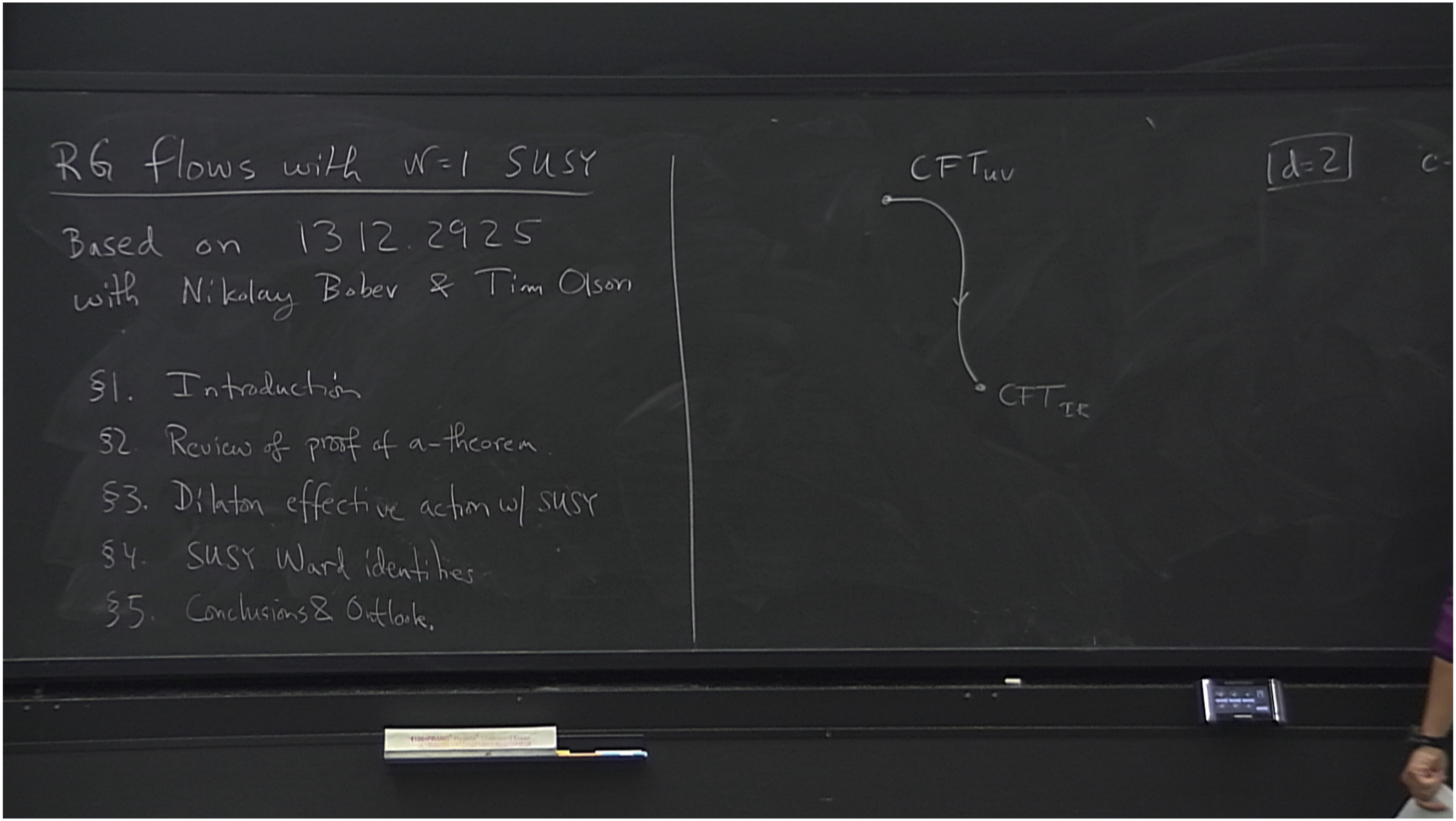
Title: RG flows with N=1 supersymmetry

Date: Apr 01, 2014 02:00 PM

URL: http://pirsa.org/14030074

Abstract: <span>The dilaton effective action plays a key role for the recent proof of the a-theorem by Schwimmer and Komargodski. In the presence of other massless modes, one may ask if this proof is affected. In particular, in renormalization group (RG) flows with N=1 supersymmetry, there is a natural massless partner of the dilaton, namely an axion field. I will discuss RG flows, the a-theorem, and the form of the N=1 supersymmetric dilaton-axion effective action and its physics.</span>


RG flows with N=1 SUSY
Based on 1312.2925
with Nikolay Bobev & Tim Olson
§1. Introduction
§2. Review of proof of a-theorem
§3. Dilaton effective action w/ SUSY
§4. SUSY Ward identities
§5. Conclusions & Outlook.
CFT_UV
CFT_IR
d=2

$CFT_{UV}$

$c(E)$ monoton decreasing

$CFT_{IR}$

$\boxed{d=2}$ c-theorem $c_{UV} > c_{IR}$

$\langle T_\mu{}^\mu \rangle = 0$ in flat bkgr.

$\langle T_\mu{}^\mu \rangle = c\,R$ trace anomaly.

central charge

Ricci scalar.

