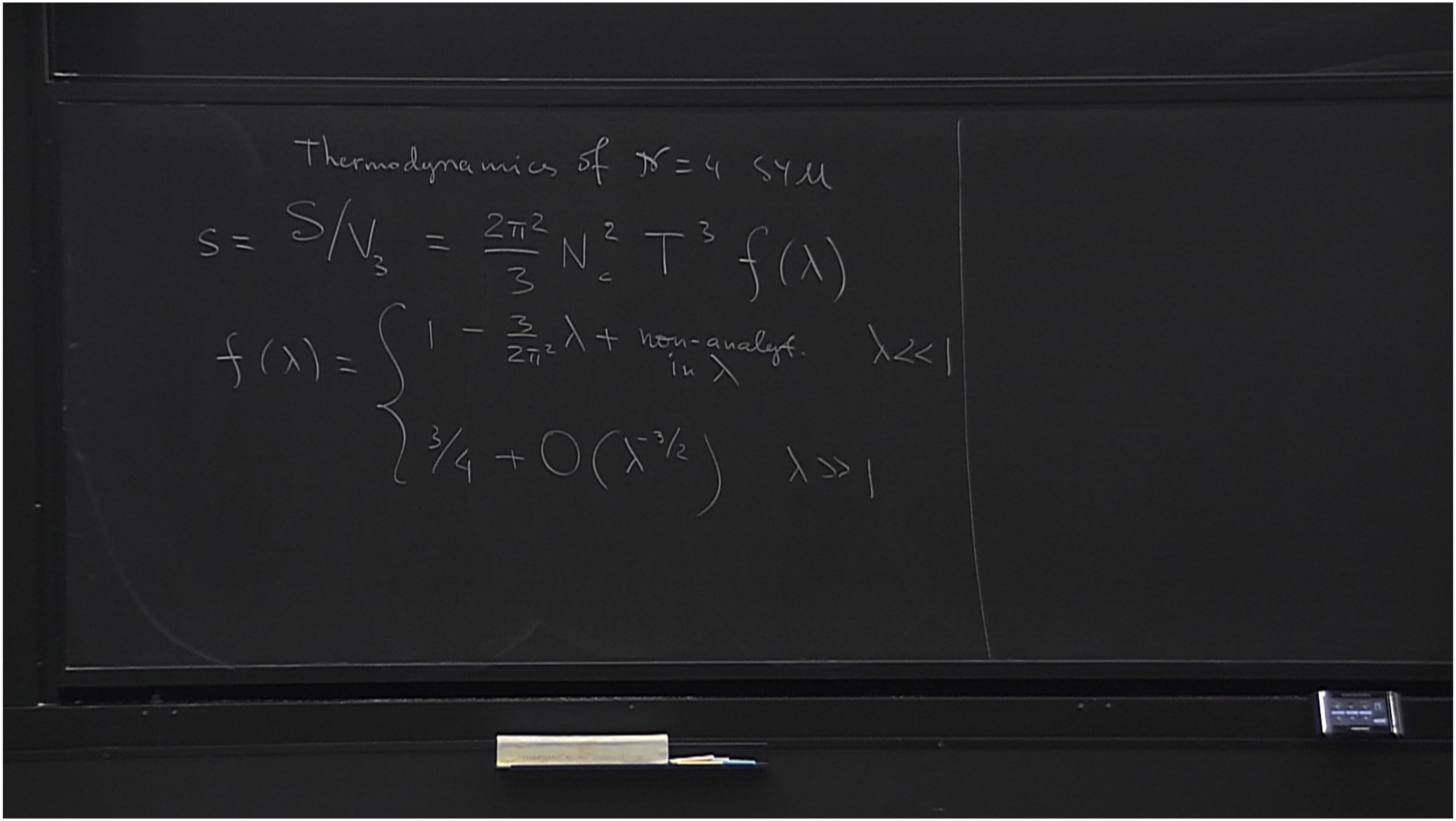
Title: 13/14 PSI - Explorations in String Theory - Lecture 10

Date: Mar 28, 2014 11:30 AM

URL: http://pirsa.org/14030070

Abstract:



Thermodynamics of $\mathcal{N}=4$ SYM

$$s = S/V_3 = \frac{2\pi^2}{3} N_c^2 \, T^3 f(\lambda)$$

$$f(\lambda) = \begin{cases} 1 - \frac{3}{2\pi^2}\lambda + \text{non-analyt. in } \lambda & \lambda \ll 1 \\ 3/4 + O(\lambda^{-3/2}) & \lambda \gg 1 \end{cases}$$

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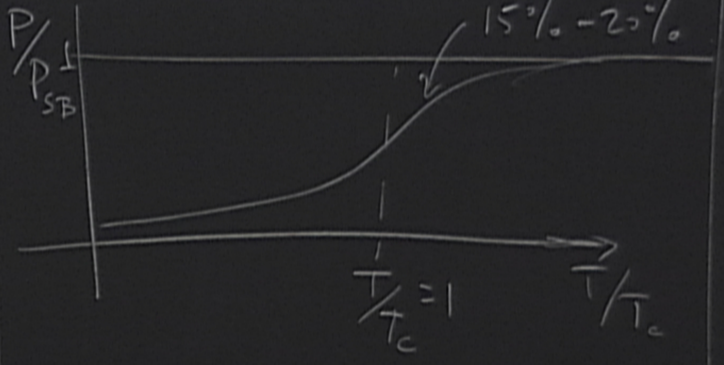
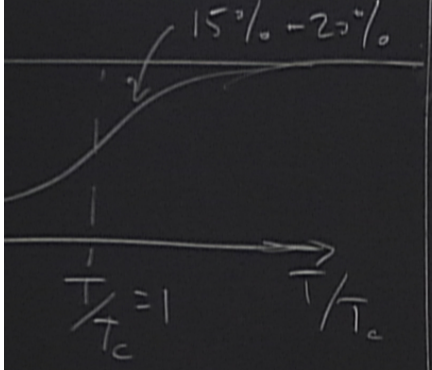




