

Title: 13/14 PSI - Explorations in String Theory - Lecture 11

Date: Mar 31, 2014 11:30 AM

URL: <http://pirsa.org/14030073>

Abstract:

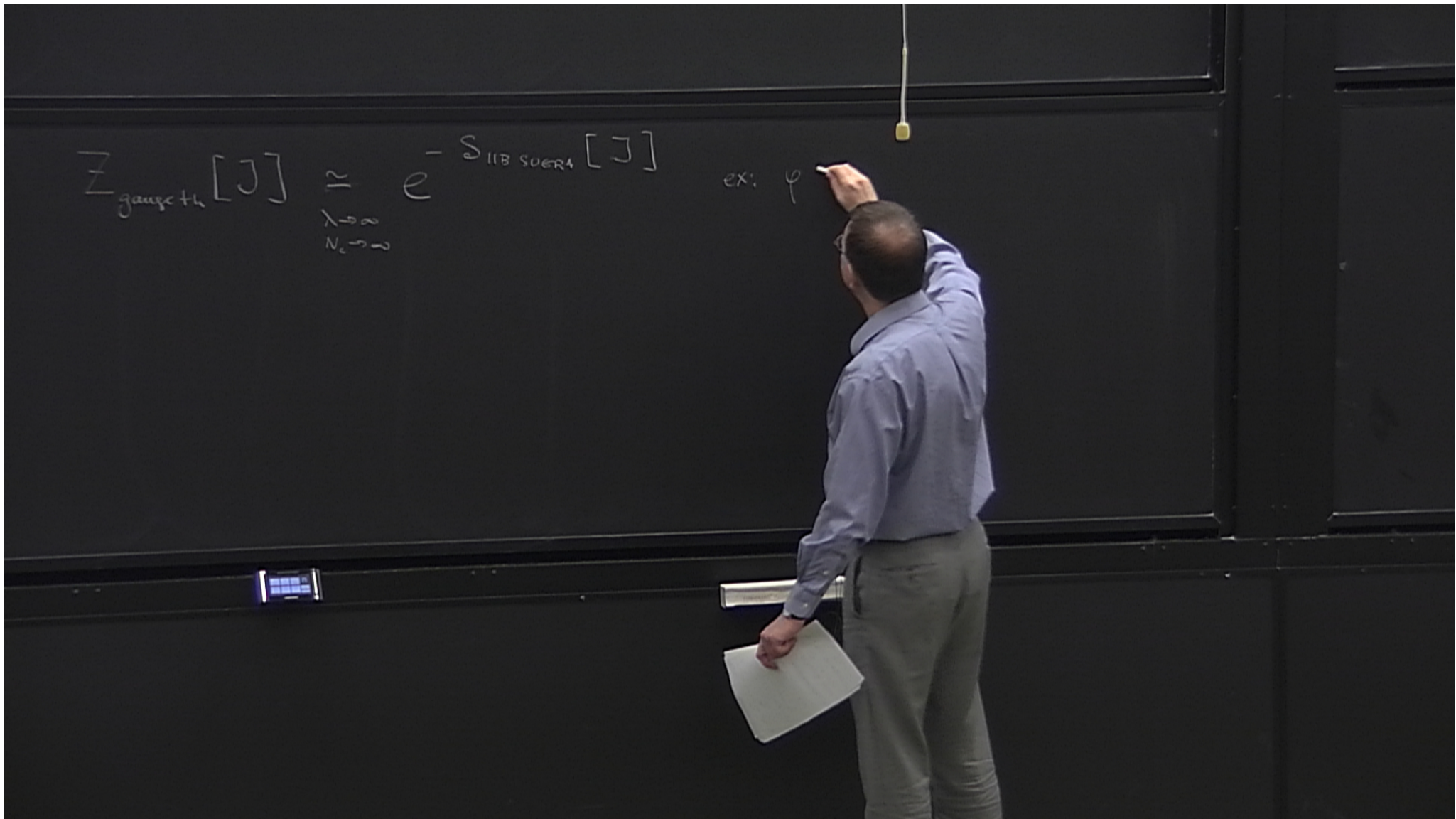
scalar field in AdS_5 :

