

Title: 13/14 PSI - Explorations in String Theory - Lecture 1

Date: Mar 17, 2014 11:30 AM

URL: <http://pirsa.org/14030059>

Abstract:

Aspects of gauge-gravity duality and holography

I. Introduction

II. Holographic tools

III. Holograph. dictionary

IV. Holography and fluid dynamics

V. Hologr. and cond. matter physics

and holography

Goal: to understand strongly interacting quantum systems in and out of equilibrium

Experiments: 1) RHIC / LHC heavy ion collisions

$$T \sim 2 \cdot 10^{12} \text{ K}$$

Pb

Au

2) cold atoms

(ultra-cold atomic gases)

(^6Li)

$$T \sim 10^{-8} \text{ K}$$

Features: these systems are:

- many body ($N \sim 10^4$)
- quantum ($\lambda_{\text{de Broglie}} \gtrsim l$)
- strongly interacting

$$\text{NR: } l \sim (V/N)^{1/3}$$

$$\text{Rel with kinetic regime: } l \sim l_{\text{mfp}}$$

$$\alpha_s(T_{\text{RHIC}}) \sim 1$$

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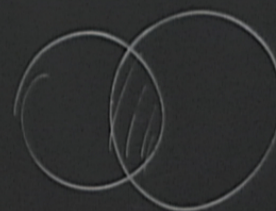
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- in loc

L_V

Evolution

- in local thermodynamic equil.

$$L_{\text{var. in } T} \gg l_{\text{mfp}}$$

Evolution of RHIC-LHC fireball": need thermodynamics and transport (kinetics)

Thermodynamics of QCD

$$\Omega = -T \ln Z_{\text{QCD}} = \Omega(T, \mu_B) = \Omega(T, \mu_B, \Lambda_{\text{QCD}}, m_q, N_c)$$

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Thermodynamics of QCD

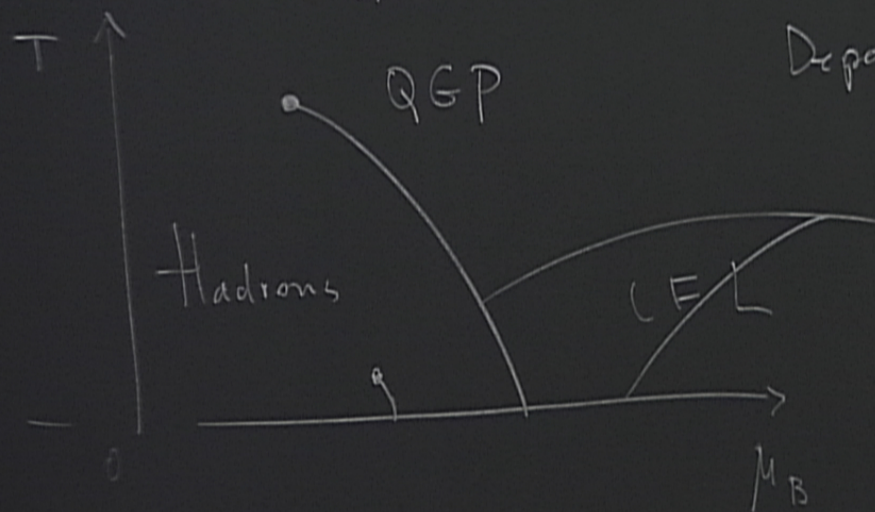
$$\Omega = -T \ln Z_{\text{QCD}} = \Omega(T, \mu_B) = \Omega(T, \mu_B, \Lambda_{\text{QCD}}, m_q, N_c)$$

$$\Omega = \mathcal{E} - TS - \mu Q = \mathcal{F} - \mu Q$$

$$P = - \left. \frac{\partial \Omega}{\partial V} \right|_{T, \mu}$$

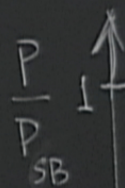
$$S = - \left. \frac{\partial \Omega}{\partial T} \right|_{V, \mu} \text{ etc}$$

Depends on N_c, m_q .



