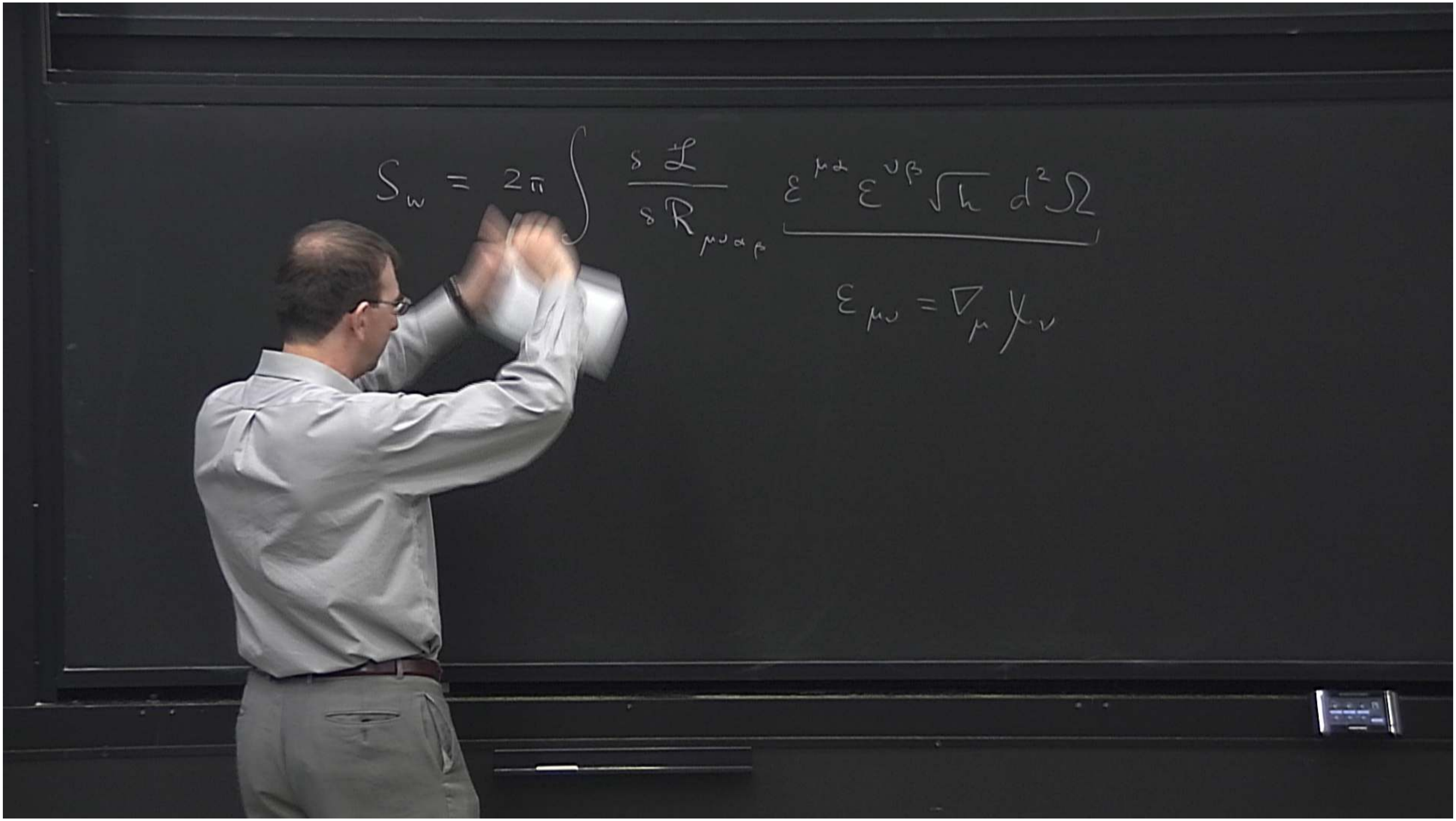
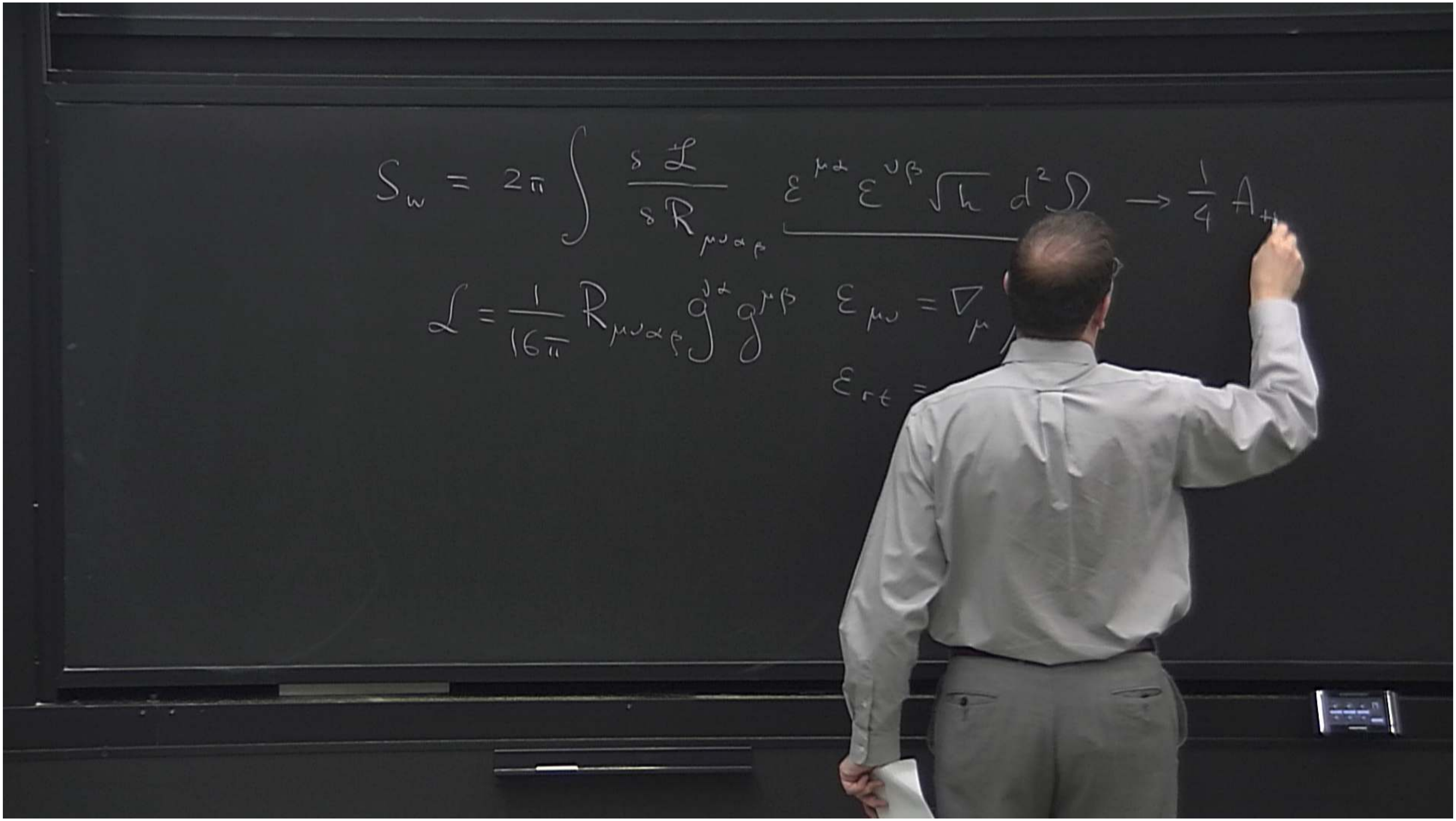
Title: 13/14 PSI - Explorations in String Theory - Lecture 5

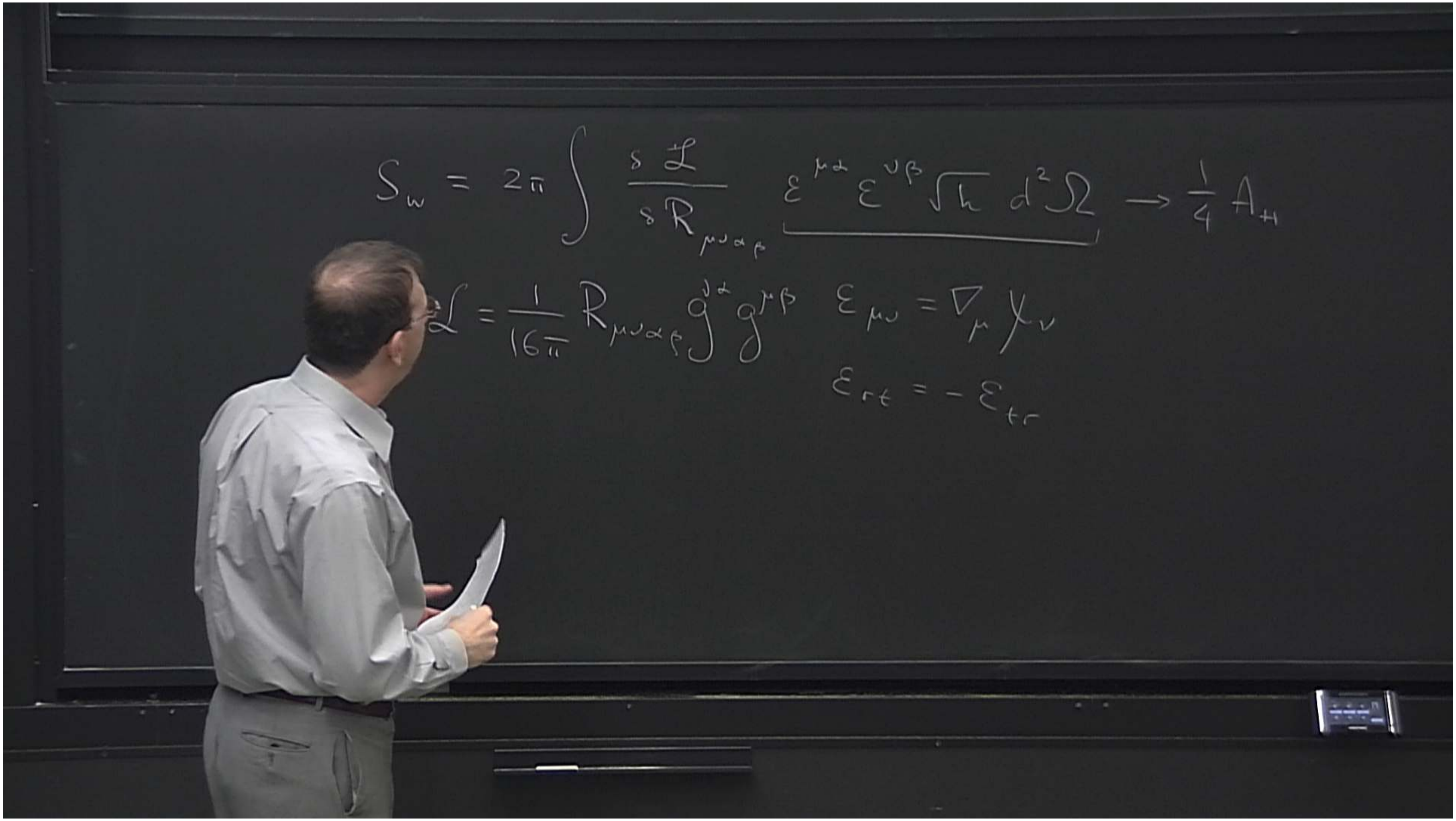
Date: Mar 21, 2014 11:30 AM

URL: http://pirsa.org/14030063

Abstract:


$S_w = 2\pi \int \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\alpha\beta}} \varepsilon^{\mu\alpha} \varepsilon^{\nu\beta} \sqrt{h}\, d^2\Omega$
$\varepsilon_{\mu\nu} = \nabla_\mu \chi_\nu$


$S_w = 2\pi \int \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\alpha\beta}} \varepsilon^{\mu\alpha} \varepsilon^{\nu\beta} \sqrt{h}\, d^2\Omega \rightarrow \frac{1}{4} A$
$\mathcal{L} = \frac{1}{16\pi} R_{\mu\nu\alpha\beta} g^{\nu\alpha} g^{\mu\beta}$
$\varepsilon_{\mu\nu} = \nabla_\mu$
$\varepsilon_{rt} =$


$S_w = 2\pi \int \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\alpha\beta}} \varepsilon^{\mu\alpha} \varepsilon^{\nu\beta} \sqrt{h}\, d^2\Omega \rightarrow \frac{1}{4} A_H$
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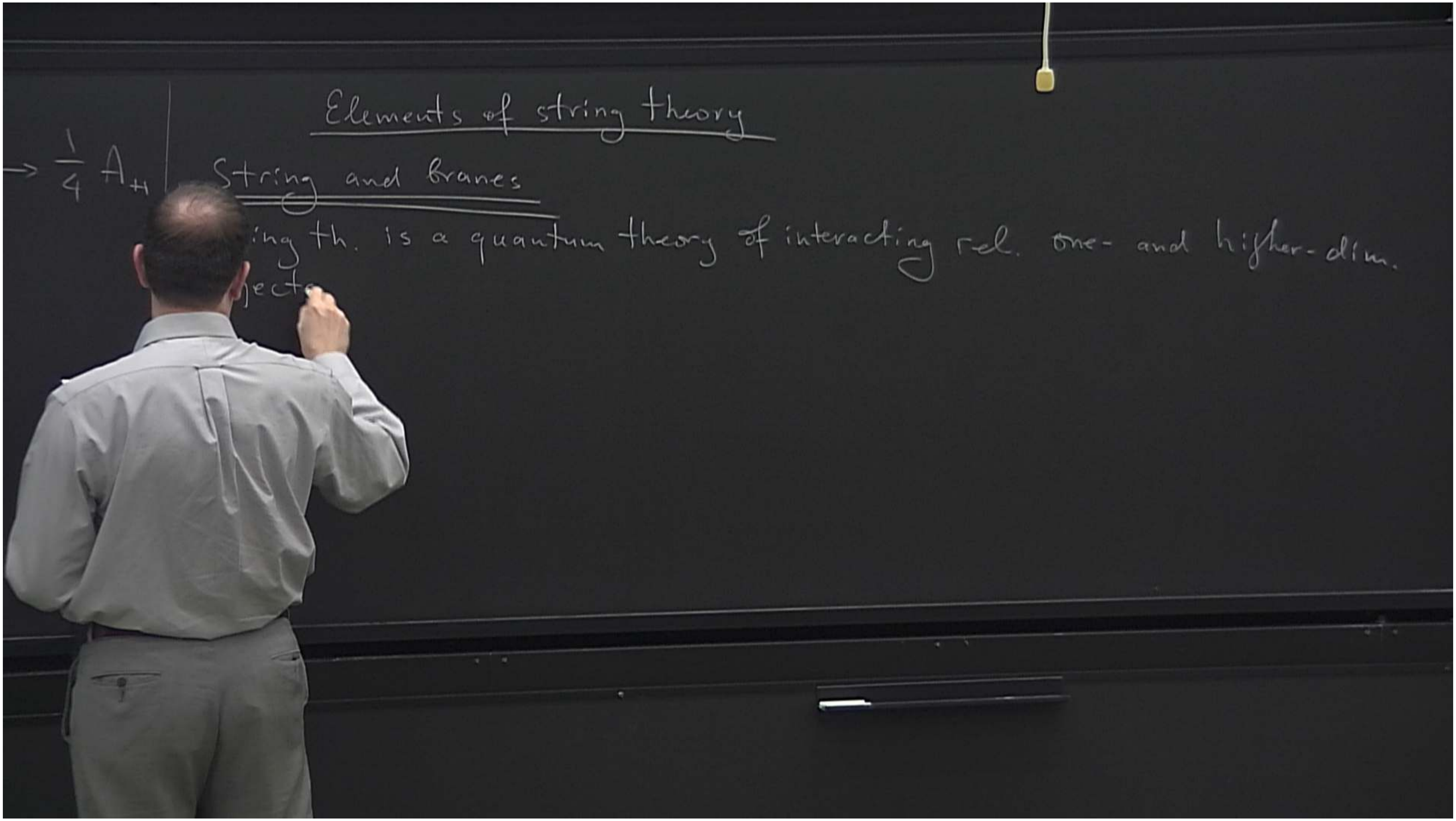
$\varepsilon_{rt} = -\varepsilon_{tr}$

$$S_w = 2\pi \int \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\alpha\beta}} \underline{\varepsilon^{\mu\alpha} \varepsilon^{\nu\beta} \sqrt{h}\, d^2\Omega} \longrightarrow \frac{1}{4} A_H$$

$$\mathcal{L} = \underline{\frac{1}{16\pi} R_{\mu\nu\alpha\beta}\, g^{\nu\alpha} g^{\mu\beta}} \qquad \varepsilon_{\mu\nu} = \nabla_\mu \chi_\nu$$

$$\varepsilon_{rt} = -\varepsilon_{tr}$$

$$+ \alpha_1 R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} + \dots$$


Elements of string theory
String and branes
ing th. is a quantum theory of interacting rel. one- and higher-dim.
jects

