

Title: 13/14 PSI - Quantum Information Review - Lecture 12

Date: Mar 04, 2014 02:00 PM

URL: <http://pirsa.org/14030025>

Abstract:

- Schumacher compression
- Entanglement concentration /
dilution
- Quantum error correction

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it suffices to code for a source
that sends $|\gamma_i\rangle$ w.p. λ_i

- Quantum error correction

Diagonalize: $\rho = \sum \lambda_i |\psi_i\rangle\langle\psi_i|$
it suffices to code for a source
that sends $|\psi_i\rangle$ w.p. λ_i

Strategy: - project onto typical subspace of $\rho^{\otimes n}$

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 $\log(\dim(\text{typical subspace}))$ qubits $\approx n S(\rho)$

- project (either part) onto the typical subspace.
 - also measure the "type class" (Hamming weight of $i_1, \dots, i_n$)
- now all strings $i_1, \dots, i_n$ have the same amplitude, and
the dimension is still close to $2^n E(1\psi)$

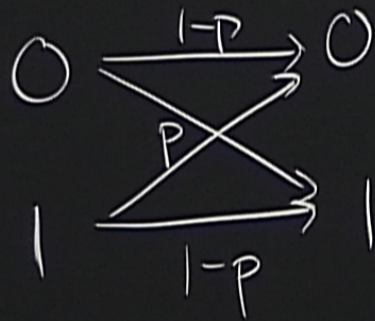
Dilation

A creates $1\psi\rangle_{AB}^{\otimes n}$ locally

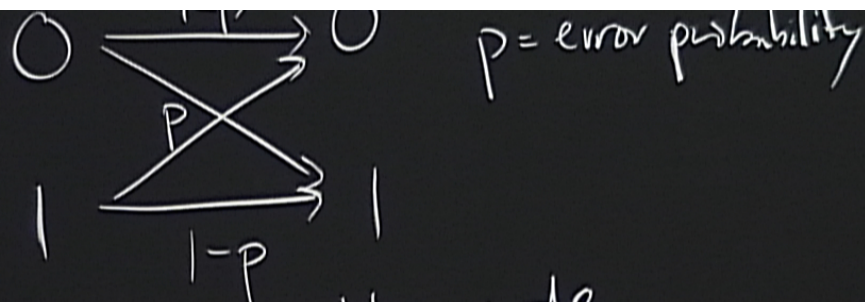
Error correction

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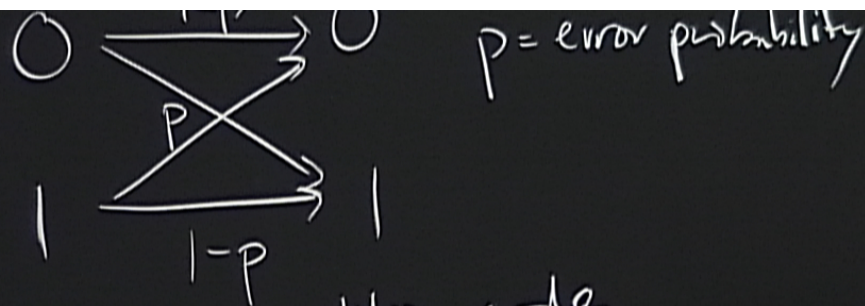
Classical case — simple error model: binary symmetric channel



p = error probability



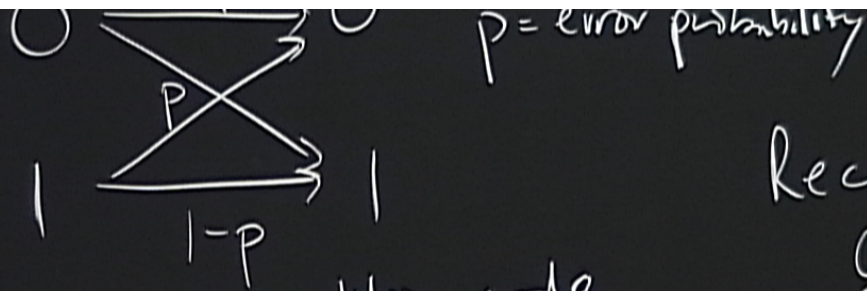
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 $0 \rightarrow 000$



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Recovery by majority voting:

$$000 \rightarrow 0$$

$$001 \rightarrow 0$$

$$010 \rightarrow 0$$

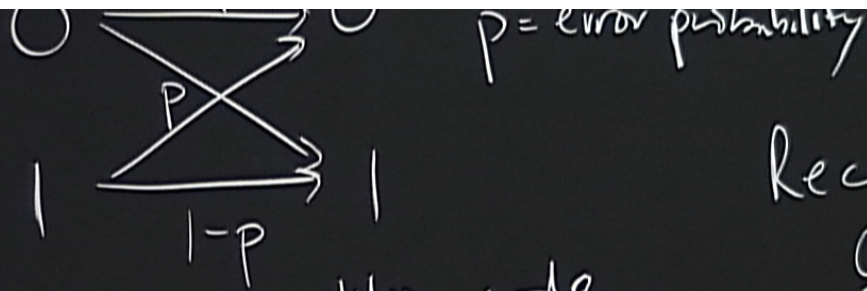
etc.

$$111 \rightarrow 1$$

$$101 \rightarrow 1$$

etc.

$$\Pr(2 \text{ or } 3 \text{ errors}) =$$



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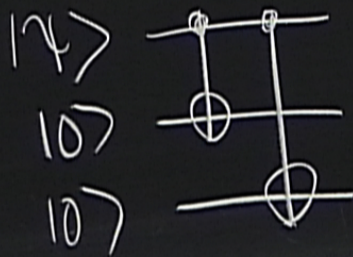
etc.

