Title: 13/14 PSI - Explorations in String Theory - Lecture 3

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Abstract:

Aspects of BH Thermodynamics

$\hbar = 1, \quad c = 1 \qquad - + + + \cdots +$

$$\boxed{ds_d^2 = -f_d(r)\,dt^2 + \frac{dr^2}{f_d} + r^2\,d\Omega^2_{d-2}}$$

where $f_d = 1 - \left(\frac{r_0}{r}\right)^{d-3}$

$d\Omega^2_{d-2}$ is the metric on $S^{d-2}_{R=1}$

Asympt. flat: for $r \to \infty$ $f_d \to 1 \to ds_d^2 \to$ Mink.$_d$

The parameter r_0 is related to mass

$$M = (d-2)\,\Omega_{d-2}\, r_0^{d-3} \Big/ 16\pi G_d \,, \quad \text{where } \Omega_{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)}$$

Surfa
a un

1 $\quad - + + + \dots +$

$$f_d = 1 - \left(\frac{r_0}{r}\right)^{d-3}$$

is the metric on $S^{d-2}_{R=1}$

k · d

$$\Omega_{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)}$$

Surface area of a unit d-2-dim sphere

G_d is d-dim Newton const.

$$S = \frac{1}{2\varkappa_d^2}\int (R - 2\Lambda)\sqrt{-g}\, d^d x + \int L_{matter}\sqrt{-g}\, d^d x$$

$$\varkappa_d^2 = 8\pi G_d \qquad \delta S = 0 \Rightarrow$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \varkappa_d^2 T_{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{2\sqrt{-g}} \frac{\delta L_{matter}}{\delta g_{\mu\nu}}$$

Taking the trace of e.o.m.

$$R = \frac{T^{\mu}_{\mu} - \Lambda d}{1 - d/2}$$

$$\boxed{R_{\mu\nu} = T_{\mu\nu} - \frac{g_{\mu\nu}}{d-2} T^{\mu}_{\mu} + g_{\mu\nu} \frac{2\Lambda}{d-2} \quad + \text{ e.o.m. of matter fields}}$$

• elem. dim analysis

since the action is dimensionless ($c=1, \hbar=1$)

$\Rightarrow \varkappa^2_d$

$g_{\mu\nu}$ is dimensionless

$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \qquad \Gamma \sim \partial g$

$R_{\mu\nu\lambda\rho} \sim \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$

$[\Gamma^{\lambda}_{\mu\nu}] = 1/L \qquad R_{\mu\nu\lambda\rho} \sim 1/L^2$

$R_{\mu\nu} \sim 1/L^2 \qquad R \sim 1/L^2$

$\forall d$!

Taking the trace of e.o.m.

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$$\boxed{R_{\mu\nu} = T_{\mu\nu} - \frac{g_{\mu\nu}}{d-2} T^{\mu}_{\mu} + g_{\mu\nu}\frac{2\Lambda}{d-2}}$$

+ e.o.m. of matter fields

• elem. dim analysis

since the action is dimensionless ($c=1, \hbar=1$)