For a d-dim CFT: $S = a f(\lambda) T^{d-1}$

$$S_{QCD} \sim F\left(\frac{T}{\Lambda_{QCD}}\right) T^3$$

$$s = \frac{\partial P}{\partial T}\Big|_{\mu} \Rightarrow \boxed{P = \frac{1}{d} a f(\lambda) T^{d}}$$

$$\mathcal{F} = -pV = E - TS$$

$$\varepsilon = Ts - p$$

$$\Rightarrow \varepsilon = \frac{d-1}{d} a \cdot f(\lambda) T^{d} = (d-1) P$$

Expected, since in CFT: $T^{\mu}{}_{\mu} = 0$

$$tr\begin{pmatrix} -\varepsilon & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix} = 0 \Rightarrow \varepsilon = (d-1)P$$

$$\Rightarrow \boxed{v_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{1}{d-1}}$$

Holographic dictionary

$$Z_{\text{gauge th.}}[J] = Z_{\text{str. th}}[J]$$

In particular,

$$Z^{\left(\substack{\lambda\to\infty\\ N_c\to\infty}\right)}_{\substack{\mathcal{N}=4\text{ SYM}\\ SU(N_c)\text{ in } d=3+1}}[J] = Z_{\substack{\text{IIB str.}\\ \text{on } AdS_5\times S^5}}[J] \simeq e^{-S_{\substack{\text{type IIB}\\ \text{SUGRA}\\ \text{on } AdS_5\times S^5}}} + \text{corrections}$$

