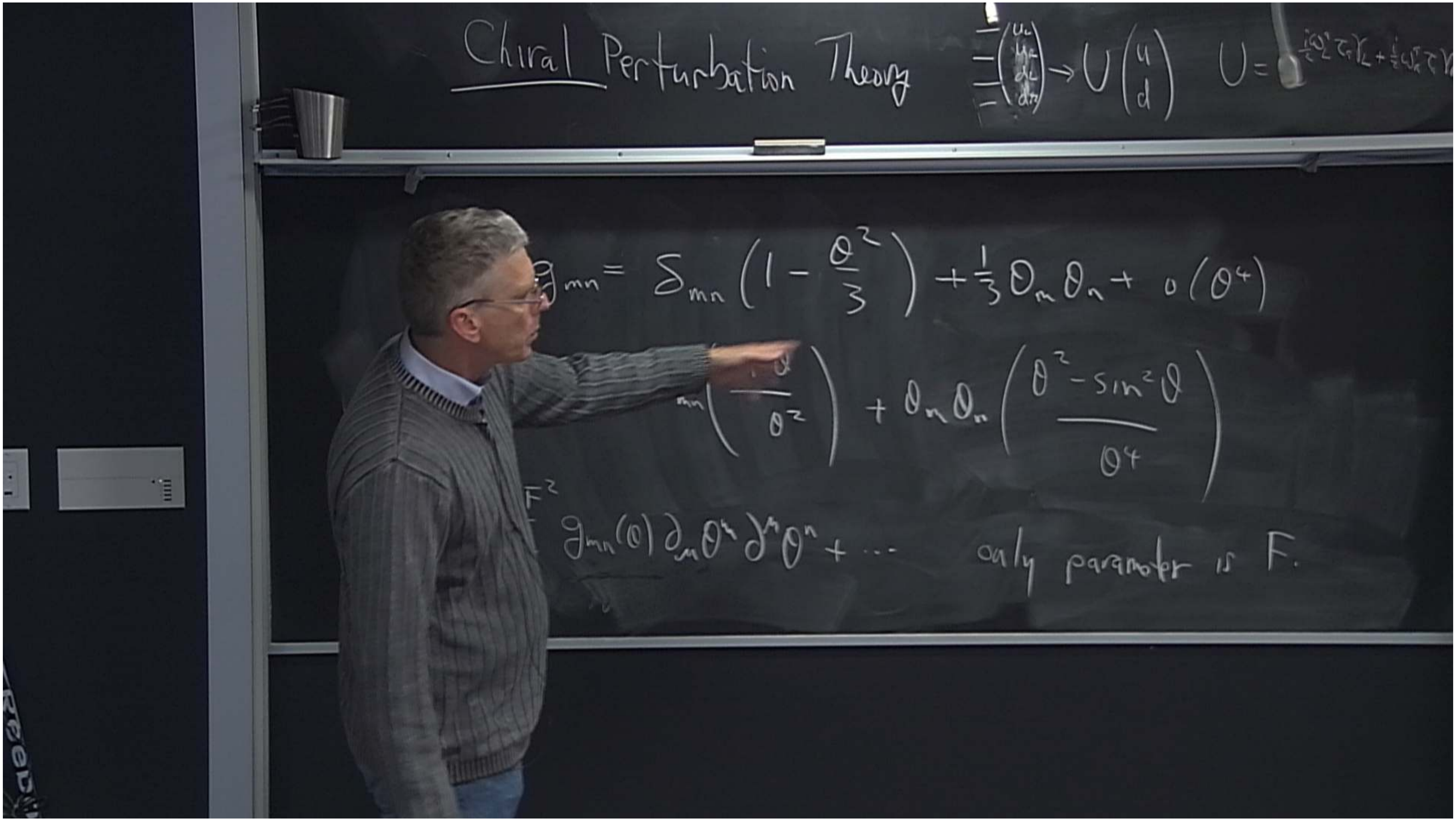

$U^{-1}\partial_\mu U = \frac{i}{2}\vec{A}_\mu \cdot \begin{pmatrix}\vec\tau & 0\\ 0 & \vec\tau\end{pmatrix} + \frac{i}{2}\vec{e}_\mu \Big($
$\vec{A}_\mu = -2\,\frac{\sin^2\left(\frac{\theta}{2}\right)}{\theta^2}\left(\vec\theta\times\partial_\mu\vec\theta\right) \simeq -\frac{1}{2}\left(\vec\theta\times\right.$
$\vec{e}_\mu = \partial_\mu\vec\theta\left(\frac{\sin\theta}{\theta}\right) - \left(\frac{\sin\theta-\theta}{\theta^3}\right)\left(\vec\theta\cdot\partial_\mu\vec\theta\right)\vec\theta$
CAUTION

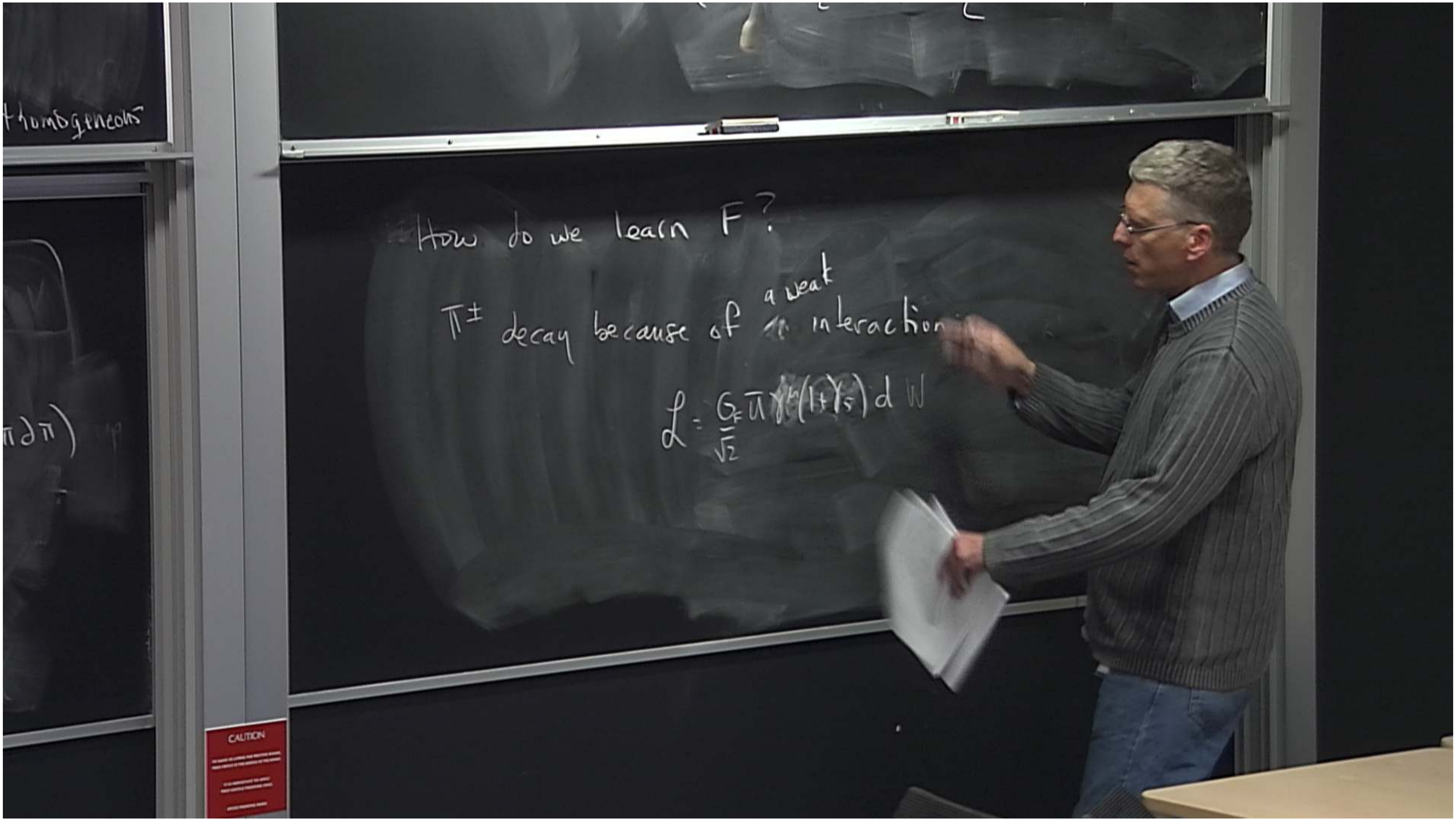

Chiral Perturbation Theory
$g_{mn} = \delta_{mn}\left(1 - \frac{\theta^2}{3}\right) + \frac{1}{3}\theta_m\theta_n + O(\theta^4)$
$= \delta_{mn}\left(\frac{\sin^2\theta}{\theta^2}\right) + \theta_m\theta_n\left(\frac{\theta^2 - \sin^2\theta}{\theta^4}\right)$
CAUTION