Zamolodchikov (1986).

$\boxed{d=4}$ $\langle T_\mu{}^\mu \rangle = c\, W^2 - a\, E_4 + \cancel{b \Box R}$

$W^2$: $(\text{Weyl})^2$

$E_4$: Euler density

$E_4 = (\text{Rie})^2 - 4(\text{Ric})^2 + R^2$

§2

Cardy (1988) a-theorem : $a_{UV} > a_{IR}$

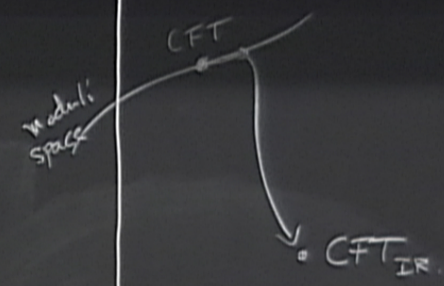
Proof in 2011 Komargodski + Schwimmer. (KS)

In this talk. $\mathcal{N}=1$ susy version of KS

$\boxed{d=4}$

## §2

Conf. spont. broken $\Rightarrow$ Goldstone "dilaton" $\tau(x)$




Low-E eff action $S_{eff}[\tau]$.

form determined by symmetries

$$\text{Weyl transforming} = \begin{cases} g_{\mu\nu}^{(x)} \to g_{\mu\nu}^{(x)} e^{2\sigma(x)} \\ \tau(x) \to \tau(x) + \sigma(x) \end{cases}$$

$$\delta_\sigma S = \int d^4x\, \sigma(x) \langle T_\mu{}^\mu \rangle$$

... version of KS.

$\boxed{d=4}$

$$S_{UV} \longrightarrow S_{IR} + S_{eff}[\tau]$$

$$\begin{array}{cccc} \delta_\sigma \big\downarrow & & \big\downarrow & \big\downarrow \\ & \langle T_\mu{}^\mu \rangle & \langle T_\mu{}^\mu \rangle & \langle T_\mu{}^\mu \rangle \\ & & & \uparrow \\ & c_{UV},\, a_{UV} & c_{IR},\, a_{IR} & \Delta c = c_{UV} - c_{IR} \\ & & & \Delta a = a_{UV} - a_{IR}. \end{array}$$

Anomaly matching argument

$$\boxed{\delta_\sigma S_{eff}(\tau) = \int d^4x\, \sigma(x) \left( \Delta c\, W^2 - \Delta a\, E_4 \right)}$$

$$\delta_\sigma S = \int d^4x\, \sigma(x) \langle T_\mu{}^\mu \rangle$$

This determines $S_{eff}[\tau]$ up to Weyl-invariant terms.

Result (2010)

$$S_{eff}[\tau] = \int d^4x \sqrt{-g} \Big[ \overbrace{(\partial\tau)^2 e^{-2\tau} + \frac{1}{6} R}^{\text{Weyl-inv.}} + \tau \left( \Delta c\, W^2 - \Delta a\, E_4 \right) + \Delta a \left( 4 G^{\mu\nu} \partial_\mu \tau \partial_\nu \tau - 4 (\partial\tau)^2 \Box\tau + 2 (\partial\tau)^4 \right) \Big]$$

$g_{\mu\nu} \to \eta_{\mu\nu}$

$$S_{eff}[\tau] = \int d^4x \left[ (\partial\tau)^2 e^{-2\tau} - 4(\partial\tau)^2 \Box\tau + 2(\partial\tau)^4 \right].$$

$e^{-\tau} = 1 - \varphi$ $\qquad$ $\varphi$ "physical dilaton" $\qquad$ $\Box\varphi = 0$

$$S_{eff} \Rightarrow (\partial\varphi)^2 + \Delta a (\partial\varphi)^4$$

$$A_4(\varphi\varphi \to \varphi\varphi) = \Delta a (s^2 + t^2 + u^2)$$

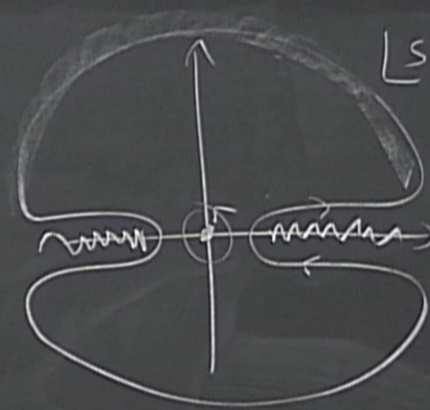
**KS:** $\dfrac{A_4(t\to 0)}{s^3}$

$s=0$ residue

$$\Delta a = \int_{s_0}^{\infty} \frac{\mathrm{Im}\, A(t\to 0)}{s^3}\, ds = \int \frac{\sigma_{tot}(s)}{s^2}\, ds > 0$$

$$\boxed{\Delta a > 0}$$

$$\boxed{a_{UV} > a_{IR}}$$



This determines $S_{eff}[\tau]$ up to Weyl-invariant terms.