$$+\ \text{Corrections}\left(\frac{1}{\lambda^{3/2}}, \frac{1}{N_c^2}\right)$$

$l_s \quad g_s$

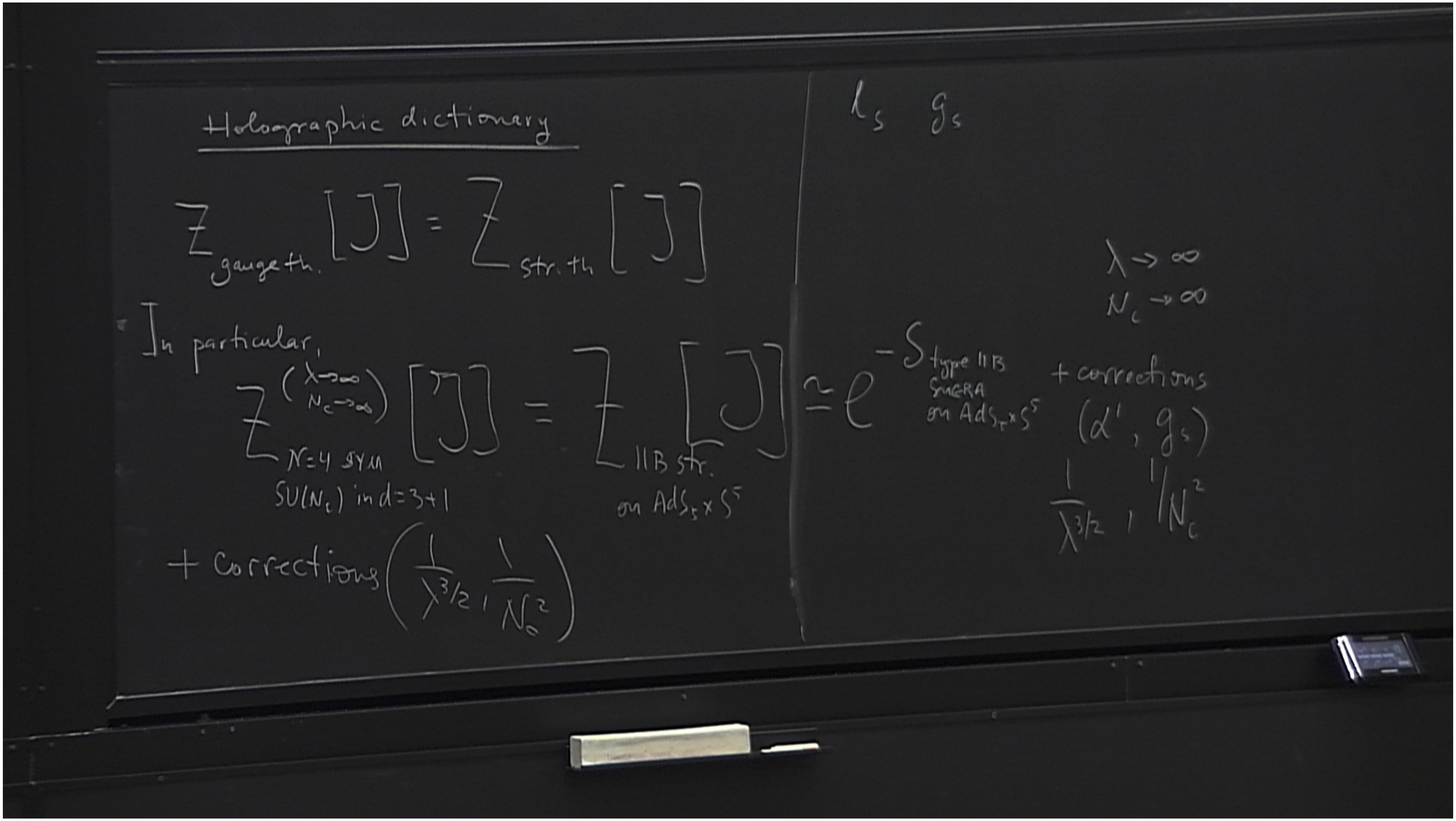
$\lambda \to \infty$

$N_c \to \infty$

+ corrections

$$(\alpha', g_s)$$

$$\frac{1}{\lambda^{3/2}}, \ 1/N_c^2$$



Holographic dictionary

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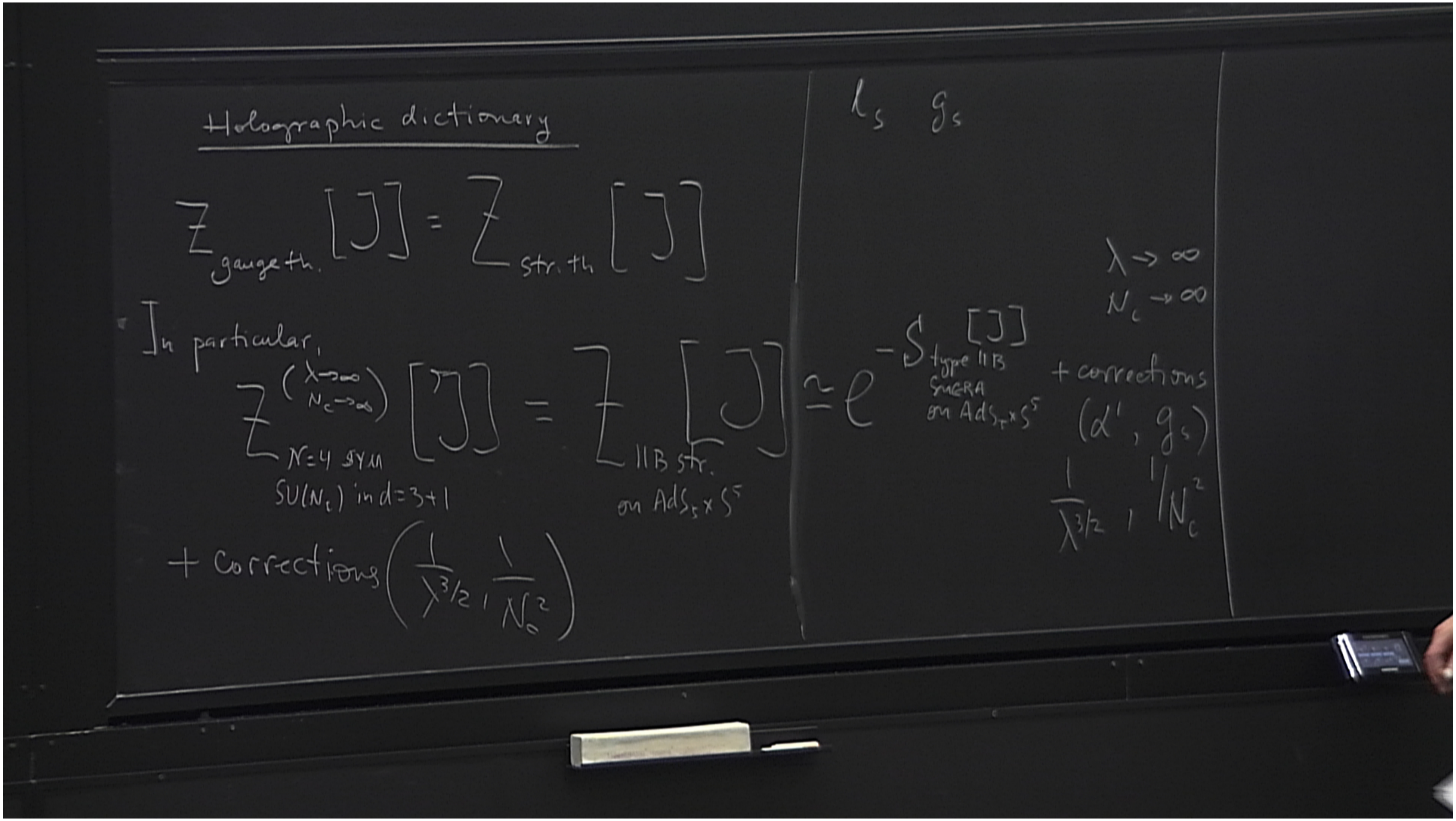
+ corrections (α', g_s)