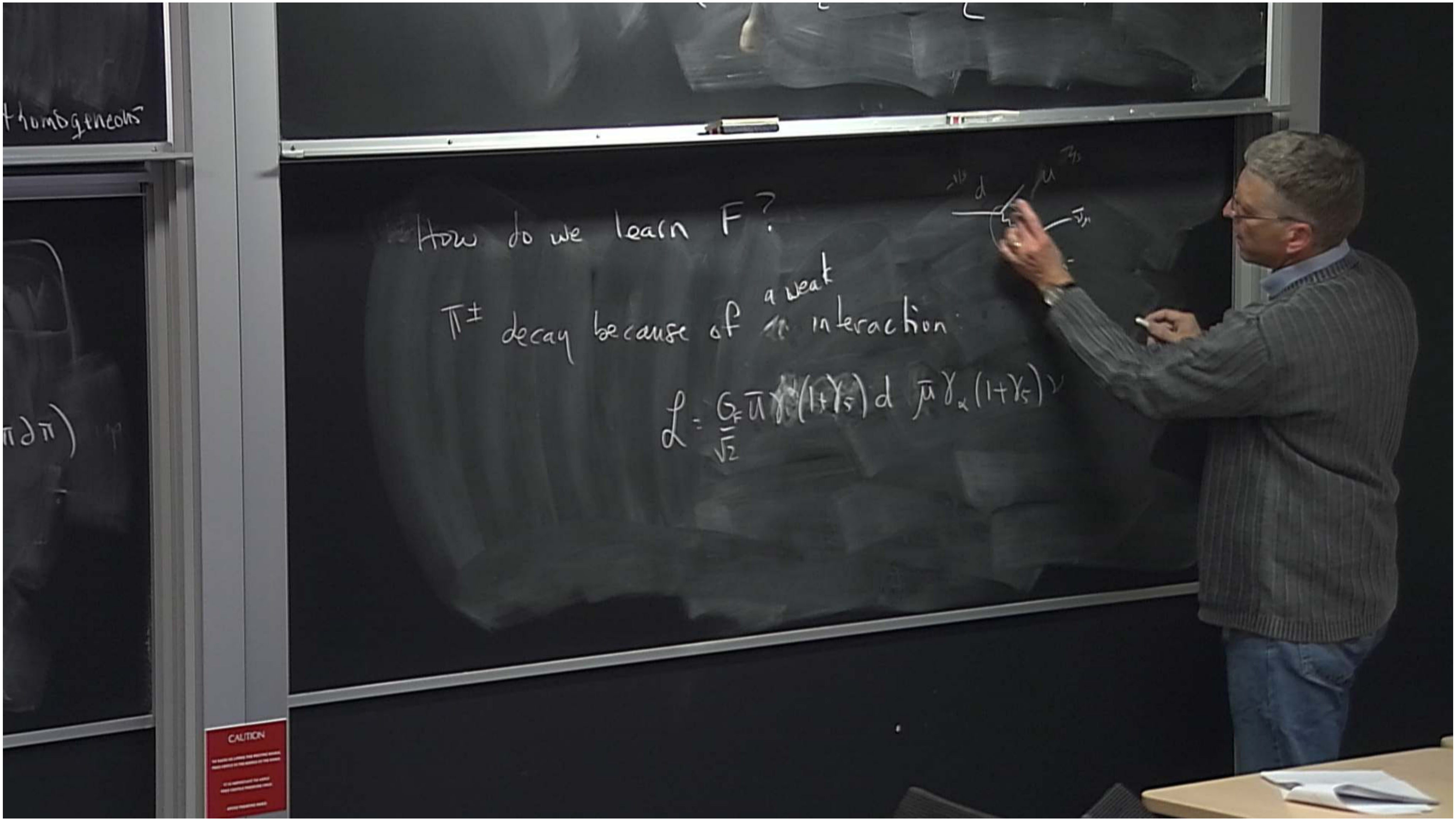


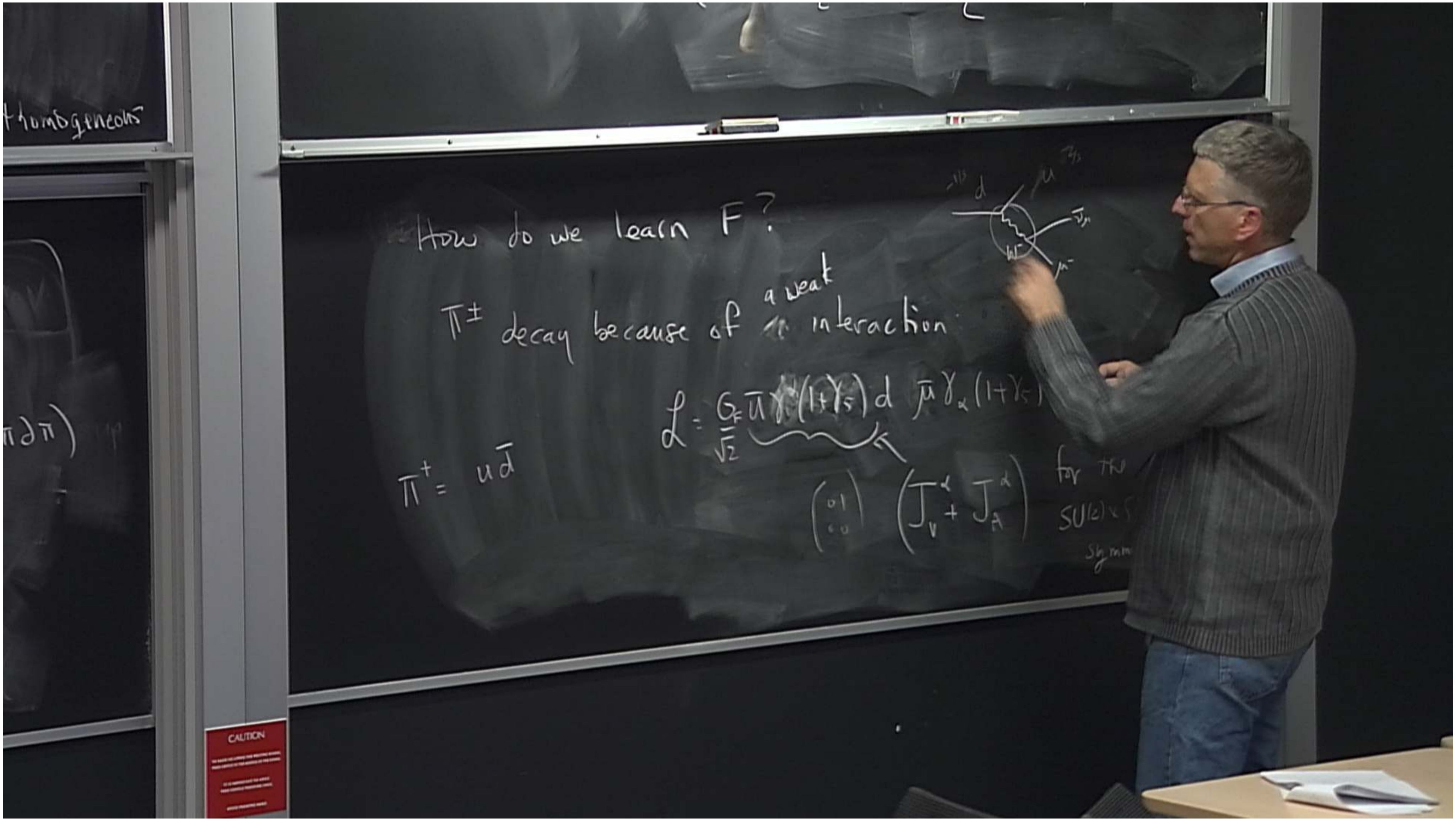
Chiral Perturbation Theory
$g_{mn} = \delta_{mn}\left(1 - \frac{\theta^2}{3}\right) + \frac{1}{3}\theta_m\theta_n + O(\theta^4)$
$= \delta_{mn}\left(\frac{\sin^2\theta}{\theta^2}\right) + \theta_m\theta_n\left(\frac{\theta^2 - \sin^2\theta}{\theta^4}\right)$
$\mathcal{L} = -\frac{F^2}{2} g_{mn}(\theta)\,\partial_\mu\theta^m\,\partial^\mu\theta^n + \cdots$

