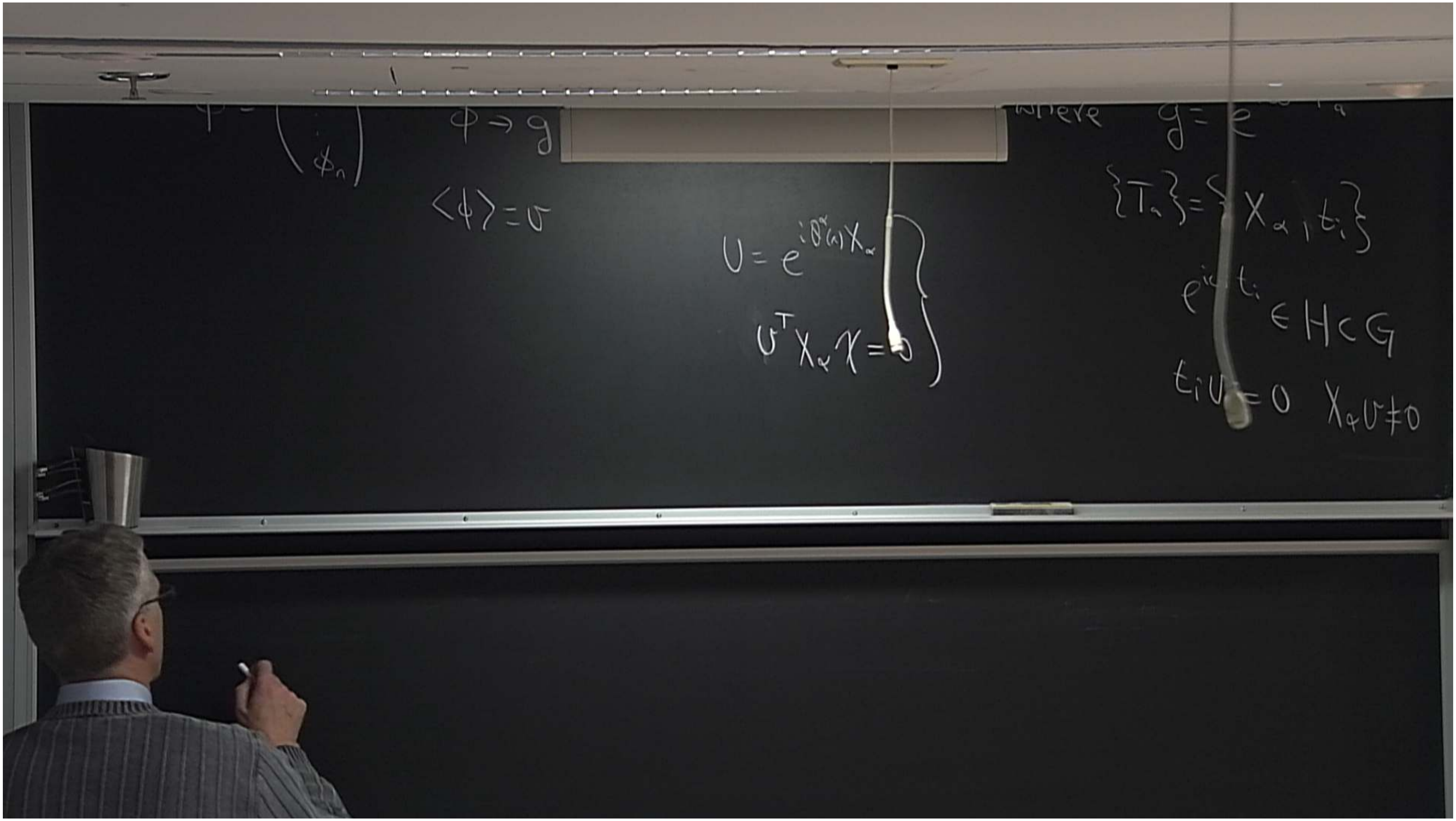
Title: Introduction to Effective Field Theories - Lecture 17

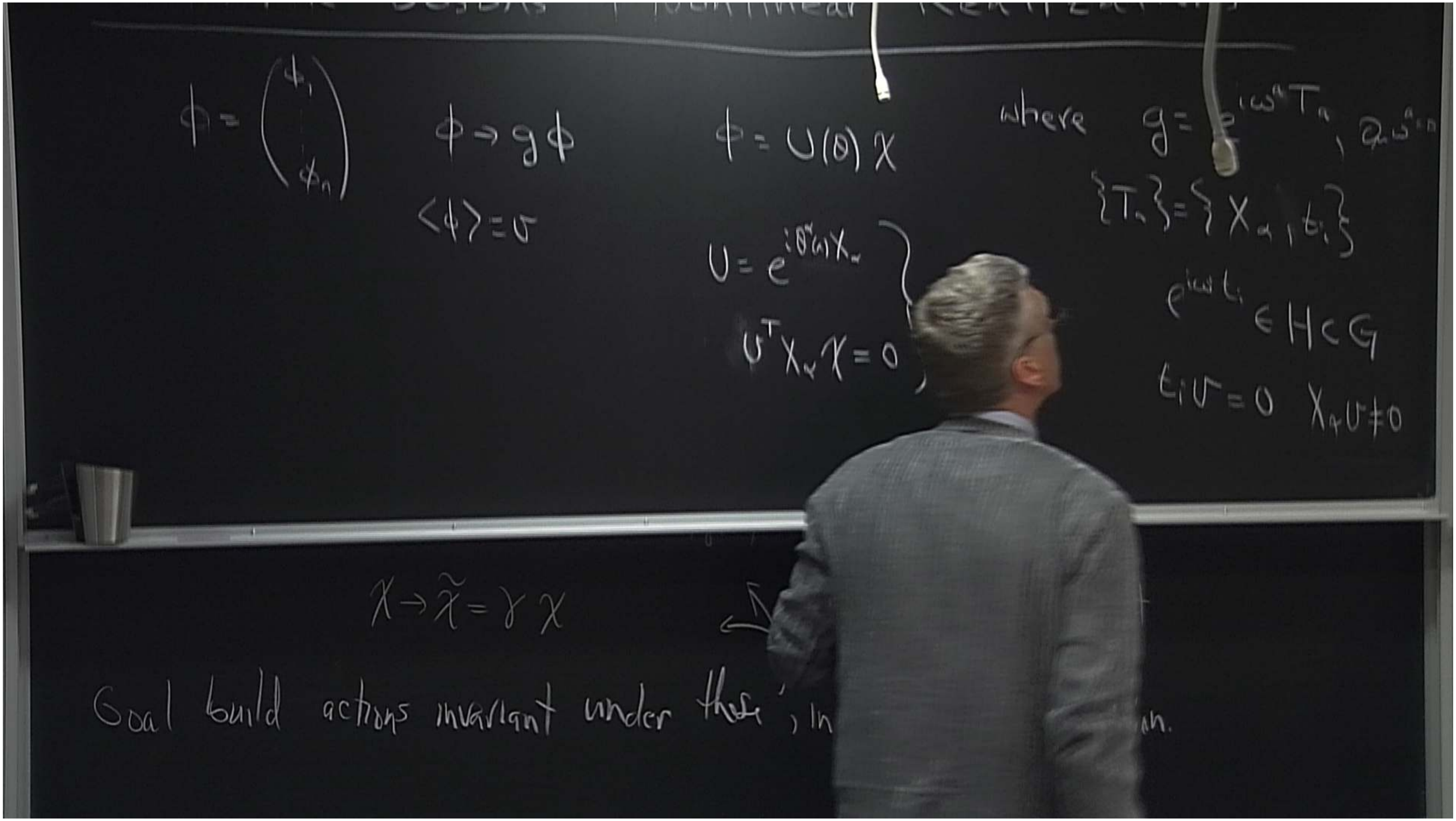
Date: Mar 14, 2014 09:00 AM

URL: http://pirsa.org/14030075

Abstract:



$\phi \to g$
$\langle \phi \rangle = v$
$U = e^{i\theta^\alpha(x) X_\alpha}$
$v^T X_\alpha \chi = 0$
where $g = e$
$\{T_a\} = \{X_\alpha, t_i\}$
$\in H \subset G$
$t_i v = 0 \quad X_\alpha v \neq 0$



ϕ = (ϕ1 … ϕn)
ϕ → gϕ
⟨ϕ⟩ = v
ϕ = U(θ) χ
U = e^{iθ^α(x)X_α}
v^T X_α χ = 0
where g = e^{iω^a T_a}, ∂_μ ω^a = 0
{T_a} = {X_α, t_i}
e^{iω t_i} ∈ H ⊂ G
t_i v = 0 X_α v ≠ 0
χ → χ̃ = γ χ
Goal build actions invariant under these

$\Lambda_4 U \neq 0$

Acting on θ, χ : $\theta^\alpha \to \theta^\alpha$, $\chi \to \chi$

$$\omega^a = \begin{Bmatrix} \omega^\alpha \\ \omega^i \end{Bmatrix} \qquad g\, U(\theta) = U(\tilde\theta)\, \gamma(\tilde\theta, g, \theta) \quad \text{where} \quad U(\tilde\theta) = e^{i\tilde\theta^\alpha X_\alpha}$$

$$\chi \to \tilde\chi = \gamma\, \chi \qquad \gamma = e^{iu^i t_i} \in H$$

Goal build actions invariant under these, in a derivative expansion.

for small θ^α: $\delta\theta^\alpha = \tilde\theta^\alpha - \theta^\alpha = \omega^\alpha - C^\alpha{}_{b\gamma}\, \omega^b\, \theta^\gamma + o(\theta^2)$ where $[T_a, T_b] = i C^d{}_{ab} T_d$.

$u^i =$

if C_{abc} completely antisymmetric $[t,t]=t$ $C_{\alpha ij}=0 \Leftrightarrow C_{i\alpha j}=0$ $[t,X]\sim X$

$\chi_\alpha U \neq 0$

Acting on θ, χ : $\theta^\alpha \to \theta^\alpha$, $\chi \to \chi$

$$\omega^a = \begin{Bmatrix} \omega^\alpha \\ \omega^i \end{Bmatrix} \qquad g\,U(\theta) = U(\tilde{\theta})\,\gamma(\tilde{\theta}, g, \theta) \quad \text{where} \quad U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$$

$$\chi \to \tilde{\chi} = \gamma\,\chi \qquad \gamma = e^{iu^i t_i} \in H$$

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for small θ^α: $\delta\theta^\alpha = \tilde{\theta}^\alpha - \theta^\alpha = \omega^\alpha - C^\alpha{}_{b\gamma}\,\omega^b\,\theta^\gamma + O(\theta^2)$ where $[T_a, T_b] = i\,C^d{}_{ab}\,T_d$.

$$u^i = -\,C^i{}_{a\beta}\,\omega^a\,\theta^\beta + O(\theta^2)$$

if C_{abc} completely antisymmetric $[t,t]=t$ $c_{aij}=0 \Leftrightarrow C_{iaj}=0$ $[t,X]\sim X$

Claim: is special case where $g=h\in H$.

$$hU(\theta)=U(\tilde{\theta})\gamma \quad \text{always has solution } \gamma=h$$

$$\text{so } U(\tilde{\theta})=hUh^{-1}$$

if C_{abc} completely antisymmetric $[t,t]=t$ $\quad c_{\alpha ij}=0 \Leftrightarrow C_{i\alpha j}=0$ $\quad [t,X]\sim X$ $\circledast$

Claim: is special case where $g=h\in H$

$$hU(\theta)=U(\tilde{\theta})\gamma \quad \text{always has solution } \gamma=h$$

so $U(\tilde{\theta})=hUh^{-1} \leftarrow \oplus$

$\rightarrow \delta\theta^{\alpha}=-C^{\alpha}{}_{i\beta}\,\omega^{i}\theta^{\beta}$

$u^{i}=\omega^{i}$

if C_{abc} completely antisymmetric $[t,t]=t$ $\quad C_{\alpha ij}=0 \Leftrightarrow C_{i\alpha j}=0$ $\quad [t,X]\sim X$ $\circledast$

Claim: is special case where $g = h \in H$.

$h\,U(\theta) = U(\tilde\theta)\,\gamma$. always has solution $\gamma = h$.

so $U(\tilde\theta) = h\,U\,h^{-1}$ $\leftarrow \circledast$

$\rightarrow \delta\theta^\alpha = -\,C^\alpha{}_{i\beta}\,\omega^i\,\theta^\beta$

$u^i = \omega^i$

when restricted to the unbroken group these transformation rules are linear + homogeneous

$\chi_\alpha U^\alpha \neq 0$

Acting on θ, χ : $\theta^\alpha \to \theta^\alpha$, $\chi \to \chi$

$$\omega^a = \begin{Bmatrix} \omega^\alpha \\ \omega^i \end{Bmatrix}$$

$g\, U(\theta) = U(\tilde{\theta})\, \gamma(\tilde{\theta}, g, \theta)$ where $U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$

$\chi \to \tilde{\chi} = \gamma\, \chi$ $\qquad \gamma = e^{iu^i t_i} \in H$

Goal build actions invariant under these, in a derivative expansion.

for small θ^α: $\delta\theta^\alpha = \tilde{\theta}^\alpha - \theta^\alpha = \omega^\alpha - C^\alpha{}_{b\gamma}\, \omega^b\, \theta^\gamma + O(\theta^2)$ where $[T_a, T_b] = i C^d{}_{ab} T_d$.

$u^i = \omega^i - C^i{}_{a\beta}\, \omega^a \theta^\beta + O(\theta^2)$

$u' = \omega'$

transformation rules are ...
+ homogeneous

Any nonderivative interaction involving χ that was H-invariant is automatically G-invariant.

if $\mathcal{L} = \chi^T \chi \to \chi^T h^T h \chi = \chi^T \chi$ for $h \in H$

then under G $\chi^T \chi \to \tilde{\chi}^T \tilde{\chi} = \chi^T \underbrace{\gamma^T \gamma}_{=1} \chi = \chi^T \chi$ inv. for free.

$u' = \omega'$

transformation rules are ...
inhomogeneous

Any nonderivative interaction involving χ that was H-invariant is automatically G-invariant.