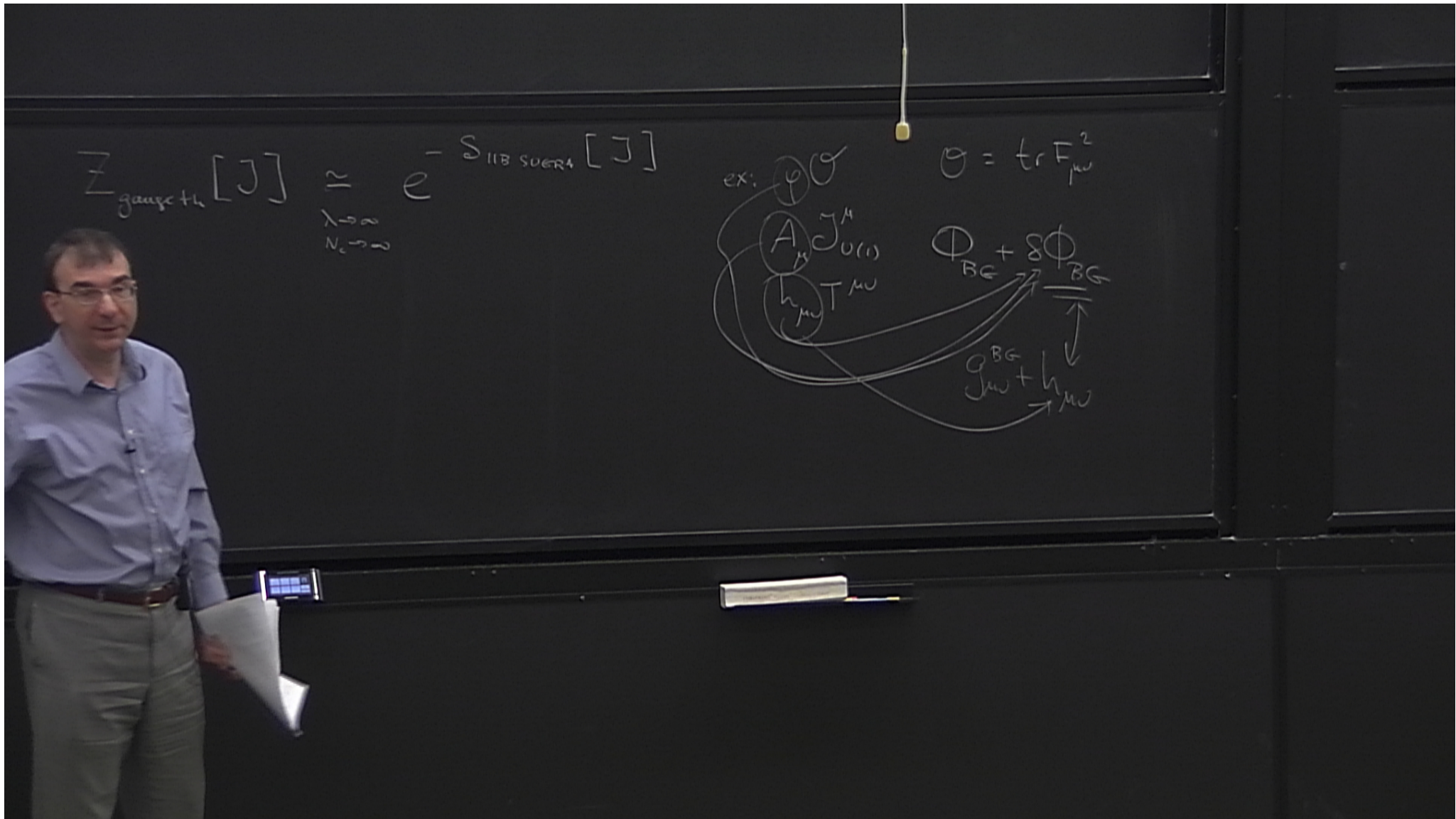
$$Z_{\text{gauge th}}[J] \simeq e^{-S}$$

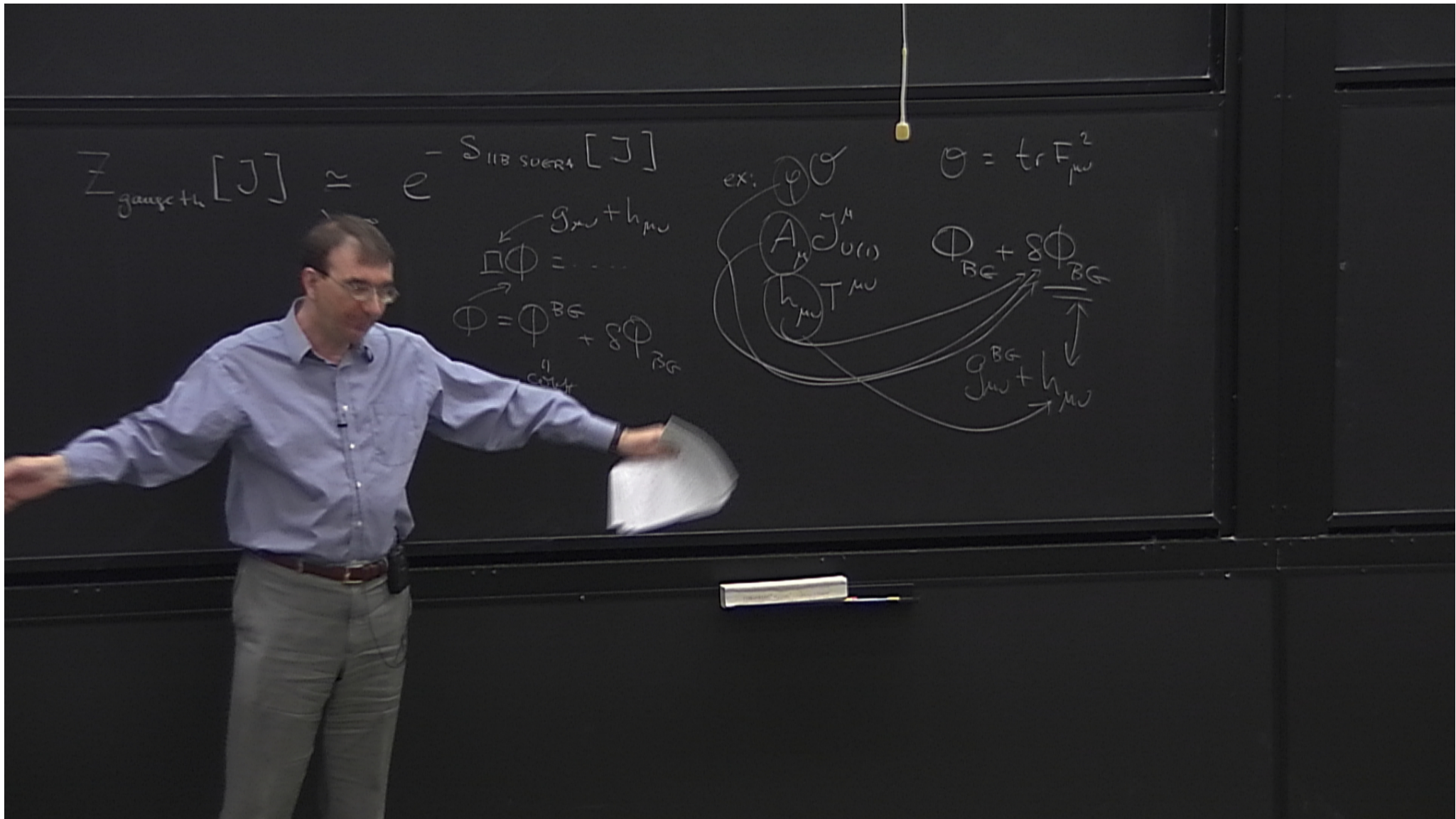
$\lambda \rightarrow \infty$
 $N_c \rightarrow \infty$