$$\Pr(2 \text{ or } 3 \text{ errors}) = p^3 + 3p^2(1-p) = O(p^2) \quad (p \ll 1)$$

- ① no cloning
- ② determining the error requires learning about the state — disturbance
- ③ need to correct a continuum of errors

①&②: find a q. code for bit flip errors

①: Copy in the comp. basis



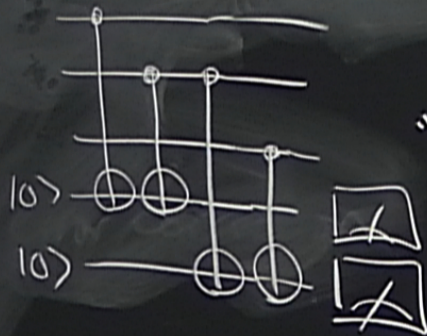
$$|0\rangle \mapsto |0_L\rangle = |000\rangle$$

$$|1\rangle \mapsto |1_L\rangle = |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|000\rangle + \beta|111\rangle$$

(2) measure the error, not the data

recovery:



"Syndrome"

00
01
10
11

no error

error on bit 3

error on bit 1

error on bit 2

→ can correct

- we almost always find ourselves in the typical subspace
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 $\log(\dim(\text{typical subspace}))$ qubits $\approx n S(p)$

③ consider a general error, $p \neq$

③ consider a general error, $\rho \mapsto \mathcal{E}(\rho)$
 can be realized by unitary interaction w/ environment:

$$|\psi\rangle_S |0\rangle_E \xrightarrow{U_{SE}} |\psi'\rangle_{SE}$$

Suppose S is a qubit.

$$\begin{aligned} |0\rangle_S |0\rangle_E &\xrightarrow{U_{SE}} \gamma_{00} |0\rangle_S |\phi_{00}\rangle_E + \gamma_{01} |1\rangle_S |\phi_{01}\rangle_E \\ |1\rangle_S |0\rangle_E &\xrightarrow{U_{SE}} \gamma_{10} |0\rangle_S |\phi_{10}\rangle_E + \gamma_{11} |1\rangle_S |\phi_{11}\rangle_E \end{aligned}$$

- project (either part) onto the typical subspace.

$$\text{so } (\alpha|0\rangle_S + \beta|1\rangle_S)|0\rangle_E \mapsto \alpha(\gamma_{00}|0\rangle_S|\phi_{00}\rangle_E + \gamma_{01}|1\rangle_S|\phi_{01}\rangle_E) \\ + \beta(\gamma_{10}|0\rangle_S|\phi_{10}\rangle_E + \gamma_{11}|1\rangle_S|\phi_{11}\rangle_E)$$

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rewrite using $|x\rangle, X|z\rangle, XZ|x\rangle$

$$|x\rangle_S|0\rangle_E \mapsto \dots + \alpha_X |x\rangle|\phi_x\rangle$$

CAUTION

DO NOT TOUCH THE BOARD WHEN THE LECTURER IS SPEAKING

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rewrite using $| \psi \rangle, X| \psi \rangle, Z| \psi \rangle, XZ| \psi \rangle$

$$| \psi \rangle_S | 0 \rangle_E \mapsto \alpha_I | \psi \rangle | \phi_I \rangle + \alpha_X X| \psi \rangle | \phi_X \rangle \\ + \alpha_Z Z| \psi \rangle | \phi_Z \rangle + \alpha_{XZ} XZ| \psi \rangle | \phi_{XZ} \rangle$$

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$$\text{so } (\alpha|0\rangle_S + \beta|1\rangle_S)|0\rangle_E \rightarrow \alpha(\gamma_{10}|0\rangle_S + \gamma_{11}|1\rangle_S)|\phi_{10}\rangle_E + \beta(\gamma_{10}|0\rangle_S + \gamma_{11}|1\rangle_S)|\phi_{11}\rangle_E)$$

rewrite using $|Y\rangle, X|Y\rangle, Z|Y\rangle, XZ|Y\rangle$

$$| \psi \rangle_S | 0 \rangle_E \mapsto \alpha_I | \psi \rangle | \phi_I \rangle + \alpha_X | \psi \rangle | \phi_X \rangle \\ + \alpha_Z | \psi \rangle | \phi_Z \rangle + \alpha_{XZ} | \psi \rangle | \phi_{XZ} \rangle$$

$$\alpha_I |\phi_I\rangle = \frac{1}{2} (\gamma_{00} |\phi_{00}\rangle + \gamma_{11} |\phi_{11}\rangle)$$

$$\text{so } (\alpha|0\rangle_S + \beta|1\rangle_S)|0\rangle_E \mapsto \alpha(\gamma_{00}|0\rangle_S|\phi_{00}\rangle_E + \gamma_{01}|1\rangle_S|\phi_{01}\rangle_E) + \beta(\gamma_{10}|0\rangle_S|\phi_{10}\rangle_E + \gamma_{11}|1\rangle_S|\phi_{11}\rangle_E)$$

rewrite using $|I\rangle, |X\rangle, |Z\rangle, |XZ\rangle$

$$|I\rangle_S|0\rangle_E \mapsto \alpha_I|I\rangle|\phi_I\rangle + \alpha_X|X\rangle|\phi_X\rangle + \alpha_Z|Z\rangle|\phi_Z\rangle + \alpha_{XZ}|XZ\rangle|\phi_{XZ}\rangle$$

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