if $\mathcal{L} = \chi^T \chi \to \chi^T h^T h \chi = \chi^T \chi$ for $h \in H$

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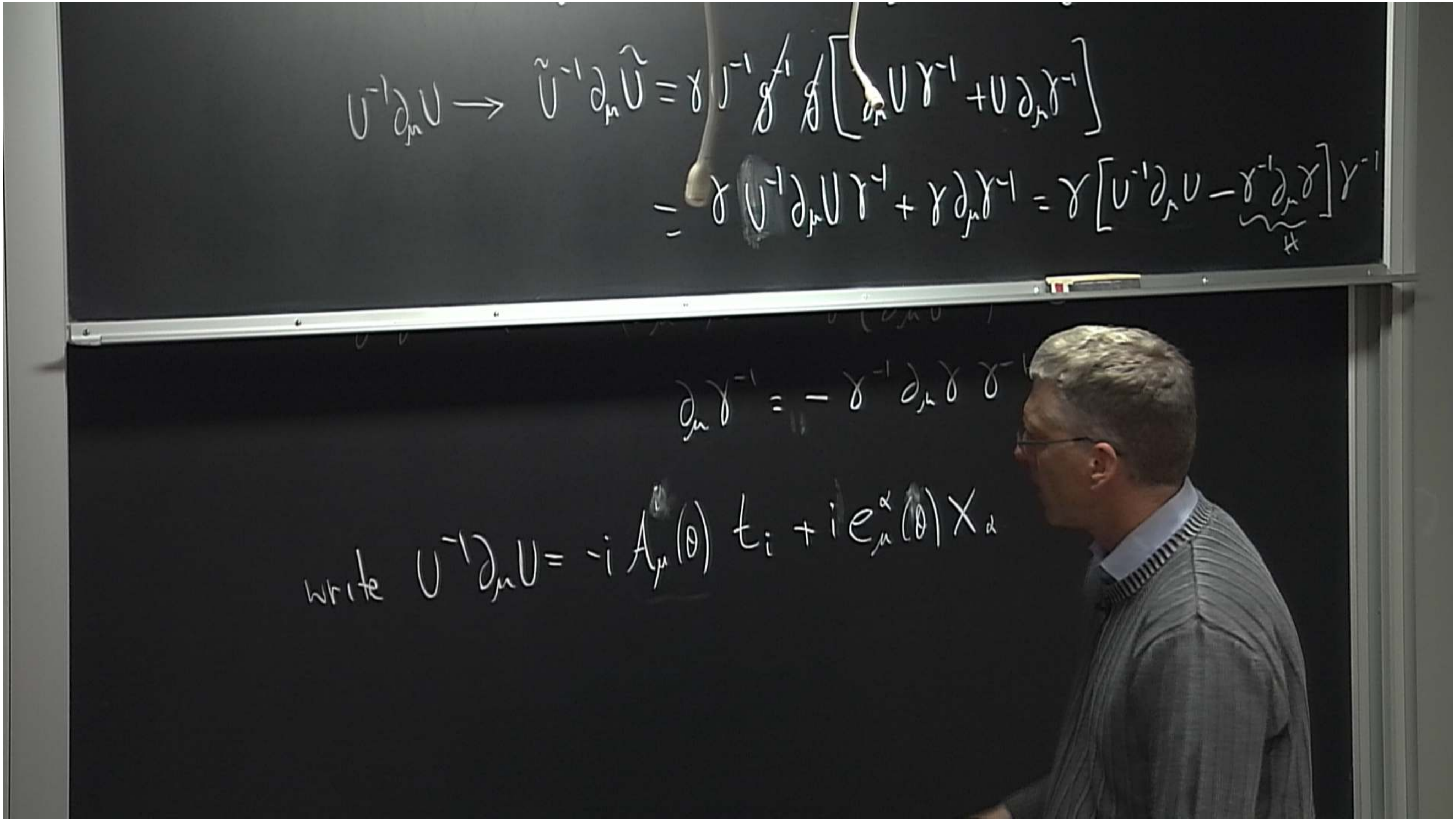
$U^{-1}\partial_\mu U \to \tilde{U}^{-1}\partial_\mu \tilde{U} = \gamma U^{-1}\cancel{g^{-1}}\cancel{g}[\partial_\mu U \gamma^{-1} + U\partial_\mu\gamma^{-1}]$
$= \gamma U^{-1}\partial_\mu U\gamma^{-1} + \gamma\partial_\mu\gamma^{-1} = \gamma[U^{-1}\partial_\mu U - \underbrace{\gamma^{-1}\partial_\mu\gamma}_{H}]\gamma^{-1}$
$\partial_\mu\gamma^{-1} = -\gamma^{-1}\partial_\mu\gamma\,\gamma^{-1}$
write $U^{-1}\partial_\mu U = -i A^i_\mu(\theta)\, t_i + i e^\alpha_\mu(\theta)\, X_\alpha$

$$U^{-1}\partial_\mu U \to \tilde{U}^{-1}\partial_\mu \tilde{U} = \gamma U^{-1} \cancel{g^{-1}} \cancel{g} \left[\partial_\mu U \gamma^{-1} + U \partial_\mu \gamma^{-1} \right]$$

$$= \gamma U^{-1}\partial_\mu U \gamma^{-1} + \gamma \partial_\mu \gamma^{-1} = \gamma \left[U^{-1}\partial_\mu U - \underbrace{\gamma^{-1}\partial_\mu \gamma}_{H} \right] \gamma^{-1}$$

write $U^{-1}\partial_\mu U = -i A^i_\mu(\theta)\, t_i + i e^\alpha_\mu(\theta)\, X_\alpha$

$$= -i A^i_\alpha(\theta)\, \partial_\mu \theta^\alpha\, t_i + i e^\alpha_\beta(\theta)\, \partial_\mu \theta^\beta\, X_\alpha$$

$$\gamma\gamma^{-1} = 1 \qquad (\partial_\mu \gamma)\gamma^{-1} + \gamma(\partial_\mu \gamma^{-1}) = 0$$

$$\partial_\mu \gamma^{-1} = -\gamma^{-1}\partial_\mu \gamma\, \gamma^{-1}.$$

write $\underbrace{U^{-1}\partial_\mu U} = -i A^i_\mu(\theta)\, t_i + i e^\alpha_\mu(\theta) X_\alpha$

$$= -i A^i_\alpha(\theta)\partial_\mu \theta^\alpha t_i + i e^\alpha_\beta(\theta)\, \partial_\mu \theta^\beta X_\alpha$$

So $A^i_\mu(\theta)\, t_i \to \gamma A^i_\mu(\theta)\, t_i \gamma^{-1} + i\gamma^{-1}\partial_\mu \gamma.$ ← gauge field transformation

$u^i = \omega^i$

transformation rules are linear + homogeneous

$$e^{\alpha}_{\mu}(\theta)\, X_{\alpha} \to \gamma\, e^{\alpha}_{\mu}(\theta)\, \gamma^{-1}$$

$u^i = \omega^i$

transformation rules are linear + homogeneous

$$e^\alpha_\mu(\theta) X_\alpha \to \gamma\, e^\alpha_\mu(\theta) X_\alpha \gamma^{-1}$$

Can build invariants from e^α_μ. eg $\mathrm{Tr}\left[e^\alpha_\mu X_\alpha e^\beta_\nu X_\beta\right] g^{\mu\nu}$.

and if $D_\mu \chi = \partial_\mu \chi - i A^\alpha_\mu(\theta) \chi$ $\qquad \chi \to \gamma \chi$

then $D_\mu \chi \to \gamma \partial_\mu \chi + (\partial_\mu \gamma) \chi$

$u^i = \omega^i$

transformation rules are linear + homogeneous

$$e_\mu^\alpha(\theta)\, X_\alpha \to \gamma\, e_\mu^\alpha(\theta) X_\alpha \gamma^{-1}$$

Can build invariants from e_μ^α : eg $\mathrm{Tr}\left[e_\mu^\alpha X_\alpha e_\nu^\beta X_\beta\right] g^{\mu\nu}$

and if $D_\mu \chi = \partial_\mu \chi - i A_\mu^i(\theta)\, t_i \chi \qquad \chi \to \gamma\chi$

then $D_\mu \chi \to \gamma \partial_\mu \chi + (\partial_\mu \gamma)\chi - i\left[\gamma A_\mu^i t_i \gamma^{-1} + \right.$

$u^i = \omega^i$

transformation rules are linear + homogeneous

$$e^\alpha_\mu(\theta) X_\alpha \to \gamma\, e^\alpha_\mu(\theta) X_\alpha \gamma^{-1}$$

Can build invariants from e_μ^α. eg $\mathrm{Tr}\left[e_\mu^\alpha X_\alpha e_\nu^\beta X_\beta\right] g^{\mu\nu}$

and if $D_\mu \chi = \partial_\mu \chi - i A^i_\mu(\theta)\, t_i \chi$ $\quad \chi \to \gamma \chi$

then $D_\mu \chi \to \gamma \partial_\mu \chi + (\partial_\mu \gamma)\chi - i\left[\gamma A^i_\mu t_i \gamma^{-1} - i \gamma^{-1} \partial_\mu \gamma\right] \gamma \chi$

$\omega' = \omega$

transformation rules are ... + homogeneous

$$e^{\alpha}_{\mu}(\theta)\, X_{\alpha} \to \gamma\, e^{\alpha}_{\mu}(\theta)\, X_{\alpha}\, \gamma^{-1}$$