$\frac{1}{\lambda^{3/2}}, \ 1/N_c^2$

+ Corrections $\left(\frac{1}{\lambda^{3/2}}, \frac{1}{N_c^2}\right)$



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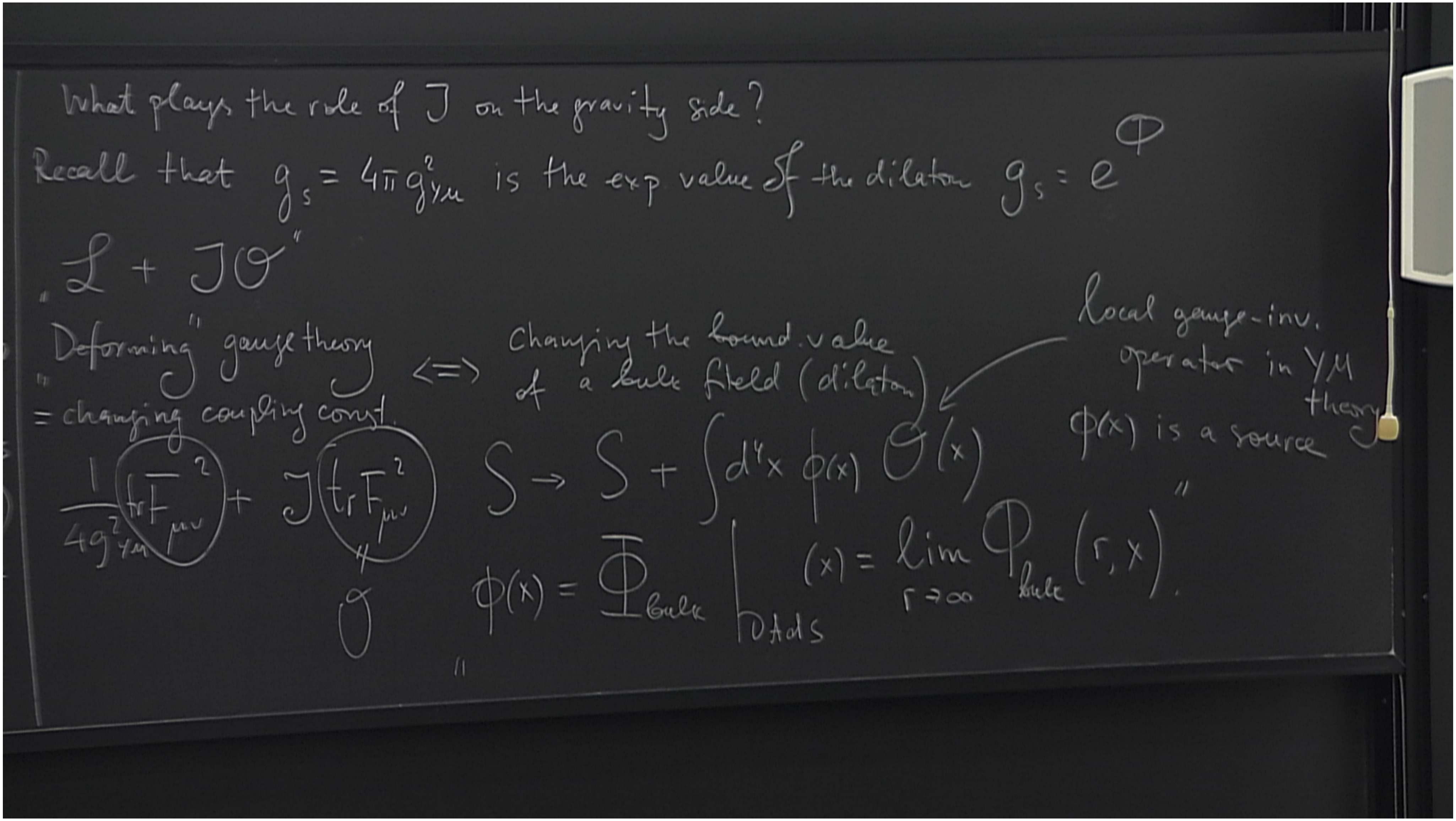
l_s g_s

$\lambda\to\infty$

$N_c\to\infty$

+ corrections (α', g_s)

$\frac{1}{\lambda^{3/2}}, 1/N_c^2$



What plays the role of J on the gravity side?

Recall that $g_s = 4\pi g_{YM}^2$ is the exp. value of the dilaton $g_s = e^{\Phi}$

„$\mathcal{L} + J\mathcal{O}$"

„Deforming" gauge theory <=> Changing the bound. value of a bulk field (dilaton)

„= changing coupling const.

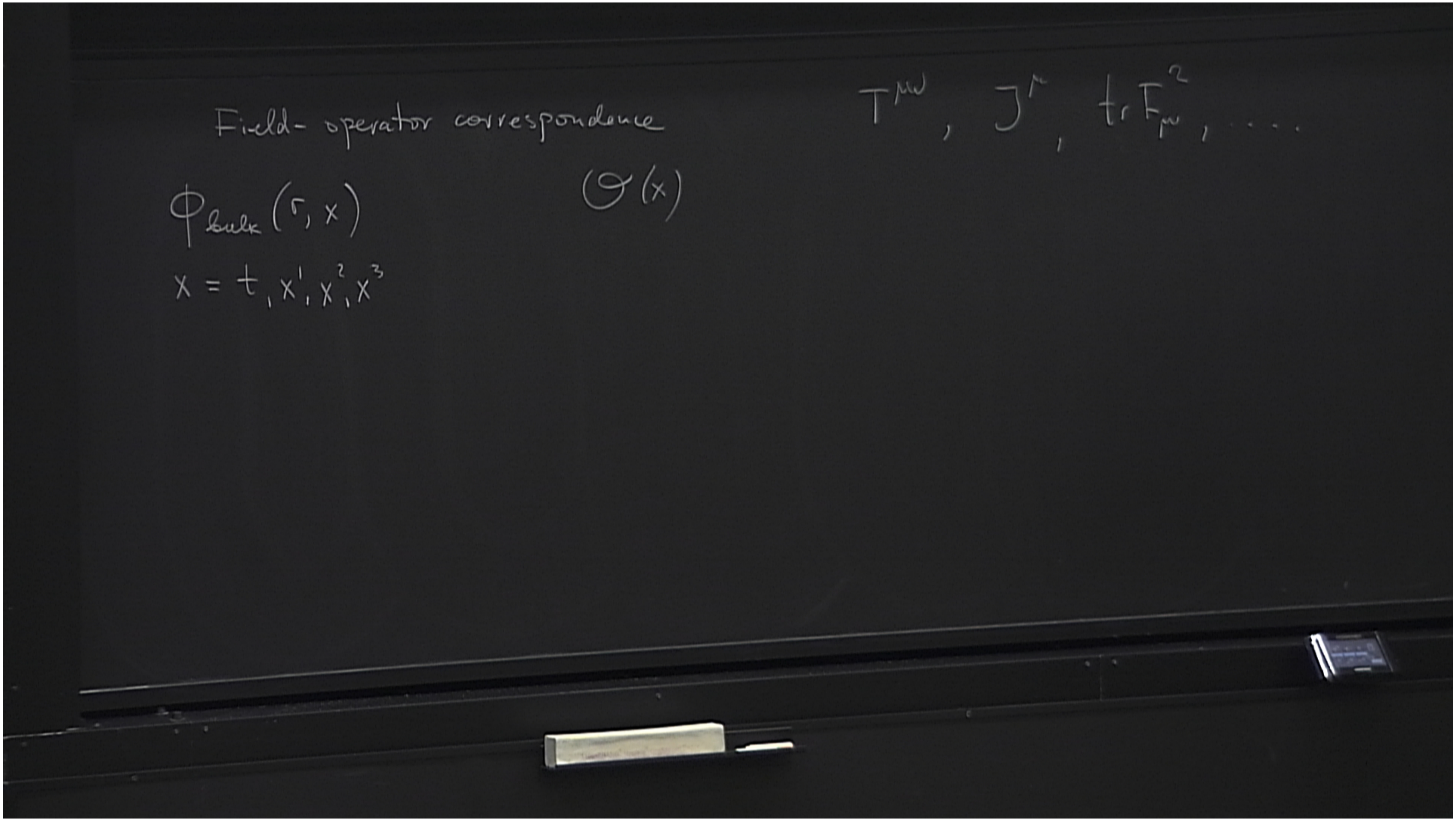
$$\frac{1}{4g_{YM}^2}\,\mathrm{tr}F_{\mu\nu}^2 + J\,\underbrace{\mathrm{tr}F_{\mu\nu}^2}_{\mathcal{O}}$$

