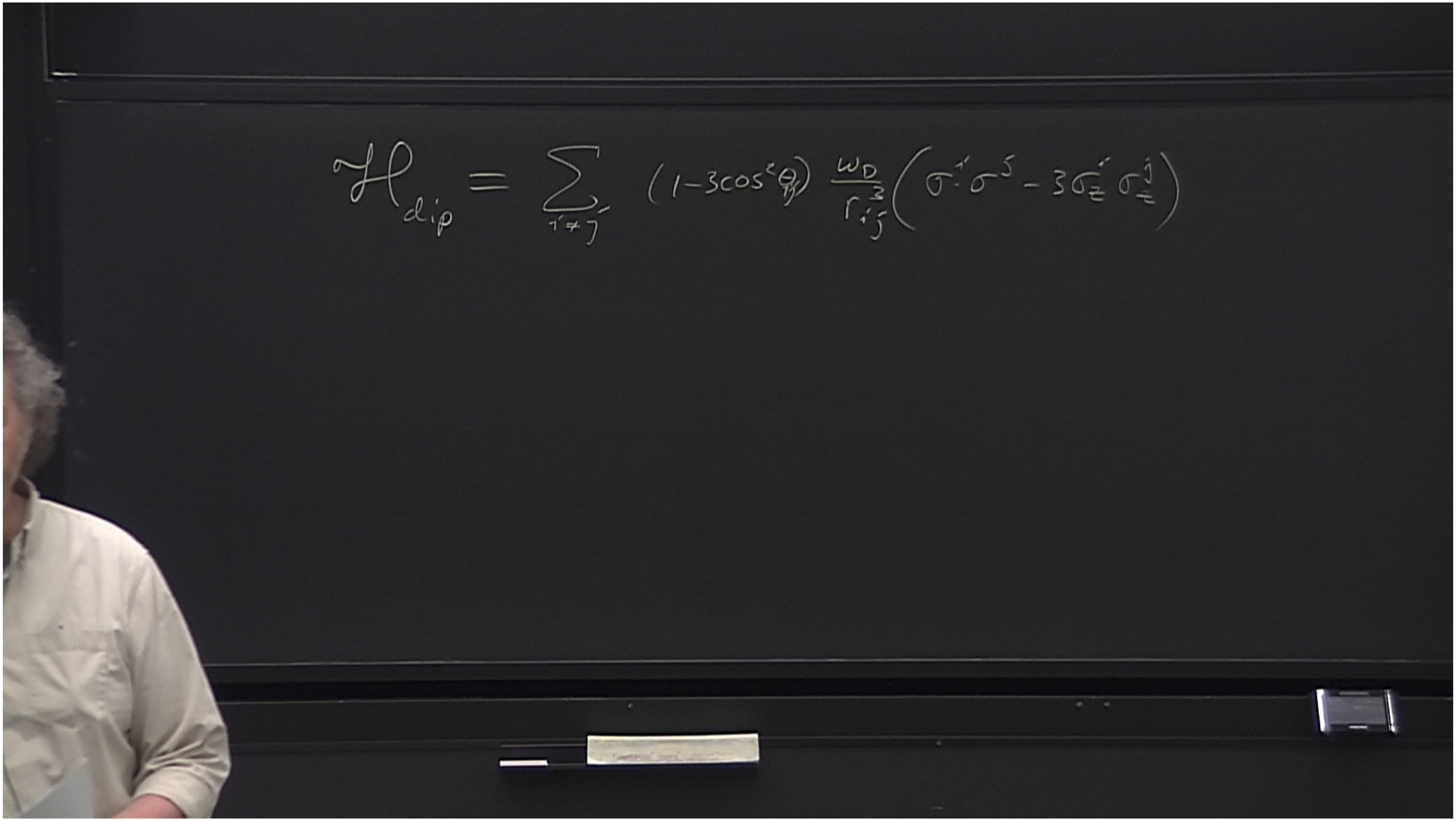
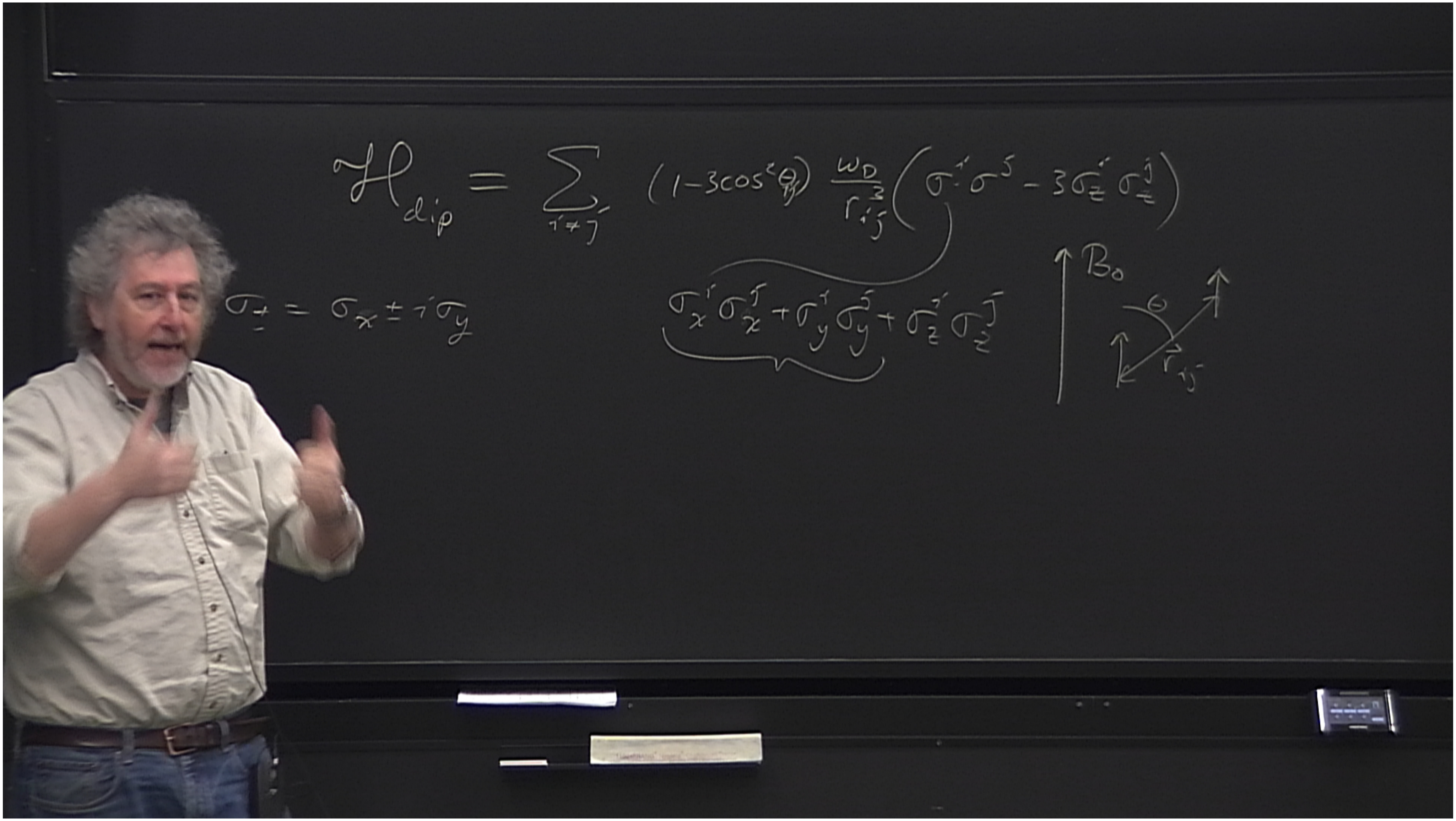
Title: 13/14 PSI - Explorations in Quantum Information - Lecture 7

Date: Mar 25, 2014 09:00 AM

URL: http://pirsa.org/14030037

Abstract:

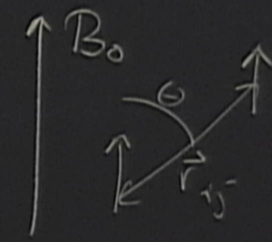

$\mathcal{H}_{dip} = \sum_{i \neq j} (1-3\cos^2\theta_{ij}) \frac{\omega_D}{r_{ij}^3} \left(\sigma^i \cdot \sigma^j - 3\sigma_z^i \sigma_z^j\right)$

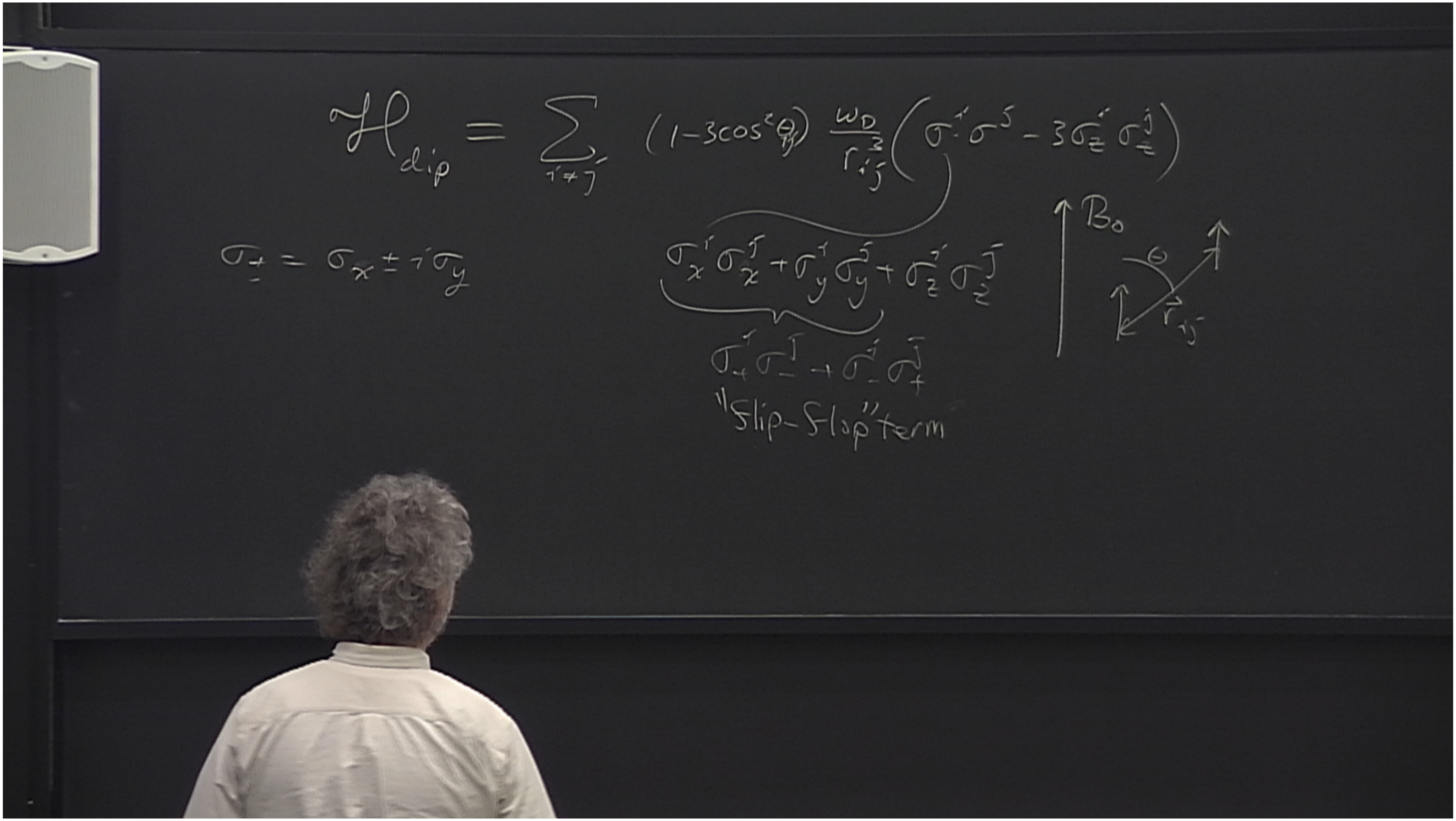

$\mathcal{H}_{dip} = \sum_{i \neq j} (1 - 3\cos^2\theta_{ij}) \frac{\omega_D}{r_{ij}^3} (\sigma^i \cdot \sigma^j - 3\sigma_z^i \sigma_z^j)$
$\sigma_\pm = \sigma_x \pm i\sigma_y$
$\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j + \sigma_z^i \sigma_z^j$
B_0
θ
$\vec{r}_{ij}$

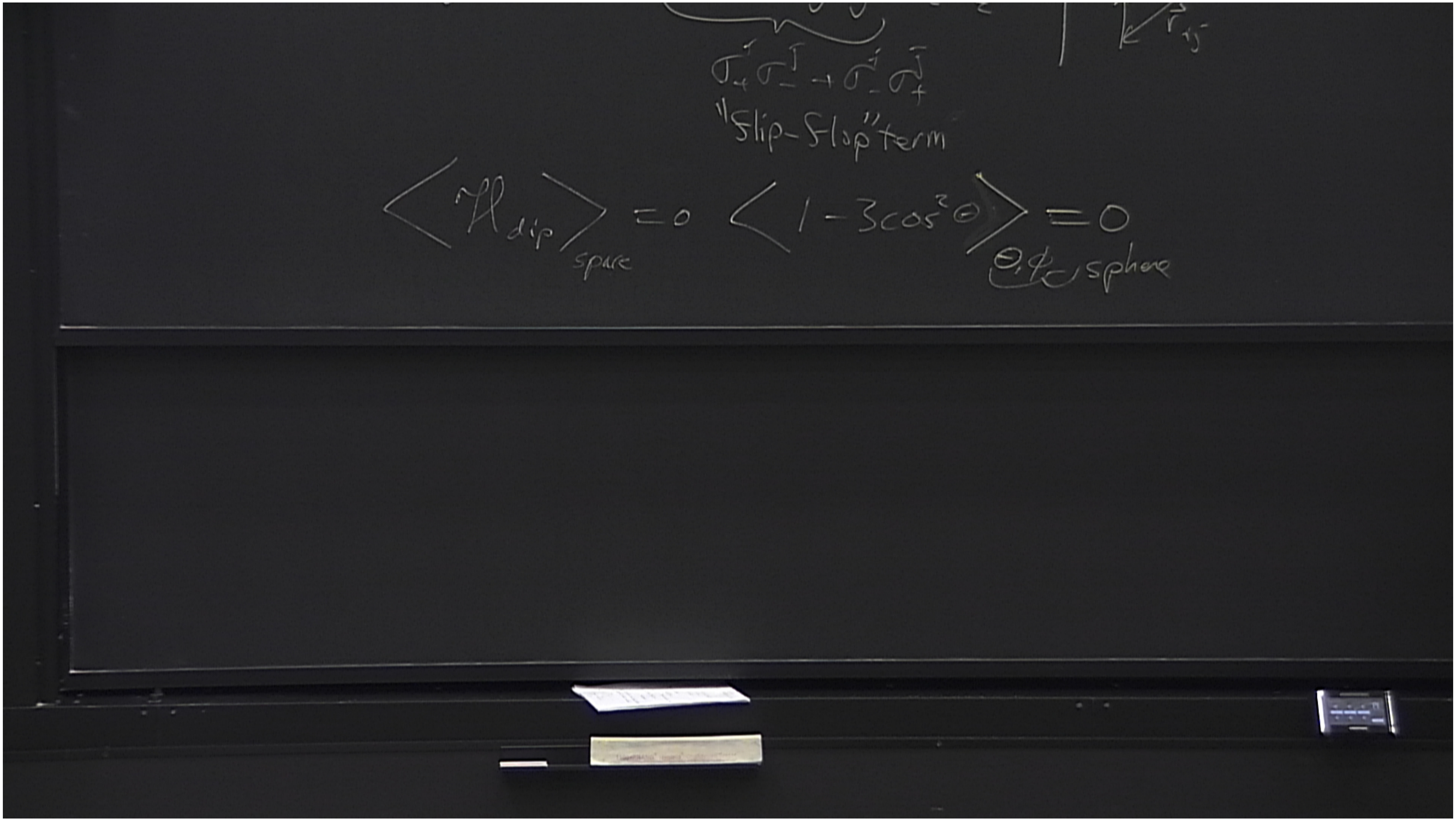
$$\mathcal{H}_{dip} = \sum_{i\neq j} (1-3\cos^2\theta_{ij}) \frac{\omega_D}{r_{ij}^3}\left(\sigma^i\cdot\sigma^j - 3\sigma_z^i\sigma_z^j\right)$$