Can build invariants from e^{α}_{μ}: eg $\mathrm{Tr}\left[e_{\mu}^{\alpha} X_{\alpha}\, e_{\nu}^{\beta} X_{\beta}\right] g^{\mu\nu}$

and if $D_{\mu}\chi = \partial_{\mu}\chi - i A^{i}_{\mu}(\theta)\, t_i\, \chi$ $\qquad \chi \to \gamma\chi$

then $D_{\mu}\chi \to \gamma\,\partial_{\mu}\chi + \cancel{(\partial_{\mu}\gamma)}\chi - i\left[\gamma A^{i}_{\mu} t_i \gamma^{-1} \cancel{- i\,\partial_{\mu}\gamma\,\gamma^{-1}}\right]\gamma\chi = \gamma D_{\mu}\chi$

$\omega' = \omega'$ transformation rules ... +homogeneous

$$e_\mu^\alpha(\theta) X_\alpha \to \gamma\, e_\mu^\alpha(\theta) X_\alpha \gamma^{-1}$$

Can build invariants from e_μ^α: eg $\mathrm{Tr}\left[e_\mu^\alpha X_\alpha e_\nu^\beta X_\beta\right] g^{\mu\nu}$

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$u' = \omega'$

transformation rules are ... + homogeneous

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Can build invariants from e^α_μ: eg $\mathrm{Tr}\left[e_\mu^\alpha X_\alpha\, e_\nu^\beta X_\beta\right] g^{\mu\nu}$

and if $D_\mu \chi = \partial_\mu \chi - i A^i_\mu(\theta)\, t_i\, \chi$ $\qquad \chi \to \gamma \chi$

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$$N_{\alpha\beta} = \mathrm{Tr}(T_\alpha T_\beta)$$

$$\gamma\gamma^{-1} = 1 \qquad (\partial_\mu \gamma)\gamma^{-1} + \gamma(\partial_\mu \gamma^{-1}) = 0$$

$$\partial_\mu \gamma^{-1} = -\gamma^{-1}\partial_\mu \gamma\, \gamma^{-1}.$$

$$g_{\alpha\beta} = N_{\gamma\delta}\, e^\gamma_\alpha e^\delta_\beta$$

write $\underbrace{U^{-1}\partial_\mu U} = -i A^i_\mu(\theta)\, t_i + i e^\alpha_\mu(\theta) X_\alpha$

$$= -i A^i_\alpha(\theta)\partial_\mu \theta^\alpha t_i + i e^\alpha_\beta(\theta)\partial_\mu \theta^\beta X_\alpha$$

$A^i_\mu(\theta)\, t_i \to \gamma A^i_\mu(\theta)\, t_i \gamma^{-1} - i(\partial_\mu \gamma)\gamma^{-1} \leftarrow$ gauge field transformation

So

$$N_{\alpha\beta} = \mathrm{Tr}(T_\alpha T_\beta)$$

$$\gamma\gamma^{-1} = 1 \qquad (\partial_\mu \gamma)\gamma^{-1} + \gamma(\partial_\mu \gamma^{-1}) = 0$$

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$$= -i A^i_\alpha(\theta)\partial_\mu \theta^\alpha\, t_i + i e^\alpha_\beta(\theta)\, \partial_\mu \theta^\beta X_\alpha$$

So $\quad A^i_\mu(\theta)\, t_i \to \gamma A^i_\mu(\theta)\, t_i \gamma^{-1} - i(\partial_\mu \gamma)\gamma^{-1} \leftarrow$ gauge field transformation

$$N_{\alpha\beta} = \mathrm{Tr}(T_\alpha T_\beta)$$

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$$\boxed{g_{\alpha\beta} = N_{\gamma\delta}\, e_\alpha^\gamma e_\beta^\delta}$$

vielbein

$$\text{write } \underbrace{U^{-1}\partial_\mu U} = -i A_\mu^i(\theta)\, t_i + i e_\mu^\alpha(\theta) X_\alpha$$

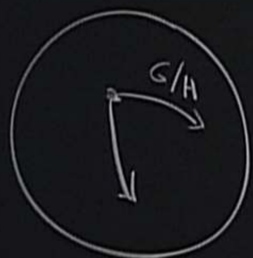
$$= -i A_\alpha^i(\theta)\partial_\mu \theta^\alpha t_i + i e_\beta^\alpha(\theta)\, \partial_\mu \theta^\beta X_\alpha$$

$$A_\mu^i(\theta)\, t_i \to \gamma A_\mu^i(\theta)\, t_i \gamma^{-1} - i(\partial_\mu \gamma)\gamma^{-1} \leftarrow \text{gauge field transformation}$$

So

$\lambda_\phi v \neq 0$

$$\mathcal{L} = -\underbrace{V(\theta)} - \frac{F^2}{2} g_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \ldots$$



V must be G-invariant $\Rightarrow V = \text{const}$

$g_{\alpha\beta}$ to have Killing vectors for G. $\delta\theta^\alpha = \xi^\alpha(\theta)$

$$ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2 \longrightarrow \partial_\mu\theta\, \partial^\mu\theta + \sin^2\theta\, \partial_\mu\phi\, \partial^\mu\phi \quad \text{for } G = SO(3),\ H = SO(2)$$

$\lambda_4 v \neq 0$

$$\mathcal{L} = -\underbrace{V(\theta)} - \frac{F^2}{2} g_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \ldots$$

V must be G-invariant $\Rightarrow V = \text{const}$

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$$ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2 \longrightarrow \partial_\mu\theta\,\partial^\mu\theta + \sin^2\theta\, \partial_\mu\phi\,\partial^\mu\phi$$

for $G = SO(3)$, $H = SO(2)$

$$\delta\theta^\alpha = \xi^\alpha(\theta)$$

$$\delta\mathcal{L} = \boxed{\frac{\partial V}{\partial\theta^\alpha}\xi^\alpha} + \frac{E^2}{2}\Big[\partial_\gamma g_{\alpha\beta}\,\xi^\gamma\,\partial_\mu\theta^\alpha\,\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\gamma\xi^\alpha\,\partial_\mu\theta^\gamma\,\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\mu\theta^\alpha\,\partial_\gamma\xi^\beta\,\partial^\mu\theta^\gamma\Big]$$

$\mathcal{L}_\xi V$

$\mathcal{L}_\xi g_{\alpha\beta}$

$$\delta\theta^\alpha = \xi^\alpha(\theta)$$

$$\delta\mathcal{L} = \boxed{\frac{\partial V}{\partial\theta^\alpha}\,\xi^\alpha} + \frac{E^2}{2}\Big[\partial_\gamma g_{\alpha\beta}\,\xi^\gamma\,\partial_\mu\theta^\alpha\,\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\gamma\xi^\alpha\,\partial_\mu\theta^\gamma\,\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\mu\theta^\alpha\,\partial_\gamma\xi^\beta\,\partial^\mu\theta^\gamma\Big]$$

$$\mathcal{L}_\xi V \qquad\qquad \mathcal{L}_\xi g_{\alpha\beta} = 0$$

$$\delta\theta^\alpha = \xi^\alpha(\theta)$$

$$\delta\mathcal{L} = \boxed{\frac{\partial V}{\partial\theta^\alpha}\xi^\alpha} + \frac{E^2}{2}\Big[\partial_\gamma g_{\alpha\beta}\,\xi^\gamma\,\partial_\mu\theta^\alpha\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\gamma\xi^\alpha\,\partial_\mu\theta^\gamma\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\mu\theta^\alpha\,\partial_\gamma\xi^\beta\,\partial^\mu\theta^\gamma\Big]$$