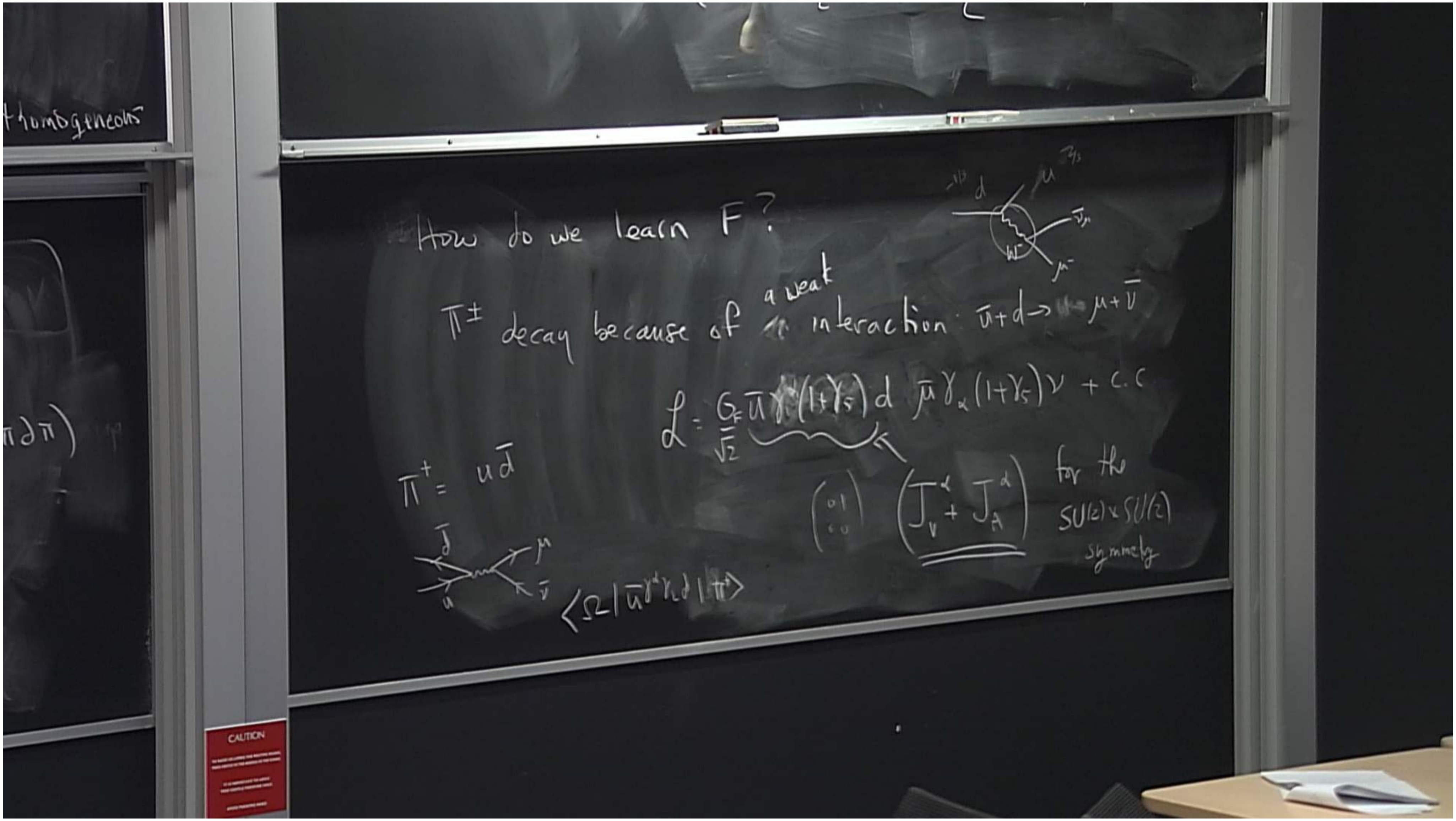

Canonical field: $\vec{\pi} = \vec{\theta} F$
$\mathcal{L} = -\frac{1}{2}\partial_\mu \vec{\pi}\cdot\partial^\mu \vec{\pi} - \frac{1}{2F^2}(\vec{\pi}\cdot\partial_\mu \vec{\pi})(\vec{\pi}\cdot\partial^\mu \vec{\pi}) + O(\pi^4 \partial\pi\partial\pi)$


Chiral Perturbation Theory
only parameter is F.


How do we learn F?
π± decay because of a weak interaction
L = G_F/√2 ū γ^μ (1+γ_5) d W
CAUTION


How do we learn F?
π± decay because of a weak interaction
$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{u} \gamma^{\alpha} (1+\gamma_5) d \; \bar{\mu} \gamma_{\alpha} (1+\gamma_5) \nu$
CAUTION


How do we learn F?
π± decay because of a weak interaction
π+ = u d̄
for the


How do we learn F?
π± decay because of a weak interaction: ū+d → μ+ν̄
L = G_F/√2 ū γ^α (1+γ5) d μ̄ γ_α (1+γ5) ν + c.c.
π+ = u d̄
J_V^α + J_A^α for the SU(2)×SU(2) symmetry
⟨Ω| ū γ^α γ5 d |π+⟩

Chiral Perturbation Theory

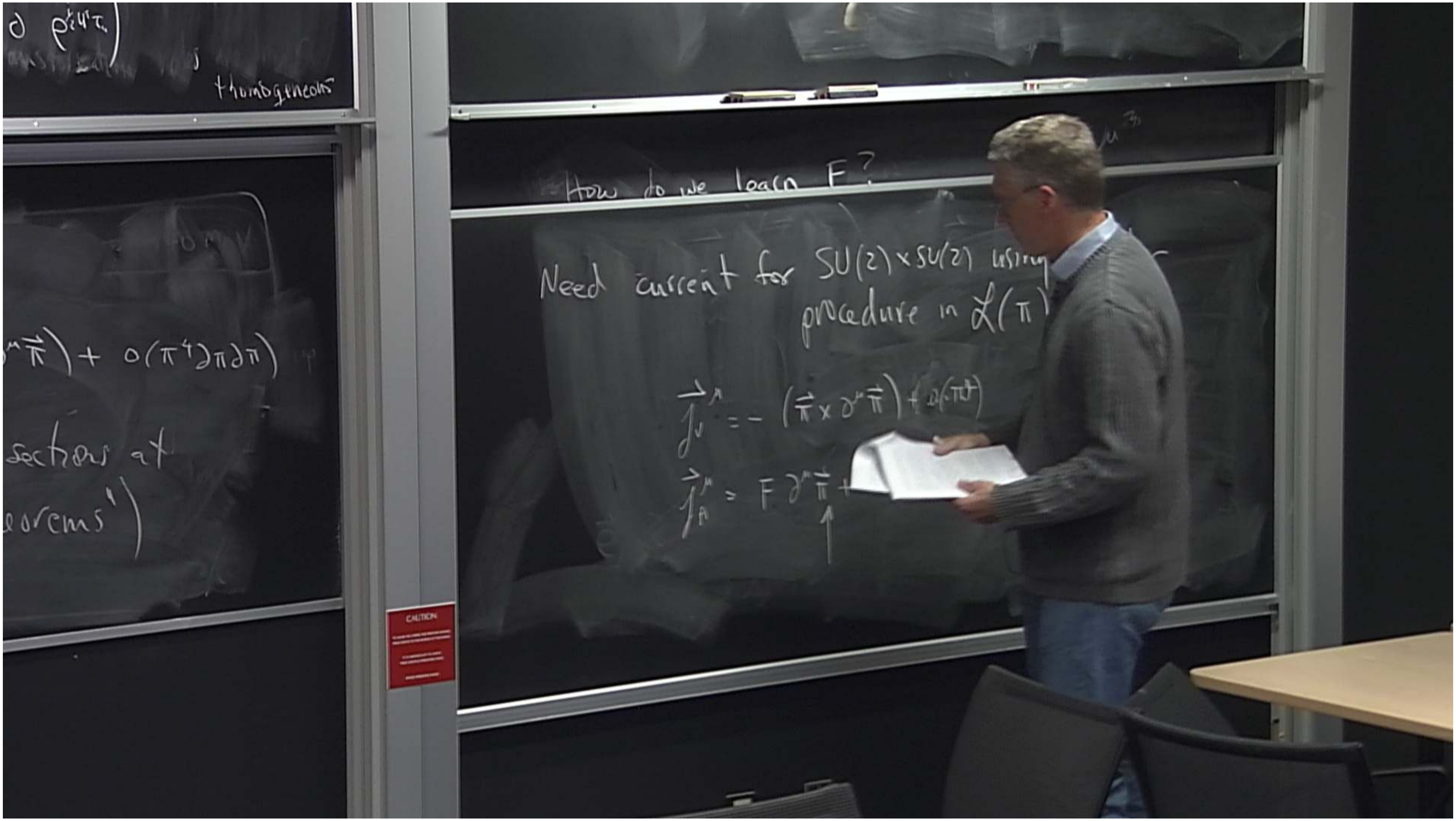
$$\begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix} \quad U = e^{\frac{i}{2}\omega_L^i \tau_i \gamma_L + \frac{i}{2}\omega_R^i \tau_i \gamma_R}$$