$\Rightarrow \kappa_d^2 \sim L^{d-2}$

$\kappa_d^2 = 8\pi G_d \sim l_p^{d-2}$

$g_{\mu\nu}$ is dimensionless

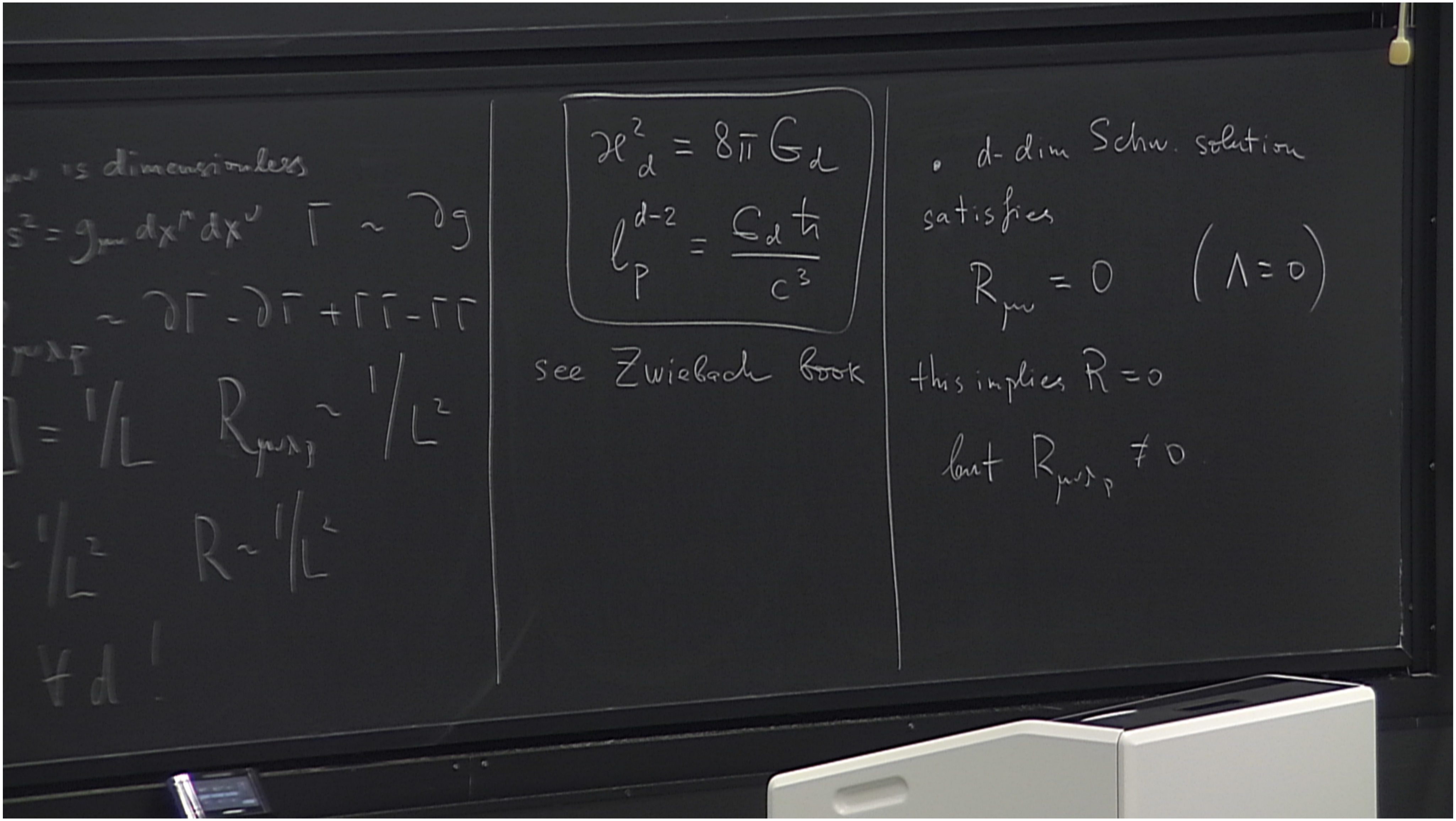
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad \Gamma \sim \partial g$

$R_{\mu\nu\lambda\rho} \sim \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$

$[\Gamma^{\alpha}_{\mu\gamma}] = 1/L \quad R_{\mu\nu\lambda\rho} \sim 1/L^2$

$R_{\mu\nu} \sim 1/L^2 \quad R \sim 1/L^2$

$\forall d$!


is dimensionless
$s^2 = g_{\mu\nu} dx^\mu dx^\nu \quad \Gamma \sim \partial g$
$\sim \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$
$= 1/L \quad R_{\mu\nu\lambda\rho} \sim 1/L^2$
$1/L^2 \quad R \sim 1/L^2$
$\forall d\,!$
$\boxed{\varkappa_d^2 = 8\pi G_d \qquad l_p^{d-2} = \frac{G_d \hbar}{c^3}}$
see Zwiebach book
• d-dim Schw. solution satisfies
$R_{\mu\nu} = 0 \quad (\Lambda = 0)$
this implies $R = 0$
but $R_{\mu\nu\lambda\rho} \neq 0$

The "true" sing. is at $r=0$

Compute curvature invar. such as the Kretschmann inv.

$$K = R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma}$$

For $d=4$ Schwarzschild:

$$K = \frac{12\, r_0^2}{r^6} \quad \left(\begin{array}{l} K \to \infty \text{ for } r \to 0 \\ K = \dfrac{12}{r_0^4} \text{ at } r = r_0 \end{array} \right)$$

in $\forall d$ $K \sim \dfrac{1}{r_0^4}$

- $g_{\mu\nu}$ is a tensor $\to$ other coord. systems such as Kruskal-Szekeres (Eddington-Finkelstein)

other coord. systems
– Szekeres
…n – Finkelstein)

- in addition to K, there are many more curv. invar.