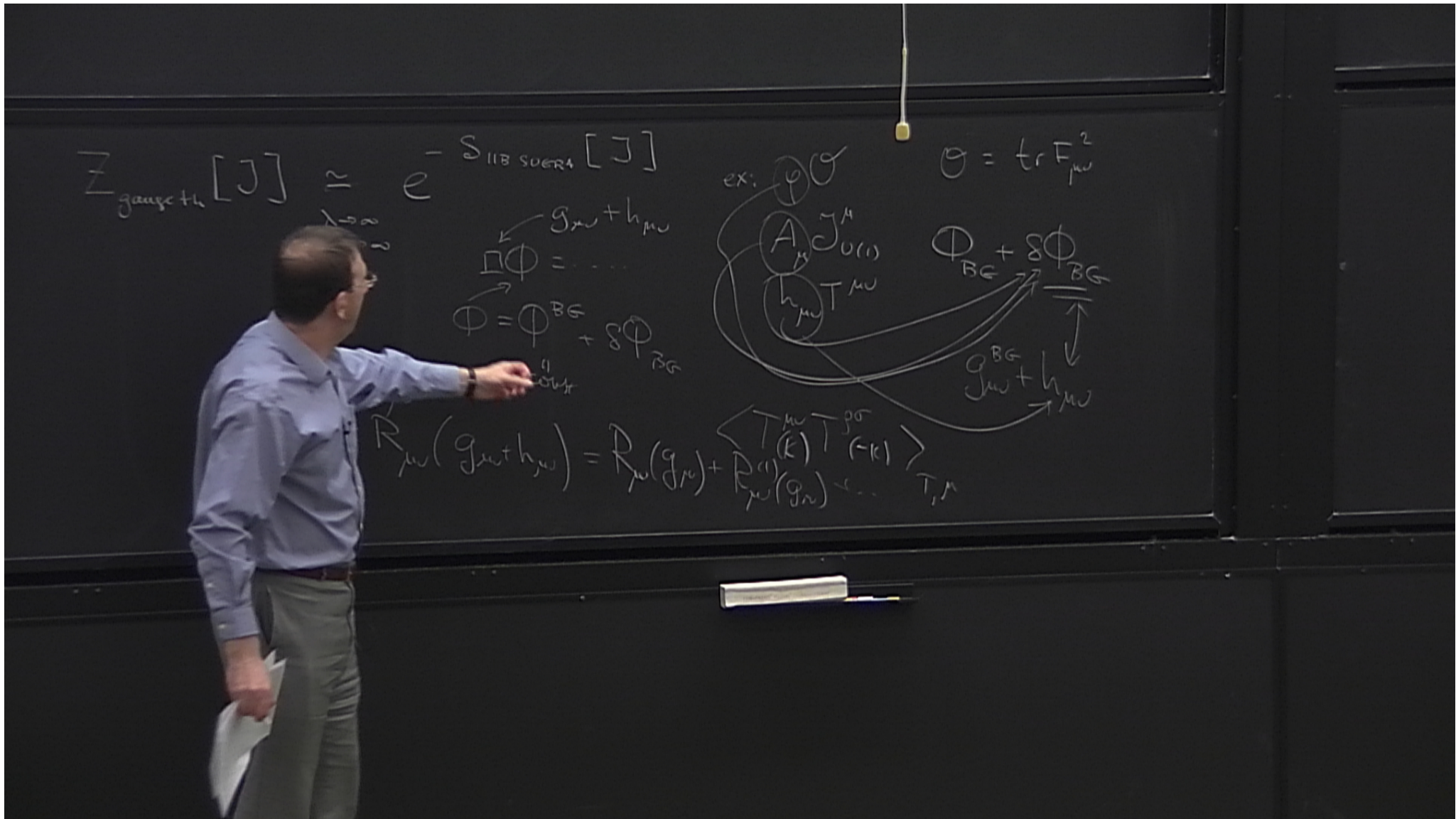
scalar field in AdS_5 :

$$Z_{\text{gauge th}}[J] \underset{\substack{\lambda \rightarrow \infty \\ N_c \rightarrow \infty}}{\approx} e^{-S_{\text{IIB SUGRA}}[J]}$$