$\frac{1}{4} A_{H}$

Elements of string theory

String and branes

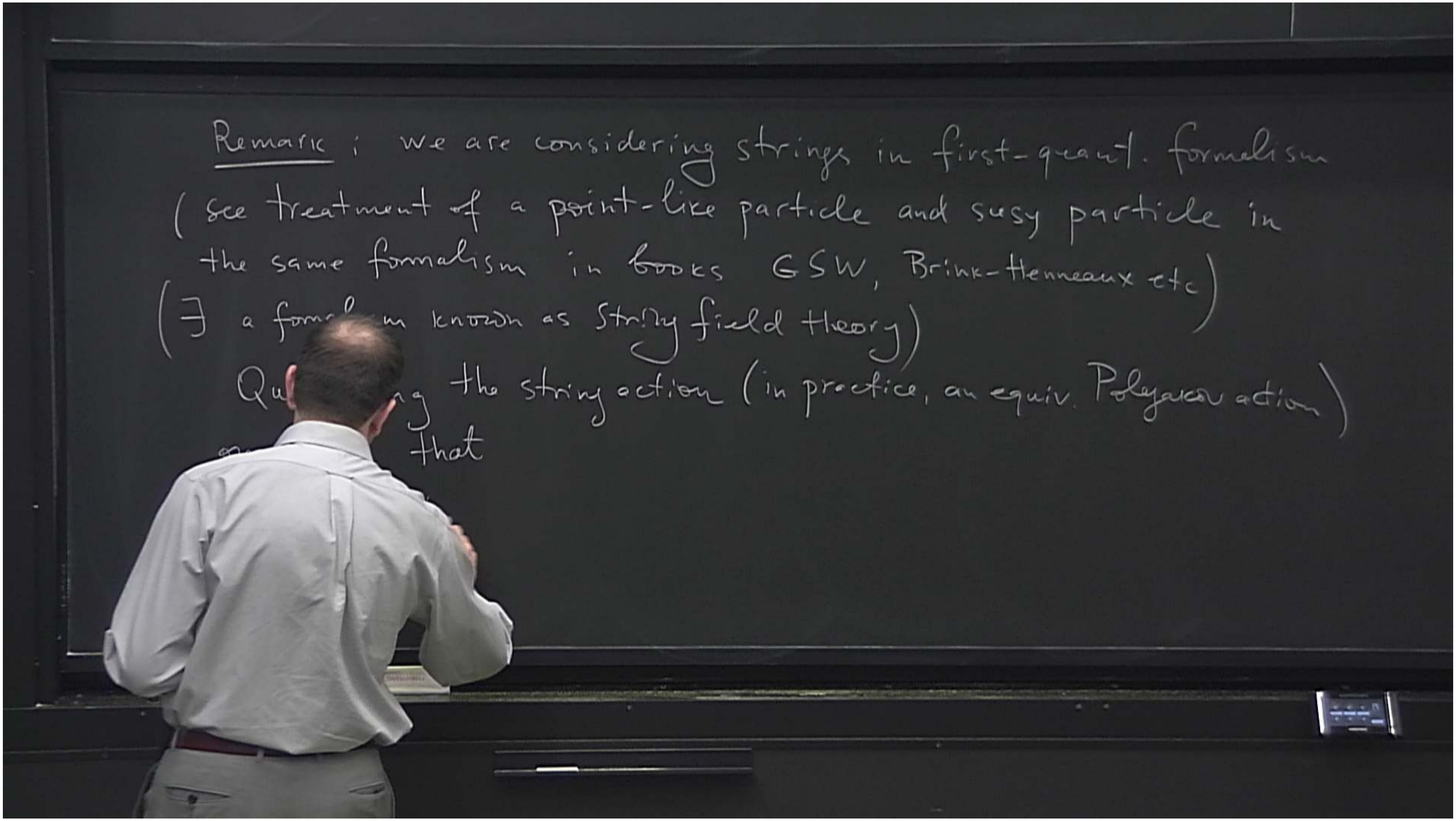
String th. is a quantum theory of interacting rel. one- and higher-dim. objects.

Fundam. param. : $T = \frac{1}{2\pi\alpha'}$; $\alpha' = l_s^2$

$$S = -T \int d^2\sigma \sqrt{-\det g}\ ,$$ where $g_{\alpha\beta} = \eta_{MN} \frac{\partial X^M}{\partial \sigma^\alpha} \frac{\partial X^N}{\partial \sigma^\beta}$ induced metric

σ^α worldsheet coord. X^M : $M = 0, \dots D-1$

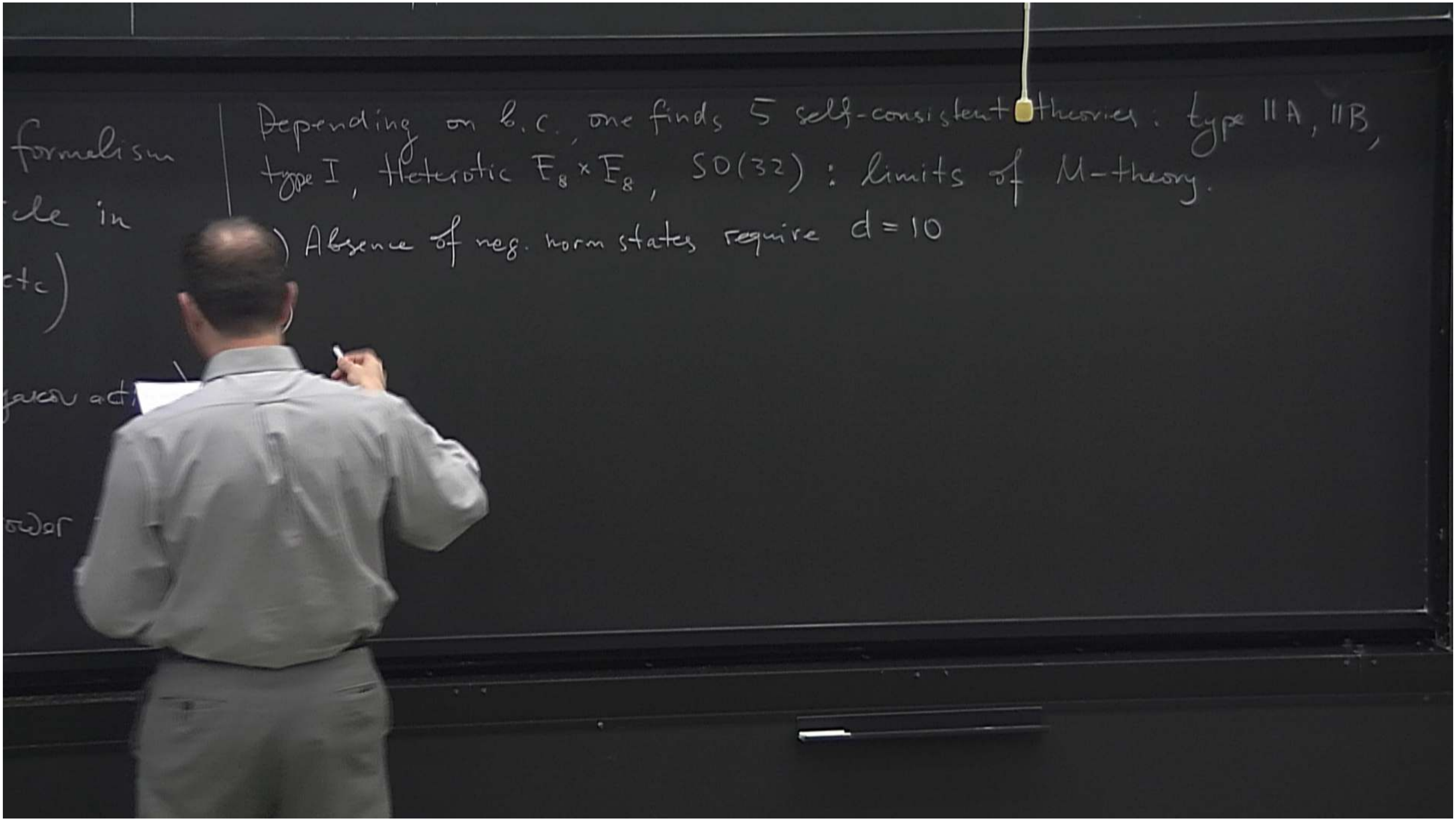
$\alpha = 0, 1$

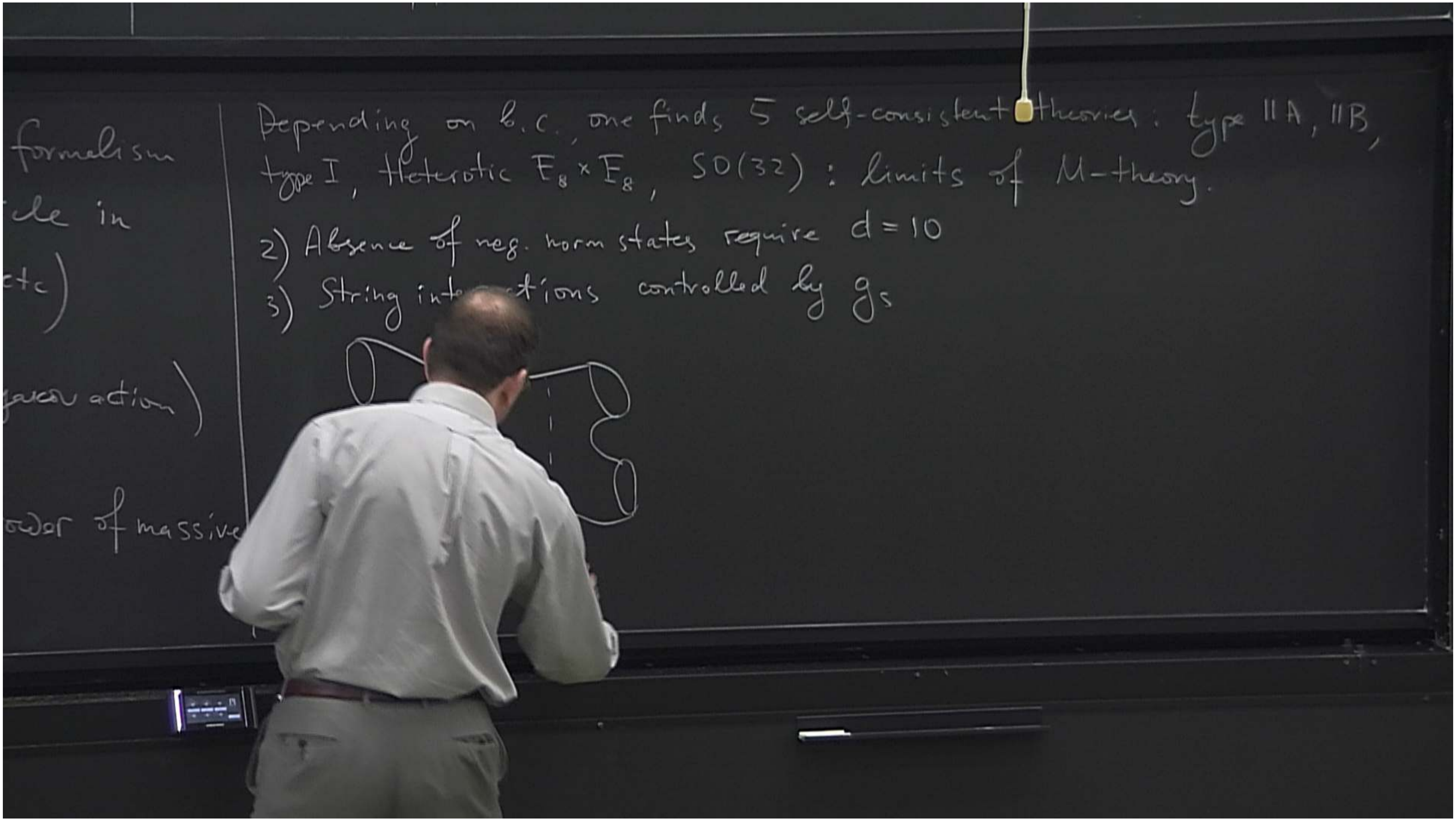

Remark: we are considering strings in first-quant. formalism
(see treatment of a point-like particle and susy particle in
the same formalism in books GSW, Brink-Henneaux etc)
known as String field theory)
the string action (in practice, an equiv. Polyakov action)
that

Remark: we are considering strings in first-quant. formalism
(see treatment of a point-like particle and susy particle in the same formalism in ~~books~~ GSW, Brink-Henneaux etc)
($\exists$ a formalism known as String field theory)

Quantizing the string action (in practice, an equiv. Polyakov action) one finds that

1) $\exists$ finite number of massless modes and an infinite tower of massive modes $m_s \sim l_s^{-1}$.


formalism
Depending on b.c., one finds 5 self-consistent theories: type IIA, IIB,
type I, Heterotic E8 × E8, SO(32): limits of M-theory.
) Absence of neg. norm states require d = 10


