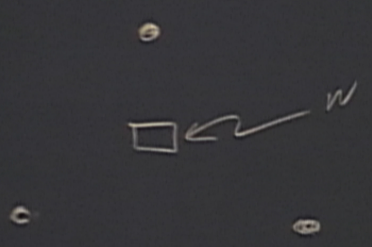
Title: 13/14 PSI - Explorations in Quantum Information - Lecture 8

Date: Mar 26, 2014 09:00 AM

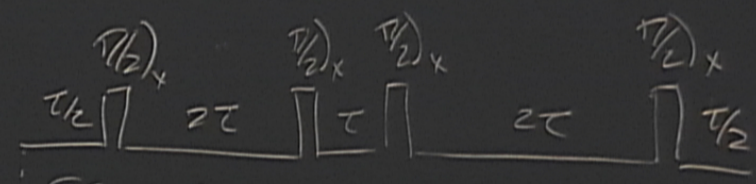
URL: http://pirsa.org/14030038

Abstract:



$$\sigma_+\sigma_- + \sigma_-\sigma_+ \quad \| \quad \sigma_+\sigma_+ + \sigma_-\sigma_-$$

$$\sigma_z\sigma_z$$



$\sigma_z\sigma_z$ | $\sigma_z\sigma_z$ $\sigma_y\sigma_y$ $\sigma_z\sigma_z$ $\sigma_y\sigma_y$ $\sigma_z\sigma_z$

$$\overline{\mathcal{H}}_{d/p} \propto \frac{1}{6\tau}\Big(6\tau\,\sigma\cdot\sigma - 3\big(2\tau\,\sigma_z\sigma_z + 4\tau\,\sigma_y\sigma_y\big)\Big)$$

$$\sigma_x\sigma_x + \sigma_y\sigma_y + \cancel{\sigma_z}\sigma_z - \cancel{\sigma_z}\sigma_z - 2\,\sigma_y\sigma_y$$

$$\sigma_x\sigma_x - \sigma_y\sigma_y$$

$$\sigma_+\sigma_- + \sigma_-\sigma_+ \;\|\; \sigma_+\sigma_+ + \sigma_-\sigma_-$$