$$S \to S + \int d^4x\, \phi(x)\, \mathcal{O}(x)$$

local gauge-inv. operator in YM theory

$\phi(x)$ is a source

„$\phi(x) = \Phi_{bulk}\Big|_{\partial AdS}(x) = \lim_{r\to\infty} \Phi_{bulk}(r,x)$".



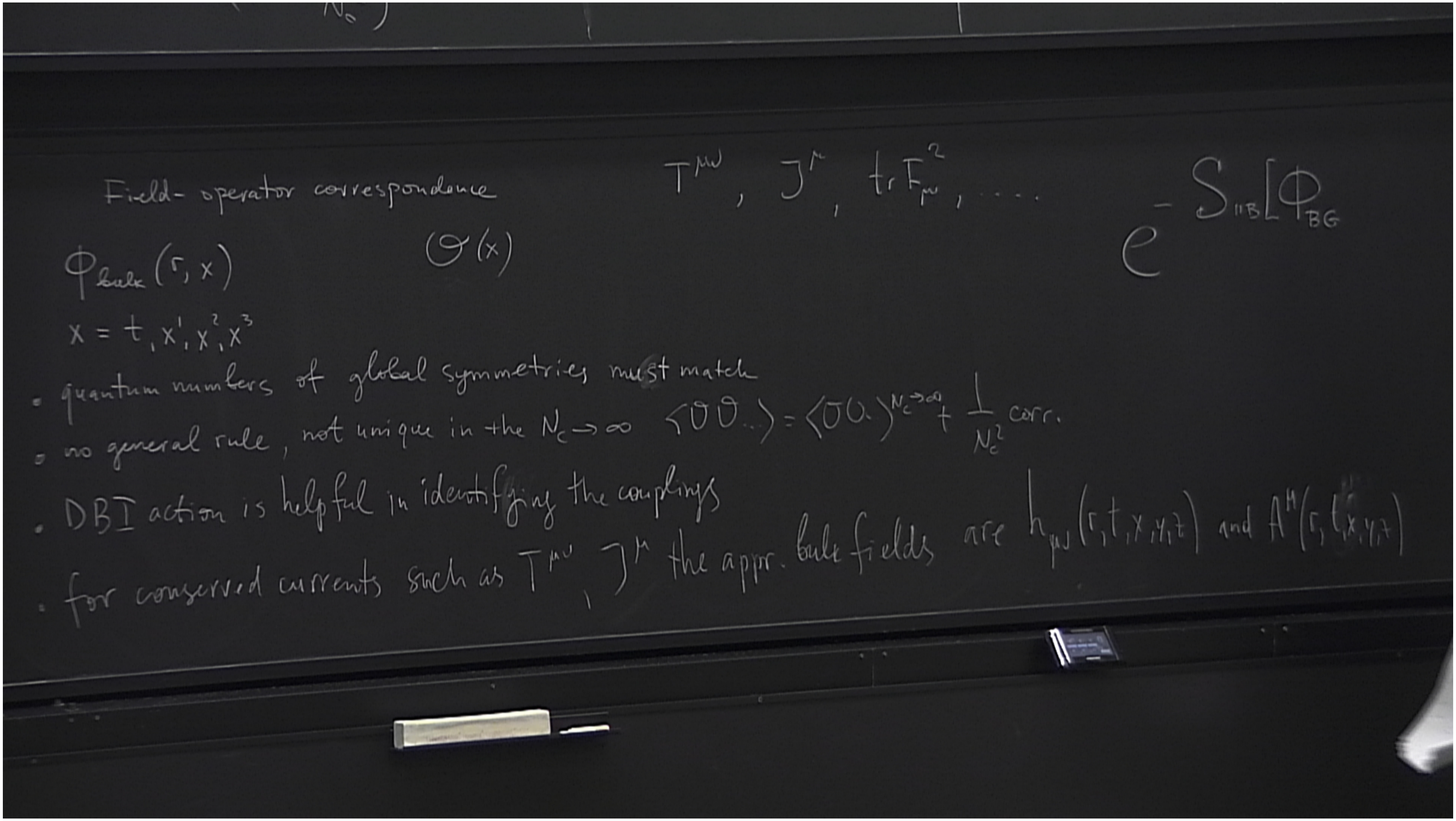
Field- operator correspondence

$T^{\mu\nu}$, J^{μ}, $\mathrm{tr}\, F_{\mu\nu}^2$, ...