formalism
cle in
etc)
jacobi action)
power of massive
Depending on b.c., one finds 5 self-consistent theories: type IIA, IIB,
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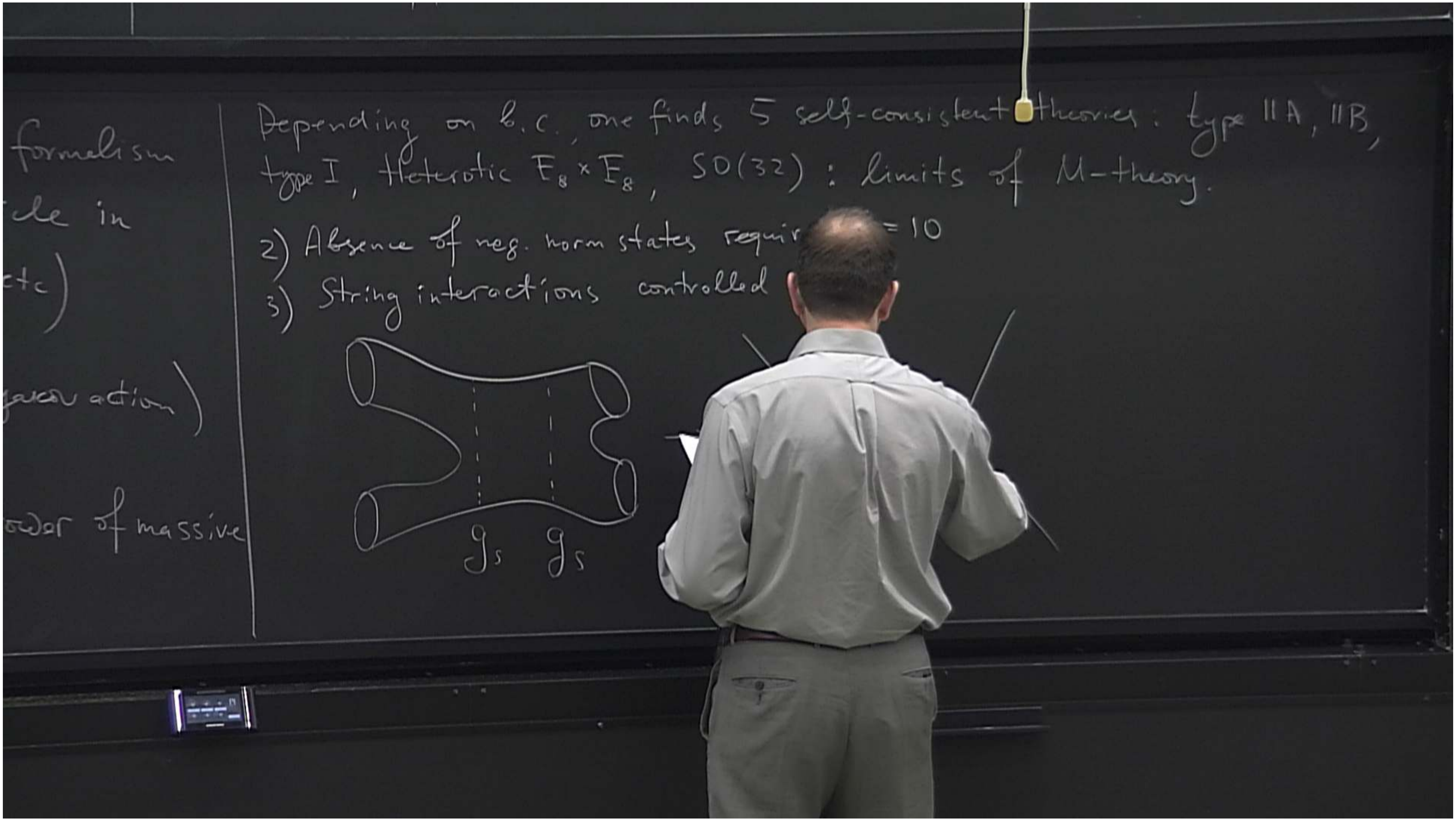
2) Absence of neg. norm states require d = 10
3) String interactions controlled by gs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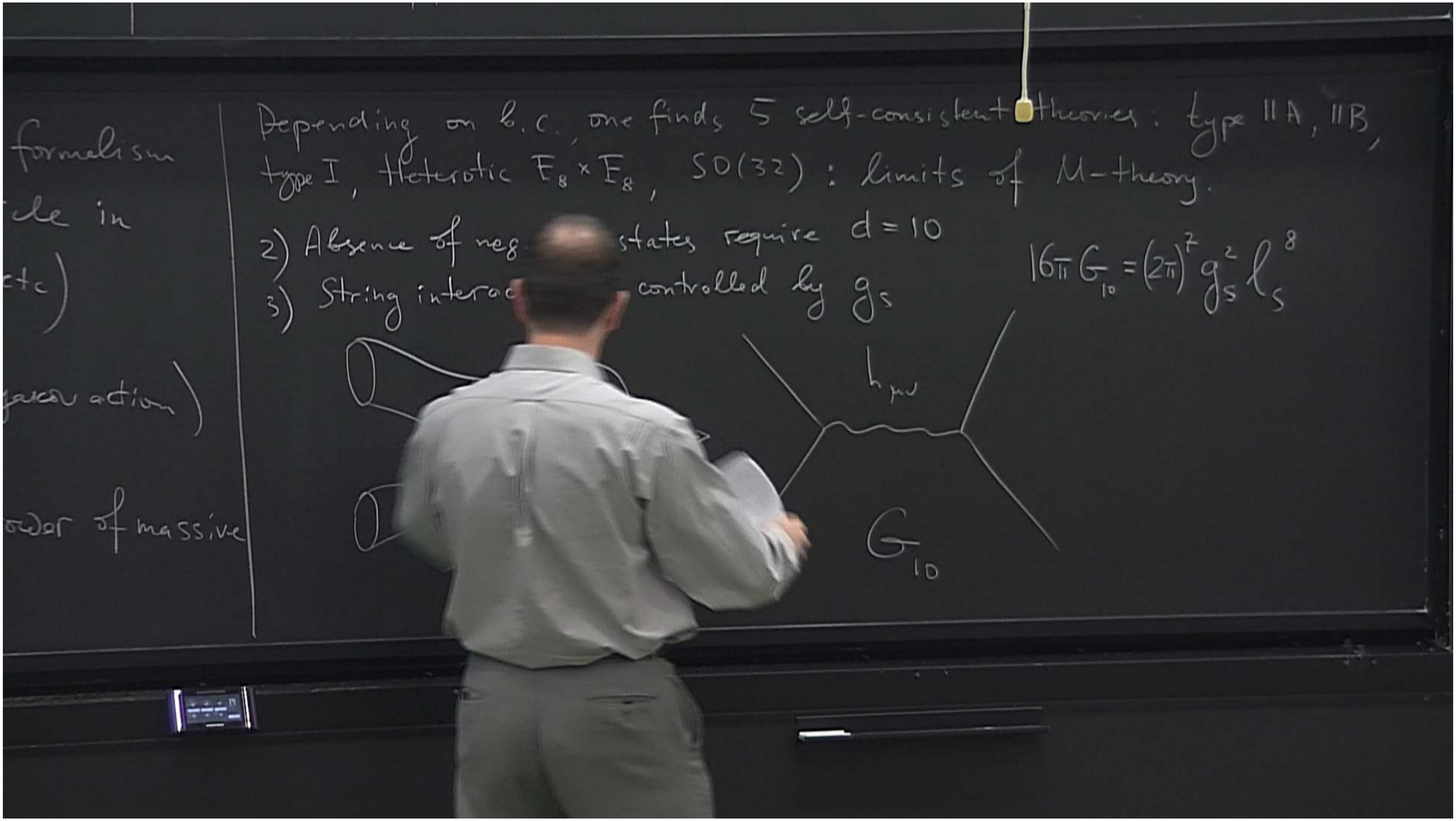
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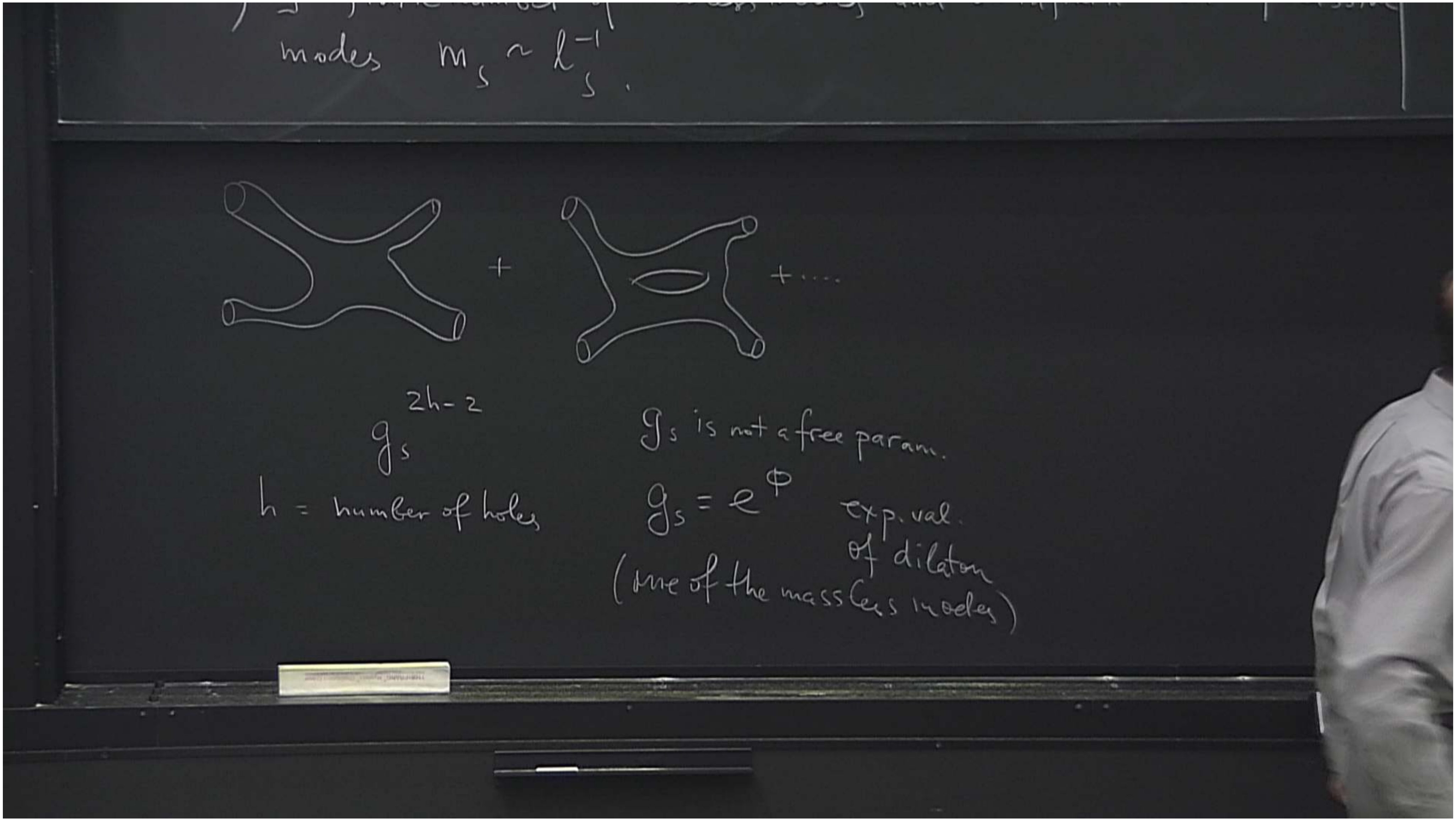
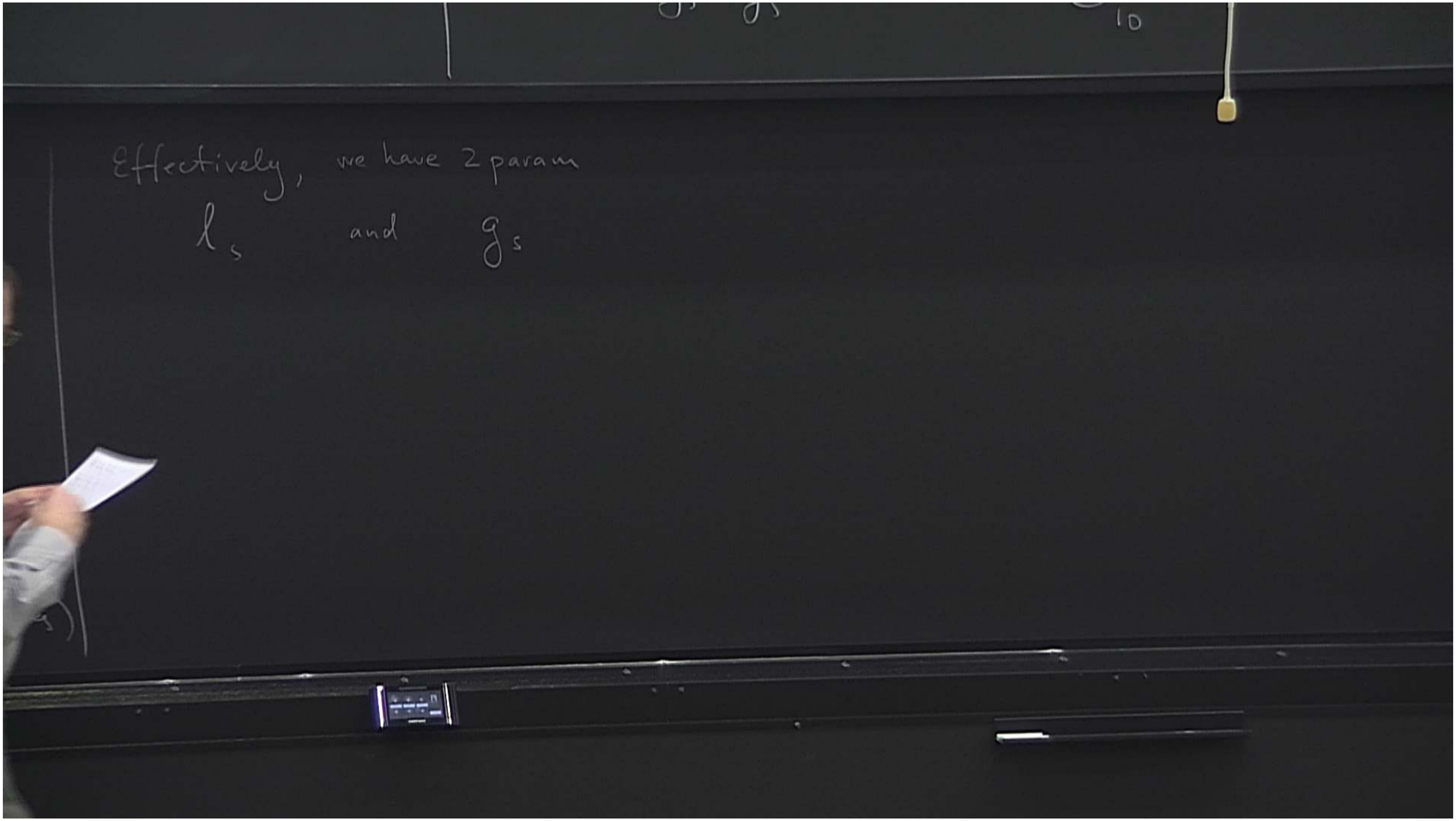
type I, Heterotic $E_8 \times E_8$, SO(32): limits of M-theory.
2) Absence of neg states require d = 10
3) String interac controlled by g_s
$16\pi G_{10} = (2\pi)^7 g_s^2 \ell_s^8$
$h_{\mu\nu}$
G_{10}
formalism
cle in
etc)
action)
of massive


modes m_s ~ l_s^-1 .
+
+ ....
g_s^2h-2
h = number of holes
g_s is not a free param.
g_s = e^φ exp. val. of dilaton
(one of the massless modes)