Result (2010)

$$S_{eff}[\tau] = \int d^4x \sqrt{-g} \Big[ \overbrace{(\partial\tau)^2 e^{-2\tau} + \tfrac{1}{6}R}^{\text{Weyl-inv.}}$$
$$+ \tau\left(\Delta c\, W^2 - \Delta a\, E_4\right)$$
$$+ \Delta a\left(4 G^{\mu\nu}\partial_\mu\tau\partial_\nu\tau - 4(\partial\tau)^2\Box\tau + 2(\partial\tau)^4\right)\Big]$$

Explicit breaking

$M \to M e^{-\tau}$ $\quad\tau$ compensator

$e^{-\tau} = 1 - \frac{\varphi}{f}$

This determines $S_{eff}[\tau]$ up to Weyl-invariant terms.

Result (2010)

$$S_{eff}[\tau] = \int d^4x \sqrt{-g} \Big[ \overbrace{(\partial\tau)^2 e^{-2\tau} + \tfrac{1}{6} R}^{\text{Weyl-inv.}}$$

$$+ \tau\left(\Delta c\, W^2 - \Delta a\, E_4\right)$$

$$+ \Delta a\left(4 G^{\mu\nu}\partial_\mu\tau\partial_\nu\tau - 4(\partial\tau)^2\Box\tau + 2(\partial\tau)^4\right)\Big]$$

Explicit breaking

$M \to M e^{-\tau}$ $\quad \tau$ compensator

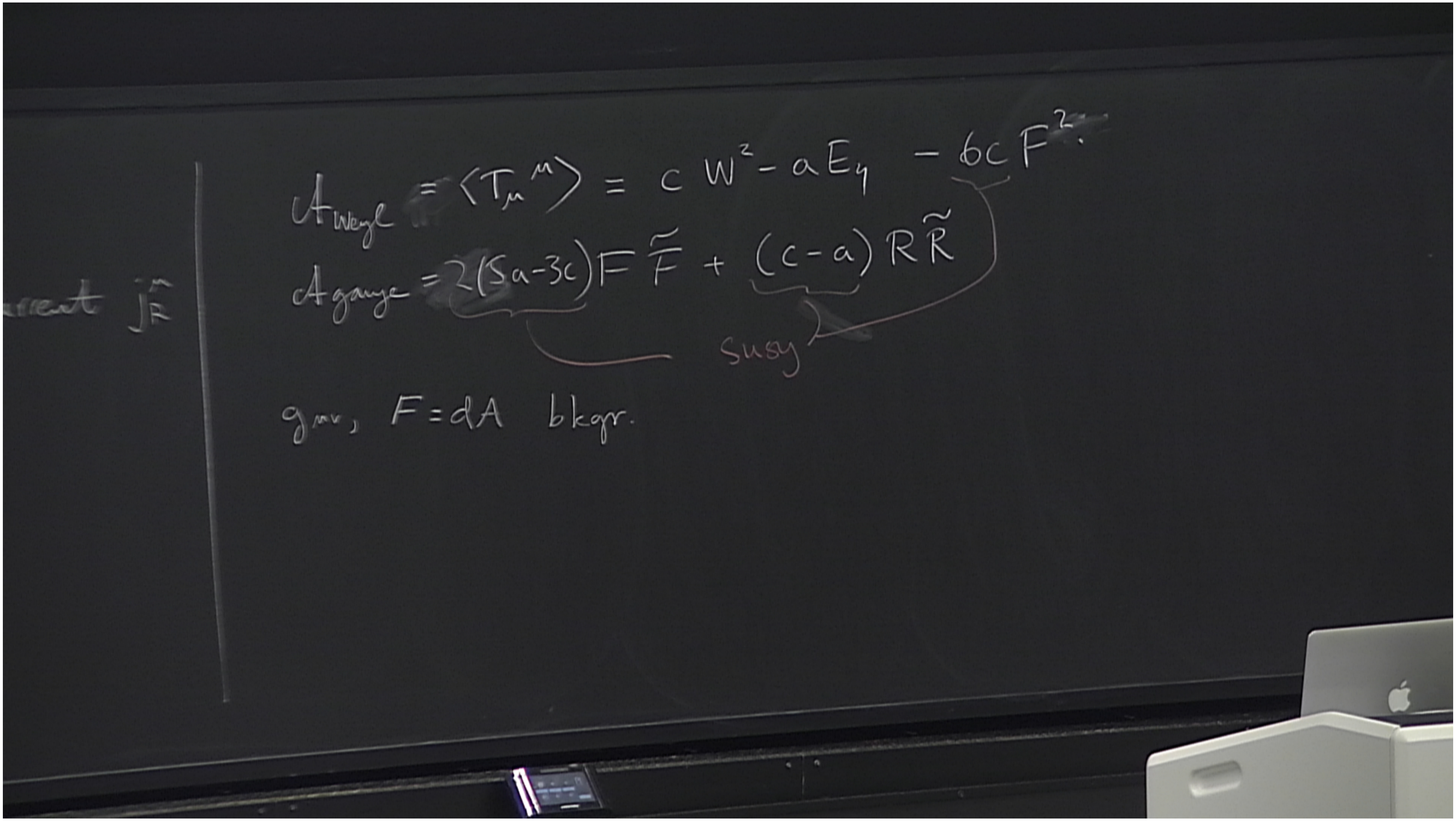
$e^{-\tau} = 1 - \frac{\varphi}{f}$ $\quad \Big| \quad \frac{M}{f} \ll 1$