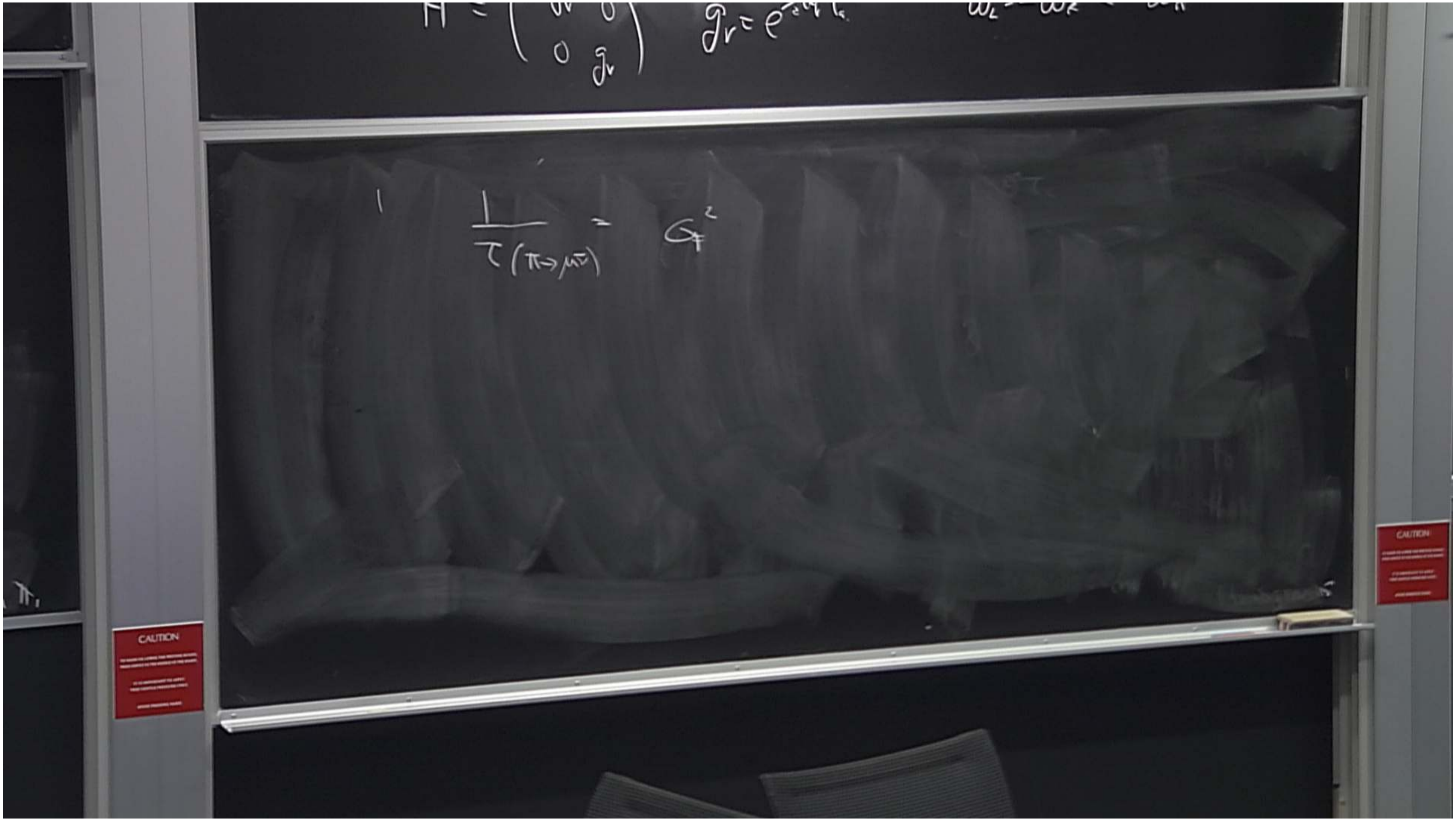
in UV theory

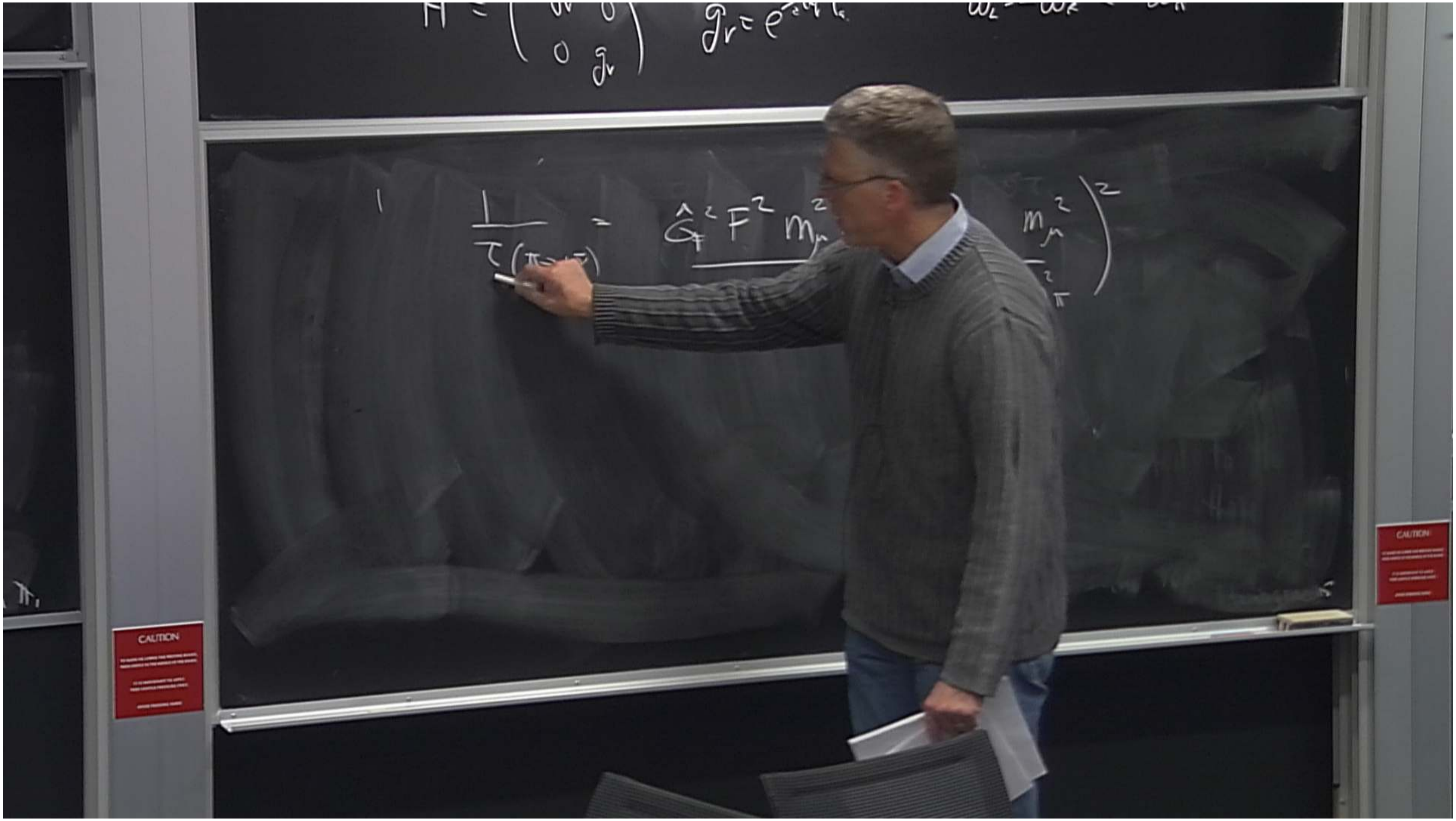
$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \underbrace{J^{\lambda}}_{} \left[\bar{\mu}\gamma_{\lambda}(1+\gamma_5)\nu\right] \to J^{\lambda} \text{ is the current for the generator } \tau_1 \pm i\tau_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

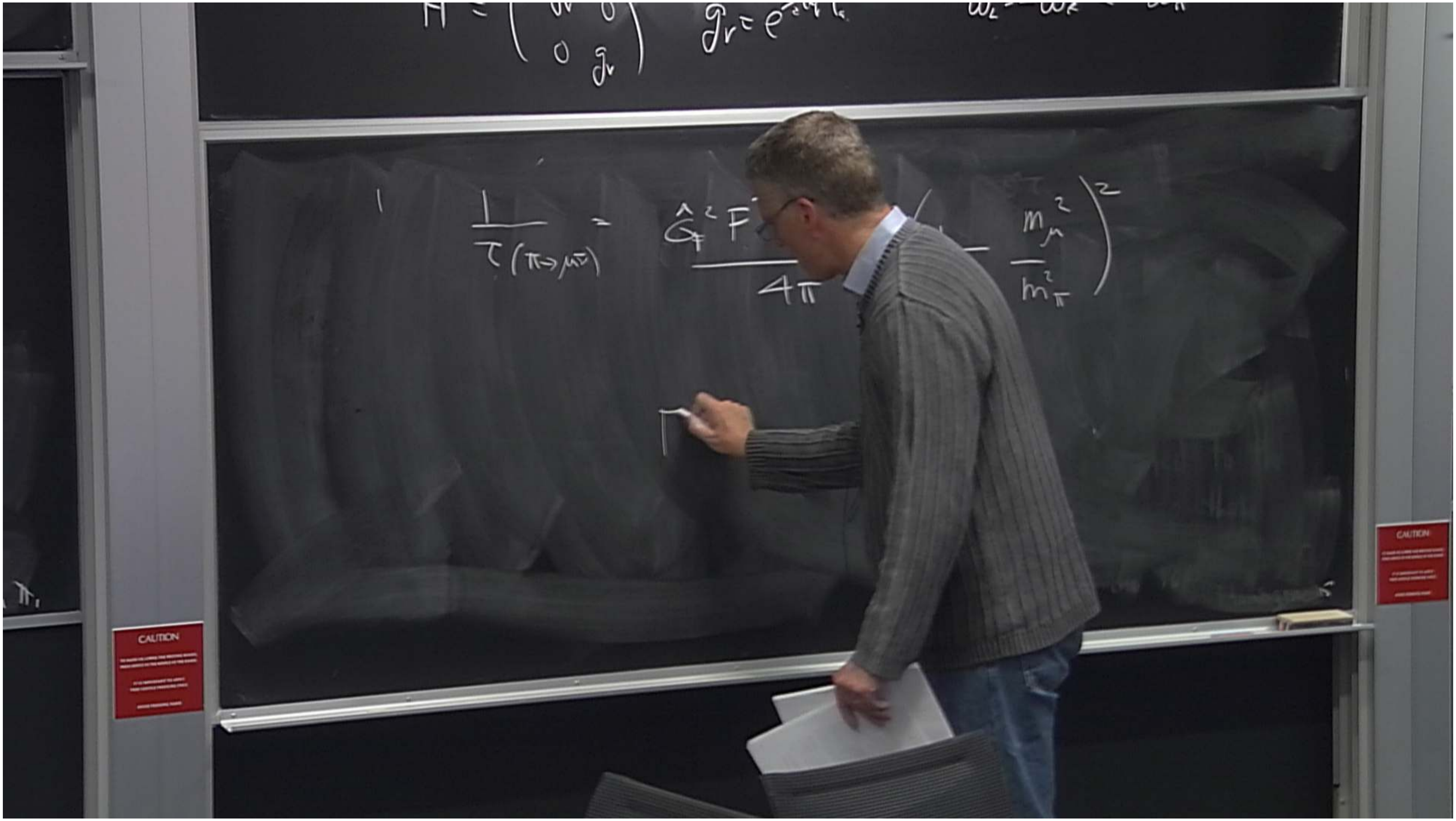
$\to$ in EFT $\quad \mathcal{L}_W = \frac{G_F}{\sqrt{2}} \mathcal{J}^{\lambda} \left[\bar{\mu}\gamma_{\lambda}(1+\gamma_5)\nu\right]$