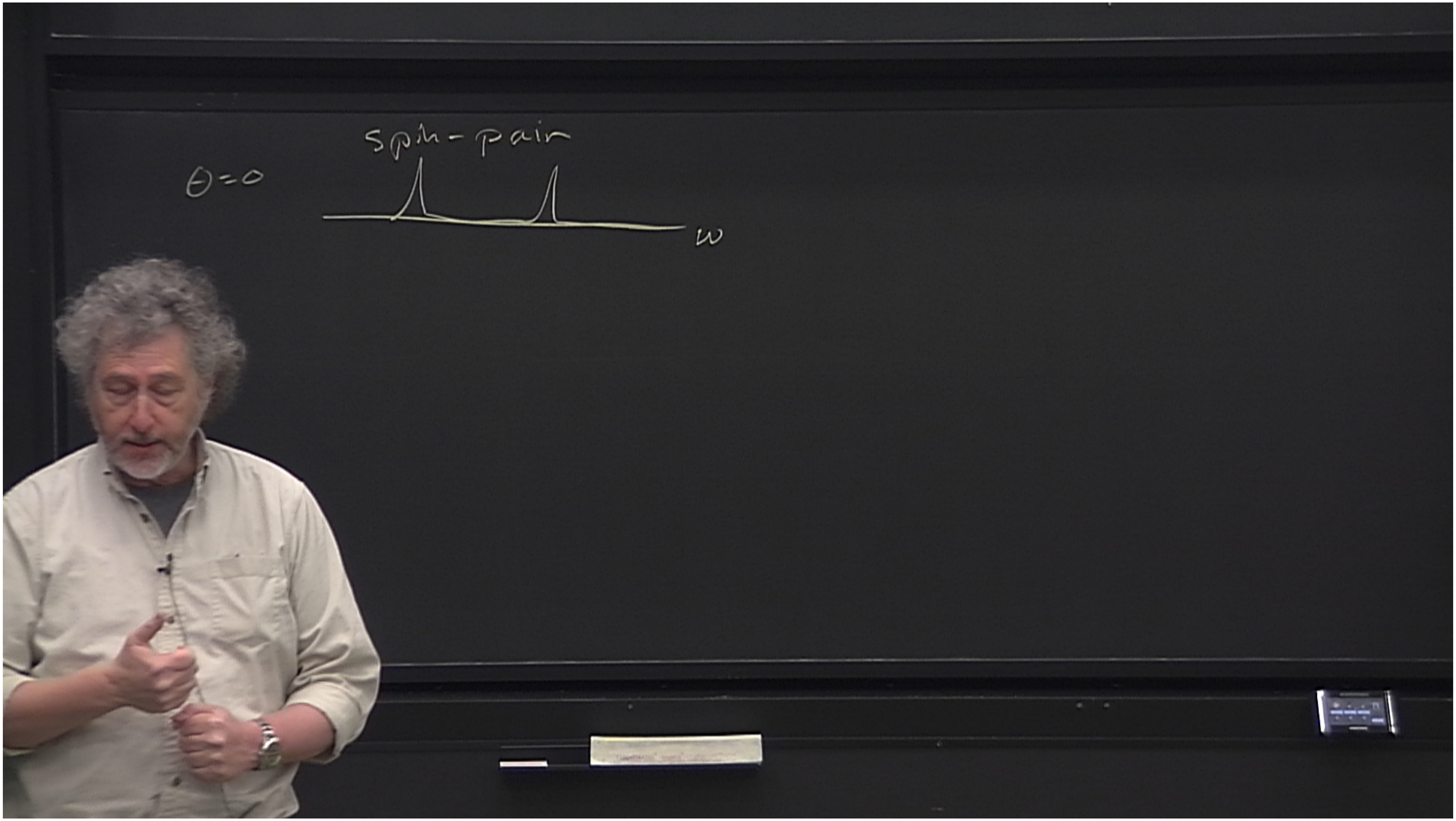
$$\sigma_\pm = \sigma_x \pm i\sigma_y$$

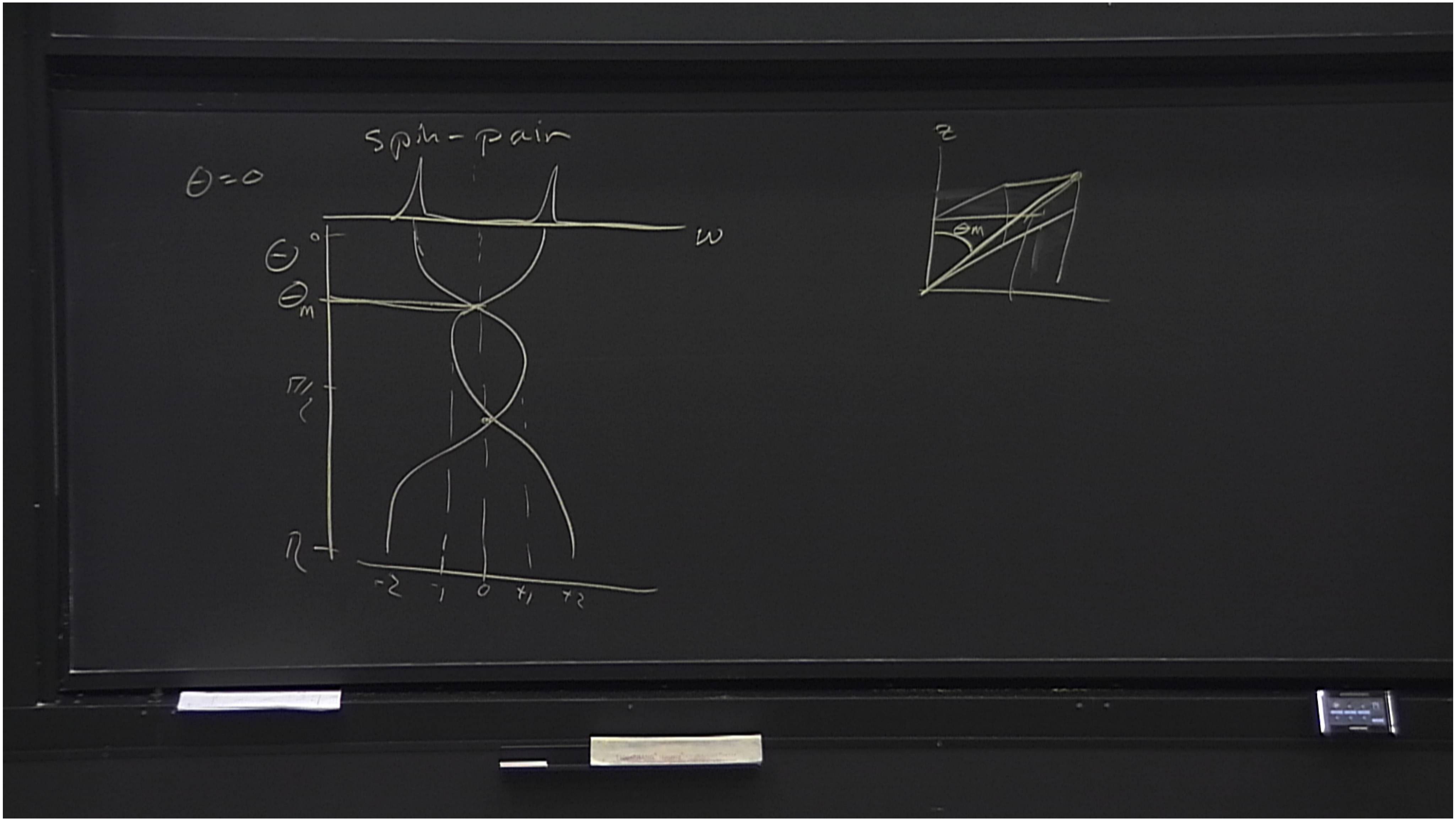
$$\sigma_x^i\sigma_x^j + \sigma_y^i\sigma_y^j + \sigma_z^i\sigma_z^j$$

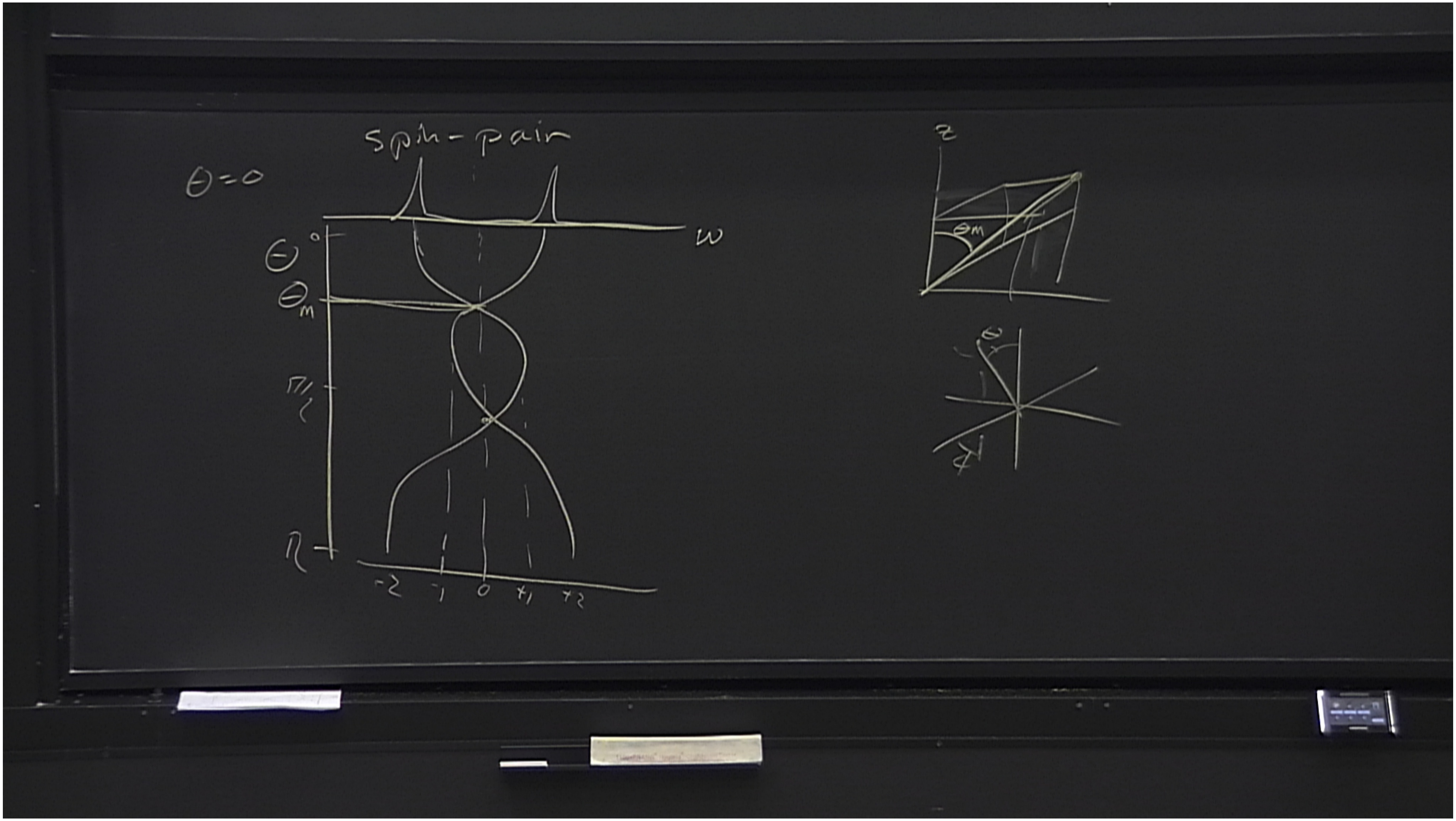



$\mathcal{H}_{dip} = \sum_{i \neq j} (1 - 3\cos^2\theta_{ij}) \frac{\omega_D}{r_{ij}^3} (\sigma^i \cdot \sigma^j - 3\sigma_z^i \sigma_z^j)$
$\sigma_\pm = \sigma_x \pm i\sigma_y$
$\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j + \sigma_z^i \sigma_z^j$
$\sigma_+^i \sigma_-^j + \sigma_-^i \sigma_+^j$
"flip-flop" term
B_0
θ
$\vec{r}_{ij}$