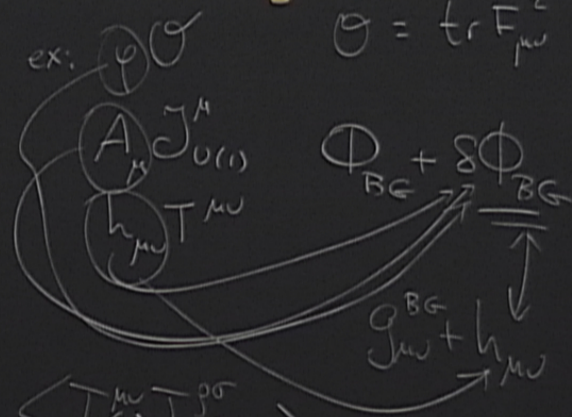




$$Z_{\text{gauge}+h}[\mathcal{J}] \underset{\lambda \rightarrow \infty}{\simeq} e^{-S_{\text{IIB super4}}[\mathcal{J}]}$$

$$\square \Phi = \dots$$

$$\Phi = \Phi_{\text{BG}} + \delta \Phi_{\text{BG}}$$



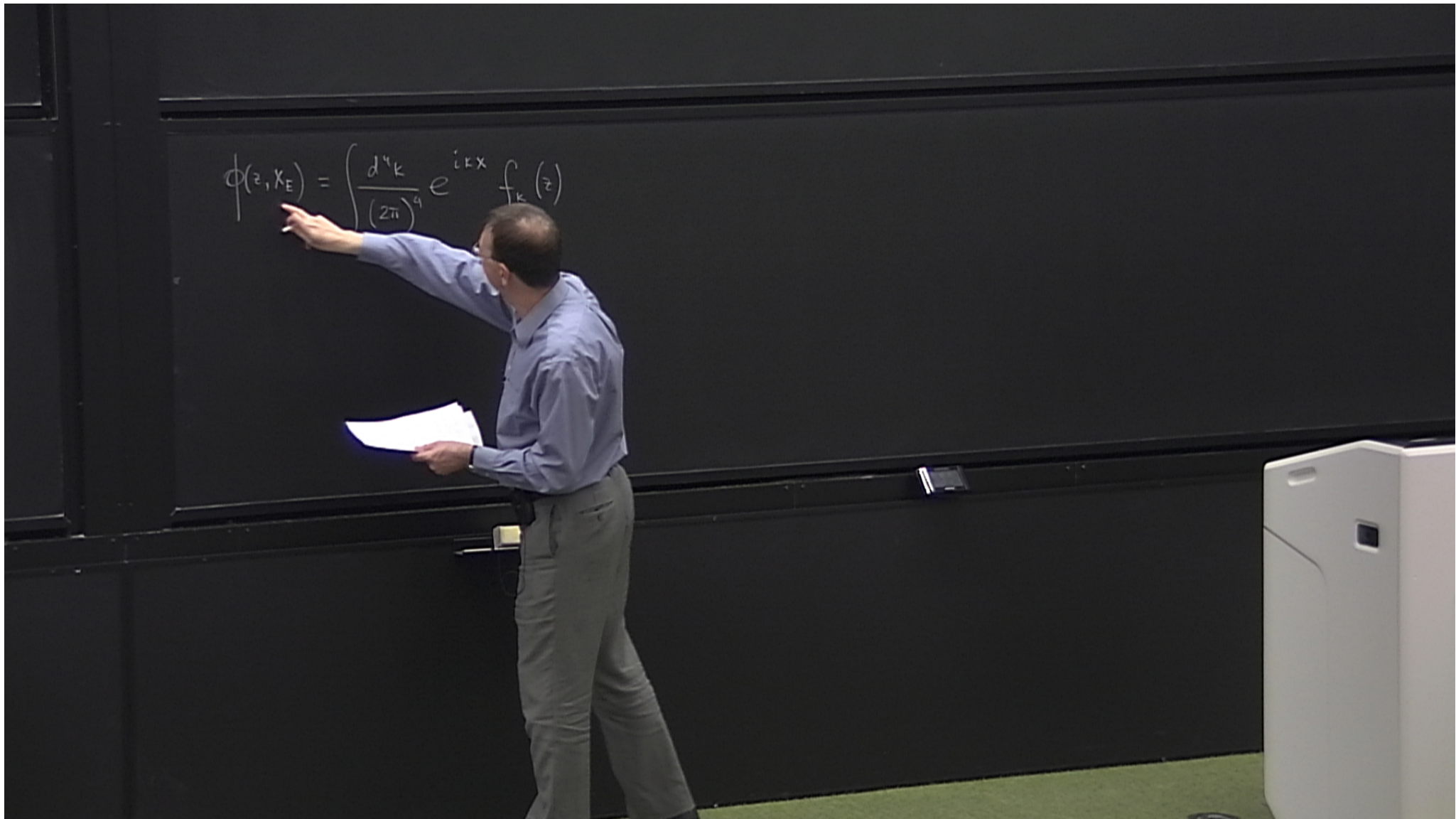
$$\Theta = \text{tr} F_{\mu\nu}^2$$

$$R_{\mu\nu}(g_{\mu\nu}+h_{\mu\nu}) = R_{\mu\nu}(g_{\mu\nu}) + \left\langle T_{\mu\nu}(k) T^{\rho\sigma}(-k) \right\rangle_{T, A}$$