Consider those w/o covar. deriv.

exercise: $d = 4$ 14

$d = 5$ 40

$\forall d$ $N = \dfrac{d(d-1)(d-2)(d+3)}{12}$

$/16\pi G_d$ Ω_{d-2} $\Gamma\left(\frac{d-1}{2}\right)$ a unit d-2-dim sphere

$T^{\mu\nu} = \frac{1}{2\sqrt{-g}} \frac{\delta \mathcal{L}_{matter}}{\delta g_{\mu\nu}}$

- in $\forall d$ $K \sim \frac{1}{r_0^4}$ at $r \sim r_0$

$\boxed{\text{• } g_{\mu\nu} \text{ is a tensor} \Rightarrow \text{other coord. systems such as Kruskal–Szekeres (Eddington–Finkelstein)}}$

- $K(r_0) \sim \frac{1}{r_0^4} \ll 1/\ell_p^4$

$\Rightarrow M \gg 1/\ell_p \equiv M_p \Rightarrow$

$\to 0$

at $r = r_0$

- in addition to K, there are many more curv. invar.

Consider those w/o covar. deriv.

exercise: $d = 4$ 14

$d = 5$ 40

$\forall d$ $N = \frac{d(d-1)(d-2)(d+3)}{12}$

near-hor. region is safely described by class. gravity (EH action)

Check of validity of grav. solution

Hawking temperature

Near-hor. region

$$f_d(r) = f'_d(r_0)(r-r_0) + O\left((r-r_0)^2\right)$$

$$f'_d(r_0) = \frac{d-3}{r_0}$$

Make a Wick rot. to Euclidean time τ

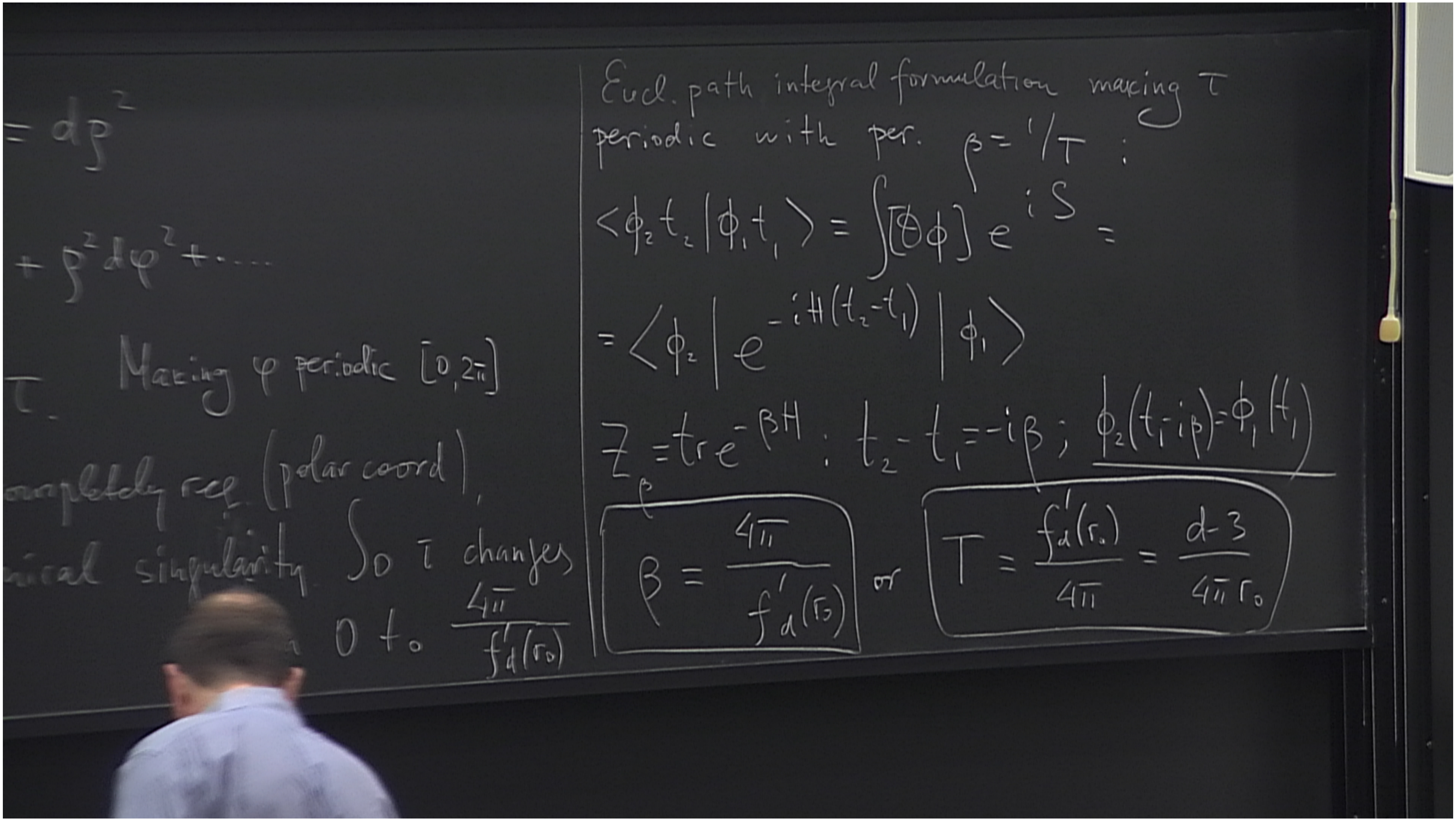
$$t \to -i\tau \quad \text{and change coord:}$$