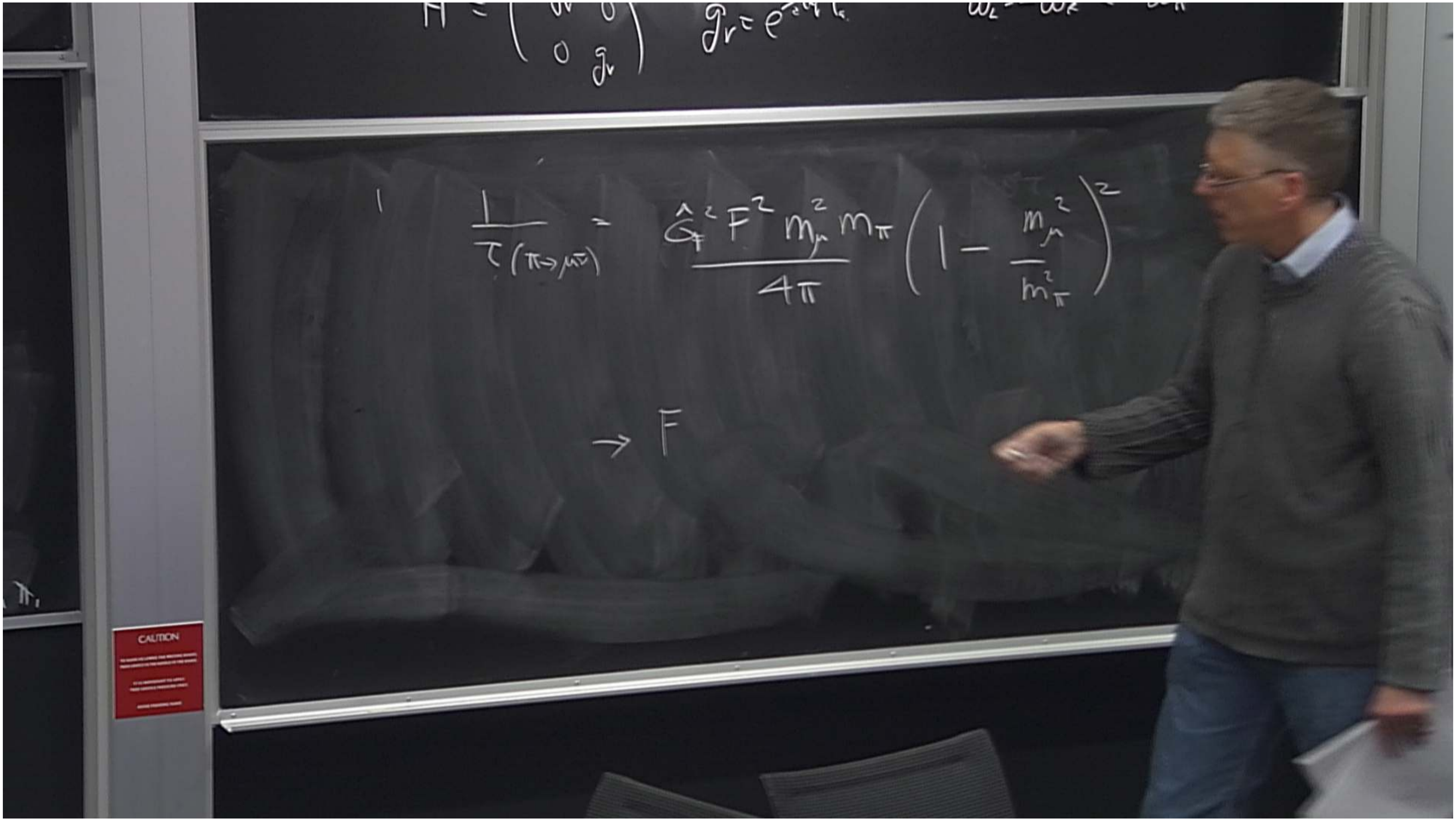
$\mathcal{J}^{\lambda}$ " " " : built from π.