$\phi_{bulk}(r, x)$

$\mathcal{O}(x)$

$x = t, x^1, x^2, x^3$



Field- operator correspondence

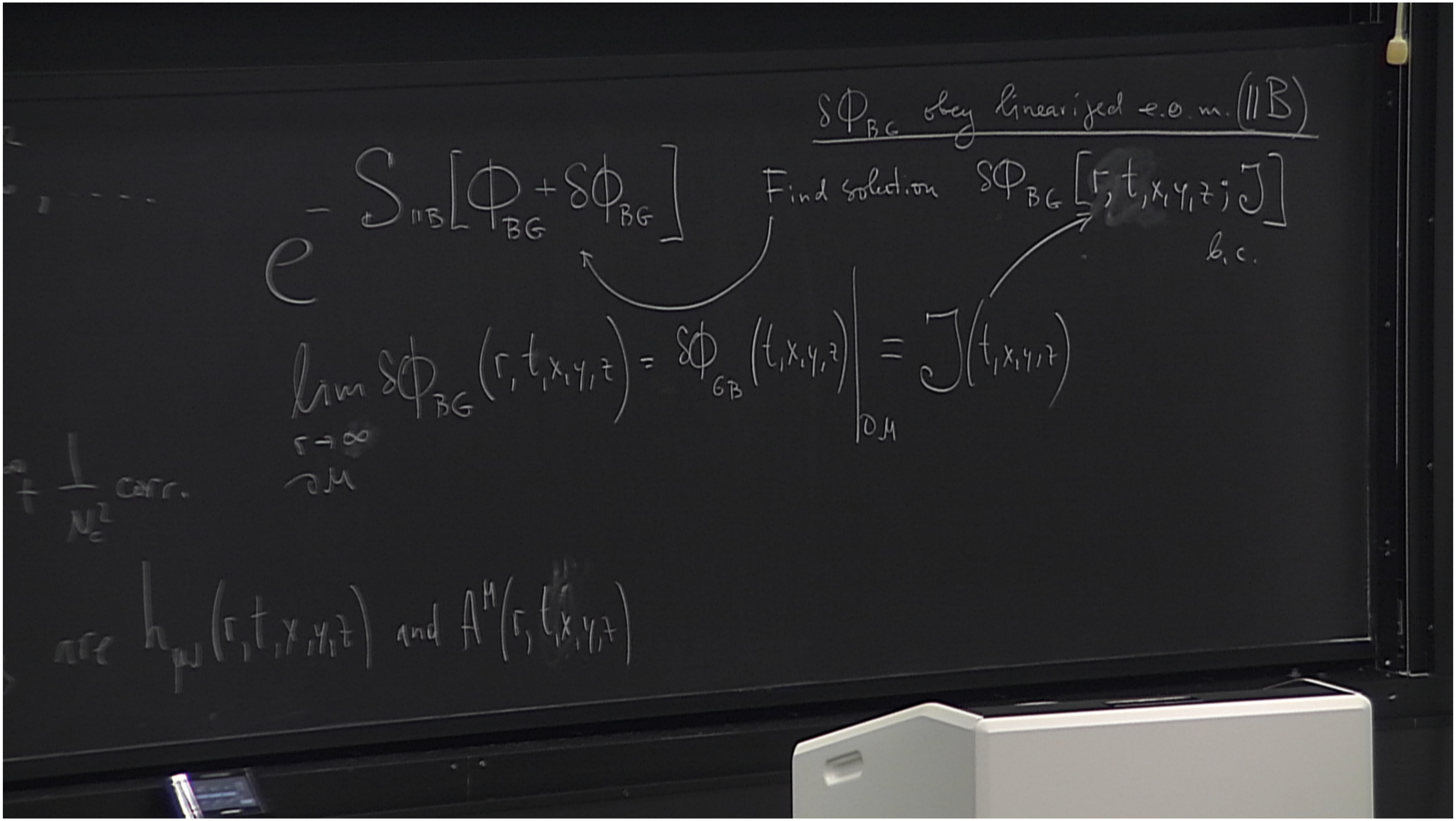
$T^{\mu\nu}$, J^μ, $\mathrm{tr}\,F_{\mu\nu}^2$, ...

$e^{-S_{IIB}[\Phi_{BG}}$

$\phi_{bulk}(r, x)$ $\quad\quad$ $\mathcal{O}(x)$

$x = t, x^1, x^2, x^3$

- quantum numbers of global symmetries must match
- no general rule, not unique in the $N_c \to \infty$ $\langle \mathcal{O}\mathcal{O}..\rangle = \langle \mathcal{O}\mathcal{O}..\rangle^{N_c\to\infty} + \frac{1}{N_c^2}$ corr.
- DBI action is helpful in identifying the couplings
- for conserved currents such as $T^{\mu\nu}$, J^μ the appr. bulk fields are $h_{\mu\nu}(r, t, x, y, z)$ and $A^\mu(r, t, x, y, z)$



