Title: 13/14 PSI - Explorations in Particle Theory - Lecture 2

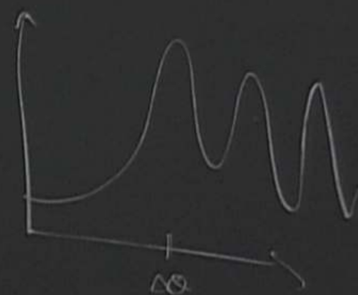
Date: Apr 08, 2014 09:00 AM

URL: http://pirsa.org/14030045

Abstract:

II Particle Abundances & Early Universe Thermodynamics

Ref: The Early Universe by Kolb & Turner Ch. 3, 5



Roadmap

1. Review of Friedmann-Robertson-Walker universe
2. Equilibrium thermo.
3. Abundances away from equilibrium

II Particle Abundances & Early Universe Thermodynamics

Roadmap

1. Review of Friedmann-Robertson-Walker universe
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Ref: The Early Universe by Kolb & Turner Ch. 3, 5

Theory: Lagrangian, spectrum

$\Downarrow$

how much of it is left today

$\Omega_{\chi^0} = \Omega_{DM}$

Things we need:

- conditions in the early universe.
  - → characteristic energy scales, densities of particles,
- rates of particle production, scattering, annihilation for our new theory → plug in conditions
  - ↳ we know how to do!

Clues for early universe:

- cosmic microwave bkd
  - → photons, 2.7 K, 400 /cm^3
- + Hubble expansion.

Universe was HOT & DENSE

- use CMB/photon "bath" as benchmark

1) FRW Universe

Homogeneity + isotropy

shape of "your" space part of space-time

very close

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

$k = 0$

scale factor.

d^3x Minkowski

(use "natural" units $\hbar = c = 1$)

$[E] = [l]^{-1} = [t]^{-1}$

Scale factor: ruler length at time t

Hubble scale: $H \equiv \dfrac{\dot{a}(t)}{a(t)}$ → related to time it takes to double size of universe.

If $k=0$

$H \propto t^{-1}$

large H = short doubling time

$a(t) = ?$

T ... related to a, t?

Energy $\Longleftrightarrow$ spacetime dynamics

(?) $T^{\mu\nu}$ (H, a)

spacetime indices

$$T^{\mu\nu} = \begin{pmatrix} \rho & & 0 & \\ 0 & -p & 0 & 0 \\ & 0 & -p & 0 \\ & 0 & 0 & -p \end{pmatrix}$$

isotropy

Eqn. of state

$p = \omega \rho$ energy density

pressure of gas

a) Relativistic matter: RADIATION

$p = \frac{1}{3}\rho$

b) Non-relativistic matter:

"MATTER"

$p = 0$

$(v=0)$

c) Vacuum energy

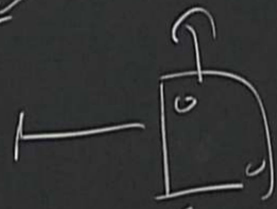
"DARK ENERGY"

$p = -\rho$

Type of "stuff" changes with a differently.

$\rho_{rad} \propto a(t)^{-4}$ ← volume factor change a^{-3}

$\rho_{matter} \propto a(t)^{-3}$ ← ∴ spatial density $\sim a^{-3}$



$\rho_{vac} \propto$ constant ←

λ stretched by expansion

↳ extra a^{-1}

suppr. energy for rad.

$\rho(t_0) = \rho_{rad}(t_0) + \rho_{mat}(t_0) + \rho_{vac}(t_0)$

$\sim a^{-4}$ $\quad \sim a^{-3}$ $\quad \sim$ const.

↑ here now vac-dom universe

∀ times we care about)

universe was rad dominated

Einstein eq: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$\Downarrow$

① $H^2 = \frac{8\pi G}{3}\rho = \left(\frac{\dot a}{a}\right)^2$

② $\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$\rho_{crit} = \frac{3H^2}{8\pi G}$

if $k=0 \Longleftrightarrow \rho_{tot} = \rho_{crit}$

Define $\Omega_i = \frac{\rho_i}{3H^2/8\pi G}$

$= \frac{\rho_i}{\rho_{crit}}$

$k=0 \Longleftrightarrow \sum_i \Omega_i = 1$

Type of "stuff" changes with a differently.