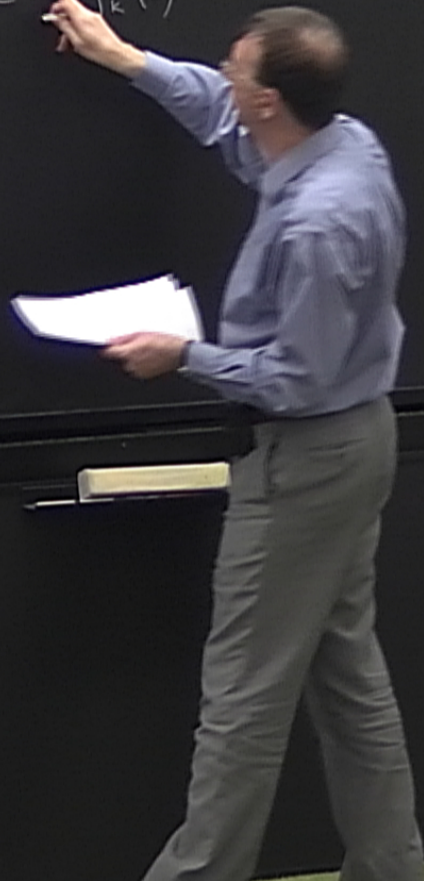
- Euclidean

$$Z_{\text{gauge th}}[J] \approx e^{-S_{\text{11B SUPER}}[J]}$$

$$m(g + h_{\text{ref}}) = R_{\text{ref}}(g)$$

A man in a light blue shirt and grey trousers stands in front of a large blackboard. He is pointing with his right hand towards a mathematical equation written on the board. In his left hand, he holds a stack of papers. The equation is written in white chalk. To the right of the man, a small electronic device is mounted on the blackboard. A white piece of equipment is visible on the far right side of the frame.
$$\phi(z, x_E) = \int \frac{d^4 k}{(2\pi)^4} e^{i k x} f_k(z)$$

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$$\phi(z, x_E) = \int \frac{d^4 k}{(2\pi)^4} e^{i \hat{x}_E \hat{k}} f_k(z) \phi_0(k)$$

$$S_E = \frac{\pi^3 L^2}{4 x_{10}^2} \int dz$$

$$\phi(z, x_E) = \int \frac{d^4 k}{(2\pi)^4} e^{i \tilde{E} \hat{x}_E} \underbrace{f_k(z)} \phi_0(k)$$

$$S_E = \frac{\pi^3 L^2}{4 x_{10}^2} \int_{\mathcal{C}} dz \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 k'}{z^2} \frac{\delta(k+k')}{z^2} \left[\partial_z f_k \partial_z f_{k'} - k k' f_k f_{k'} + \frac{m^2 L^2}{z^2} f_k f_{k'} \right] \phi_0(k) \phi_0(k')$$

$$\phi(z, k_E) = \int \frac{d^4 k}{(2\pi)^4} e^{i \vec{k} \vec{x}_E} \underbrace{f_k(z)} \phi_0(k)$$

$$S_E = \frac{\pi^3 L^8}{4\chi_{10}^2} \int_{\mathcal{E}} dz \int \frac{d^4 k d^4 k'}{(2\pi)^4} \frac{\delta(k+k')}{z^2} \left[\partial_z f_k \partial_z f_{k'} - k k' f_k f_{k'} + \frac{m^2 L^2}{z^2} f_k f_{k'} \right] \phi_0(k) \phi_0(k')$$

The e.o.m.

$$f_k'' - \frac{3}{z} f_k' - \left(k_E^2 + \frac{m^2 L^2}{z^2} \right) f_k = 0 \Rightarrow$$