$\underline{\delta\Phi_{BG} \text{ obey linearized e.o.m. (IIB)}}$

$$e^{-S_{IIB}\left[\Phi_{BG}+\delta\Phi_{BG}\right]}$$

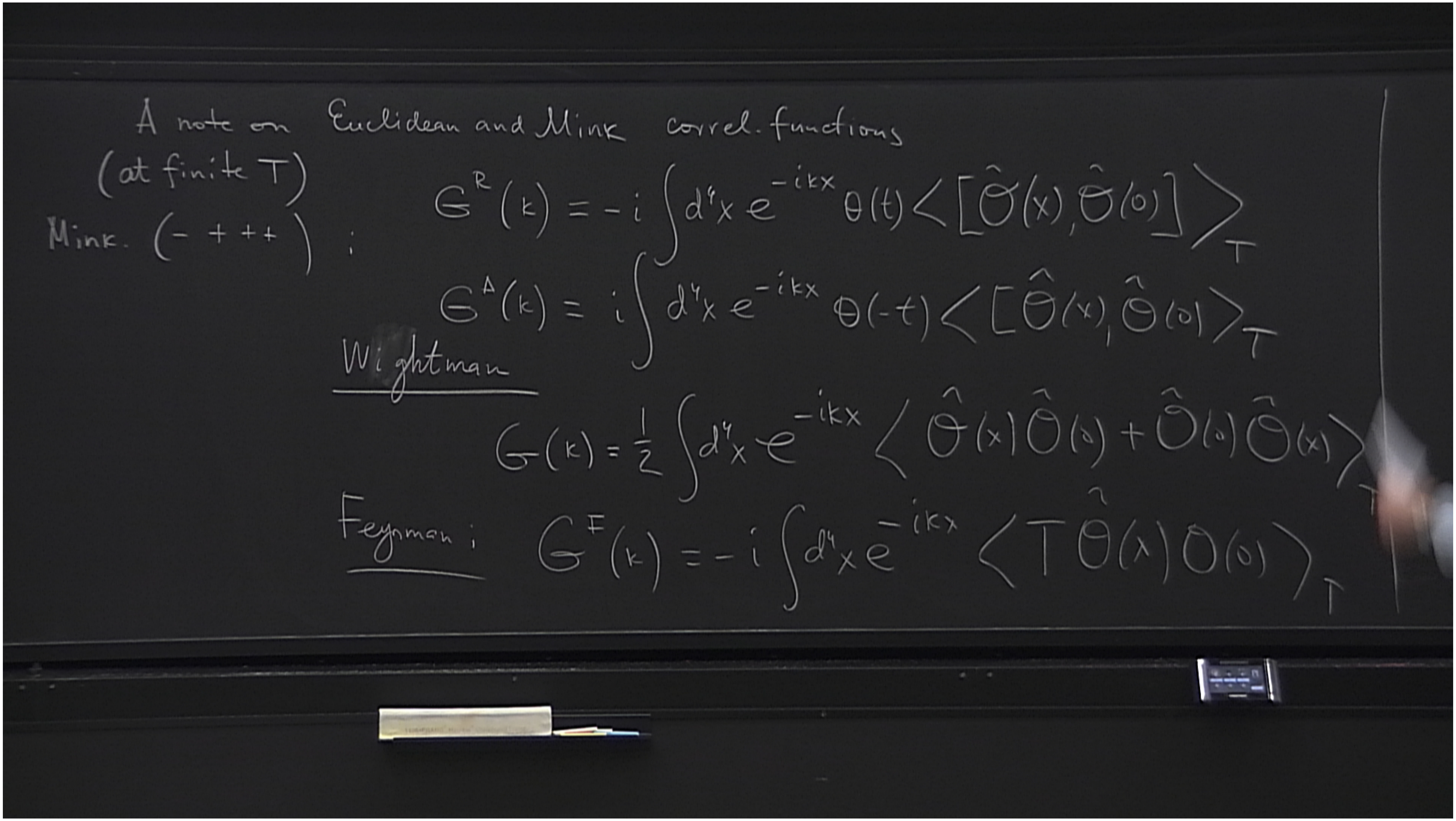
Find solution $\delta\Phi_{BG}\left[r,t,x,y,z;J\right]$

b.c.

$$\lim_{\substack{r\to\infty\\ \partial M}} \delta\Phi_{BG}(r,t,x,y,z) = \delta\Phi_{GB}(t,x,y,z)\Big|_{\partial M} = J(t,x,y,z)$$

$+\frac{1}{N_c^2}$ corr.

are $h_{\mu\nu}(r,t,x,y,z)$ and $A^{\mu}(r,t,x,y,z)$



A note on Euclidean and Mink correl. functions

(at finite T)

Mink. $(- + + +)$:

$$G^R(k) = -i\int d^4x\, e^{-ikx}\,\theta(t)\left\langle\left[\hat{O}(x),\hat{O}(0)\right]\right\rangle_T$$

$$G^A(k) = i\int d^4x\, e^{-ikx}\,\theta(-t)\left\langle\left[\hat{O}(x),\hat{O}(0)\right\rangle_T$$

Wightman

$$G(k) = \frac{1}{2}\int d^4x\, e^{-ikx}\left\langle\hat{O}(x)\hat{O}(0)+\hat{O}(0)\hat{O}(x)\right\rangle_T$$

Feynman: $G^F(k) = -i\int d^4x\, e^{-ikx}\left\langle T\hat{O}(x)O(0)\right\rangle_T$

$)(x)\rangle_T$

T

See: Birrell, Davies; De Witt

$$G^F(k) = \frac{1}{2}\left(G^R + G^A\right) - i\, G$$

Using spectral repr. of G^R, G (see e.g. LL vol IX, le Bellac)

$$G(k) = -\coth\frac{\omega}{2T}\, \mathrm{Im}\, G^R(k)$$

$$G^F(k) = \mathrm{Re}\, G^R + i \coth\frac{\omega}{2T}\, \mathrm{Im}\, G^R$$

$$G^A(k) = G^R(-k) = \left(G^R(k)\right)^*$$

Feynman: $G^F(k) = -i\int d^4x\, e^{-ikx}\langle T\,\mathcal{O}(x)\,\mathcal{O}(0)\rangle$

$T \to 0$: $G^F = \mathrm{Re}\, G^R + i\, \mathrm{sgn}\,\omega\, \mathrm{Im}\, G^R$

Euclidean: $G^E(k_E) = \int d^4x_E\, e^{-ik_E x_E}\langle T_E\, \hat{\mathcal{O}}(x_E)\,\hat{\mathcal{O}}(0)\rangle$

$T \neq 0$: Matsubara propagator, defined at $\omega_E = 2\pi T n$ (~~boson~~)

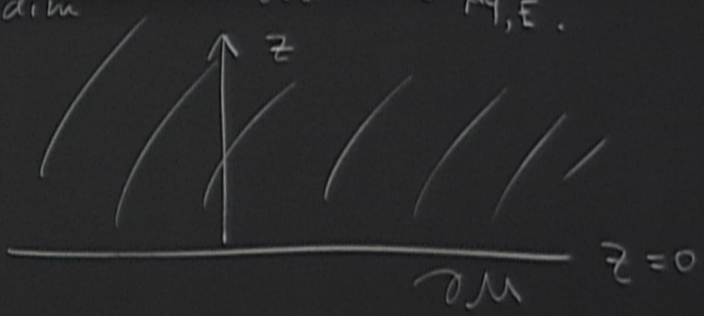
$$G^R(2\pi i T_n, \vec{k}) = -G^E(2\pi T_n, \vec{k})$$

example : massless scalar field in AdS_5

1) Euclidean AdS_5 : $ds_5^2 = \frac{L^2}{z^2}\left(dz^2 + d\vec{x}_E^2\right)$

4 dim

$-dt^2 \to d\cancel{x}_{4,E}^2$.