$\rho_{rad} \propto a(t)^{-4}$

$\rho_{matter} \propto a(t)^{-3}$

$\rho_{vac} \propto$ constant

volume factor change a^{-3}

spatial density $\sim a^{-3}$

λ stretched by expansion

extra a^{-1} suppr. energy for rad.

$\rho_{tot} = \rho_{rad}(t_0) + \rho_{mat}(t_0) + \rho_{vac}(t_0)$

$\sim a^{-4}$ $\sim a^{-3}$ $\sim$ const.

here now vac-dom universe

$\forall$ times we care about, universe was rad dominated

Rad. dominated:

$$a(t) \propto \sqrt{t}$$

$$H(t) = \frac{1}{2t} \propto a^{-2}$$

② Equilibrium in the Early Universe

→ use stat mech

implicit def. of equilibrium

$$\text{Eq} = \rightleftarrows \quad \text{rates are same}$$

No change in time.

Δ Hubble expansion violates equilibrium!

Instead, define quasi-steady state.

Consider time scales that are

1) long compared to reactions between particles

AND

2) short compared to H^{-1}

Use Grand Canonical Ensemble.

↳ $f(\vec{p}, \vec{x}, t)$

$\quad\quad\quad\quad\quad\;\;\,$↳ very slowly state.

between

$$f_i^{eq}(E) = \left[e^{(E-\mu_i)/T_i} \underset{\text{fermions}}{\overset{\text{bosons}}{\mp}} 1 \right]^{-1}$$

T_i characterizes the energy i

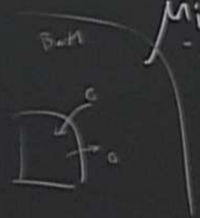
μ_i ... the number of the species i

→ particle # changing rxns are fast!

$E = mc^2$.

↳ $\mu \to n_i - n_{\bar{i}}$

matter-antimatter asymmetry



Define:

i) particle i density

$$E_i = \sqrt{p_i^2 + m_i^2} = \text{function of } p$$

\# of possible configs

$$\boxed{n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(\vec{p})}$$

$$\boxed{\rho_i = g_i \int \frac{d^3p}{(2\pi)^3} E(\vec{p}) f_i(\vec{p})}$$

Assume $\mu/T \ll 1$

$\mu/T \lesssim 10^{-10}$

$$n_i(T) = \begin{cases} \underbrace{\left\{ \begin{matrix} 1 \\ 3/4 \end{matrix} \right\}}_{\text{bos/ferm.}} g_i \dfrac{\zeta(3)}{\pi^2} T^3 & T \gg m_i \\[2ex] g_i \left(\dfrac{m_i T}{2\pi} \right)^{3/2} \underbrace{e^{-m_i/T}}_{\text{exp suppression.}} & T \ll m_i \end{cases}$$

Convenient to define T_γ as "THE" temperature $T = T_\gamma$.

Rad-dominated:

$$\boxed{\rho = \frac{\pi^2}{30} g_* T^4} \qquad g_* = \sum_{i \in \text{bos}} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{i \in \text{ferm}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4$$

relativistic species.

Assume $\mu/T \ll 1$

$\mu/T \lesssim 10^{-10}$

bos/ferm.

$$n_i(T) = \begin{cases} \left\{ 1, \tfrac{3}{4} \right\} g_i \dfrac{\zeta(3)}{\pi^2} T^3 & T \gg m_i \\ g_i \left(\dfrac{m_i T}{2\pi} \right)^{3/2} \underbrace{e^{-m_i/T}}_{\text{exp suppression}} & T \ll m_i \end{cases}$$

$\bar{M}_{Pl} = \dfrac{M_{Pl}}{\sqrt{8\pi}}$

$G = \dfrac{1}{M_{Pl}^2}$

$\rho \sim a^{-4}$ rad.

If g_r is const

↳ don't change # relativistic species.

$$\boxed{a \sim T^{-1}}$$

$$\boxed{H = \frac{1.66\sqrt{g_r}}{M_{Pl}} T^2}$$

Convenient: define T_γ as "THE" temperature $T = T_\gamma$.