## §3. SUSY:

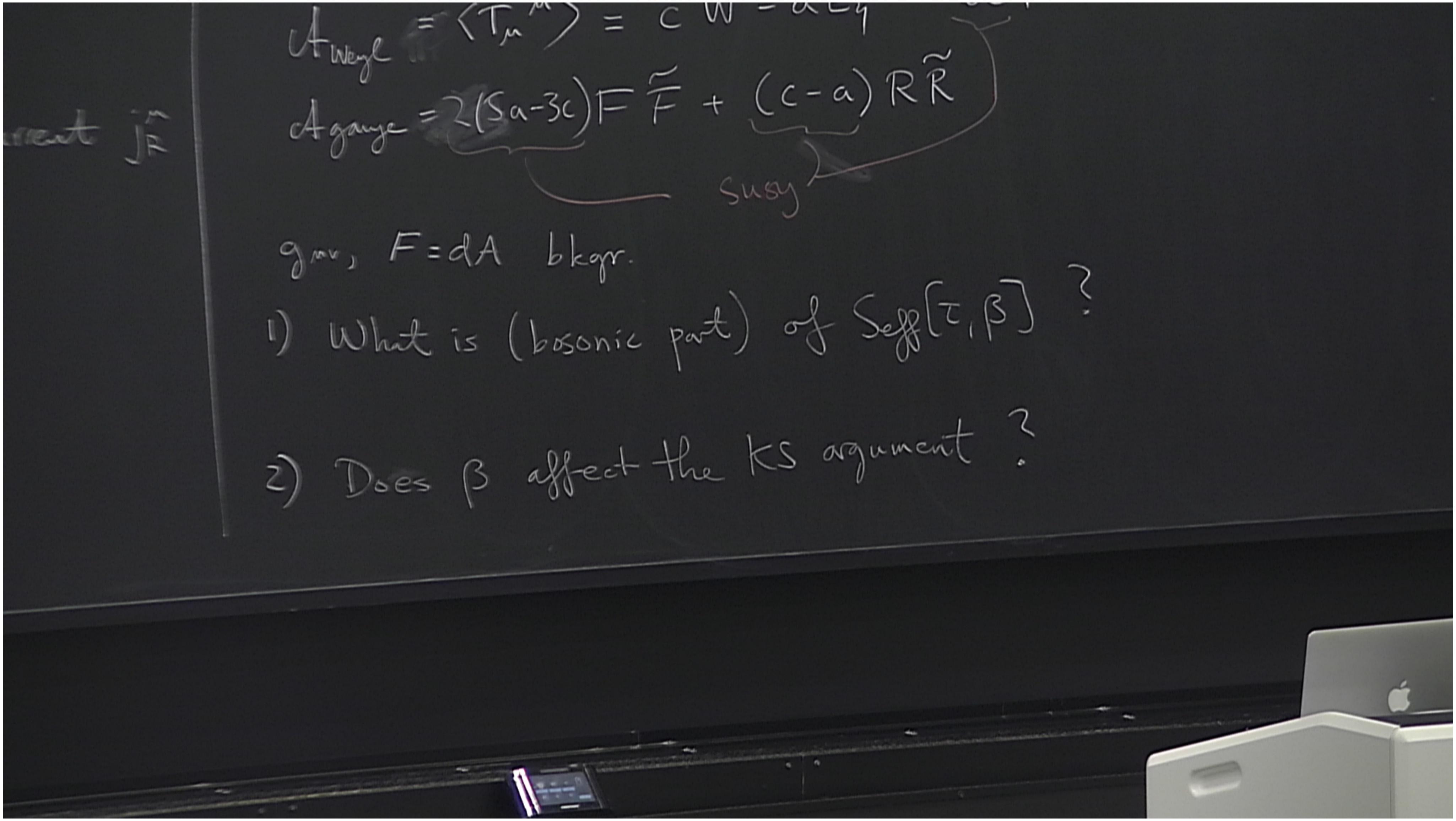
$T_{\mu\nu}$ same supermultiplet as $U(1)_R$ current $j^\mu_R$

$$\begin{array}{ccc} \cancel{\text{conf}} & + & \cancel{U(1)_R} \\ \downarrow & & \downarrow \\ \tau(x) & & \beta(x) \end{array}$$

$\mathcal{N}=1$ superfields $\Phi = \tau(x) + i\beta(x)$



rrent $j_F^\mu$
$\mathcal{A}_{Weyl} = \langle T_\mu{}^\mu \rangle = c\, W^2 - a\, E_4 \;\; - 6c\, F^2$
$\mathcal{A}_{gauge} = 2(5a-3c)\, F\tilde{F} + (c-a)\, R\tilde{R}$
susy
$g_{\mu\nu}$, $F = dA$ bkgr.