$\mathcal{L}_\xi V$

$\mathcal{L}_\xi g_{\alpha\beta} = 0$

$$\delta\theta^\alpha = \xi^\alpha(\theta)$$

$$\delta\mathcal{L} = \boxed{\frac{\partial V}{\partial\theta^\alpha}\xi^\alpha} + \frac{E^2}{2}\Big[\partial_\gamma g_{\alpha\beta}\,\xi^\gamma\,\partial_\mu\theta^\alpha\,\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\gamma\xi^\alpha\,\partial_\mu\theta^\gamma\,\partial^\mu\theta^\beta + g_{\alpha\beta}\,\partial_\mu\theta^\alpha\,\partial_\gamma\xi^\beta\,\partial^\mu\theta^\gamma\Big]$$

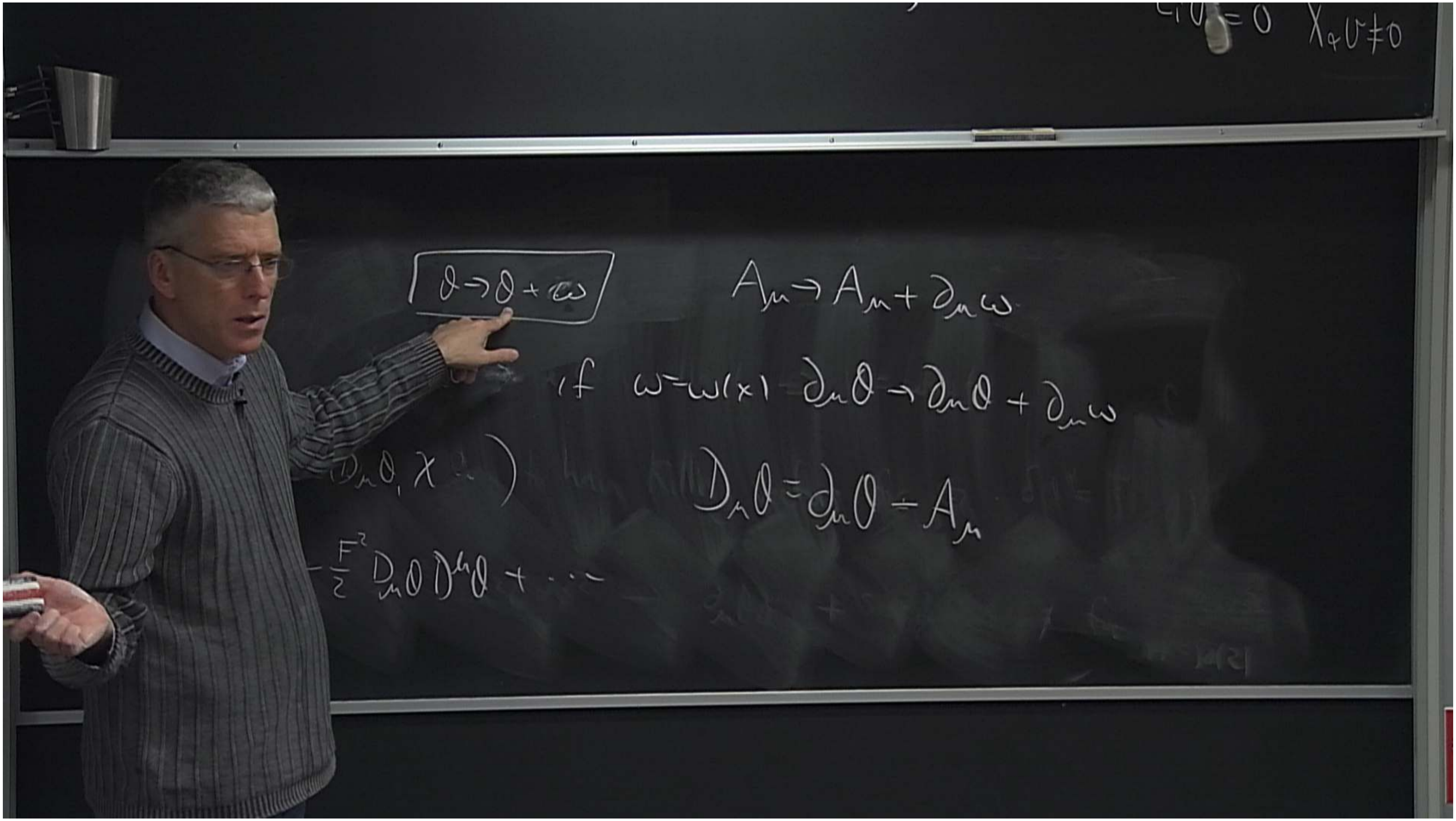
$$\mathcal{L}_\xi V$$

$$\mathcal{L}_\xi g_{\alpha\beta} = 0$$

$\lambda_\alpha v^\alpha \neq 0$

$(\omega_\alpha^{\ i} J_\alpha^{\ i})$

$\Gamma^\alpha_{\beta\gamma}$


$X_4 v \neq 0$
$\theta \to \theta + \omega$
$A_\mu \to A_\mu + \partial_\mu \omega$
if $\omega = \omega(x)$ $\partial_\mu \theta \to \partial_\mu \theta + \partial_\mu \omega$
$D_\mu \theta = \partial_\mu \theta - A_\mu$
$\frac{F^2}{2} D_\mu \theta D^\mu \theta + \dots$

$X_4 v \neq 0$

$$\mathcal{L} = -\frac{F^2}{2}\partial_\mu\theta\,\partial^\mu\theta \qquad J^\mu = F^2\partial^\mu\theta$$

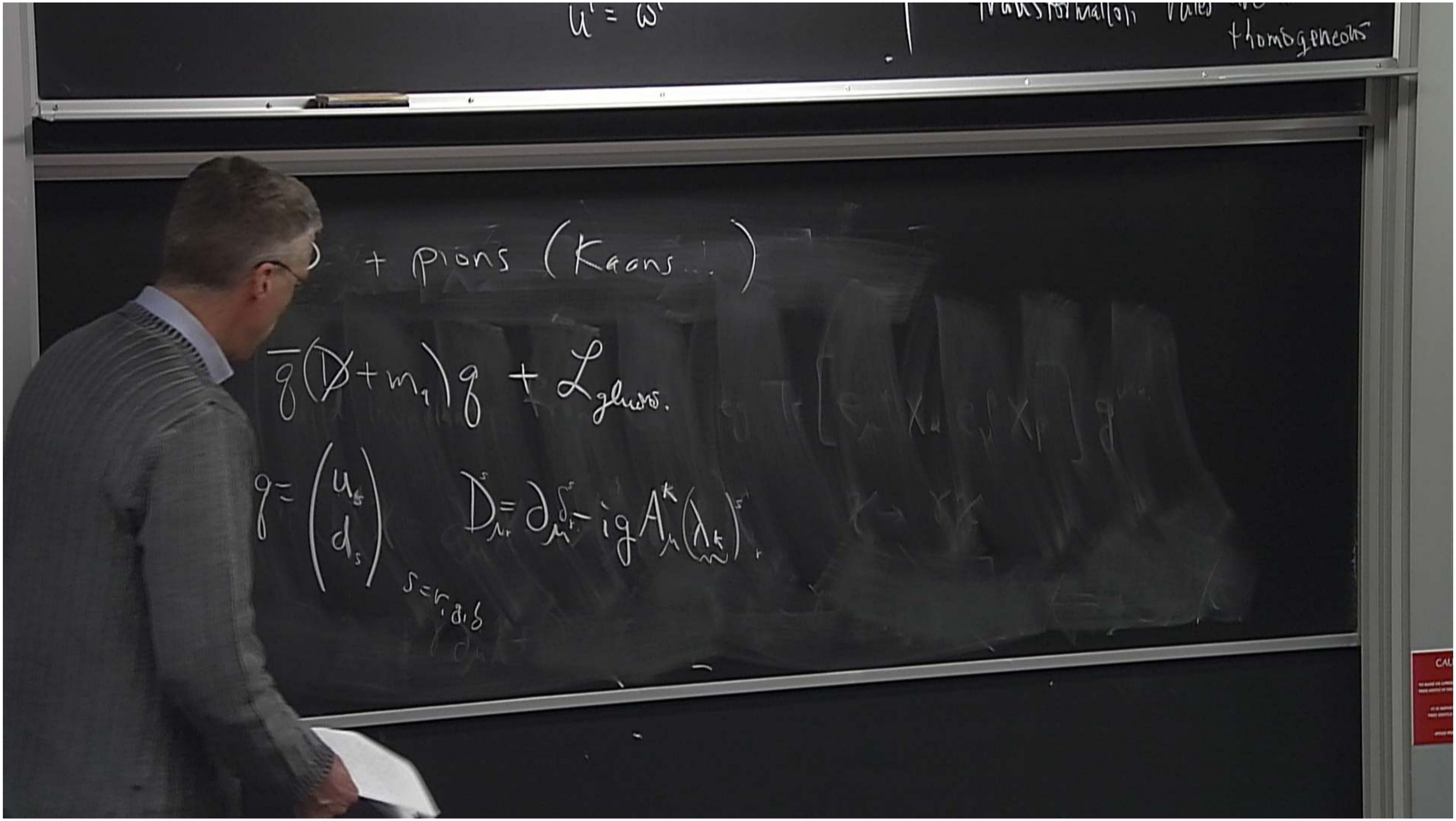
$$\boxed{\theta \to \theta + \omega} \qquad A_\mu \to A_\mu + \partial_\mu\omega$$