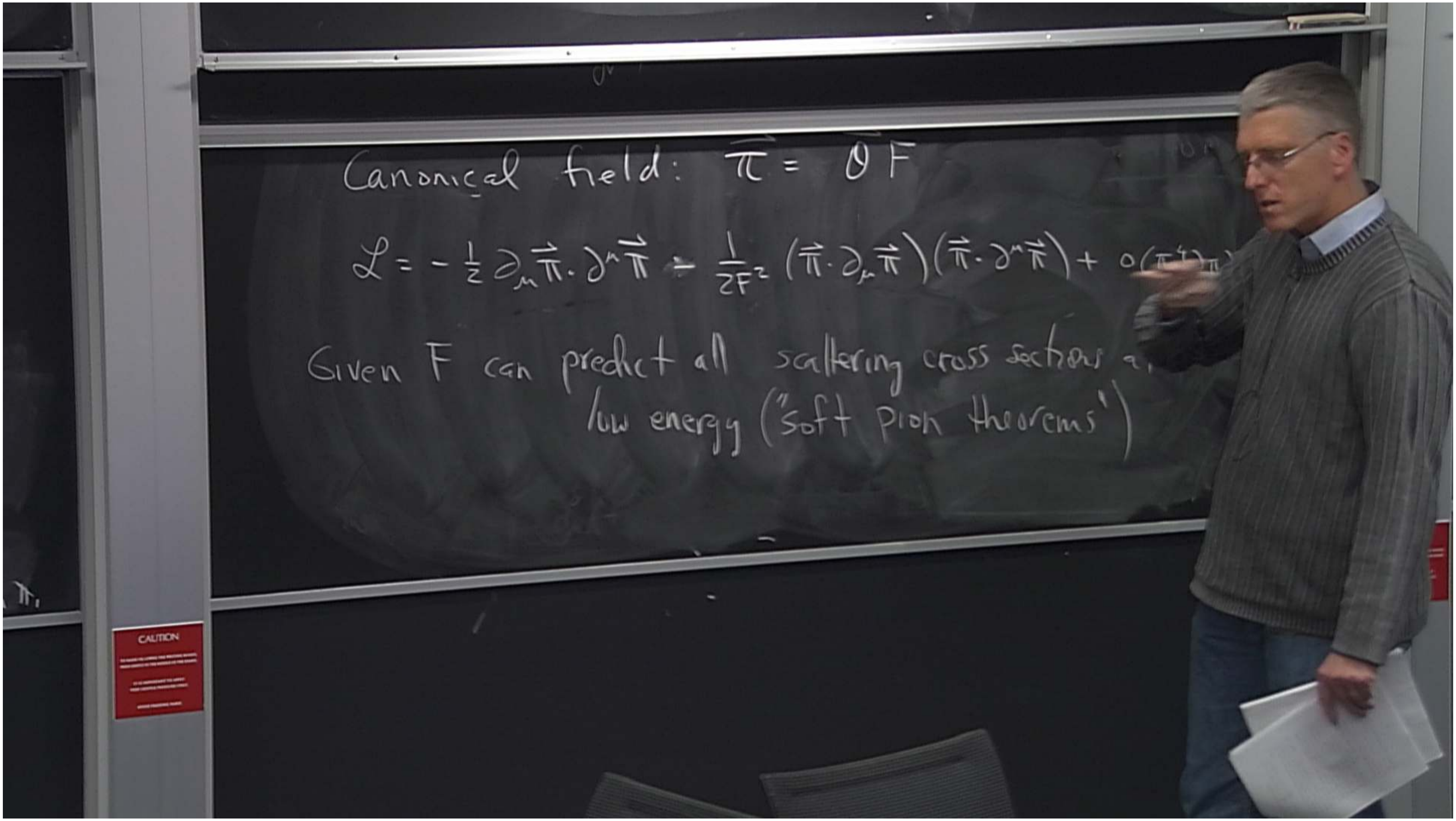

How do we learn F?
Need current for SU(2) x SU(2)
procedure in L(π)
CAUTION

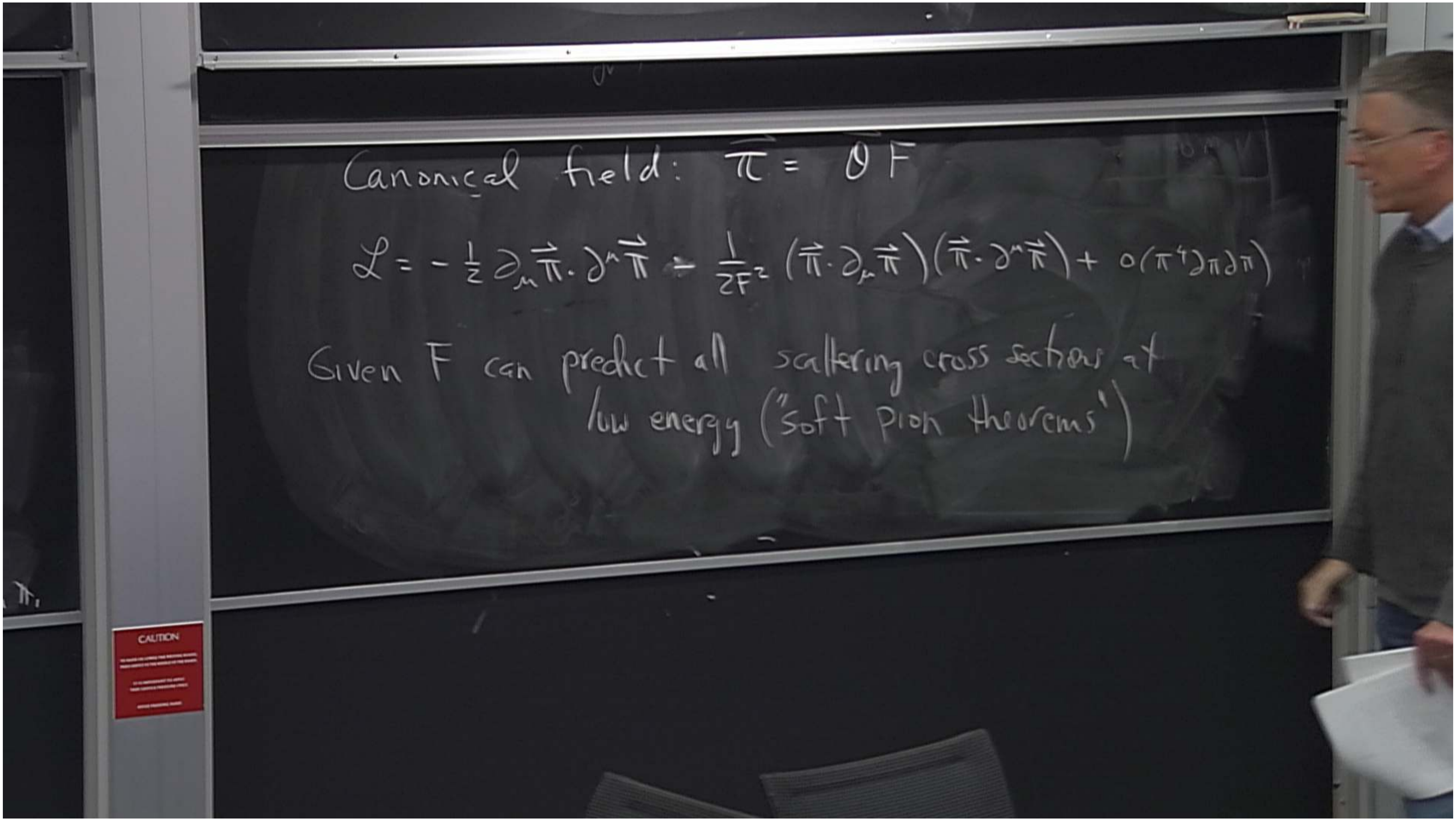

$\frac{1}{\tau(\pi \to \mu \bar{\nu})} = G_F^2$
CAUTION
CAUTION




CAUTION
CAUTION


1/τ(π→μν̄) = Ĝ² F² m_μ² m_π / 4π (1 − m_μ²/m_π²)²
→ F
CAUTION


Canonical field: $\vec{\pi} = \theta F$
$\mathcal{L} = -\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2F^2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})(\vec{\pi}\cdot\partial^\mu\vec{\pi}) + O($
Given F can predict all scattering cross sections a
low energy ("soft pion theorems")
CAUTION


Canonical field: $\vec{\pi} = \theta F$
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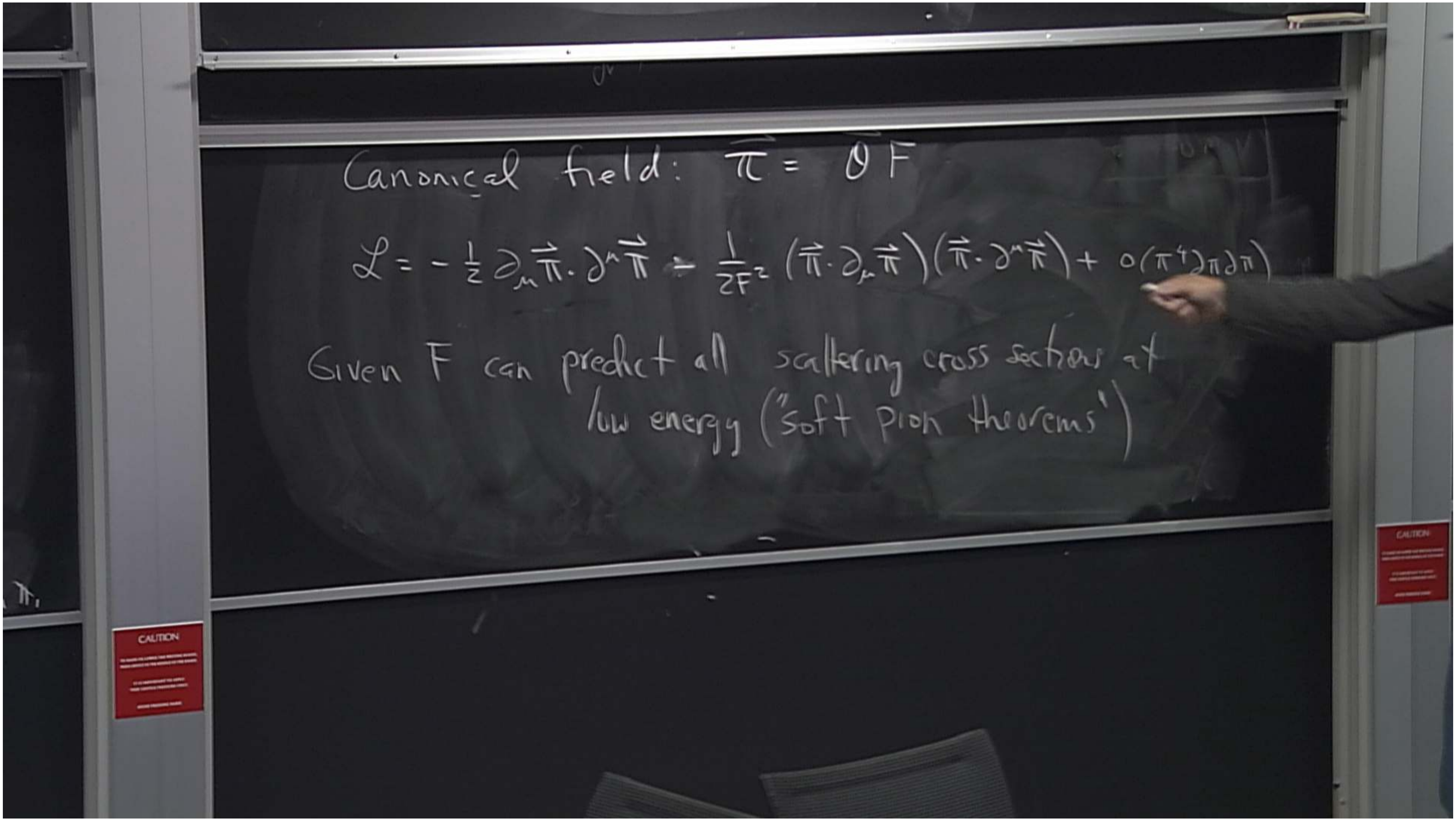
Given F can predict all scattering cross sections at low energy ("soft pion theorems")