$\mathcal{A}_{Weyl} = \langle T_\mu{}^\mu \rangle = c W - a E_4$
$\mathcal{A}_{gauge} = 2(5a-3c) F \tilde{F} + (c-a) R \tilde{R}$
susy
$g_{\mu\nu}$, $F = dA$ bkgr.
1) What is (bosonic part) of $S_{eff}[\tau, \beta]$ ?
2) Does $\beta$ affect the KS argument ?

$$= \langle T_\mu{}^\mu \rangle = c\, W^2 - a E_4 - 6c F^2 .$$

$$= 2(5a-3c) F\tilde{F} + (c-a) R\tilde{R}$$

susy

$F = dA$ bkgr.

What is (bosonic part) of $S_{eff}[\tau, \beta]$ ?

Does $\beta$ affect the KS argument ?

$\beta$ pseudo-scalar

$\Rightarrow \tau\tau \to \tau\tau$ not affected. ✓

**Superspace formalism gives effective action:**

$$
\begin{aligned}
S_0 = & -f^2 \int d^4x \sqrt{-g}\, e^{-2\tau} \left( \frac{1}{2}(\nabla\tau)^2 + \frac{1}{12} R + \frac{1}{2} (\nabla\beta - A)^2 \right) \\
& + \int d^4x \sqrt{-g} \Big[ \Delta c\, \tau\, W^2 - \Delta a\, \tau\, E_4 - 6\, \Delta c\, \tau\, (F_{\mu\nu})^2 \\
& \qquad + \beta \left( 2\, (5\, \Delta a - 3\, \Delta c)\, F^{\mu\nu} \widetilde{F}_{\mu\nu} + (\Delta c - \Delta a)\, R^{\mu\nu\rho\sigma} \widetilde{R}_{\mu\nu\rho\sigma} \right) \Big] \\
& + 8\, \Delta a \int d^4x \sqrt{-g} \left( \left[ R^{\mu\nu} A_\nu - \frac{1}{6} R\, A^\mu + A^2 A^\mu \right] \nabla_\mu \beta - A^\mu A^\nu \nabla_\mu \nabla_\nu \tau \right) \\
& + 2\, \Delta a \int d^4x \sqrt{-g} \Big\{ \Big[ \left( R + 2 A^2 \right) g^{\mu\nu} - 2 \left( R^{\mu\nu} + 2 A^\mu A^\nu \right) \Big] \nabla_\mu \tau\, \nabla_\nu \tau \\
& \qquad + \Big[ \left( \frac{1}{3} R - 2 A^2 \right) g^{\mu\nu} - 2 \left( R^{\mu\nu} + 2 A^\mu A^\nu \right) \Big] \nabla_\mu \beta\, \nabla_\nu \beta + 8\, A^\nu \nabla^\mu \beta\, \nabla_\nu \nabla_\mu \tau \Big\} + \ldots
\end{aligned}
$$

Schwimmer-Theisen (2010)

+... stands for terms with higher powers of the fields.

*Note:* limit without $U(1)_R$ reproduces $S_{eff}$ with just dilaton.

<u>Weyl</u> $g_{\mu\nu} \to g_{\mu\nu} e^{2\sigma}$ , $\tau \to \tau + \sigma$

<u>gauge</u> $A_\mu \to A_\mu - \partial_\mu \alpha$ , $\beta \to \beta + \alpha$

Weyl: $g_{\mu\nu} \to g_{\mu\nu} e^{2\sigma}$, $\tau \to \tau + \sigma$

gauge: $A_\mu \to A_\mu - \partial_\mu \alpha$, $\beta \to \beta + \alpha$

$$\delta_\sigma S_{eff} = \int d^4x \; \sigma(x) \, \mathcal{A}_{Weyl}$$

$$\delta_\alpha S_{eff} = \int d^4x \; \alpha(x) \, \mathcal{A}_{gauge}$$

$$\underline{\text{Weyl}}:\quad g_{\mu\nu} \to g_{\mu\nu} e^{2\sigma}, \quad \tau \to \tau + \sigma$$

$$\underline{\text{gauge}}:\quad A_\mu \to A_\mu - \partial_\mu \alpha, \quad \beta \to \beta + \alpha$$

$$\delta_\sigma S_{eff} = \int d^4x \; \sigma(x)\, \mathcal{A}_{Weyl}$$

$$\delta_\alpha S_{eff} = \int d^4x \; \alpha(x)\, \mathcal{A}_{gauge}$$

$$S_{eff} = \underbrace{S_{WZ} + S_{inv}}_{\mathcal{N}=1 \text{ SUSY}}$$

$$S_{WZ} = \int d^4x \left\{ \tau \left( ac\, W^2 - \right.\right.$$

$$
\begin{aligned}
S_{WZ} = \int d^4x \Big\{ & \tau \left( \Delta c\, W^2 - \Delta a\, E_4 - 6 \Delta c\, F^2 \right] \\
& + \beta \left[ 2 (5 \Delta c - 3 \Delta a) F \tilde{F} + (\Delta a - \Delta c) R \tilde{R} \right] \\
& - \Delta a \left[ 4 G^{\mu\nu} \partial_\mu \tau \partial_\nu \tau - 2 (\partial \tau)^2 \left[ 2 \Box \tau - (\partial \tau)^2 \right] \right] \Big\} + \dots
\end{aligned}
$$

$$S_{WZ} = \int d^4x \Big\{ \tau \big( \Delta c\, W^2 - \Delta a\, E_4 - 6\Delta c\, F^2 \big] + \beta \big[ 2(5\Delta c - 3\Delta a) F\tilde{F} + (\Delta a - \Delta c) R\tilde{R} \big] - \Delta a \big[ 4 G^{\mu\nu}\partial_\mu\tau\,\partial_\nu\tau - 2(\partial\tau)^2 [2\Box\tau - (\partial\tau)^2] \big] \Big\} + \dots$$