Effectively, we have 2 param
l_s and g_s

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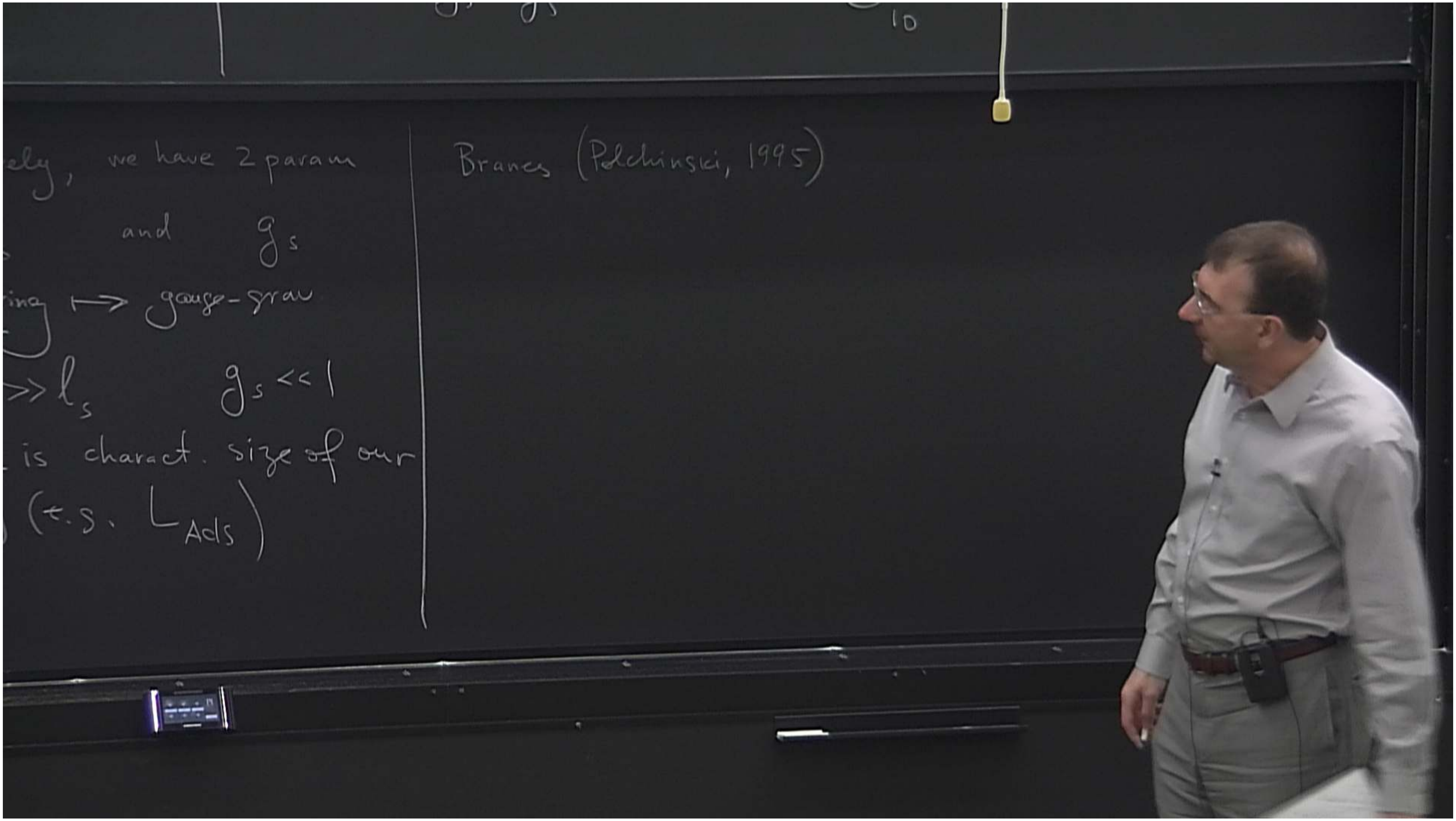
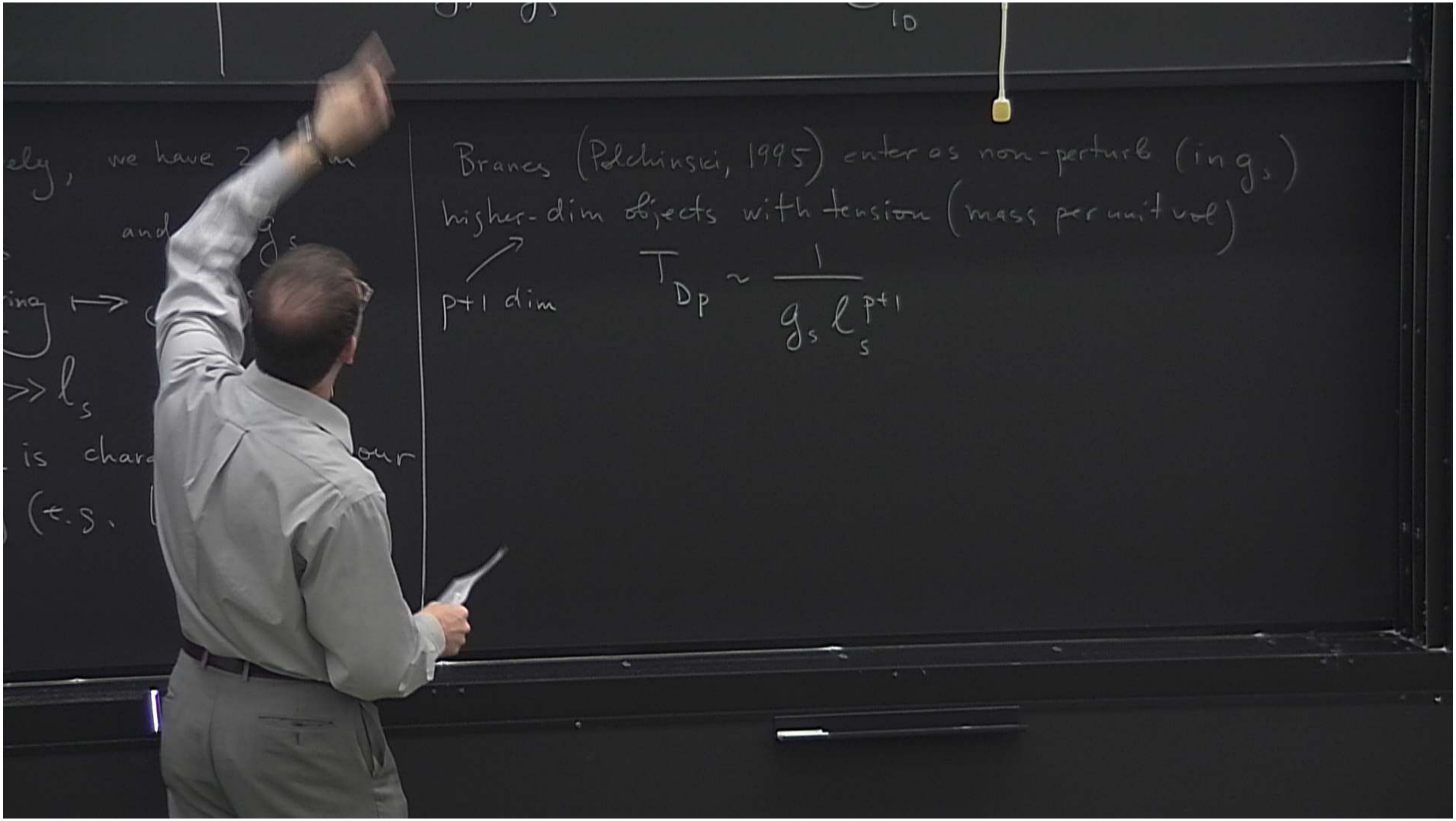
l_s and g_s

Gauge-string $\mapsto$ gauge-grav

$L \gg l_s$ $\quad$ $g_s \ll 1$

where L is charact. size of our geometry (e.g. L_{AdS})


Branes (Polchinski, 1995)
we have 2 param
and gs
gauge-grav
gs << 1
is charact. size of our
(e.g. LAdS)


Branes (Polchinski, 1995) enter as non-perturb (in gs)
higher-dim objects with tension (mass per unit vol)
p+1 dim
TDp ~ 1 / gs ls^p+1

...ely, we have 2 param

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$\gg l_s$ $\quad g_s \ll 1$

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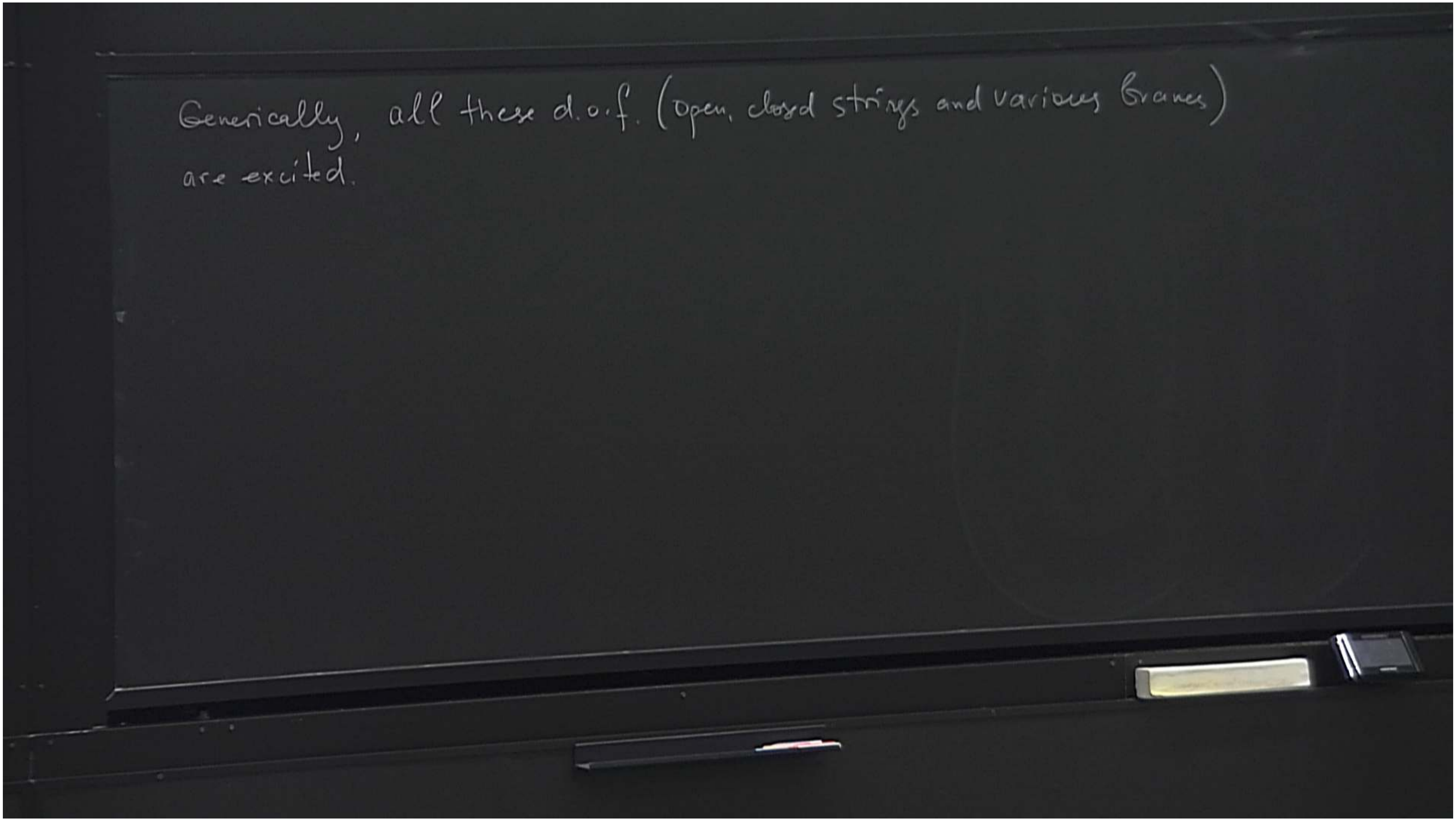
Branes (Polchinski, 1995) enter as non-perturb (in g_s) higher-dim objects with tension (mass per unit vol)

p+1 dim $\quad T_{Dp} \sim \frac{1}{g_s \, l_s^{p+1}}$

Dp branes are „topol. defects" with p+1 dim world volume on which open strings can end (and move freely along the world volume)


Generically, all these d.o.f. (open, closed strings and various Branes)
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and g_s