Canonical field: $\vec{\pi} = \theta F$
$\mathcal{L} = -\frac{1}{2}\partial_\mu \vec{\pi}\cdot\partial^\mu \vec{\pi} - \frac{1}{2F^2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})(\vec{\pi}\cdot\partial^\mu\vec{\pi}) + O(\pi^4 \partial\pi\partial\pi)$
Given F can predict all scattering cross sections at
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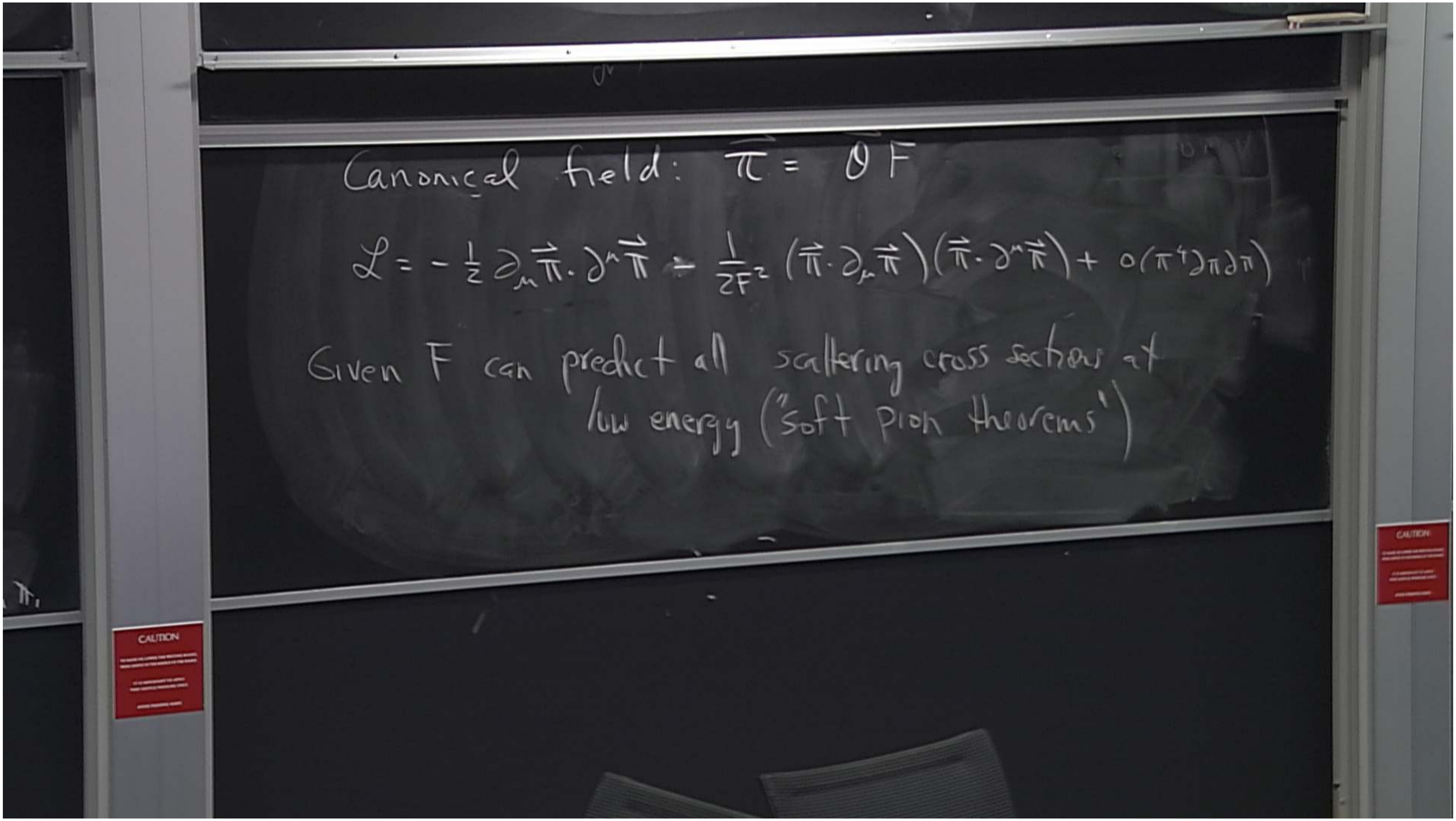
Canonical field: $\vec{\pi} = \theta F$
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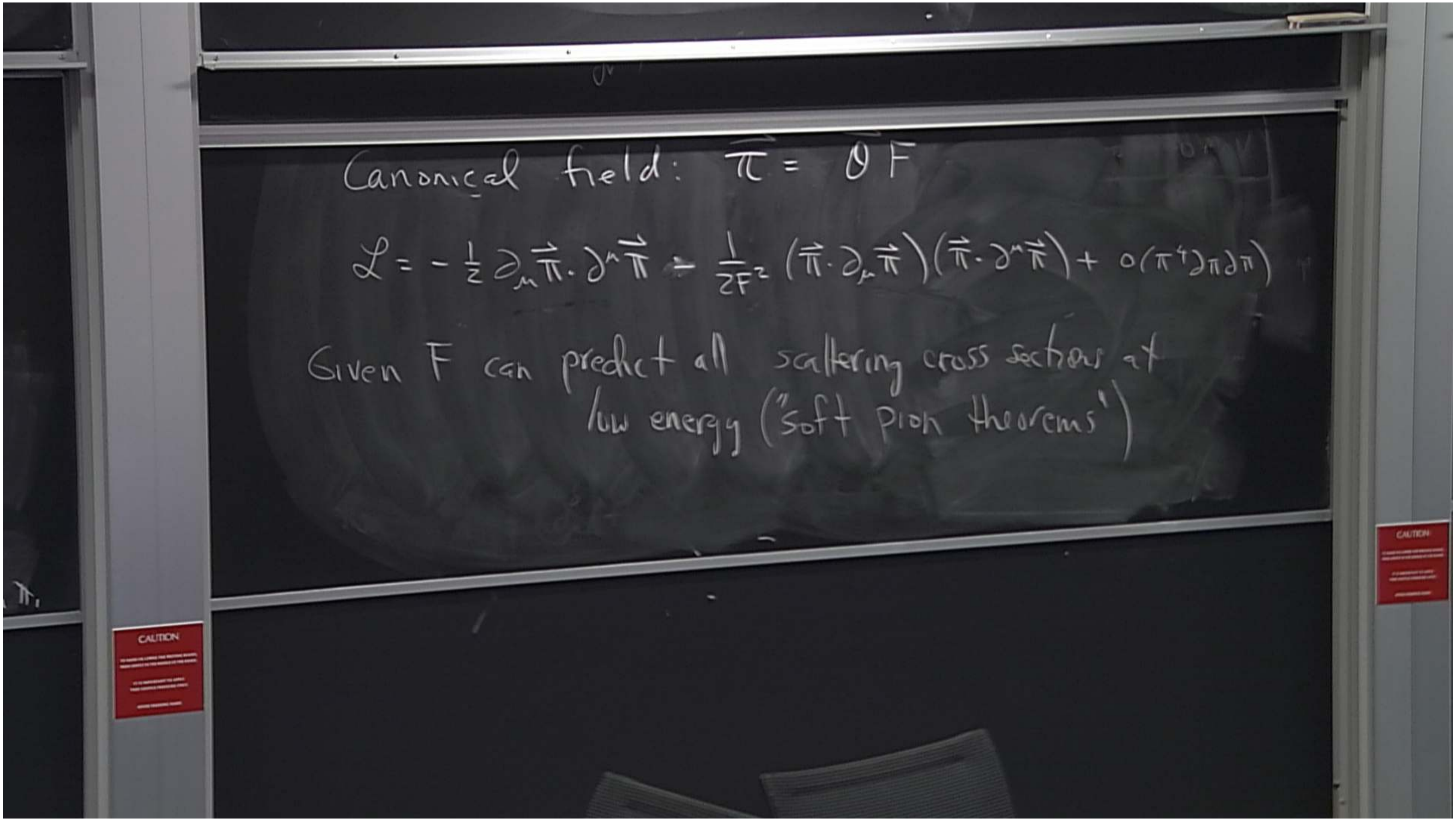
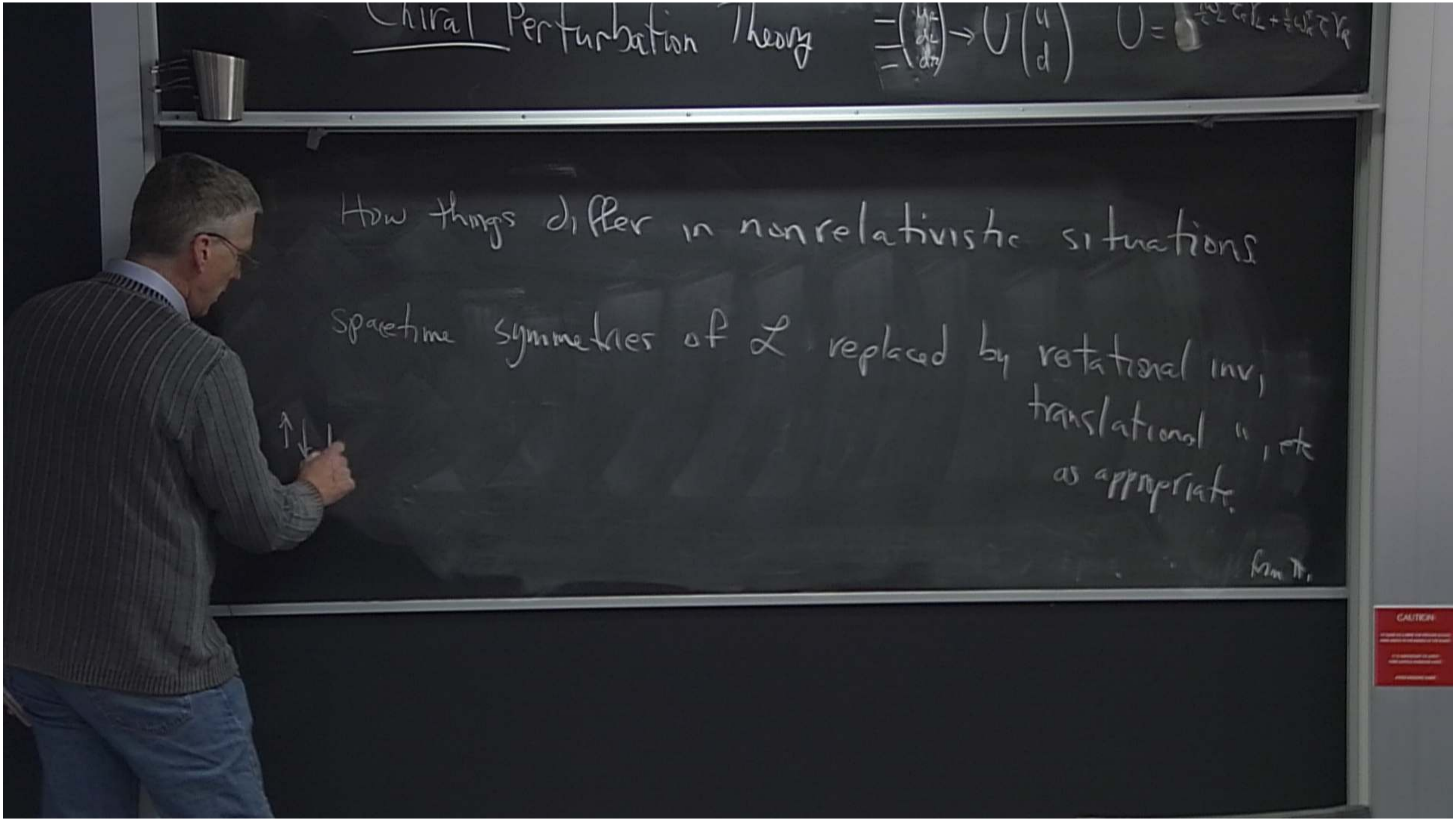
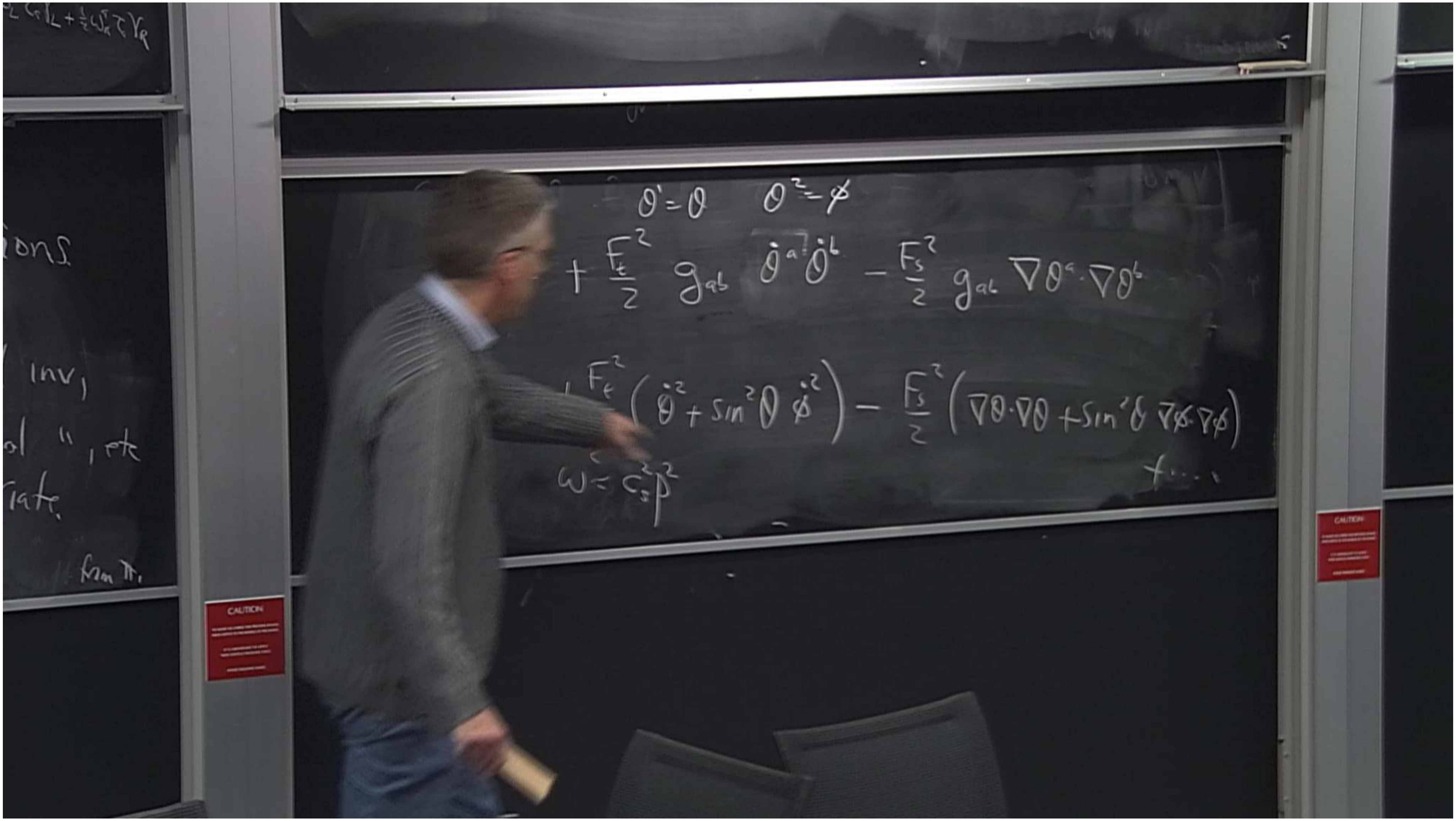
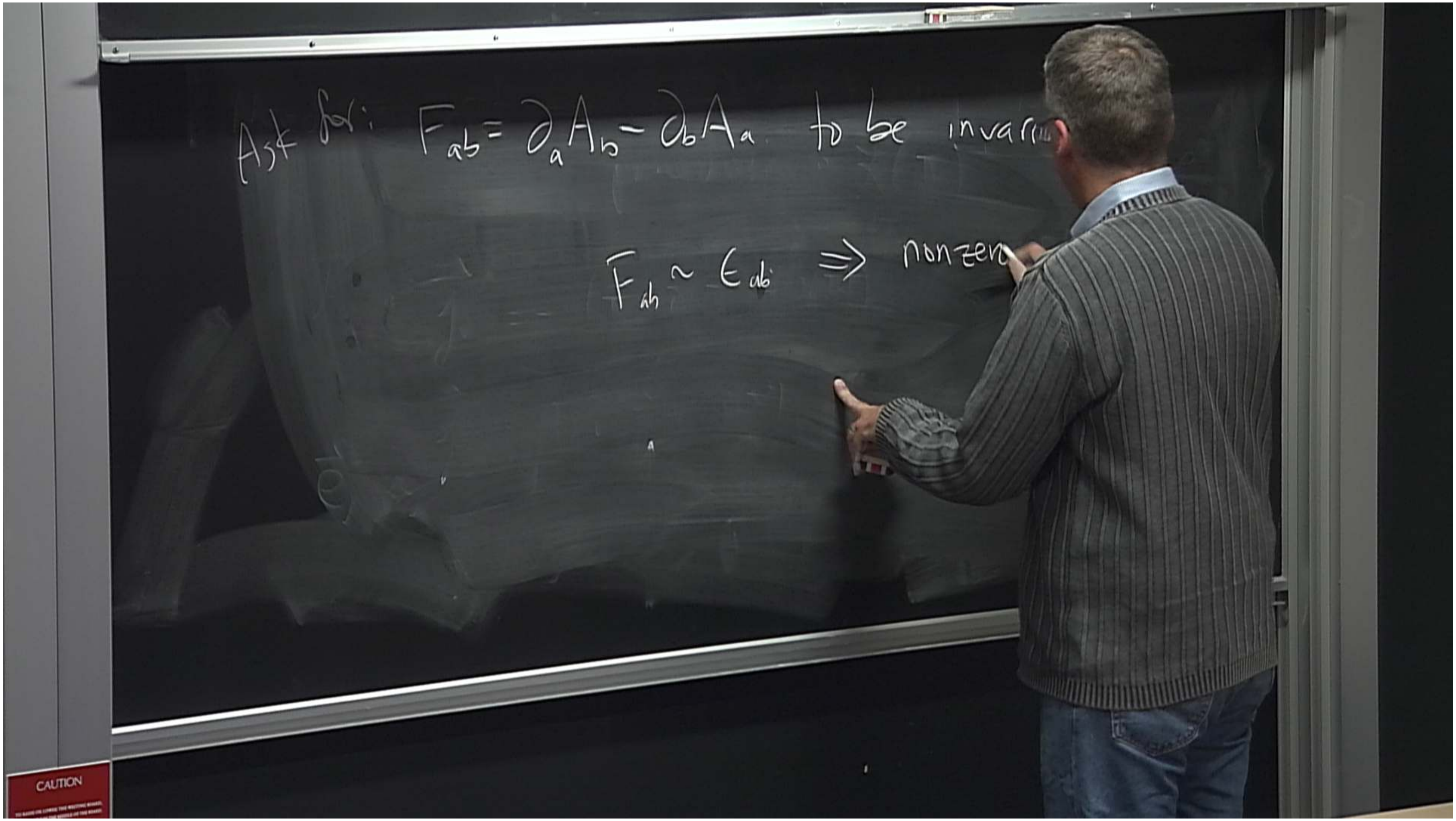
Given F can predict all scattering cross sections at low energy ("soft pion theorems")


Canonical field: $\vec{\pi} = \theta F$
$\mathcal{L} = -\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2F^2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})(\vec{\pi}\cdot\partial^\mu\vec{\pi}) + O(\pi^4\partial\pi\partial\pi)$
Given F can predict all scattering cross sections at
low energy ("soft pion theorems")


Chiral Perturbation Theory
How things differ in nonrelativistic situations
Spacetime symmetries of $\mathcal{L}$ replaced by rotational inv,
translational ", etc
as appropriate.


θ¹=θ   θ²=φ
+ Fₜ²/2 g_ab θ̇ᵃ θ̇ᵇ − Fₛ²/2 g_ab ∇θᵃ·∇θᵇ
Fₜ²/2 (θ̇² + sin²θ φ̇²) − Fₛ²/2 (∇θ·∇θ + sin²θ ∇φ·∇φ)
ω² = cₛ²p²
ions
inv,
" , etc
rate.
CAUTION


Ask for: $F_{ab} = \partial_a A_b - \partial_b A_a$ to be invar
$F_{ab} \sim \epsilon_{ab} \Rightarrow$ nonzer