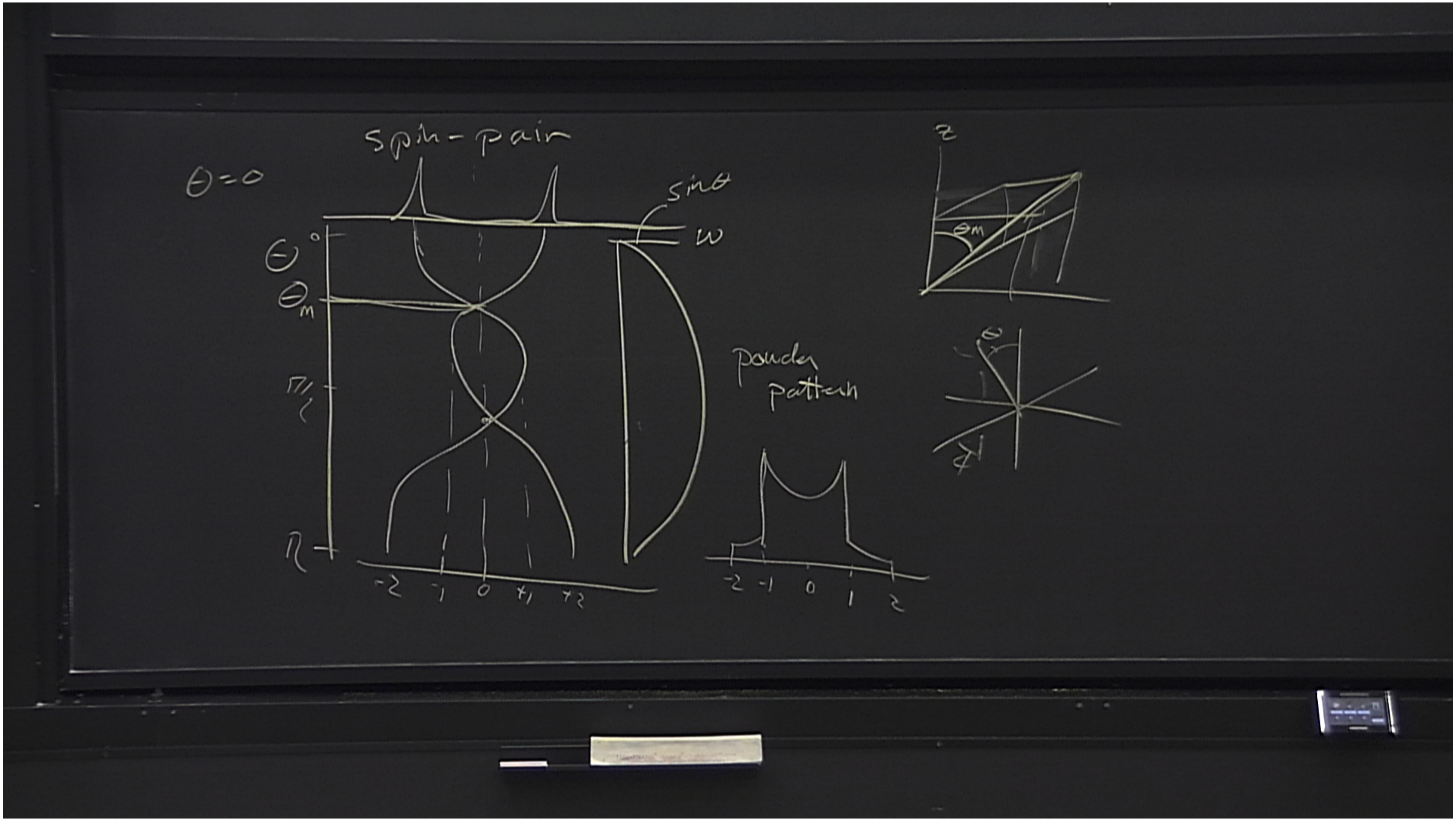

"flip-flop" term
⟨H_dip⟩_space = 0
⟨1 − 3cos²θ⟩ = 0
θ,φ ∈ sphere


spin-pair
θ=0
ω

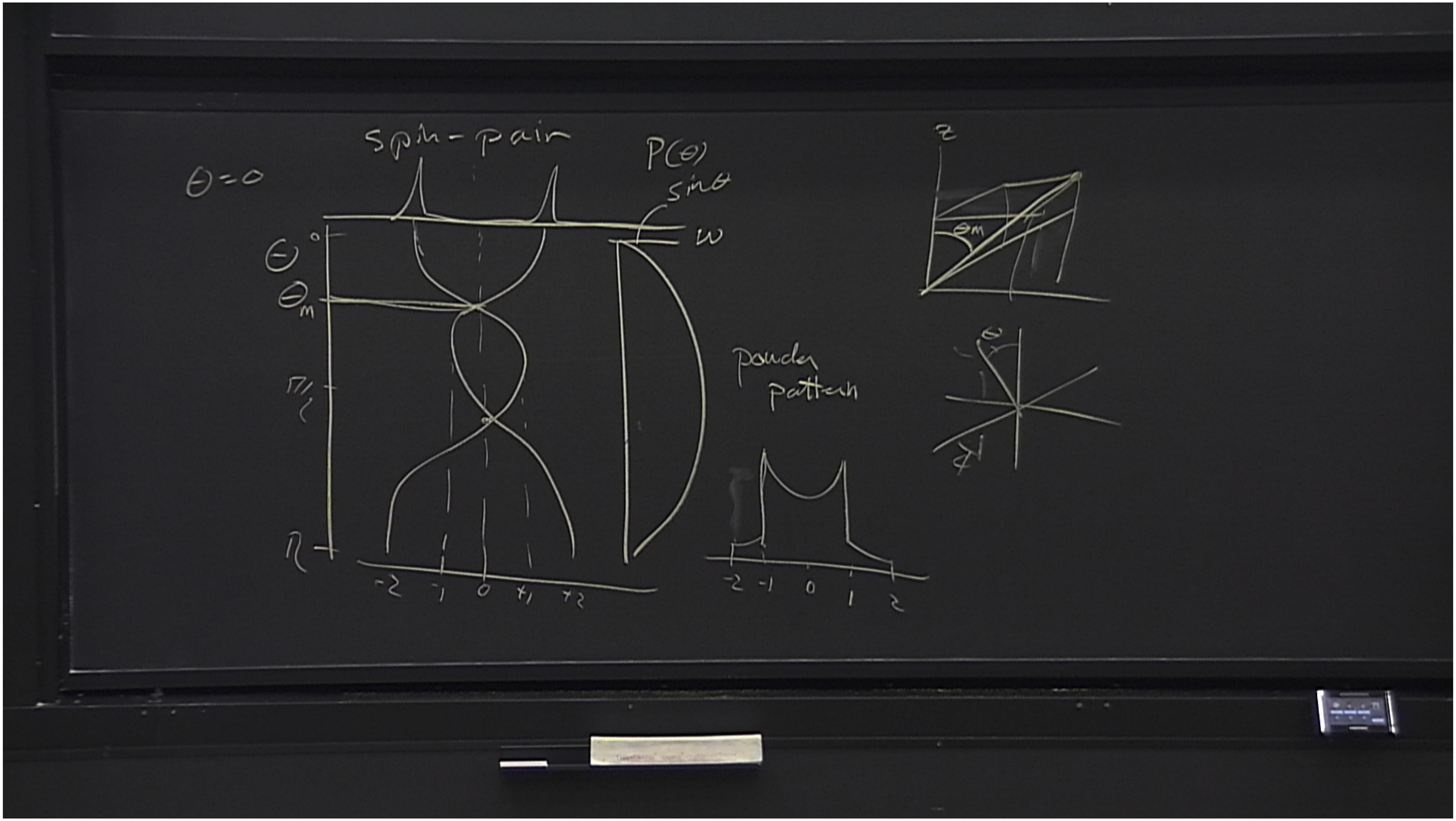

θ=0
sph-pair
ω
Θ
0
Θm
π/2
π
-2
-1
0
+1
+2
z
Θm



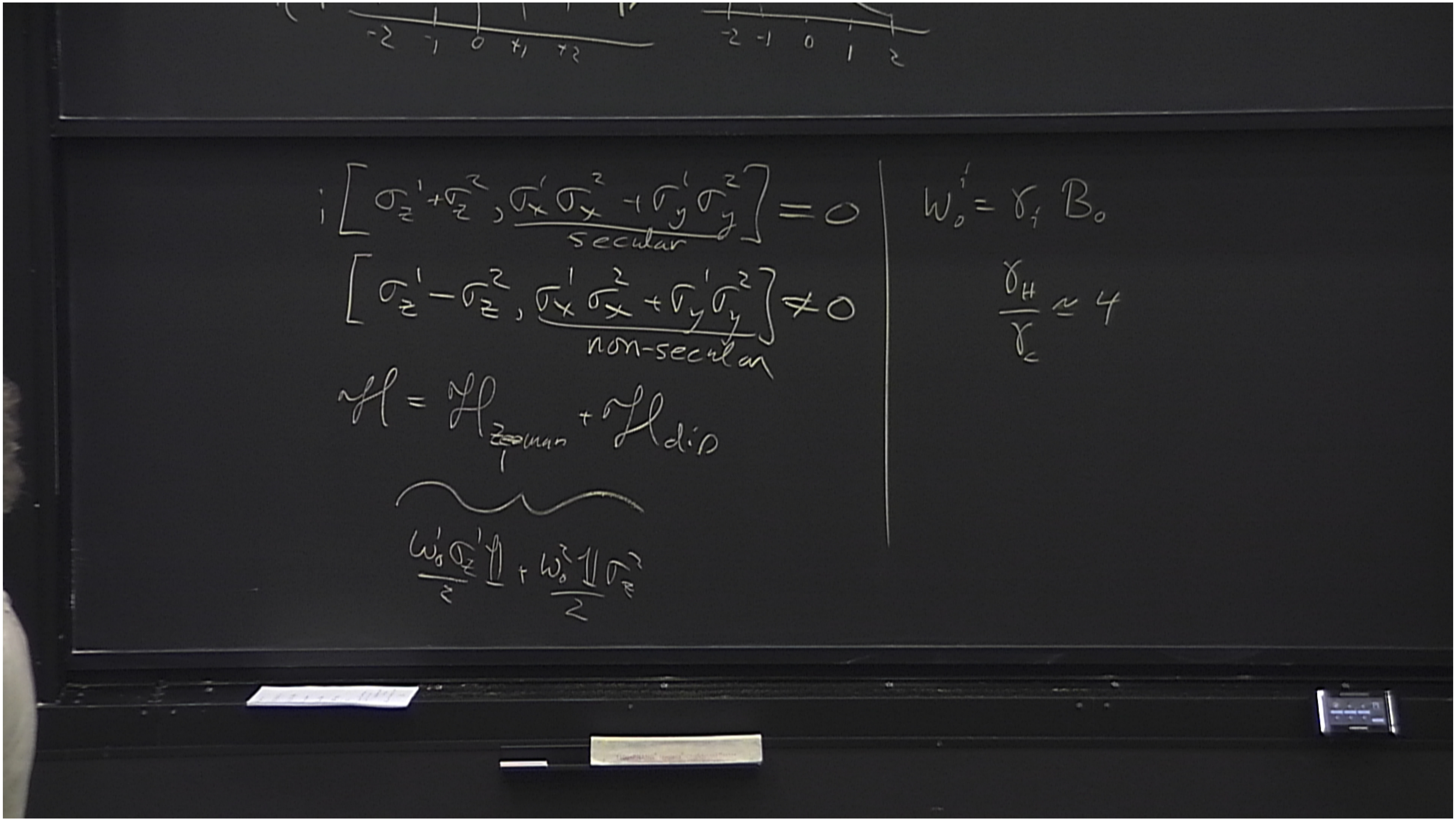
spin-pair
θ=0
ω
θ
0
θm
π/2
π
-2
-1
0
+1
+2
z
θm
θ

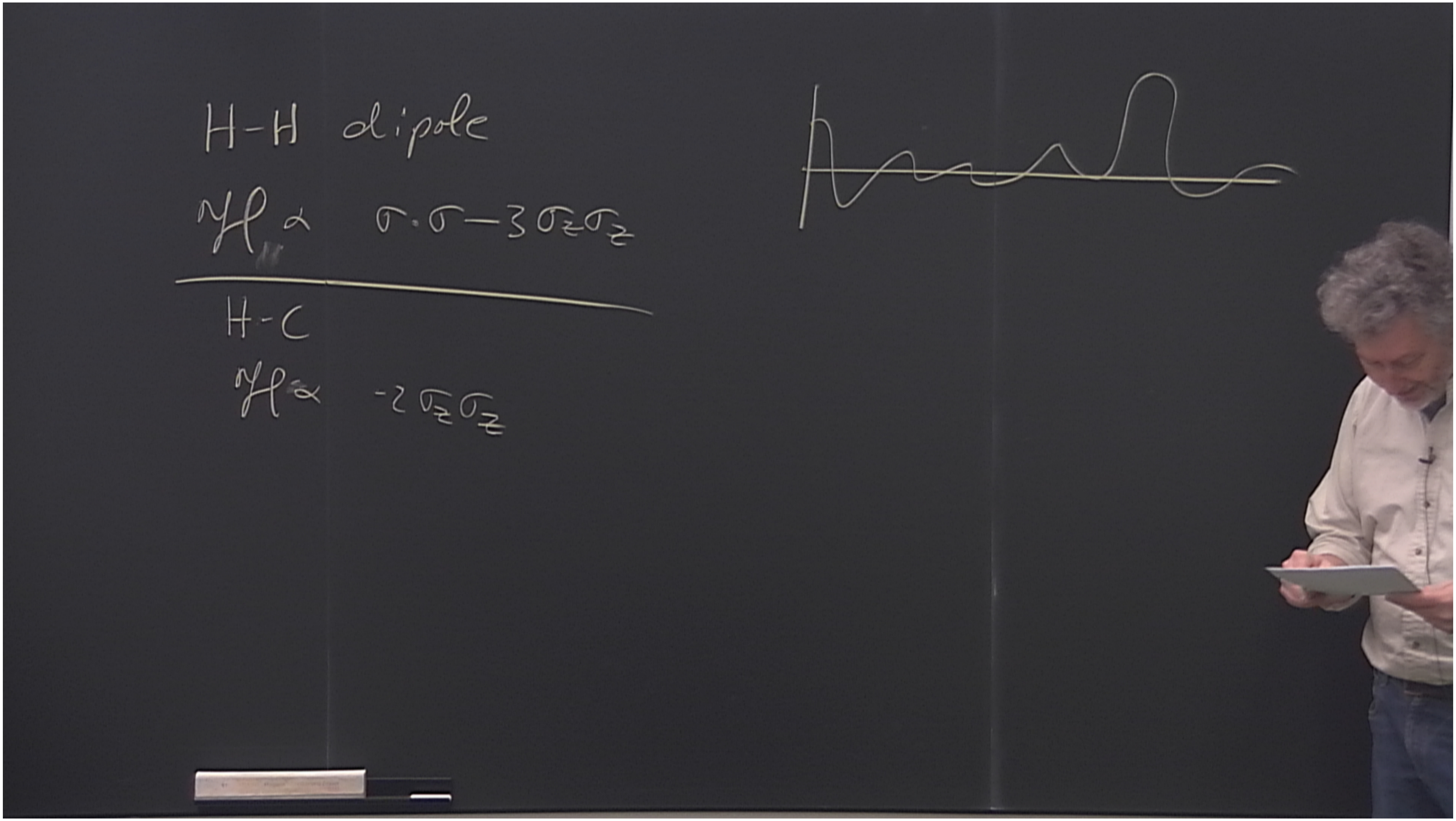


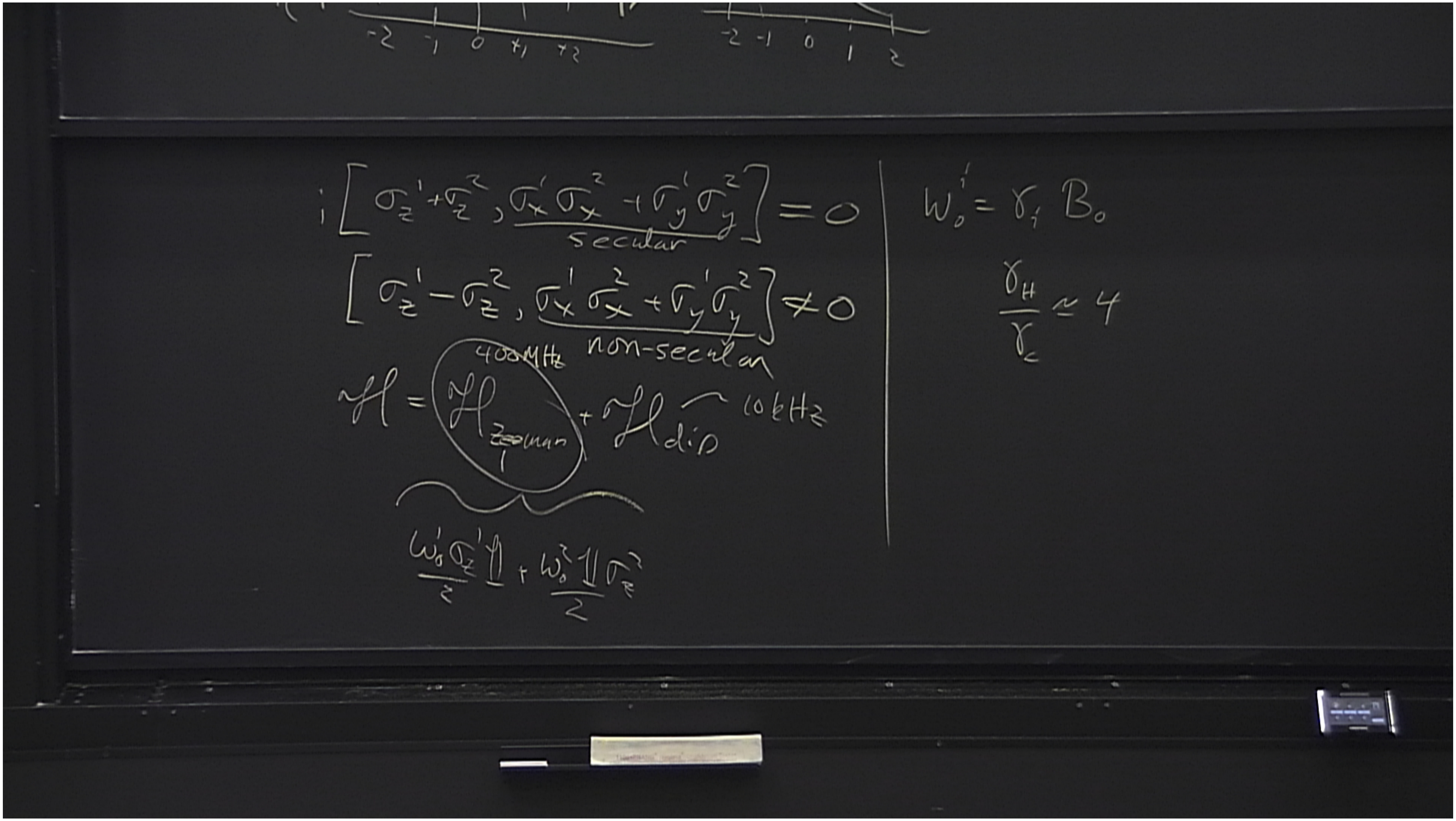
spin-pair
θ=0
sinθ
ω
θ
θ_m
π/2
π
-2 -1 0 +1 +2
powder
pattern
-2 -1 0 1 2
z
θ_m
θ