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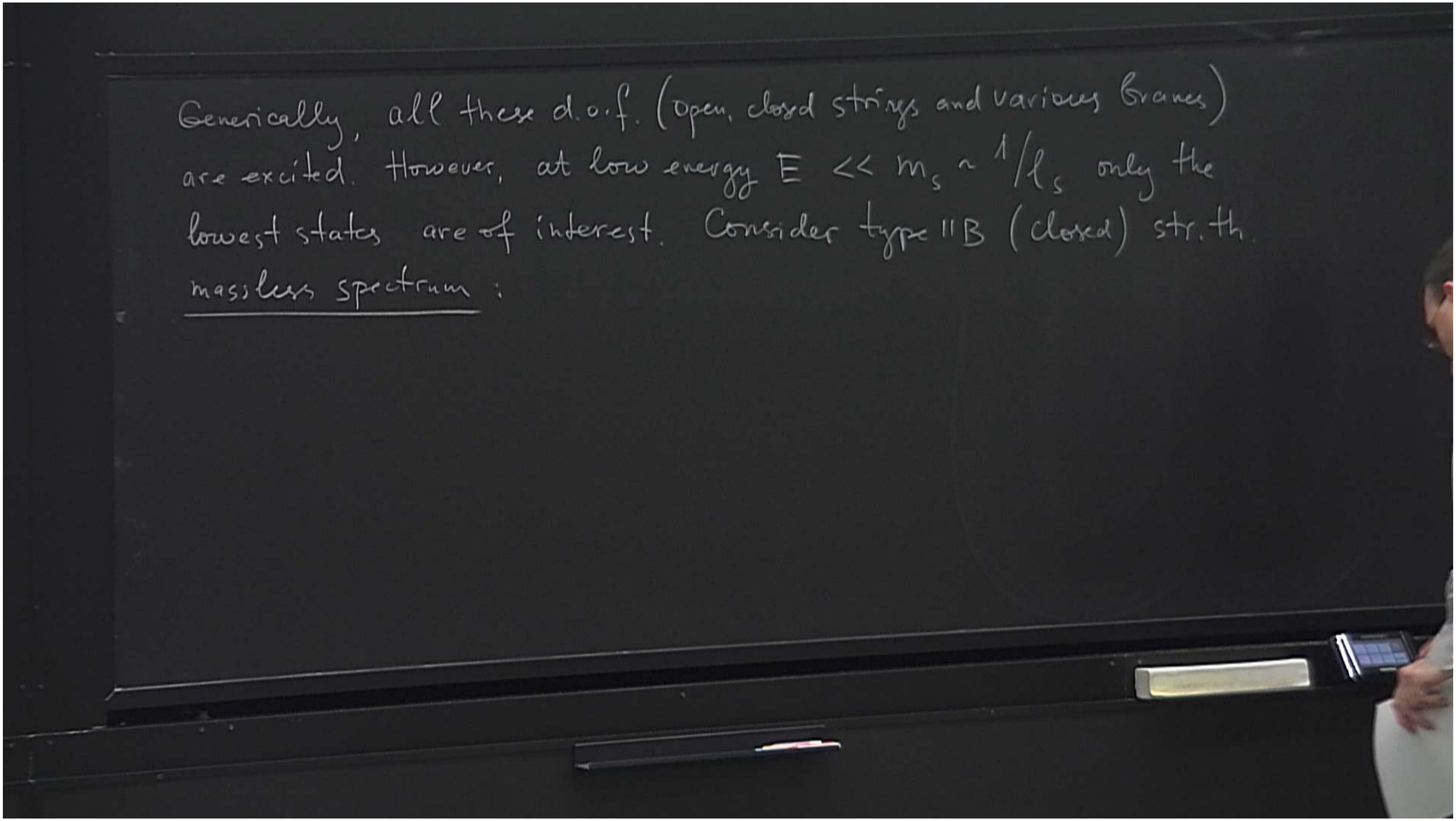
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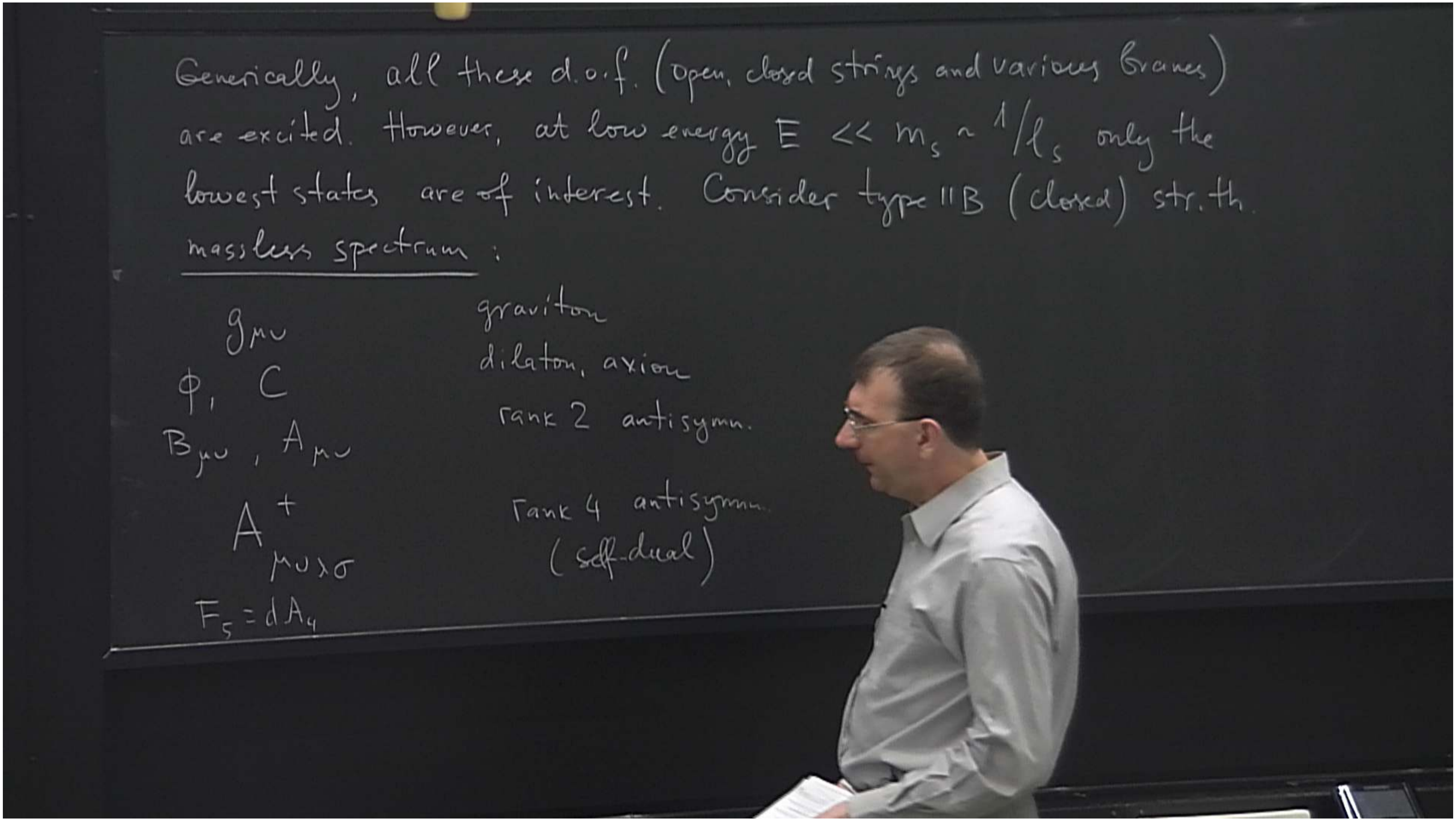
massless spectrum :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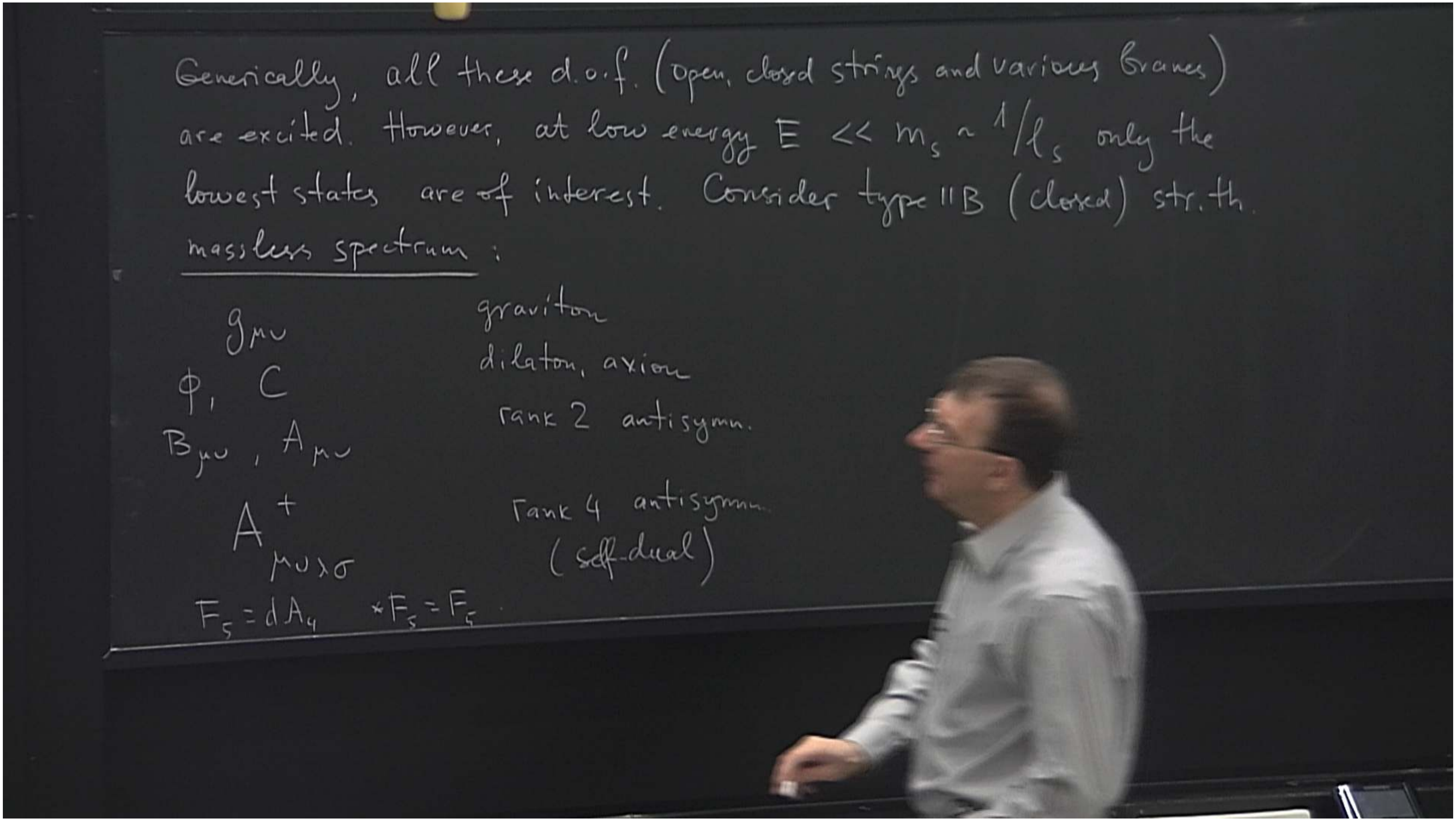
$g_{\mu\nu}$ graviton
ϕ, C dilaton, axion
$B_{\mu\nu}$, $A_{\mu\nu}$ rank 2 antisymm.
$A^{+}_{\mu\nu\lambda\sigma}$ rank 4 antisymm. (self-dual)
$F_5 = dA_4$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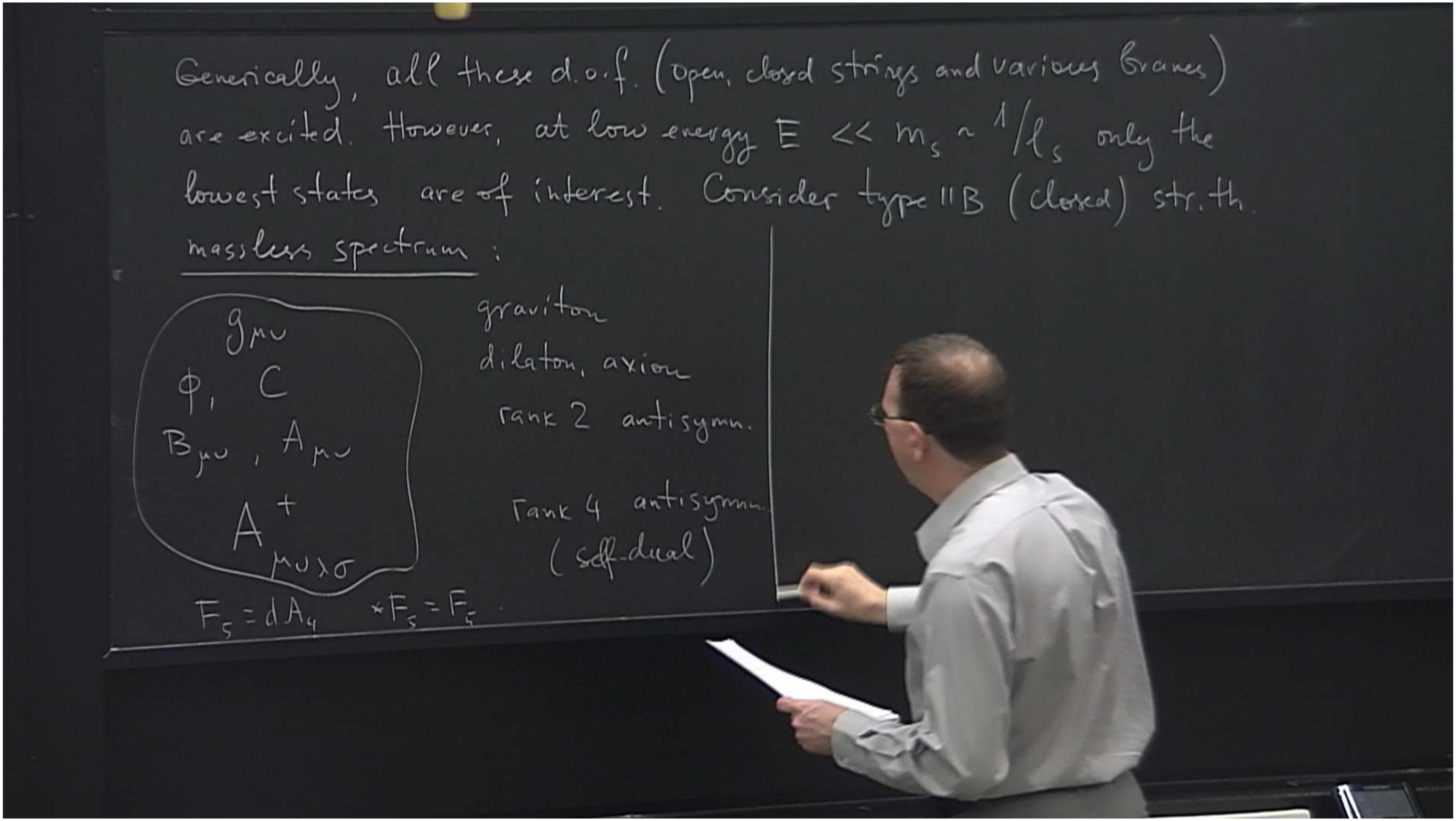
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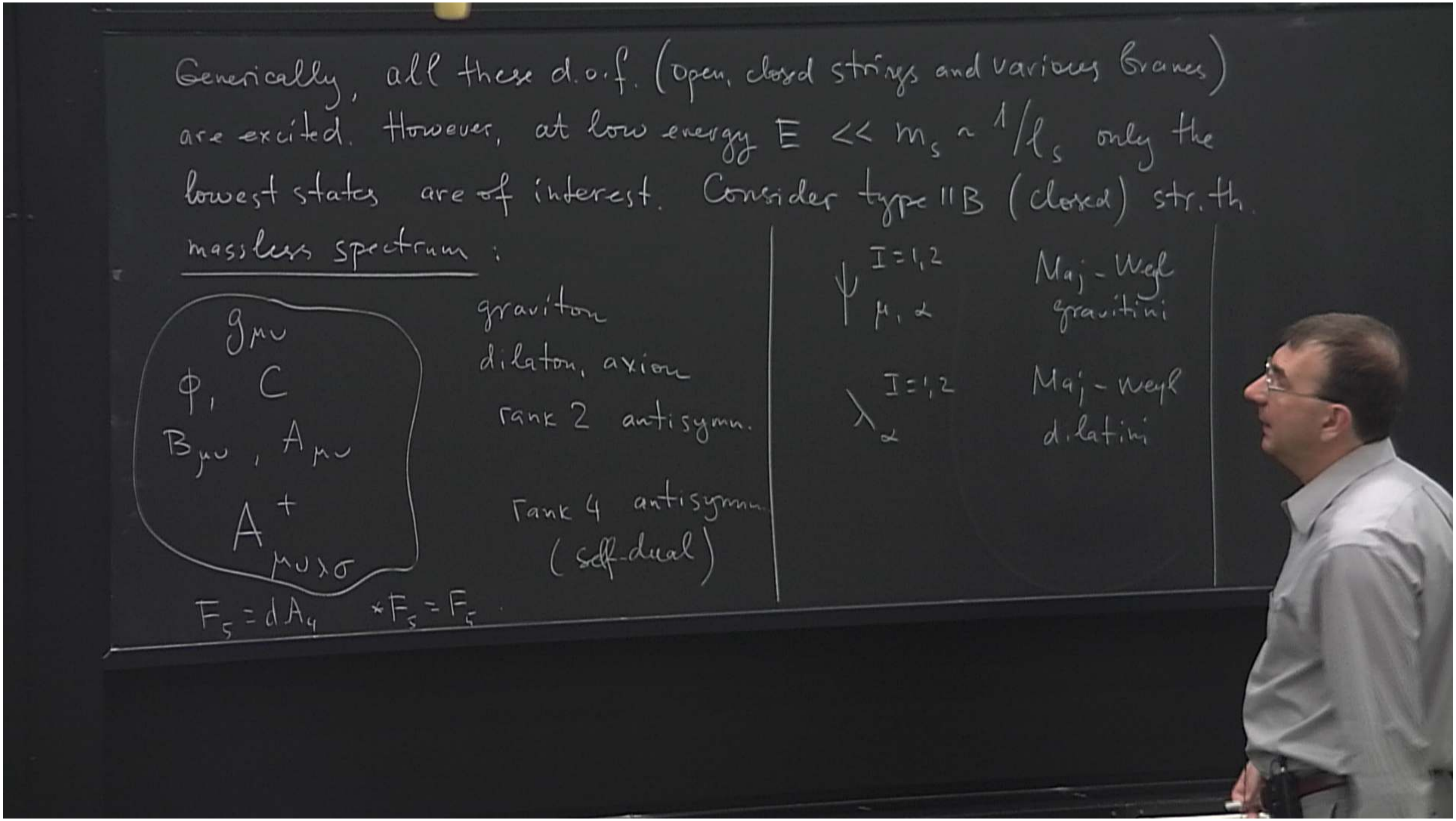
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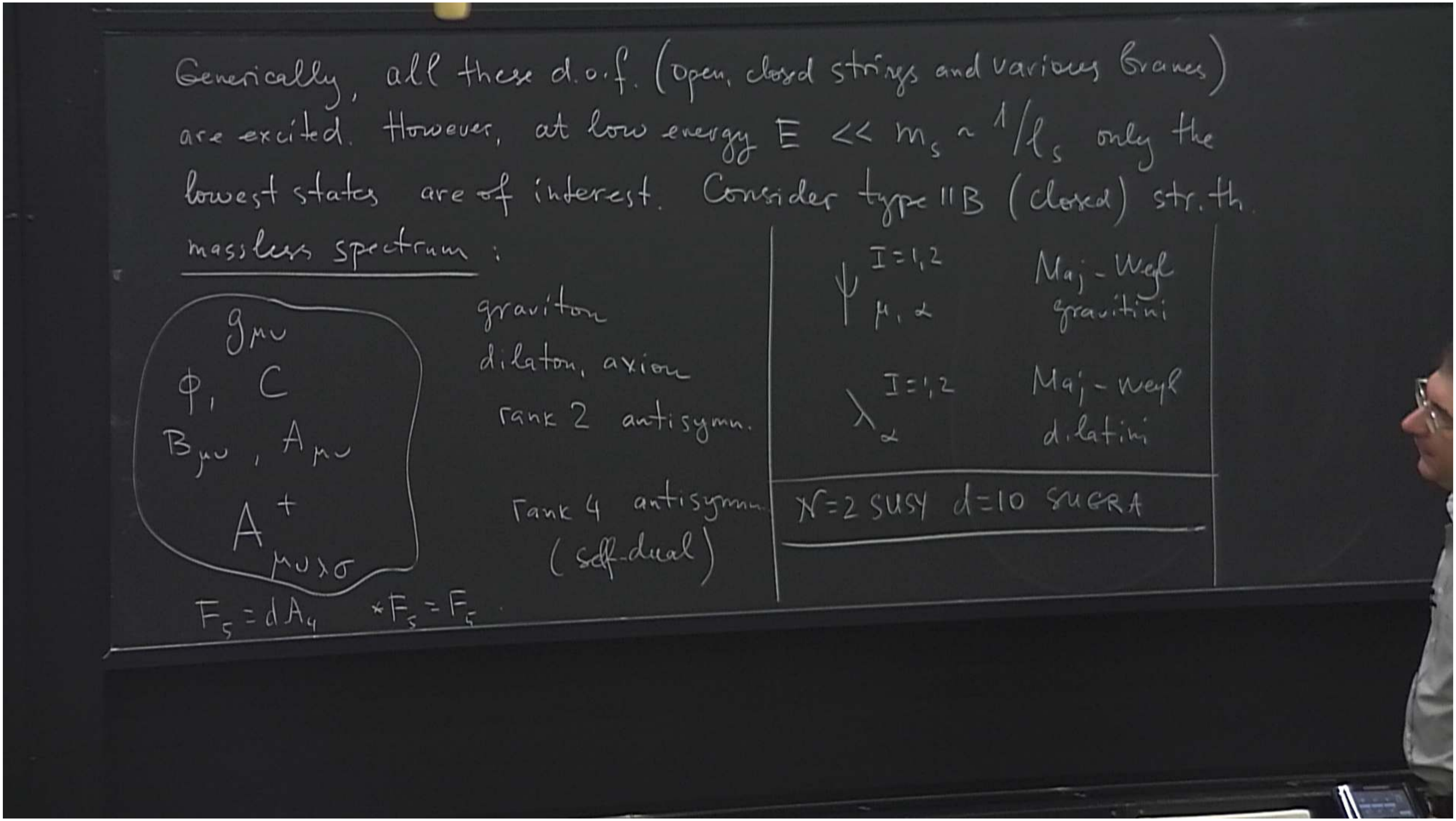
ψ^{I=1,2}_{μ, α}
Maj - Weyl
gravitini
λ^{I=1,2}_α
Maj - Weyl
dilatini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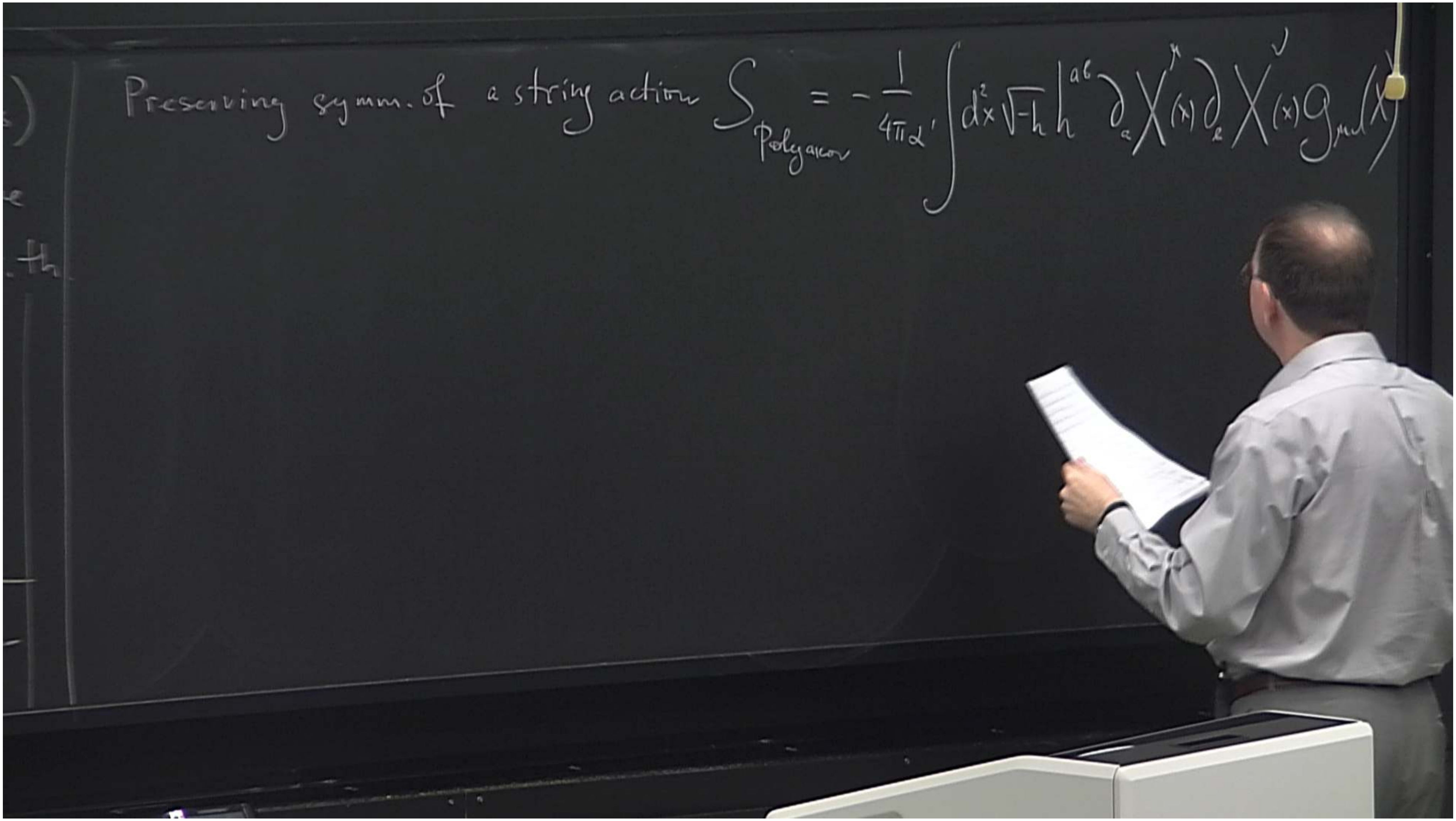
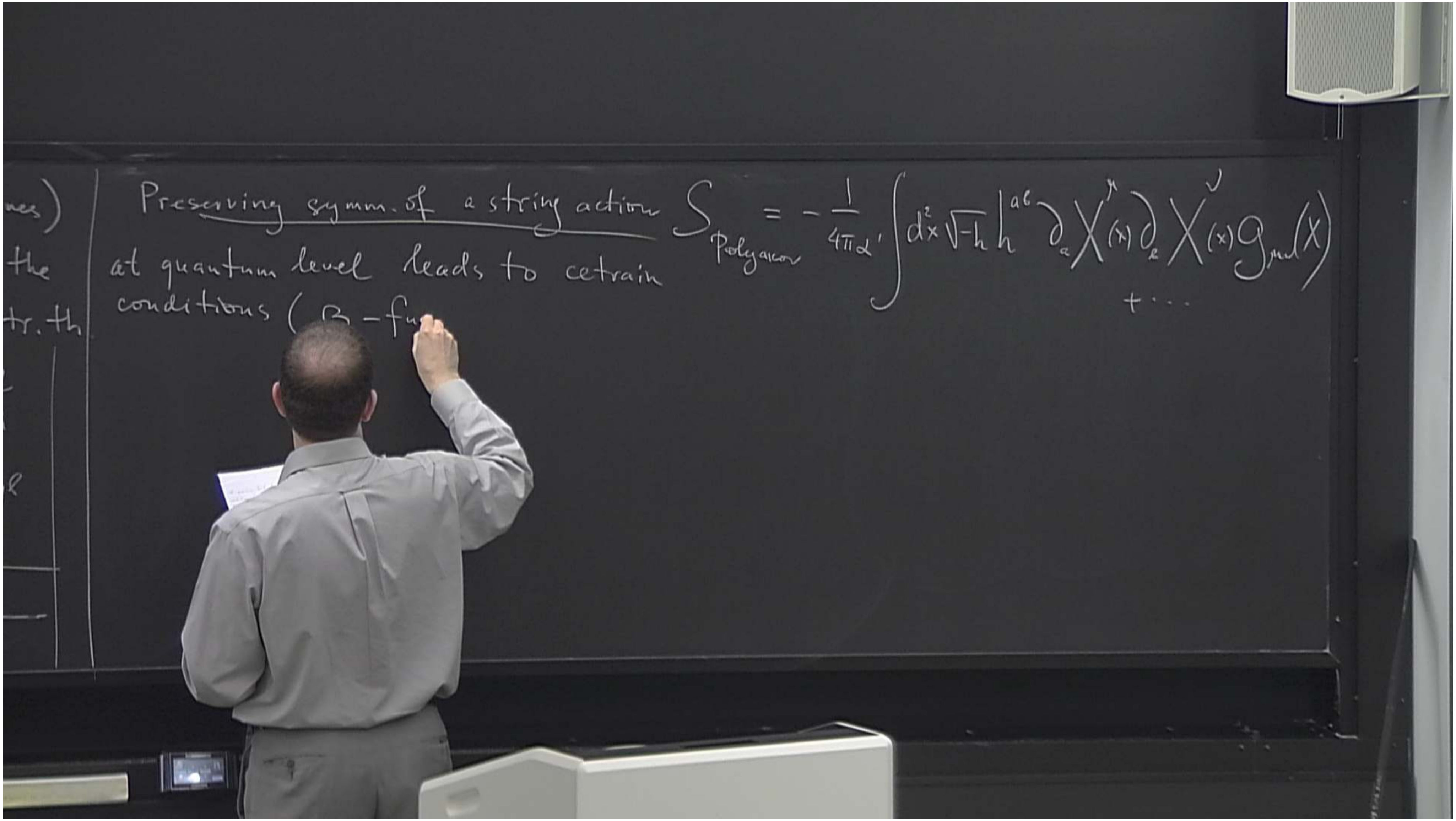
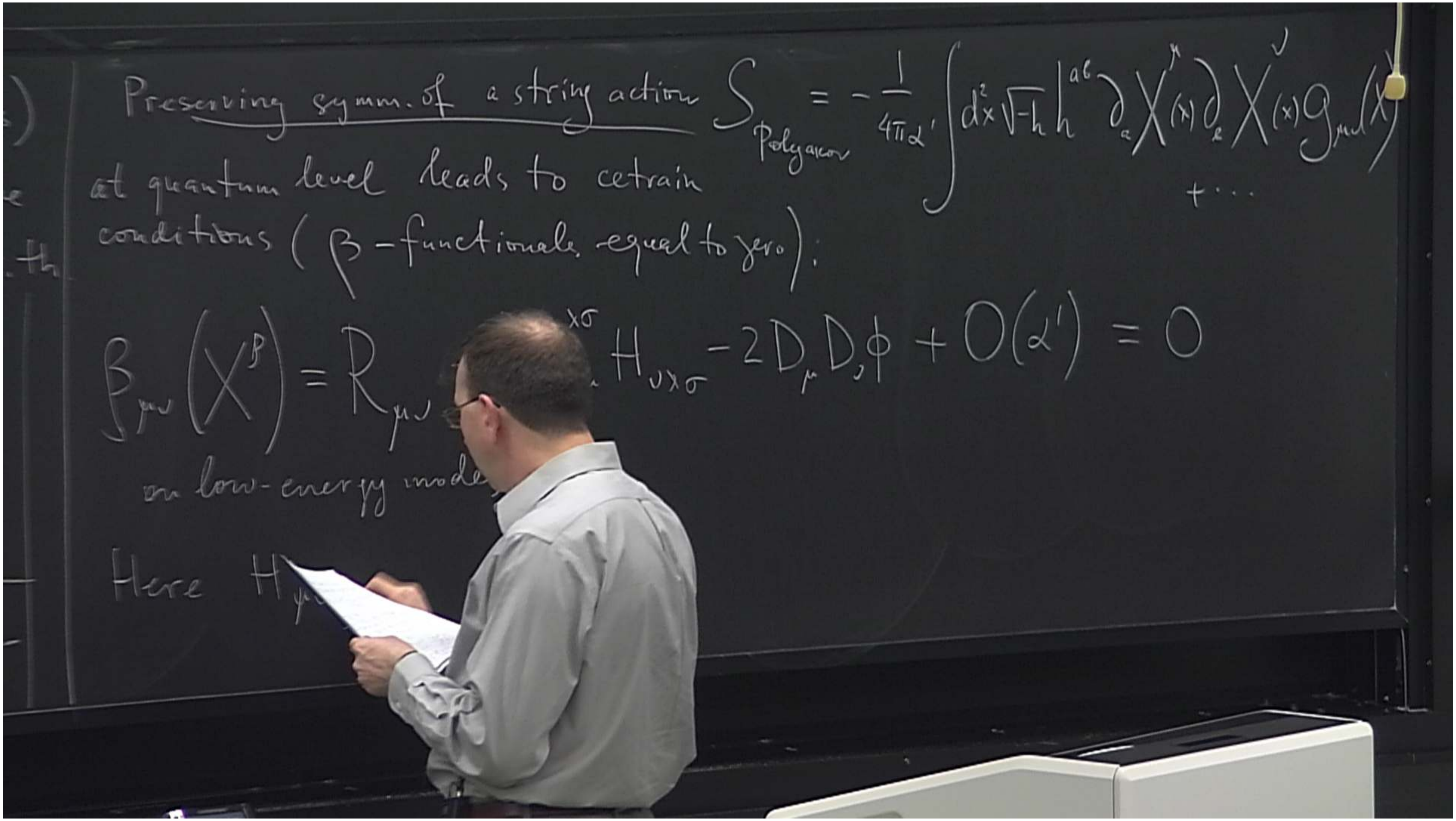
ψ^{I=1,2}_{μ,α} Maj-Weyl gravitini
λ^{I=1,2}_α Maj-Weyl dilatini
N=2 SUSY d=10 SUGRA