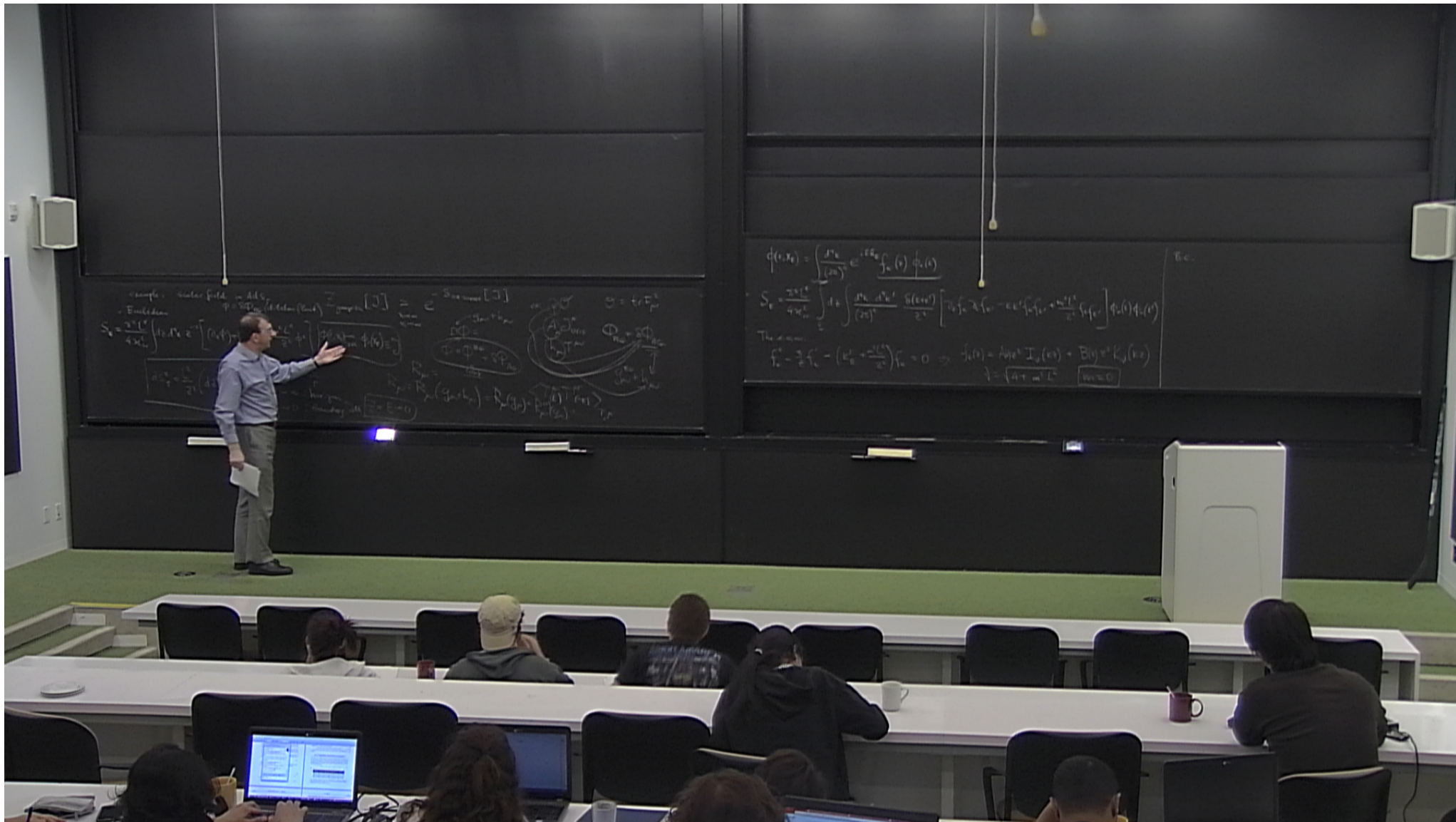
$$\phi(z, k_E) = \int \frac{d^4 k}{(2\pi)^4} e^{i \vec{k} \vec{x}_E} \underbrace{f_k(z)}_{\phi_0(k)}$$

$$S_E = \frac{\pi^3 L^8}{4\chi_{10}^2} \int_{\mathcal{E}} d\vec{z} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 k'}{(2\pi)^4} \frac{\delta(k+k')}{z^2} \left[\partial_z f_k \partial_z f_{k'} - k k' f_k f_{k'} + \frac{m^2 L^2}{z^2} f_k f_{k'} \right] \phi_0(k) \phi_0(k')$$

The e.o.m.

$$f_k'' - \frac{3}{z} f_k' - \left(k_E^2 + \frac{m^2 L^2}{z^2} \right) f_k = 0 \Rightarrow f_k(z) = A(k) z^2 I_\nu(kz) + B(k) z^2 K_\nu(kz)$$

$$\nu = \sqrt{4 + m^2 L^2}$$



$$ds^2 = \frac{1}{z^2} (dz^2 + d\vec{x}_E^2)$$

$z \rightarrow \infty$: horizon

$z \rightarrow 0$: boundary

$$\mathcal{M} \left[\bar{z} = \epsilon \rightarrow 0 \right]$$

$$R_{\mu\nu} = R_{\mu\nu}(g_{\mu\nu} + h_{\mu\nu}) = R_{\mu\nu}(g_{\mu\nu}) + R_{\mu\nu}^{(1)}(g_{\mu\nu}) + \dots$$

The action (on shell!) reduces to

$$S_E = \frac{\pi^3 L^8}{4\kappa_{10}^2} \int \frac{d^4k d^4k'}{(2\pi)^8} (2\pi)^4 \delta^{(4)}(k+k') \phi_0(k) \phi_0(k') \left(\frac{g^{\mu\nu}(z) \partial_\mu f_\nu(z)}{z^3} \right)$$

$$ds^2 = \frac{1}{z^2} (dz^2 + d\vec{x}_E^2)$$

$z \rightarrow \infty$: horizon
 $z \rightarrow 0$: boundary

$$R_{\mu\nu} = R_{\mu\nu}(g_{\mu\nu} + h_{\mu\nu}) = R_{\mu\nu}(g_{\mu\nu}) + R_{\mu\nu}^{(1)}(g_{\mu\nu}) + \dots$$