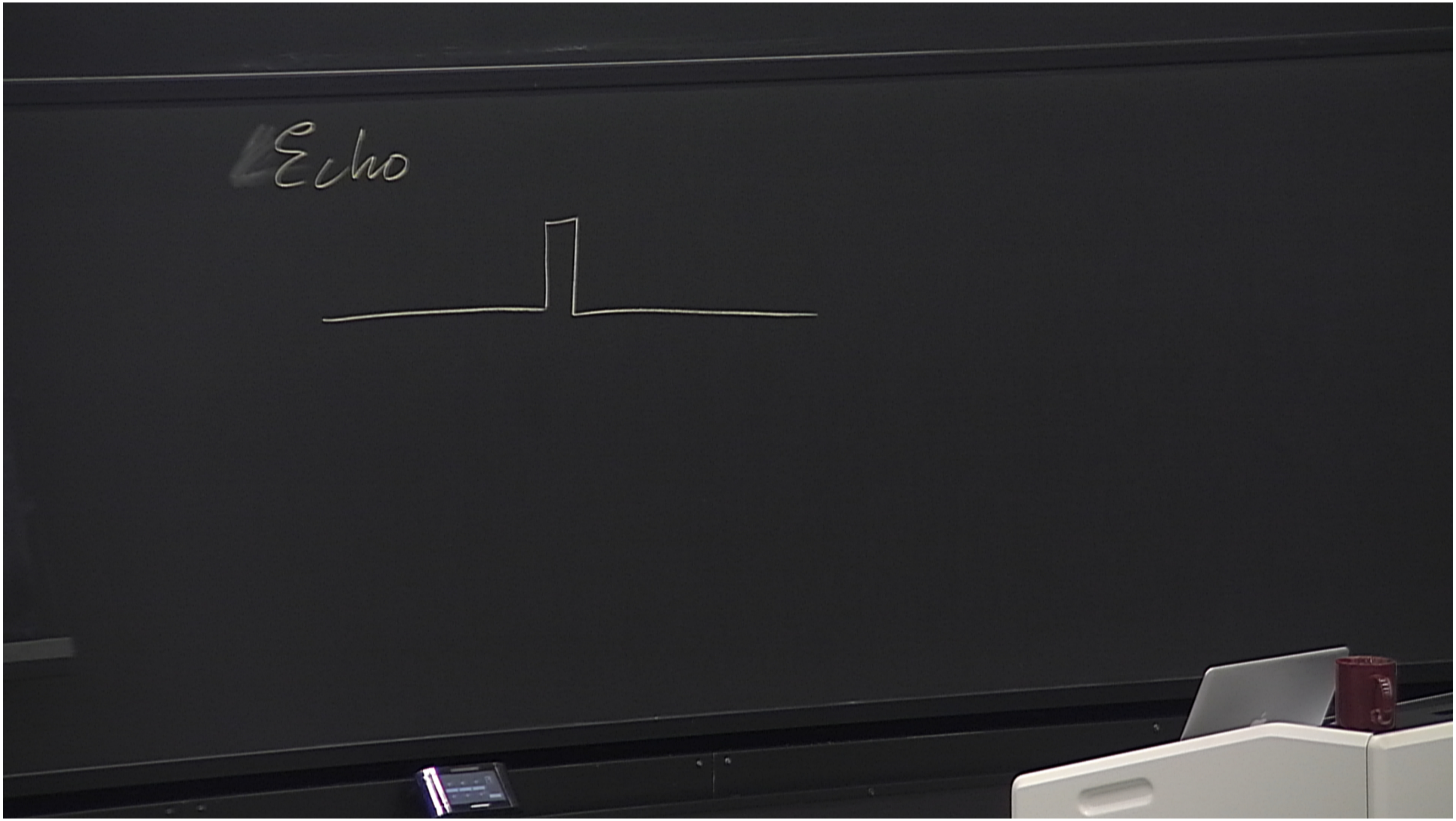


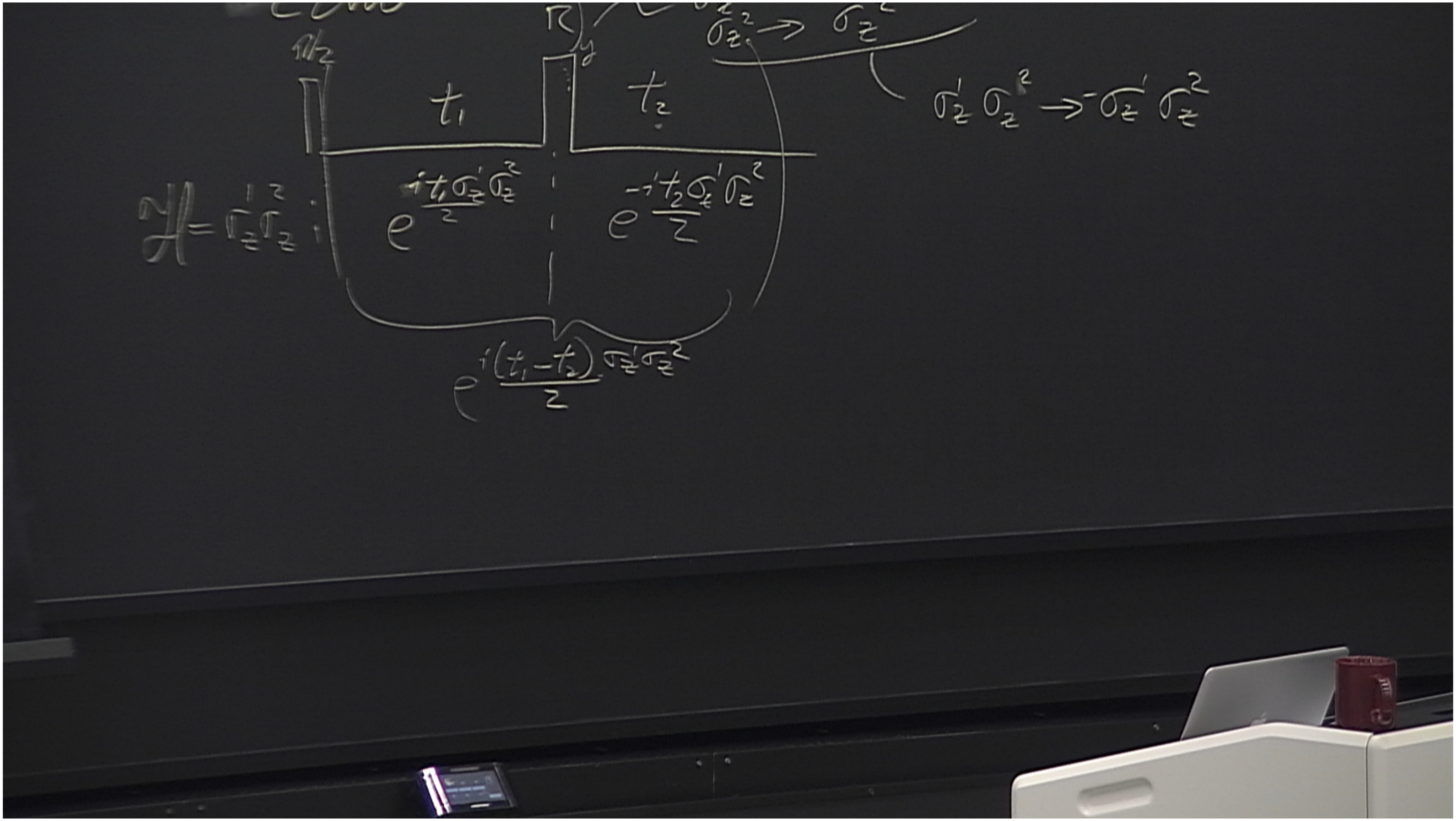
spin-pair
P(θ)
sin θ
ω
θ=0
Θ
θm
π/2
π
-2
-1
0
+1
+2
powder pattern
-2
-1
0
1
2
z
θm
θ

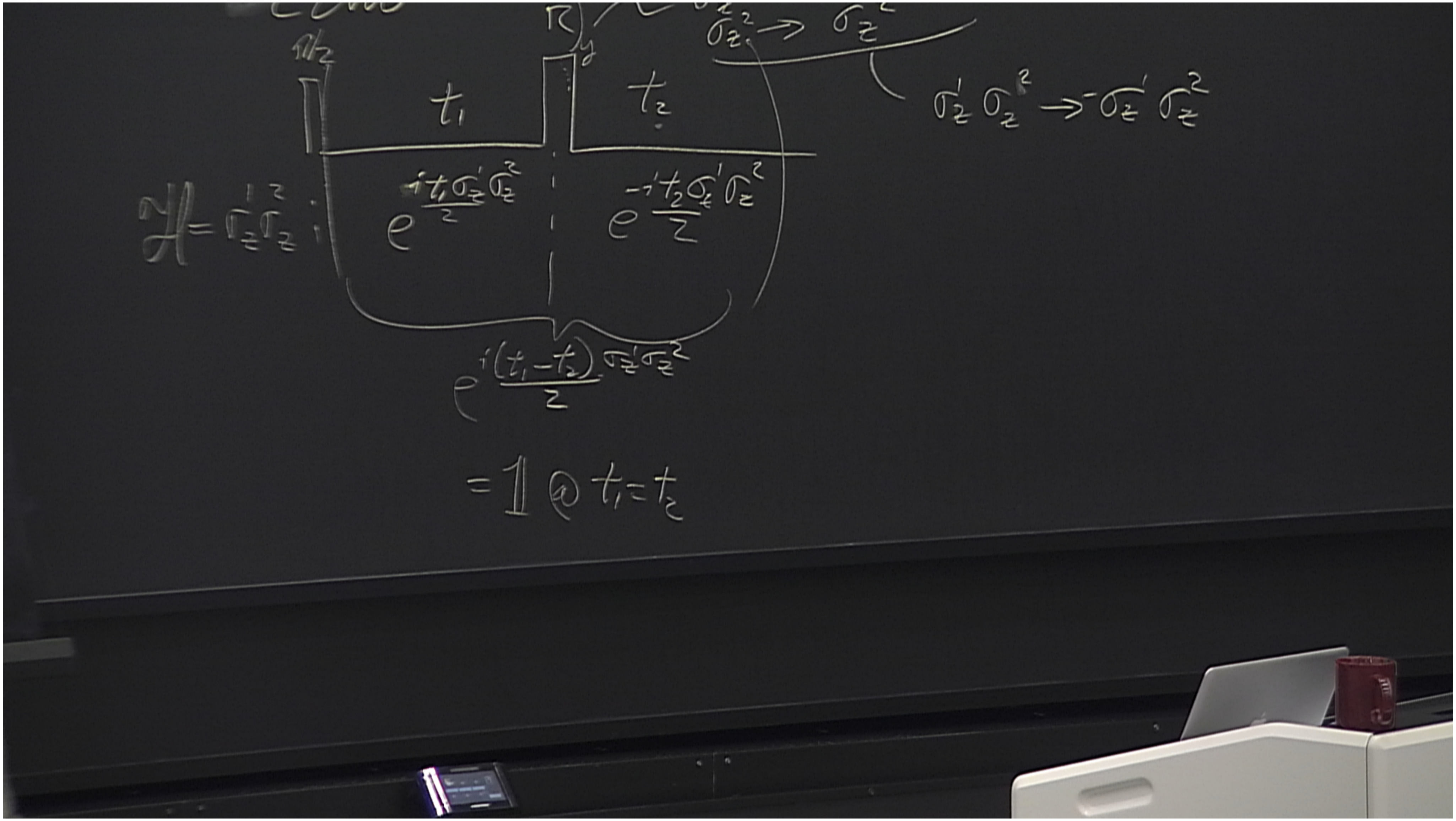

-2 -1 0 +1 +2
-2 -1 0 1 2
$i[\sigma_z^1+\sigma_z^2, \sigma_x^1\sigma_x^2+\sigma_y^1\sigma_y^2] = 0$
secular
$[\sigma_z^1-\sigma_z^2, \sigma_x^1\sigma_x^2+\sigma_y^1\sigma_y^2] \neq 0$
non-secular
$\mathcal{H} = \mathcal{H}_{Zeeman} + \mathcal{H}_{dip}$
$\frac{\omega_0^1\sigma_z^1 \mathbb{1}}{2} + \frac{\omega_0^2 \mathbb{1}\sigma_z^2}{2}$
$\omega_0^i = \gamma_i B_0$
$\frac{\gamma_H}{\gamma_C} \simeq 4$