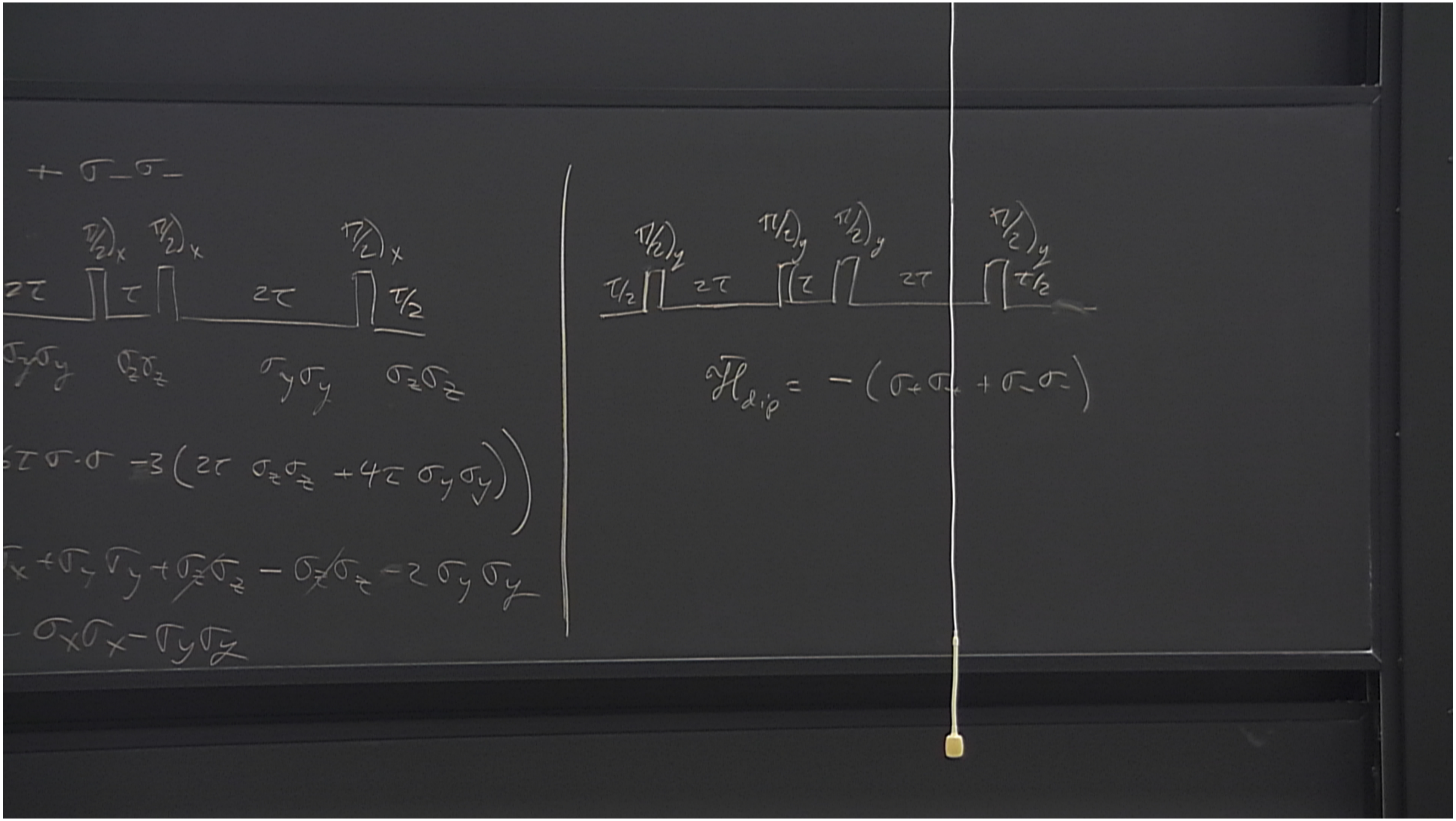
$$\sigma_z\sigma_z$$

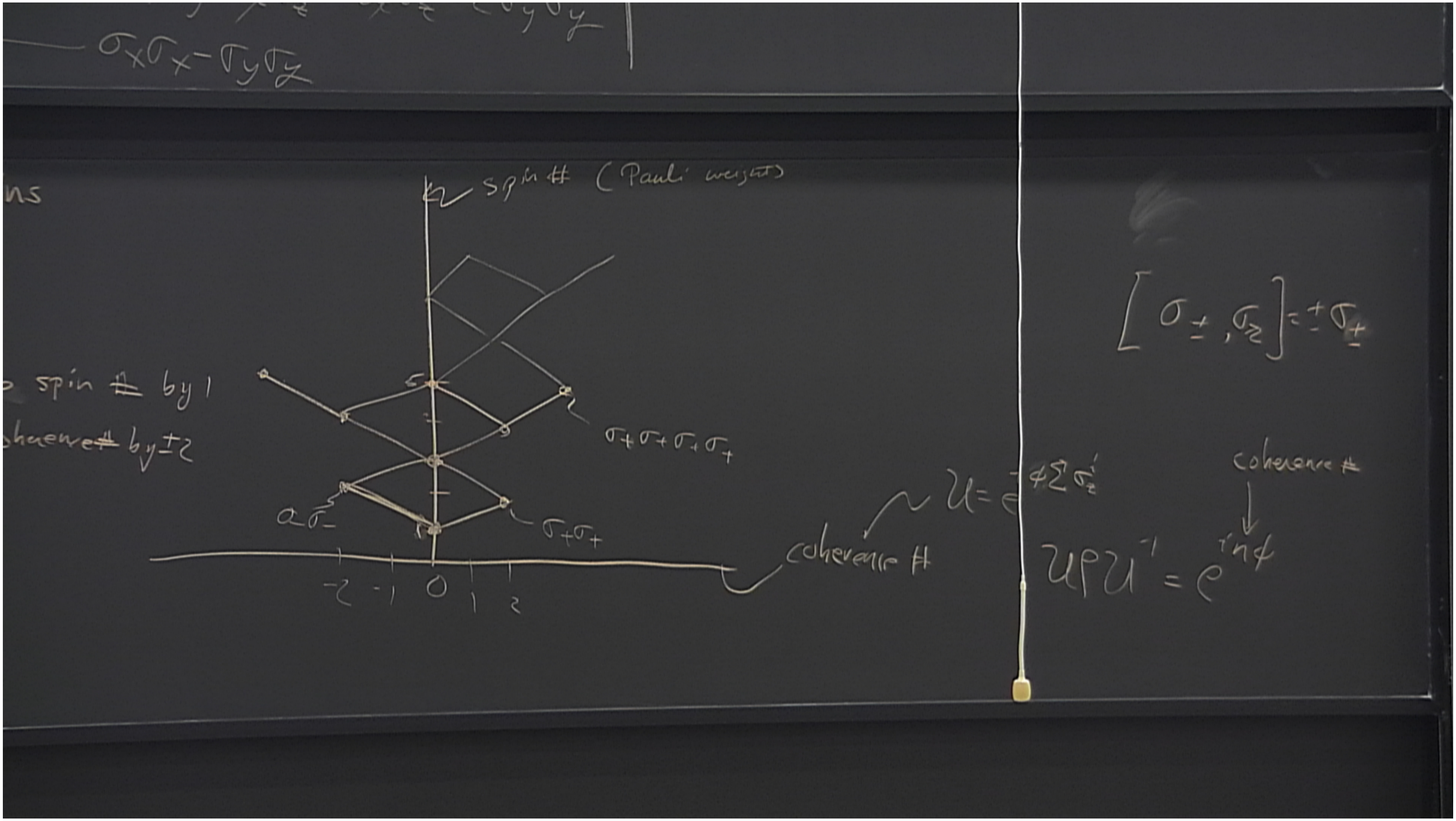
	$(\pi/2)_x$		$(\pi/2)_x$		$(\pi/2)_x$		$(\pi/2)_x$	
	$\tau/2$		2τ		τ		2τ	$\tau/2$
$\sigma_z\sigma_z$	$\sigma_z\sigma_z$		$\sigma_y\sigma_y$		$\sigma_z\sigma_z$		$\sigma_y\sigma_y$	$\sigma_z\sigma_z$

$$\overline{\mathcal{H}}_{dip} \propto \frac{1}{6\tau}\left(6\tau\,\sigma\cdot\sigma - 3\left(2\tau\,\sigma_z\sigma_z + 4\tau\,\sigma_y\sigma_y\right)\right)$$

$$\sigma_x\sigma_x + \sigma_y\sigma_y + \cancel{\sigma_z\sigma_z} - \cancel{\sigma_z\sigma_z} - 2\,\sigma_y\sigma_y$$

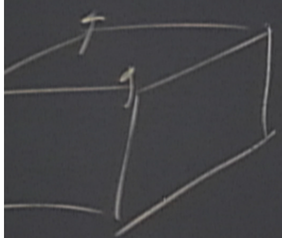
$$\sigma_x\sigma_x - \sigma_y\sigma_y$$


$+\ \sigma_-\sigma_-$
$\pi/2)_x$ $\pi/2)_x$ $\pi/2)_x$
2τ τ 2τ $\tau/2$
$\sigma_z\sigma_y$ $\sigma_z\sigma_z$ $\sigma_y\sigma_y$ $\sigma_z\sigma_z$
$6\tau\,\sigma\cdot\sigma - 3\left(2\tau\ \sigma_z\sigma_z + 4\tau\ \sigma_y\sigma_y\right)\Big)$
$\sigma_x + \sigma_y\sigma_y + \cancel{\sigma_z}\sigma_z - \cancel{\sigma_z}\sigma_z - 2\,\sigma_y\sigma_y$
$- \sigma_x\sigma_x - \sigma_y\sigma_y$
$\pi/2)_y$ $\pi/2)_y$ $\pi/2)_y$ $\pi/2)_y$
$\tau/2$ 2τ τ 2τ $\tau/2$
$\tilde{\mathcal{H}}_{dip} = -\left(\sigma_+\sigma_+ + \sigma_-\sigma_-\right)$


$\sigma_x\sigma_x - \sigma_y\sigma_y$
spin # (Pauli weights)
spin # by 1
coherence # by ±2
$\sigma_+\sigma_+\sigma_+\sigma_+$
$\sigma_+\sigma_+$
$a\sigma_-$
-2 -1 0 1 2
coherence #
$[\sigma_\pm, \sigma_z] = \pm\sigma_\pm$
coherence #
$U = e^{-i\phi\sum \sigma_z^i}$
$U\rho U^{-1} = e^{in\phi}$