The action (on shell!) reduces to

$$S_E = \frac{\pi^3 L^8}{4\kappa_{10}^2} \int \frac{d^4k d^4k'}{(2\pi)^8} (2\pi)^4 \delta^{(4)}(k+k') \phi_0(k) \phi_0(k') \cdot \left(\frac{f_{k'}(z)}{f_k(z)} \right)$$

$$ds^2 = \frac{L^2}{z^2} (dz^2 + d\vec{x}_E^2)$$

$$z = \frac{L}{r}$$

$$z \rightarrow \infty : \text{horizon}$$

$$z \rightarrow 0 : \text{boundary}$$

$$R_{\mu\nu} = R_{\mu\nu}(g_{\mu\nu} + h_{\mu\nu}) = R_{\mu\nu}(g_{\mu\nu}) + R_{\mu\nu}^{(1)}(g_{\mu\nu}) + \dots$$

$$R_{\mu\nu}^{(1)}(g_{\mu\nu}) = -\frac{1}{2} \nabla_\mu \nabla_\nu h + \dots$$

The action (on shell!) reduces to

$$S_E = \frac{\pi^3 L^8}{4 \gamma \epsilon_{10}^2} \int \frac{d^4 k d^4 k'}{(2\pi)^8} (z\pi)^4 \delta^{(4)}(k+k') \phi_0(k) \phi_0(k') \cdot \left(\frac{f_{k'}(z) \partial_z f_k(z)}{z^3} \right) \Bigg|_{\epsilon}^{\infty} \equiv \left(\frac{1}{z^3} \phi_0(k) \phi_0(k') \mathcal{F}(z, k, k') \right) \Bigg|_{\epsilon}^{\infty}$$

$$ds^2 = \frac{L^2}{z^2} (dz^2 + d\vec{x}_E^2)$$

$z = \frac{L}{r}$
 $z \rightarrow \infty$: horizon
 $z \rightarrow 0$: boundary

$$\mathcal{M} \left[\bar{z} = \varepsilon \rightarrow 0 \right]$$

$$R_{\mu\nu} = R_{\mu\nu}(g_{\mu\nu} + h_{\mu\nu}) = R_{\mu\nu}(g_{\mu\nu}) + R_{\mu\nu}^{(1)}(g_{\mu\nu}) + \dots$$

$$g_{\mu\nu} + h_{\mu\nu} \rightarrow g_{\mu\nu}$$

The action (on shell!) reduces to

$$S_E = \frac{\pi^3 L^8}{4 \pi^2} \left(\frac{d^4 k d^4 k'}{(2\pi)^8} \right) (z\pi)^4 \delta^{(4)}(k+k') \phi_0(k) \phi_0(k') \cdot \left(\frac{f_{k'}(z) \partial_z f_k(z)}{z^3} \right) \Bigg|_{\varepsilon}^{\infty} \equiv \int \frac{d^4 k d^4 k'}{(2\pi)^8} \phi_0(k) \phi_0(k') \mathcal{F}(z, k, k') \Bigg|_{\varepsilon}^{\infty}$$

The 2-pt function $\hat{\Theta}$ coupled to ϕ is

$$\langle \hat{\Theta}$$

We find $\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle = \frac{N_c^2}{8\pi^2} (2\pi)^4 \delta^{(4)}(k+k') \left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right]$, where $[...] = \lim_{\varepsilon \rightarrow 0}$ and discarding contact terms.

$$\frac{f'_k(\varepsilon)}{\varepsilon^3} = -\frac{k^2}{2\varepsilon^2} - \frac{\gamma_E k^4}{4} - \frac{k^4}{4}$$

We find $\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle = \frac{N_c^2}{8\pi^2} (2\pi)^4 \delta^{(4)}(k+k') \left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right]$, where $\left[\dots \right] = \lim_{\varepsilon \rightarrow 0}$ and discarding contact terms.

$$\frac{f'_k(\varepsilon)}{\varepsilon^3} = -\frac{k^2}{2} - \frac{E^2 k^4}{4} - \frac{k^4 \ln \varepsilon}{4} - \frac{k^4 \ln k/2}{4} + O(\varepsilon^2) \quad \varepsilon \rightarrow 0$$

We find $\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle = \frac{N_c^2}{8\pi^2} (2\pi)^4 \delta^{(4)}(k+k') \left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right]$, where $\left[\dots \right] = \lim_{\varepsilon \rightarrow 0}$ and discarding contact terms.

$$\frac{f'_k(\varepsilon)}{\varepsilon^3} = -\frac{k^2}{2\varepsilon^2} - \frac{\gamma_E k^4}{4} - \frac{k^4 \ln \varepsilon}{4} - \frac{k^4}{8} + O(\varepsilon^2) \quad \varepsilon \rightarrow 0$$

$$\left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right] = -\frac{k^4}{8} \ln k^2$$

We find $\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle = \frac{N_c^2}{8\pi^2} (2\pi)^4 \delta^{(4)}(k+k') \left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right]$, where $\left[\dots \right] = \lim_{\varepsilon \rightarrow 0}$ and discarding contact terms.

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Prior: add const

$$\left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right] = -\frac{k^4}{8} \ln k^2$$

We find $\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle = \frac{N_c^2}{8\pi^2} (2\pi)^4 \delta^{(4)}(k+k') \left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right]$, where $\left[\dots \right] = \lim_{\varepsilon \rightarrow 0}$ and discarding contact terms.

$$\frac{f'_k(\varepsilon)}{\varepsilon^3} = -\frac{k^2}{2\varepsilon^2} - \frac{\gamma_E k^4}{4} - \frac{k^4 \ln \varepsilon}{4} - \frac{k^4 \ln k/2}{4} + O(\varepsilon^2) \quad \varepsilon \rightarrow 0$$

Prior: add counterterms to the action.
Holographic renormalization

$$\left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right] = -\frac{k^4}{8} \ln k^2$$