Preserving symm. of a string action


Preserving symm. of a string action
at quantum level leads to cetrain
conditions (β - fu
S_Polyakov = - 1/(4πα') ∫ d²x √-h h^ab ∂_a X^μ(x) ∂_b X^ν(x) g_μν(X)
+ . . .


Preserving symm. of a string action
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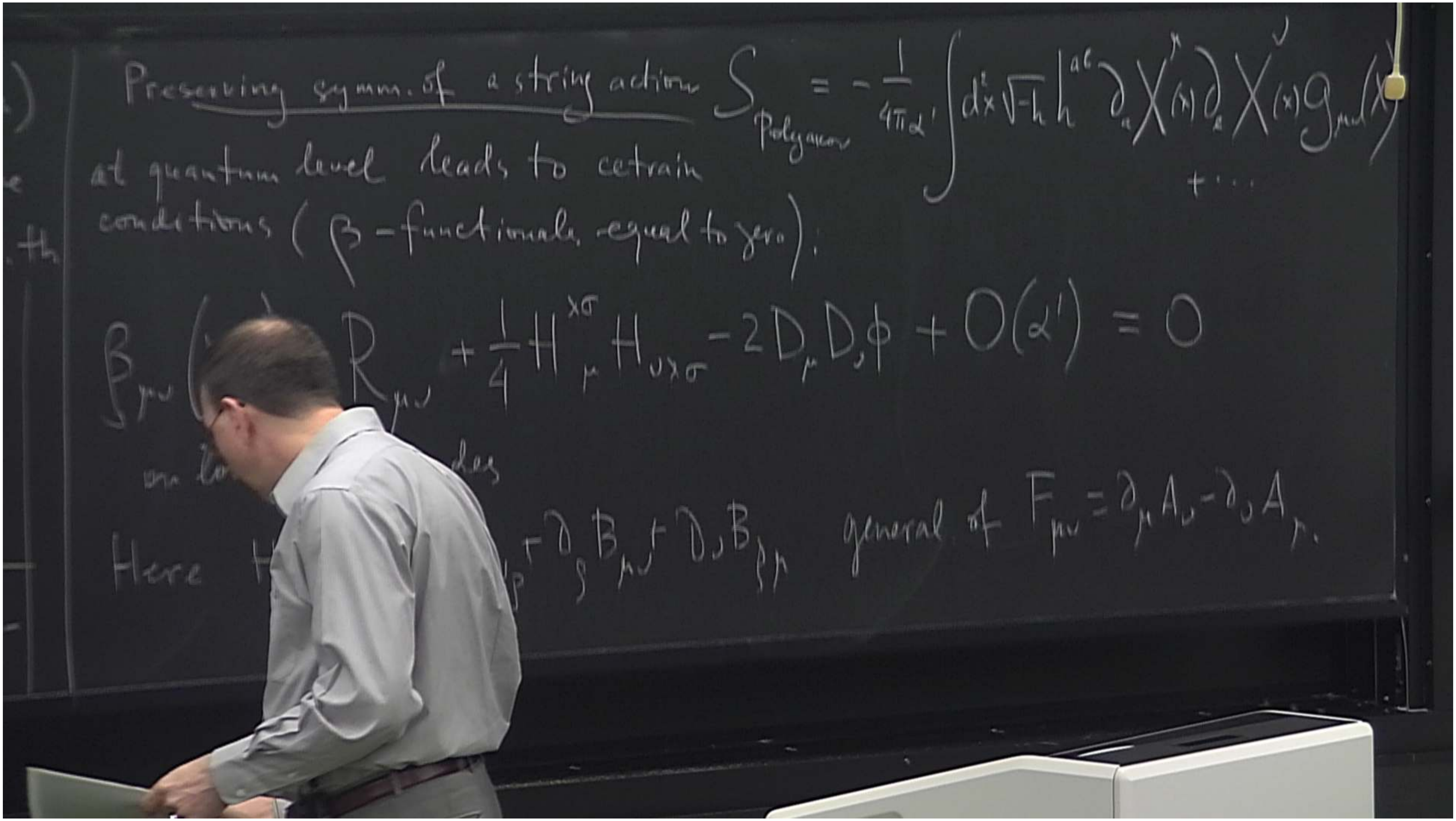
conditions (β-functionals equal to zero):
+ ...
on low-energy mode
Here


Preserving symm. of a string action
at quantum level leads to cetrain
conditions (β - functionals equal to zero):
general of

Preserving symm. of a string action $S_{Polyakov} = -\frac{1}{4\pi\alpha'}\int d^2x \sqrt{-h}\, h^{ab} \partial_a X^\mu(x) \partial_b X^\nu(x) g_{\mu\nu}(X)$

