

Title: 13/14 PSI - Explorations in Particle Theory - Lecture 2

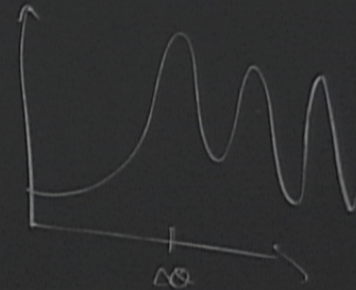
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Abstract:

II Particle Abundances
of Early Universe
Thermodynamics

Ref: The Early Universe
by Kolb & Turner
Ch. 3, 5



Roadmap

- ① Review of Friedmann-Robertson-Walker universe
- ② Equilibrium thermo.
- ③ Abundances away from equilibrium

II Particle Abundances & Early Universe Thermodynamics

Roadmap

- 1 Review of Friedmann-Robertson-Walker universe
- 2 Equilibrium thermo.
- 3 Abundances away from equilibrium

Ref: The Early Universe
by Kolb & Turner
Ch. 3, 5

Theory: Lagrangian,
spectrum

⇓
how much of it is
left today
 $\Omega_{\chi^2} = \Omega_{DM}$

Things we need:

- conditions in the early universe.
→ characteristic energy scales, densities of particles.
- rates of particle production, scattering, annihilation for our new theory → plug in conditions
↳ we know how to do!

Clues for early universe:

- cosmic microwave bkd
→ photons, 2.7 K, $400/\text{cm}^3$
- + Hubble expansion

Universe was HOT & DENSE
• use CMB / photon "bath" as benchmark

1) FRW Universe

Homogeneity + isotropy

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

↑ scale factor

shape of "your" space-time

↓ $d\Omega^2$ Minkowski

very close
 $k = 0$

(use "natural" units: $\hbar = c = 1$)

$$[E] = [L]^{-1} = [t]^{-1}$$

Scale factor: ruler length at time t

Hubble scale: $H \equiv \frac{\dot{a}(t)}{a(t)} \rightarrow$ related to time it takes to double size of universe.

If $k=0$

$$H \propto t^{-1}$$

large t = short doubling time

$$a(t) = ?$$

T ... related to a, t ?

Energy \leftrightarrow spacetime dynamics

$$\begin{pmatrix} ? \\ T_{\mu\nu} \end{pmatrix}$$

$$(H/a)$$

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

↑ isotropy

Egn. of state

$$P = w \rho$$

↑ energy density

pressure of gas

a) Relativistic matter: RADIATION

$$P = \frac{1}{3} \rho$$

b) Non-relativistic matter

"MATTER"

$$P = 0 \quad (v=0)$$

c) Vacuum energy

"DARK ENERGY"

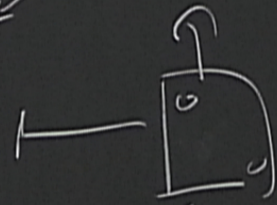
$$P = -\rho$$

Type of 'stuff' changes with a differently.

$$\rho_{\text{rad}} \propto a(t)^{-4}$$

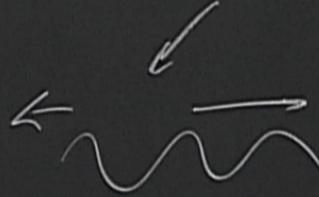
Volume factor change a^{-3}

$$\rho_{\text{matter}} \propto a(t)^{-3}$$



spatial density $\propto a^{-3}$

$$\rho_{\text{vac}} \propto \text{constant}$$



λ stretched by expansion

$$\rho(t) = \rho_{\text{rad}}(t) + \rho_{\text{mat}}(t) + \rho_{\text{vac}}(t)$$

$\sim a^{-4}$

$\sim a^{-3}$

$\sim \text{const.}$

\downarrow extra a^{-1}
suppr. energy for rad.

X times we care about

universe was rad dominated

\uparrow here now
vac-dom universe

Einstein eq: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

\Downarrow

$$\textcircled{1} H^2 = \frac{8\pi G}{3} \rho \approx \left(\frac{\dot{a}}{a}\right)^2$$

$$\textcircled{2} \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$$

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}$$

$$\text{if } k=0 \iff \rho_{\text{tot}} = \rho_{\text{crit}}$$

Define $\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}}$
 $= \frac{\rho_i}{3H^2/8\pi G}$

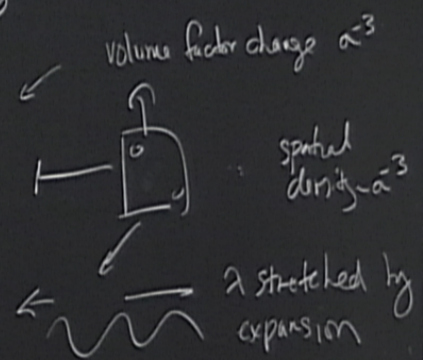
$$k=0 \iff \sum_i \Omega_i = 1$$

Type of "stuff" changes with a differently.