Not SUSY

Need to fix $S_{inv.}$ ← Weyl + gauge inv.

$$\hat{g}_{\mu\nu} = g_{\mu\nu} e^{-2\tau}$$

$$\hat{A}_\mu = A_\mu - \partial_\mu\beta$$

$$O(\partial^2) \quad \sqrt{\hat{g}}\,\hat{R} \to \tfrac{1}{6} R + (\partial\tau)^2 e^{-2\tau}$$

$$\sqrt{\hat{g}}\,\hat{g}^{\mu\nu}\hat{A}_\mu \hat{A}_\nu \xrightarrow{\eta_{\mu\nu}} (\partial\beta)^2 e^{-2\tau}$$

**Weyl+gauge invariants at 4-derivative order:** $\sqrt{-\hat{g}}W_i$

$$W_1 \equiv \hat{W}^2\,, \qquad W_2 \equiv \hat{R}^2\,,$$
$$W_3 \equiv (A-\nabla\beta)_\mu\, \hat{\nabla}^\mu \hat{R}\,, \qquad W_4 \equiv \left(\hat{\nabla}^\mu (A-\nabla\beta)_\mu\right)^2\,,$$
$$W_5 \equiv \hat{g}^{\mu\nu}\,(A-\nabla\beta)_\mu\, \hat{\Box}\,(A-\nabla\beta)_\nu\,, \qquad W_6 \equiv \hat{R}^{\mu\nu}\,(A-\nabla\beta)_\mu\,(A-\nabla\beta)_\nu\,,$$
$$W_7 \equiv \hat{R}\,\hat{g}^{\mu\nu}\,(A-\nabla\beta)_\mu\,(A-\nabla\beta)_\nu\,, \qquad W_8 \equiv \left(\hat{g}^{\mu\nu}\,(A-\nabla\beta)_\mu\,(A-\nabla\beta)_\nu\right)^2\,,$$
$$W_9 \equiv \hat{g}^{\mu\nu}\,(A-\nabla\beta)_\mu\,(A-\nabla\beta)_\nu\,\hat{\nabla}^\lambda (A-\nabla\beta)_\lambda\,.$$

Classified up to total derivatives, Bianchi identity etc:

Bianchi identity implies $\hat{R}^{\mu\nu}\,\hat{\nabla}_\mu (A-\nabla\beta)_\nu = \hat{\nabla}_\mu\Big(\hat{R}^{\mu\nu}\,(A-\nabla\beta)_\nu\Big) - \frac{1}{2}W_3.$

$$S_{\rm inv} = \int d^4x\,\sqrt{-\hat{g}}\left[-\frac{f^2}{2}\left(\frac{\hat{R}}{6} + \hat{g}^{\mu\nu}\,(A-\nabla\beta)_\mu\,(A-\nabla\beta)_\nu\right) + \sum_{i=1}^{9}\gamma_i\,W_i + \mathcal{O}(\nabla^6)\right]$$

**Superspace formalism gives effective action:**

$$
\begin{aligned}
S_0 = &-f^2 \int d^4x\,\sqrt{-g}\,e^{-2\tau}\Big(\frac{1}{2}(\nabla\tau)^2 + \frac{1}{12}R + \frac{1}{2}(\nabla\beta - A)^2\Big) \\
&+ \int d^4x\,\sqrt{-g}\,\Big[\,\Delta c\,\tau\,W^2 - \Delta a\,\tau\,E_4 - 6\,\Delta c\,\tau\,(F_{\mu\nu})^2 \\
&\qquad + \beta\,\Big(2\,(5\,\Delta a - 3\,\Delta c)\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + (\,\Delta c - \Delta a\,)\,R^{\mu\nu\rho\sigma}\,\widetilde{R}_{\mu\nu\rho\sigma}\Big)\Big] \\
&+ 8\,\Delta a \int d^4x\,\sqrt{-g}\,\Big(\Big[R^{\mu\nu}A_\nu - \frac{1}{6}R\,A^\mu + A^2\,A^\mu\Big]\,\nabla_\mu\beta - A^\mu\,A^\nu\,\nabla_\mu\nabla_\nu\tau\Big) \\
&+ 2\,\Delta a \int d^4x\,\sqrt{-g}\,\Big\{\Big[\Big(R + 2\,A^2\Big)g^{\mu\nu} - 2\,\Big(R^{\mu\nu} + 2\,A^\mu\,A^\nu\Big)\Big]\nabla_\mu\tau\,\nabla_\nu\tau \\
&\quad + \Big[\Big(\frac{1}{3}R - 2\,A^2\Big)g^{\mu\nu} - 2\,\Big(R^{\mu\nu} + 2\,A^\mu\,A^\nu\Big)\Big]\nabla_\mu\beta\,\nabla_\nu\beta + 8\,A^\nu\nabla^\mu\beta\,\nabla_\nu\nabla_\mu\tau\Big\} + \ldots
\end{aligned}
$$

Schwimmer-Theisen (2010)

+... stands for terms with higher powers of the fields.