$\sigma_x\sigma_x - \sigma_y\sigma_y$

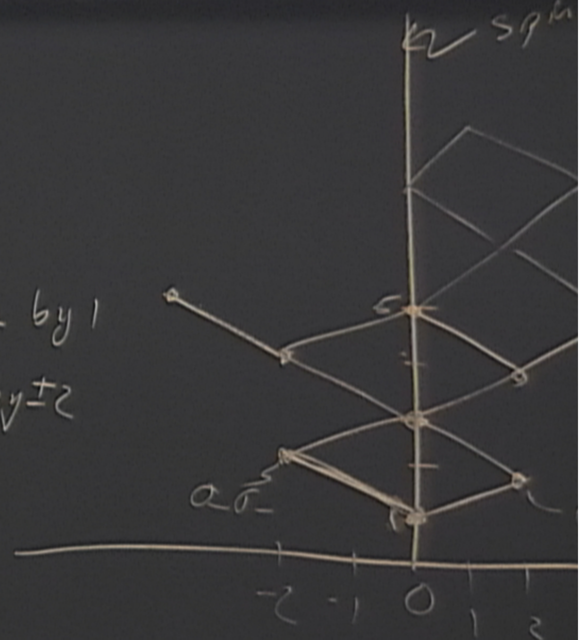
Coherence time vs "#" of spins

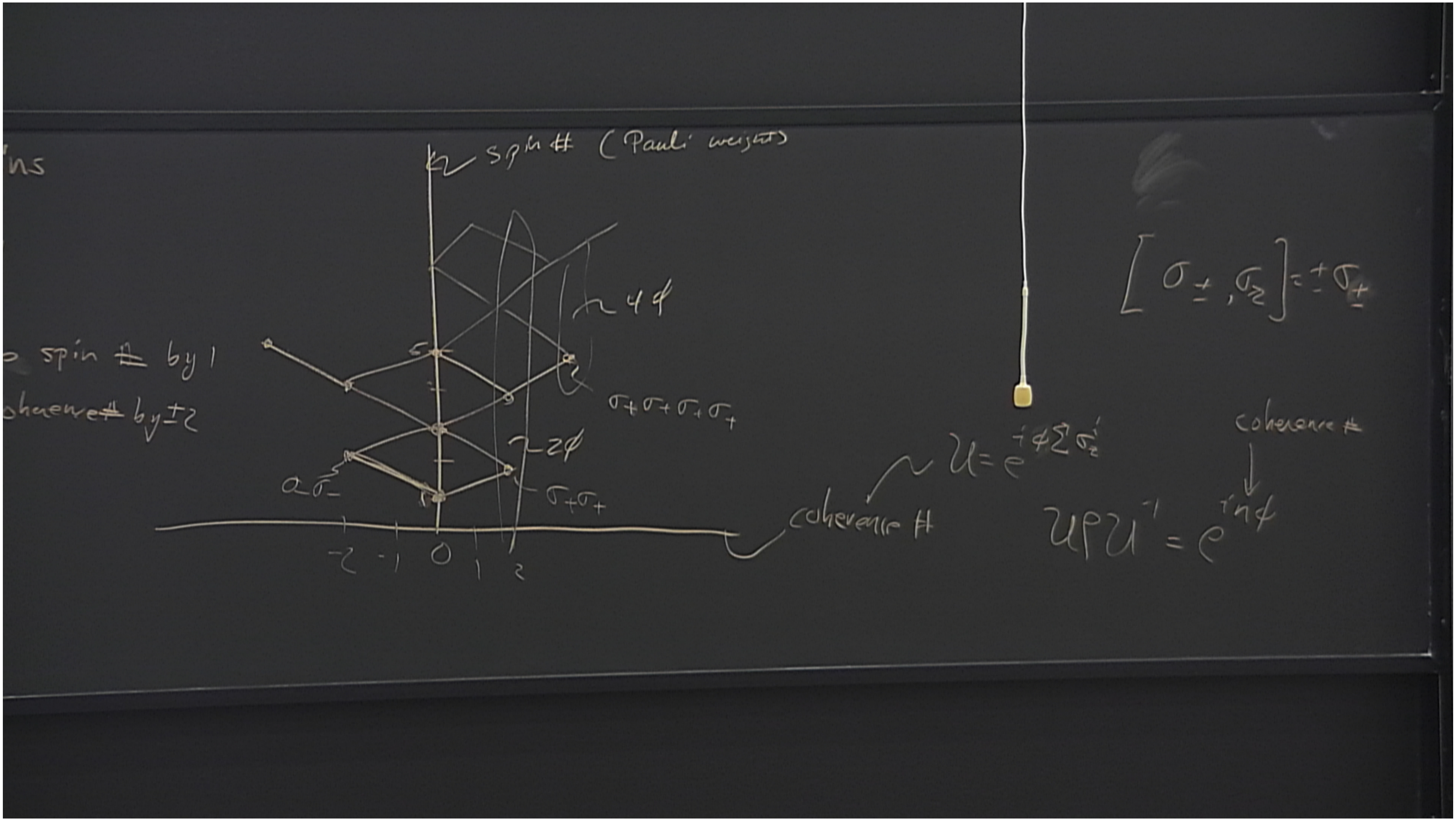


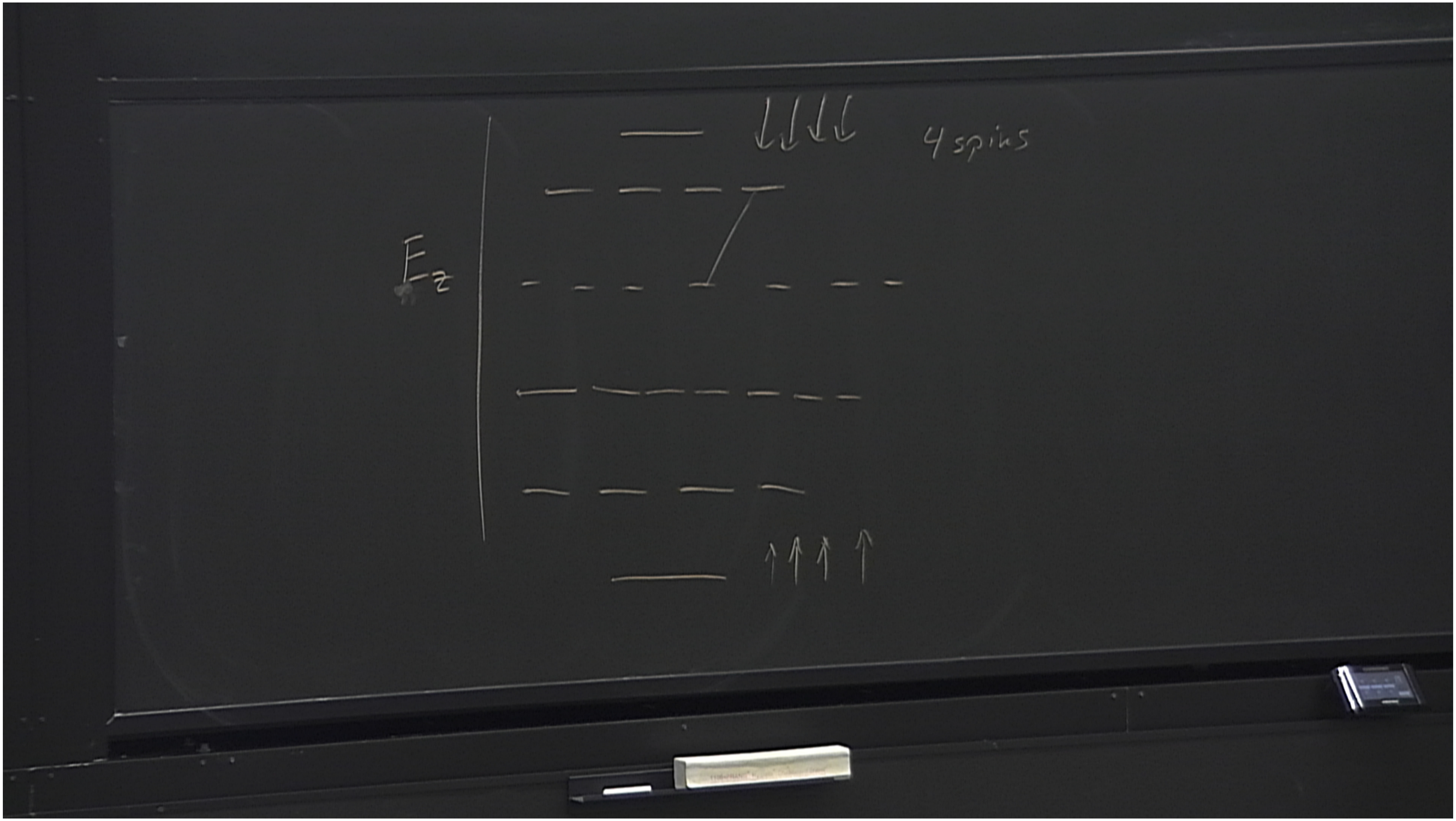
$$\rho_{og} = \mathbb{1}\otimes\mathbb{1