We find $\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle = \frac{N_c^2}{8\pi^2} (2\pi)^4 \delta^{(4)}(k+k') \left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right]$, where $\left[\dots \right] = \lim_{\varepsilon \rightarrow 0}$ and discarding contact terms.

$$\frac{f'_k(\varepsilon)}{\varepsilon^3} = -\frac{k^2}{2\varepsilon^2} - \frac{\gamma_E k^4}{4} - \frac{k^4 \ln \varepsilon}{4} - \frac{k^4 \ln k/2}{4} + O(\varepsilon^2) \quad \varepsilon \rightarrow 0$$

Prior: add counterterms to the action.
Holographic renormalization

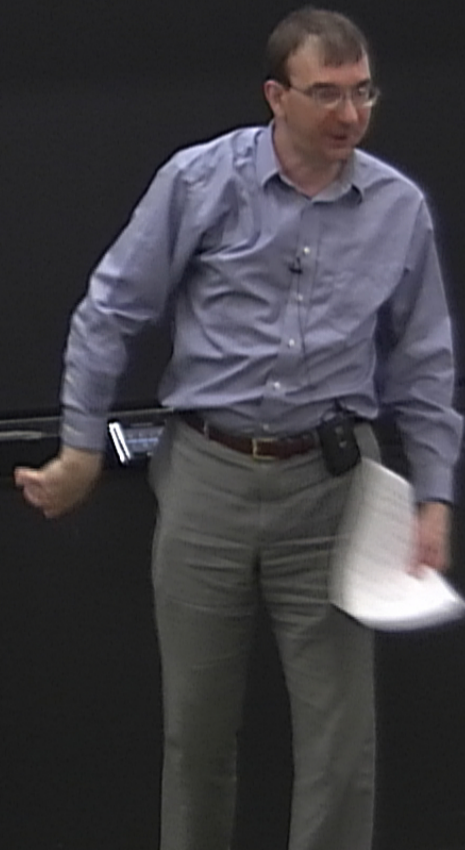
$$\left[\frac{f'_k(\varepsilon)}{\varepsilon^3} \right] = -\frac{k^4}{8} \ln k^2$$

$$\langle \hat{\Theta}(x) \hat{\Theta}(y) \rangle$$

$$\delta(k-q)$$

$$\delta'(k-q)$$

$$\langle \hat{\Theta}(k), \hat{\Theta}(k') \rangle_E = - \frac{N_c^2}{64\pi^2} k_E^4 \ln k_E^2 (2\pi)^4 \delta^{(4)}(k+k')$$



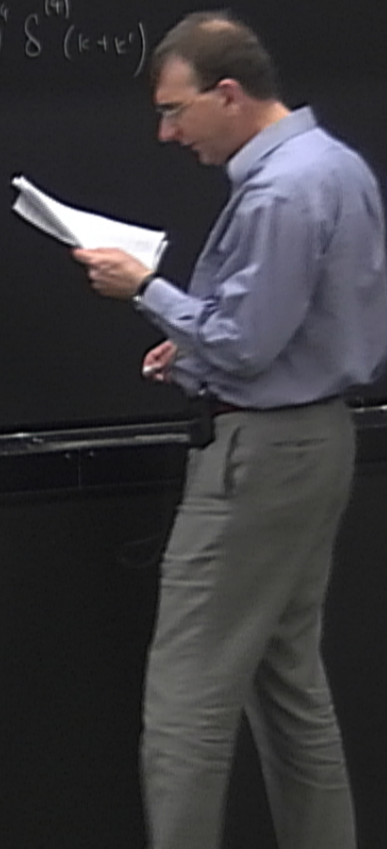
$$\langle \hat{\Theta}(x) \hat{\Theta}(y) \rangle$$

$$\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle_E = - \frac{N_c^2}{64\pi^2} k_E^4 \ln k_E^2 (2\pi)^4 \delta^{(4)}(k+k')$$

✓ Gubser, Klebanov, Polyakov 1998

Witten 1998.

$$\langle \hat{\Theta}(x) \hat{\Theta}(y) \rangle_E \sim \frac{N_c^2}{|x-y|^8}$$



$$\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle$$

$$\langle \hat{\Theta}(k) \hat{\Theta}(k') \rangle_E = - \frac{N_c^2}{64\pi^2} k_E^4 \ln k_E^2 (2\pi)^4 \delta^{(4)}(k+k')$$

✓ Gubser, Klebanov, Polyakov 1998

Witten 1998.

$$\langle \hat{\Theta}(x) \hat{\Theta}(y) \rangle_E \sim \frac{N_c^2}{|x-y|^8}$$

In general, expect $\langle \hat{\Theta}(x) \hat{\Theta}(y) \rangle \sim \frac{1}{|x-y|^{2\Delta}}$

$$\text{For } \hat{\Theta} = \text{tr } F_{\mu\nu}^2 \quad [\hat{\Theta}] \sim \frac{1}{L^4} \sim \frac{1}{L^{\Delta_0}} \Rightarrow \langle \hat{\Theta}(x) \hat{\Theta}(y) \rangle \sim \frac{1}{|x-y|^{2\Delta_0}}$$

$$\Delta_0 : \Delta(\lambda)$$

$$\lim_{\lambda \rightarrow 0} \Delta(\lambda) = \Delta_0$$

The AdS/CFT result confirms that $\Delta(\lambda \rightarrow \infty) = \Delta(\lambda \rightarrow 0) = 4$