*Note:* limit without $U(1)_R$ reproduces $S_{eff}$ with just dilaton.

## SUSY Ward identities

N=1 SUSY action on creation/annihilation operators:

$$\begin{aligned}
[Q,\zeta] &= [p|\,\lambda\,, & [Q^\dagger,\lambda] &= |p\rangle\,\zeta\,,\\
[Q,\lambda] &= 0\,, & [Q^\dagger,\zeta] &= 0\,,\\
[Q,\bar\zeta] &= 0\,, & [Q^\dagger,\bar\lambda] &= 0\,,\\
[Q,\bar\lambda] &= [p|\,\bar\zeta\,, & [Q^\dagger,\bar\zeta] &= |p\rangle\,\bar\lambda\,,
\end{aligned}$$

$$p_\mu\,(\bar\sigma^\mu)^{\dot a b} = -|p\rangle^{\dot a}[p|^b$$

spinor helicity formalism

$$\langle pq\rangle = \langle p|_{\dot a}|q\rangle^{\dot a}$$

Match to ST. : $\underbrace{\gamma_6 = -6\gamma_7 = 2\gamma_8 = -4\Delta a}$, other $\gamma_i = 0$.

## §4. SUSY WI

chiral massless scalar $\zeta$ in $\mathcal{N}=1$ SUSY.

$$\left.\begin{array}{l} \delta_\varepsilon \zeta = \varepsilon \lambda \\ \text{free-field exp} \end{array}\right\} \quad a^{(+)} \text{ transf under Susy}$$

## SUSY Ward identities

N=1 SUSY action on creation/annihilation operators:

$$\begin{aligned} [Q,\zeta] &= [p|\,\lambda\,, & [Q^\dagger,\lambda] &= |p\rangle\,\zeta\,,\\ [Q,\lambda] &= 0\,, & [Q^\dagger,\zeta] &= 0\,,\\ [Q,\bar\zeta] &= 0\,, & [Q^\dagger,\bar\lambda] &= 0\,,\\ [Q,\bar\lambda] &= [p|\,\bar\zeta\,, & [Q^\dagger,\bar\zeta] &= |p\rangle\,\bar\lambda\,, \end{aligned}$$

$$p_\mu\,(\bar\sigma^\mu)^{\dot a b} = -|p\rangle^{\dot a}[p|^b$$

spinor helicity formalism

$$\langle pq\rangle = \langle p|_{\dot a}|q\rangle^{\dot a}$$

SUSY Ward identities:

$$0 = \langle \mathbf{0}|\,[Q^\dagger,\lambda\,\zeta\,\zeta\,\zeta]\,|\mathbf{0}\rangle = \langle \mathbf{0}|\,[Q^\dagger,\lambda]\,\zeta\,\zeta\,\zeta\,|\mathbf{0}\rangle = |p_1\rangle\,\langle \mathbf{0}|\,\zeta\,\zeta\,\zeta\,\zeta\,|\mathbf{0}\rangle$$

so $\boxed{\mathcal{A}_4(\zeta\,\zeta\,\zeta\,\zeta) = 0}$

## SUSY Ward identities

N=1 SUSY action on creation/annihilation operators:

$$\boxed{\begin{aligned} [Q,\zeta] &= [p|\,\lambda\,, & [Q^\dagger,\lambda] &= |p\rangle\,\zeta\,,\\ [Q,\lambda] &= 0\,, & [Q^\dagger,\zeta] &= 0\,,\\ [Q,\bar\zeta] &= 0\,, & [Q^\dagger,\bar\lambda] &= 0\,,\\ [Q,\bar\lambda] &= [p|\,\bar\zeta\,, & [Q^\dagger,\bar\zeta] &= |p\rangle\,\bar\lambda\,, \end{aligned}}$$

$$p_\mu\,(\bar\sigma^\mu)^{\dot a b} = -|p\rangle^{\dot a}[p|^b$$

spinor helicity formalism

$$\langle pq\rangle = \langle p|_{\dot a}|q\rangle^{\dot a}$$

SUSY Ward identities:

$$0 = \langle \mathbf{0}|\,[Q^\dagger,\lambda\,\zeta\,\zeta\,\zeta]\,|\mathbf{0}\rangle = \langle \mathbf{0}|\,[Q^\dagger,\lambda]\,\zeta\,\zeta\,\zeta\,|\mathbf{0}\rangle = |p_1\rangle\,\langle \mathbf{0}|\,\zeta\,\zeta\,\zeta\,\zeta\,|\mathbf{0}\rangle$$