Two important plots: (Lat QCD)

I. Pressure vs T



$$P_{SB} = \frac{8\pi^2}{45} \left(1 + \frac{21}{32} N_f \right) T^4 + O(m^2 T^2) \leftarrow$$

Two important plots: (Lat QCD)

I. Pressure vs T

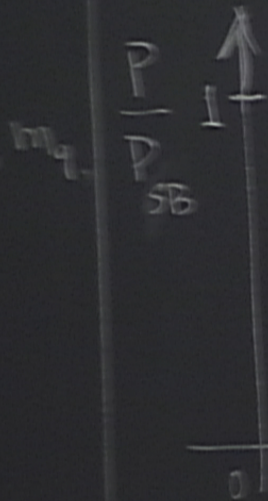
$\frac{P}{P_{SB}}$ ↑

0 → T/T_c

$$P_{SB} = \frac{8\pi^2}{45} \left(1 + \frac{21}{32} N_f \right) T^4 + O(m^2 T^2)^c \leftarrow \text{ideal gas of quarks/gluons}$$

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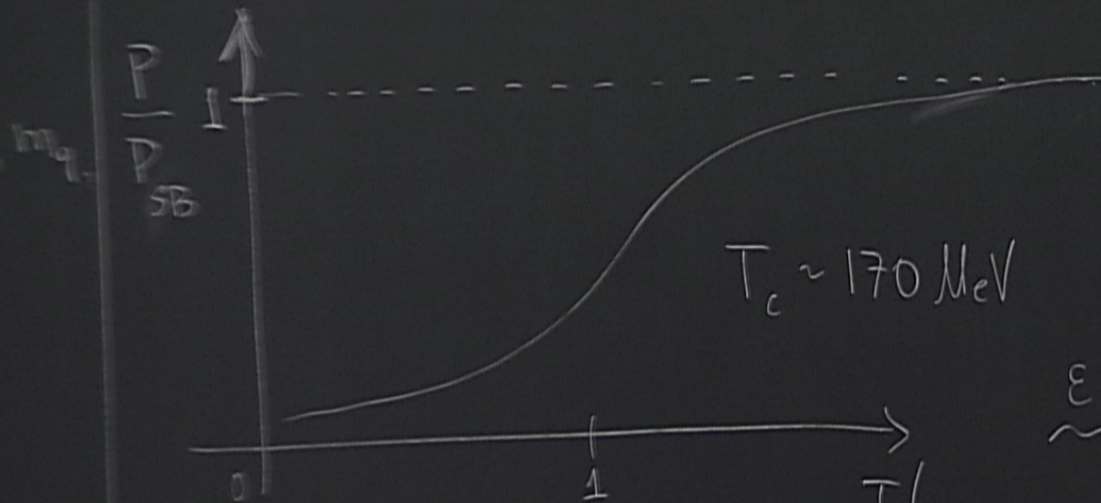