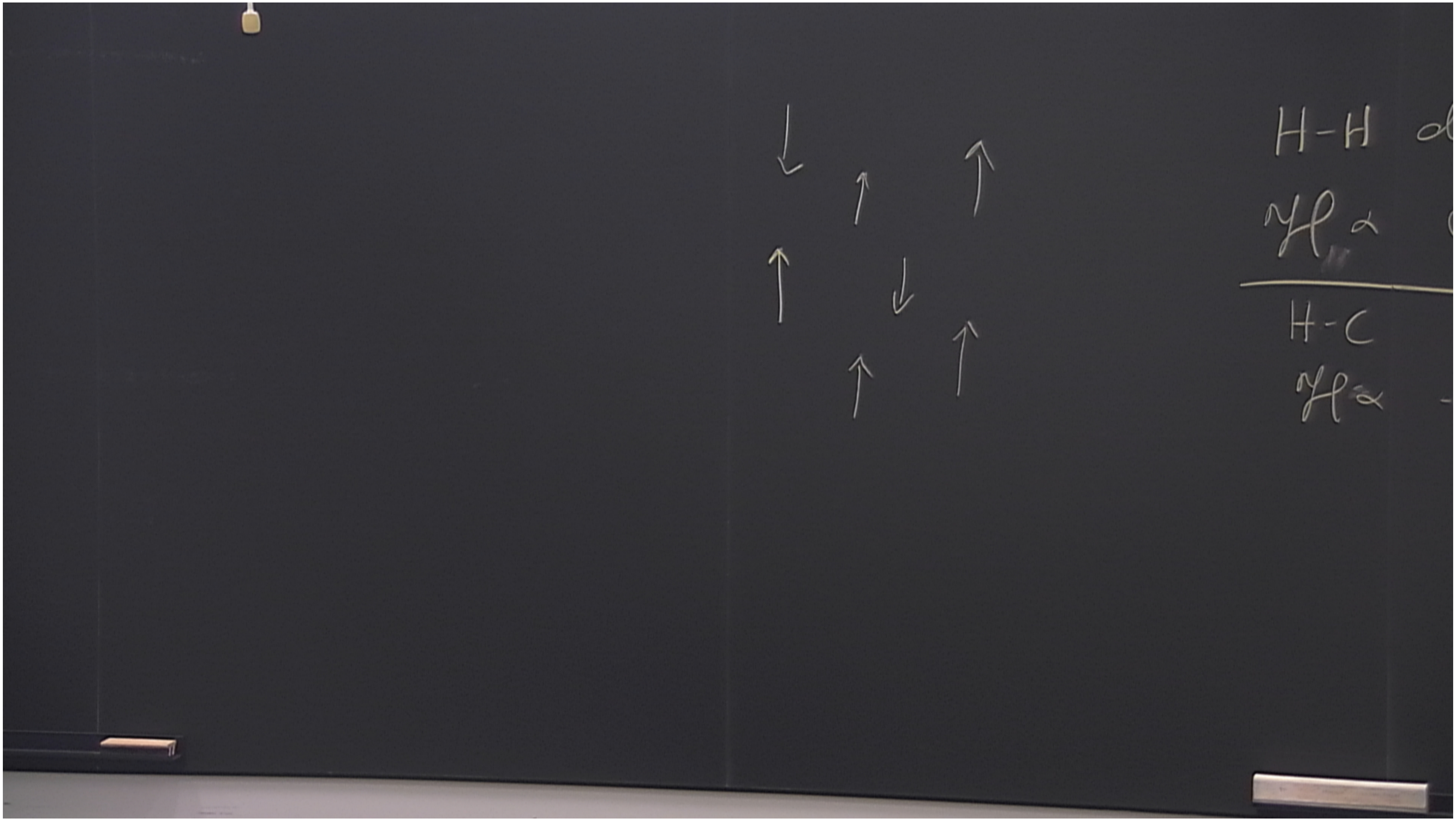

H-H dipole
ℋ ∝ σ·σ − 3σzσz
H-C
ℋ ∝ −2σzσz

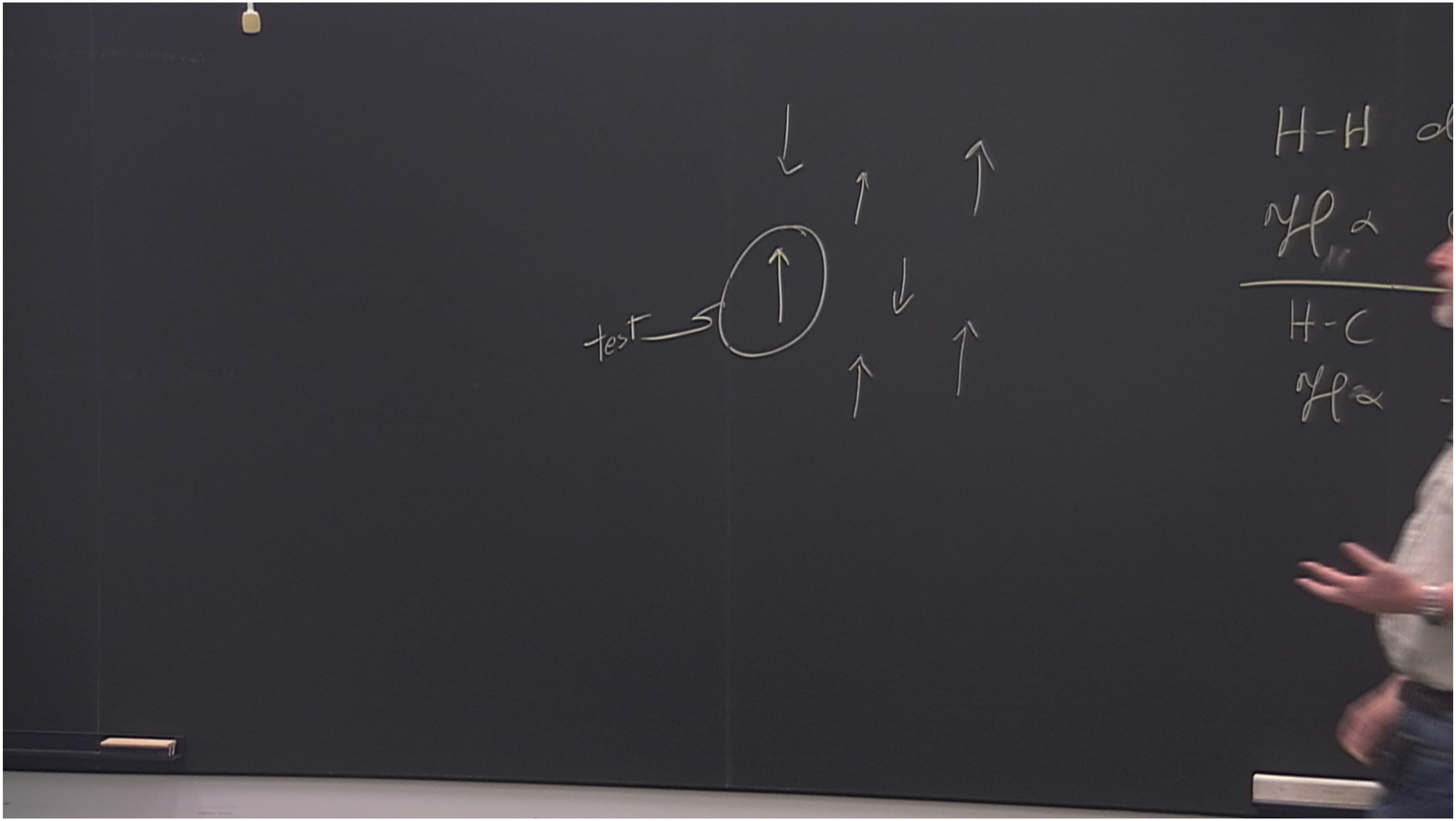

-2 -1 0 +1 +2
-2 -1 0 1 2
$i[\sigma_z^1+\sigma_z^2, \sigma_x^1\sigma_x^2+\sigma_y^1\sigma_y^2]=0$
secular
$[\sigma_z^1-\sigma_z^2, \sigma_x^1\sigma_x^2+\sigma_y^1\sigma_y^2]\neq 0$
non-secular
400 MHz
$\mathcal{H} = \mathcal{H}_{Zeeman} + \mathcal{H}_{dip}$
10 kHz
$\frac{\omega_0^1\sigma_z^1 \mathbb{1}}{2} + \frac{\omega_0^2 \mathbb{1}\sigma_z^2}{2}$
$\omega_0^i = \gamma_i B_0$
$\frac{\gamma_H}{\gamma_c} \simeq 4$


Echo

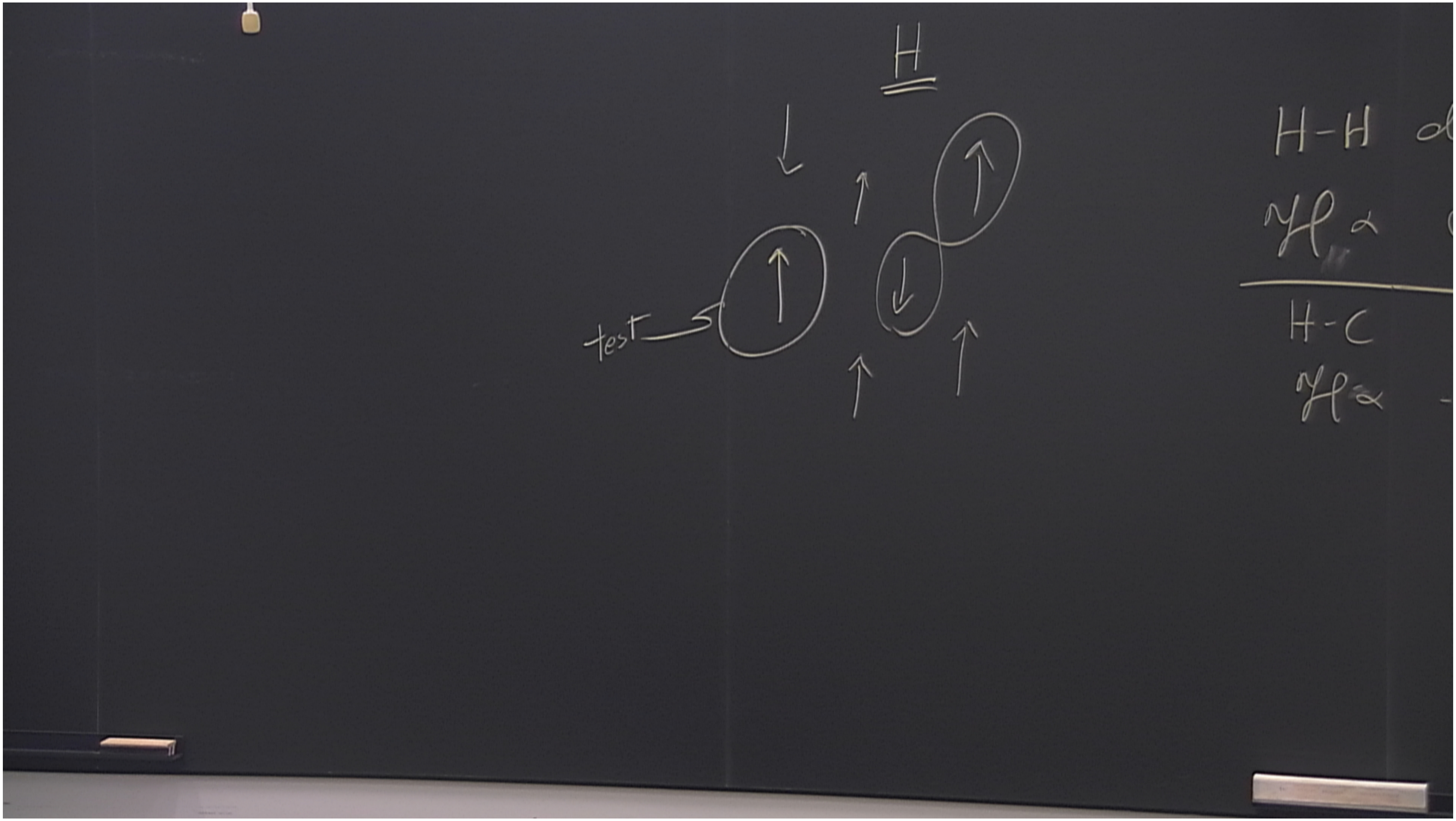

t1
t2

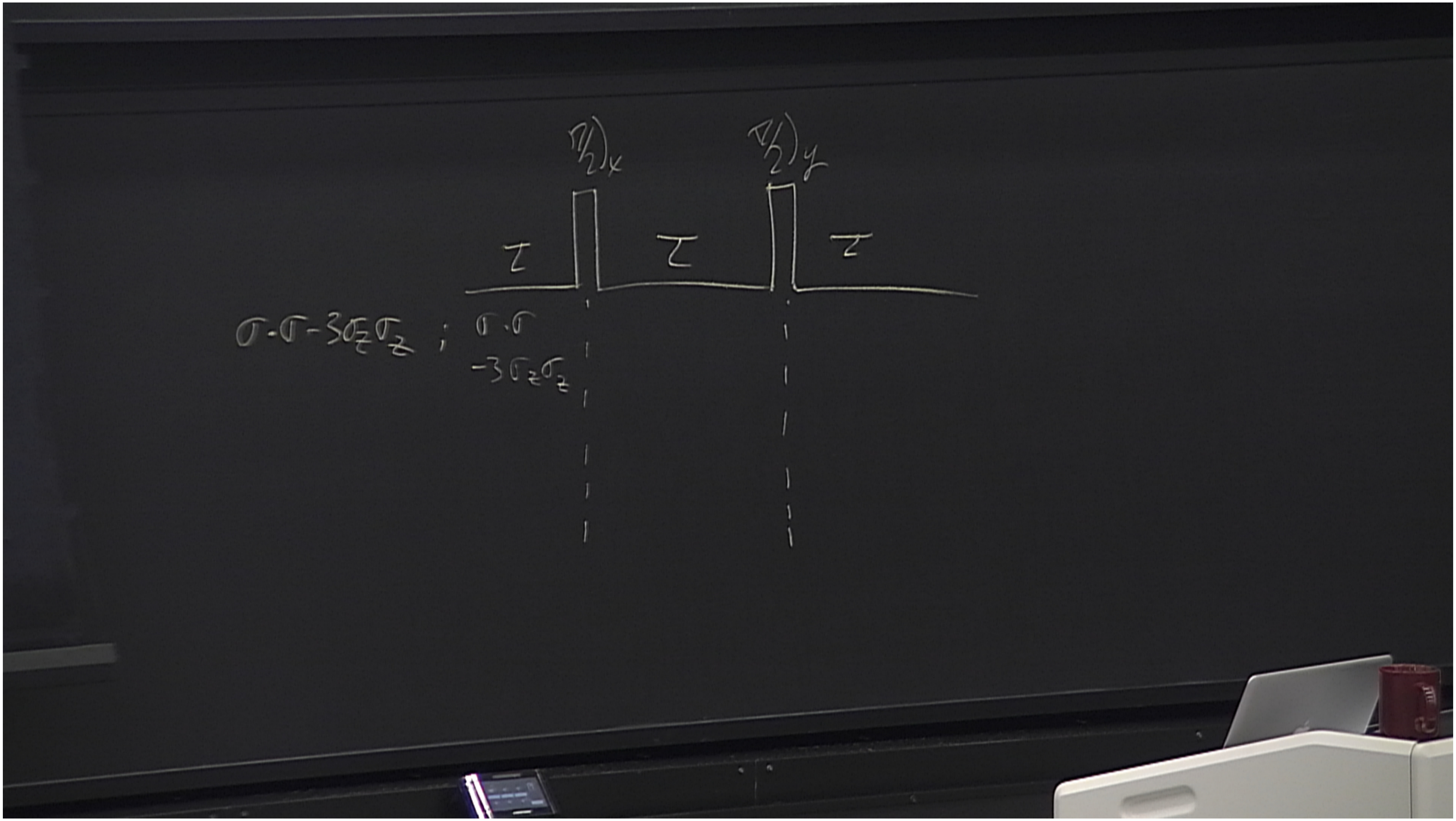

$\pi/2$
t_1
t_2
$\sigma_z^2 \to \sigma_z^2$
$\sigma_z^1 \sigma_z^2 \to -\sigma_z^1 \sigma_z^2$
$\mathcal{H} = \sigma_z^1 \sigma_z^2$
$e^{\frac{-i t_1 \sigma_z^1 \sigma_z^2}{2}}$
$e^{\frac{-i t_2 \sigma_z^1 \sigma_z^2}{2}}$
$e^{\frac{i(t_1 - t_2)}{2} \sigma_z^1 \sigma_z^2}$
$= \mathbb{1}$ @ $t_1 = t_2$