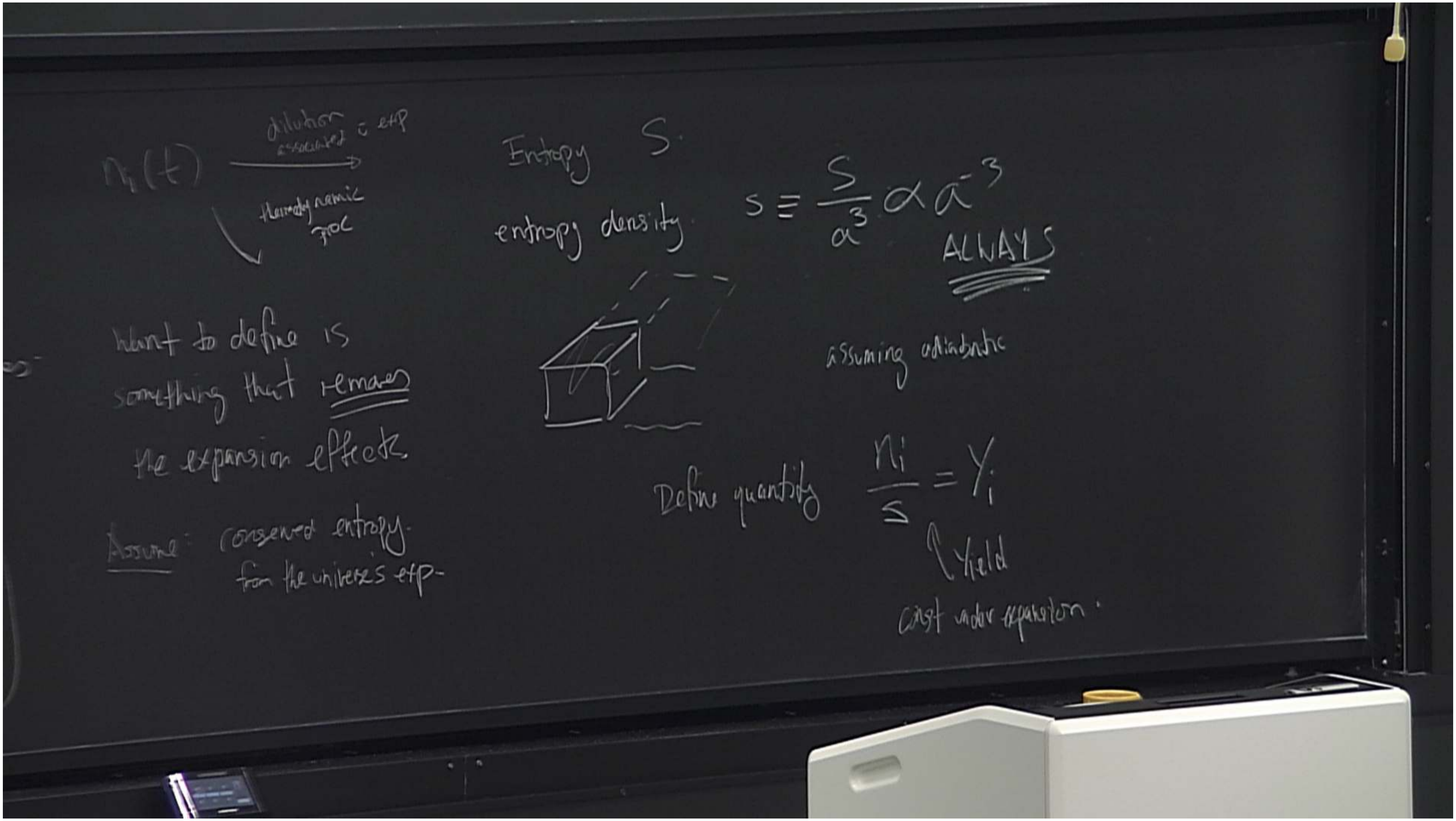
Rad-dominated

$$\boxed{\rho = \frac{\pi^2}{30} g_r T^4}$$

$$g_r = \sum_{i \in \text{bos}} g_i \left(\frac{T_i}{T} \right)^4 + \sum_{i \in \text{ferm}} \frac{7}{8} g_i \left(\frac{T_i}{T} \right)^4$$

relativistic species

$n_i(t)$



n_i(t)
dilution
thermodynamic proc
Want to define is
something that remains
the expansion effects
Assume: conserved entropy.
from the universe's exp.
Entropy S.
entropy density.
s ≡ S/a^3 ∝ a^-3
ALWAYS
assuming adiabatic
Define quantity n_i/s = Y_i
Yield
const under expansion.