$\partial_\mu\theta \to$ if $\omega = \omega(x)$ $\partial_\mu\theta \to \partial_\mu\theta + \partial_\mu\omega$

$$\mathcal{L} = \mathcal{L}(D_\mu\theta, \chi \ldots) \qquad D_\mu\theta = \partial_\mu\theta - A_\mu = -A_\mu \text{ when } \theta = 0$$

"unitary gauge"

$$= -\frac{F^2}{2}\underbrace{D_\mu\theta\, D^\mu\theta}_{A_\mu A^\mu} + \ldots$$


+homogeneous
+ pions (Kaons . . .)
q̄(D̸ + m_q)q + L_gluons.
s = r, g, b

thomogeneous

pions mass ~ 140 MeV

QCD + pions (Kaons ...)

$$\mathcal{L} = \bar{q}(\not{D} + m_q)q + \boxed{\mathcal{L}_{gluons}} \quad \Lambda_{QCD} \simeq 200 \text{ MeV} \longrightarrow \text{masses } 500 \text{ meV} \to \text{up}$$

$$q = \begin{pmatrix} u_s \\ d_s \end{pmatrix}_{s=r,g,b} \qquad D^s_{\mu r} = \partial_\mu \delta^s_r - igA^k_\mu \left(\frac{\lambda_k}{2}\right)^s_r$$

+ homogeneous

QCD + pions (Kaons ...)

$\boxed{\text{pions mass} \sim 140 \text{ MeV}}$

$$\mathcal{L} = \bar{q}(\not{D} + m_q)q + \boxed{\mathcal{L}_{gluons}}$$

$\Lambda_{QCD} \simeq 200$ MeV $\longrightarrow$ masses 500 meV $\rightarrow$ up

$$q = \begin{pmatrix} u_s \\ d_s \end{pmatrix} \quad s = r, g, b \qquad D^{s}_{\mu r} = \partial_\mu \delta^{s}_{r} - i g A^{k}_{\mu} (\lambda_k)^{s}_{r}$$

m_u, m_d mass few

homogeneous

QCD + pions (Kaons ...)

$\boxed{\text{pions mass} \sim 140 \text{ MeV}}$

$$\mathcal{L} = \bar{q}(\not{D} + m_q) q + \boxed{\mathcal{L}_{gluons}}$$

$\Lambda_{QCD} \simeq 200$ MeV $\longrightarrow$ masses 500 MeV $\to$ up

$$q = \begin{pmatrix} u_s \\ d_s \end{pmatrix} \qquad s = r, g, b$$

$$D_{\mu r}^{s} = \partial_\mu \delta_r^s - i g A_\mu^k (\lambda_k)^s_r$$

$m_u \simeq 5$ MeV or less

$m_d \simeq 9$ MeV "

$m_s \sim 170$ MeV

$$m_u, m_d \ll \Lambda_{QCD}$$

$$\mathcal{L} \simeq \mathcal{L}_0 + \mathcal{L}_m \qquad \mathcal{L}_0 = \underbrace{\bar{q} \not{D} q + \mathcal{L}_{gluon}}_{\Lambda_{QCD}}$$