}\otimes\cdots\,\sigma_z^{i}\otimes\cdots\,\mathbb{1}$$

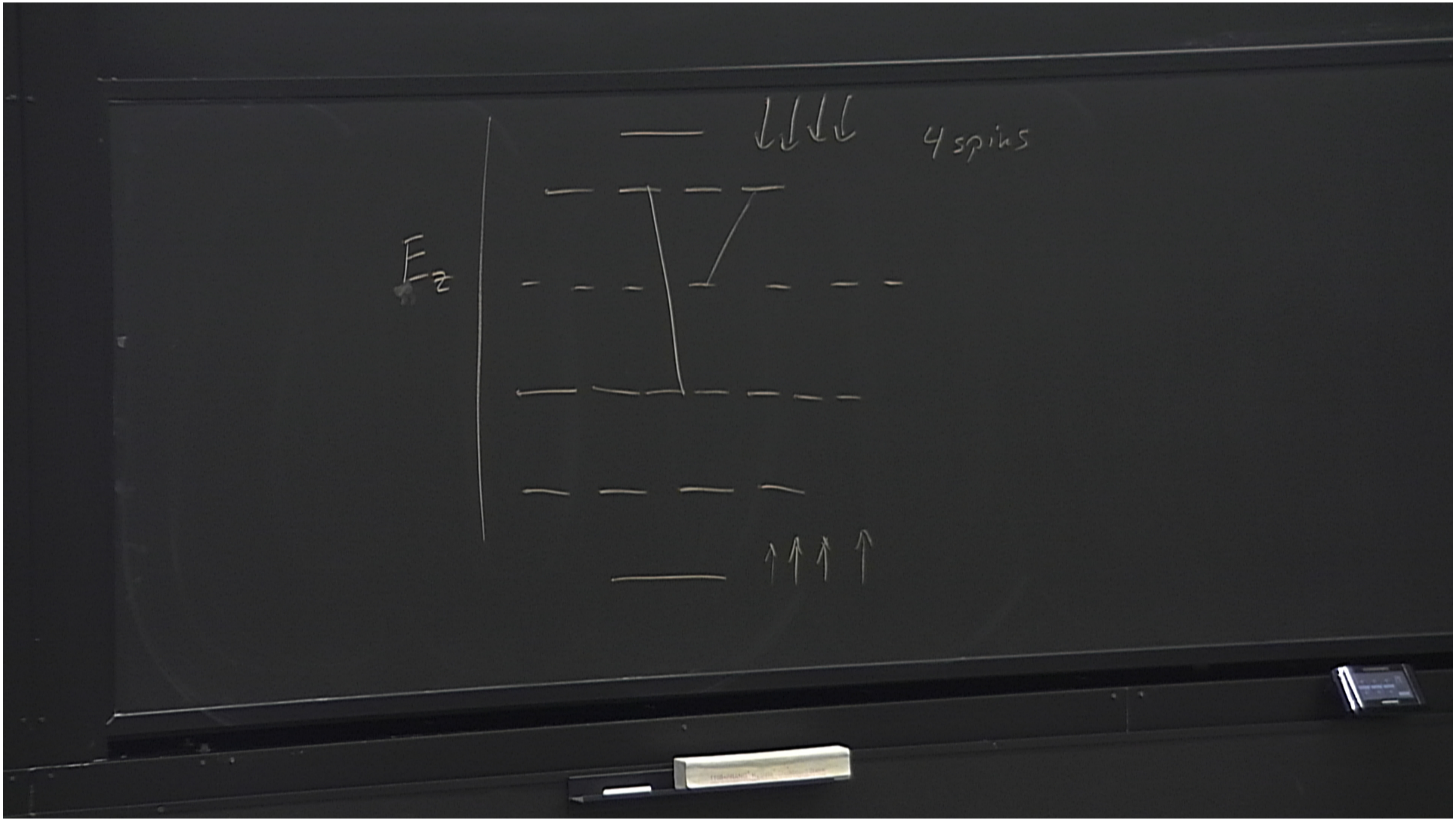
$\sigma_+\sigma_+ + \sigma_-\sigma_-$ $\begin{cases} \text{~~changes~~ spin \# by 1} \\ \text{changes coherence\# by } \pm 2 \end{cases}$