$\rho_{\text{rad}} \propto a(t)^{-4}$

$\rho_{\text{matter}} \propto a(t)^{-3}$

$\rho_{\text{vac}} \propto \text{constant}$



$$\rho_{\text{tot}} = \rho_{\text{rad}}(t) + \rho_{\text{mat}}(t) + \rho_{\text{vac}}(t)$$

$\sim a^{-4}$ $\sim a^{-3}$ $\sim \text{const.}$

X times we care about universe was radiation dominated

↑ here now
 vdw-der universe
 vector \dot{a}
 suppr. energy for rad.

Rad. dominated:

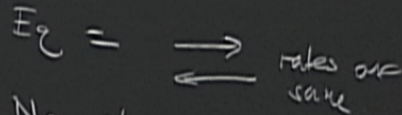
$$a(t) \propto \sqrt{t}$$

$$H(t) = \frac{1}{2t} \propto a^{-2}$$

② Equilibrium in the
Early Universe

→ use stat mech

implicit def. of equilibrium



No change \bar{u} time.

* Hubble expansion violates
equilibrium!

Instead, define quasi-steady state.

Consider time scales that are

1) long compared to reactions between
particles

AND

2) short compared to H^{-1}

Use Grand Canonical Ensemble.

$$\hookrightarrow f(\vec{p}, \vec{x}, t) \quad \leftarrow \text{very slowly}$$

$$f_i^{eq}(E) = \left[e^{(E-\mu_i)/T_i} \pm 1 \right]^{-1}$$

bosons
↑
fermions

Define

particle i density

$$E_i = \sqrt{p_i^2 + m_i^2} = \text{function of } p$$

of possible states

$$n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(\vec{p})$$

$$Q_i = g_i \int \frac{d^3p}{(2\pi)^3} E(\vec{p}) f_i(\vec{p})$$

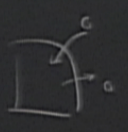
T_i characterizes the energy i

μ_i ... the number of the species i

→ particle # changing rxns are fast!

$$E = mc^2$$

↳ $\mu \rightarrow n_i - n_{\bar{i}}$
matter-antimatter asymmetry



Assume $M/T \ll 1$

$$M/T \lesssim 10^{10}$$

$$n_i(T) = \begin{cases} \left\{ \begin{matrix} 1 \\ -1/4 \end{matrix} \right\} g_i \frac{\zeta(s)}{\pi^2} T^{-3} & T \gg m_i; \\ g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} & T \ll m_i; \end{cases}$$

48 suppression

Convenient: define T_d as the temperature $T = T_d$.
 Rad-dominated:

$$\rho = \frac{\pi^2}{30} g_r T^4$$

$$g_r = \sum_{\text{le bos}} g_i \left(\frac{T_i}{T} \right)^4 + \sum_{\text{ferm}} \frac{7}{8} g_i \left(\frac{T_i}{T} \right)^4$$

relative spectra

Assume $M/T \ll 1$

$$M/T \approx 10^{10}$$

$$n_i(t) = \left\{ \begin{array}{l} \left\{ \frac{1}{s^{3/4}} \right\} g_i \frac{\zeta(3)}{\pi^2} T^3 \quad T \gg m_i \\ g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} \quad T \ll m_i \end{array} \right.$$

no/ferm
 $\zeta(3)$
 $T \gg m_i$
 $T \ll m_i$
 exp suppression

$$g = \frac{1}{M_{pl}^2}$$

$\rho \sim a^{-4}$ rad.
 If g_r is const
 \hookrightarrow don't change # relativistic species.

$n_i(t)$

Convenient: defining
 Rad-dominated

T_r as "THE" temperature $T = T_r$

$$\rho = \frac{\pi^2}{30} g_r T^4$$

$$g_r = \sum_{\text{fer}} g_i \left(\frac{T_i}{T} \right)^4 + \sum_{\text{rel}} \frac{7}{8} g_i \left(\frac{T_i}{T} \right)^4 \leftarrow \text{relativistic species}$$

$$a \sim T^{-1}$$

$$H = \frac{1.66 \sqrt{g_r} T^2}{M_{pl}}$$

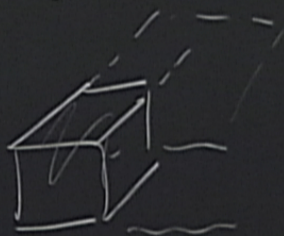
$n_i(t)$ $\xrightarrow{\text{dilution associated w/ exp}}$
 \searrow thermodynamic ρ

Entropy S
 entropy density

$$s \equiv \frac{S}{\alpha^3} \propto \alpha^{-3}$$

ALWAYS

want to define is something that remains the expansion effects



assuming adiabatic

Assume: conserved entropy from the universe's exp.

Define quantity

$$\frac{n_i}{s} = Y_i$$

\uparrow yield
 const under expansion