$$\mathcal{L}_m = \bar{q} m q$$

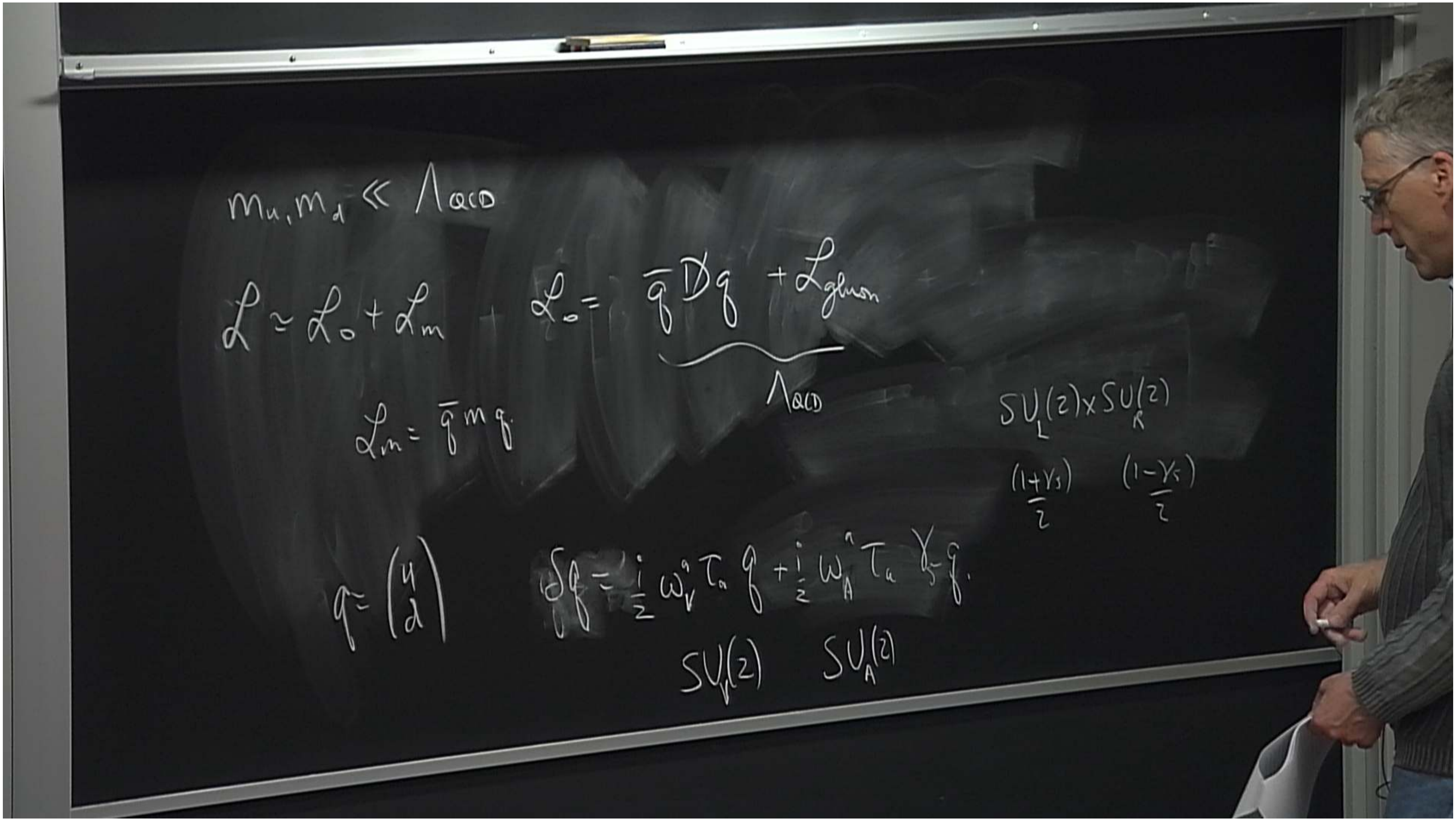
$$\delta q = i$$

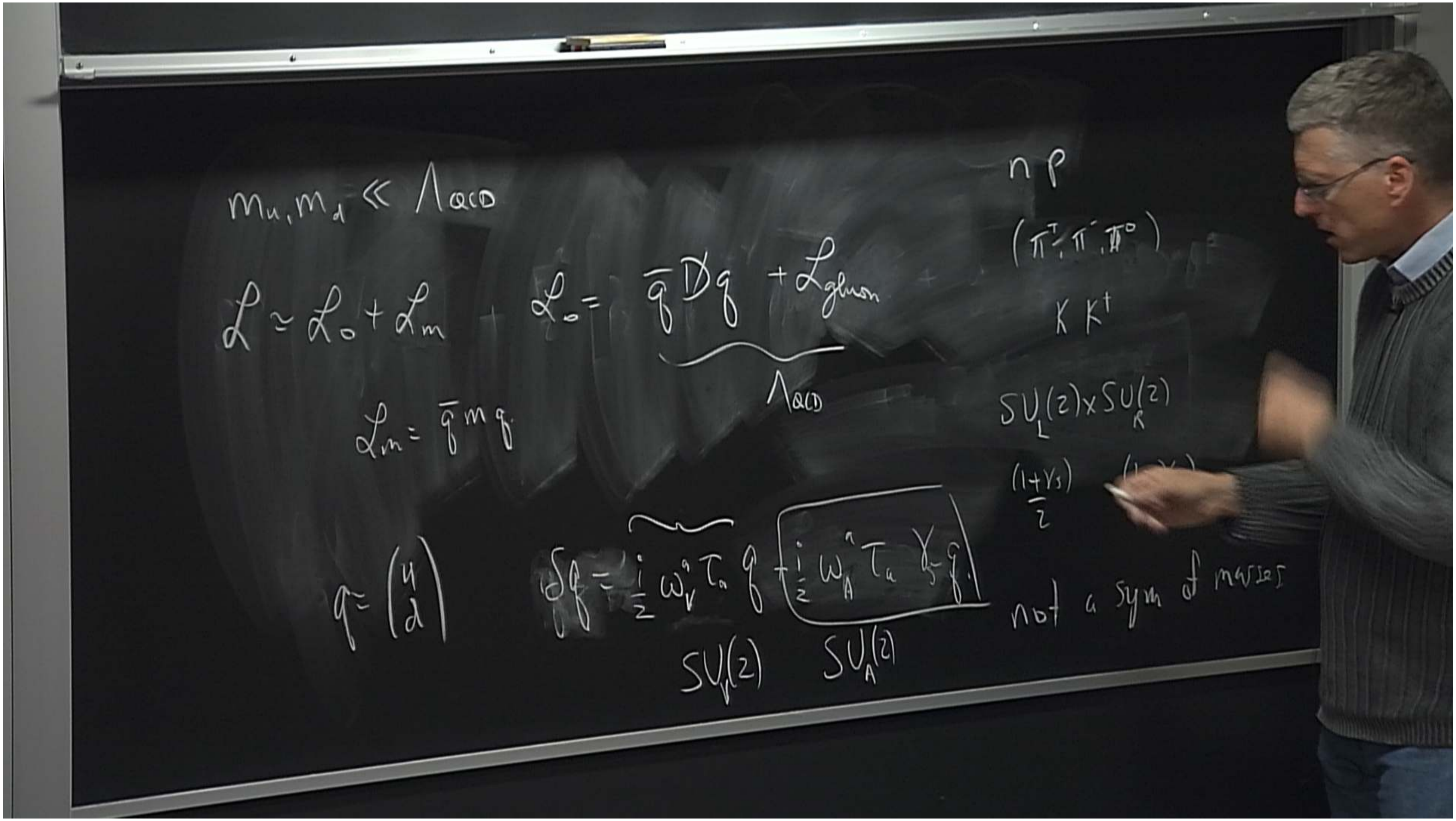
$$m_u, m_d \ll \Lambda_{QCD}$$

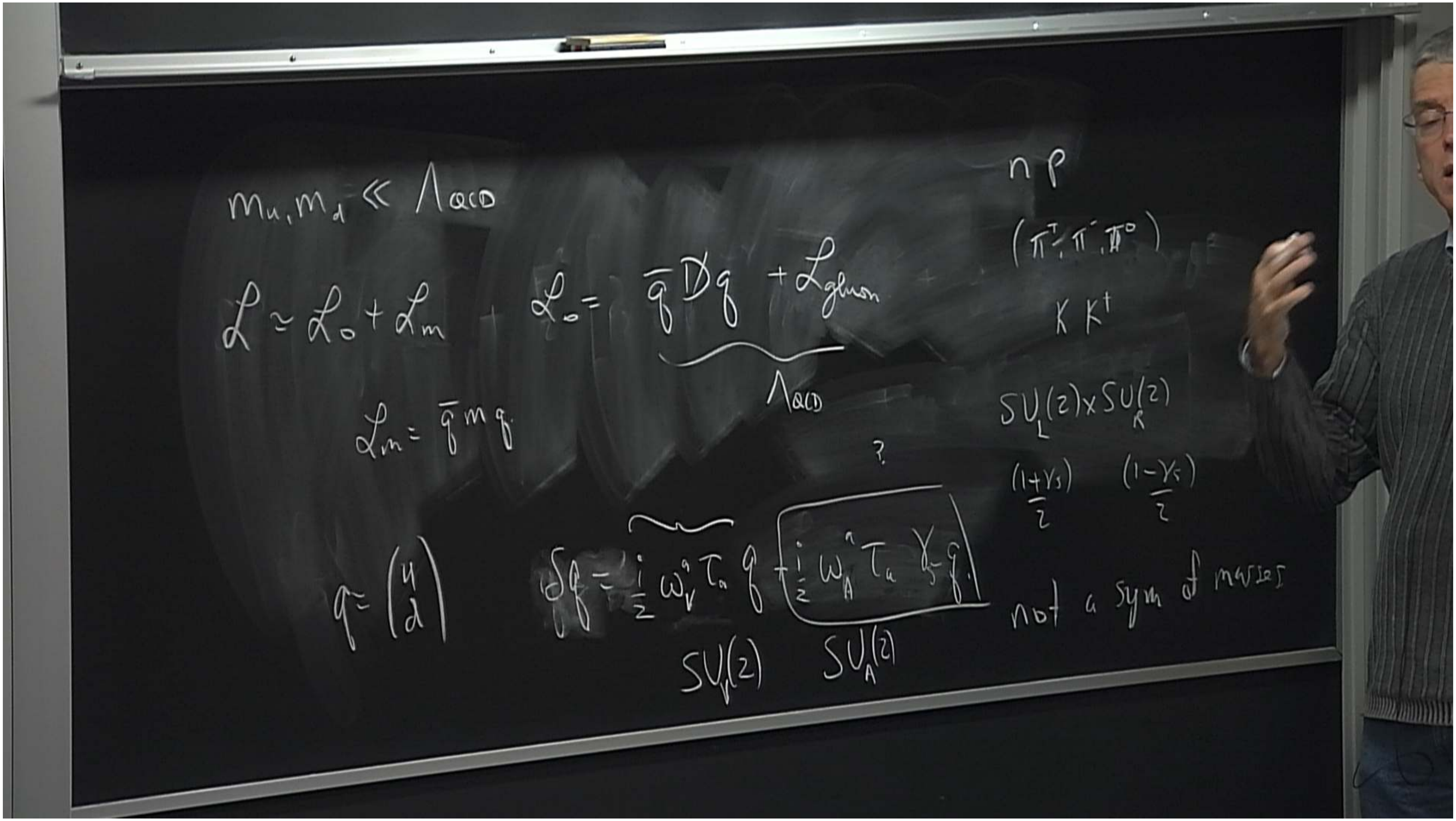
$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_m \qquad \mathcal{L}_0 = \underbrace{\bar{q} \not{D} q + \mathcal{L}_{gluon}}_{\Lambda_{QCD}}$$