$+ \dots$

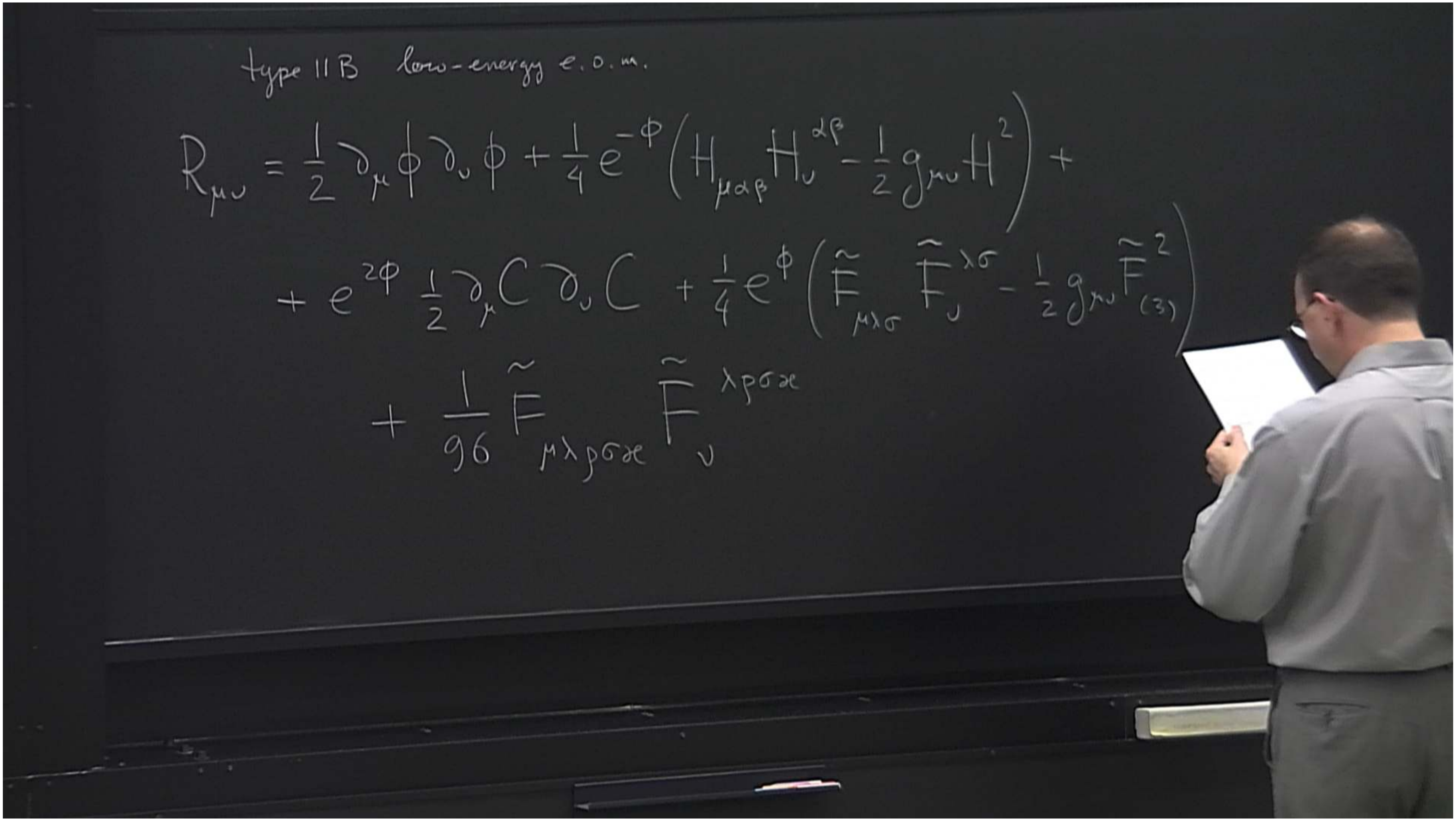
at quantum level leads to cetrain conditions (β-functionals equal to zero):

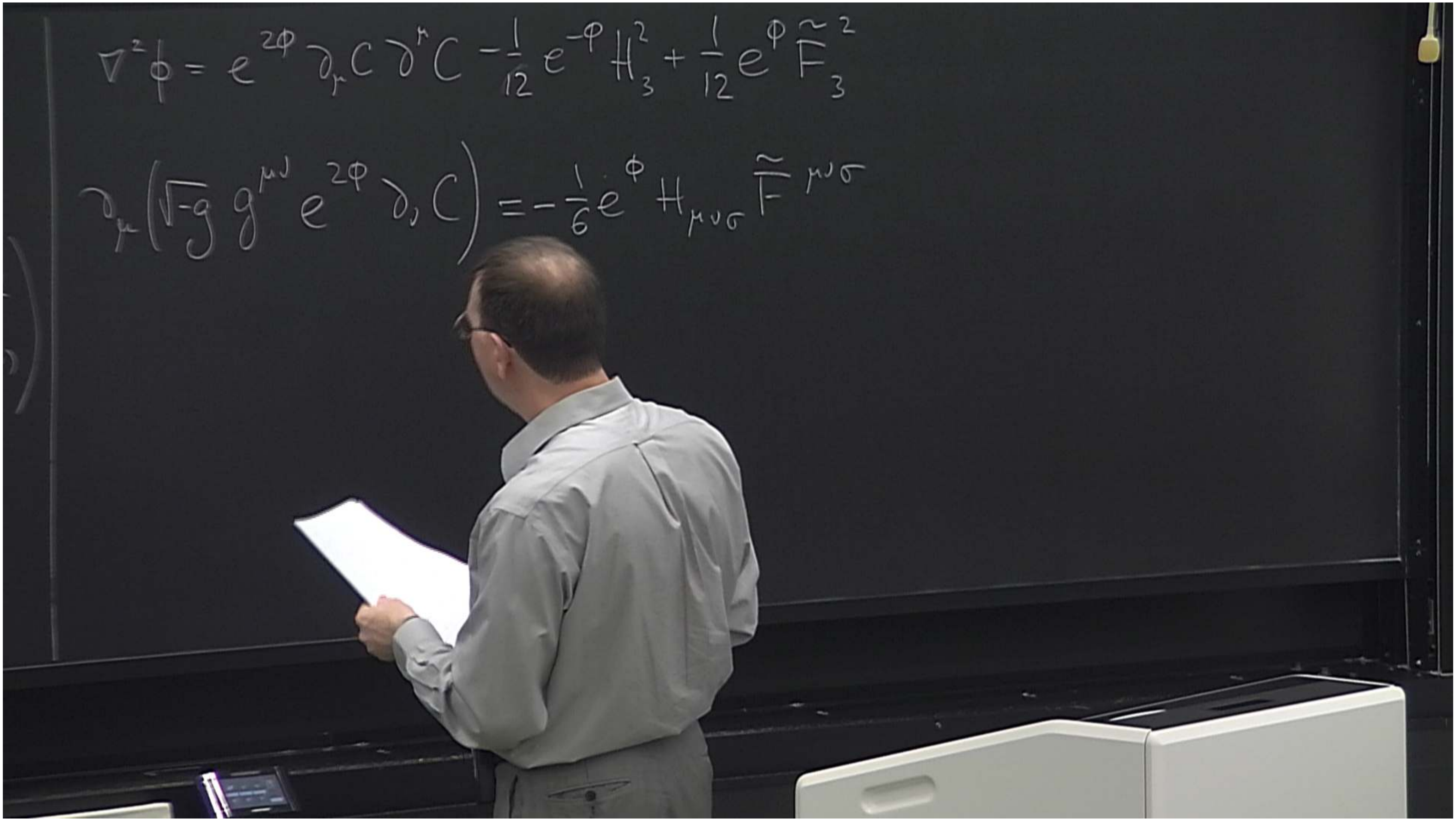
$$\beta_{\mu\nu}(X^\rho) = R_{\mu\nu} + \frac{1}{4} H^{\lambda\sigma}_{\mu} H_{\nu\lambda\sigma} - 2 D_\mu D_\nu \phi + O(\alpha') = 0$$

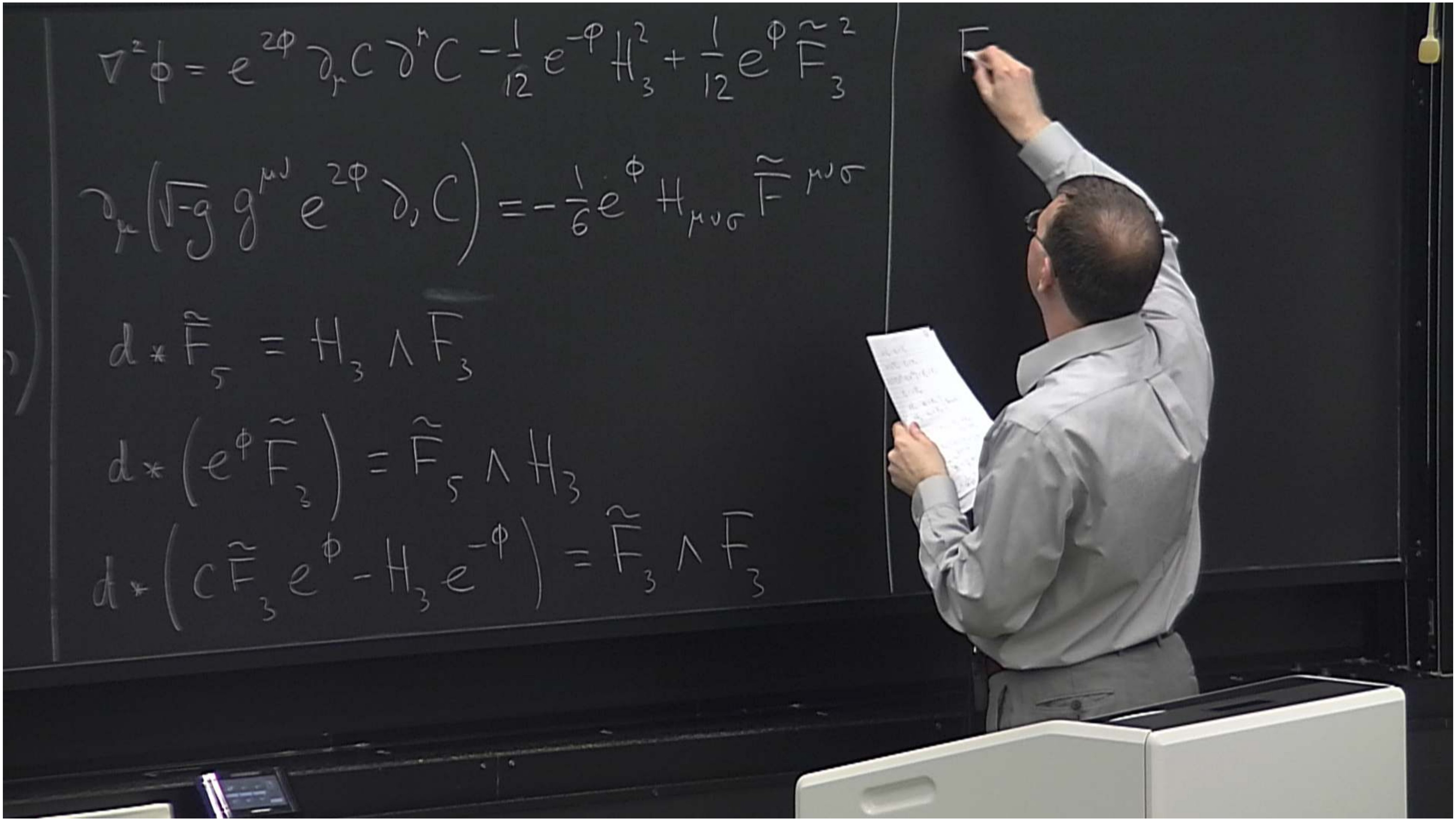
on low-energy modes

Here $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}$ general. of $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.


type IIB low-energy e.o.m.
$R_{\mu\nu} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi$


type IIB low-energy e.o.m.
$R_{\mu\nu} = \frac{1}{2}\partial_\mu \phi \partial_\nu \phi + \frac{1}{4} e^{-\phi}\left(H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} - \frac{1}{2} g_{\mu\nu} H^2\right) +$
$+ e^{2\phi} \frac{1}{2} \partial_\mu C \partial_\nu C + \frac{1}{4} e^{\phi}\left(\tilde{F}_{\mu\lambda\sigma} \tilde{F}_\nu{}^{\lambda\sigma} - \frac{1}{2} g_{\mu\nu} \tilde{F}^2_{(3)}\right)$
$+ \frac{1}{96} \tilde{F}_{\mu\lambda\rho\sigma\varkappa} \tilde{F}_\nu{}^{\lambda\rho\sigma\varkappa}$