$$\underline{\underline{\epsilon = pc}} \quad (\text{for } mc^2 \ll pc)$$

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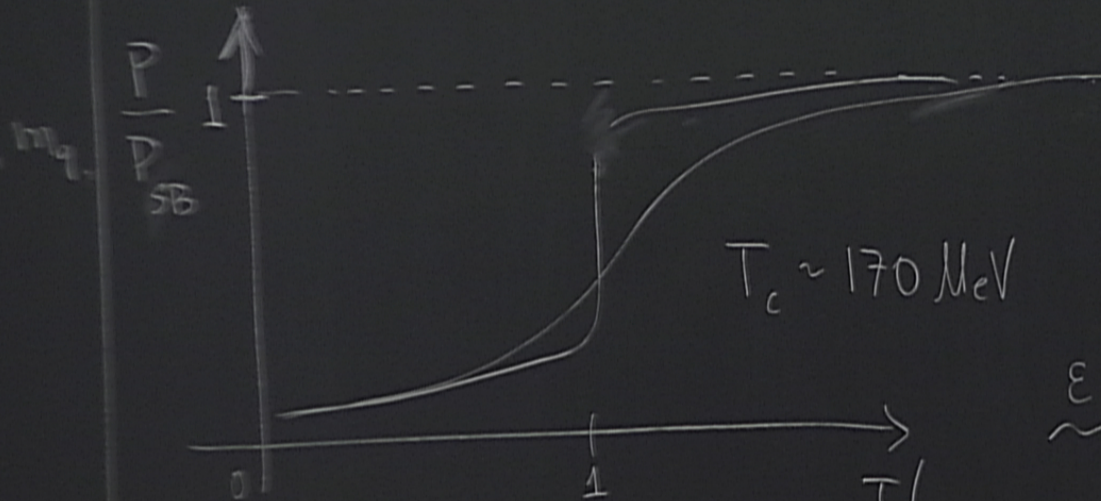


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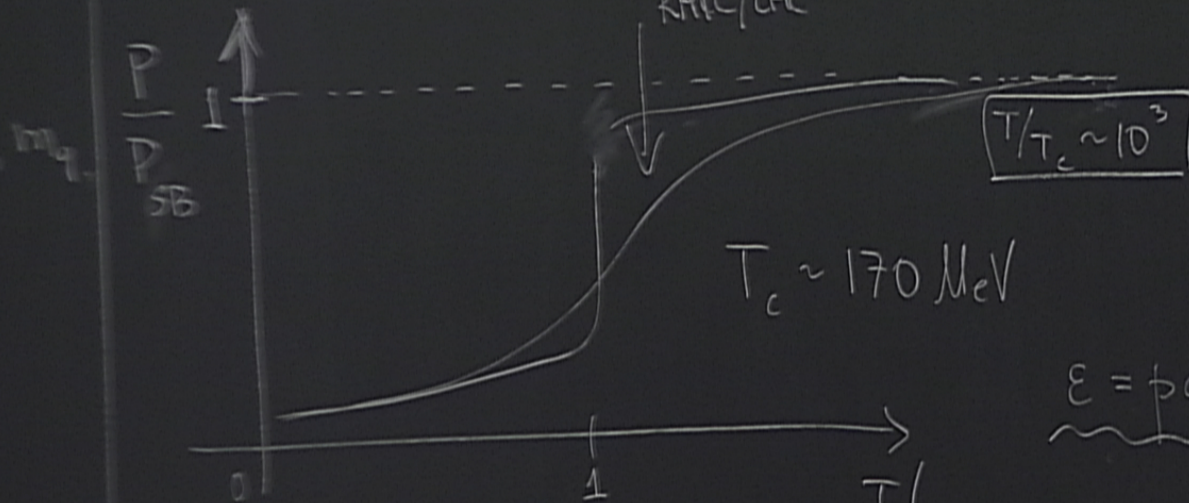
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RHIC/LHC : operate at $\frac{T}{T_c} \sim 2-4$