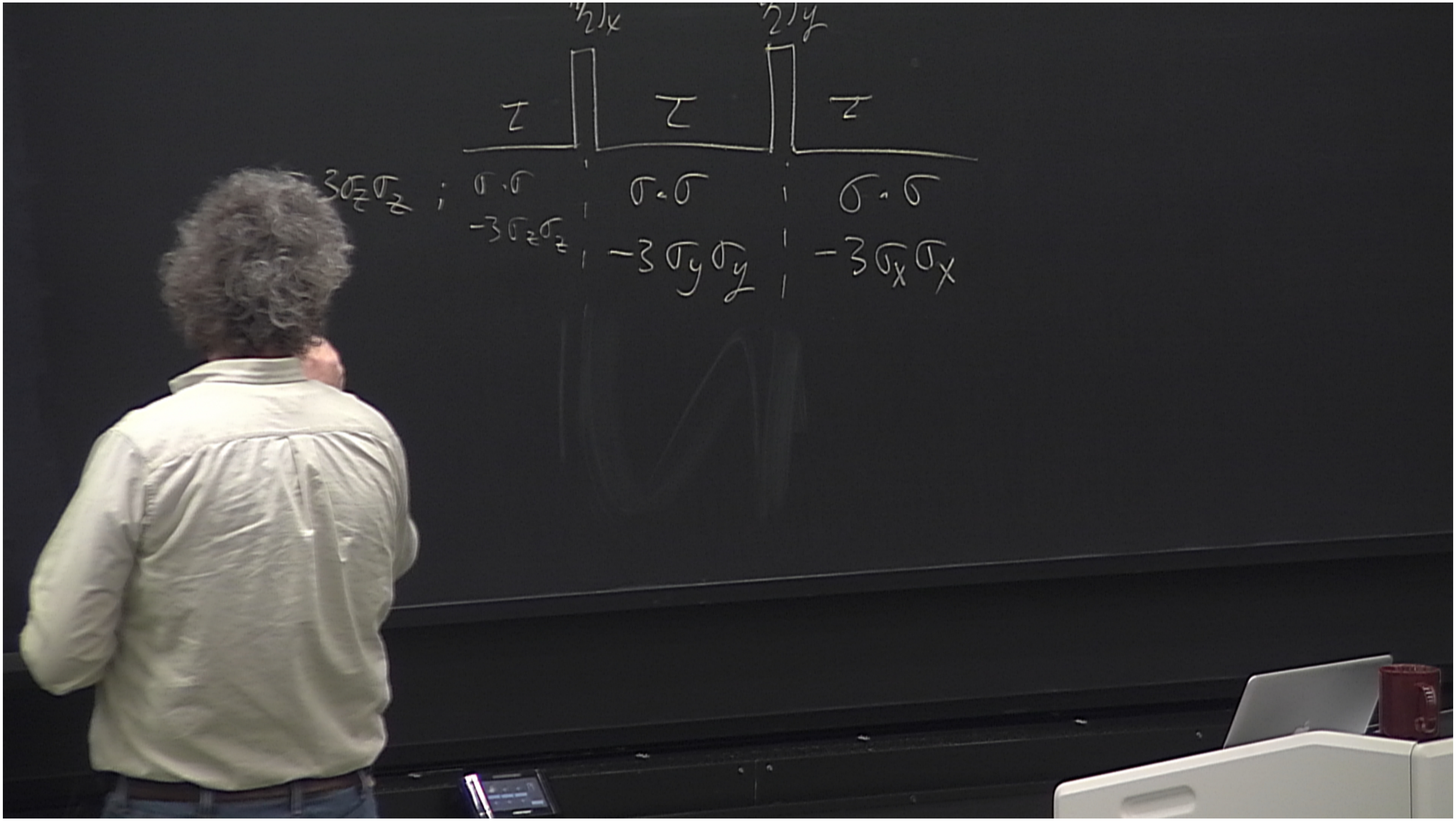

H-H
H-C



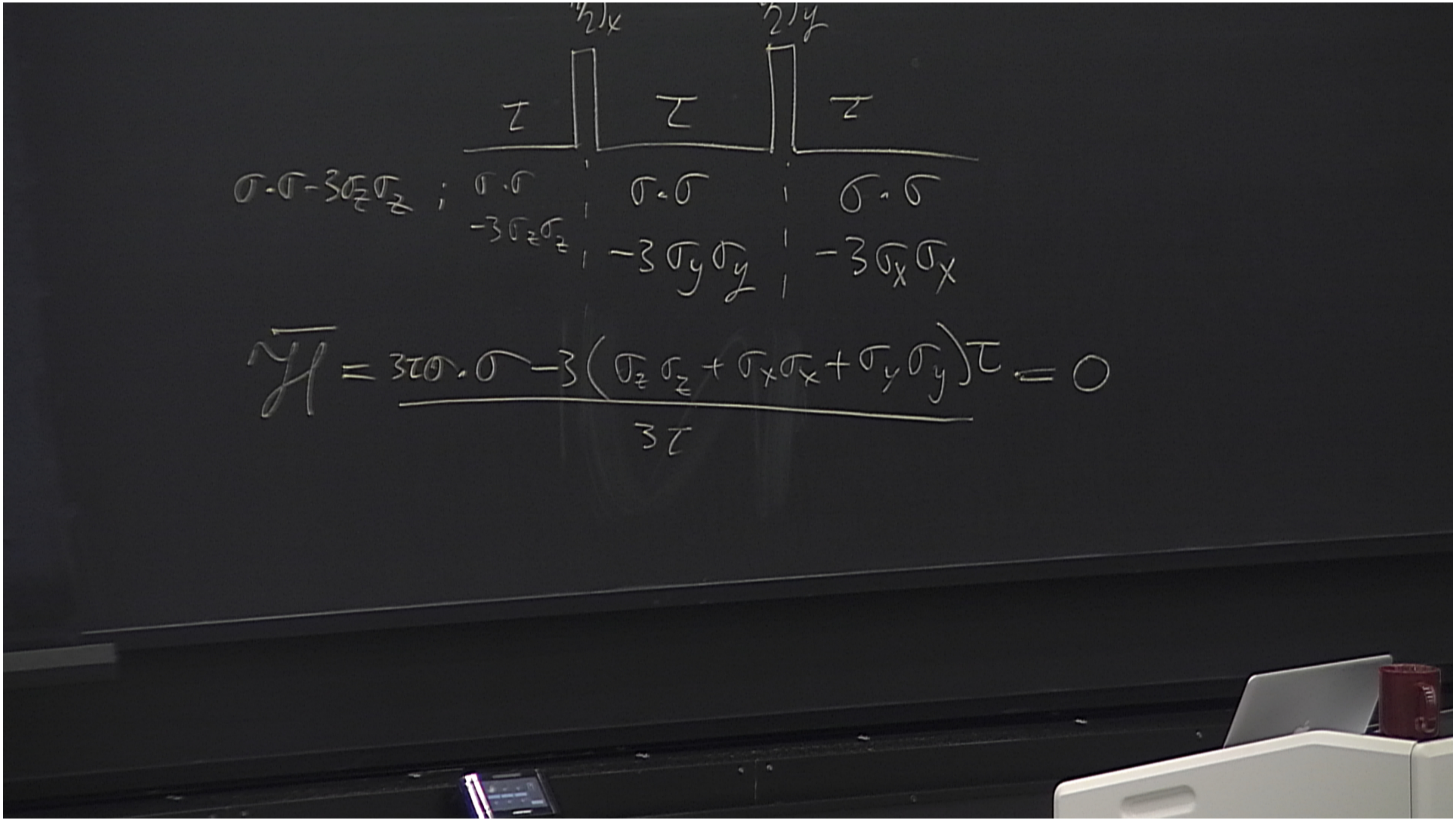
test
H-H
H-C

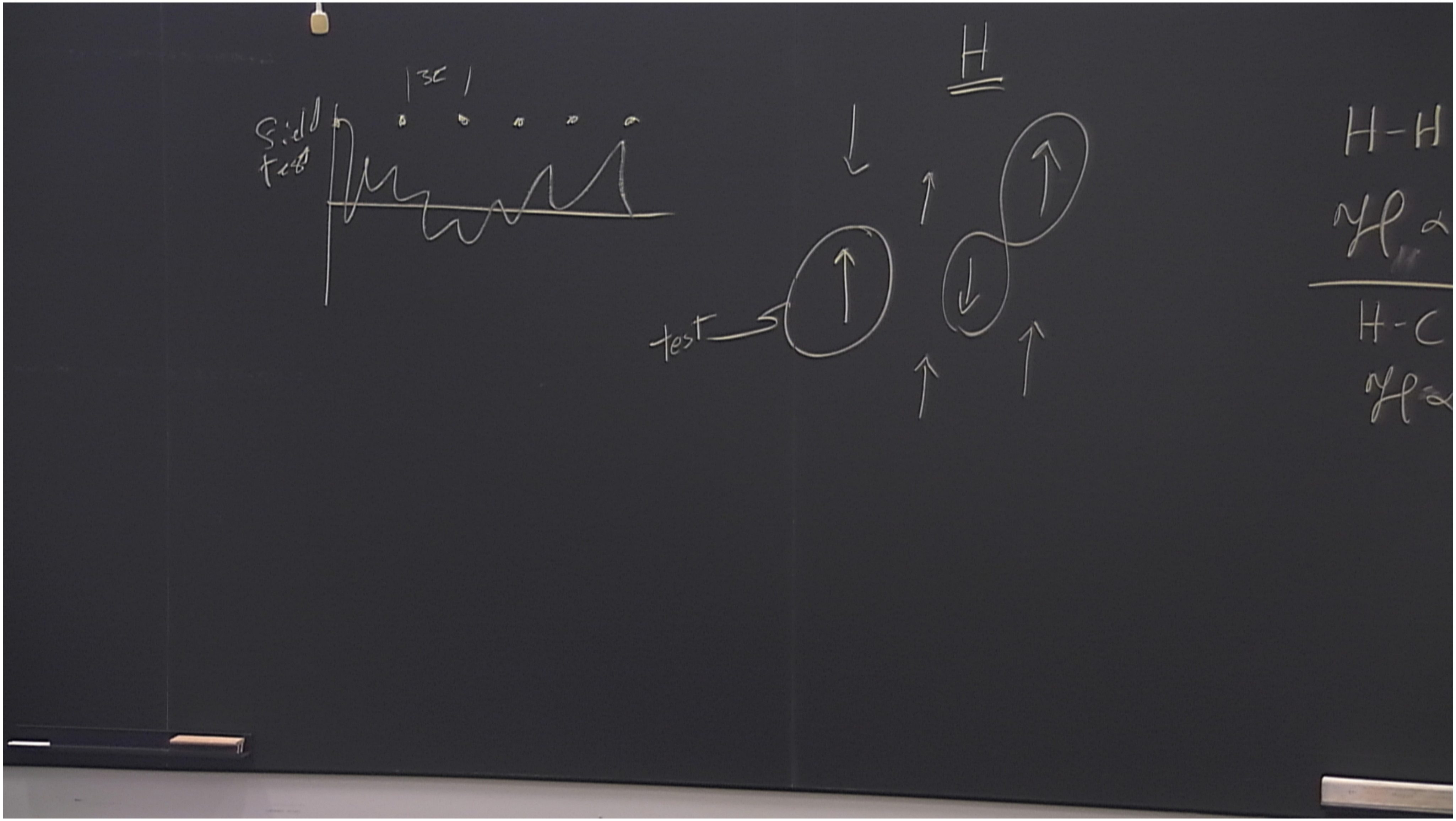

H
test
H-H
H-C

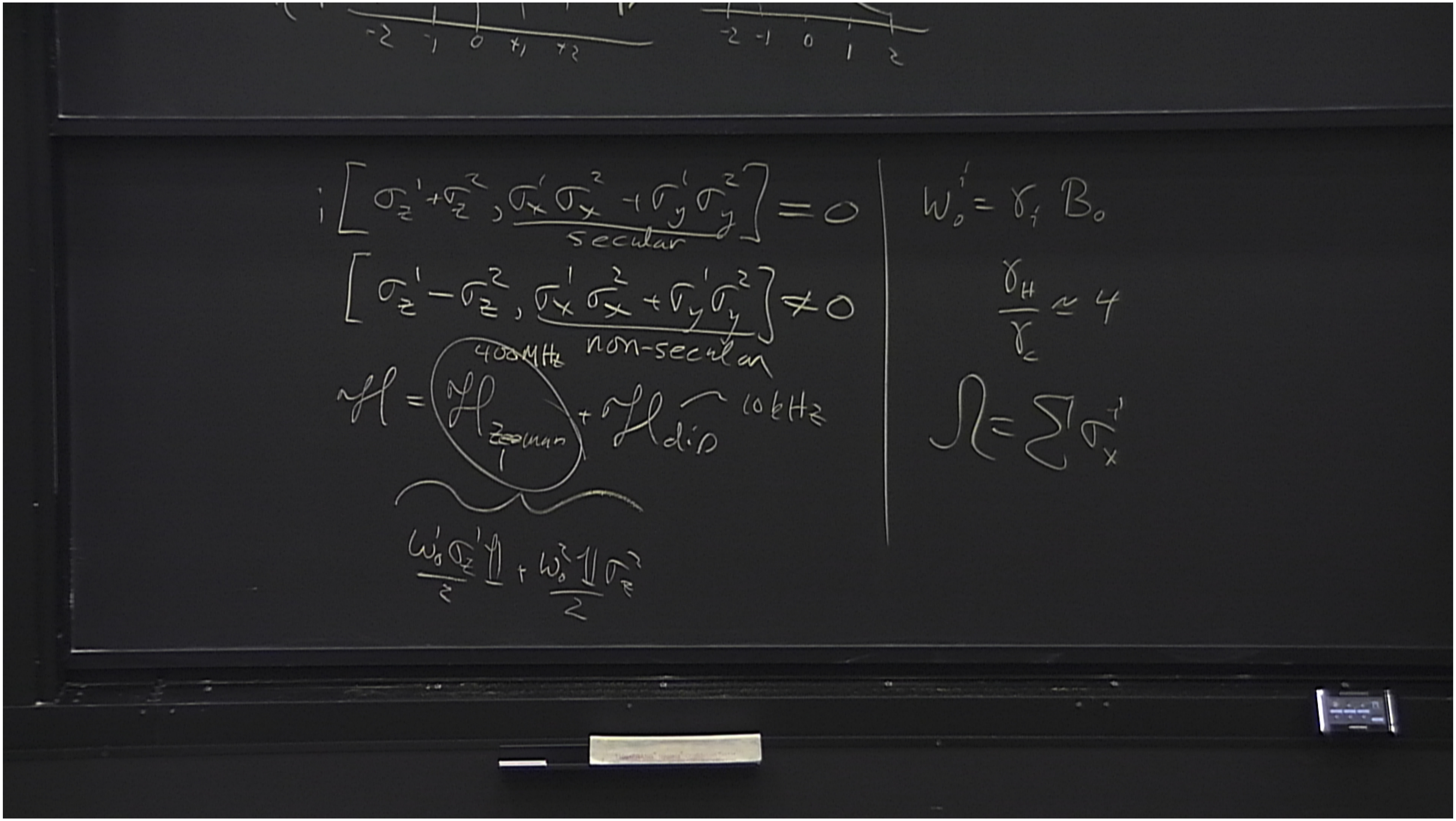

π/2)x
π/2)y
τ
τ
τ
σ·σ−3σzσz ; σ·σ
−3σzσz

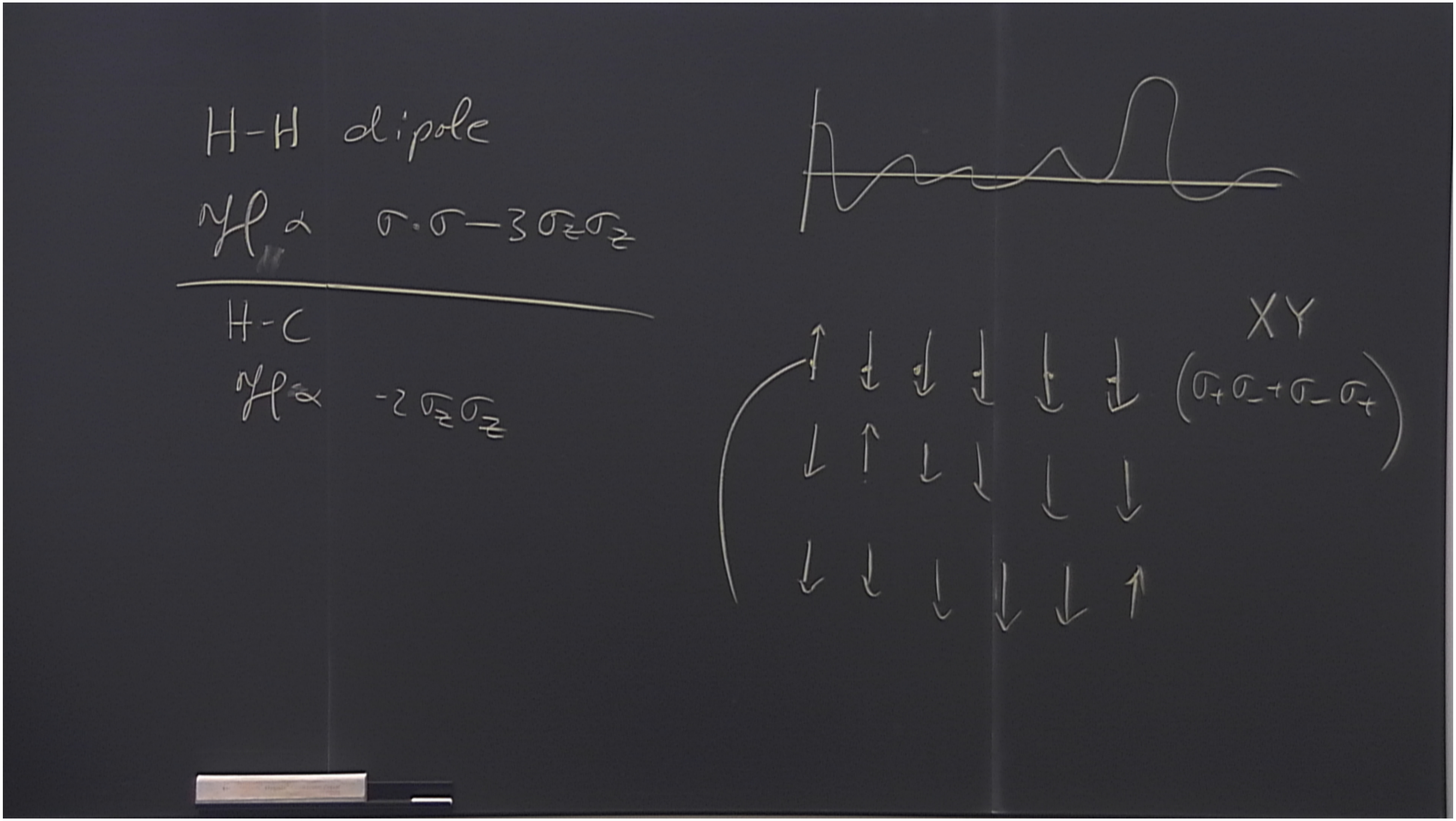


τ
τ
τ
3σzσz ;
σ·σ −3σzσz
σ·σ −3σyσy
σ·σ −3σxσx


σ·σ − 3σzσz ; σ·σ −3σzσz
σ·σ −3σyσy
σ·σ −3σxσx
H = 3τσ·σ − 3(σzσz + σxσx + σyσy)τ / 3τ = 0


H
test
H-H
H-C

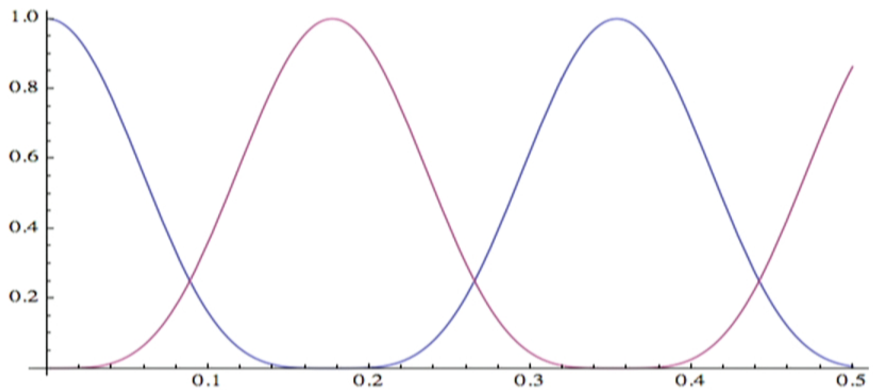

-2 -1 0 +1 +2
-2 -1 0 1 2
i[σz¹+σz², σx¹σx² + σy¹σy²] = 0
secular
[σz¹−σz², σx¹σx² + σy¹σy²] ≠ 0
non-secular
400MHz
H = H Zeeman + H dip
10kHz
ω0¹σz¹𝟙/2 + ω0²𝟙σz²/2
ω0¹ = γi B0
γH/γc ≃ 4
Ω = Σ σx^i