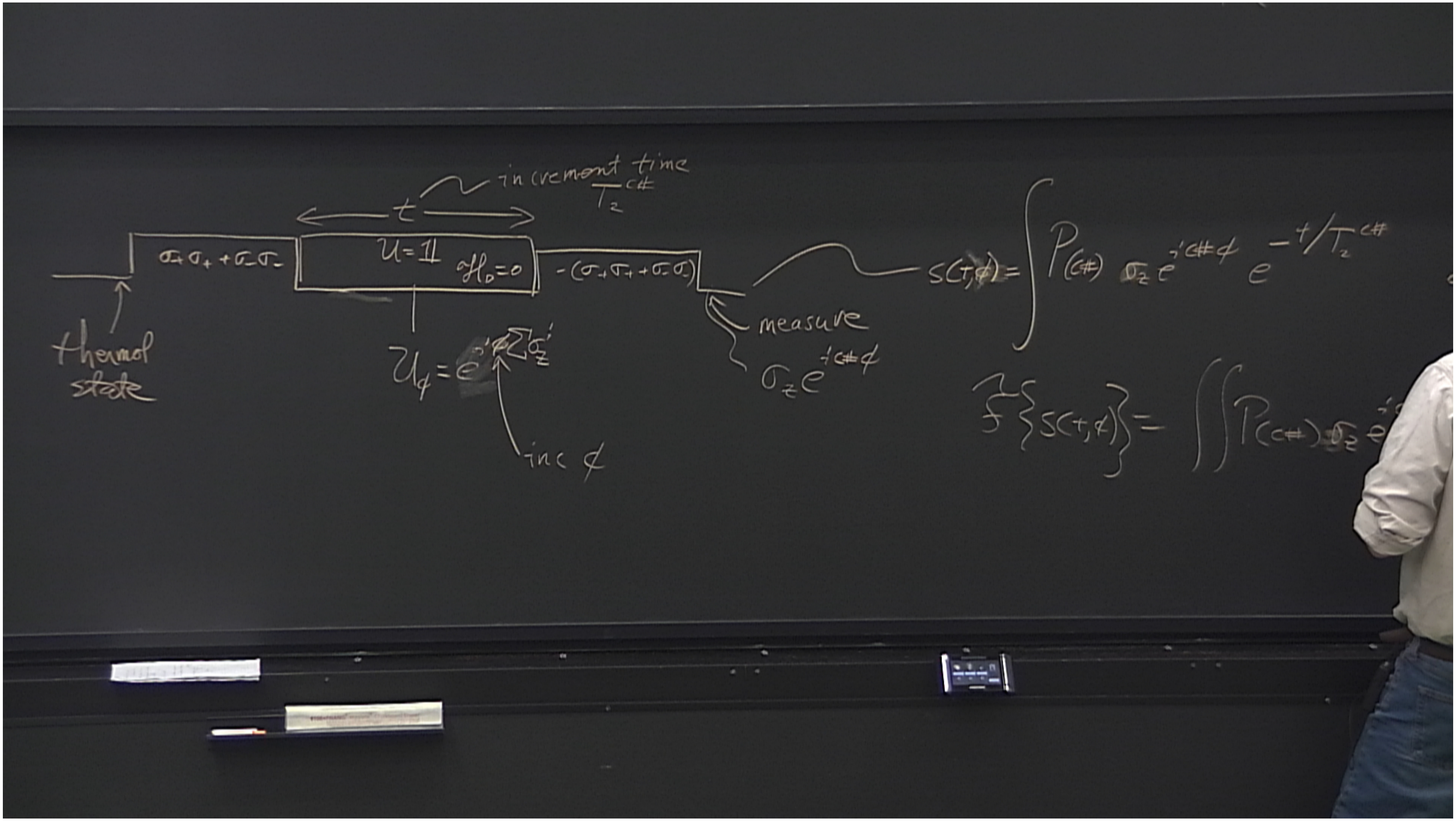
coherence# < spin #

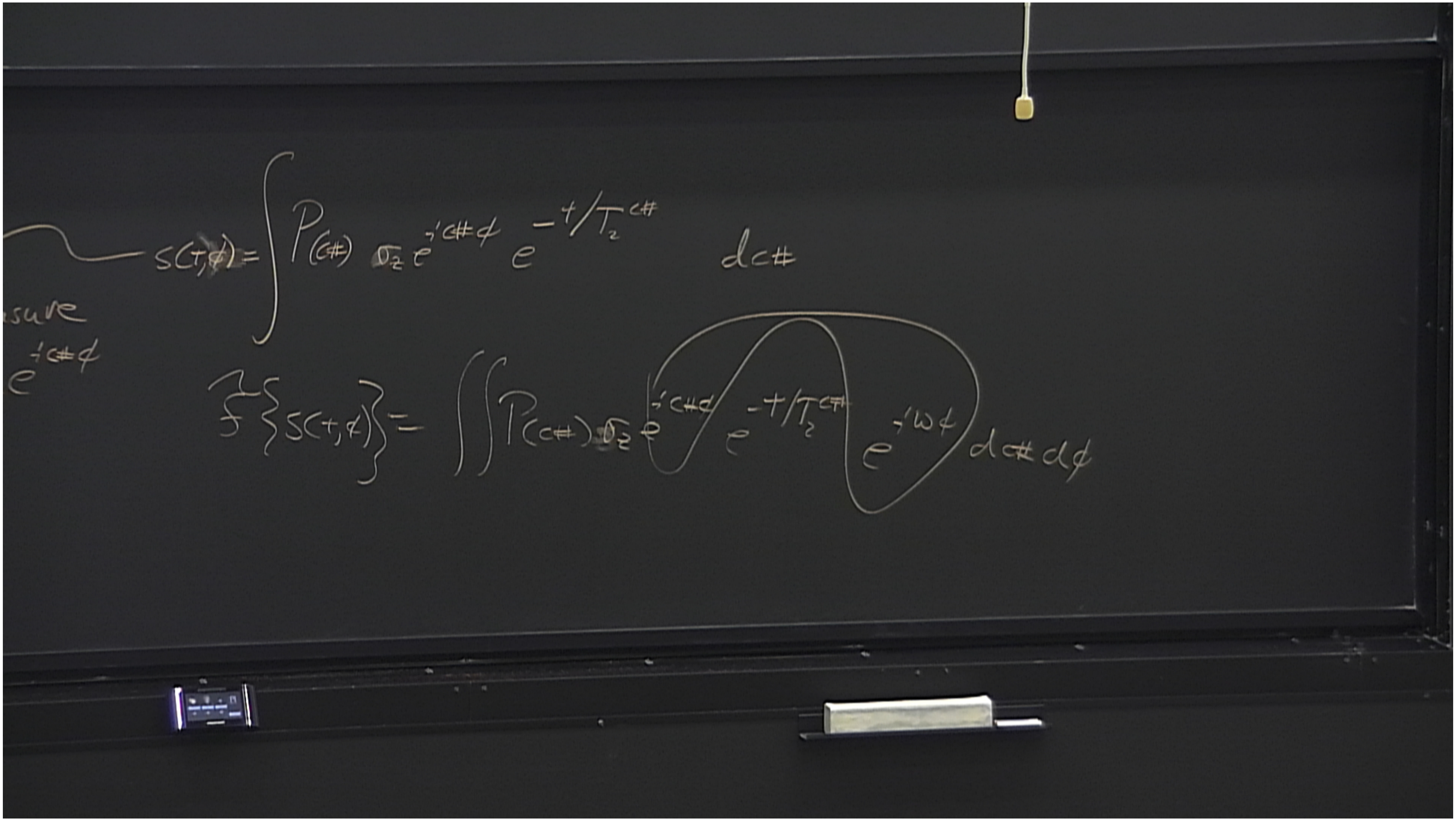


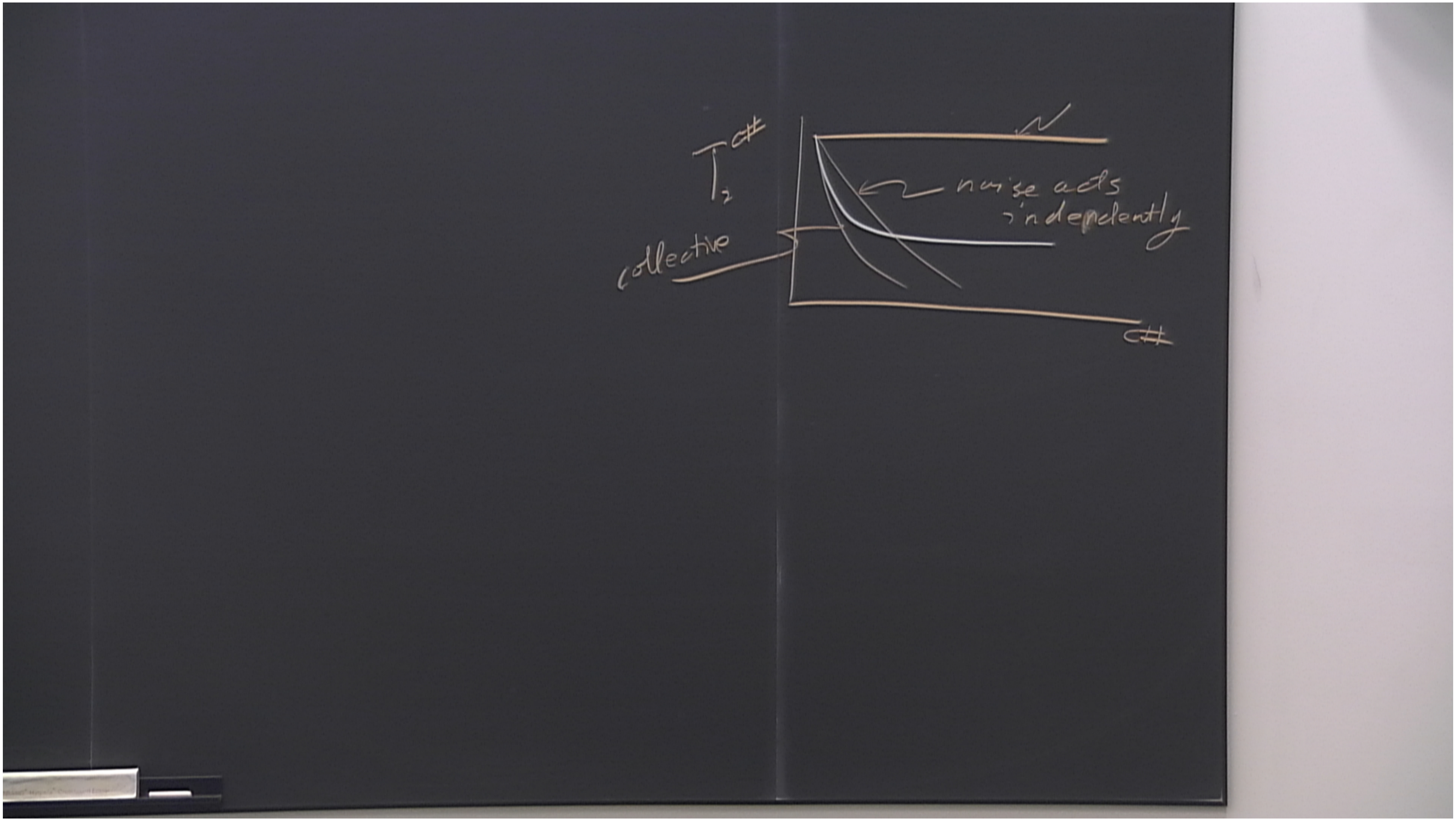

spin # (Pauli weights)
~4φ
spin # by 1
coherence # by ±2
σ+σ+σ+σ+
~2φ
σ−
σ+σ+
coherence #
-2 -1 0 1 2
[σ±, σz] = ±σ±
coherence #
U = e^{-iφΣσz^i}
UρU^{-1} = e^{-inφ}


4 spins
E_z


4 spins
E_z


increment time
$T_2^{c\#}$
t
$\sigma_+\sigma_+ + \sigma_-\sigma_-$
$U = \mathbb{1}$
$\mathcal{H}_D = 0$
$-(\sigma_+\sigma_+ + \sigma_-\sigma_-)$
thermal state
$U_\phi = e^{i\phi \sum \sigma_z^i}$
inc ϕ
measure
$\sigma_z e^{i c\# \phi}$
$S(t,\phi) = \int P_{(c\#)}\, \sigma_z\, e^{i c\# \phi}\, e^{-t/T_2^{c\#}}$
$\mathcal{F}\{S(t,\phi)\} = \iint P_{(c\#)}\, \sigma_z\, e^{i}$


sure
$e^{-ic\#\phi}$
$s(t,\phi) = \int P(c\#)\, \sigma_z\, e^{-ic\#\phi}\, e^{-t/T_2^{c\#}}\, dc\#$
$\mathcal{F}\{s(t,\phi)\} = \iint P(c\#)\, \sigma_z\, e^{-ic\#\phi}\, e^{-t/T_2^{c\#}}\, e^{-i\omega\phi}\, dc\#\, d\phi$