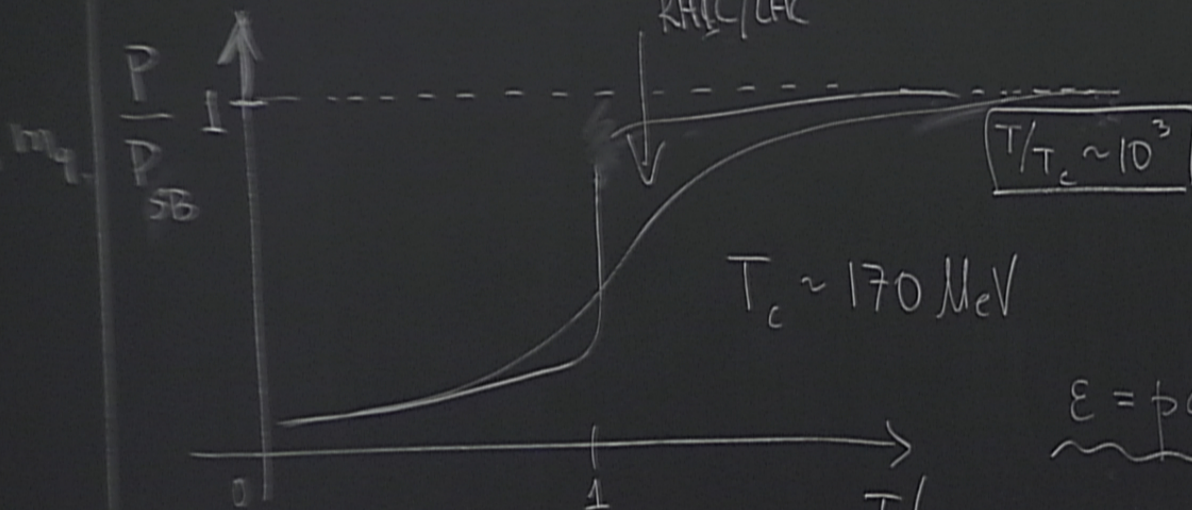
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I. Pressure vs T



$$P/P_{SB} \left(\text{at } T \sim T_{RHIC} \right) \sim 0.75 - 0.8$$

(not far from id)

Does not work for transport (visco
due to strong dep. on coupling)

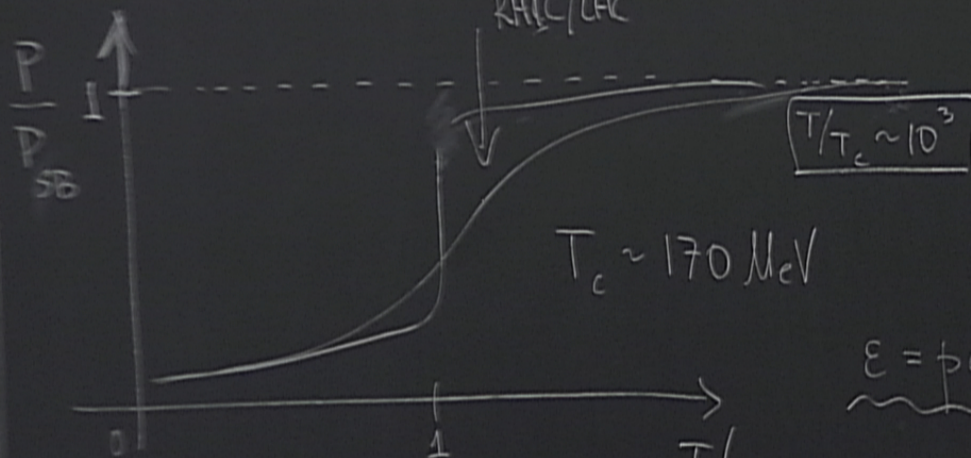
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Two important plots: (Lat QCD)

RHIC/LHC: operate at $\frac{T}{T_c} \sim 2-4$

I. Pressure vs T



$$P/P_{SB} \left(\text{at } T \sim T_{RHIC} \right) \sim 0.75-0.8$$

(not far from ideal gas)

Does not work for transport (viscosity etc)
due to strong dep. on coupling

$$\underline{\underline{\epsilon = pc}} \quad (\text{for } mc^2 \ll pc)$$

$$P_{SB} = \frac{8\pi^2}{45} \left(1 + \frac{21}{32} N_f \right) T^4 + O(m^2 T^2) \leftarrow \text{ideal gas of quarks/gluons}$$