H-H dipole
$\mathcal{H} \propto \sigma\cdot\sigma - 3\sigma_z\sigma_z$
H-C
$\mathcal{H} \propto -2\sigma_z\sigma_z$
XY
$(\sigma_+\sigma_- + \sigma_-\sigma_+)$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Tue 9:56 AM dcory

spin chain ex.nb

```
P3[t_] := U3[t] . ρ3 . U3[-t]

Ω3[t_] := Tr[ρ3 . P3[t]]

Ω3e[t_] := Tr[ρ3e . P3[t]]

Plot[{Ω3[t], Ω3e[t]}, {t, 0, .5}]
```

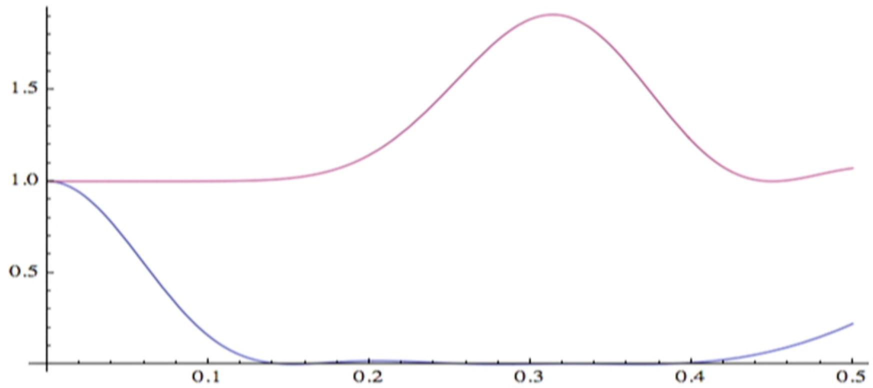


$$\frac{-\pi + 2\pi 1}{4\sqrt{2}\,\pi} \text{ // N}$$

0.176777

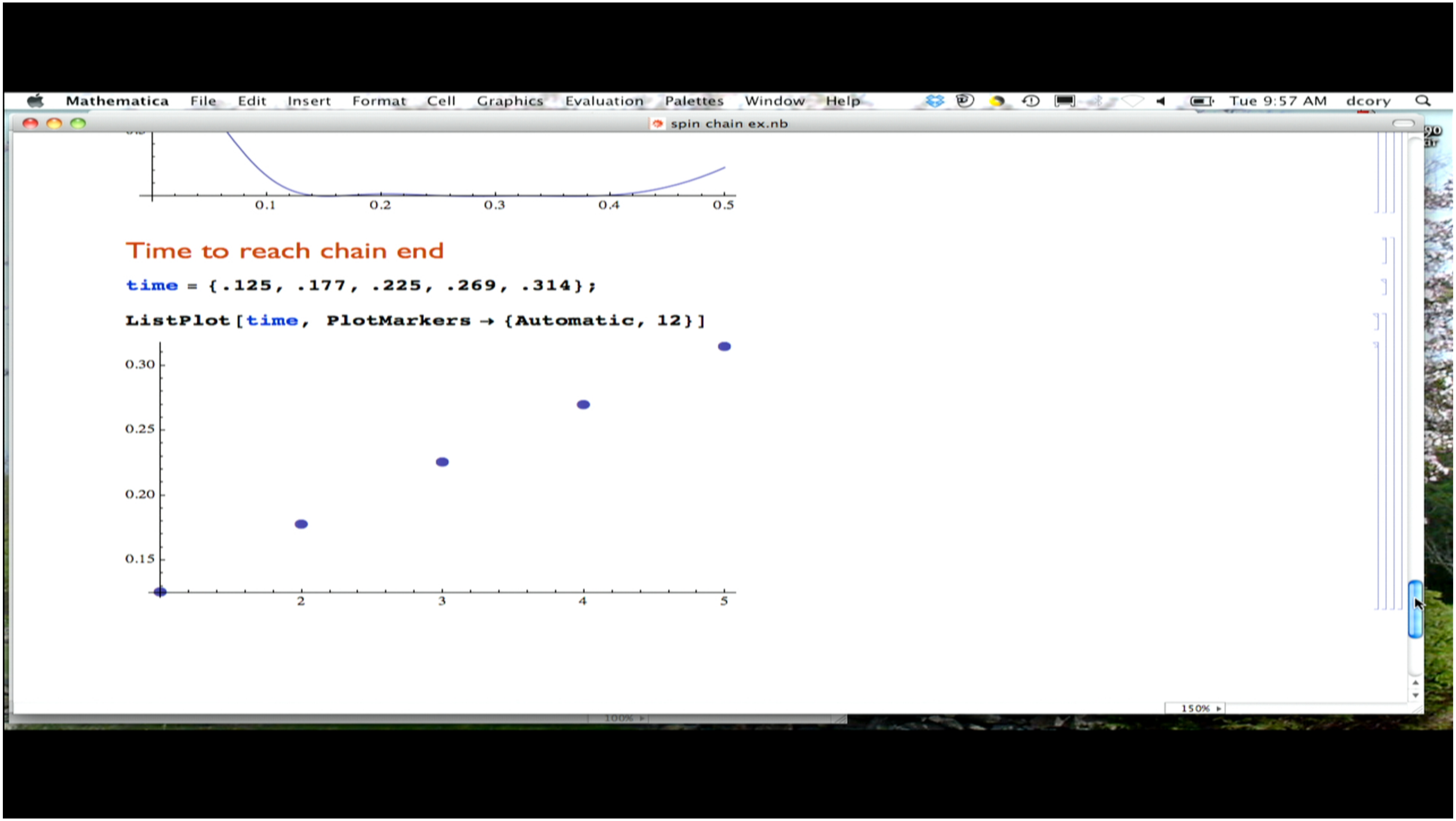
```
Ω3e[0.17677669529663687`]
```

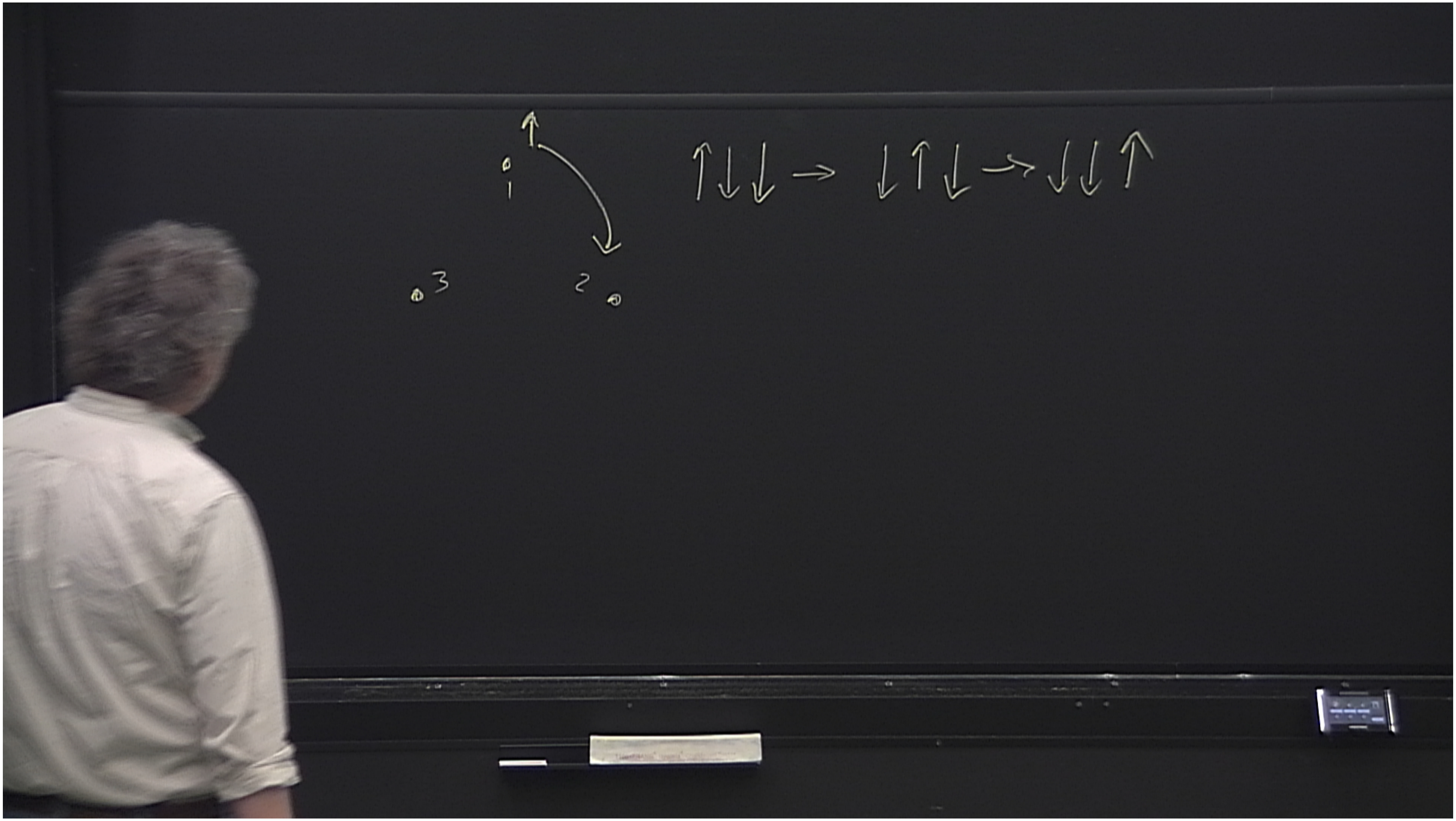
```
U6[t_] := MatrixExp[I t H6]

P6[t_] := U6[t] . ρ6 . U6[-t]

Ω6[t_] := Tr[ρ6 . P6[t]]

Ω6e[t_] := Tr[ρ6e . P6[t]]

Plot[{Ω6[t], Ω6e[t] + 1}, {t, 0, .5}]
```