$$\frac{dr^2}{f'_d(r_0)(r-r_0)} = d\rho^2$$

$$\Rightarrow ds_d^2 = d\rho^2 + \rho^2 d\varphi^2 + \dots$$

$$\varphi \equiv \frac{f'_d(r_0)}{2}\tau.$$

Making φ periodic $[0, 2\pi]$ means ds_d^2 is completely reg. (polar coord), otherwise $\exists$ conical singularity. So τ changes from 0 to $\frac{4\pi}{f'_d(r_0)}$


$= d\rho^2$
$+ \rho^2 d\varphi^2 + \dots$
τ. Making φ periodic $[0, 2\pi]$
ompletely reg (polar coord),
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0 to $\frac{4\pi}{f_d'(r_0)}$
Eucl. path integral formulation making τ periodic with per. $\beta = 1/T$:
$\langle \phi_2 t_2 | \phi_1 t_1 \rangle = \int [\mathcal{D}\phi]\, e^{iS} =$
$= \langle \phi_2 | e^{-iH(t_2 - t_1)} | \phi_1 \rangle$
$Z_\beta = \mathrm{tr}\, e^{-\beta H}$: $t_2 - t_1 = -i\beta$; $\phi_2(t_1 - i\beta) = \phi_1(t_1)$
$\boxed{\beta = \frac{4\pi}{f_d'(r_0)}}$ or $\boxed{T = \frac{f_d'(r_0)}{4\pi} = \frac{d-3}{4\pi r_0}}$

For $d=4$: $T_H = \frac{1}{4\pi r_0} = \frac{1}{8\pi G M}$ $\left(r_0 = 2GM \text{ in } d=4 \right)$

Observe: dM

Horizon Area:

$$A = \int \sqrt{g_\Omega}\, d\Omega$$

$$A_d = r_0^{d-2}\, \Omega_{d-2}$$

Introduce BH entropy: $S = \frac{A_d}{4 G_d}$

For d-dim Schw. $S = \frac{r_0^{d-2}\, \Omega_{d-2}}{4 G_d}$

For $d=4$ $\quad S = 4\pi G M^2, \quad T = \frac{1}{8\pi G M} \Rightarrow$

$$d\mathcal{M} = \frac{d-3}{16\pi G_d r_0}\, dA = \frac{d-3}{4\pi r_0}\, d\left(\frac{A}{4 G_d}\right) \Rightarrow dE = T\,dS \text{ in TD}$$

For

with:

$$\boxed{T = \frac{d-3}{4\pi r_0}\,\frac{\hbar c}{k_B}} \qquad \boxed{S = \frac{k_B c^3 A}{4 G_d \hbar}}$$

$$l_p^{d-2} = \frac{G_d \hbar}{c^3} \qquad \boxed{S = \frac{k_B A_d}{4\, l_p^{d-2}}}$$

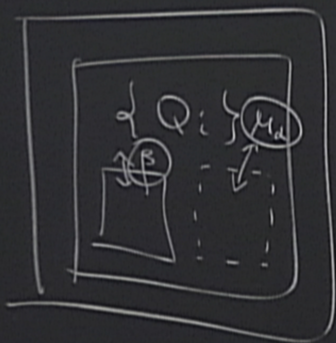
Four Laws of BH Mechanics (Bardeen, Carter, Hawking, 1973)

In $d = 4$, a stationary asympt. flat BH uniq. characterized by its mass M, angular mom J, charge Q

- $\exists$ exotic exceptions in $d=4$ and less exotic in $d>4$.

This is similar to TD equilibrium

$T\,dS$ in TD



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