II. Conformal anomaly : $\frac{\epsilon - 3P}{T^4}$ vs T/T_c

$$\langle T_{\mu\nu} \rangle = \begin{pmatrix} \epsilon & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

loc. tr. u^μ

$$T_{\mu\nu} =$$

$T \gg T_c$: ideal gas $\epsilon = 3P$

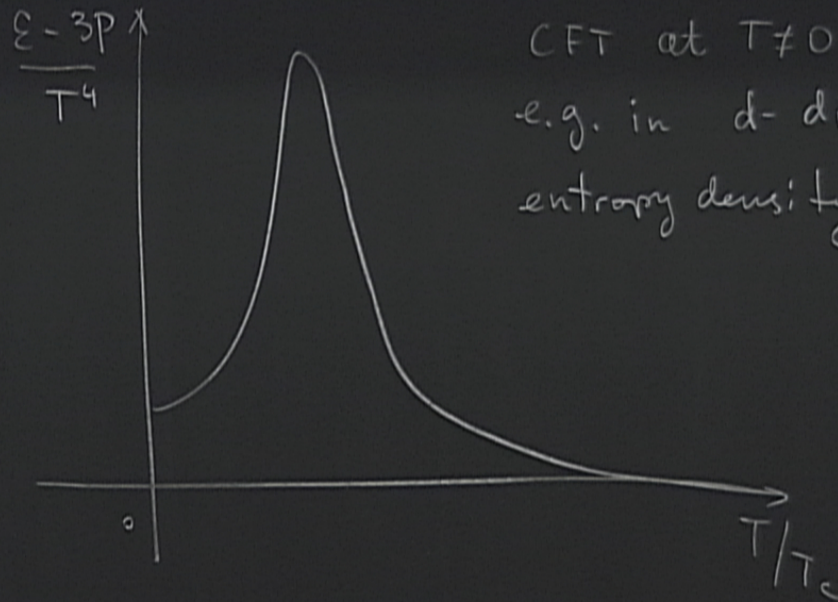
$$\frac{\epsilon - 3P}{T^4}$$



vs T/T_c

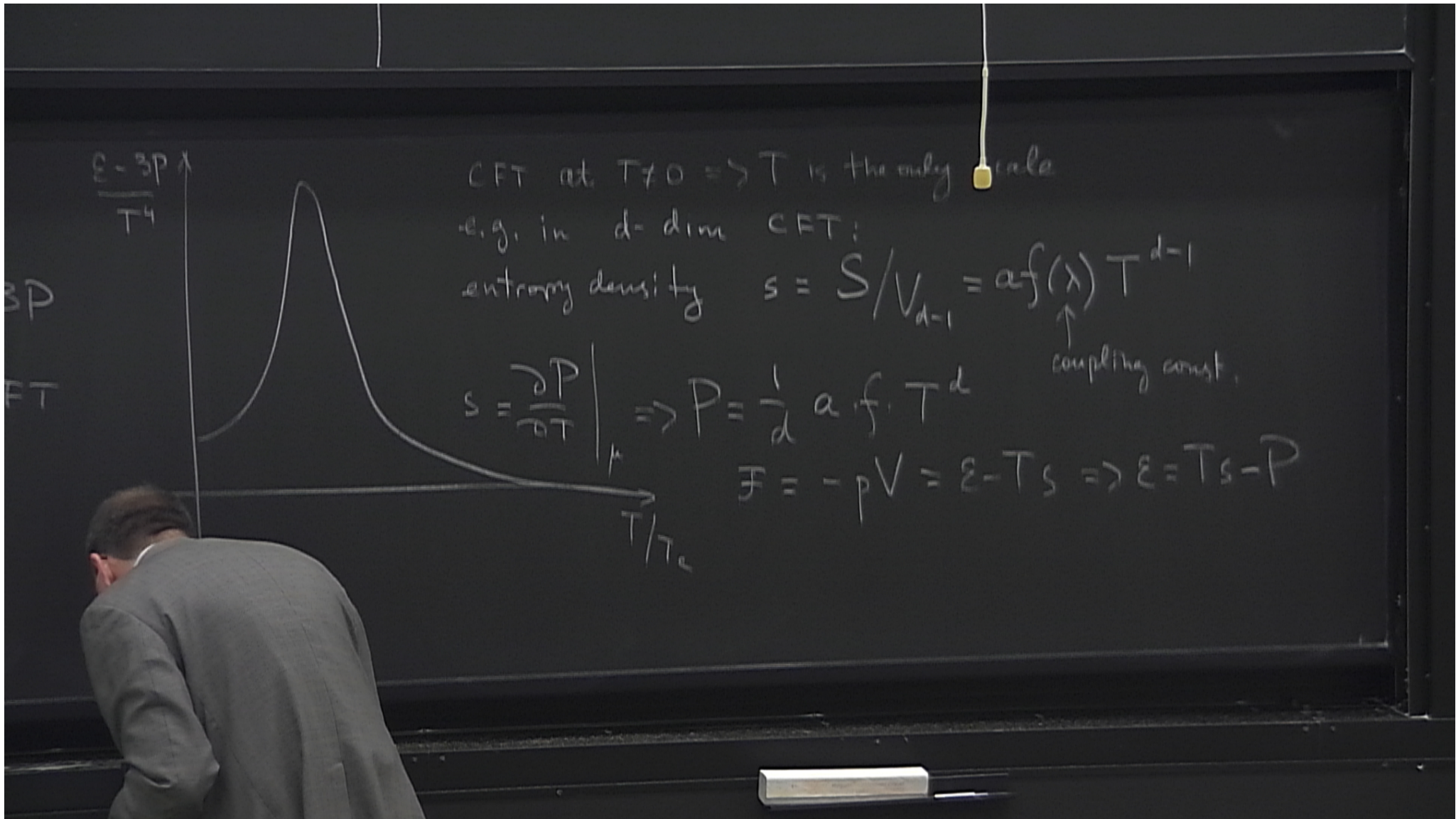
ideal gas $\epsilon = 3P$

also true for CFT
at any coupling.



CFT at $T \neq 0 \Rightarrow T$ is the only scale
e.g. in d -dim CFT:

entropy density $s = S/V_{d-1} = a f(\lambda) T$