$T_2^{c\#}$
noise acts
independently
collective
c#

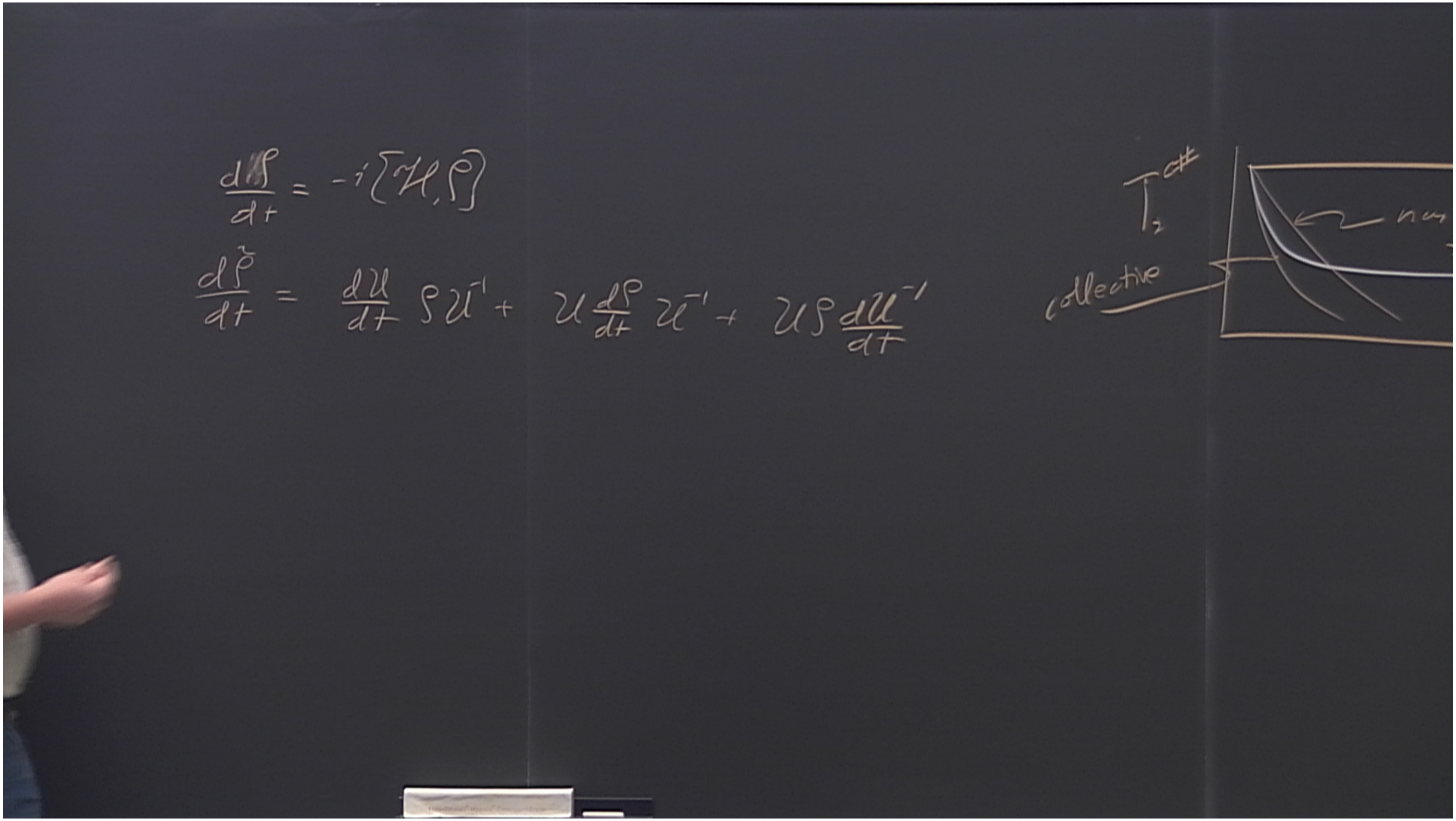

$B_0, \hat{z}$
x
B_1
$\cos(\omega_T t)$

$$\mathcal{H}_z = \underbrace{\frac{\omega_0}{2}\sigma_z}_{\text{Zeeman}} + 2\frac{\omega_1}{2}\cos(\omega_+ t)\,\sigma_x$$

decompose

$\sigma_x \to -\sigma_y$ $\qquad$ $\sigma_x \to \sigma_y$

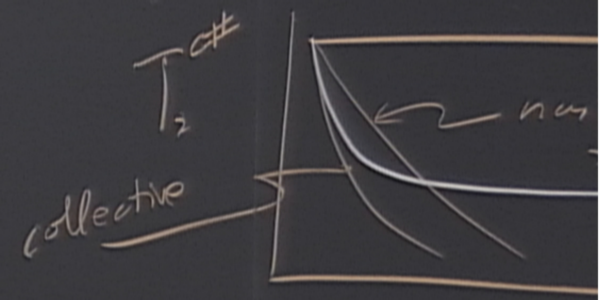
$$e^{i\frac{\omega_+ t}{2}\sigma_z}\,\sigma_x\, e^{-i\frac{\omega_+ t}{2}\sigma_z} + e^{-i\frac{\omega_+ t}{2}\sigma_z}\,\sigma_x\, e^{+i\frac{\omega_+ t}{2}\sigma_z}$$


$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
$\frac{d\tilde{\rho}}{dt} = \frac{d\mathcal{U}}{dt}\rho\,\mathcal{U}^{-1} + \mathcal{U}\frac{d\rho}{dt}\mathcal{U}^{-1} + \mathcal{U}\rho\frac{d\mathcal{U}^{-1}}{dt}$
T_2
collective

$$\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\frac{d\tilde{\rho}}{dt} = \frac{d\mathcal{U}}{dt}\rho\,\mathcal{U}^{-1} + \mathcal{U}\frac{d\rho}{dt}\mathcal{U}^{-1} + \mathcal{U}\rho\frac{d\mathcal{U}^{-1}}{dt}$$

$$\boxed{\frac{d\tilde{\rho}}{dt} = -i[\tilde{\mathcal{H}} - \mathcal{H}_t, \tilde{\rho}]}$$



$$\mathcal{H}_z = \underbrace{\frac{\omega_0}{2}\sigma_z}_{\text{Zeeman}} + 2\frac{\omega_1}{2}\cos(\omega_t t)\,\sigma_x \Longrightarrow \left(\frac{\omega_0-\omega_t}{2}\right)\sigma_z + \frac{\omega_1}{2}\sigma_x + e^{-i\frac{2\omega_t t}{2}\sigma_z}\sigma_x e^{+i\frac{2\omega_t t}{2}\sigma_z}$$