example: **massless** scalar field in AdS_5

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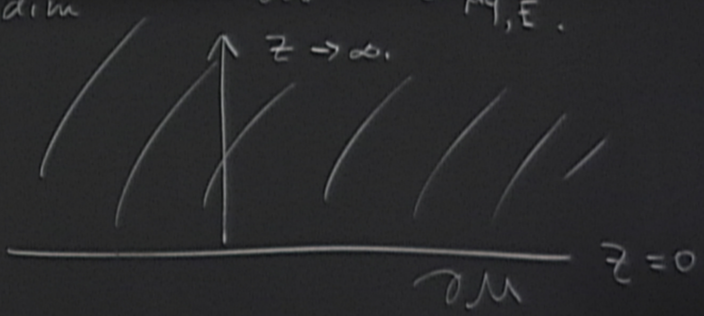
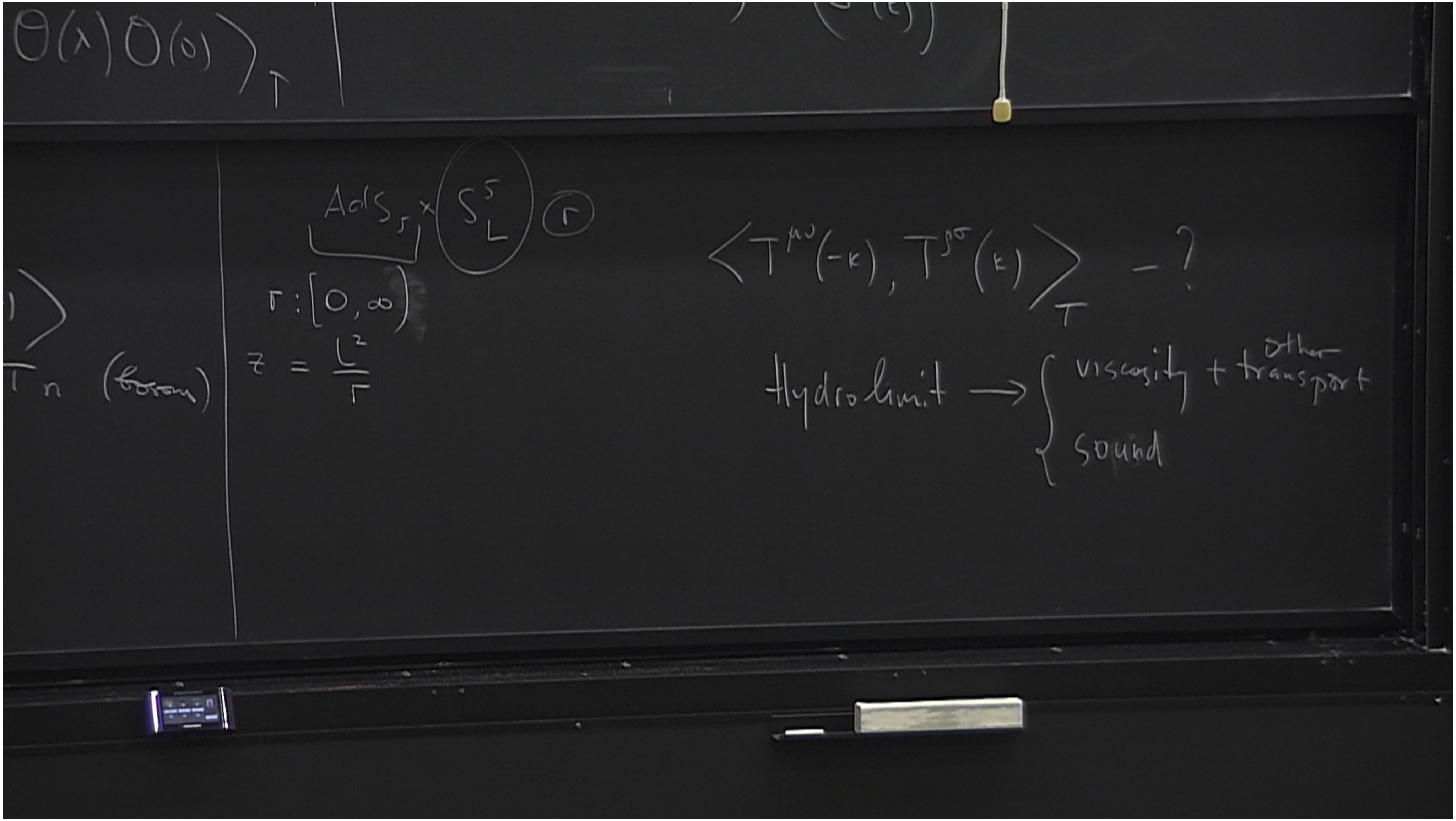


$\mathcal{J} \longleftrightarrow \delta\phi$ coupled to $\mathrm{tr}\, F_{\mu\nu}^2$

Φ_{BG} = const.

$\phi_{BG} + \delta\phi_{BG} \to$ insert into action

e.o.m.



$\Theta(x)\,\Theta(0)\rangle_T$

$\rangle$

$T\; n$ (~~boson~~)

$\underbrace{AdS_5}\times \boxed{S^5_L}$ ⓡ

$r : [0,\infty)$

$z = \frac{L^2}{r}$

$\langle T^{\mu\nu}(-k), T^{\rho\sigma}(k)\rangle_T$ — ?

Hydro limit → $\begin{cases}\text{viscosity + other transport}\\ \text{sound}\end{cases}$