CFT at $T \neq 0 \Rightarrow T$ is the only scale

e.g. in d -dim CFT:

entropy density $s = S/V_{d-1} = a f(\lambda) T^{d-1}$
↑
coupling const.

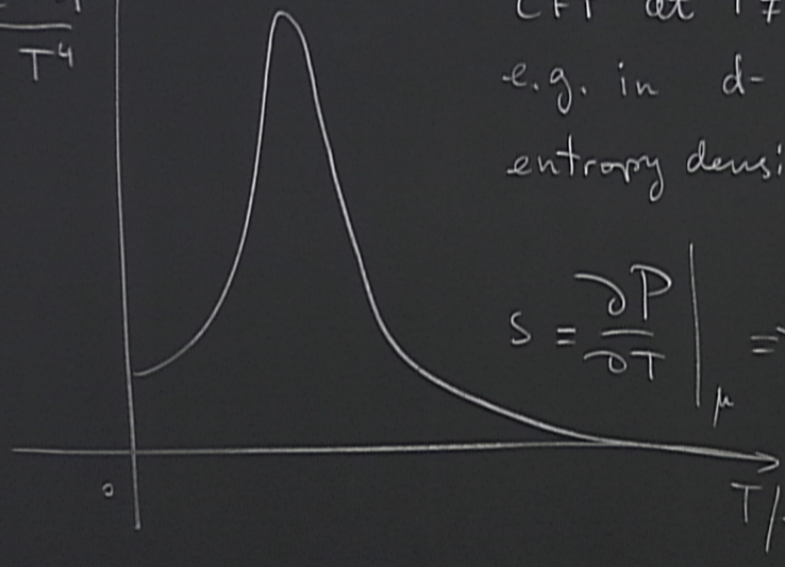
$$s = \left. \frac{\partial P}{\partial T} \right|_{\mu} \Rightarrow P = \frac{1}{d} a f \cdot T^d$$

$$\mathcal{F} = -pV = \mathcal{E} - Ts \Rightarrow \mathcal{E} = Ts - P$$

$$\frac{\varepsilon - 3P}{T^4}$$

BP

FT



CFT at $T \neq 0 \Rightarrow T$ is the only scale

e.g. in d -dim CFT:

entropy density $s = S/V_{d-1} = a f(\lambda) T^{d-1}$

↑
coupling const.