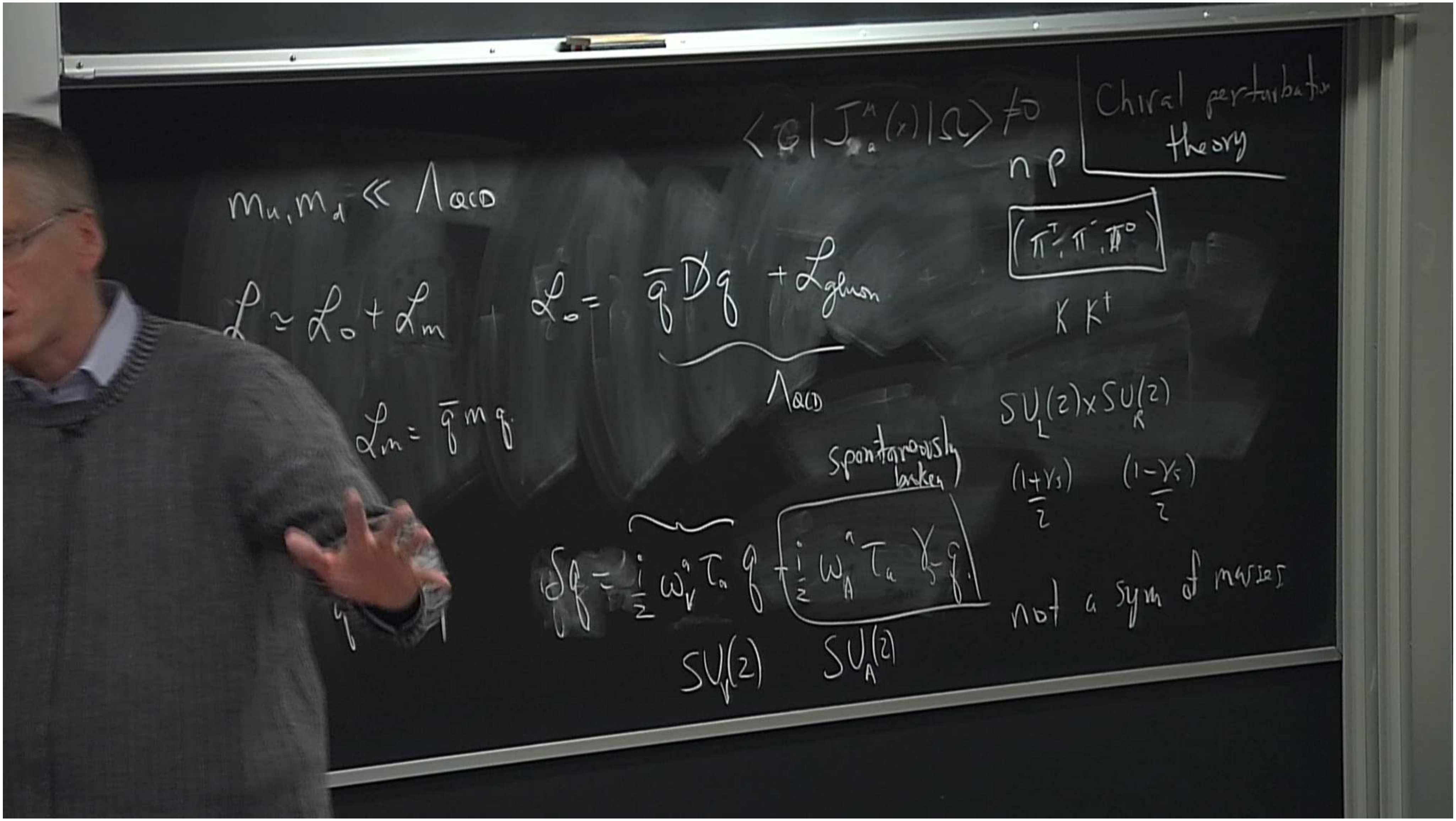
$$\mathcal{L}_m = \bar{q} m q$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \delta q = \frac{i}{2} \omega_V^a \tau_a q + \frac{i}{2} \omega_A^a \tau_a \gamma_5 q.$$


$m_u, m_d \ll \Lambda_{QCD}$
$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_m$
$\mathcal{L}_0 = \underbrace{\bar{q} \not{D} q + \mathcal{L}_{gluon}}_{\Lambda_{QCD}}$
$\mathcal{L}_m = \bar{q} m q$
$SU_L(2) \times SU_R(2)$
$\frac{(1+\gamma_5)}{2}$ $\frac{(1-\gamma_5)}{2}$
$q = \begin{pmatrix} u \\ d \end{pmatrix}$
$\delta q = \frac{i}{2} \omega_V^a \tau_a q + \frac{i}{2} \omega_A^a \tau_a \gamma_5 q.$
$SU_V(2)$ $SU_A(2)$


$m_u, m_d \ll \Lambda_{QCD}$
$\mathcal{L} \simeq \mathcal{L}_0 + \mathcal{L}_m$
$\mathcal{L}_0 = \bar{q} \not{D} q + \mathcal{L}_{gluon}$
Λ_{QCD}
$\mathcal{L}_m = \bar{q} m q$
$q = \begin{pmatrix} u \\ d \end{pmatrix}$
$\delta q = \frac{i}{2} \omega_V^a \tau_a q + \frac{i}{2} \omega_A^a \tau_a \gamma_5 q$
$SU_V(2)$ $SU_A(2)$
n p
(π^+, π^-, π^0)
K K^+
$SU_L(2) \times SU_R(2)$
$\frac{(1+\gamma_5)}{2}$
not a sym of masses


$m_u, m_d \ll \Lambda_{QCD}$
$\mathcal{L} \simeq \mathcal{L}_0 + \mathcal{L}_m$
$\mathcal{L}_0 = \underbrace{\bar{q} \not{D} q + \mathcal{L}_{gluon}}_{\Lambda_{QCD}}$
$\mathcal{L}_m = \bar{q} m q$
?
$q = \begin{pmatrix} u \\ d \end{pmatrix}$
$\delta q = \frac{i}{2} \omega_V^a \tau_a q + \boxed{\frac{i}{2} \omega_A^a \tau_a \gamma_5 q}$
$SU_V(2)$ $SU_A(2)$
n p
(π^+, π^-, π^0)
K K^+
$SU_L(2) \times SU_R(2)$
$\frac{(1+\gamma_5)}{2}$ $\frac{(1-\gamma_5)}{2}$
not a sym of masses


$m_u, m_d \ll \Lambda_{QCD}$
$\mathcal{L} \simeq \mathcal{L}_0 + \mathcal{L}_m$
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Λ_{QCD}
$\mathcal{L}_m = \bar{q} m q$
$\langle G | J^\mu_a(x) | \Omega \rangle \neq 0$
Chiral perturbation theory
$n \; p$
(π^+, π^-, π^0)
$K \; K^+$
$SU_L(2) \times SU_R(2)$
$\frac{(1+\gamma_5)}{2}$
$\frac{(1-\gamma_5)}{2}$
spontaneously broken
$\delta q = \frac{i}{2} \omega_V^a \tau_a q + \frac{i}{2} \omega_A^a \tau_a \gamma_5 q$
$SU_V(2)$
$SU_A(2)$
not a sym of masses