decompose

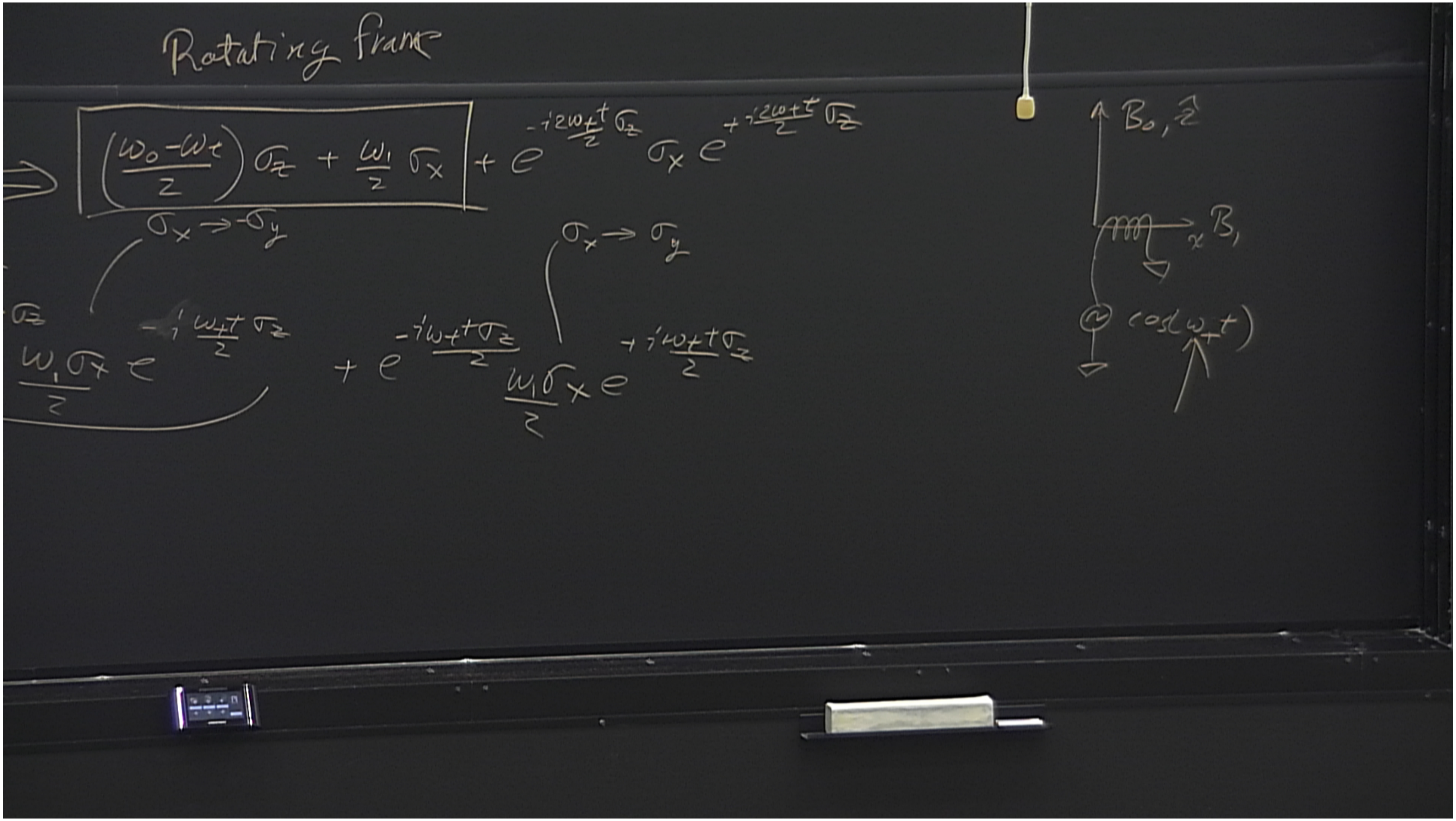
$\sigma_x \to -\sigma_y$ $\qquad$ $\sigma_x \to \sigma_y$

$$e^{i\frac{\omega_t t}{2}\sigma_z}\frac{\omega_1\sigma_x}{2}e^{-i\frac{\omega_t t}{2}\sigma_z} + e^{-i\frac{\omega_t t\sigma_z}{2}}\frac{\omega_1\sigma_x}{2}e^{+i\frac{\omega_t t\sigma_z}{2}}$$

move to interaction frame.

$$\mathcal{U}_t = e^{i\frac{\omega_t t\sigma_z}{2}}$$

$$\tilde{\rho} = \mathcal{U}\rho\,\mathcal{U}^{-1} \qquad \mathcal{H}_t = \frac{\omega_t\sigma_z}{2}$$


Rotating frame
$\Rightarrow \boxed{\left(\frac{\omega_0-\omega_t}{2}\right)\sigma_z + \frac{\omega_1}{2}\sigma_x} + e^{-i\frac{2\omega_t t}{2}\sigma_z}\sigma_x e^{+i\frac{2\omega_t t}{2}\sigma_z}$
$\sigma_x \to -\sigma_y$
$\sigma_x \to \sigma_y$
σ_z
$\frac{\omega_1\sigma_x}{2} e^{-i\frac{\omega_t t}{2}\sigma_z} + e^{-i\frac{\omega_t t\sigma_z}{2}}\frac{\omega_1\sigma_x}{2}e^{+i\frac{\omega_t t\sigma_z}{2}}$
$B_0, \hat{z}$
x B_1
$\cos(\omega_t t)$

Rotating frame

$$\Rightarrow \boxed{\left(\frac{\omega_0-\omega_t}{2}\right)\sigma_z + \frac{\omega_1}{2}\sigma_x} + e^{-i\frac{2\omega_t t}{2}\sigma_z}\,\sigma_x\, e^{+i\frac{2\omega_t t}{2}\sigma_z}$$

$\sigma_x \to -\sigma_y$

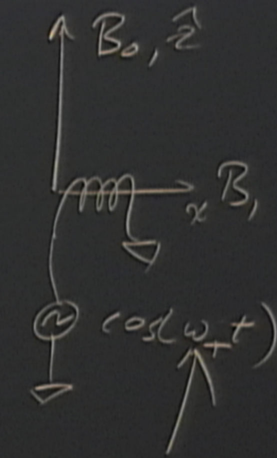
$\sigma_x \to \sigma_y$

Rotating frame app.

$\Rightarrow 0$

σ_z

$$\frac{\omega_1 \sigma_x}{2} e^{-i\frac{\omega_t t}{2}\sigma_z} + e^{-i\frac{\omega_t t\sigma_z}{2}}\frac{\omega_1\sigma_x}{2} e^{+i\frac{\omega_t t\sigma_z}{2}}$$



$$-i[\mathcal{H},\rho]$$

$$\frac{dU}{dt}\rho U^{-1} + U\frac{d\rho}{dt}U^{-1} + U\rho\frac{dU^{-1}}{dt}$$

$$\boxed{= -i[\tilde{\mathcal{H}} - \mathcal{H}_t, \tilde{\rho}]}$$