$$s = \left. \frac{\partial P}{\partial T} \right|_{\mu} \Rightarrow P = \frac{1}{d} a \cdot f \cdot T^d$$

$$\mathcal{F} = -pV = \varepsilon - Ts \Rightarrow \varepsilon = Ts - P$$

$$\varepsilon = \frac{d-1}{d} a \cdot f \cdot T^d = (d-1) P$$

anomaly : $\frac{\epsilon - 3P}{T^4} \propto T/T_c$

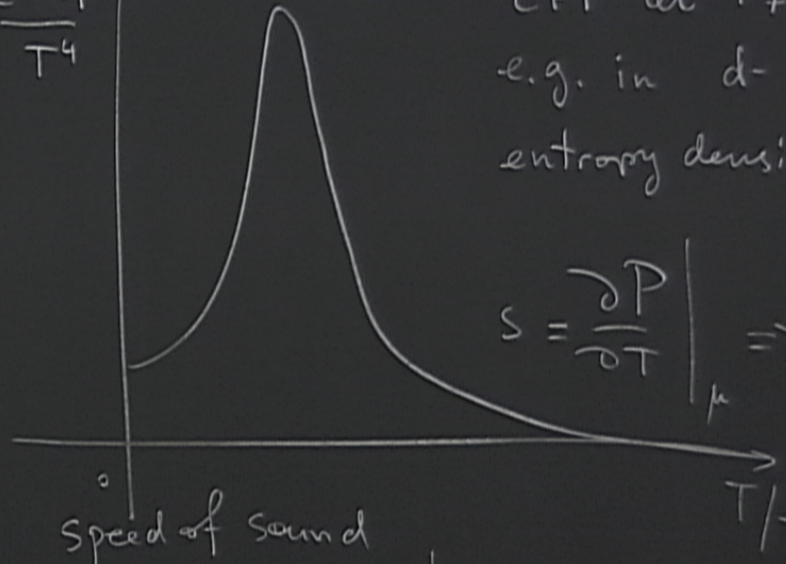
$T \gg T_c$: ideal gas $\epsilon = 3P$

also true for CFT
at any coupling.

For CFT: $T_\mu^\mu = 0$

$\text{tr} \begin{pmatrix} -\epsilon & & \\ & P & \\ & & P & \\ & & & P \end{pmatrix} = 0 \Rightarrow \epsilon = (d-1)P \Rightarrow$
(EOS)

$\frac{\epsilon - 3P}{T^4}$



speed of sound

$v_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{1}{d-1}$

CFT at $T \neq 0 \Rightarrow$
e.g. in d -dim
entropy density s

$s = \left. \frac{\partial P}{\partial T} \right|_\mu \Rightarrow P = \frac{1}{d} \epsilon$

$$\left(\begin{smallmatrix} P \\ P \\ P \end{smallmatrix} \right) = 0 \Rightarrow \varepsilon = (d-1)P \Rightarrow \text{speed of sound} \quad v_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{1}{d-1} \quad \text{with } T_c \quad \varepsilon = \frac{d-1}{d} T_c \mu$$

In theories with internal scale(s), e.g. Λ_{QCD} , the result is more complicated

$$S = a T^{d-1} F\left(\frac{\Lambda}{T}\right)$$

$$F \rightarrow 1 \text{ at } T \rightarrow \infty.$$