so $\boxed{\mathcal{A}_4(\zeta\,\zeta\,\zeta\,\zeta) = 0}$

$$0 = \langle \mathbf{0}|\,[Q^\dagger,\bar\zeta\,\lambda\,\zeta\,\zeta]\,|\mathbf{0}\rangle = |p_1\rangle\,\langle \mathbf{0}|\,\bar\lambda\,\lambda\,\zeta\,\zeta\,|\mathbf{0}\rangle + |p_2\rangle\,\langle \mathbf{0}|\,\bar\zeta\,\zeta\,\zeta\,\zeta]\,|\mathbf{0}\rangle$$

$$\boxed{\mathcal{A}_4(\bar\zeta\,\zeta\,\zeta\,\zeta) = 0}$$

## SUSY Ward identities - results

$$\mathcal{A}_4(\zeta\zeta\zeta\zeta) = \mathcal{A}_4(\bar\zeta\bar\zeta\bar\zeta\bar\zeta) = 0\,,$$

$$\mathcal{A}_4(\bar\zeta\zeta\zeta\zeta) = \mathcal{A}_4(\zeta\bar\zeta\zeta\zeta) = \ldots = \mathcal{A}_4(\bar\zeta\bar\zeta\bar\zeta\zeta) = 0\,.$$

While $\mathcal{A}_4(\bar\zeta\bar\zeta\zeta\zeta)$ can be non-vanishing.

Now write $\zeta = \varphi + i\xi$ . Then

$$\boxed{\begin{aligned} \mathcal{A}_4(\varphi\varphi\varphi\varphi) &= \mathcal{A}_4(\xi\xi\xi\xi)\,, \\ \mathcal{A}_4(\varphi\varphi\varphi\varphi) &= \mathcal{A}_4(\varphi\varphi\xi\xi) + \mathcal{A}_4(\varphi\xi\varphi\xi) + \mathcal{A}_4(\varphi\xi\xi\varphi)\,. \end{aligned}}$$

Action with only dilaton interactions cannot be SUSY!

**N=1 SUSY dilaton effective action**

- We matched

$$S = S_{\mathrm{WZ}} + S_{\mathrm{inv}}$$

to Schwimmer+Theisen result $\gamma_6 = -6\,\gamma_7 = 2\,\gamma_8 = -4\,\Delta a$

- Take *trivial bkgr* limit to test via SUSY WI's:

$$S_0 = \int d^4x \Big\{ -\frac{f^2}{2}\, e^{-2\tau} \Big[(\partial\tau)^2 + (\partial\beta)^2\Big] + 2\,\Delta a\,\Big[2\,\Box\tau\big((\partial\tau)^2 - (\partial\beta)^2\big) + 4\,\Box\beta\,(\partial\tau\cdot\partial\beta) - 4\,(\partial\tau\cdot\partial\beta)^2 - \big((\partial\tau)^2 - (\partial\beta)^2\big)^2\Big] + \mathcal{O}(\partial^6)\Big\}\,.$$

- *Rewrite*, noting: $Z \equiv e^{-(\tau + i\,\beta)} \quad \Rightarrow \quad |\partial Z|^2 = e^{-2\tau}\Big((\partial\tau)^2 + (\partial\beta)^2\Big)$

$$\boxed{S_0 = \int d^4x \left\{ -\frac{f^2}{2}\left|\partial Z\right|^2 + 2\Delta a\left[ -\left(\frac{\partial Z}{Z}\right)^2 \frac{\Box \overline{Z}}{\overline{Z}} - \left(\frac{\partial \overline{Z}}{\overline{Z}}\right)^2 \frac{\Box Z}{Z} + \left|\frac{\partial Z}{Z}\right|^4\right] + \mathcal{O}(\partial^6)\right\}}$$

## N=1 SUSY dilaton effective action

- Expand in "physical modes"

dilaton  axion

$$Z = 1 - \frac{\zeta}{f}, \qquad \zeta = \varphi + i\,\xi$$

$$\boxed{S_0 \to \int d^4x \left\{ -\frac{1}{2}\Big((\partial\varphi)^2 + (\partial\xi)^2\Big) + \frac{2\,\Delta a}{f^4}\left[\Big((\partial\varphi)^2 - (\partial\xi)^2\Big)^2 + 4\,(\partial\varphi\cdot\partial\xi)^2\right]\right\}}$$

Plus terms that vanish on the EOM: $\Box\varphi = 0\,, \quad \Box\xi = 0$

- 4-pt amplitudes **obey** the SUSY WI's, e.g.:

$$\mathcal{A}_4(\varphi\,\varphi\,\varphi\,\varphi) = \frac{4\Delta a}{f^4}(s^2 + t^2 + u^2) \ = \mathcal{A}_4(\xi\,\xi\,\xi\,\xi)$$

$$\mathcal{A}_4(\varphi\,\varphi\,\xi\,\xi) = \frac{4\Delta a}{f^4}(-s^2 + t^2 + u^2)$$

## §5 Outlook

- a-thm in $d=6$? [1205.3994]  GJMS op.
- dilaton eff. action in any $d$. [1209.3424] w/ Tim Olson.
- odd-$d$. 3d $f$-theorem $\begin{cases} \text{E.E. proof ✓} \\ \text{field theory?} \end{cases}$
- flow across dimension. M5 branes 2d Riemann

  $6d \downarrow 4d$.  $\mathcal{N}=1$ SUSY