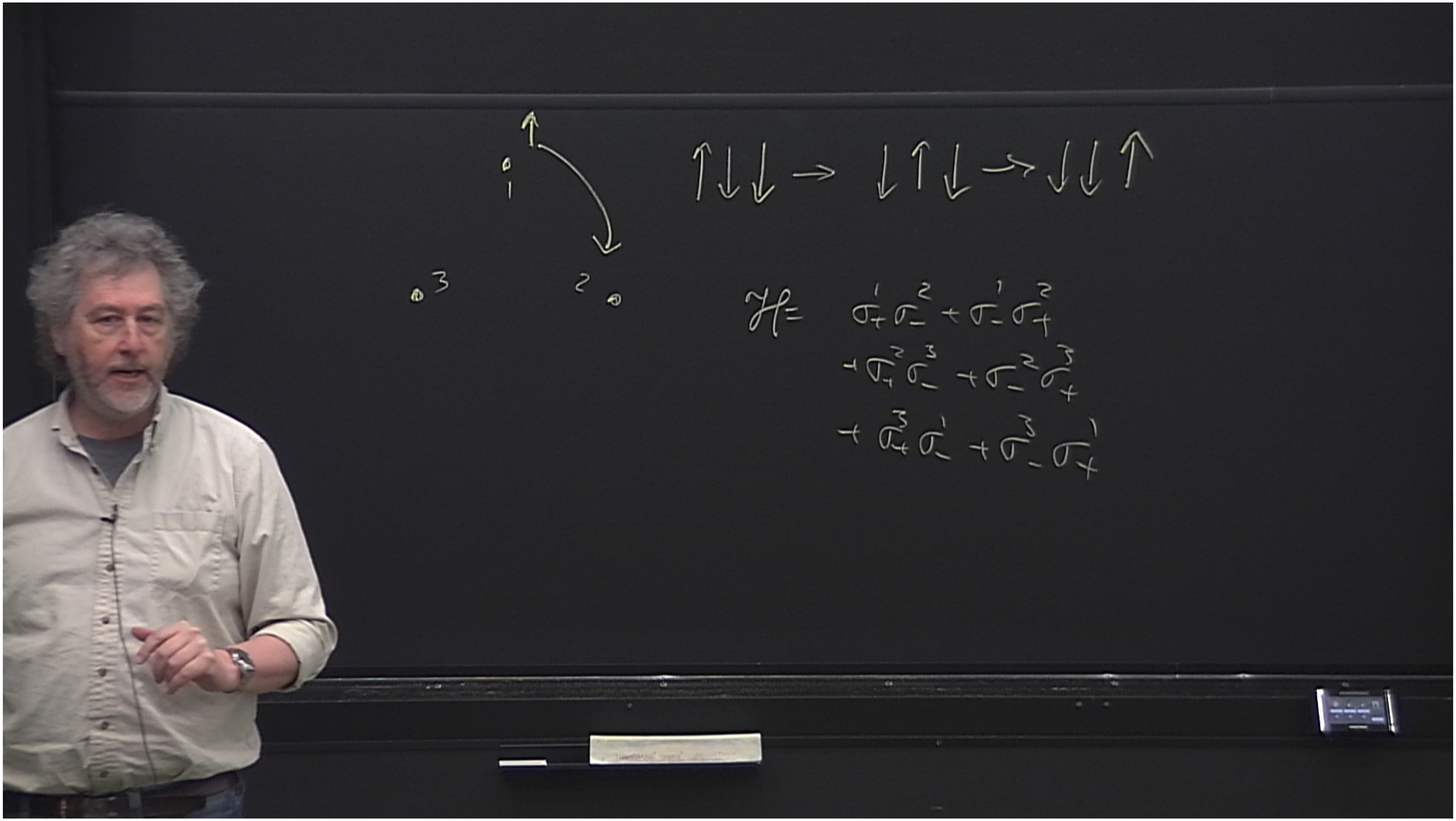


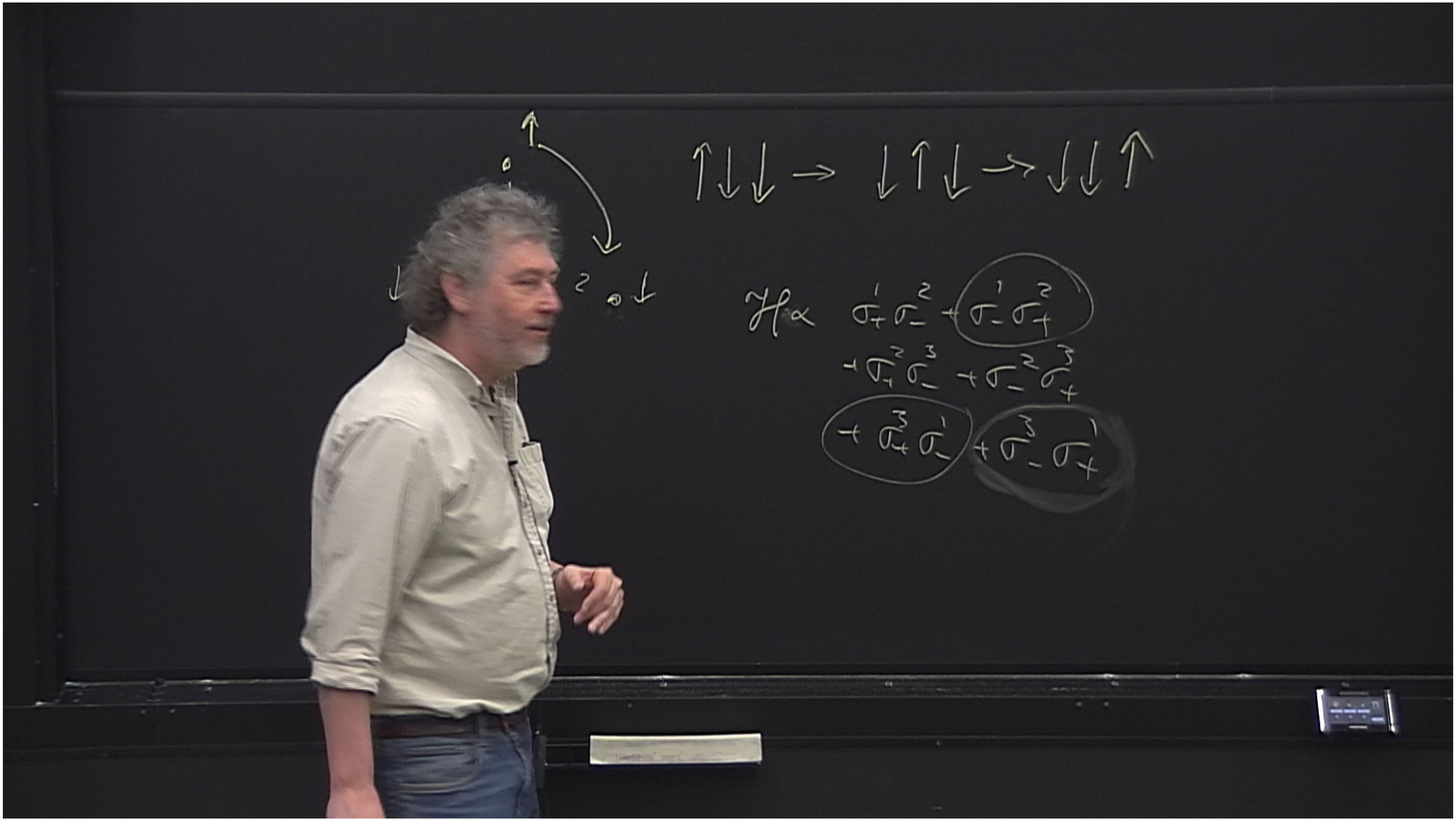
Time to reach chain end

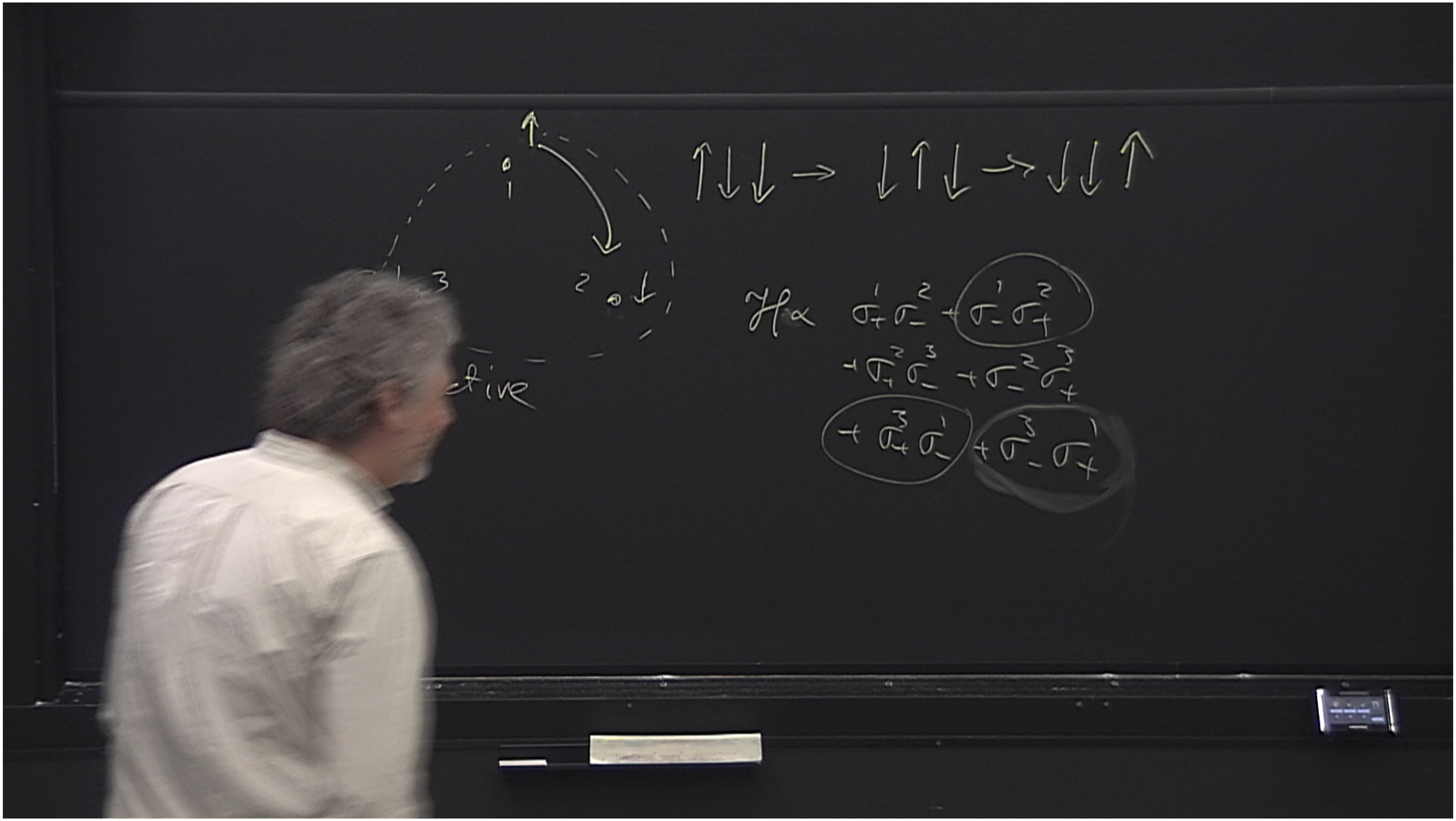
```
time = {.125, .177, .225, .269, .314};

ListPlot[time, PlotMarkers → {Automatic, 12}]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tue 9:57 AM dcory
spin chain ex.nb
0.1 0.2 0.3 0.4 0.5
Time to reach chain end
time = {.125, .177, .225, .269, .314};
ListPlot[time, PlotMarkers → {Automatic, 12}]
0.30
0.25
0.20
0.15
2 3 4 5
150%


1
3
2


3
2
1
↑↓↓ → ↓↑↓ → ↓↓↑
H = σ+1 σ-2 + σ-1 σ+2
+ σ+2 σ-3 + σ-2 σ+3
+ σ+3 σ-1 + σ-3 σ+1





Wolfram CDF Player File Edit Window Help Tue 10:04 AM dcory

Chern4.cdf

Wolfram CDF Player

This document is published under FreeCDF terms »

spin up state ↑↓↓. Since this is a cyclic system there are two ways to get to the state we wish to avoid ↓↓↑ (the desired outcome of the Hamitonian is ↓↑↓):

- we can reach the state in one "step" via the operator $\sigma_-^1 \sigma_+^3$

or

- we can reach this in two steps in the desired direction. We set the phases in the Hamiltonian so that these destructively interfere. Now the desired dynamics is realized. Note the two paths must end up being π out of phase.

```
Ht[ω_] := ω * (KroneckerProduct[
        KroneckerProduct[σp, σm], PauliMatrix[0]] +
         KroneckerProduct[
        KroneckerProduct[σm, σp], PauliMatrix[0]] +

    Exp[I π / 2] KroneckerProduct[
         KroneckerProduct[PauliMatrix[0], σp], σm] +
        Exp[-I π / 2] KroneckerProduct[
         KroneckerProduct[PauliMatrix[0], σm], σp] +

    KroneckerProduct[
        KroneckerProduct[σp, PauliMatrix[0]], σm] +
        KroneckerProduct[
        KroneckerProduct[σm, PauliMatrix[0]], σp])
```

- **operator**

```
Ut[ω_, t_] := MatrixExp[-I Ht[ω] t]

UIt[ω_, t_] := MatrixExp[I Ht[ω] t]
```

The operator is peridic in time with a periodicity of tω=2/Sqrt[3]

```
Ut[π, 0] // N // MatrixForm
```

1.	0.	0.	0.	0.	0.	0.	0.
0.	1.	0.	0.	0.	0.	0.	0.
0.	0.	1.	0.	0.	0.	0.	0.
0.	0.	0.	1.	0.	0.	0.	0.
0.	0.	0.	0.	1.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.

125%

Wolfram CDF Player File Edit Window Help Tue 10:04 AM dcory

Chern4.cdf

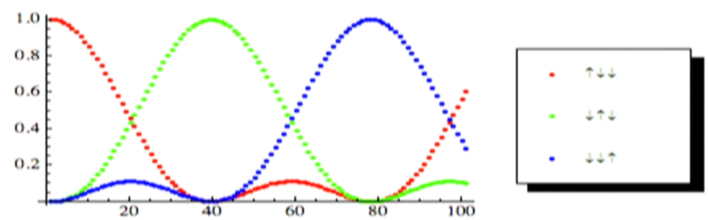
Wolfram CDF Player

This document is published under FreeCDF terms »

```
0.  0.  0.  0. + 1. i  0.  0.  0.  0.
0.  0.  0.  0.         0.  0.  0.  1.
```

- **evolution**

```
ρ0 = KroneckerProduct[
      KroneckerProduct[Ep, Em], Em];

ρ[t_] := Ut[π, t] . ρ0 . UIt[π, t]

ListPlot[{Table[Re[M1[t]], {t, 0, 1, 0.01}], Table[Re[M2[t]], {t, 0, 1, 0.01}],
  Table[Re[M3[t]], {t, 0, 1, 0.01}]}, PlotStyle → {Red, Green, Blue},
 PlotLegend → {"↑↓↓", "↓↑↓", "↓↓↑"}, LegendPosition → {1.1, -0.4}]
```



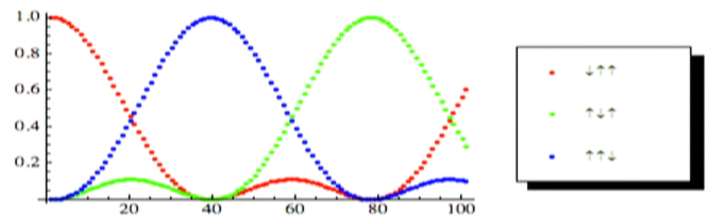
So yes we have the dynamics we want, a rotation of spin.

- **evolution in 2-up manifold**

```
ρ2 = KroneckerProduct[
      KroneckerProduct[Em, Ep], Ep];

ρ2t[t_] := Ut[π, t] . ρ2 . UIt[π, t]

ListPlot[{Table[Re[Tr[O4 . ρ2t[t]]], {t, 0, 1, 0.01}],
  Table[Re[Tr[O5 . ρ2t[t]]], {t, 0, 1, 0.01}], Table[Re[Tr[O6 . ρ2t[t]]], {t, 0, 1, 0.01}]},
 PlotStyle → {Red, Green, Blue}, PlotLegend → {"↓↑↑", "↑↓↑", "↑↑↓"},
```

125%

```
Table[Re[Tr[O5 . ρ2t[t]]], {t, 0, 1, 0.01}], Table[Re[Tr[O6 . ρ2t[t]]], {t, 0, 1, 0.01}]},
PlotStyle → {Red, Green, Blue}, PlotLegend → {"↓↑↑", "↑↓↑", "↑↑↓"},
LegendPosition → {1.1, -0.4}]
```



So yes we have the dynamics we want, a rotation of spin. Notice in comparison to the earlier plot that spin up and spin down move in oposite directions as we desire.

- **eigenstructure**

```
Eigenvalues[Ht[ω]]
```

$\{-\sqrt{3}\,\omega, -\sqrt{3}\,\omega, \sqrt{3}\,\omega, \sqrt{3}\,\omega, 0, 0, 0, 0\}$

Note, two of the eigenvalues are trivially zero since the all up and all down states do not evolve.
As expected each spin polarized circulating state corresponds to a qubit.

```
vect[ω_] := Eigenvectors[Ht[ω]]

Conjugate[vect[ω]] [[5]] . vect[ω] [[5]] // N

1.
```

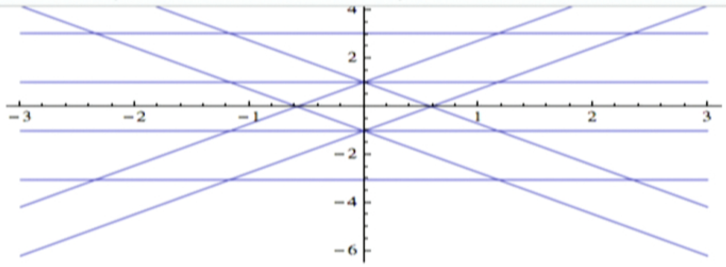
Add a Zeeman term to split the degeneracy

Wolfram CDF Player File Edit Window Help Tue 10:06 AM dcory

Chern4.cdf

Wolfram CDF Player

This document is published under FreeCDF terms »



Note that the original Hamiltonian frequency gives the rate of moving around the ring, this is the horizontal axis and is like the momentum axis. The Zeeman frequency (the slider) lifts the degeneracies between the 2-up and 2-down manifolds.

To open a gap at these crossings we add a term to the Hamiltonian that flips all three spins (swaps 2-up to 2-down).

Add an anti-crossing at the degeneracy

```
Hza[ω_, ωz_, ωr_] := ωr * (KroneckerProduct[
        KroneckerProduct[PauliMatrix[1], PauliMatrix[0]], PauliMatrix[0]] + KroneckerProduct[
        KroneckerProduct[PauliMatrix[0], PauliMatrix[1]], PauliMatrix[0]] +
    KroneckerProduct[
        KroneckerProduct[PauliMatrix[0], PauliMatrix[0]], PauliMatrix[1]]) +

  ωz * (KroneckerProduct[
        KroneckerProduct[PauliMatrix[3], PauliMatrix[0]], PauliMatrix[0]] + KroneckerProduct[
        KroneckerProduct[PauliMatrix[0], PauliMatrix[3]], PauliMatrix[0]] +
    KroneckerProduct[
        KroneckerProduct[PauliMatrix[0], PauliMatrix[0]], PauliMatrix[3]]) +
```

125%

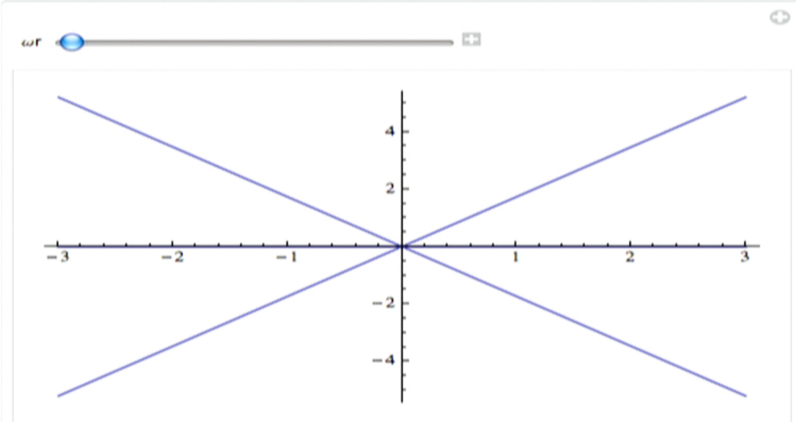
Wolfram CDF Player File Edit Window Help Tue 10:06 AM dcory

Chern4.cdf

Wolfram CDF Player

This document is published under FreeCDF terms »

```
Manipulate[Plot[Sort[Table[energya2[ω, 0, ωr] [[n]], {n, 1, 8}]], {ω, -3, 3}],
 {ωr, 0, 2}, SaveDefinitions → True]
```



■ **A different gauge?**

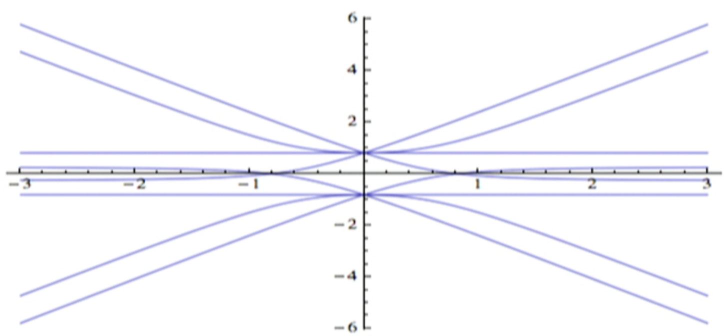
There are multiple solutions for the phases in the Hamiltonian that result in the same cycle. They are simply related to each other.

```
Ht2[ω_, a_, b_] := ω * (KroneckerProduct[
        KroneckerProduct[σp, σm], PauliMatrix[0]] +
         KroneckerProduct[
        KroneckerProduct[σm, σp], PauliMatrix[0]] +

    Exp[I a] KroneckerProduct[
         KroneckerProduct[PauliMatrix[0], σp], σm] +
```

125%

```
Manipulate[Plot[Sort[Table[energya2[ω, 0, ωr] [[n]], {n, 1, 8}]], {ω, -3, 3}],
 {ωr, 0, 2}, SaveDefinitions → True]
```

ωr



- **A different gauge?**

There are multiple solutions for the phases in the Hamiltonian that result in the same cycle. They are simply related to each other.

```
Ht2[ω_, a_, b_] := ω * (KroneckerProduct[
        KroneckerProduct[σp, σm], PauliMatrix[0]] +
         KroneckerProduct[
        KroneckerProduct[σm, σp], PauliMatrix[0]] +

    Exp[I a] KroneckerProduct[
```