$\nabla^2\phi = e^{2\phi}\partial_\mu C\,\partial^\mu C - \frac{1}{12}e^{-\phi}H_3^2 + \frac{1}{12}e^{\phi}\tilde{F}_3^2$
$\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}e^{2\phi}\partial_\nu C\right) = -\frac{1}{6}e^{\phi}H_{\mu\nu\sigma}\tilde{F}^{\mu\nu\sigma}$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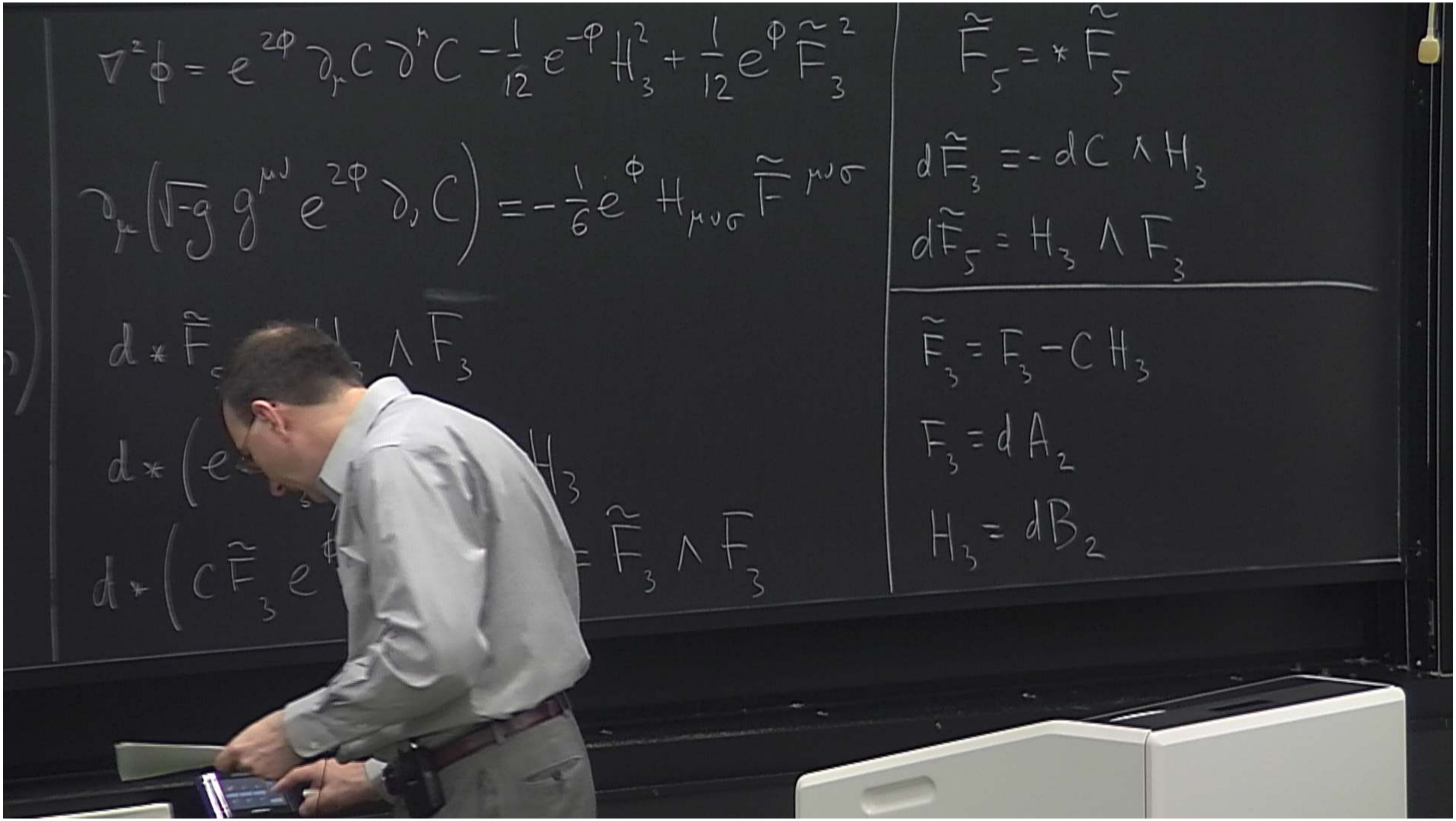
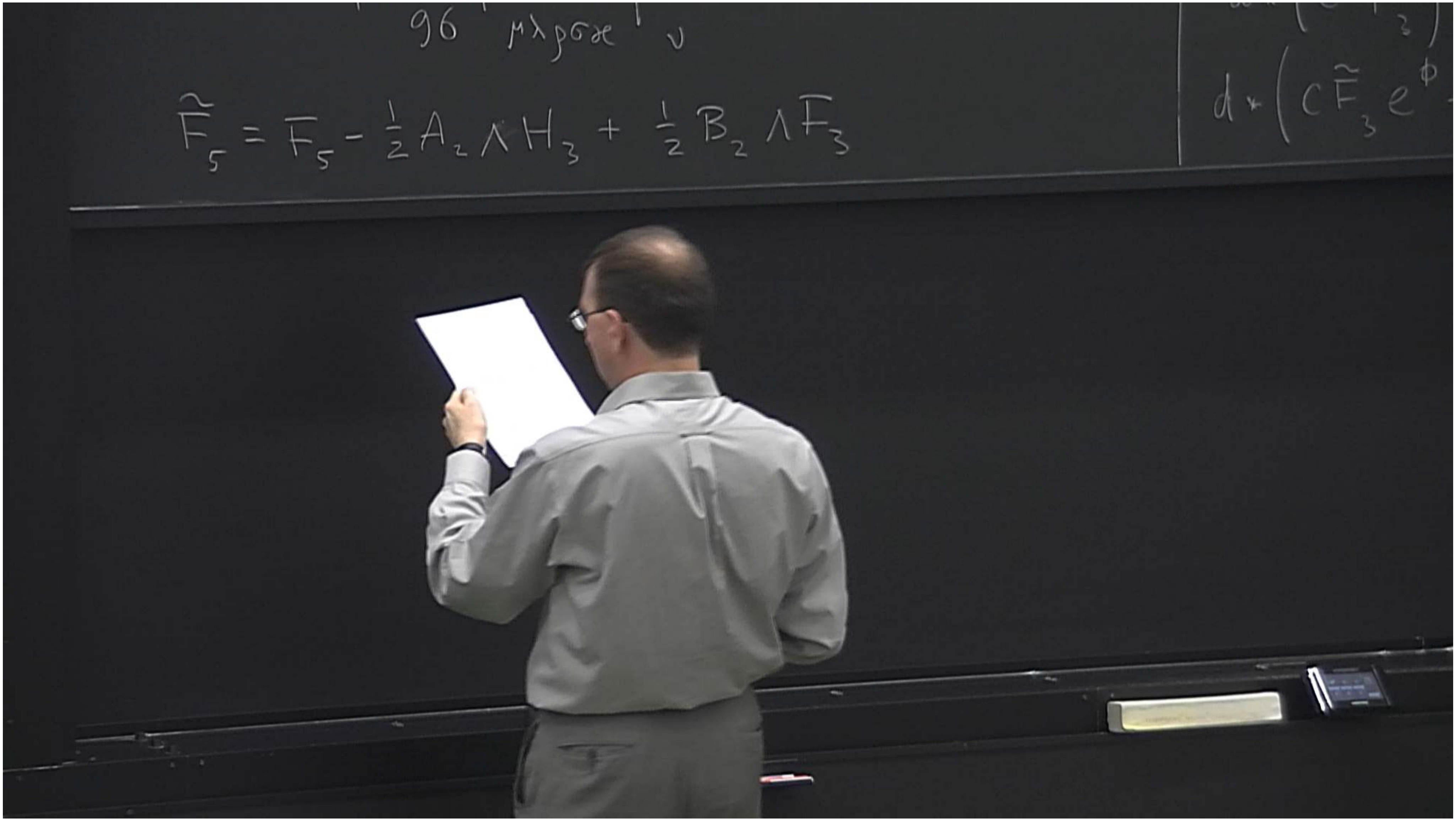
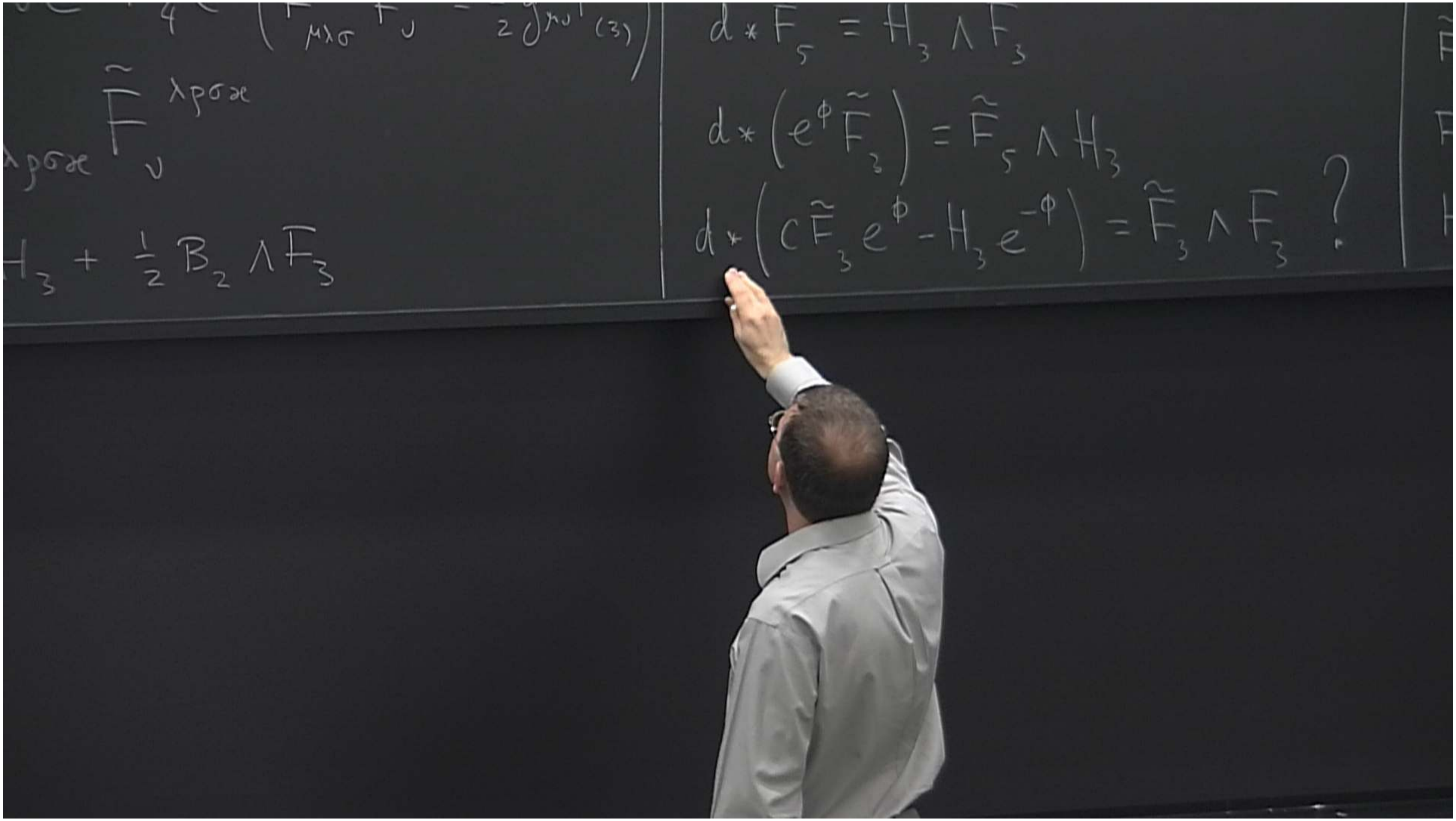
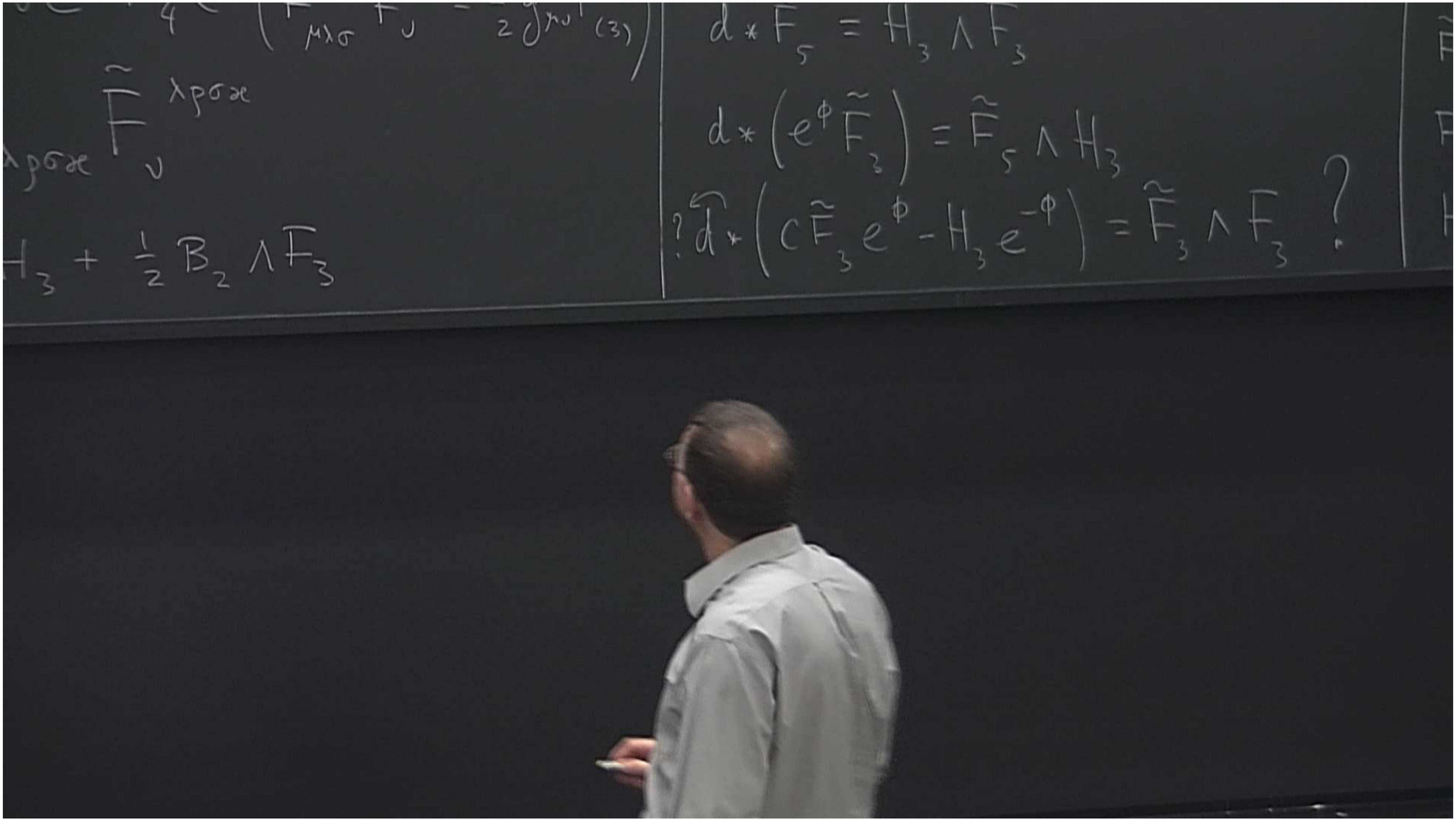
$d * \tilde{F}_5 = H_3 \wedge F_3$
$d * \left(e^{\phi} \tilde{F}_3 \right) = \tilde{F}_5 \wedge H_3$
$d * \left(C \tilde{F}_3 e^{\phi} - H_3 e^{-\phi} \right) = \tilde{F}_3 \wedge F_3$
F



$\nabla^2\phi = e^{2\phi}\partial_\mu C\,\partial^\mu C - \frac{1}{12}e^{-\phi}H_3^2 + \frac{1}{12}e^{\phi}\tilde{F}_3^2$
$\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}e^{2\phi}\partial_\nu C\right) = -\frac{1}{6}e^{\phi}H_{\mu\nu\sigma}\tilde{F}^{\mu\nu\sigma}$
$\tilde{F}_5 = *\tilde{F}_5$
$d\tilde{F}_3 = -dC \wedge H_3$
$d\tilde{F}_5 = H_3 \wedge F_3$
$\tilde{F}_3 = F_3 - C H_3$
$F_3 = dA_2$
$H_3 = dB_2$


$\tilde{F}_5 = F_5 - \frac{1}{2} A_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$


$\tilde{F}^{\lambda\rho\sigma\varkappa}$
$H_3 + \frac{1}{2} B_2 \wedge F_3$
$d * F_5 = H_3 \wedge F_3$
$d * (e^{\phi} \tilde{F}_3) = \tilde{F}_5 \wedge H_3$
$d * (c \tilde{F}_3 e^{\phi} - H_3 e^{-\phi}) = \tilde{F}_3 \wedge F_3$?


d * F_5 = H_3 ∧ F_3
d * (e^φ F̃_3) = F̃_5 ∧ H_3
?d * (C F̃_3 e^φ - H_3 e^-φ) = F̃_3 ∧ F_3 ?
H_3 + 1/2 B_2 ∧ F_3

$$\tilde{F}_5 = F_5 - \frac{1}{2} A_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$$

$$? \, d * \left(C \tilde{F}_3 e^{\Phi} \right.$$

$$F_p^2 = F_{M_1 \dots M_p} F^{M_1 \dots M_p}$$

$$\phi = \frac{1}{p!} \phi_{i_1 \dots i_p} dx^{i_1} \wedge \dots \wedge dx^{i_p}$$

$$* \phi = \frac{1}{(d-p)!} \frac{1}{p!} \varepsilon_{i_1 \dots i_d} \sqrt{|g|} \, \phi_{j_1 \dots j_p} g^{i_1 j_1} \cdots g^{i_p j_p} dx^{i_{p+1}} \wedge \dots \wedge dx^{i_d}$$

$$+ e^{2\phi}\,\frac{1}{2}\partial_\mu C\,\partial_\nu C + \frac{1}{4}e^{\phi}\left(\tilde{F}_{\mu\lambda\sigma}\tilde{F}_\nu{}^{\lambda\sigma} - \frac{1}{2}g_{\mu\nu}\tilde{F}^2_{(3)}\right)$$

$$+ \frac{1}{96}\tilde{F}_{\mu\lambda\rho\sigma\kappa}\tilde{F}_\nu{}^{\lambda\rho\sigma\kappa}$$

$$\tilde{F}_5 = F_5 - \frac{1}{2}A_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$$

$$g_{\mu\nu} = g^{BG}_{\mu\nu} + h_{\mu\nu}$$

$$\Phi = \Phi^{BG} + \varphi$$

$$C = C^{BC} + c$$

$$d * \tilde{F}_5 = H \ldots$$

$$d * \left(e^{\phi}\tilde{F}_3\right) \ldots$$

$$?\, d * \left(C\tilde{F}_3 e^{\phi} \ldots\right.$$

$$F_p^2 = F_{M_1\ldots M_p}F^{M_1\ldots M_p}$$

$$\phi = \frac{1}{p!}\phi_{i_1\ldots i_p}\,dx^{i_1}\wedge\ldots\wedge dx^{i_p}$$

$$*\phi = \frac{1}{(d-p)!}\frac{1}{p!}\varepsilon_{i_1\ldots i_d}\sqrt{|g|}\,\phi_{j_1\ldots j_p}\,g^{i_1 j_1}\cdots g^{i_p j_p}\,dx^{i_{p+1}}\wedge\ldots\wedge dx^{i_d}$$